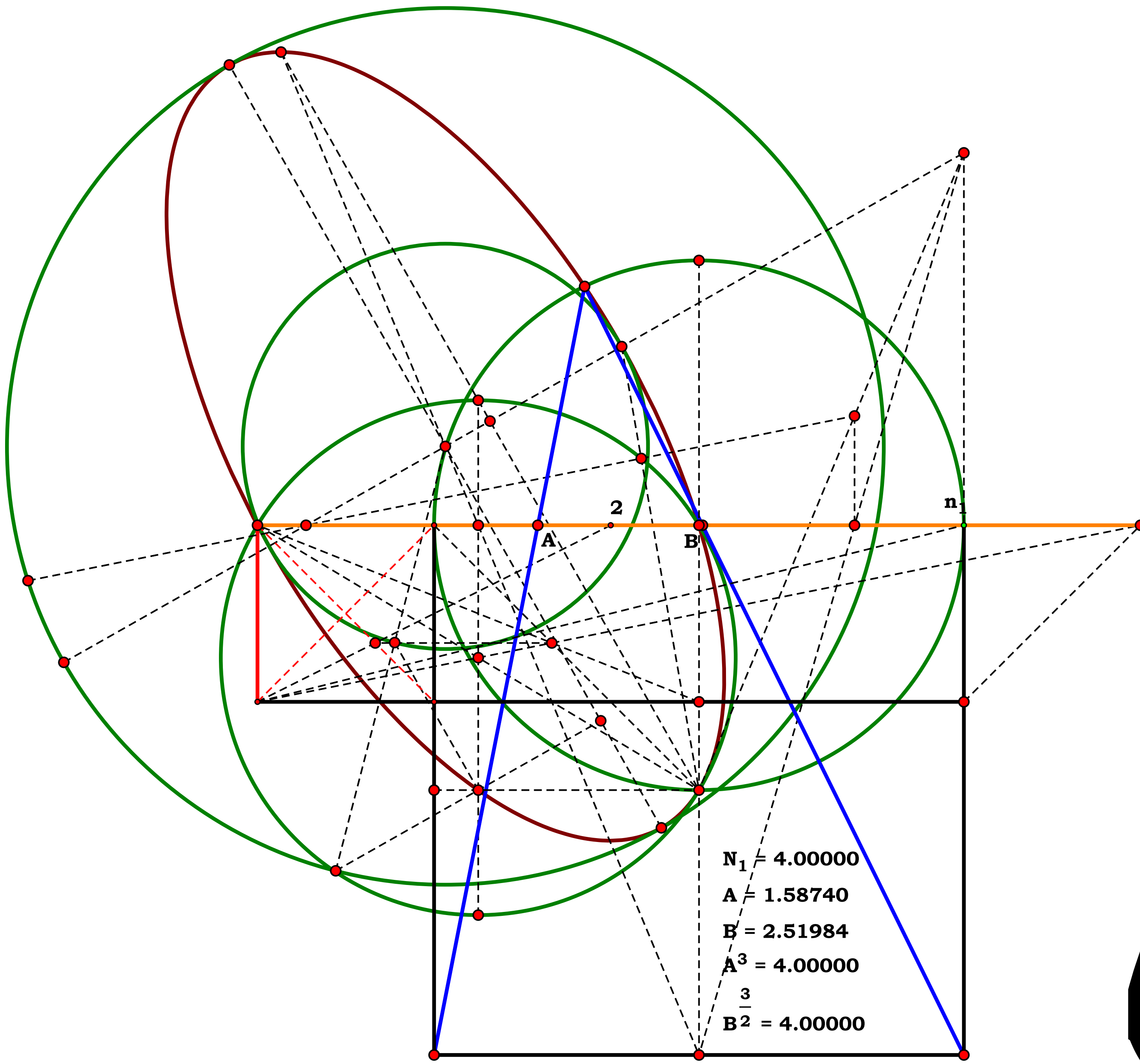
The Delian Quest

2021

John 312



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John Clark

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Literacy

The process I used to do this work is actually described by Plato. Plato was a grammar teacher and one of the most fundamental concepts you have to realize, as Plato demonstrated in his dialogs, especially *Parmenides* and *Theaetetus*, is that all grammar systems are methods of binary recursion which he called *Dialectic*, words by two's. One starts with names and naming conventions based on this binary, and from there, one has to keep track of their names, build a dictionary with them, and from the items named, discover their relationships. This work is my first attempt, to write factual human literature demonstrating the pairing of our grammar systems to say the same things.

But fundamental to Plato's teaching in grammar is the point being made by the Judeo-Christian Scripture, that all a mind can do, by biological and physical fact, is read and write. We read and write with our whole body, mind and soul. Grammar affords us the ability to predict the results of any behavior, any complexity, and as one will see in this work, compute the results of any given number of variables, free from computational time: output is concurrent with the input. In short, not even the fastest computers today can match Geometry for speed and accuracy: And when one can comprehend the metaphors of the Book, one will realize by doing the math, there are four, and only four, basic systems of grammar, Common Grammar, Arithmetic, Algebra and Geometry: these four make a Grammar Matrix which defines literacy and mental competence. We learn to guide our whole behavior using a binary matrix in order to do our own

work, as Plato would say, or again, our biologically defined job. This Matrix was put into metaphors of the JCS.

This is a grammar book, a helper in learning how to read and write, or as I am want to say, learning how to say what you see.

There is no Trigonometry here, which is not a formal grammar, there is no Cartesian Geometry here, which is not a formal grammar, and there is no Calculus here, which is not a formal grammar. However, what is here is proof that the doctrines promoted by these previously mentioned are provably wrong. Binary recursion produces four, and only four, categories of grammar. Every formal system of grammar lays out, step by step, the pairing of our original Universal Binary with mental and computational behavior.

What is the Delian Quest?

One should, at least attempt, to find an intelligible concept commensurate with certain perceptible things. For example, the *Delian Problem*, as you probably know, is about the duplication of a cube, or one can say, the manipulation of a three dimensional object so as to produce a given product: in this case, a cube of twice the volume of one given. What is stated is a request for a perceptible result, not in terms of an estimate, or one satisfying some pre-determined precision, but exactly, perfectly. However, one can look at the problem intelligibly and metaphorically, in terms of the definition of what a mind, or again, what man is to become: a symbolic information processor in order to maintain and promote life. A mind, after all, when functional, is the most powerful life support system possible for any form of life. With it one can virtualize the environment and predict results favorable to the continuance of that life. Put into what people call a religious metaphor: "*The Testimony of Jesus is the Spirit of Prophecy*." Grammar systems, when functional, allow us to predict the

results of any number of givens. Thus, if one is smart enough, they see reasoning, judgment, and prophecy as synonyms.

Intelligibly then, one is looking for the very same result one finds in the metaphor, *The Father, the Son, and the Holy Spirit are One*. If one is paying attention, we are being presented with another three dimensional request. What is the meaning of that request?

We, as a mind, are evolving to become the most powerful life support system possible, i.e. the most fit for survival. What this means is that we are evolving to become masters of the perceptible through a mind. The foundation of mind is memory. Memory is a virtualization of perception and thus memory, is the foundation of what is called the Intelligible. Intelligence is the ability to manipulate memory for survival. Intelligence is founded upon what is called pattern recognition, or which Plato called the ability to see the *similar idea in the many examples*. There is no intelligible, no other idea, which demonstrates intelligence, than the recognition of an idea which covers all perceptions. That idea will then eventually end up with many names set into a phrase such as *a unit*, *a thing*. This phrase, comprising of two words, is what Plato called *Dialectic*, or what we call today, *Binary*. Every thing is defined as a binary relationship between a thing's form, shape, limits, etc., and a thing's relative difference also known as material and material difference: the stuff a thing is made of.

As the Universe is made up of every thing, one can say that the Universe is the product of Binary Recursion. Now, if one were parsing information correctly, they could reduce the story of Adam and Eve into a single metaphorical sentence which denotes how a functional mind works such as: *Adam and Eve are a Conjugate Binary Pair, which by Complete Induction and Deduction produce the human race*. Fundamental intelligence will eventually arrive at a simple provable fact, as a computer

can produce all of its output as the result of binary recursion, so can any functional mind.

Our mind is defined to master the Universal, the concept of binary and binary recursion, by learning from the particulars in our environment. Thus, it comes naturally, to the more intelligent of a species, to attempt to learn how every particular thing can be judged by that Universal. For example, the metaphor of the Father, Son and Holy Spirit can be transformed into exactly what we are, a mind responsible for a given product such that the product is true. Our Father, or teacher is perception: We, as the student, or Son of perception learn behavior from those perceptions. Thus, both metaphors, Duplication of the Cube, and what is called a religious quest, simply become, Perception determines conception; conception determines will; which is a description of every life support system of a living organism. In short, be it a scientific problem or a mystical problem, our mind sees but one object, one problem to categorize them under: a simple biological fact. This is an example of what Plato meant by the similar idea in the many examples, or again, the definition of any thing, or again, a unit. *Dialectic* is a term Plato used to denote binary recursion.

A mind is one of a group of life support systems of the body within which we reside; as such it has a well-defined, biologically determined, job to perform and well-defined, physically determined, means of doing that job. As the definition of a thing, aka, a unit, is a binary expression, our job is to learn, all the days of our life, correct and true binary recursion in order to maintain and promote life. Therefore, this work is simply part of my work, given to me, given to us all, by simple biological fact.

When I was very young, I became aware of a problem with humanity as a whole. The human mind is not exactly functional, and people are prone to a life of pointless and bizarre behavior. I, personally, was not being educated by our social systems, and had spent a great deal of time looking for teachers in books.

I was quickly approaching my 40's when I decided to try and learn some Geometry. In order to keep myself motivated, I chose to try and solve a problem called impossible to solve. The greatest minds in history had tried to solve it and gave up on it, therefore, I figured that since it was unsolvable, I would never have an excuse for leaving off my study of geometry. Unsolvable problems, like unrequited love, is one of the best carrots on a stick for our own dumb ass.

Somehow, I started with exactly the right figure to pursue. And somehow, writing equations to figures came naturally to me and it certainly had nothing to do with that ridiculous so called Cartesian Geometry, although of great utility for mechanics, I do not call it a pure mathematic. In geometry, a ruler is not allowed unless one produces one as capable of doing the math by the geometric figure. One does not set up rulers to measure where they will physically put a point. There is no precision in that.

Ten years down the road, I realized that I did solve the problem but that its solution was trivial compared to what I discovered along the way. I had written equations to figures for so long, I started learning how to write figures to equations until I had laid the foundation for *Basic Analog Mathematics*, or BAM for short. One actually draws the blueprint for computation where the output is concurrent with the input, i.e. no processing time. One can do all of one's logical and analogical processing using simple geometry as exampled in BAM and all of it quite independent of processing time. What put me onto this were apparently aliens, or so I assume, when they deliberately demonstrated the effect while helping to save my life from a very stupid decision I made while driving.

So called Euclidean Geometry, is just a grammar system, like any other, derived from binary recursion, i.e. the recursion of a simple unit. What this means is that it is wholly impossible to claim any other kind of geometry unless one's mind is dysfunctional and incapable of comprehending the fallacies it introduces. As every grammar is a product which expresses simple binary recursion, one cannot claim a different geometry for it would have to be based on something other than the recursion of a unit, i.e. it would leave no math by which to proof it, nor would it be based on the Universal Binary which defines our physical reality. Every form of true mathematics is the result of binary recursion and although this is fundamental to mathematics, it is surprising how many so called mathematicians cannot apply this first principle by which to judge their own words. Non-Euclidean Geometries, every one of them, are based on simple fallacies which their proponents cannot grasp.

The two main focal points of this early study of geometry which was constantly on my mind were that I was learning how to say what I saw. The second was how to establish a unit from which all the rest of the equations were derived. The fact is, that unit has to be expressed in the figure itself, either as one of the segments or a proportion of one of the segments. These two focal points of mind remain as the motivation for my study which will lead to a better understanding of the result, Basic Analog Mathematics or more formal, *Basic Analog Grammar*.

One of the things I discovered while going back over this project is that some work done and finished in the last update were never updated in the resulting product, i.e. overlooked, for example, the very first series of plates.

Grammar and Naming Conventions

Binary recursion affords us exactly four categories of grammar: Common Grammar, Arithmetic, Algebra and Geometry. This means that biologically

we are afforded a Grammar Matrix by which to process information. This matrix not only allows us to formulate verification by cross-checking, but also allows us to acquire the maximum utility from our experiences. However, the arts to do these processes have been greatly neglected because the human race is still very much mentally incompetent. Plato, himself, suggested using geometry as an aid to follow the concepts presented in common grammar, however, that work, *Parmenides* has been, by their own admission, over the heads of so called Platonic scholars. One has to learn not only how to correctly construct a figure, but also learn how to pair it with logical grammars. For example, the Arithmetic Naming Convention gives us names which we call numbers. A number is just a name in arithmetic grammar. If one is a complete idiot, they claim that there are different kinds of numbers which come about because of how one uses numbers. Every book on math I have ever read was written by someone who can be judged, by simple grammatical fact, as illiterate.

The Algebraic Naming Convention affords us letters. When we first establish a correspondence between the Arithmetic Naming Convention in our write-up of a figure, we are simply using Algebra as synonyms for Arithmetic. When we convert the Algebraic from being determined by the Arithmetic, which uses a standard naming outside the figure, to the figure as establishing the unit. One will find when they do this, they will see relationships in the figure not revealed by Arithmetic as given in the raw.

In many of the plates I accompany the write up with, one can see the results.

I will go over the previous introductory material for this work, and let it follow this introductory addendum. I will also add an essay addressing objectives in pairing the analogic of geometry to logical grammars such as common grammar, arithmetic and algebra.

All in all, one should develop a very firm objective belief, it is wholly impossible to predict the results of our own behavior, or fulfill the promise

of intelligence, when factually, there is not one correct grammar book on the planet, nor is simple binary recursion being taught anywhere as the foundation of the Grammar Matrix, we are evolving to master. Everywhere on this planet, from the simplest minds to those claimed to be genius; mankind is pre-literate and falls short of the very first principle which defines intelligence, the recognition of simple binary recursion. I, personally, find it odd, that one can even use a computer every day, even program it to perform its functions, and not realize that every thing it does, is the result of binary recursion which means that every possible system of grammar it can parrot, is also the product of binary recursion, or as Plato said, there are two, and only two, parts of speech recursively applied to produce all systems of grammar. The only thing a mind can do, is learn to read and write. Until there is a social recognition and use of our Grammar Matrix as the foundation of our behavior, mankind cannot be said to be more than an illiterate fool, no matter how many awards we give our self to celebrate that very same stupidity.

We are constantly told every moment of our life that our survival hinges on the mastery of simple binary recursion, and it is not a sign of intelligence when one does not recognize it.

The Delian Quest is more than a simple book, more than a single perceptible problem, it is our biological imperative, our Universal Problem, intelligence via the intelligible. Although my book shows that I solved for a particular ability to do cube roots exactly in geometry, it may only faintly help in achieving our Universal Quest, to have dominion over our environment.

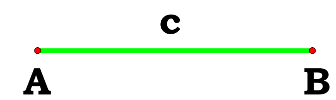
The Simile in Multis

Or the Magic of Metaphor

Saturday, September 4, 2021

The greatest form of ignorance is thinking that you know what you provably do not know: Plato. Almost everyone believes that they have the ability to think and reason with some measure of truth, when this is provably not true. These people will not suffer being corrected as they defend themselves over what is true: they suffer the more extreme conditions of mental illness, yet they are also, the most common type of human today.

The ability to comprehend grammar is a biological given. We denote this ability as linguistic functionality of a mind, or any information processor. In the Judeo-Christian Scripture, it is called the light which is the manner in which Plato put the sun in the cave metaphor of the *Republic*. In that metaphor, Plato was admitting that he, himself, was a prophet as defined in the JCS. That metaphor refers to the fact that all information processing, the life of man, is a physical fact which is based on the definition of a thing as a relative constrained by correlatives, i.e. just like the metaphor of Adam and Eve, it is a binary relationship which Plato called *Dialectic*, language by two's. This binary is an intelligible which we put into the perceptible using symbols by which we construct grammar systems in order to do our biologically defined job. The difficulty of comprehending the binary metaphor can be gleaned from a history of responses to Plato's dialog called *Parmenides*. In that dialog Plato suggests to the reader to draw a line segment to follow it. The line segment is a binary, or in the grammar of geometry, the *First Principle*. We can only name the correlatives and the relatives of a binary construct: these are called nouns and verbs.

The unit binary in geometry, 

, which gives us the simple sentence and our primitive equation. AB are correlatives, while c is the relative. This is the simplest binary analog example. If one looks to so called *Set Theory* and *Venn Diagrams*, one may learn why their writers always fall into contradictions, as Plato and Euclid noted, you do not start diagramming with a circle, but with a simple segment. If you cannot understand *Parmenides*, you cannot comprehend grammar and binary recursion. To understand this binary, we have to realize that A and B are the shape, limits, or container of c. A, B and c, individually are not things, they are parts of a thing. To put this into a signed number, it would be AcB for Common Grammar and 12c for Arithmetic, however, this name denotes a particular thing. In geometry, if we use the measurement function, it would look like AcB = 12c, this means that A = 0, B = 12 and c is the coordinate system of reference called linear distance. In this mode, we are simply making synonyms between Common Grammar names and Arithmetic names. If we now take AcB and divide it by 12c, we get $\frac{AB}{12}$ and since they are synonyms, we get $\frac{AB}{12}$ = 1: or we are denoting that AB in Grammar is the same name as 12 in arithmetic. We have paired Common Grammar to Arithmetic, or again, made the two names synonyms.

This is a straightforward transform back to simple counting. When we can do this, we can now comprehend that not only is common grammar and arithmetic simple methods of counting, all grammar systems are, or in other words, every possible system of grammar is effected by simple binary recursion based on the simple definition of a thing, and even our own biology for any particular sense can either abstract a things material difference, or make us aware of a things limits, or boundaries. Binary recursion not only allows us to address memory, but also to manage and

manipulate memory in order for us to do our biologically defined job, to have life, and to have it more abundantly.

The true comprehension of the intelligible unit covers the whole of the perceptible and the intelligible. Binary recursion can only produce a binary result and not the gibberish common to the intellectuals of the world today.

So, before we become too lost, we have to put our binary into a definition of a thing, or unit.

Definition: A thing is any relative constrained by correlatives, or in simple terms, a thing is comprised of a shape with some material in that shape. A thing is a binary construct. The material is not the shape, nor is the shape material. A noun is a container for verbs.

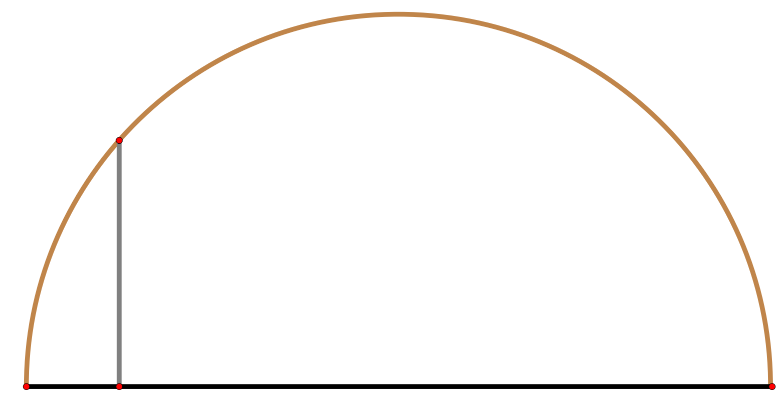
Stupidity is when one takes a noun, which has absolutely no meaning at all for its paradigm is a container, not a material difference, and claims that the particular noun is a container for perceptions, verbs, that they have never acquired, one achieves functional schizophrenia, a schizoid population. What a person knows, retains in memory, is proportional to their own experiences.

When one studies *Parmenides* by Plato, they will begin to see the confusion in their own mind because the untrained mind confuses this distinction and tends to call the parts of a thing, things of which they are parts, i.e., gibberish. The ability to keep track of the intelligible is what Confucius meant by being aware of the truth of things. We have to always keep in mind that a thing is a binary, and this divides words into two, and only two, parts of speech, nouns and verbs: these are in a binary relationship and every grammar, correctly taught, is aimed at teaching simple binary recursion. Today, everyone simply uses words as a caveman uses a club; they spend a lifetime playing the shell game with words. There is no dictionary of common grammar today, which holds to the first

principles of binary recursion in grammar, i.e., every one of them is written by the functionally illiterate.

Let us take a brief and simple look at metaphor, how it works and what it is.

Let us take a simple grammar system, arithmetic and name a few things with it.

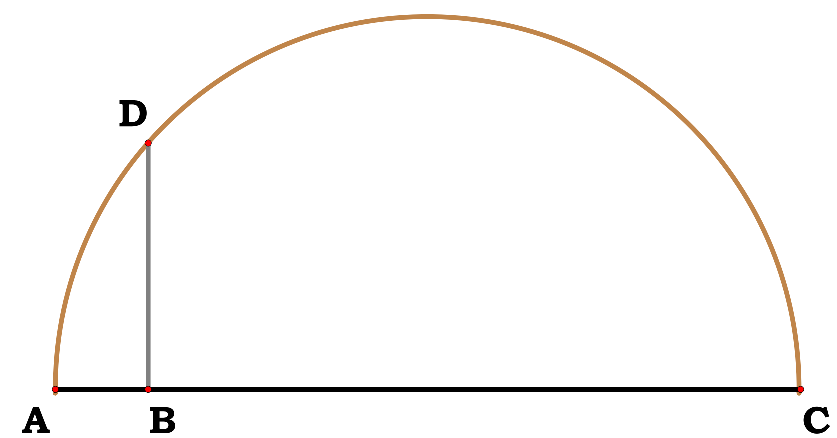


It is just a simple thing, perhaps a tunnel with an upright interior wall someone has sketched out for us. Let us take a moment to think about a common phrase used by some so called intellectuals, *self-evident*, and *self-evident truth*. We have a distinction between the perceptible and the intelligible. Which can we actually share with anyone? Can an intelligible ever be *self evident*? Can a perceptible ever be an intelligible, and vice versa? Which is absolute? Do we all live in the same reality? Do we all have the same memory sets and ability to virtualize our environment? Can we all perceive the same things? But can we all transform those perceptions into the intelligible of memory as everyone else? Which is objective, the perceptible or intelligible, and which is subjective. Which is sharable, and which is not?

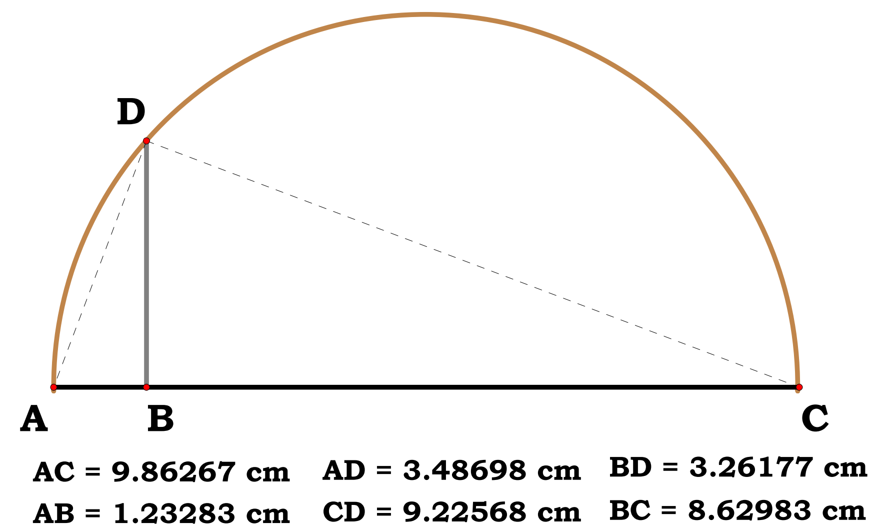
Think about the word Truth. Is it perceptible or intelligible? Does one look for truth in the perceptible, claiming as many do to be on a search for truth… and in perceptible things to boot? Is that indicative of the least bit

of intelligence? Truth is the state of being true. Can one thing, in of itself be true? Does anyone, in their right mind, go around claiming, for each particular thing, that it is not different from itself? When told that truth is within, do we start dissecting brains to find it? I know plenty of intellectuals claim that by dissecting the brain they can find language, but then they paid a great deal of money to formalize their insanity. If you cannot paper train a dog, can you have paper trained intellectuals?

We are given perceptible objects, self-evident objects, which any bug, dog looking to piss on a wall, or weed can in some measure perceive, but it is not in the least bit intelligible. We want to mentally manipulate this thing in our mind, virtualize it, and we do that with the aid of grammar systems. Our sketch of the tunnel was our first grammar system. Let us start naming its parts.

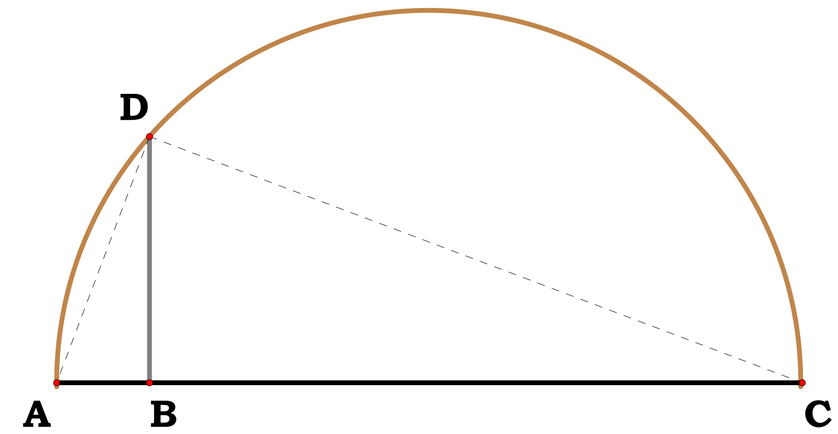


I have used the alphabet which is used in the English Common Grammar system to name the limits of each part of our little tunnel. Algebra also uses alphabets in its system of grammar. So, now we have paired two grammar systems, common grammar which is a logic and the geometric figure which is an analogic. Let us pair arithmetic synonyms to our common grammar names.



Well, we find that our tunnel is kind of small, perhaps on a Hollywood scale for some movie. We have all these arithmetic names established as synonyms with common grammar names. And we also see, that this pairing is traditionally elliptical which should not be occurring in a formal system of grammar. This ellipses does not tell us much about the relationships between the parts, that is, ones we cannot see with our eyes. We are getting a hint that there is something which is not self-evident, but which only names can show us is there.

So far, we have taken our common grammar and paired it with our arithmetic grammar, but something is lacking. Our common grammar alphabet can name anything perceptible or intelligible, however, our arithmetic pairing is specific to a predefined system of measure, in this case centimeters. How do we make a pairing with a common grammar and a universal name? We perform an arithmetic operation, division. If we divide any of the two arithmetic names, we find that the particular now becomes a universal because the particular unit of measure now becomes a simple ratio.



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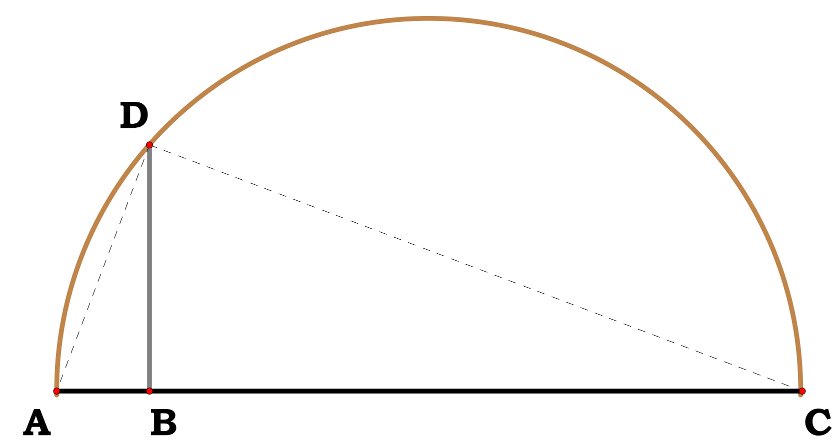

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We can now transform the arithmetic pairing with common grammar to a true algebraic pairing. Now we have three universal grammatical expressions, the common grammar, the arithmetic and the algebraic, each telling the reader that the geometric figure is, itself, a universal expression.

So, even though our program is wrong in the way it establishes it assignments, we can fix it by using an operation with the symbols.

We have six different arithmetic names which we can denominate as a unit for the figure. We can try all six, and see what it tells us, but right away we see we have patterns but we also have done something else, something very intelligible, so most do not get it. The simple arithmetic convention of names is comprised of two names, a noun and a verb. Ten cats, five fish, four centimeters. It is a verb noun system. Some call it signed numbers which does not make any sense at all because even numbers are signs, or symbols. Arithmetic, when correctly comprehended, provides us with simple sentences of verb and noun when paired to common grammar. However, when we ratio one name to another, the name for the material difference disappears. Length, weight, color, etc., that is names for some material difference, cancels out as unity. We have gone from the particular to the universal, or metaphor. We have found the similar idea in the many examples when we remove the particular, we get a universal. And if we keep going, we get equations wholly independent of any particular system of grammar or *coordinate system of reference*: keep this in mind when reading the pseudo intellectual Einsteins of history. These equations are just as valid for dirt as for jelly fish.

The fact that every thing in the Universe is the result of binary recursion also tells us that every thing in the Universe can be compared as ratios. Long grammatical names can be reduced to a much smaller set of ratios.

What we have done, is through grammar, brought to life the intelligible which is not at all perceptible. We slowly learn to go from the simple arithmetic one-to-one correspondence of names to points of perception, or arithmetic identity, to geometric, or metaphorical, identity. Most today are, however, either wholly unaware that there are two types of identity, or binary identity, or they are trying to figure it out. Identity, is itself, a binary conceptual unit. This process allows us to deal with information in the

simplest possible terms, not as particulars, as our senses produce for us, but in the intelligible which only the intelligent can comprehend. The very fact that so called intellectuals today are inventing particular words, and pretend grammar systems as fast as they can, is simply due to the fact that they are illiterate. Binary recursion produces exactly four categories of grammar, common grammar, arithmetic, algebra and geometry, and as it is binary, there is no such thing as a theory of grammar, a theory of numbers, or countless other hot air theories of the pseudo-intellectuals of our day.

Most people, denote themselves as this or that, they make themselves a horde in a box, a collection of empty nouns, gender, religion, politics, when factually, we are simply an information processor working at all times with virtual information through Language turned into four specific grammar systems. Language is Universal and Intelligible, while Grammar is Particular and Perceptible. One cannot teach Language but one can teach, providing they have learnt it, grammar systems. Not having a standard for Language, however, what is called grammar systems today are provably not much more than gibberish.

Plato tried to keep Aristotle close, but it was clear to him, while Aristotle was claiming that metaphor has no place in reasoning, Plato was teaching it as fundamental to thought. As Plato said, you can teach some people for an incredibly long time, but they will never get it. We have the letter of the Law, which is based on arithmetic constructs, but we have the intent of the Law which is metaphorical, something most if not every, judge in the world would deny.

The algebraic equation, when correctly written, is independent of any particular application, just like every other grammar.

A grammar matrix makes full use of the absolute and the relative for the express purpose of information management. Mysticism rules the world today, but there is usually someone who spends their life denying its

validity, but never banishing it from their own mind. One of the greatest and most destructive myth of all is that words, symbols, have meaning, and when great fun by Plato was made of that idea in *Cratylus*, his whole point was missed. If words had, in of themselves, any meaning at all, then it is a fact, they would be wholly useless for information processing. The indexing system is not the indexed perceptions. Meaning is the motivation to effect one's behavior, like hunger, pain, thirst, pleasure, which no word, in the history of the Universe, has ever felt. To ascribe meaning to words is anthropomorphic, the signs of a savage. How is it that even today, one can be sued for things of no meaning while actual things done by our governing bodies go by with a blessing? We mean to do things, which means that meaning is a synonym for planned behavior, which, has not come to fruition for a linguistically functional species on this planet.

Faith is the substance of the things you had hoped for, the evidence of those things not seen, thus faith is prophecy, the ability to virtualize information to be used and is used to bring about the best state of life for one's self, and one's environment. But, which can never be said too much, mankind does not even know what a grammar system is yet, nor does he teach it.

Tell me the world is not sick. Sickness is a degraded state of the body, during evolution a mind is in a degraded state of functionality.

A sign of a very weak mind is confusing a thing with its virtualization, a theme Plato exampled a number of times. Our virtualizations are intelligible, they are not nor ever can be the perceptible, however, one can consider them as maps, thus prophecy, predicting behavior, is map making. The map is not the territory. The intelligible is not the perceptible. We do not eat the recipe, we use the recipe to make a meal. But, no matter how many times you say it, you find a world eating paper, the very same paper they wipe with.

Therefore, we are back at the beginning, the cave paintings, the hand-made tools, all of these are the evolution of linguistic ability for the mind can do nothing else but learn to read and write. Analogic, the pairing of behavior to intelligible concepts, was our first and shall be our last, system of grammar. With a simple stroke of the hand to produce a segment, we have recognized the binary foundation of Geometry, which, the fools of today do not recognize, means there is one, and only one, geometry, just like there is one, and only one, common grammar, one and only one arithmetic, and one, and only one algebra. Each of these comprise a symbol set, and the method of recursively applying those symbols to count data.

Arithmetic reasoning is based on a one-to-one correspondence, one limit to one material difference. We use arithmetic to accumulate data from perception. Geometric, or again, metaphorical reasoning, is based on the one to many, the simile in multis, the one idea from the many examples, and this is called pattern recognition. How is it, that today, we have our so-called intellectuals spouting pattern recognition with their mouth, while on the other hand, claiming that there are an infinite number of grammar systems? Sounds like a damned brain dead fool to me.

Intelligence is the ability to construct standards of information processing which can be applied to all data. Ask your computer, does it use binary recursion to do it all, while on the other hand morons like Microsoft claims that there are countless systems of grammar? Do you want to read tomorrow, the files you make today, or do you want Microsoft to tell you that you are no longer allowed access to your own files? Is Microsoft, or any other corporation, educational, religious, or governmental the standard of information processing, or is simple binary recursion which is common to all and yet independent of all as well?

Adam and Eve are a Conjugate Binary Pair, whereby Compete Induction and Deduction, produce the human race. Words without wisdom is the condom which prevents the birth of man.

An illiterate species cannot reliably produce a literate computer. In my following work on *Basic Analog Mathematics*, you may learn how to draw one which is faster, and more accurate than any computer produced today, providing you have the intelligence to comprehend the work. Wow, instead of simply Geometry, I can call it Transcendental Transfinite Tit Tweaking Hand Computation! Or T-T-T-THC for short: it is legal you know, my stoned government said so.

Naming Convention

Saturday, September 4, 2021

Universally

Every possible grammar is a method of utilizing binary for memory manipulation. And every possible grammar is effected by complete induction and deduction of the unit. A unit, is a universal conjugate binary pair of a relative constrained by correlatives.

At the foundation of every possible grammar is the intelligible unit to formulate each particular grammar; the symbol set it uses and the method by which those symbols are grouped to express names is simply a recursion of that unit; these are the naming convention by which each grammar is based and every proof can resolve the entire chain of usage back to the convention of names, or what Plato called, the First Principle Parts of that grammar, which is the convention of names that grammar is based on, logical and analogical.

Fundamentally, a grammar is an indexing system, which is something you should take away from the above. We can call a grammar system an indexing system, or a memory mapping system but we can never say, if we are sane, that the grammar, or its product, is the data, or information, our actual experiences. Grammar is simply a tool afforded to us by Language. It is the Ultimate Tool.

These two elements of grammar, symbol sets and how we apply recursion to them, which are standards of human behavior for a correct system of grammar and are the standard for sapient psychology. As the human race is still primitive, it has not standardized this yet; close, but still not rational. It cannot be said that the human race is sapient or even civil, not by factual definition. Psychology is commensurate with the principles of grammar which are functionally resident in a mind as

grammar systems, and currently man is claiming ignorance of both. This amounts to calling himself a brainless fool and like a fool, he is happy with that result.

Our two intelligible elements of a unit, or thing, can be expressed as no difference between such and such, which is Arithmetic, and a difference, which is Geometric. For example, we compose symbol sets with standard behaviors of the hand. If I wanted to compose a symbol set which was absolute to those two elements, it would be geometry, stop, go stop, for a line segment. In Logics, we use a many to one in terms of behavior to formulate symbol and the methods to manipulate them. Thus the symbol sets are composed of many behaviors to form letters, numbers, etc. The symbolic convention in Arithmetic requires a one-to-one correspondence in how those symbols can be grouped or indexed to form names. The second element is how we index or group those letters to form particular names for the particular examples of the elements we experience in the things around us. These names must be kept in a dictionary, but there are no standard recognized dictionaries today except, when achieved in the coding particular programs which keeps them from crashing. There is no such care taken in human social behavior.

Common Grammar does not have an ordered system of indexing these, Arithmetic does, while Algebra combines both of the indexing methods used in common grammar and arithmetic for its system. In algebra, the names for the parts of things are unordered, however, the operations, our use of those names, are arithmetically ordered.

Geometry is the remaining grammar which starts with an arithmetic, one-to-one correspondence between the intelligible unit and the behavior of the hand. This means that Geometry is composed of two, and only two, symbols; point and segment, one is perceptible, the line, and one is intelligible, the point. A point which can be within a range is called its locus. When someone using the grammar is stupid, the ascribe relative

differences to points, as if they are perceptible objects instead of boundaries. They have moving points, and an infinite number of them to comprise a line, a plane, and even space itself. They are wholly ignorant of their insanity. When we make something visible, and call it a point, we are not denoting the perceptible, but something intelligible which these people do not *get*. A circle is just such a locus. Just like every other grammar, geometry functions by complete induction and deduction of a unit. The loci of a process is a simple way of saying things like, for ever P from N to X, etc. When one is simple minded, they actually believe that points move, or that one can ascribe motion to them; but an absolute is never a relative. It is not professional, save by way of conversation, to claim a point moves: such phrases are colloquialisms. One may say it moves only as a colloquialism and claiming a colloquialism in a formal write-up may not be professional and should always be pointed out as such in order not to confuse children.

When one is simple, or is conveying geometry to the simple minded, one does not expect them to comprehend the intelligible, even so called geniuses never did. So, we say, in those cases, for those minds, that geometry is effected by straightedge and compass. This is akin to saying that essays are written with pen or pencil, and paper. We express ourself to the simple minded in terms of the perceptible, but it should always be followed by a more exacting intelligible expression as simple minded people often never even imagine a distinction between perceptible descriptions and intelligible definitions. If you explain it to them, then they have a chance to eventually comprehend something intelligible. If you do not, then you forget that most of us are lazy.

Every grammar system forms names by using names, as a process of recursion, thus we are always seeking names by manipulating names; this is called reasoning, finding the equation, or solving for an equation or describing some thing or process, or a recipe and even instructions. As

grammar is a process of virtualization, the actual product will factually always be a virtual representation, or metaphor. Names, recursively used, produce only names, even in geometry. The first names, however, are symbols which one learns to pair with their own hand, or method of expression such as speech. These name one's own behavior specifically for grammar basics, units of behavior to effect a grammar.

Mystics teach and preach that reality is determined by names when in fact, names are determined by the intelligible mapping of reality. The map does not produce any reality at all. We may produce things by following a map, but the map has not ability, no motivation, to do anything at all. Names, in of themselves, have absolutely no meaning. There is nothing, in all of creation, which is a product of itself. From the Pope to the multiple PhD holders, men preach mysticism about names. Man is very much still a simple minded savage in the universe. The assignment of memory to names is an intelligible standard of mental behavior which is still not taught today; man is still proto-linguistic. The claim that the meaning of a set of symbols is derived from those symbols is the most basic self-referential fallacy possible and this fallacy is still the foundation of a great deal of social education. This is simply delusion and insanity, a schizophrenic result produced during evolution. A mind has to evolve out of schizophrenia, out of delusion, out of mysticism.

Currently there are no correct grammar books, no correct educational systems, and no correct governments on the earth and the only one that can change that is that person we have slept with every day of our life. Did you know that this makes it wholly impossible not to have spent the greater part of our life not sleeping with a whore? Wow, now I need a shrink. Is that a self-referential fallacy, or simply a tautology?

Casually, we call something an angle, which means what Euclid said it did; things which are angled or again, in some respect proportional other than simply 1. Angle is not a noun until it is defined in terms of our naming

convention. Euclid showed how to do this, but simple minded people, view the angle as if it were a crotch and they describe it as such, the meeting of two legs when it is factually a ratio. Simple minded people, who apparently never consider the obvious, never ponder how it is, in plane geometry, that we ever never have anything more than two dimensions. The word dimension means binary, mentioning by the two-elements of a thing. By recursion of this unit, we say one dimension, meaning a single binary unit, two dimension, meaning 2, or two binary units etc. Thus a one-dimensional object in geometry is a simple segment. One mentions the points, or limits, and one mentions the relative difference called a line for linearity. A line can represent any relative whatsoever. It is completely metaphorical or to be more correct, since words have no meaning, we, ourselves, employ it metaphorically, or always intelligibly. So, when I say grammars are metaphorical, in truth, it means that if we are intelligent, we employ them metaphorically.

Any particular thing can have any number of units to describe it. We make this *having* possibly as part of grammar. This is one distinction between the perceptible and the intelligible, but if we are not idiots, we do not say that each particular thing exists in so many dimensions, when dimension refers to a naming convention established by Language and expressed in grammars.

Another thing to consider, in the working with things. If I slice and dice a unit, this is geometric, or deduction. When I add unit to unit, this is induction. There is no mystery, save for the ignorant, between induction and deduction. Deduction is Geometric, while induction is Arithmetic. They are not types of reasoning, but types of behavior in regard to things and our use of grammar. How can it ever be possible, when all of information processing is afforded by complete induction and deduction of a unit, to now have this doubled except by mystics and the ignorant? Is reasoning different from itself? Inductive reasoning, deductive reasoning,

positive and negative reasoning, are phrases for those who play with words, but are wholly devoid of intelligence. If, when I turn my computer on, it does something today differently than yesterday, I need to fix it, or junk it. For example, I have a raid 5 system for the boob tube which today has a red light on that will not go away. So far, raid 5 means I have lost no data. So, I have to back it up before I pull and replace the defective drive before further failure makes it impossible. This means I will not be able to watch reruns and I might become emotionally damaged if I cannot hear Walter tell me about flying monkeys.

Due to human simplicity brought about by our evolution, people confuse the name of a thing with the convention of names all of the time. For example, mystics teach that there are such things as real numbers, whole numbers, rational numbers, irrational numbers, imaginary numbers positive and negative numbers; all of which confuse a name and a naming convention with some particular use of it. This is mysticism in action. How many times can someone read Plato, and learn that the relative difference between terms, (such as lines and points) cannot be predicated of each other? It is wholly impossible for a name to be irrational; names are how we rationalize, or name. When you use a name constructed in an arithmetic convention of names to name a geometric process, then it is not the result which is irrational, it is the user who claims that induction is equal to deduction. We recursively name our only two working convents in binary; which was once put as, the point, or limit, or arithmetic, is that which has no part, or geometric; etc. My point is, a lot of teaching goes into making children remember half-baked, delusional rubbish. Teaching is supposed to unconfused children, not habituate them to it. I do know that forcing children to repeat rubbish as part of their social structure does cause mental damage and it is part of our social structure today.

Deduction, aka Geometric reasoning, or proportional, or again metaphorical processing has a very decided effect on memory requirements, it effects a kind of memory compression. Things are not grouped Arithmetically, one-to-one, but in accordance with some system of measure, of which there are actually few. It is also called thinking in accordance with the definition of a thing. Reasoning is factually geometric, or metaphorical when a mind is functional. It is wholly impossible to reason manipulating names arithmetically unless one has infinite memory, and infinite patients as one can do nothing with that information. Arithmetic is a method of assigning names, not manipulating them. I use the terms Arithmetic and Geometric, in this respect, in accordance with the original convention of name assignment, not the operations on the resulting names; every grammar system makes available the use of both for operations on the names created by these conventions.

Particularly

I have always looked at the Delian Quest as a unique type of novel. But in the writing of that novel, one can say that all the work to this point is sketched out, not in a finished format. During the work, I was learning about naming conventions, and I was very aware of it, but now, in the finishing of this work, I have to lay down what I understand of that convention. For example, we have induction and deduction. Deduction is parsing what we already have, where induction is using what we have to acquire more. In geometry, it can look like this.

I can start naming relative difference, or the part, in terms of some other given system of arithmetic naming, or I can start by naming it simply as 1, making it the unit. If I start with two things, I have to name them relative to some other standard, or I can name one or the other relative and the remaining one as a proportion to it. This will produce results which look

different but that difference is wholly determined by the naming convention. When one renders a definition from a chain of reasoning, the resulting definitions appear different, yet that difference is wholly determined by the naming convention we started with.

I can name AB as 1, which means that point C is an induction, in ratio to AB and likewise if CB is named 1, then AC is a ratio to it. If I name AC as 36 and CB as 14, I am using an external unit and the process is inductive. If I name AC 1, the CB is a ratio to AC then I am doing a deductive process upon the names.

Induction and Deduction

One of the things one should ultimately arrive at in the distinction between induction and deduction, and how one writes up a plate.

Arithmetic equality relies on the Arithmetic system of grammar and so one will always have numbers which are no more than arithmetic names. The Geometric system of grammar relies on proportion which does not have arithmetic names, it has proportion. We learn proportion by using Arithmetic names, but proportion is actually independent of them. Every grammar uses both Arithmetic equality and Geometric equality; the one is not, nor ever can be the other, a relative is never an absolute and this fact has everything to do with Law. When primitive people write Laws, they mistake the Arithmetic with the Geometric, the absolute with the relative.

For example; 2 is an Arithmetic Name, however, in common grammar we can distinguish between two different operations it can name; two cats, or twice denied. Two cats, or twice the standard by which a thing is determined to be a cat, or twice denied, two judgments based on one or two standards, or units of judgment.

Now if you are really stupid, you Cantor your speech claiming that there are two kinds of numbers, Cardinal and Ordinal. It is wholly impossible to

have two kinds of numbers as a number is no more than an arithmetic name. So, the Cantor's of grammar are mythologist, it is not long before they start multiplying how many kinds of numbers one has, just like those who claim that certain names can effect magical spells and inCantations (sic).

Thus, there are no irrational numbers, there are results which can be put into arithmetic names, and results which can only be put into geometric names. We have a grammar matrix to function by because we have two elements of every thing to name and thus we formulate four distinct systems of grammar all of which use the unit, but simply express that unit using four distinct behaviors to construct the symbol sets and the methods those symbols are manipulated.

Arithmetic is for particular examples, for example, assigning names, while Geometric is for the universal, or every member of a class. Judgment is then, and always has been, Geometric, or proportional, or again, metaphorical. Line upon line; Precept upon precept.

Thus, the simple minded manipulate names arithmetically, the more complex a mind is, the more, as Plato noted, that mind sees and uses, the similar idea in the many examples, or metaphorically, proportionally, just as the Bible is written.

A correct grammar book is a book using Geometry, proportional, metaphorical, reasoning. A geometry book has to use the grammar matrix, all four grammar systems.

Notes

And so to be complete with the demonstrations, I should example each of these choices and how it changes the APPEARANCE of the definition; not to mention cleaning up my past write-ups which were rather awkward. I may not clean them all up or catch everything as I have no help in these projects of mine. Not many people actually consider that the only path to

salvation for mankind is learning how to do, and doing, the work of the mind, our own work. It is a learned process which is currently not even taught. I do not call anyone a teacher of a thing who is ignorant of what we are and why we are. We are simply another life support system of the body with a well defined job to do and well defined means of doing it. Unfortunately, this well defined thing is not often discovered, nor is it discoverable by the blind, or mentally handicapped which is just what happens during evolution.

When starting out in exploring, one may ponder these issues, but since one is running to learn, they often get put aside until one gets to where one is going, and then one has to clean up the mess in the end, just like building anything.

If one has ever seen one or more of the skits, the *Anal Carpenter*, it is something to consider.

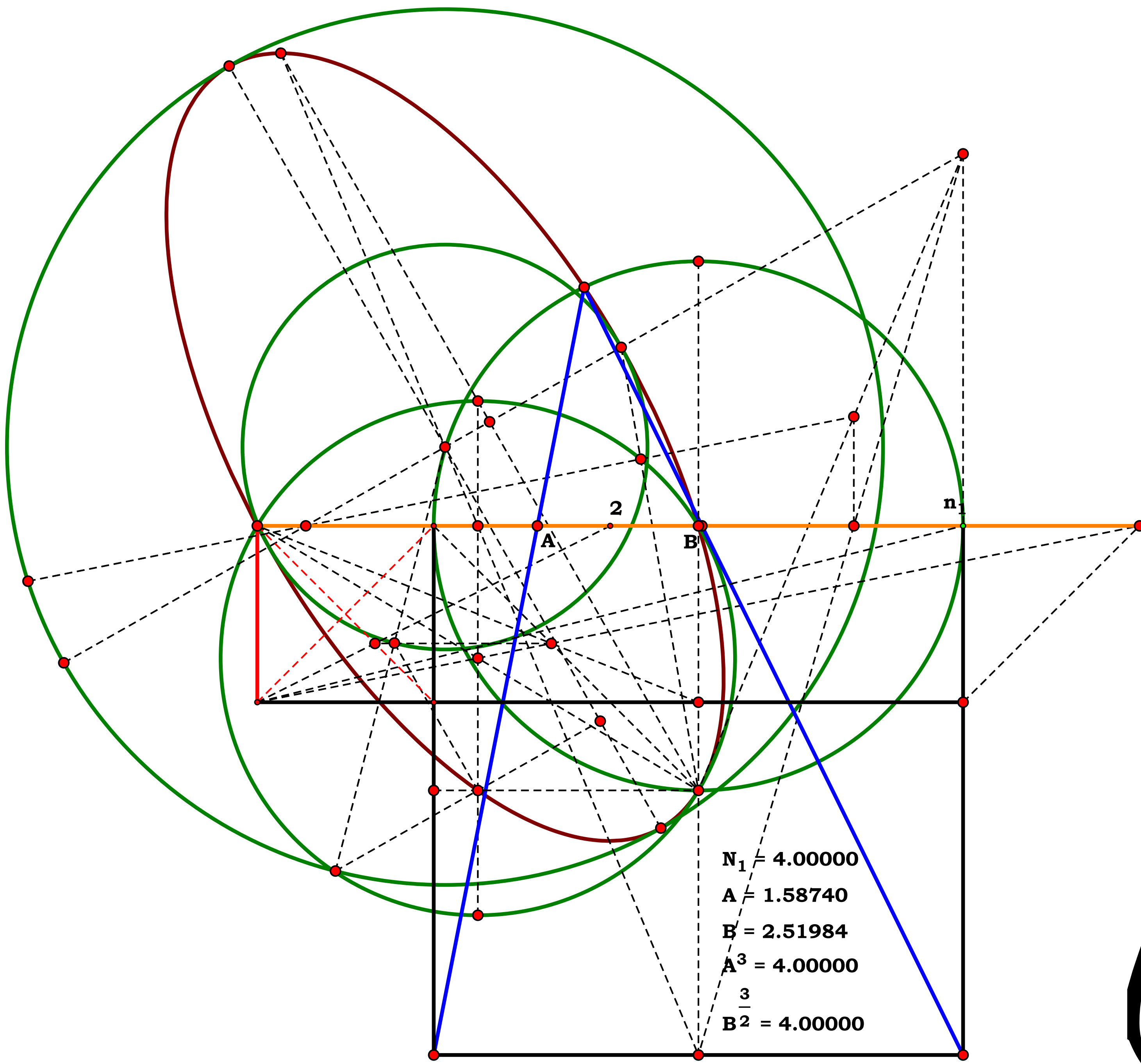
Thus, in terms of the naming conventions in geometry, there are no actual options for mistake, the range of options arrive in the basket of the logical system of grammar we use to pair with geometry. We can name things in the simple arithmetic, implying some standard unit, of which there are many, or we can name every thing in terms of proportion. We then have 2 square ways to name our geometric elements in grammar and I will example these throughout the work. The equations will look different for each one of these ways, and how they are mixed in the write-up, but the final equations should be true to the choice of the naming conventions used at the start.

So, I have a lot of work in this final version, the conclusion of my Delian Quest as a particular Novel, but as a living behavior, it can never end until I, myself, expire.

The four horseman, four ways we ride to measure the four corners of the earth are simply four naming conventions used in four grammar

systems or the single grammar matrix of the virtual reality in our mind as an image of God, or reality itself.

I can also say that the Delian Quest, like all initial investigations and learning, displays a lot of thrashing about. This leaves open a final work, which is demonstrated in a highly organized fashion. BAM, or BAG is more on that lines, but I have in mind a much shorter work which I currently call Hominid's search for the Holy Grail. In this work, the end is already a given. However, I do not think anyone suspects that a single equation can denote the whole of grammatical manipulation, which it does.

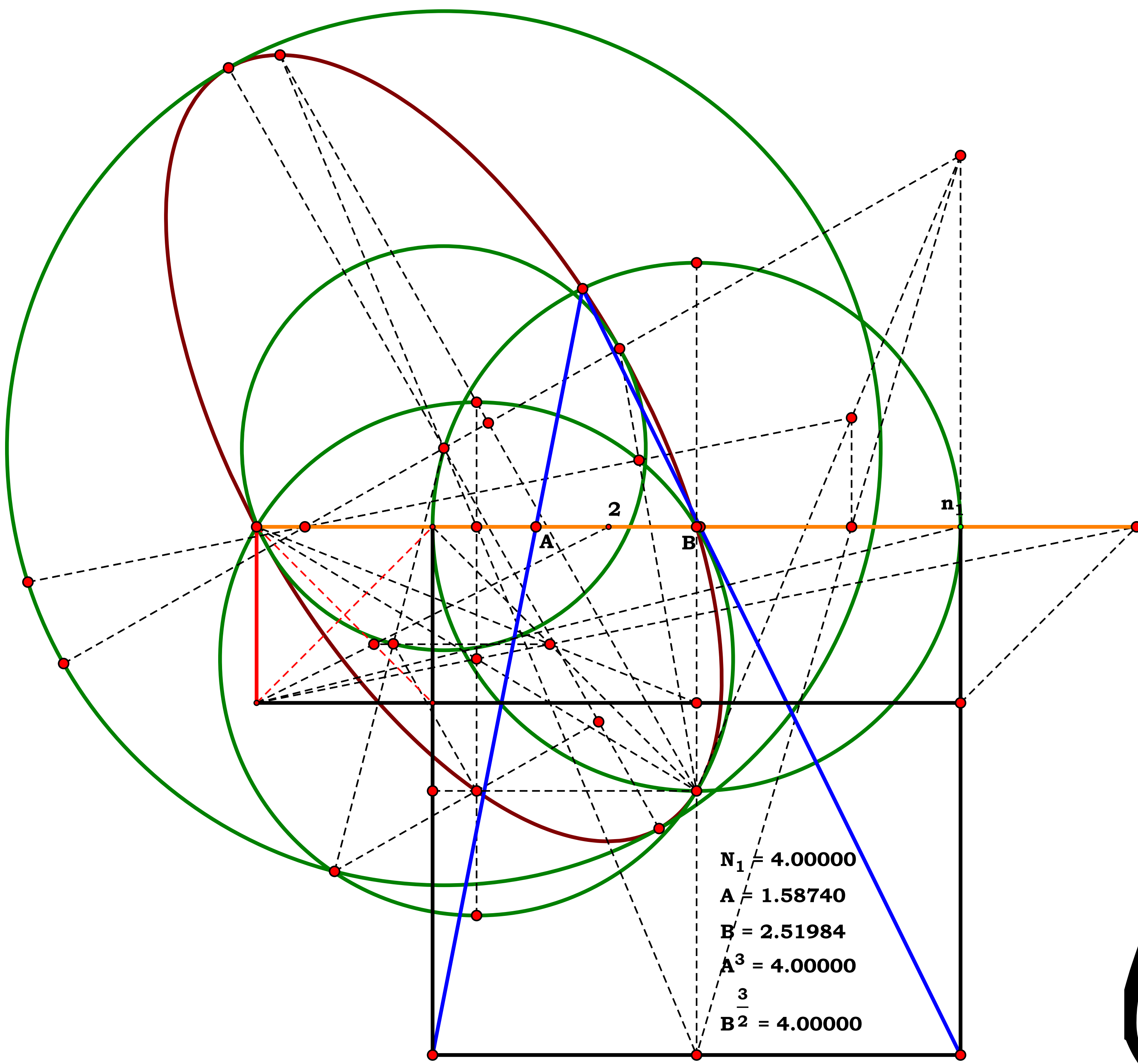


The Delian Quest by Year

John Clark

The Delian Quest 1989

John Clark



The Delian Quest: Original Submission

Sunday, January 19, 2020

The following is a copy of the statistics of a document written long ago. I wrote it using Ami Pro, Windows Write, or Word 2, at one time had Word 6 save it another time. I eventually made Word 2003 my standard word processor. This is the first figure I started with and it eventually led not only to the Delian solution but to Basic Analog Mathematics and my understand that every possible grammar is a binary expression.

Filename: DELIAN.DOC

Directory: E:\My Documents\1989

Template: C:\Users\John\AppData\Roaming\Microsoft\Templates\Normal.dot

Title:

Subject:

Author: John J. Clark

Keywords:

Comments:

Creation Date: 5/9/1992 4:32:00 PM

Change Number: 26

Last Saved On: 10/6/1992 7:23:00 AM

Last Saved By: John J. Clark

Total Editing Time: 339 Minutes

Last Printed On: 10/2/2019 10:29:00 AM

As of Last Complete Printing

Number of Pages: 19

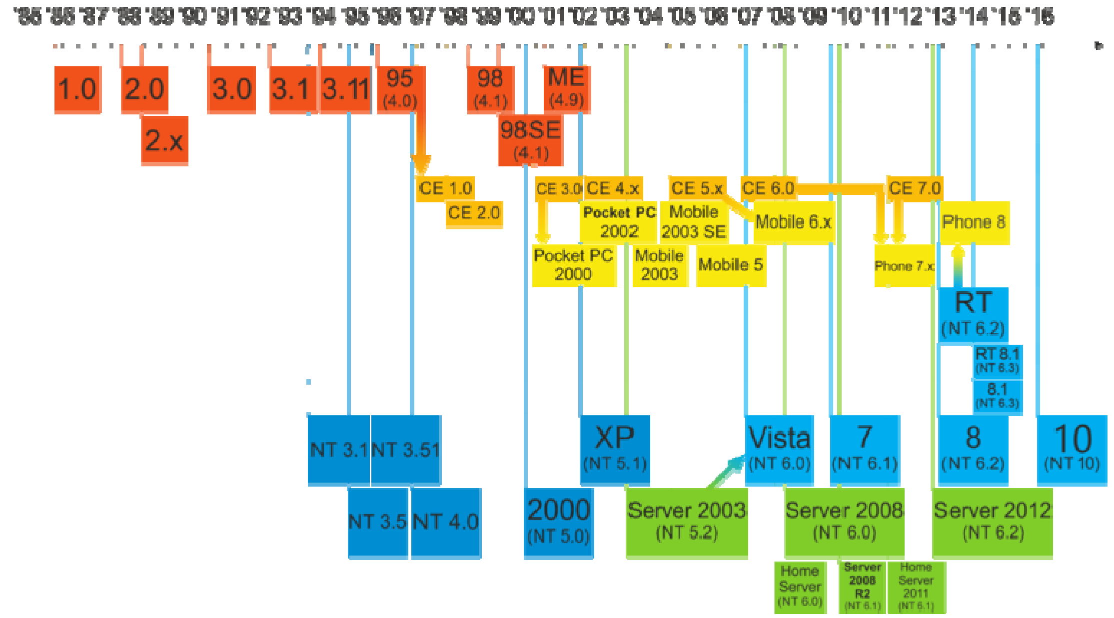
Number of Words: 2,019 (approx.)

Number of Characters: 9,553 (approx.)

What it tells me is that I wrote it using an operating system prior to Windows 3 and may have originally written using Ami Pro which I quickly abandoned when I purchased Word. The formatting, however, leans towards Word 2. It tells me that I started learning geometry at about the age of 38 never having taken the topic in school. It was doing my drawing by hand at the time, the first drawing program I used was TommyCad. I later found Geometer's Sketchpad. Early on, I was using Word 2 to construct data table's which I would save to a text document picked up by QBasic to do the math, and then pass it back to Word. I wrote macro's for that. I did that until Mathcad sent me an invitation to buy that program and have used it ever since.

I suspect Mathcad sent me the offer as Microsoft approached me first to become a beta tester, probably based on the machine I had just purchased which ran at a blazing 25mhz, which I later dropped out of because of the stupid way they ran the program, the original version of Dos 6 completely ate my hard drive and I lost a lot of work. I was using one of those sewing machine boxed computers at the time to carry it to and from work as I worked at G.M. I was the first G.M. factory worker to carry a pc in and out of the plant which led to a Union settlement allowing employees to bring their own computers in and out.

The following table is from Wikipedia.



I wrote the letter as I believed that a real geometer should work on the problem of cube duplication from this starting point as I have never even had geometry in school. However, I was surprised to find that these publications only expected such letters from professors instead of factory workers. No matter, once I had the combination of a drawing program and Mathcad, I could get more involved in simply learning. When I got these two together, I started putting some of my original drawing's on paper into a formal digital diary.

I have posted my work on AOL, personal free websites, even shareware CD rom's, mostly because I was looking for a kindred study flame, which never happened.

Before the Delian Quest, I pondered the issue of *Who wrote the Book of John*? I found that all evidence points to the wife of Christ, Mary and that enough still remains in the text to make a good case of it as it was deliberately altered to hide the fact. There is evidence to support the idea that Christ started to perform at his own wedding, which has a root in Law. His wife evidently became a companion prophet. There is also evidence that Peter had his eye on her which never panned out.

Then I decided to tackle the Name of the Beast 666. The depth of that solution evolved over years. After these, I decided to find another impossible problem to solve when I ran into the Delian Problem. My studies started in my childhood by putting before myself specific questions. Then for a long time just reading any book I could come up with. Then I started targeting so called impossible problems. My original approach to the Bible was a result of a sign post in a lucid dream. My first reaction was that it was rubbish, but it quickly faded when I realized, rather quickly, it was using words in a manner I had never seen before, it was deliberately testing the reader. Later, I found another writer who wrote in a similar fashion, Plato.

All of this tells me that I have been in a state of cognitive dissonance before I started my work on the Number of His Name and geometry as I should have been dead prior to this, before I came back to Michigan.

At this time, I am want to do another revision of the Delian Quest and have decided to put the opening as I had done before the 2015 release.

THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is, as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefore this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

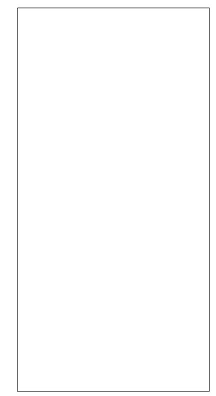


Plate 1

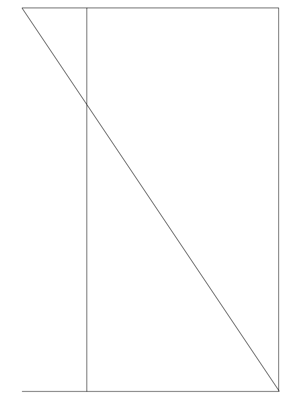


Plate 2

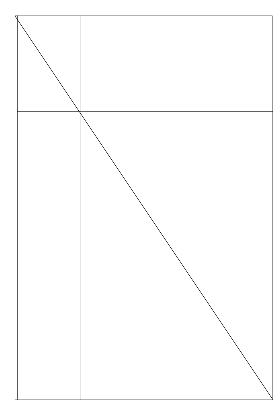


Plate 3

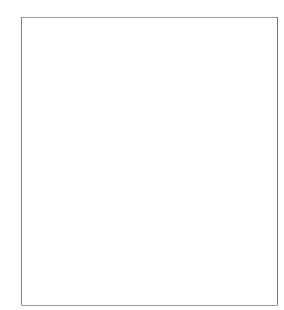


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

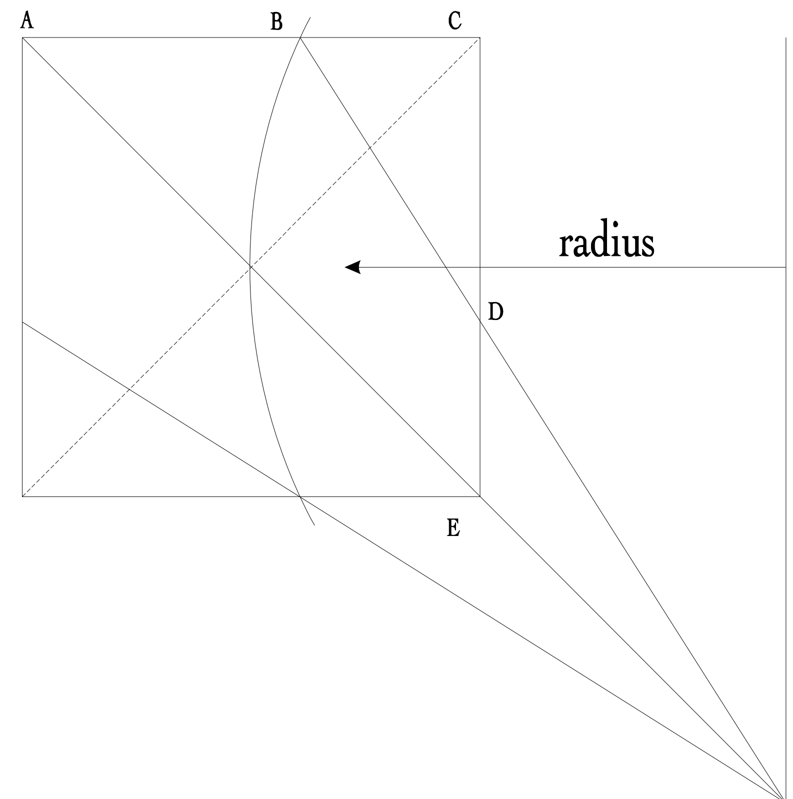
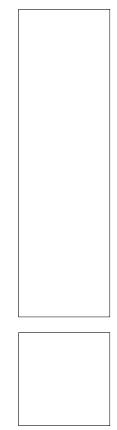
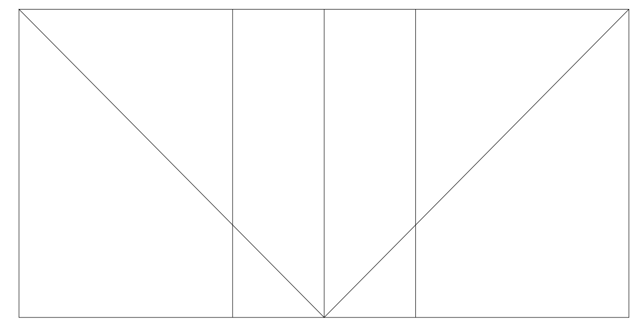


Plate 5 AB = CD. BC = DE

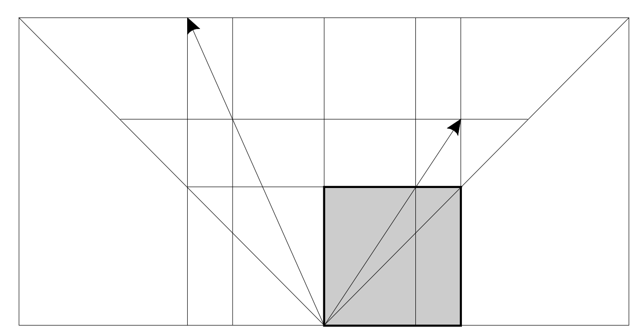
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



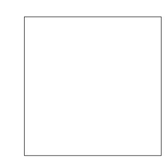
P.6



P.7

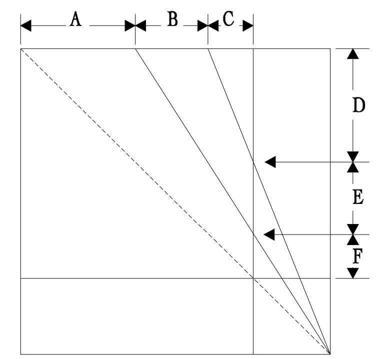


P.8

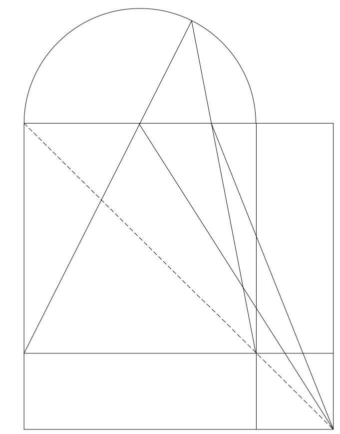


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

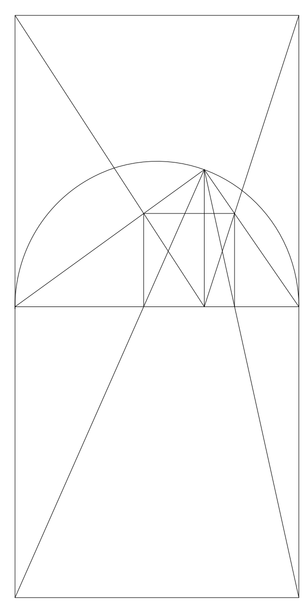


P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

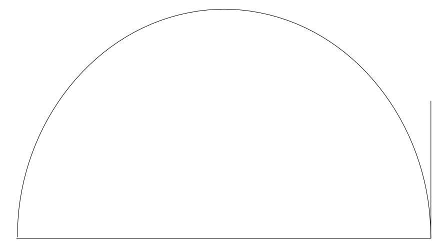


Plate 13

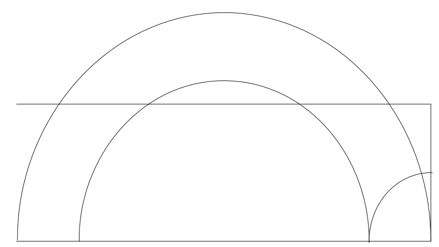


Plate 14

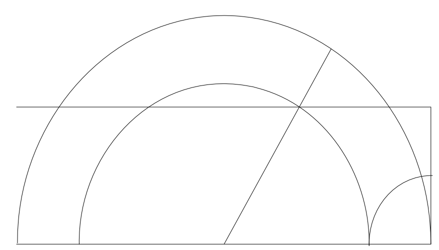


Plate 15

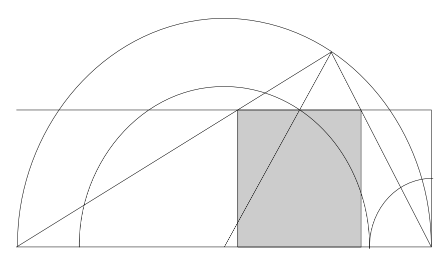
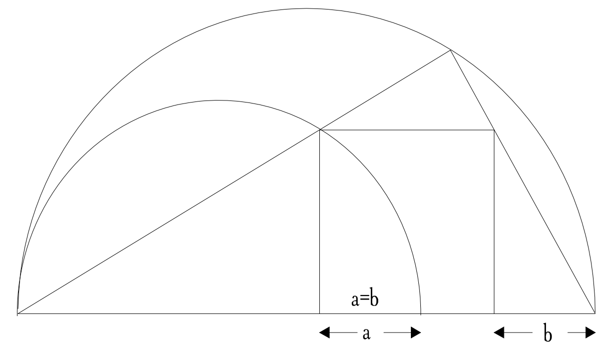
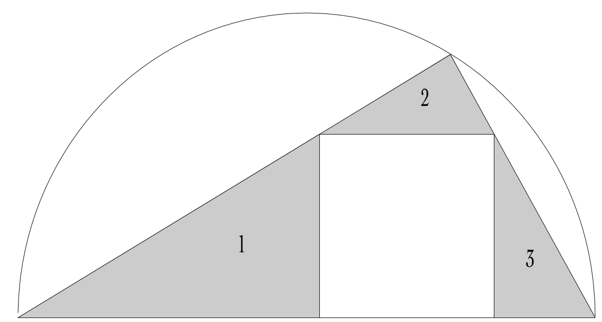


Plate 16

In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.

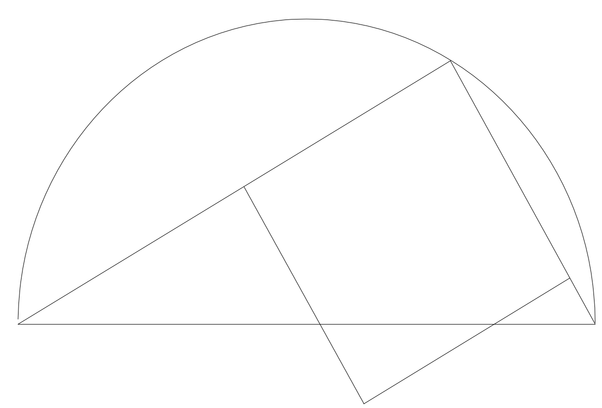


In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"



P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."

P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.



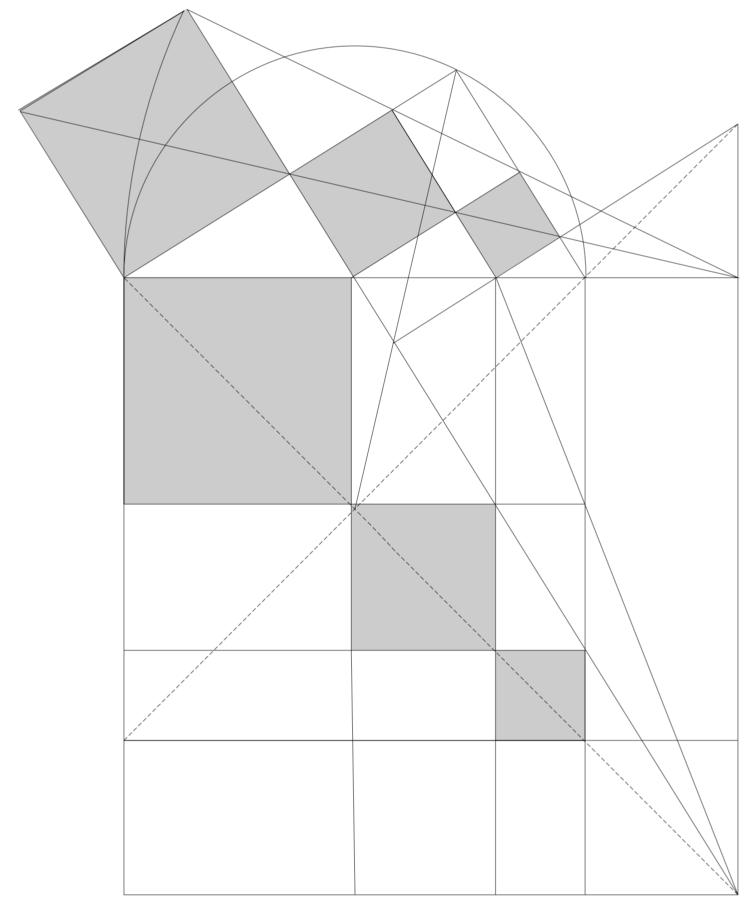
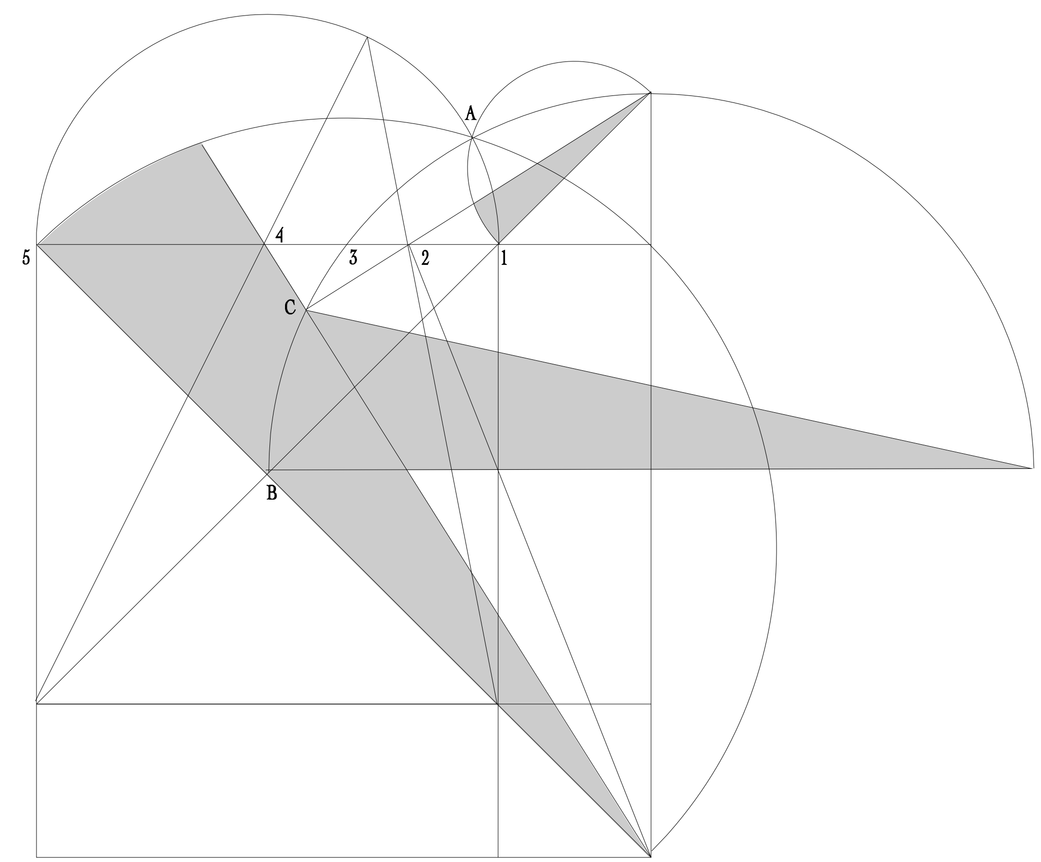


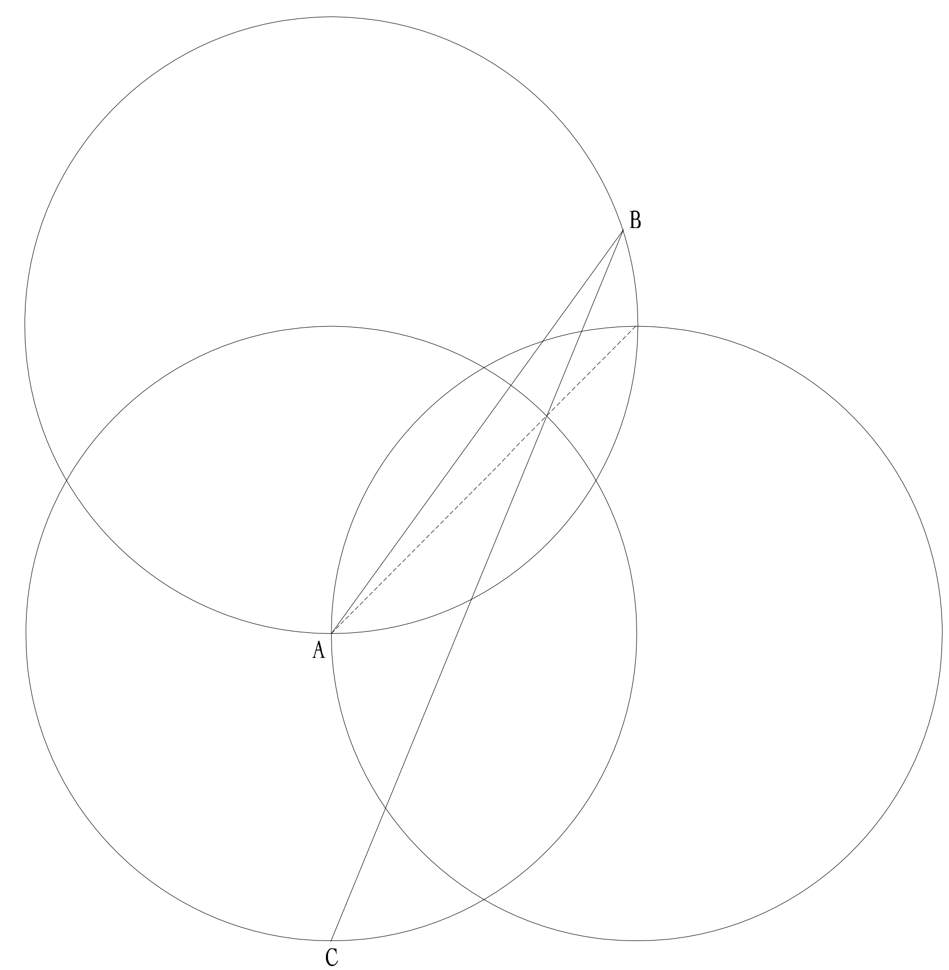
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.



All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.



How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

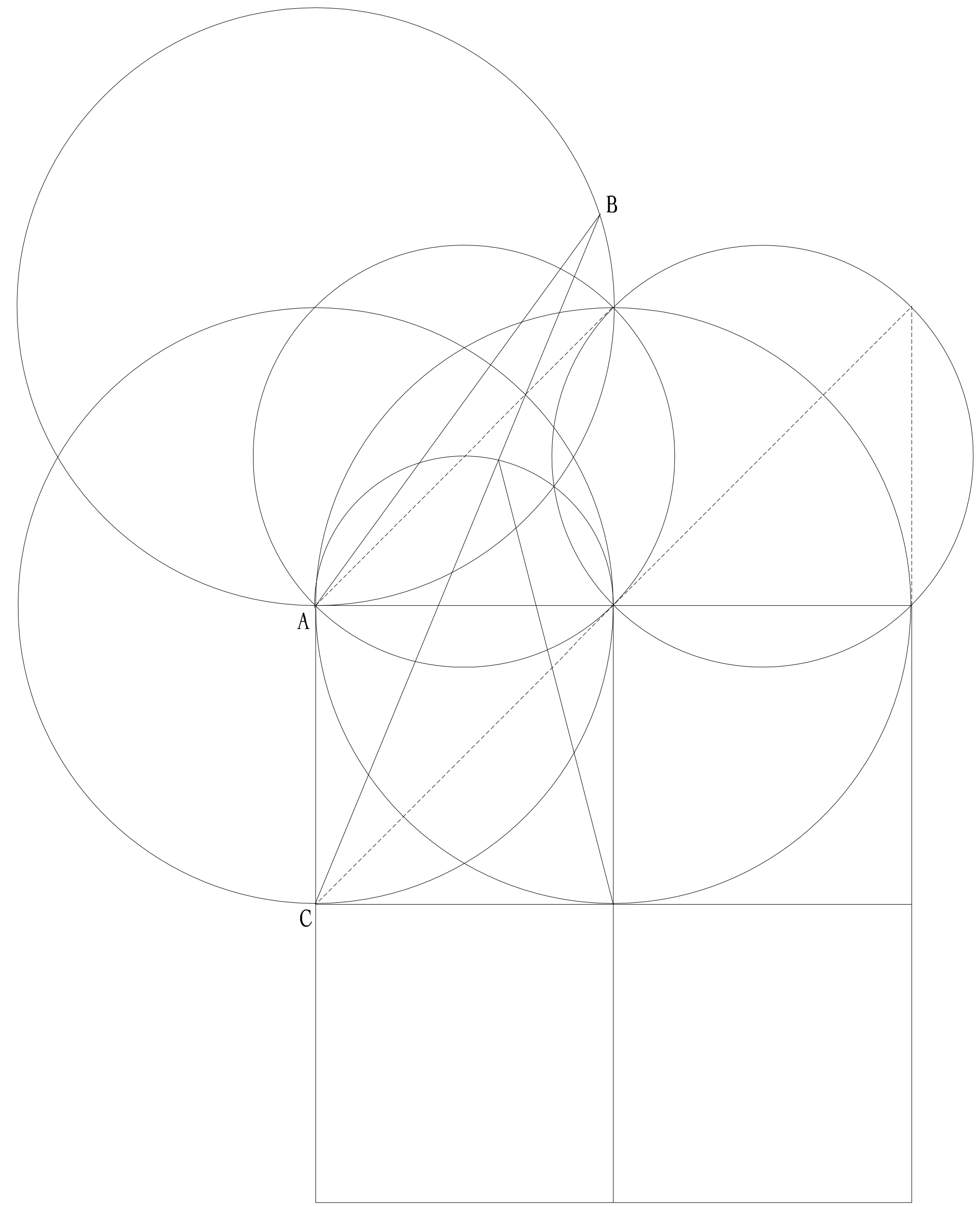
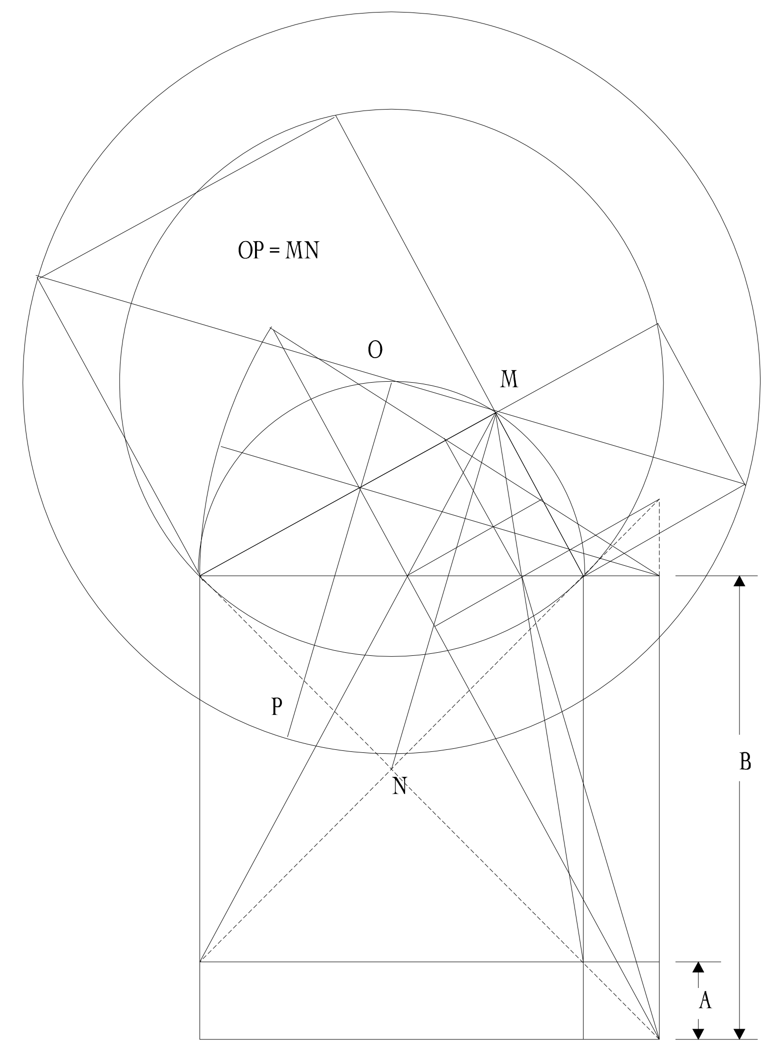


Plate 23

On Plate 24 the radius for the circle OP is given by MN.



One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of B^2A (if you have missed it, the figure gives both roots, A^2B and B^2A) there is a series of intersects, (three of them). When these intersects form a line parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

The following is a list of returns for publication of the previous material. The first one is interesting in that the writer claims not to have understood the preceding document.

GEOMETRIAE DEDICATA

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F. D. VELDKANW
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Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

American Mathematical Society

P.O. Box 6248, Providence, Rhode Island 02940 USA Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

Location:
201 Charles **Street**
Providence, **RI**
02904

December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitied "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Matheniatics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

American Mathematical Society

Roger E. Howe
Bulletin
Editorial Committee

Department of Mathematics
Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches Institut
Technische Universitiit MUnchen
Arcisstr. 21

D-8000 MUnchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".

Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H.-J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:

You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.

All together the article in the given version is not understandable.

JOURNAL OF GEOM TRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,

Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.

We are very sorry that we could not be of any help to you.

Sincerely yours,
H.-J. Kroll

(This one is a form letter.)

smf société mathématique de france

paris, le

BULLETIN

n. réf. a l'attention de

v. réf.

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Delian solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

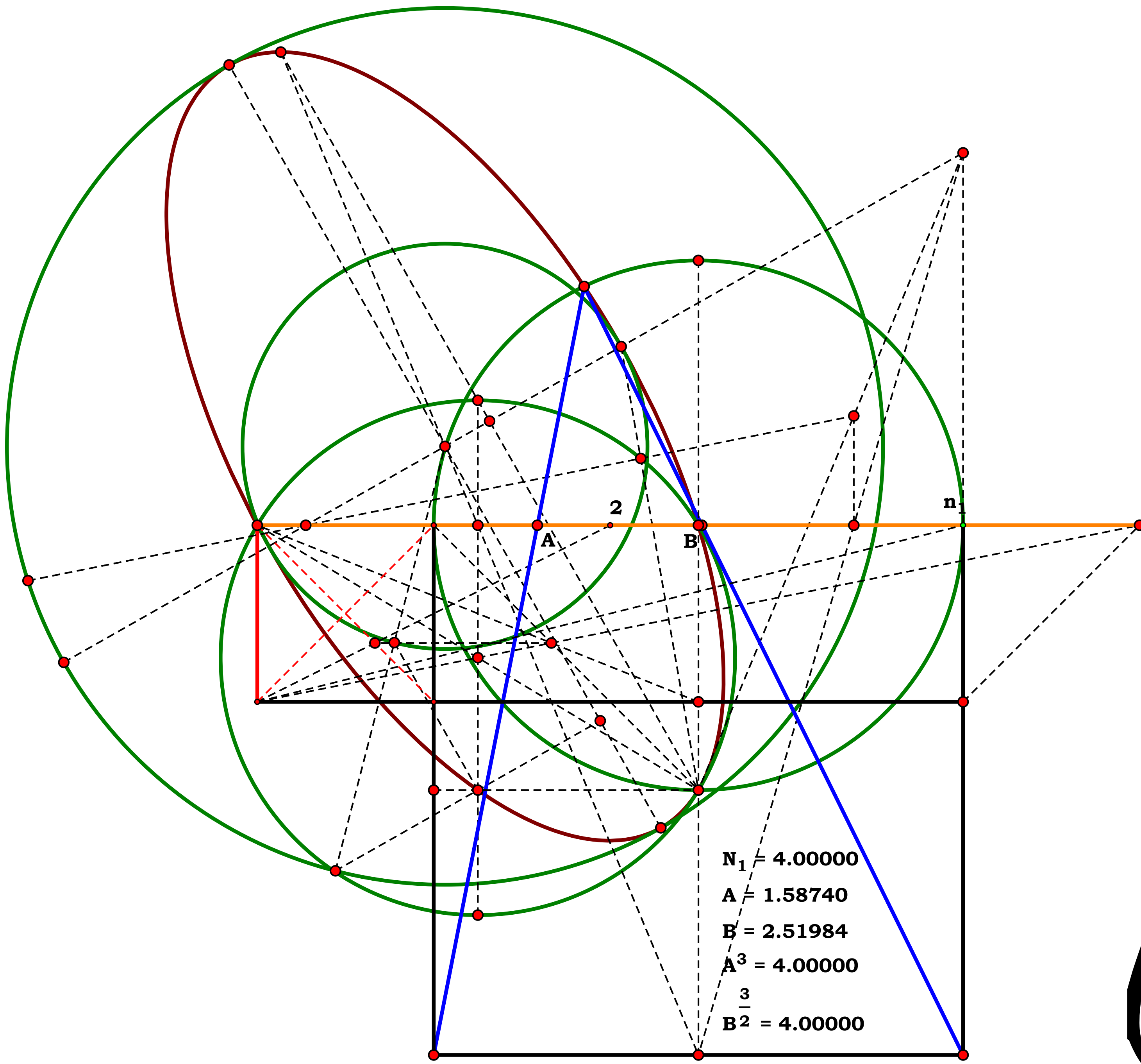
Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA

Directeur de la Publication

P.J. : Manuscrit

I was expecting at least some type of guidance, or cogent response, other than, after stating that I was not a geometer, everyone insisted on rubbing it in by calling me a professor, which I found very rude.



The Delian Quest 1992

John Clark

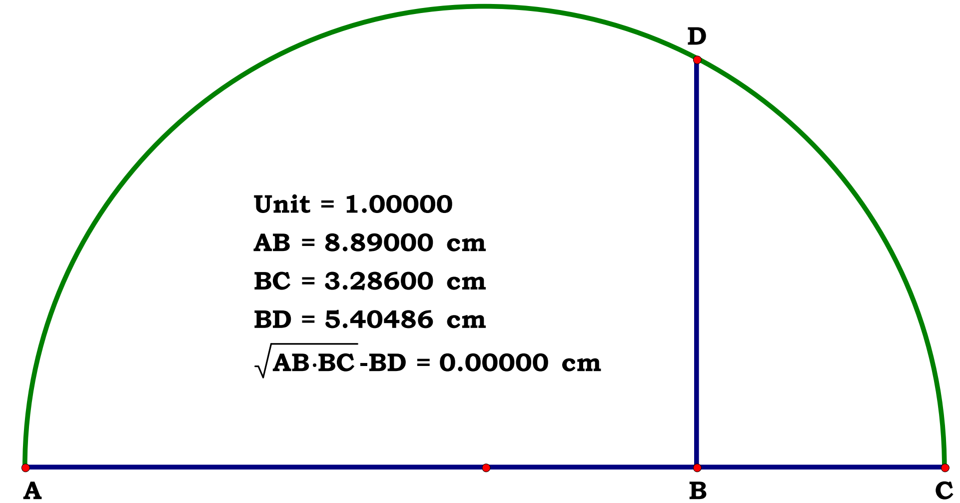
Unit := 1

Given.

AB := 8.8900 $\quad$ $N_1 := 2$

BC := 3.28600 $\quad$ $N_2 := 4$

A Duplicate Ratio



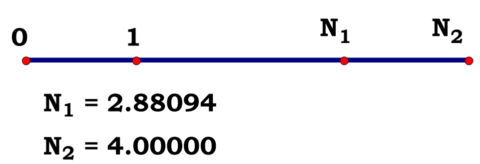
062092R1

Descriptions.

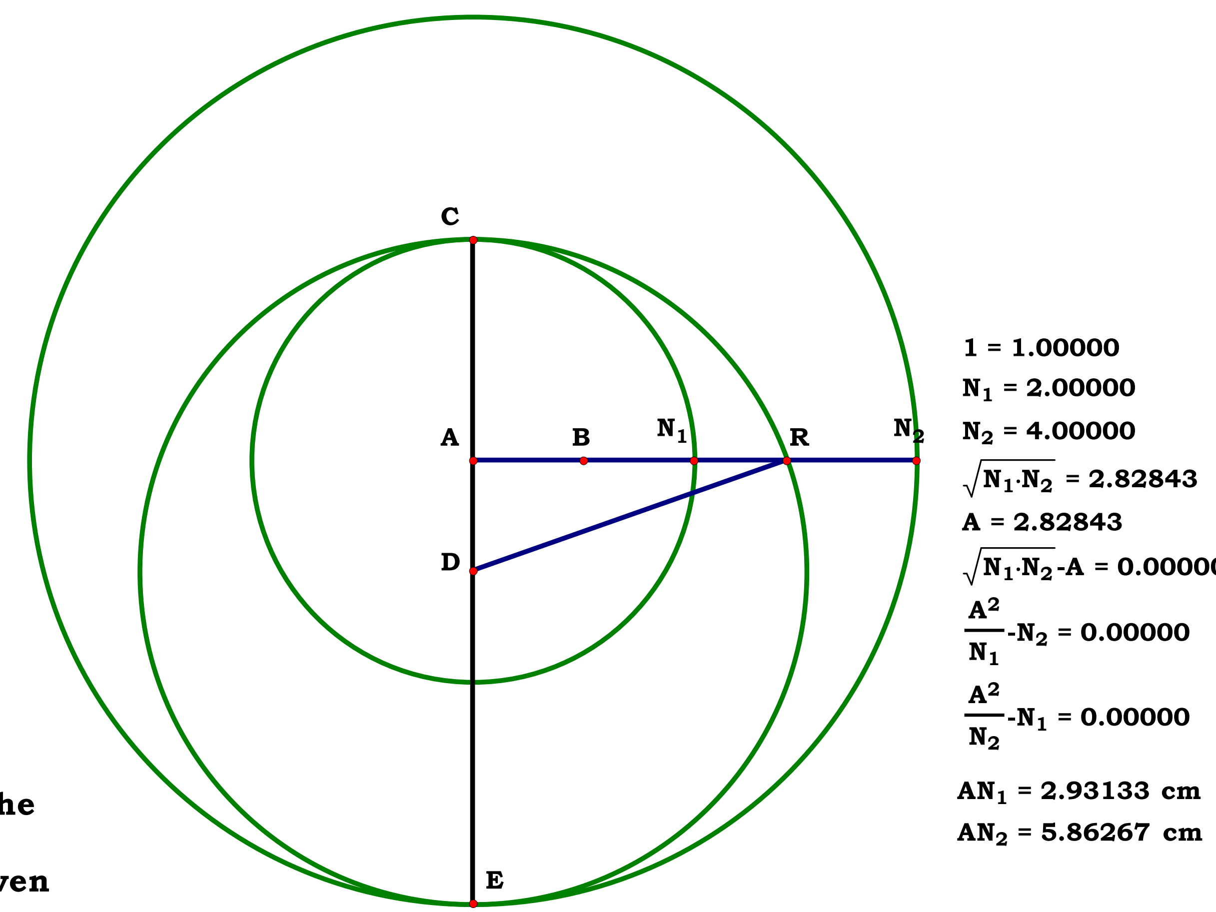
There are many ways to take the square root of any two differences, this is one of them. It is a very old figure, one can find it in Euclid's *Elements*. One can then say, that the *Delian Quest* starts with a given, The *Elements* of Euclid. What many do not realize, is the foundation of that work, the concept that Geometry is just another binary grammar system, traces back to Plato.

$BD := \sqrt{AB \cdot BC} \qquad BD = 5.404863$

Now, let us fold BA over BC and make what is called a numbered line.



As the first figure is a given, all we have to do to show how to take the square root of two numbers on a number line is simply unfold it. I am going to unfold it to the perpendicular to the point of origin. And now we can take advantage of all the simple resulting equations for the two numbers and the result.



$\sqrt{N_1 \cdot N_2} = 2.828427 \qquad R := \sqrt{N_1 \cdot N_2}$

Definitions.

$$R - \sqrt{N_1 \cdot N_2} = 0 \qquad \frac{R^2}{N_1} - N_2 = 0 \qquad \frac{R^2}{N_2} - N_1 = 0$$

The numbered line is not a new idea. Numbers are just names developed as the Arithmetic Naming Convention. We can also use Common Grammar to name our points as has been done for thousands of years. A line with the points given names has always been a part of formal Geometry.

Unit := 1
Given.
$N_1 := 4$
$N_2 := 2$

A Duplicate Ratio

062092R2

Descriptions.

It really does not make much of a difference if unfold I fold my figure, certainly not in the result. The next question is, can I take a third thing and put it proportionally at point A?

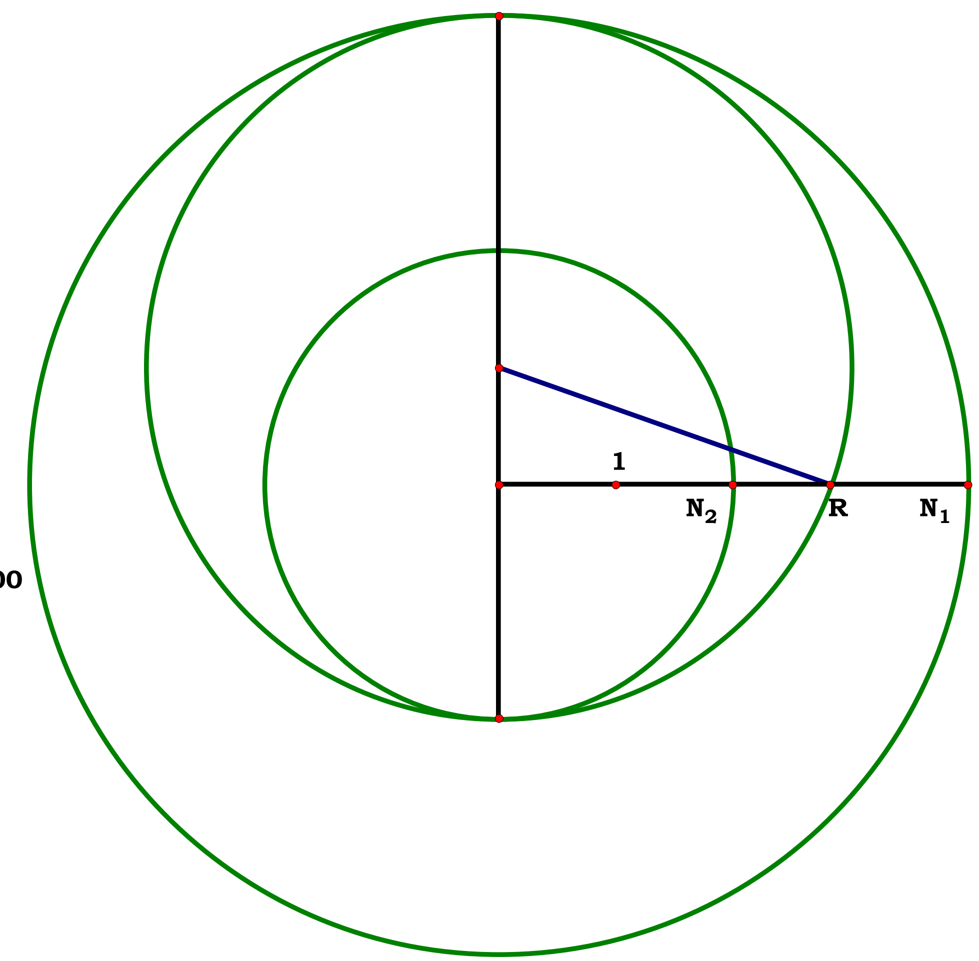
$$R := \sqrt{N_1 \cdot N_2}$$

Definitions.

$$\sqrt{N_1 \cdot N_2} = 2.828427$$

$$\frac{R^2}{N_1} - N_2 = 0 \qquad \frac{R^2}{N_2} - N_1 = 0$$

$1 = 1.00000$
$N_1 = 4.00000$
$N_2 = 2.00000$
$\sqrt{N_1 \cdot N_2} = 2.82843$
$R = 2.82843$
$\sqrt{N_1 \cdot N_2} - R = 0.00000$
$\frac{R^2}{N_1} - N_2 = 0.00000$
$\frac{R^2}{N_2} - N_1 = 0.00000$



Unit := 1

Given.

$N_1 := 4$

$N_2 := 2$

A Duplicate Ratio

062092R3

Descriptions.

To add any number of differences, proportionally to the first two given, we simply take half of it and project from the root of the first two to find our two radii from the center which will place that third difference on the line All the while, we se wwe have be producing duplicate ratios. It then follows that any number proportional ratios is going to depend on the square root of term pairs.

The whole exercise is then, given two differences and then find the point of similarity from which they are set into this proportional series.

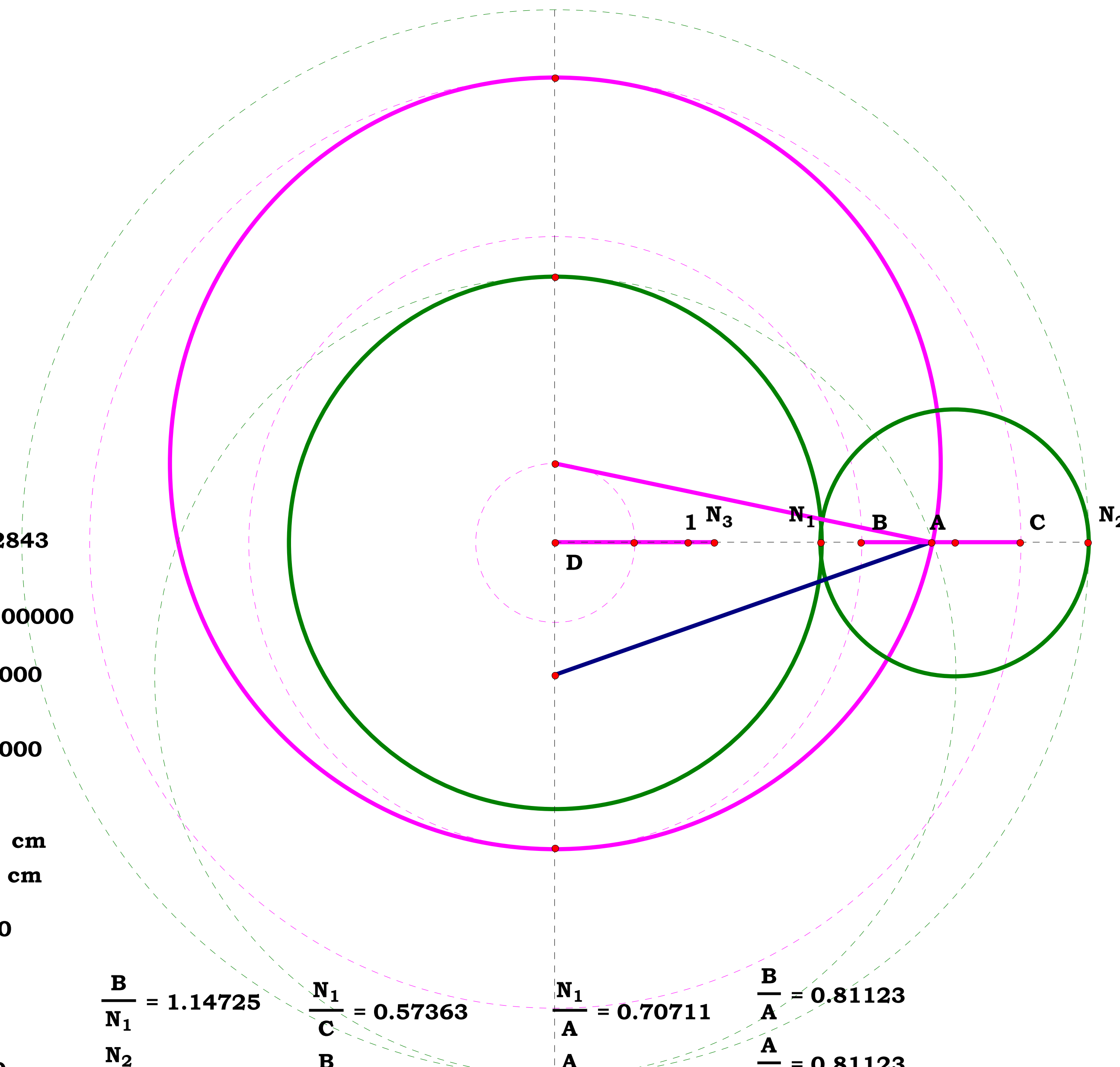
$A := \sqrt{N_1 \cdot N_2}$

Definitions.

$\sqrt{N_1 \cdot N_2} = 2.828427$

$A - \sqrt{N_1 \cdot N_2} = 0$

$\frac{A^2}{N_1} - N_2 = 0 \quad \frac{A^2}{N_2} - N_1 = 0$



$1 = 1.00000$

$N_1 = 2.00000$

$N_2 = 4.00000$

$\sqrt{N_1 \cdot N_2} = 2.82843$

$A = 2.82843$

$\sqrt{N_1 \cdot N_2} - A = 0.00000$

$\frac{A^2}{N_1} - N_2 = 0.00000$

$\frac{A^2}{N_2} - N_1 = 0.00000$

$N_3 = 1.19210$

$BC = 2.42850$ cm

$D1 = 2.03717$ cm

$\frac{BC}{D1} = 1.19210$

$B = 2.29450$

$C = 3.48660$

$\frac{N_1}{N_2} = 0.50000$

$\frac{B}{N_1} = 1.14725 \quad \frac{N_1}{C} = 0.57363 \quad \frac{N_1}{A} = 0.70711 \quad \frac{B}{A} = 0.81123$

$\frac{N_2}{C} = 1.14725 \quad \frac{B}{N_2} = 0.57363 \quad \frac{A}{N_2} = 0.70711 \quad \frac{A}{C} = 0.81123$

062092R4

Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.

Descriptions.

$AC := N_1 + BC$

$AF := N_1 \quad AG := AC \quad AJ := \sqrt{AF \cdot AG}$

$AL := \dfrac{DE}{2} \quad JL := \sqrt{AJ^2 + AL^2}$

$AD := JL - AL \quad AE := JL + AL$

$\dfrac{N_1}{AD} - \dfrac{AE}{AC} = 0 \quad \dfrac{AD}{AJ} - \dfrac{AJ}{AE} = 0 \quad \dfrac{N_1}{AJ} - \dfrac{AJ}{AC} = 0$ etc.,

Definitions

$AC = 3.74206 \quad AJ = 1.666383 \quad AL = 0.525$

$AD = 1.222129 \quad AE = 2.272129$

Unit, external

Given.

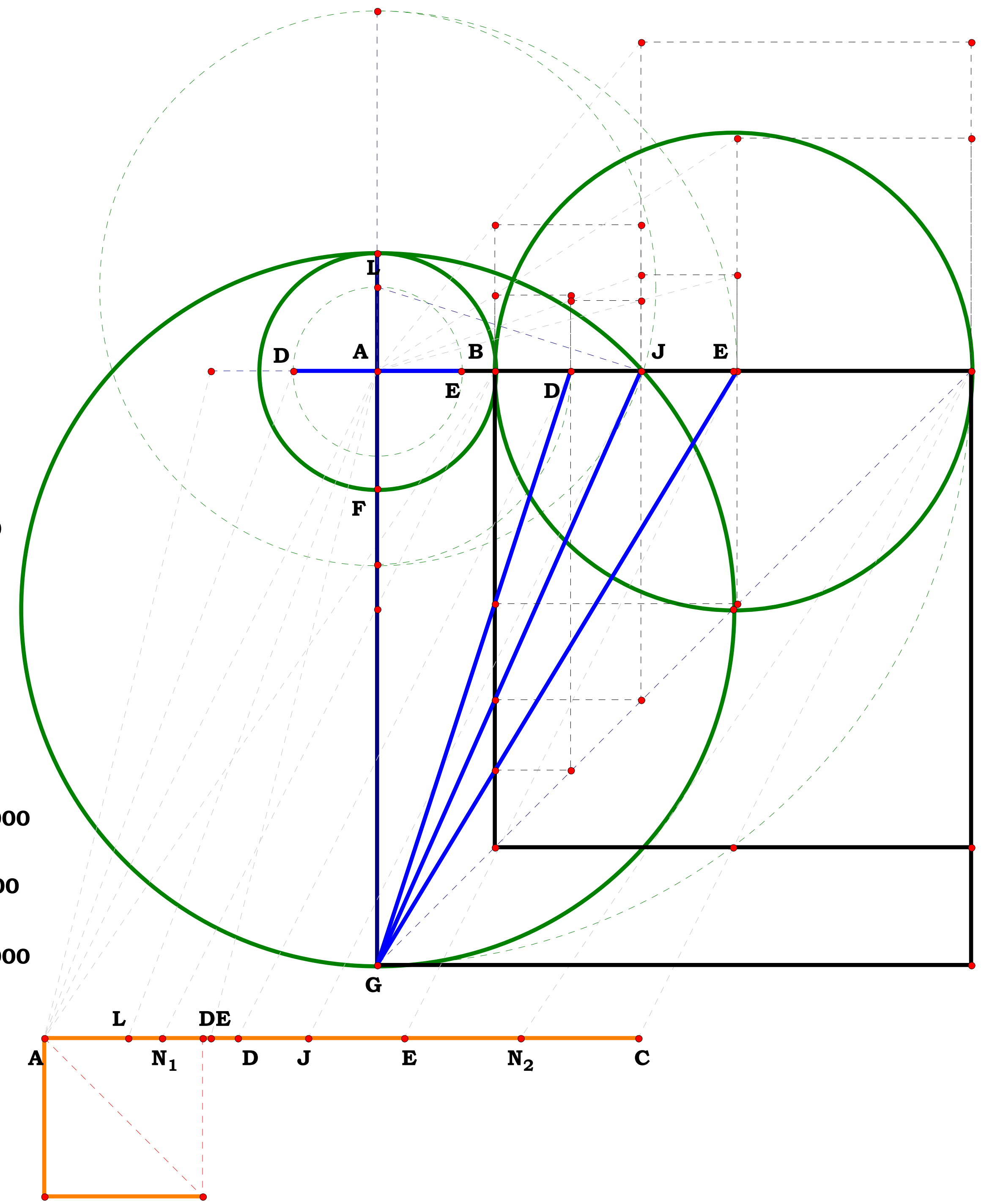
$N_1 := .74206$

$N_2 := 3 \quad BC := N_2$

$N_3 := 1.05 \quad DE := N_3$

A Duplicate Ratio

$N_1 = 0.74206$
$N_2 = 3.00000$
$DE = 1.05000$
$X/Y = 0.35000$
$X = 7.00000$
$Y = 20.00000$
$C = 3.74206$
$J = 1.66638$
$L = 0.52500$
$D = 1.22212$
$E = 2.27212$
$\dfrac{N_1}{D} - \dfrac{E}{C} = 0.00000$
$\dfrac{D}{J} - \dfrac{J}{E} = 0.00000$
$\dfrac{N_1}{J} - \dfrac{J}{C} = 0.00000$



Definitions

$$AC - \left(N_1 + N_2\right) = 0$$

$$AF - N_1 = 0$$

$$AG - \left(N_1 + N_2\right) = 0$$

$$AJ - \sqrt{N_1 \cdot \left(N_1 + N_2\right)} = 0$$

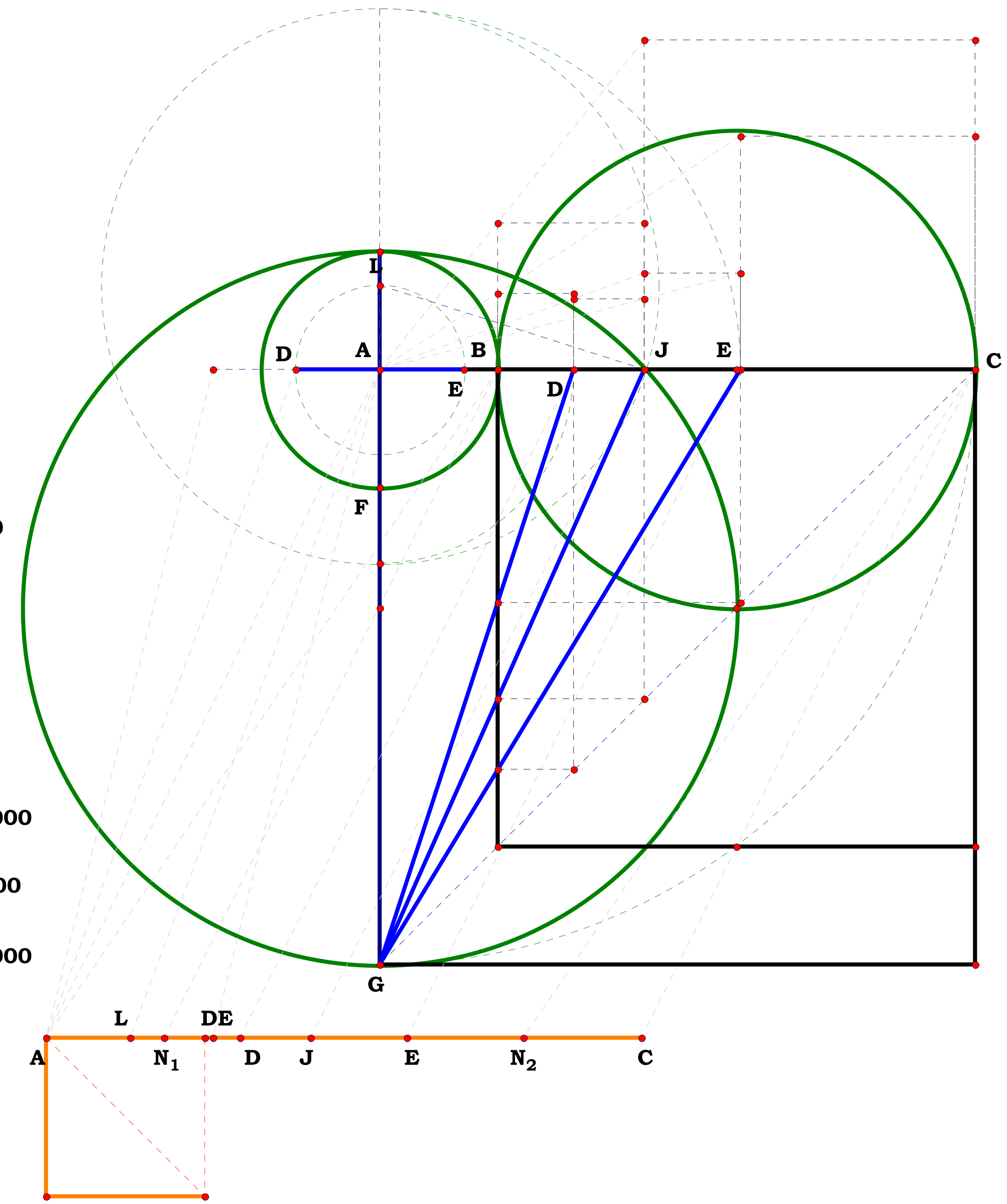
$$AL - \frac{N_3}{2} = 0$$

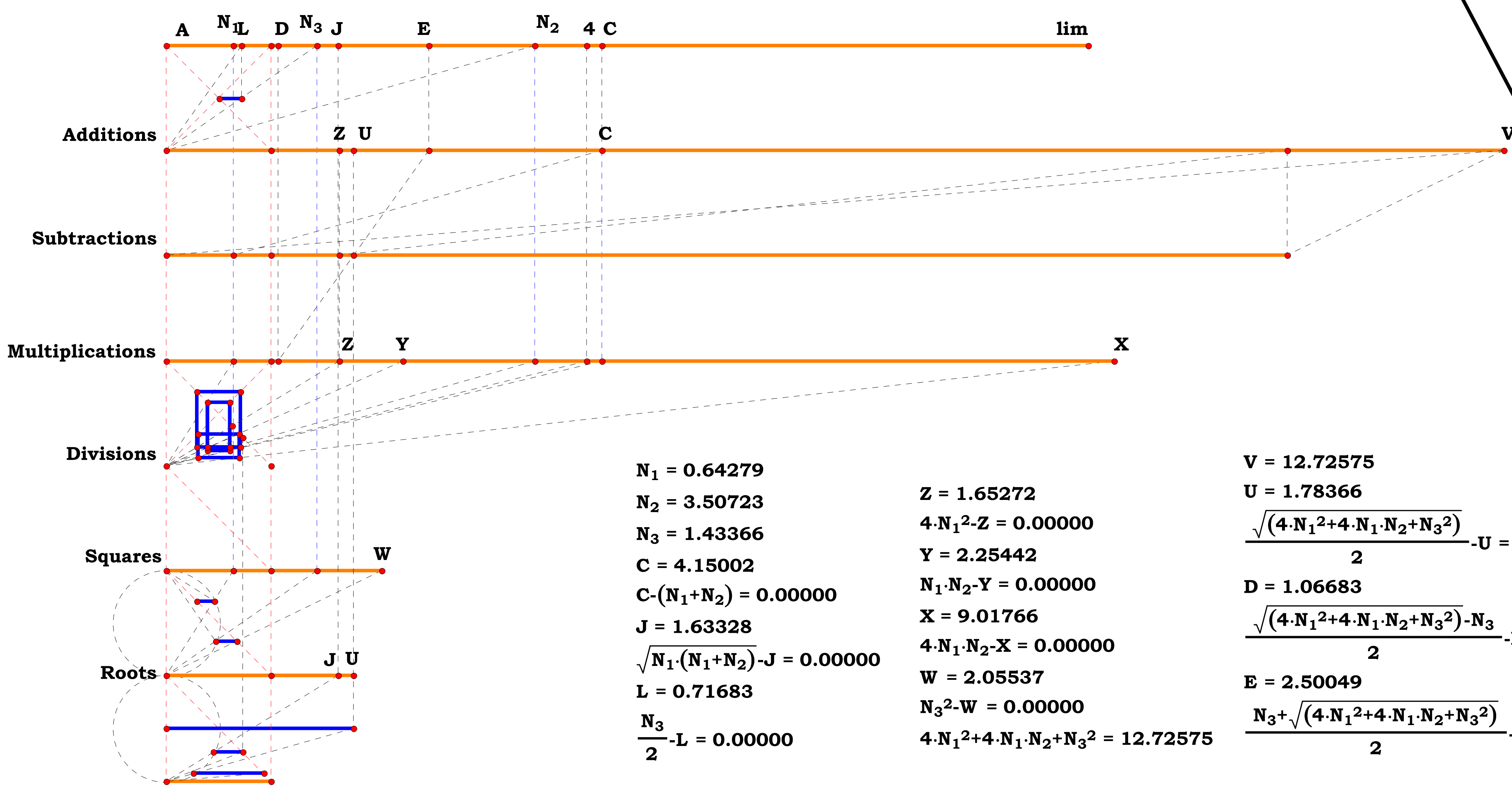
$$JL - \frac{\sqrt{\left(4 \cdot N_1{}^2 + 4 \cdot N_2 \cdot N_1 + N_3{}^2\right)}}{2} = 0$$

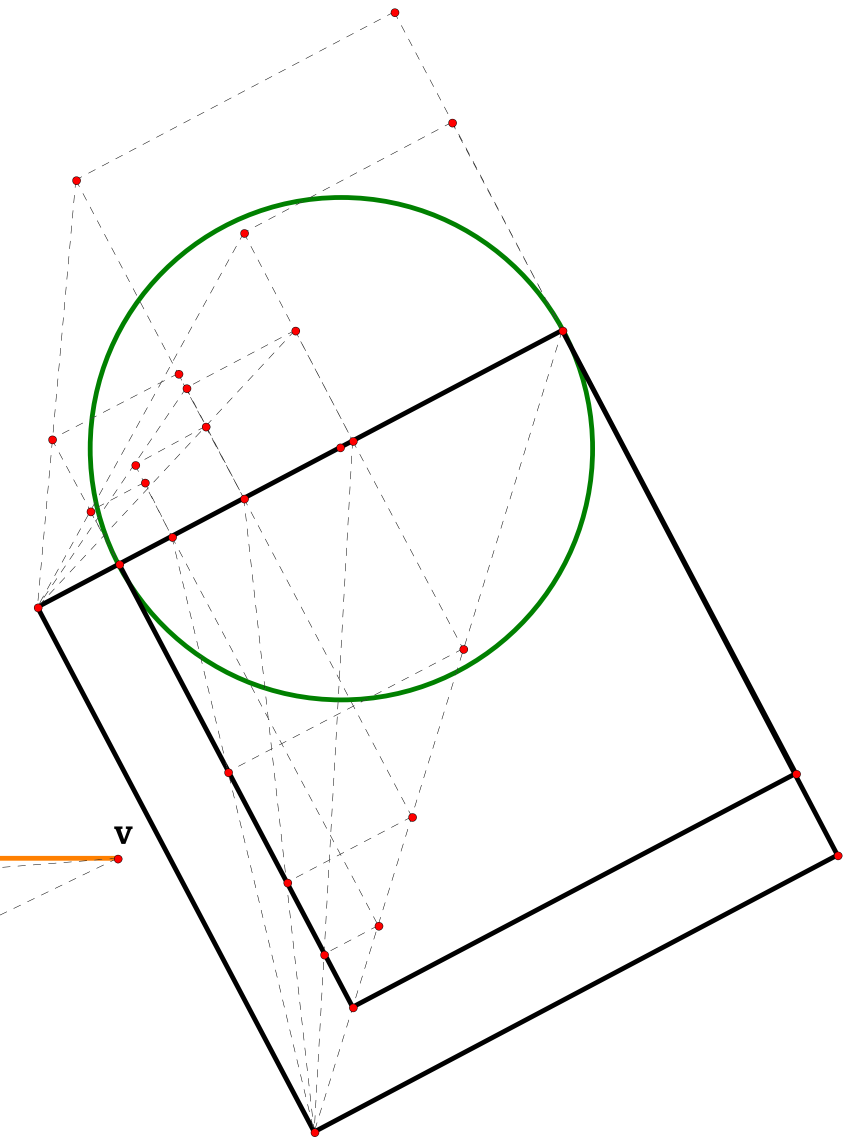
$$AD - \frac{\sqrt{4 \cdot N_1{}^2 + 4 \cdot N_1 \cdot N_2 + N_3{}^2} - N_3}{2} = 0$$

$$AE - \frac{N_3 + \sqrt{4 \cdot N_1{}^2 + 4 \cdot N_1 \cdot N_2 + N_3{}^2}}{2} = 0$$

$N_1 = 0.74206$
$N_2 = 3.00000$
$DE = 1.05000$
$X/Y = 0.35000$
$X = 7.00000$
$Y = 20.00000$
$C = 3.74206$
$J = 1.66638$
$L = 0.52500$
$D = 1.22212$
$E = 2.27212$
$\frac{N_1}{D} - \frac{E}{C} = 0.00000$
$\frac{D}{J} - \frac{J}{E} = 0.00000$
$\frac{N_1}{J} - \frac{J}{C} = 0.00000$







$N_1 = 0.64279$

$N_2 = 3.50723$

$N_3 = 1.43366$

$C = 4.15002$

$C-(N_1+N_2) = 0.00000$

$J = 1.63328$

$\sqrt{N_1 \cdot (N_1+N_2)}-J = 0.00000$

$L = 0.71683$

$\frac{N_3}{2}-L = 0.00000$

$Z = 1.65272$

$4 \cdot N_1{}^2-Z = 0.00000$

$Y = 2.25442$

$N_1 \cdot N_2-Y = 0.00000$

$X = 9.01766$

$4 \cdot N_1 \cdot N_2-X = 0.00000$

$W = 2.05537$

$N_3{}^2-W = 0.00000$

$4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2 = 12.72575$

$V = 12.72575$

$U = 1.78366$

$\frac{\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}}{2}-U = 0.00000$

$D = 1.06683$

$\frac{\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}-N_3}{2}-D = 0.00000$

$E = 2.50049$

$\frac{N_3+\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}}{2}-E = 0.00000$

062092R5

Unit.
$AB := 1$
Given.
$N_1 := 3.98280$ $BC := N_1$
$N_2 := .81091$ $DE := N_2$

A Duplicate Ratio

Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.

Definitions

$AC := 1 + N_1$ $AC = 4.9828$

$AF := AB$ $AF = 1$

$AG := (AB + N_1)$ $AG = 4.9828$

$AJ := \sqrt{(N_1 + AB)}$ $AJ = 2.232219$

$AL := \dfrac{N_2}{2}$ $AL = 0.405455$

$JL := \dfrac{\sqrt{\left(N_2^{\,2} + 4 \cdot N_1 + 4\right)}}{2}$ $JL = 2.268743$

$AD := \dfrac{\sqrt{\left(N_2^{\,2} + 4 \cdot N_1 + 4\right)} - N_2}{2}$ $AD = 1.863288$

$AE := \dfrac{N_2 + \sqrt{\left(N_2^{\,2} + 4 \cdot N_1 + 4\right)}}{2}$ $AE = 2.674198$

$\dfrac{AB}{AD} - \dfrac{AE}{AC} = 0$ $\dfrac{AD}{AJ} - \dfrac{AJ}{AE} = 0$

$\dfrac{AB}{AJ} - \dfrac{AJ}{AC} = 0$

$\dfrac{AB}{AD}-\dfrac{AE}{AC} = 0.00000$

$\dfrac{AD}{AJ}-\dfrac{AJ}{AE} = 0.00000$

$\dfrac{AB}{AJ}-\dfrac{AJ}{AC} = 0.00000$

AB = 1.80433 cm	AB = 1.00000
AD = 3.36199 cm	AD = 1.86329
AJ = 4.02767 cm	AJ = 2.23222
AE = 4.82514 cm	AE = 2.67420
AC = 8.99063 cm	AC = 4.98280
AL = 0.73158 cm	AL = 0.40545
AF = 1.80433 cm	AF = 1.00000
JL = 4.09357 cm	JL = 2.26874

$AC-(1+N_1) = 0.00000$

$AF-AB = 0.00000$

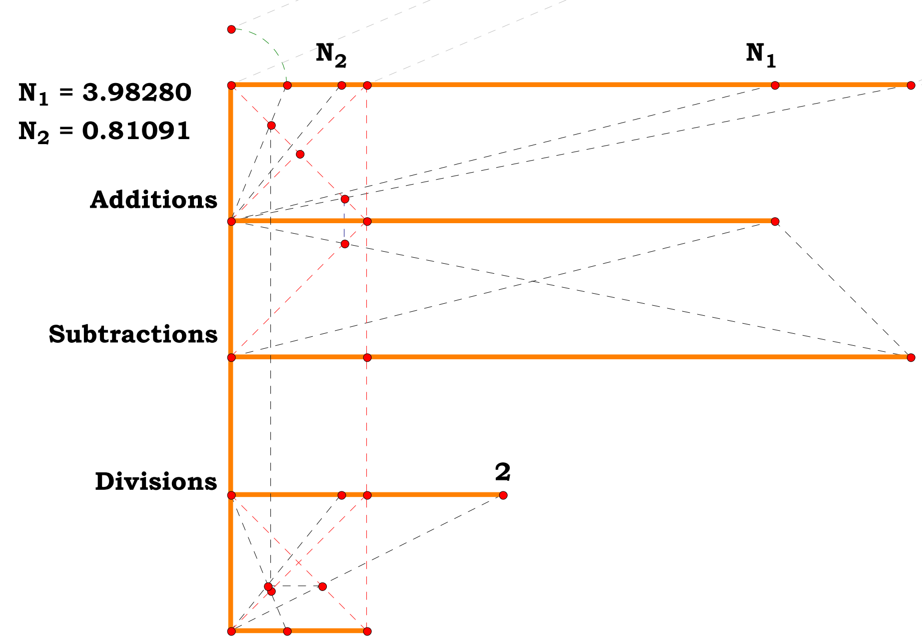
$AJ-\sqrt{N_1+AB} = 0.00000$

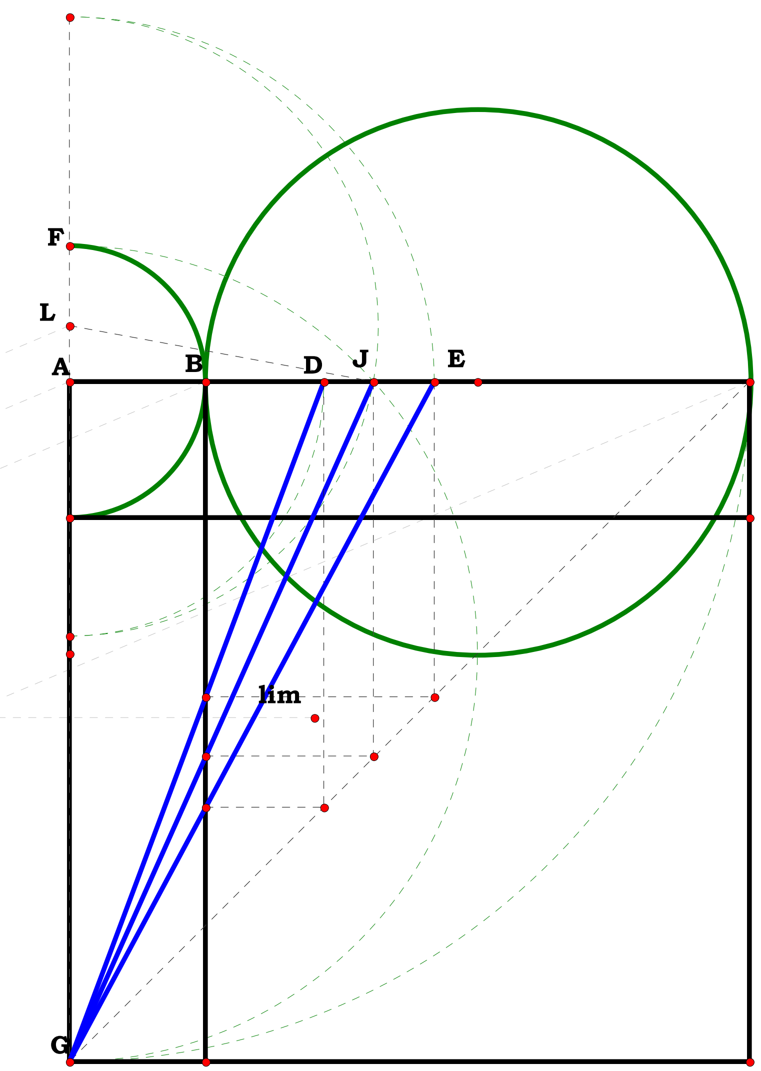
$AL-\dfrac{N_2}{2} = 0.00000$

$JL-\dfrac{\sqrt{N_2^2+4 \cdot N_1+4}}{2} = 0.00000$

$AD-\dfrac{\sqrt{N_2^2+4 \cdot N_1+4}-N_2}{2} = 0.00000$

$AE-\dfrac{N_2+\sqrt{N_2^2+4 \cdot N_1+4}}{2} = 0.00000$

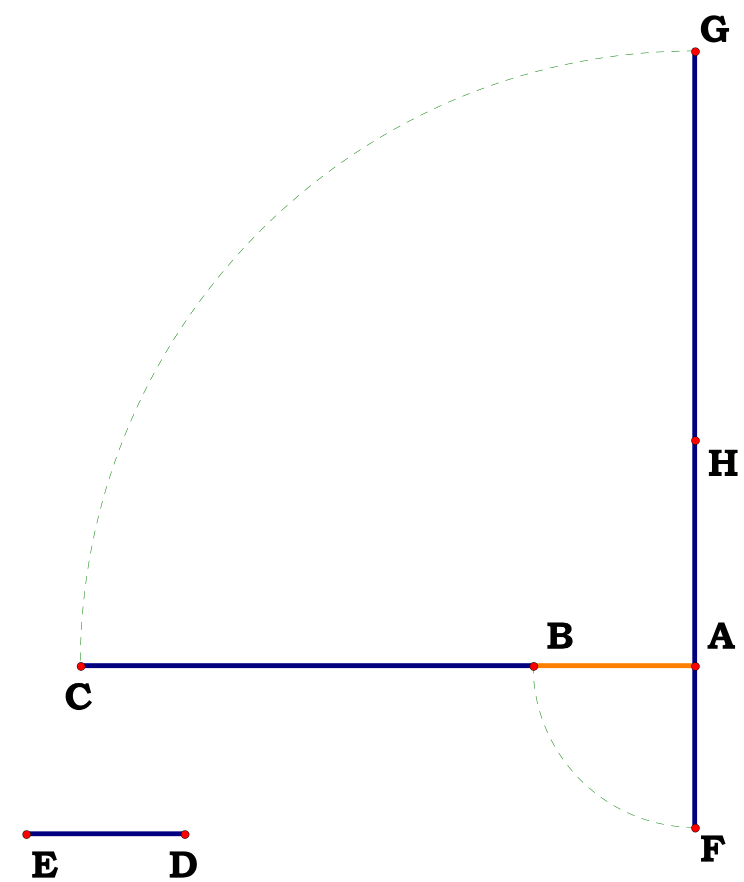


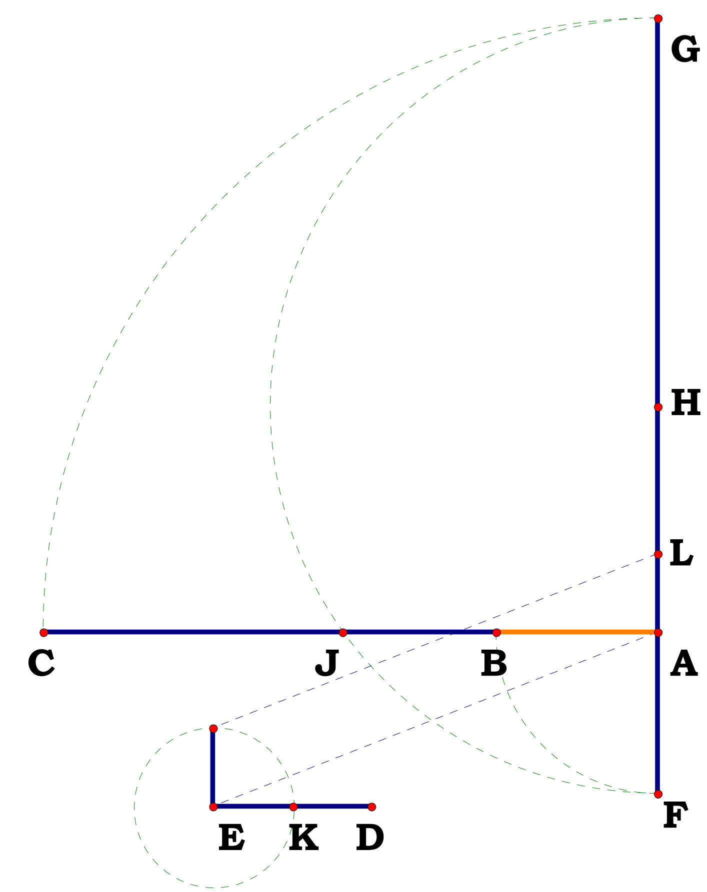


062092R6

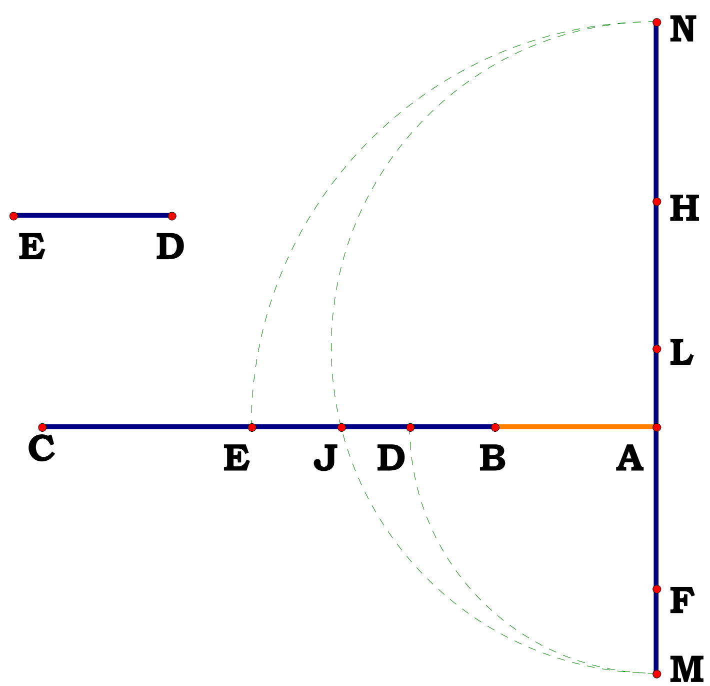
A Duplicate Ratio

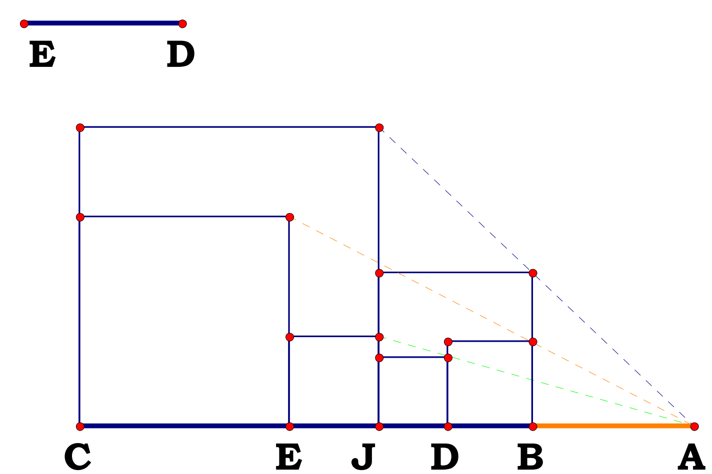
We may now be ready to come to an outline of the whole affair.





062092R6





AB = 2.13783 cm
AD = 3.25040 cm
AJ = 4.16849 cm
AE = 5.34590 cm
AC = 8.12800 cm

$\frac{AB}{AD} = 0.65771$

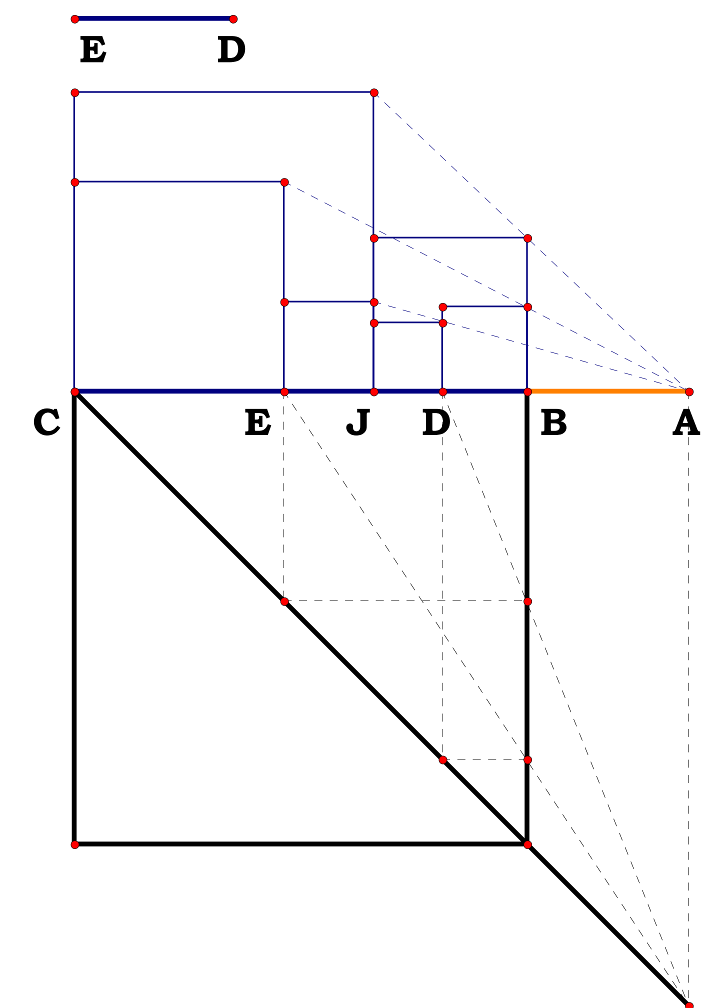
$\frac{AE}{AC} = 0.65771$

$\frac{AD}{AJ} = 0.77975$

$\frac{AJ}{AE} = 0.77975$

$\frac{AB}{AJ} = 0.51286$

$\frac{AJ}{AC} = 0.51286$



062092R6

Starting with a simple given, we will end up prepared to formulat Basic Analog Mathematics, i.e., write Geometric Figures which can compute any mathematical and any logical result which, as the output is concurrent with the input, independent of time, that is, process information in no time whatsoever, i.e., computation independent of time. And it is all the result of binary recursion. Every possible grammar is the product of binary recursion, and, as one can plainly see, it is possible to produce one's results, quite independent of time. Therefore, the ability to predict the future, using binary recursion, is not only possible, one can say, it is factually proven. Our biologically defined job, to learn to predict the results of any number of givens is a proven fact and provably possible.

BAG for 062092

$N_1 = 4.27962$

$N_2 = 2.48336$

$AB = 1.00000$ $\quad AF = 1.00000$

$AC = 5.27962$ $\quad AG = 5.27962$

$AJ = 2.29774$

$\sqrt{AB \cdot AG} - AJ = 0.00000$

$AL = 1.24168$

$JL = 2.61178$

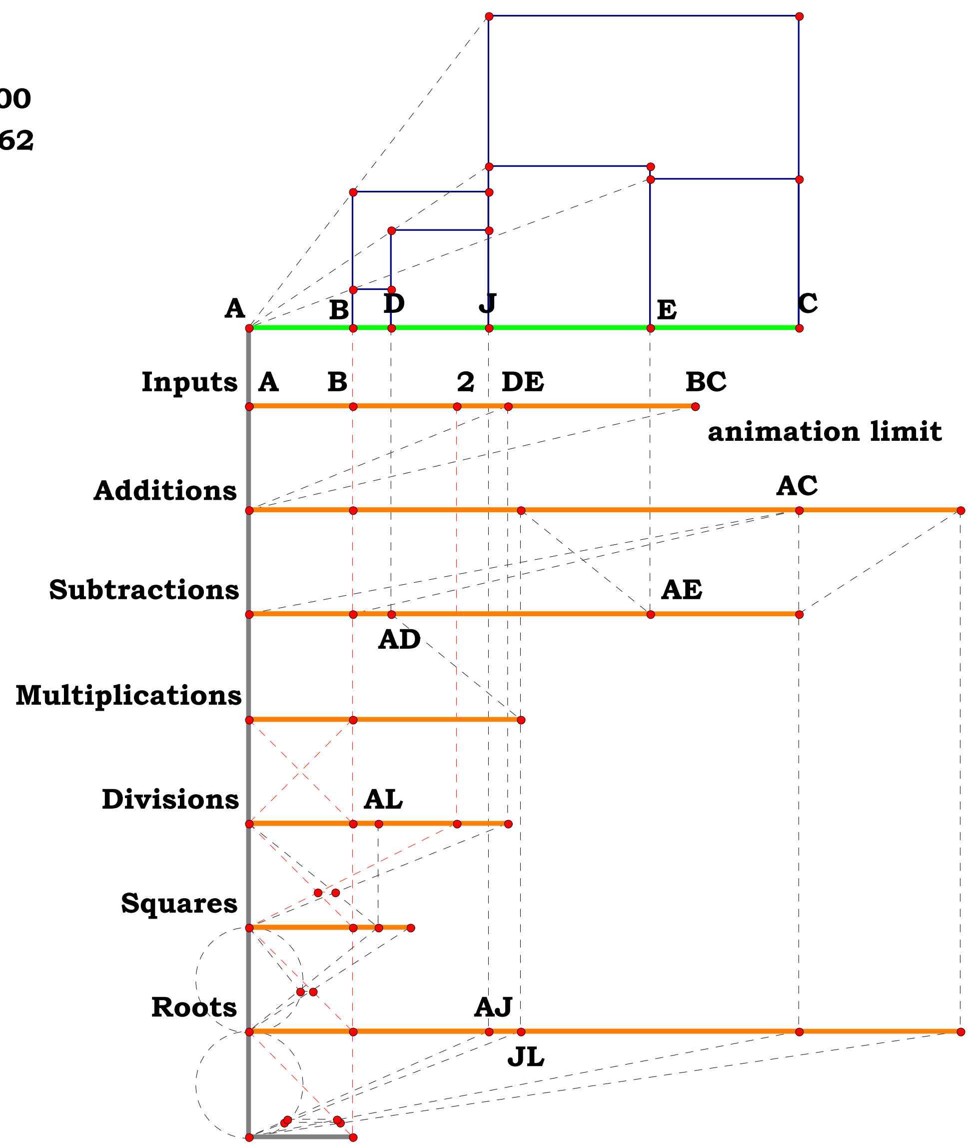
$\sqrt{AJ^2 + AL^2} - JL = 0.00000$

$AD = 1.37010$

$AD - JL - AL = 0.00000$

$AE = 3.85346$

$AE - (JL + AL) = 0.00000$



$$\frac{AB}{AD} - \frac{2}{\sqrt{N_2^2 + 4 \cdot N_1 + 4} - N_2} = 0.00000$$

$$\frac{AE}{AC} - \frac{N_2 + \sqrt{N_2^2 + 4 \cdot N_1 + 4}}{2 \cdot N_1 + 2} = 0.00000$$

$$\frac{AD}{AJ} - \frac{\sqrt{N_2^2 + 4 \cdot N_1 + 4} - N_2}{2 \cdot \sqrt{N_1 + 1}} = 0.00000$$

$$\frac{AJ}{AE} - \frac{2 \cdot \sqrt{N_1 + 1}}{N_2 + \sqrt{N_2^2 + 4 \cdot N_1 + 4}} = 0.00000$$

$$\frac{AB}{AJ} - \frac{1}{\sqrt{N_1 + 1}} = 0.00000$$

$$\frac{AJ}{AC} - \frac{1}{\sqrt{N_1 + 1}} = 0.00000$$

081292

Given AB, how close is BJ to the cube root of AB taken as a sphere?

Unit.

$AB := 1$

Given.

$N_1 := AB$

$$CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}$$

Descriptions.

$$BH := \sqrt{2\cdot AB^2} \qquad CG := \frac{AB^2}{BH}$$

$$AG := \sqrt{CG^2 + (AB + CG)^2} \qquad DG := CG\cdot\frac{2AB}{AG}$$

$$GJ := \sqrt{AB^2 - DG^2} \qquad AE := \frac{(AB + CG)\cdot(AG + GJ)}{AG}$$

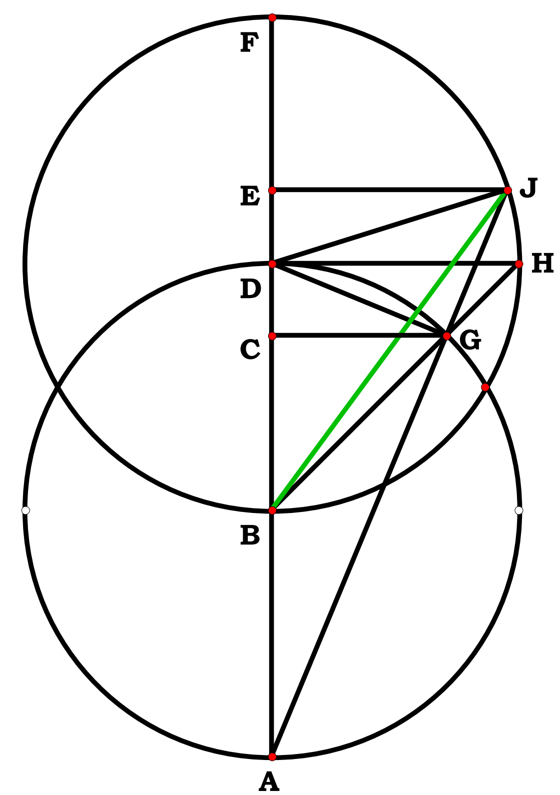
$$EJ := \frac{CG\cdot AE}{AB + CG} \qquad BJ := \sqrt{EJ^2 + (AE - AB)^2}$$

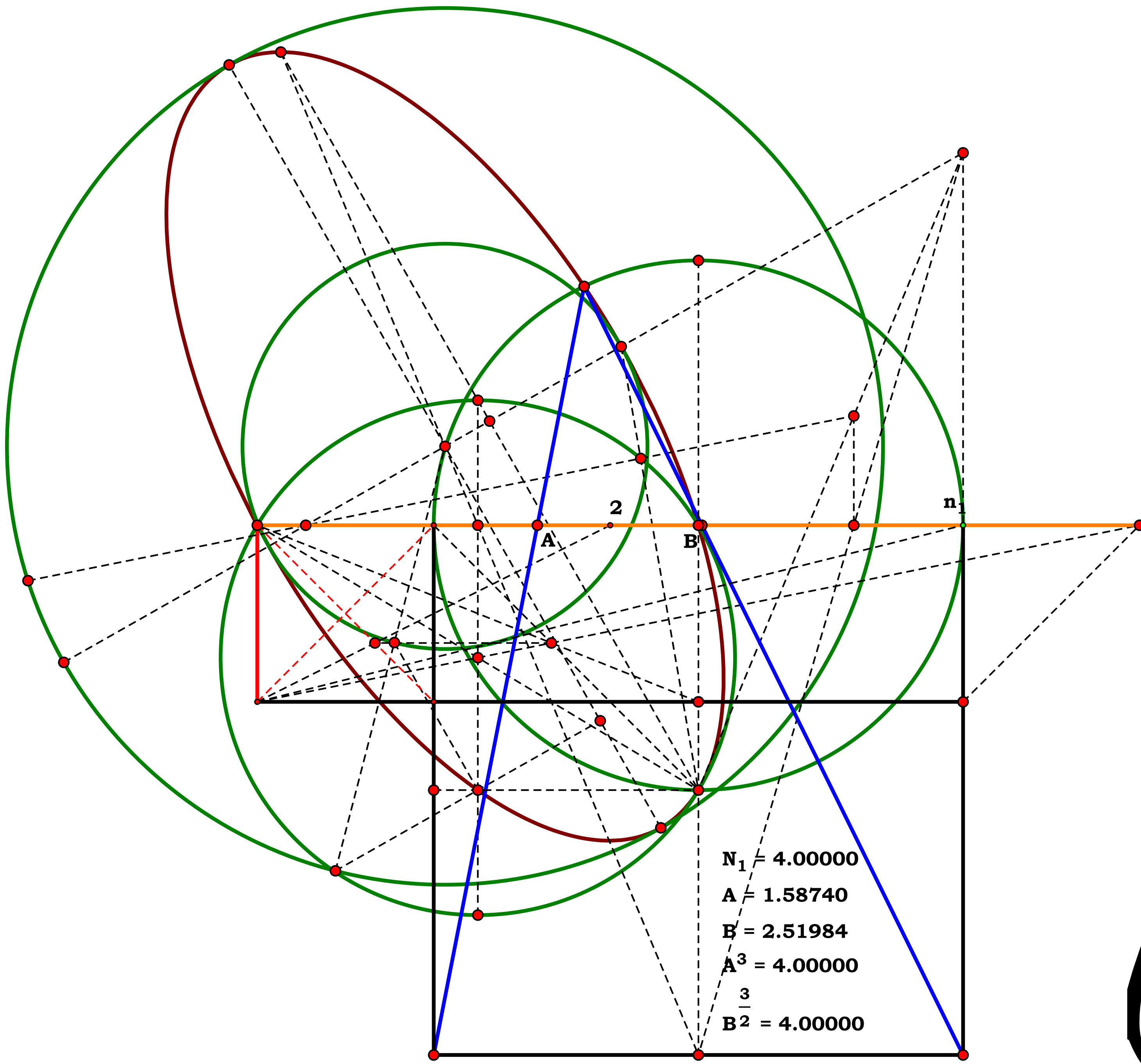
$$\frac{BJ}{\left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}} = 1.000943 \qquad BJ - \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}} = 0.00152$$

Definitions.

$$BJ - N_1\cdot\sqrt{\sqrt{2} + 2^{\frac{1}{4}}} = 0$$

Rusty Cube of a Sphere





The Delian Quest 1993

John Clark

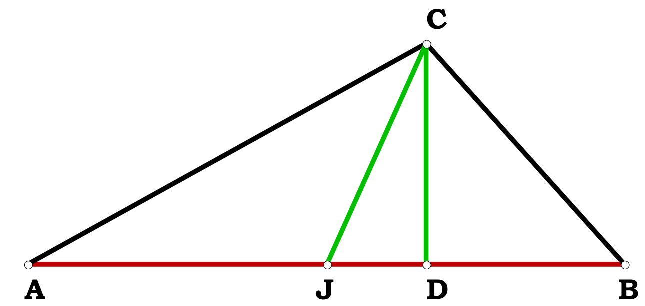
010893A

Pythagoras Revisited

$AB := 7.89517$

$AC := 6.02581$

$BC := 3.92697$



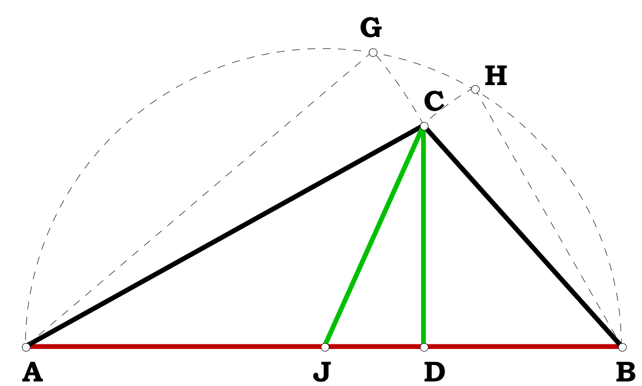
Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.

Descriptions.

$AE := \frac{AC^2}{AB} \qquad BF := \frac{BC^2}{AB} \qquad EF := AB - (AE + BF) \qquad DE := \frac{EF}{2}$

$AD := AE + DE \qquad BD := AB - AD \qquad CD := \sqrt{AC^2 - AD^2}$

$AJ := \frac{AB}{2} \qquad DJ := AD - AJ \qquad CJ := \sqrt{CD^2 + DJ^2}$



$S_1 := AB$

$S_2 := AC$

$S_3 := BC$

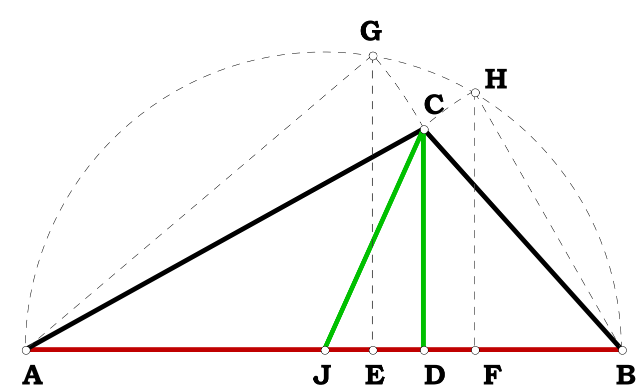
Definitions.

$EF - \frac{AB^2 - AC^2 - BC^2}{AB} = 0 \qquad DE - \frac{AB^2 - AC^2 - BC^2}{2 \cdot AB} = 0$

$AD - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \qquad BD - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$

$DJ - \frac{\sqrt{\left(AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \qquad CJ - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$

$CD - \frac{\sqrt{[(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)]}}{2 \cdot AB} = 0$



$EF - \frac{{S_1}^2 - {S_2}^2 - {S_3}^2}{S_1} = 0 \qquad DE - \frac{{S_1}^2 - {S_2}^2 - {S_3}^2}{2 \cdot S_1} = 0$

$AD - \frac{{S_1}^2 + {S_2}^2 - {S_3}^2}{2 \cdot S_1} = 0 \qquad BD - \frac{{S_1}^2 - {S_2}^2 + {S_3}^2}{2 \cdot S_1} = 0$

$DJ - \frac{\sqrt{\left({S_2}^2 - {S_3}^2\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot {S_2}^2 - {S_1}^2 + 2 \cdot {S_3}^2}}{2} = 0$

$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0$

010893B

Given.

$W := 6 \quad Y := 8$

$X := 20 \quad Z := 15$

Unit.

$$AB := \frac{X}{X}$$

Pythagoras Revisited

Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.

Descriptions.

$$AD := \frac{Y}{Z} \qquad CD := \frac{W}{X} \qquad AC := \sqrt{AD^2 + CD^2}$$

$$BD := AB - AD \qquad BC := \sqrt{BD^2 + CD^2}$$

$$AE := \frac{AC^2}{AB} \qquad BF := \frac{BC^2}{AB} \qquad EF := AB - (AE + BF) \qquad DE := \frac{EF}{2}$$

$$AJ := \frac{AB}{2} \qquad DJ := AD - AJ \qquad CJ := \sqrt{CD^2 + DJ^2}$$

Definitions.

$$AD - \frac{Y}{Z} = 0 \qquad CD - \frac{W}{X} = 0 \qquad AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$$

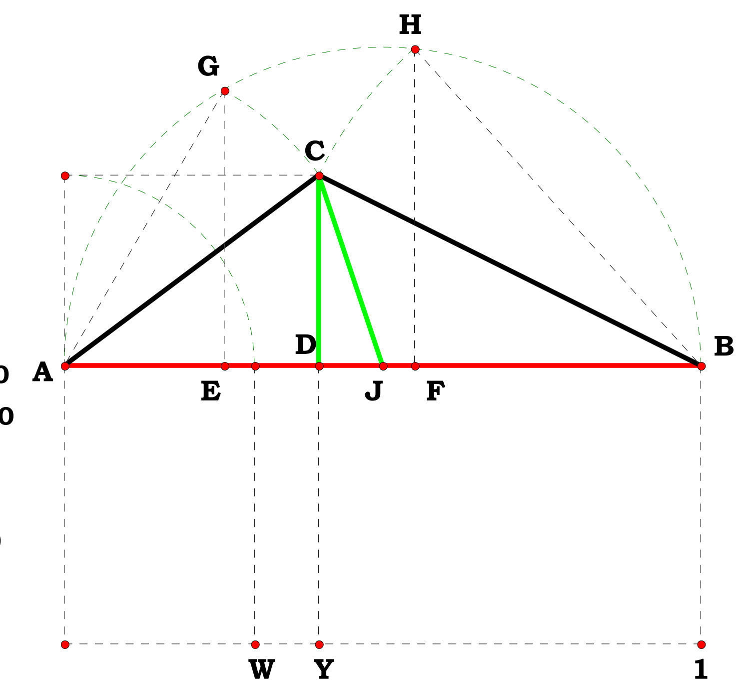
$$BD - \frac{Z - Y}{Z} = 0 \qquad BC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{X \cdot Z} = 0$$

$$AE - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} = 0 \qquad BF - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}{X^2 \cdot Z^2} = 0$$

$$EF - \frac{2 \cdot \left(X^2 \cdot Y \cdot Z - W^2 \cdot Z^2 - X^2 \cdot Y^2\right)}{X^2 \cdot Z^2} = 0 \qquad DE - \frac{\left(X^2 \cdot Y \cdot Z - W^2 \cdot Z^2 - X^2 \cdot Y^2\right)}{X^2 \cdot Z^2} = 0$$

$$AJ - \frac{1}{2} = 0 \qquad DJ - \frac{2 \cdot Y - Z}{2 \cdot Z} = 0$$

$$CJ - \frac{\sqrt{4 \cdot W^2 \cdot Z^2 + 4 \cdot X^2 \cdot Y^2 - 4 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{2 \cdot X \cdot Z} = 0$$



Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.40000
Y = 6.00000
Z = 15.00000
AC = 0.50000
BD = 0.60000
BC = 0.67082
AE = 0.25000
BF = 0.45000
EF = 0.30000
CJ = 0.31623

$$\frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} - AC = 0.00000$$

$$\frac{Z-Y}{Z} - BD = 0.00000$$

$$\frac{\sqrt{((W^2 \cdot Z^2 + X^2 \cdot Y^2) - 2 \cdot X^2 \cdot Y \cdot Z) + X^2 \cdot Z^2}}{X \cdot Z} - BC = 0.00000$$

$$\frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} - AE = 0.00000$$

$$\frac{((W^2 \cdot Z^2 + X^2 \cdot Y^2) - 2 \cdot X^2 \cdot Y \cdot Z) + X^2 \cdot Z^2}{X^2 \cdot Z^2} - BF = 0.00000$$

$$\frac{2 \cdot (X^2 \cdot Y \cdot Z - W^2 \cdot Z^2 - X^2 \cdot Y^2)}{X^2 \cdot Z^2} - EF = 0.00000$$

$$\frac{\sqrt{((4 \cdot W^2 \cdot Z^2 + 4 \cdot X^2 \cdot Y^2) - 4 \cdot X^2 \cdot Y \cdot Z) + X^2 \cdot Z^2}}{2 \cdot X \cdot Z} - CJ = 0.00000$$

Unit is external.

Given.

$N_1 := 3$

$N_2 := 4$

The curve AK is derived from the cube root figure as demonstrated.

Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?

Divide a segment twice such that the mean segment is the root of the extreems.

060393A

Descriptions.

Exploring The Curve AK

$AG := N_1 \qquad AC := \dfrac{AG}{N_2}$

$GF := \dfrac{AG}{3} \qquad FM := \sqrt{GF \cdot (AG - GF)}$

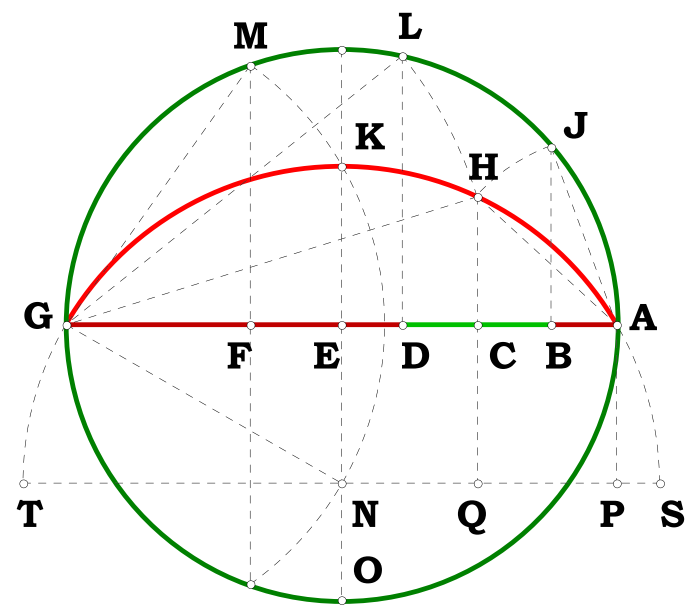
$GM := \sqrt{GF^2 + FM^2} \qquad ST := 2 \cdot GM \qquad EN := \sqrt{GM^2 - \left(\dfrac{AG}{2}\right)^2}$

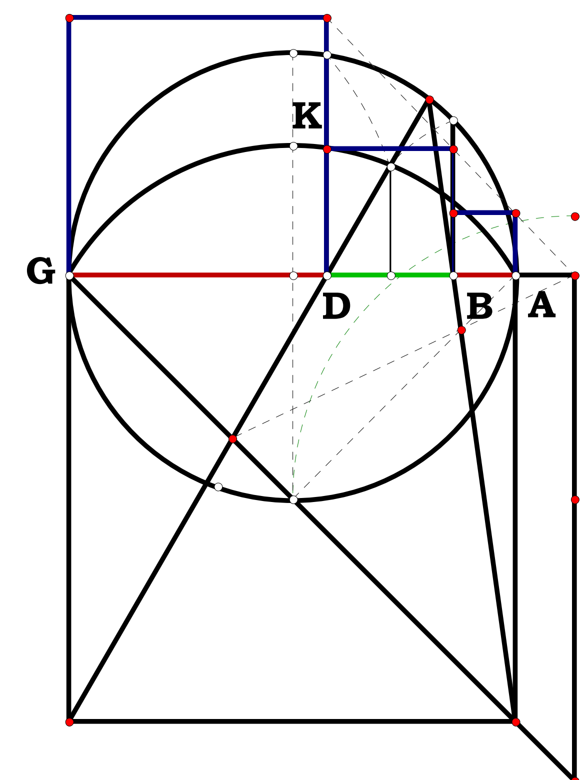
$PS := \dfrac{ST - AG}{2} \qquad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$

$CH := HQ - EN \qquad AH := \sqrt{AC^2 + CH^2} \qquad GH := \sqrt{(AG - AC)^2 + CH^2}$

$AB := \dfrac{AH^2}{AG} \qquad DG := \dfrac{GH^2}{AG} \qquad BD := AG - (AB + DG)$

$BD - \sqrt{AB \cdot DG} = 0 \qquad AB = 0.348612 \qquad BD = 0.802776 \qquad DG = 1.848612$





Definitions.

$AC - \dfrac{N_1}{N_2} = 0 \qquad GF - \dfrac{N_1}{3} = 0 \qquad FM - \dfrac{\sqrt{2} \cdot N_1}{3} = 0 \qquad GM - \dfrac{\sqrt{3} \cdot N_1}{3} = 0 \qquad ST - \dfrac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0 \qquad EN - \dfrac{N_1}{\sqrt{12}} = 0$

$PS - \dfrac{N_1 \cdot (2 \cdot \sqrt{3} - 3)}{6} = 0 \qquad HQ - \dfrac{N_1 \cdot \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12}}{N_2 \cdot \sqrt{12}} = 0 \qquad CH - \dfrac{N_1 \cdot \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0 \qquad AH - \dfrac{N_1 \cdot \sqrt{N_2 - \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} + 6}}{\sqrt{6 \cdot N_2}} = 0$

$GH - \dfrac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0 \qquad AB - \dfrac{N_1 \cdot \left(N_2 - \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} + 6\right)}{6 \cdot N_2} = 0 \qquad DG - \dfrac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} - 6\right)}{6 \cdot N_2} = 0$

$BD - \dfrac{N_1 \cdot \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0 \qquad BD^2 - \dfrac{2 \cdot N_1^{\,2} \cdot \left(6 \cdot N_2 + N_2^{\,2} - N_2 \cdot \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^{\,2}} = 0 \qquad AB \cdot DG - \dfrac{2 \cdot N_1^{\,2} \cdot \left(6 \cdot N_2 + N_2^{\,2} - N_2 \cdot \sqrt{N_2^{\,2} + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^{\,2}} = 0$

Unit.

$AG := 1$

Given.

$N_1 := 4$

060393B

The curve AK is derived from the cube root figure as demonstrated.

Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?
Divide a segment twice such that the mean segment is the root of the extreems.

Exploring The Curve AK

Descriptions.

$AC := \frac{AG}{N_1}$ $\quad GF := \frac{AG}{3}$ $\quad FM := \sqrt{GF \cdot (AG - GF)}$

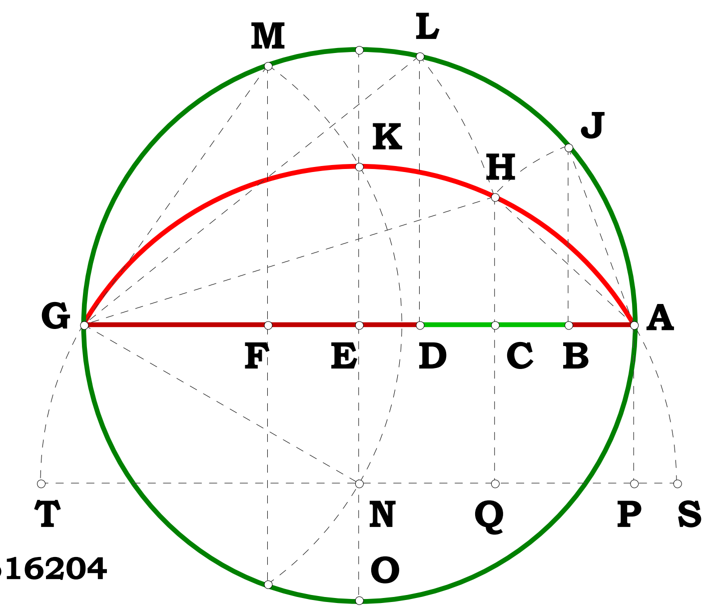
$GM := \sqrt{GF^2 + FM^2}$ $\quad ST := 2 \cdot GM$ $\quad EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$

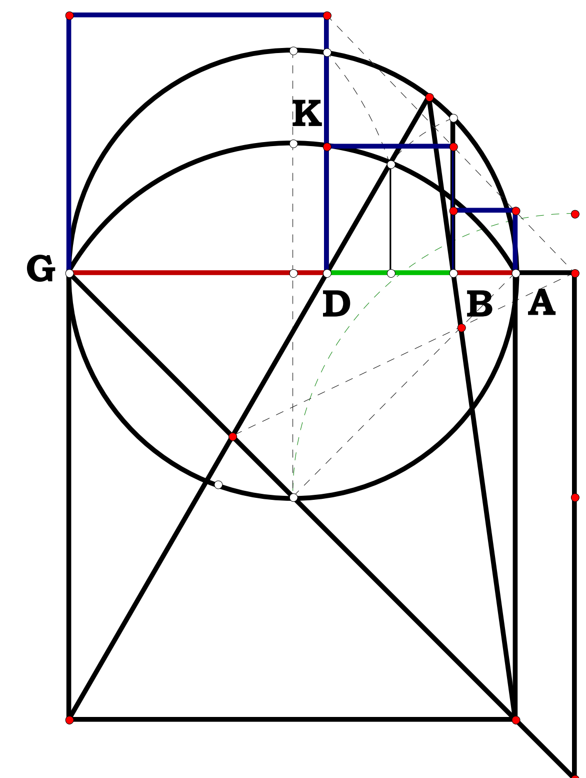
$PS := \frac{ST - AG}{2}$ $\quad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$

$CH := HQ - EN$ $\quad AH := \sqrt{AC^2 + CH^2}$ $\quad GH := \sqrt{(AG - AC)^2 + CH^2}$

$AB := \frac{AH^2}{AG}$ $\quad DG := \frac{GH^2}{AG}$ $\quad BD := AG - (AB + DG)$

$BD - \sqrt{AB \cdot DG} = 0$ $\quad AB = 0.116204$ $\quad BD = 0.267592$ $\quad DG = 0.616204$





Definitions.

$AC - \frac{1}{N_1} = 0$ $\quad GF - \frac{1}{3} = 0$ $\quad FM - \frac{\sqrt{2}}{\sqrt{9}} = 0$ $\quad GM - \frac{1}{\sqrt{3}} = 0$ $\quad ST - \frac{2 \cdot \sqrt{3}}{3} = 0$

$EN - \frac{1}{\sqrt{12}} = 0$ $\quad PS - \left(\frac{\sqrt{3}}{3} - \frac{1}{2}\right) = 0$ $\quad HQ - \frac{\sqrt{N_1^2 + 12 \cdot N_1 - 12}}{\sqrt{12 \cdot N_1}} = 0$ $\quad CH - \frac{\left(\sqrt{N_1^2 + 12 \cdot N_1 - 12} - N_1\right) \cdot \sqrt{3}}{6 \cdot N_1} = 0$

$AH - \frac{\sqrt{N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} + 6}}{\sqrt{6 \cdot N_1}} = 0$ $\quad GH - \frac{\sqrt{7 \cdot N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} - 6}}{\sqrt{6 \cdot N_1}} = 0$ $\quad AB - \frac{N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} + 6}{6 \cdot N_1} = 0$

$DG - \frac{7 \cdot N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} - 6}{6 \cdot N_1} = 0$ $\quad BD - \frac{\sqrt{N_1^2 + 12 \cdot N_1 - 12} - N_1}{3 \cdot N_1} = 0$ $\quad BD - \frac{\sqrt{2 \cdot \left(N_1^2 + 6 \cdot N_1 - 6\right) - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 12 \cdot N_1 - 12}}}{3 \cdot N_1} = 0$

060793A

Unit is external.

Given.

$AD := 2.17506$ $\quad AB := 3.14654$ $\quad AC := 1.74732$

$BD := 2.61333$ $\quad CD := 1.38168$

Two Triangles with a Common Side.

$$CG := \frac{\sqrt{(AD + CD + AC)\cdot(-AD + CD + AC)\cdot(AD - CD + AC)(AD + CD - AC)}}{2\cdot AD}$$

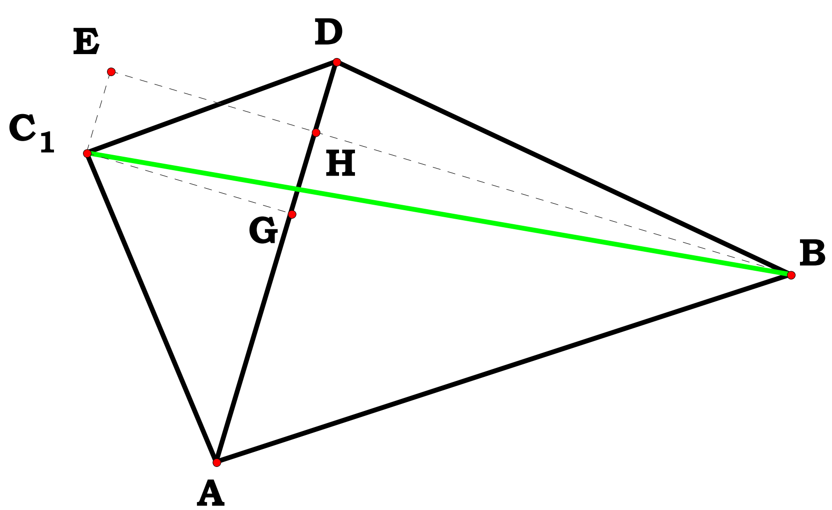
$$BH := \frac{\sqrt{(AD + AB + BD)\cdot(-AD + AB + BD)\cdot(AD - AB + BD)(AD + AB - BD)}}{2\cdot AD}$$

Descriptions.

Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

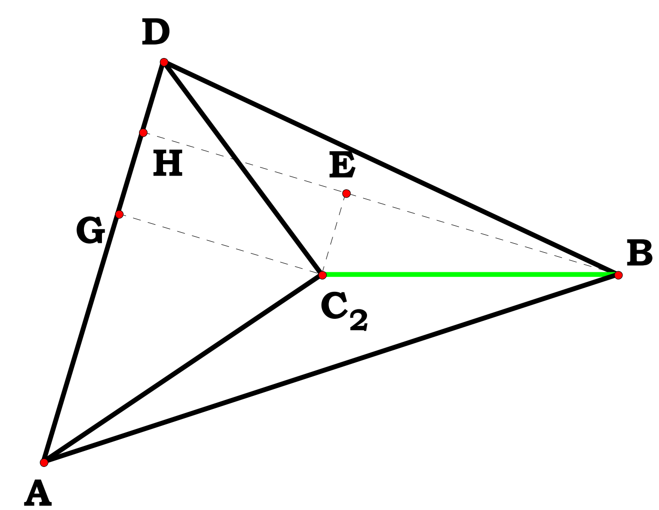
Greatest Disance:



$$BC_1 := \sqrt{GH^2 + (CG + BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH - CG)^2}$$

Least Distance



$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + AB^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)} \ldots + -AD^4 - AB^2\cdot AC^2 + AB^2\cdot AD^2 + AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2 + AB^2\cdot AD^2 - AD^4 - AB^2\cdot AC^2 \ldots + -\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}}}{2\cdot AD} = 0$$

060793B

Unit

$AB := 1$

Given.

$AD := 2.17506$ $\quad AC := 1.74732$

$BD := 2.61333$ $\quad CD := 1.38168$

Two Triangles with a Common Side.

Descriptions.

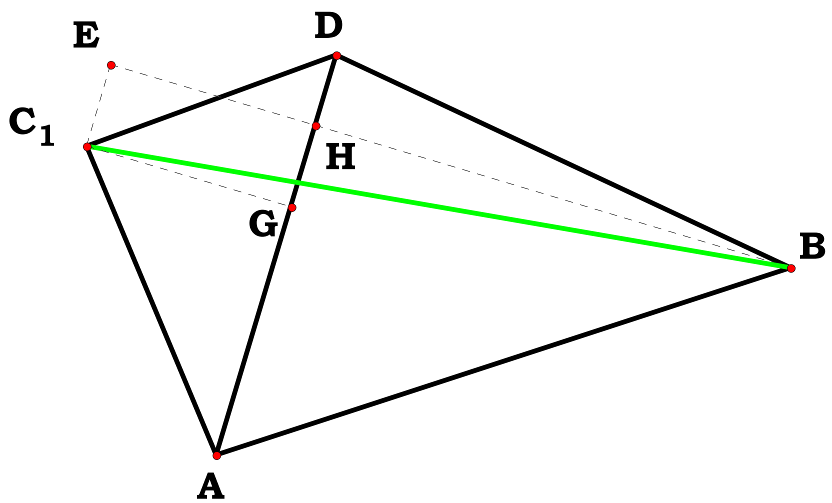
Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$CG := \frac{\sqrt{(AD+CD+AC)\cdot(-AD+CD+AC)\cdot(AD-CD+AC)(AD+CD-AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD+BD-1)\cdot(AD+BD+1)\cdot(AD-BD+1)\cdot(BD-AD+1)}}{2\cdot AD}$$

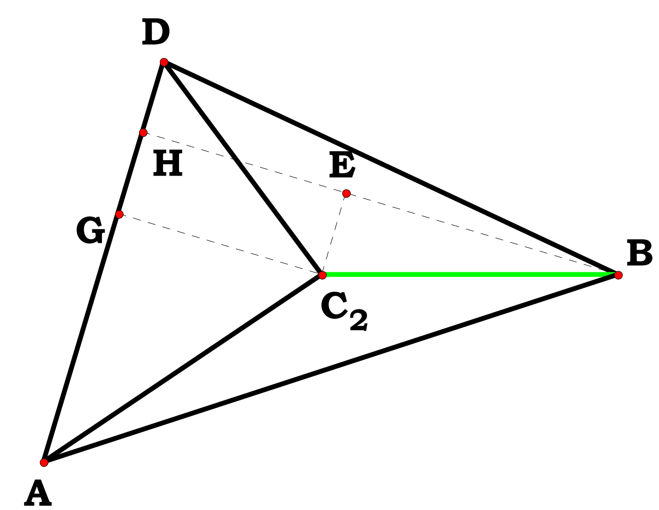
Greatest Disance:



$$BC_1 := \sqrt{GH^2+(CG+BH)^2}$$

$$BC_2 := \sqrt{GH^2+(BH-CG)^2}$$

Least Distance



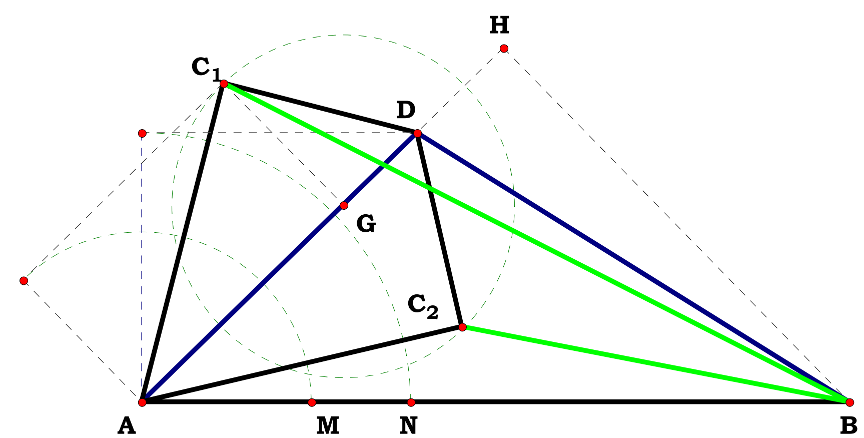
$$AG := \frac{AD^2+AC^2-CD^2}{2\cdot AD} \qquad AH := \frac{AD^2+1^2-BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\begin{array}{l}(AC\cdot AD)^2+(AC\cdot BD)^2-AC^2-AD^4+(AD\cdot BD)^2+(AD\cdot CD)^2-(BD\cdot CD)^2+AD^2+CD^2 \;\ldots\\ +\sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)\cdot(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}\end{array}}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{\begin{array}{l}(AC\cdot AD)^2+(AC\cdot BD)^2-AC^2-AD^4+(AD\cdot BD)^2+(AD\cdot CD)^2-(BD\cdot CD)^2+AD^2+CD^2 \;\ldots\\ +-\sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}\end{array}}}{2\cdot AD} = 0$$



Unit = 1.00000
XY = 0.24024
X = 3.20765
Y = 13.35191

XY = 0.38041
X = 7.60821
Y = 20.00000

AB = 9.36083 cm
AD = 5.09446 cm
AC_1 = 4.34404 cm
BD = 6.73584 cm
C_1D = 2.63736 cm
AG = 3.71664 cm
AH = 6.69425 cm

BC_1 = 9.28248 cm
BC_2 = 5.22560 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AD}{AB} = 0.54423$

$\frac{AC_1}{AB} = 0.46407$

$\frac{BD}{AB} = 0.71958$

$\frac{C_1D}{AB} = 0.28174$

$\frac{AG}{AB} = 0.39704$

$\frac{AH}{AB} = 0.71513$

(1+AD)-BD = 0.82465
(1-AD)+BD = 1.17535
(AD-1)+BD = 0.26381
1+AD+BD = 2.26381

(AC+AD)-CD = 0.72655
(AC-AD)+CD = 0.20158
(AD-AC)+CD = 0.36191
AC+AD+CD = 1.29004

AB = 1.00000
AD = 0.54423
AC = 0.46407
CD = 0.28174
BD = 0.71958
AG = 0.39704
AH = 0.71513

$\frac{(AD^2+AC^2)-CD^2}{2\cdot AD}-AG = 0.00000$

$\frac{(AD^2+1)-BD^2}{2\cdot AD}-AH = 0.00000$

$BC_1 = 0.99163$

$BC_2 = 0.55824$

$(((((AC\cdot AD)^2+(AC\cdot BD)^2)-AC^2-AD^4)+(AD\cdot BD)^2+(AD\cdot CD)^2)-(BD\cdot CD)^2)+AD^2+CD^2 = 0.38355$

$\sqrt{((1+AD)-BD)\cdot((1-AD)+BD)\cdot((AD-1)+BD)\cdot(1+AD+BD)\cdot((AC+AD)-CD)\cdot((AC-AD)+CD)\cdot((AD-AC)+CD)\cdot(AC+AD+CD)} = 0.19895$

$$\frac{\sqrt{2}\cdot\sqrt{((((((AC\cdot AD)^2+(AC\cdot BD)^2)-AC^2-AD^4)+(AD\cdot BD)^2+(AD\cdot CD)^2)-(BD\cdot CD)^2)+AD^2+CD^2)+\sqrt{((1+AD)-BD)\cdot((1-AD)+BD)\cdot((AD-1)+BD)\cdot(1+AD+BD)\cdot((AC+AD)-CD)\cdot((AC-AD)+CD)\cdot((AD-AC)+CD)\cdot(AC+AD+CD)}}}{2\cdot AD}-BC_1 = 0.00000$$

$$\frac{\sqrt{2}\cdot\sqrt{((((((AC\cdot AD)^2+(AC\cdot BD)^2)-AC^2-AD^4)+(AD\cdot BD)^2+(AD\cdot CD)^2)-(BD\cdot CD)^2)+AD^2+CD^2)-\sqrt{((1+AD)-BD)\cdot((1-AD)+BD)\cdot((AD-1)+BD)\cdot(1+AD+BD)\cdot((AC+AD)-CD)\cdot((AC-AD)+CD)\cdot((AD-AC)+CD)\cdot(AC+AD+CD)}}}{2\cdot AD}-BC_2 = 0.00000$$

Unit

$AB := 1$

Given.

$AD := 2.17506 \qquad AC := 1.74732$

060793C

$BD := 2.61333 \qquad CD := 1.38168$

Descriptions.

Let the two triangles ABD and ACD be given.

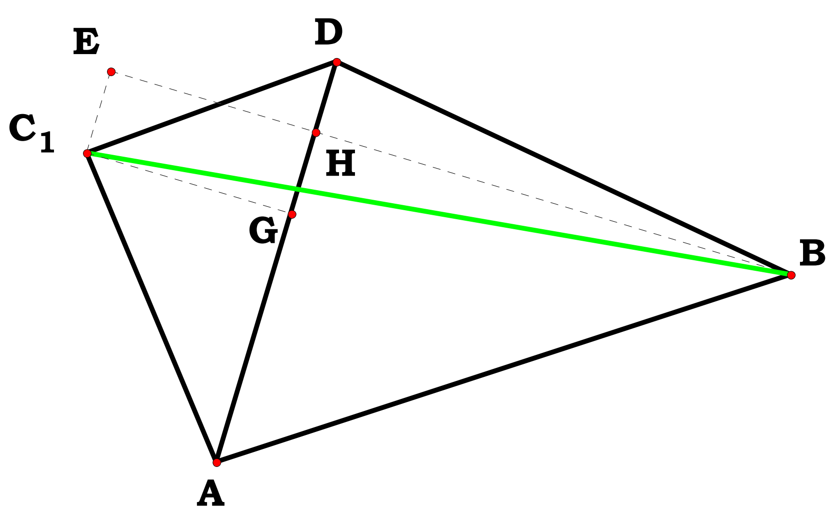
Given two triangles with a common side, find the difference between their free vertices from opposing sides.

Two Triangles with a Common Side.

$$CG := \frac{\sqrt{(AD+CD+AC)\cdot(-AD+CD+AC)\cdot(AD-CD+AC)(AD+CD-AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD+BD-1)\cdot(AD+BD+1)\cdot(AD-BD+1)\cdot(BD-AD+1)}}{2\cdot AD}$$

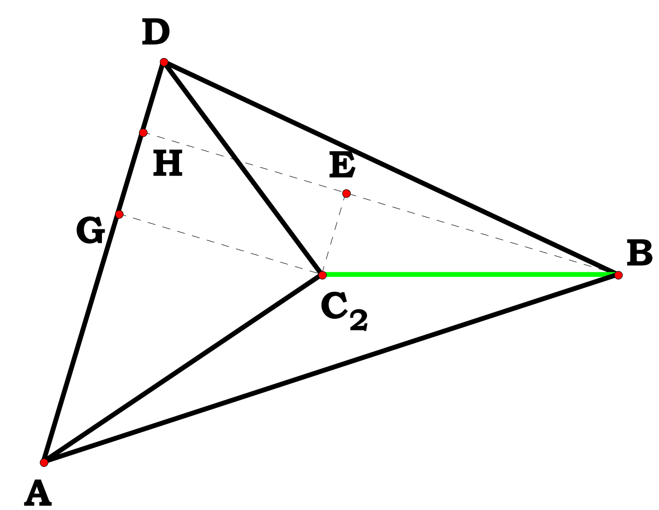
Greatest Disance:



$$BC_1 := \sqrt{GH^2 + (CG+BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH-CG)^2}$$

Least Distance



$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + 1^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)}\cdot\sqrt{(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}\ \ldots \\ +\ AC^2\cdot AD^2 + AC^2\cdot BD^2 - AC^2 - AD^4 + AD^2\cdot BD^2 + AD^2\cdot CD^2 + AD^2 - BD^2\cdot CD^2 + CD^2}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 - AC^2 - AD^4 + AD^2\cdot BD^2 + AD^2\cdot CD^2 + AD^2 - BD^2\cdot CD^2 + CD^2\ \ldots \\ +\ -\sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)}\cdot\sqrt{(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}}}{2\cdot AD} = 0$$

060793D

Unit

$AB := 1$

Given.

$Z := 10 \qquad U := 5$

$V := 2 \qquad W := 3 \qquad X := 7$

Descriptions.

Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$AM := \frac{U}{Z} \quad AN := \frac{V}{Z} \quad AO := \frac{W}{Z} \quad AP := \frac{X}{Z} \quad AD := \sqrt{AN^2 + AM^2}$$

$$BN := AB - AN \quad BD := \sqrt{BN^2 + AM^2} \quad AC := \sqrt{AP^2 + AO^2}$$

$$DH := AP - AD \quad CD := \sqrt{DH^2 + AO^2} \quad AG := \frac{AD^2 + AB^2 - BD^2}{2 \cdot AD}$$

$$BG := \frac{\sqrt{(AD + BD - AB)\cdot(AD - BD + AB)\cdot(BD - AD + AB)\cdot(AD + AB + BD)}}{2 \cdot AD}$$

$$BJ := BG + AO \quad BF := BG - AO \quad AH := AD + DH \quad CJ := AH - AG$$

$$BC := \sqrt{BJ^2 + CJ^2} \quad BE := \sqrt{BF^2 + CJ^2}$$

Definitions.

$$AM - \frac{U}{Z} = 0 \quad AN - \frac{V}{Z} = 0 \quad AO - \frac{W}{Z} = 0 \quad AP - \frac{X}{Z} = 0 \quad AD - \frac{\sqrt{U^2 + V^2}}{Z} = 0$$

$$BN - \frac{Z - V}{Z} = 0 \quad BD - \frac{\sqrt{U^2 + V^2 - 2 \cdot V \cdot Z + Z^2}}{Z} = 0 \quad AC - \frac{\sqrt{W^2 + X^2}}{Z} = 0 \quad DH - \frac{X - \sqrt{U^2 + V^2}}{Z} = 0 \quad CD - \frac{\sqrt{U^2 + V^2 + W^2 + X^2 - 2 \cdot X \cdot \sqrt{U^2 + V^2}}}{Z} = 0$$

$$AG - \frac{2 \cdot V \cdot Z}{2 \cdot Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BG - \frac{U}{\sqrt{U^2 + V^2}} = 0 \quad BJ - \frac{U \cdot Z + W \cdot \sqrt{U^2 + V^2}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BF - \frac{U \cdot Z - W \cdot \sqrt{U^2 + V^2}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad AH - \frac{X}{Z} = 0 \quad CJ - \frac{X \cdot \sqrt{U^2 + V^2} - V \cdot Z}{Z \cdot \sqrt{U^2 + V^2}} = 0$$

$$BC - \frac{\sqrt{2 \cdot Z \cdot (U \cdot W - V \cdot X) \cdot \sqrt{U^2 + V^2} + \left(W^2 + X^2 + Z^2\right)\cdot\left(U^2 + V^2\right)}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BE - \frac{\sqrt{\left(W^2 + X^2 + Z^2\right)\cdot\left(U^2 + V^2\right) - 2 \cdot Z \cdot (U \cdot W + V \cdot X) \cdot \sqrt{U^2 + V^2}}}{Z \cdot \sqrt{U^2 + V^2}} = 0$$

Two Triangles with a Common Side.

Unit = 1.00000
M = 0.50000
U = 5.00000
Z = 10.00000

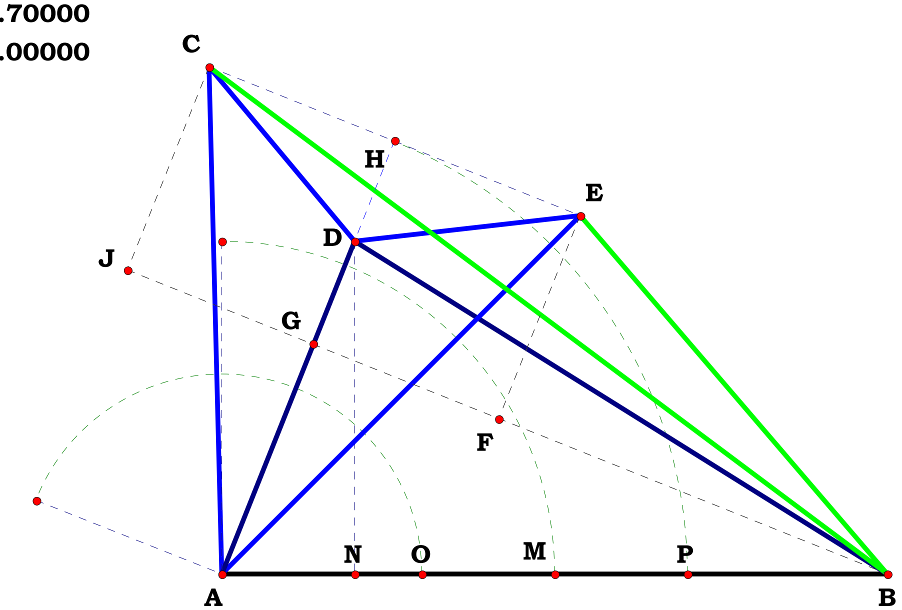
N = 0.20000
V = 2.00000

O = 0.30000
W = 3.00000

P = 0.70000
X = 7.00000

AB = 1.00000
AC = 0.76158
AD = 0.53852
BD = 0.94340
CD = 0.34070
AG = 0.37139

BG = 0.92848
BJ = 1.22848
BF = 0.62848
BC = 1.27167
BE = 0.70920



Given.

$AB := 3$

$AC := 1$

Given any value N_1, any other value, N_2, greater than twice the square root of N_1 can be divided such that the resulting pair of values equals N_1.

Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.

060993A Rectangular Roots.

Descriptions.

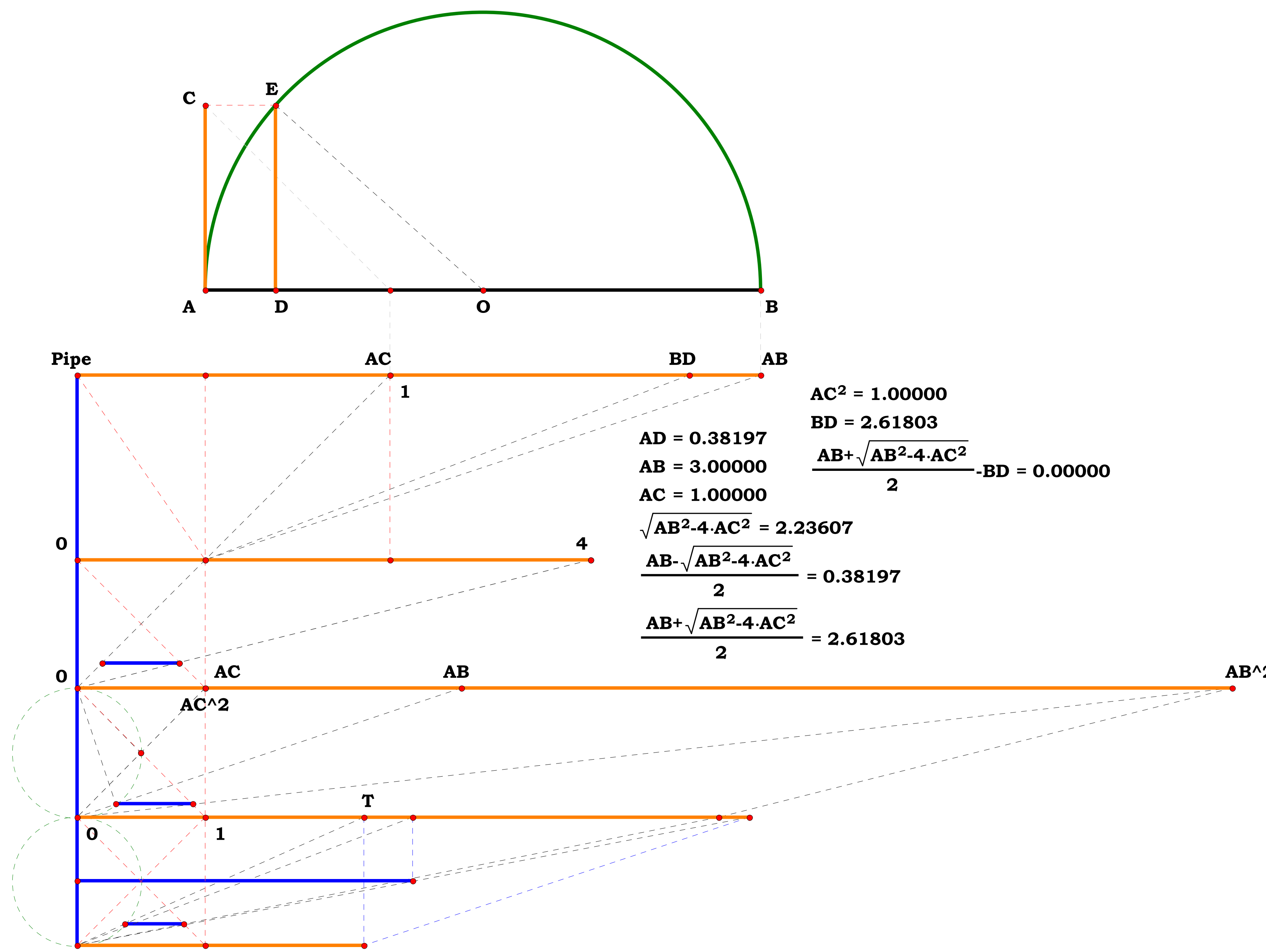
$DE := AC \quad EO := \frac{AB}{2} \quad DO := \sqrt{EO^2 - DE^2} \quad BD := EO + DO$

$AD := AB - BD \quad AD \cdot BD - AC^2 = 0$

Definitions.

$AD - \frac{AB - \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \quad AD = 0.381966$

$BD - \frac{AB + \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \quad BD = 2.618034 \quad AD + BD = 3$



060993B Rectangular Roots.

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 8$

Given any value N_1, any other value, N_2, greater than twice the square root of N_1 can be divided such that the resulting pair of values equals N_1.

Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.

Descriptions.

$$AC := \frac{X}{Y} \qquad DE := AC \qquad EO := \frac{AB}{2}$$

$$DO := \sqrt{EO^2 - DE^2} \qquad BD := EO + DO$$

$$AD := AB - BD \qquad AD \cdot BD - AC^2 = 0$$

Definitions.

$$AC - \frac{X}{Y} = 0 \qquad DE - \frac{X}{Y} = 0 \qquad EO - \frac{1}{2} = 0$$

$$DO - \frac{\sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0 \qquad BD - \frac{Y + \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0$$

$$AD - \frac{Y - \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0$$

$$\frac{Y - \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} \cdot \frac{Y + \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} - AC^2 = 0 \qquad \frac{X^2}{Y^2} - AC^2 = 0$$

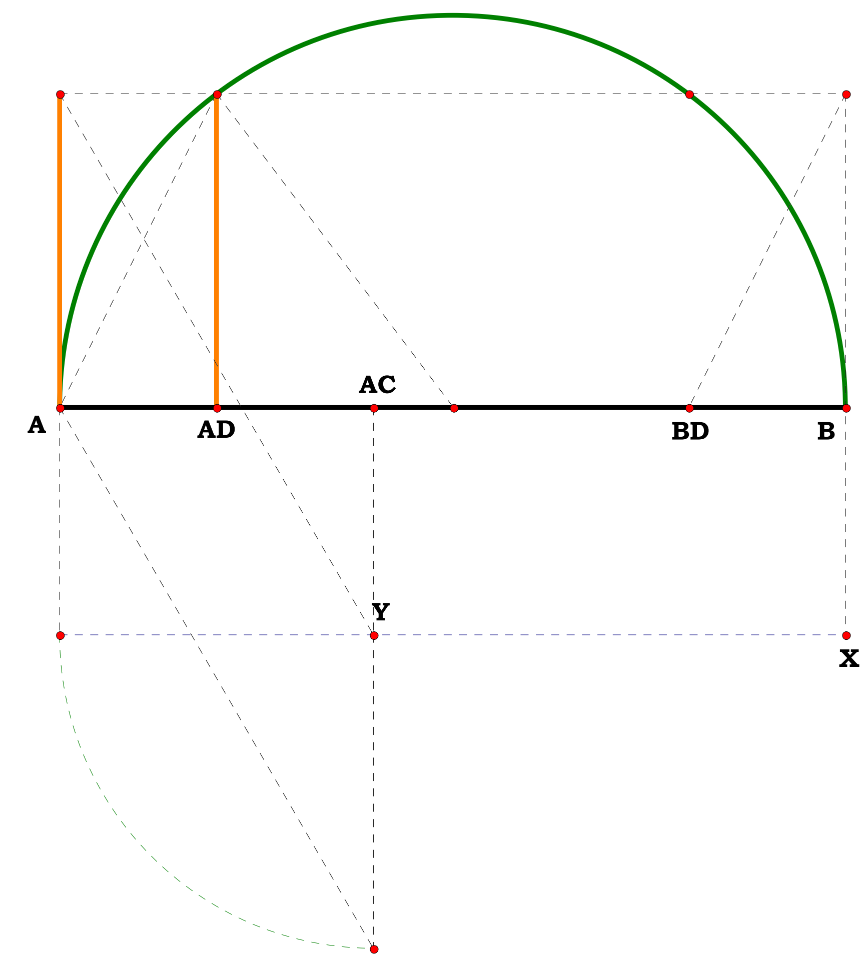
XY = 0.40000
X = 8.00000
Y = 20.00000

AD = 0.20000
AC = 0.40000
BD = 0.80000
AB = 1.00000

Unit = 1.00000

$$\frac{Y-\sqrt{Y^2-4 \cdot X^2}}{2 \cdot Y} - AD = 0.00000$$

$$\frac{Y+\sqrt{Y^2-4 \cdot X^2}}{2 \cdot Y} - BD = 0.00000$$



062193A Pyramid of Ratios I

Pyramid of Rations 1

If you just draw the figure, and measure and compare the numbers, you start to see it. Now one should simply take it and all its treasures.

Definitions for the unit we use for division. This will give us a Cardinal result. The buttons will put the N1 or 2 on a number, but you have to define its actual ratio to the figure.

Hide Points

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

Unit = 1.00000

X = 0.40000

Y = 0.30000

01 = 2.32038 cm

ON_1 = 9.28153 cm

ON_2 = 6.96115 cm

$\frac{ON_1}{01} = 4.00000$

$N_1 = 4.00000$

$\frac{ON_2}{01} = 3.00000$

$N_2 = 3.00000$

AC = 16.40759 cm

AF = 6.56304 cm

$\frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 2.50000$

$\frac{AC}{AF} = 2.50000$

$\frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} - \frac{AC}{AF} = 0.00000$

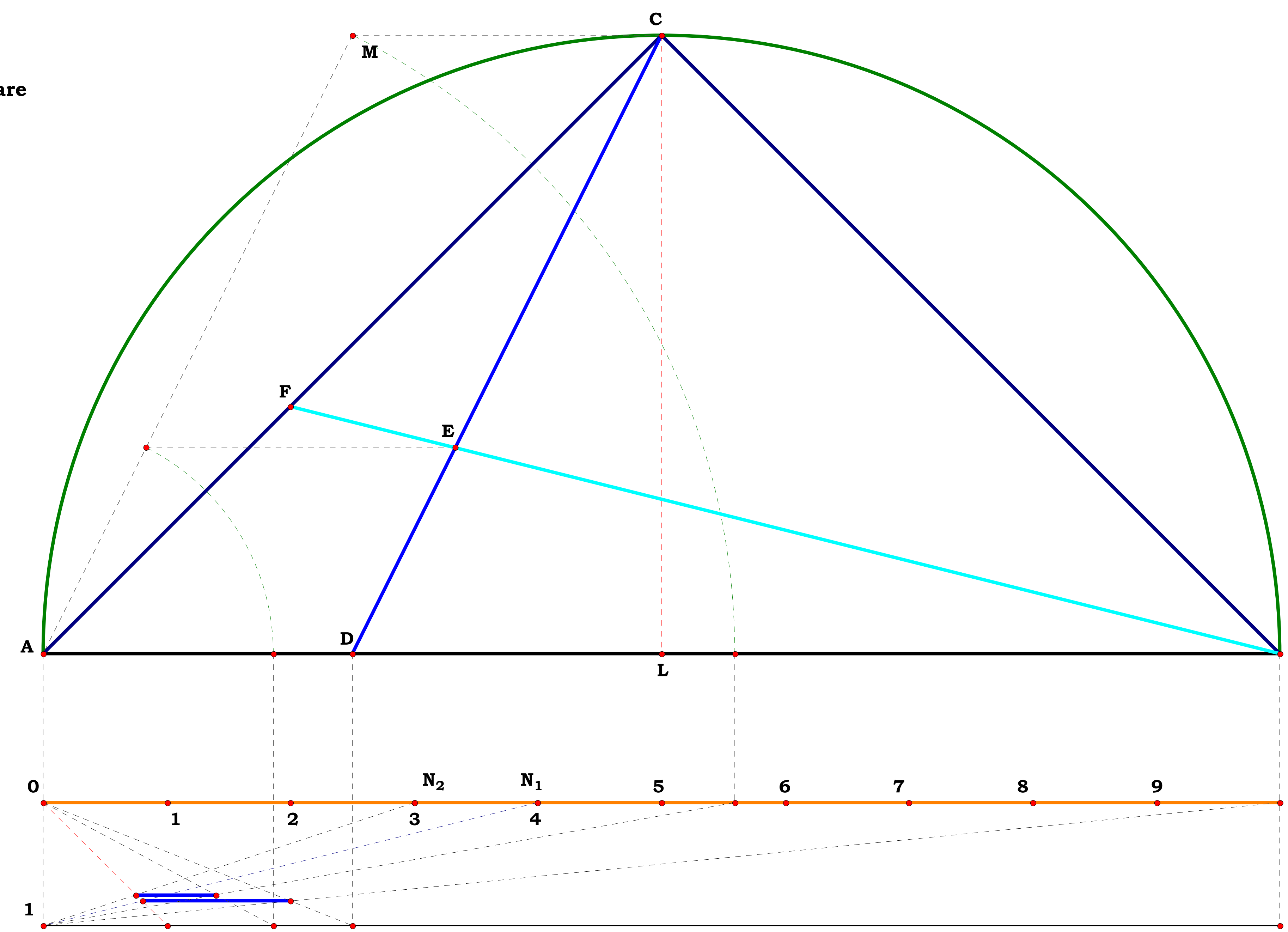
BF = 19.13437 cm

EF = 3.18906 cm

$\frac{BF}{EF} = 6.00000$

$\frac{N_1 \cdot N_2}{N_2 - 1} = 6.00000$

$\frac{N_1 \cdot N_2}{N_2 - 1} - \frac{BF}{EF} = 0.00000$



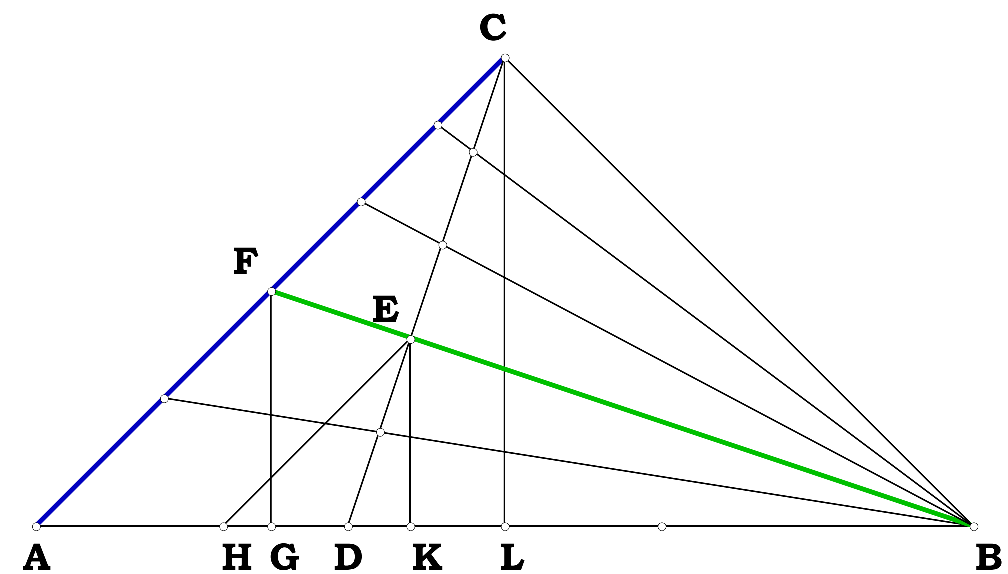
062193B

Unit.

Given.

Pyramid of Ratios I

Divide AB by N1 then divide CD by N2, what are BF/EF and AC/AF?



Descriptions.

Definitions.

$N1 := 3 \quad N2 := 5 \quad \delta := 1 .. N2$

$\delta =$

1
2
3
4
5

$AB := 1 \quad AD := \dfrac{AB}{N1} \quad AL := \dfrac{AB}{2}$

$DL := AL - AD \quad AC := \sqrt{\dfrac{AB^2}{2}}$

$CL := AL \quad CD := \sqrt{DL^2 + CL^2}$

$DE_\delta := \dfrac{CD \cdot \delta}{N2} \quad DK_\delta := \dfrac{DL \cdot DE_\delta}{CD} \quad AK_\delta := AD + DK_\delta \quad BK_\delta := AB - AK_\delta \quad EK_\delta := \dfrac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \quad HK_\delta := \dfrac{AL \cdot DK_\delta}{DL} \quad BH_\delta := BK_\delta + HK_\delta \quad EH_\delta := \dfrac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \dfrac{EH_\delta \cdot AB}{BH_\delta} \quad BF_\delta := \dfrac{BE_\delta \cdot AB}{BH_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \dfrac{BF_\delta}{EF_\delta}, 0\right) =$

3.75
5
7.5
15
0

$\text{if}\left(N2 - \delta, \dfrac{N1 \cdot N2}{N2 - \delta}, 0\right) =$

3.75
5
7.5
15
0

$\dfrac{AC}{AF_\delta} =$

3.666667
2
1.444444
1.166667
1

$\dfrac{(N1 - 1) \cdot N2 + \delta}{N1 \cdot \delta} =$

3.666667
2
1.444444
1.166667
1

This was my original write up, long ago, and it takes advantage of some functions of Mathcad. For the final product of The Delian Quest, it will become a series of demonstrations. However, the root figure does not change, nor what it does.

I never had Geometry in school, nor did I ever do well in algebra because like everything else, those who teach it really do not know what in the hell they are doing. Too much mythology, and too much insistence on traditional carping. I am a simple person and I like things simple, honest, and organized.

062193C

Unit.

$AB := 1$

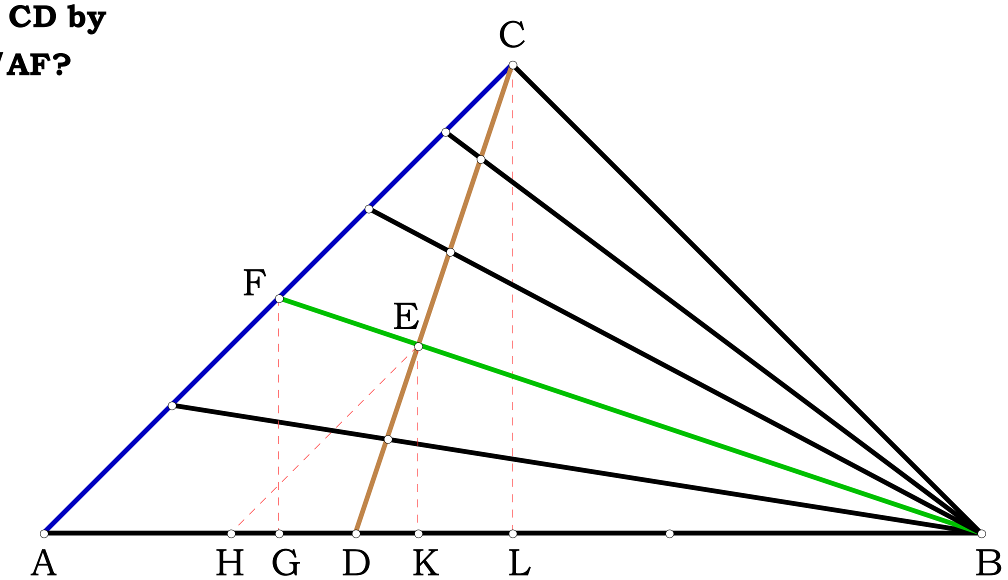
Given.

$N_1 := 4$

$N_2 := 2$

Pyramid of Ratios I

Divide AB by N_1 then divide CD by N_2, what are BF/EF and AC/AF?



Descriptions.

$AD := \frac{AB}{N_1}$ $\quad AL := \frac{AB}{2}$ $\quad AC := \sqrt{2 \cdot AL^2}$ $\quad CL := AL$ $\quad DL := AL - AD$

$CD := \sqrt{DL^2 + CL^2}$ $\quad DE := \frac{CD}{N_2}$ $\quad EK := \frac{CL \cdot DE}{CD}$ $\quad DK := \frac{DL \cdot EK}{CL}$

$AK := AD + DK$ $\quad BK := AB - AK$ $\quad BE := \sqrt{BK^2 + EK^2}$

$HK := \frac{AL \cdot EK}{CL}$ $\quad BH := BK + HK$ $\quad EH := \frac{AC \cdot EK}{CL}$ $\quad AF := \frac{EH \cdot AB}{BH}$

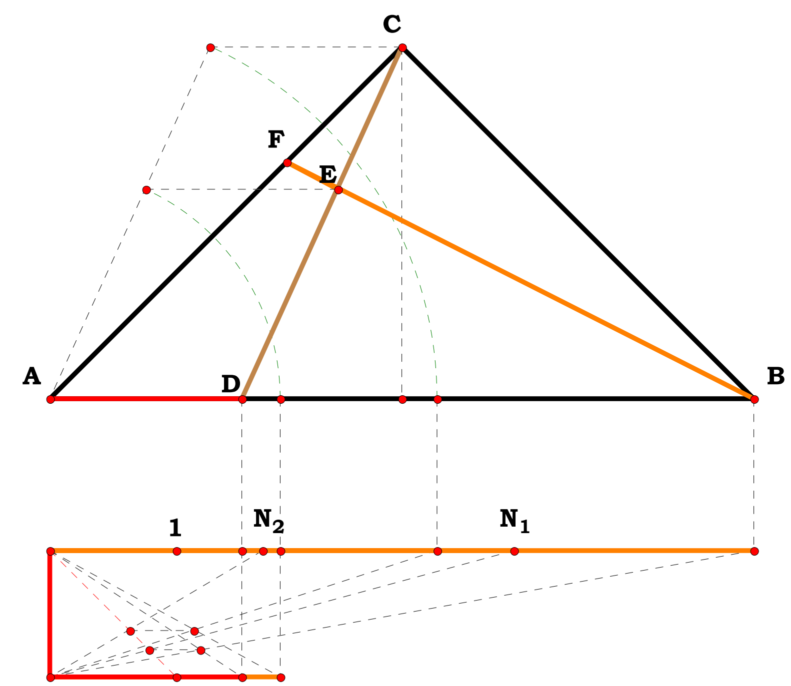
$BF := \frac{BE \cdot AB}{BH}$ $\quad EF := BF - BE$

$\frac{BF}{EF} = 8$ $\quad \frac{AC}{AF} = 1.75$ $\quad \frac{CD}{DE} = 2$

Definitions.

$$\frac{AC}{AF} - \frac{N_1 \cdot N_2 - N_2 + 1}{N_1} = 0$$

$$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0$$



$\frac{BF}{EF} = 9.08799$ $\quad \frac{N_1 \cdot N_2}{N_2 - 1} = 9.08799$

$\frac{AC}{AF} = 1.49057$ $\quad \frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 1.49057$

$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0.00000$ $\quad \frac{AC}{AF} - \frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 0.00000$

062193D Pyramid of Ratios I

Pyramid of Ratios I

The buttons on the macro are divided into two columns, one for the number of divisions I want for a given unit, and the other a particular point in that range. This means that if I set the points equal to what I am working with, I do not have to draw anything further to play with the line to examine it. Thus, althoght N1 and N2 are working with two different lengths of line, one always constant and the other variable, it is transparent in the the equation. We are dividing each of them in a Cardinal fashion, just rendering an arithmetic name for its point in a ratio of that particular line. And, this allows me to write up the equations for the figure which reduce, in the end, to w, x, y and z and always giving an exact answer no matter what original system of measurement one started with, inches, meters, or pixels, or even mystical particals of quantum dust. A thing, is a thing, is a thing, which is always independent of any convention of names and what one is looking for, is a simple, universal method of notation without using Einstein's rubbrer bands and crazy glue.

Just the fact that one is working with four names in the equation tells me that I have two different things using two different systems of measurement, each one bringing itself into the equation as it is.

Y -> 1, Y -> 2, Y -> 3, Y -> 4, Y -> 5, Y -> 6, Y -> 7, Y -> 8, Y -> 9, Y -> 10, Y -> 11, Y -> 12, Y -> 13, Y -> 14, Y -> 15, Y -> 16, Y -> 17, Y -> 18, Y -> 19, Y -> 20

X -> 0, X -> 1, X -> 2, X -> 3, X -> 4, X -> 5, X -> 6, X -> 7, X -> 8, X -> 9, X -> 10, X -> 11, X -> 12, X -> 13, X -> 14, X -> 15, X -> 16, X -> 17, X -> 18, X -> 19, X -> 20

Show Points

Y -> 1, Y -> 2, Y -> 3, Y -> 4, Y -> 5, Y -> 6, Y -> 7, Y -> 8, Y -> 9, Y -> 10, Y -> 11, Y -> 12, Y -> 13, Y -> 14, Y -> 15, Y -> 16, Y -> 17, Y -> 18, Y -> 19, Y -> 20

X -> 0, X -> 1, X -> 2, X -> 3, X -> 4, X -> 5, X -> 6, X -> 7, X -> 8, X -> 9, X -> 10, X -> 11, X -> 12, X -> 13, X -> 14, X -> 15, X -> 16, X -> 17, X -> 18, X -> 19, X -> 20

Show Points

$$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1} = 1.53333$$

$$\frac{X \cdot (W-Z)+Y \cdot Z}{Y \cdot W} = 1.53333$$

$$\frac{AC}{AF} = 1.53333$$

$$\frac{N_1 \cdot N_2}{N_2-1} = 12.50000$$

$$\frac{Y \cdot Z}{X \cdot (Z-W)} = 12.50000$$

$$\frac{BF}{EF} = 12.50000$$

AC = 7.80151 cm

AF = 5.08794 cm

BF = 8.25996 cm

EF = 0.66080 cm

Unit = 1.00000

X = 4.00000

Y = 20.00000

$\frac{Y}{X} = 5.00000$

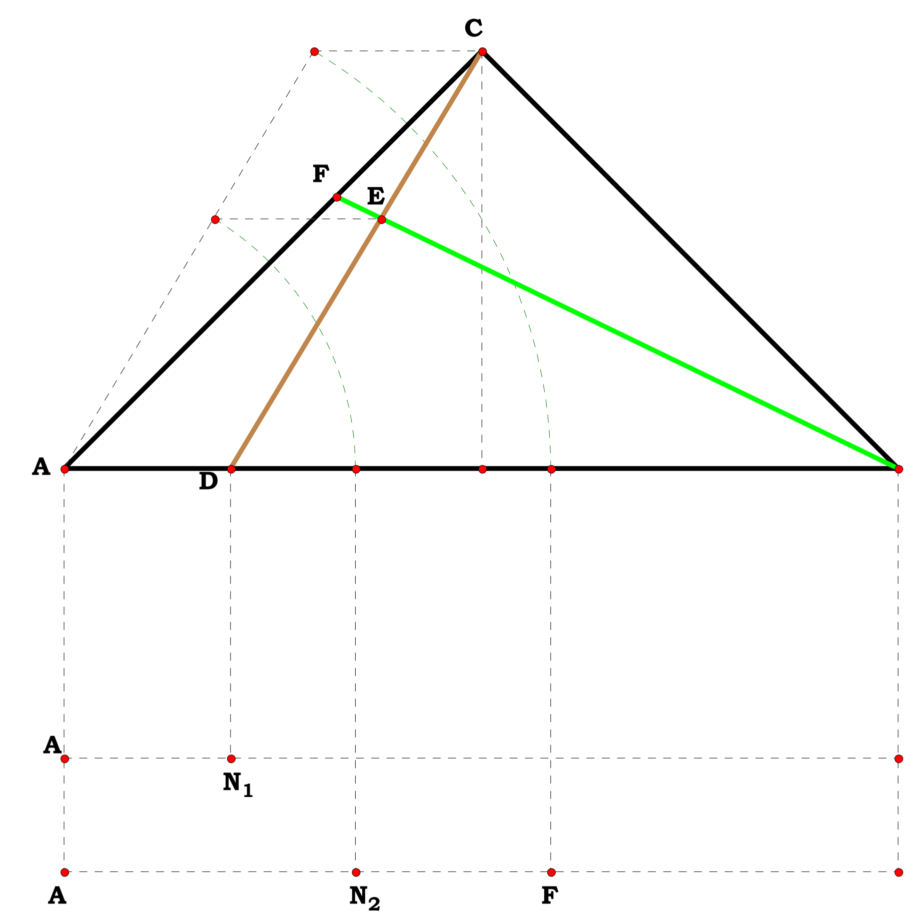
$N_1 = 5.00000$

W = 6.00000

Z = 10.00000

$\frac{Z}{W} = 1.66667$

$N_2 = 1.66667$



062193E

Unit.

$AB := 1$

Given.

$W := 6$

$X := 4$

$Y := 20$

$Z := 10$

Descriptions.

$N_1 := \frac{Y}{X} \qquad N_2 := \frac{Z}{W}$

$AD := \frac{AB}{N_1} \qquad AL := \frac{AB}{2} \qquad AC := \sqrt{2 \cdot AL^2} \qquad CL := AL \qquad DL := AL - AD$

$CD := \sqrt{DL^2 + CL^2} \qquad DE := \frac{CD}{N_2} \qquad EK := \frac{CL \cdot DE}{CD} \qquad DK := \frac{DL \cdot EK}{CL}$

$AK := AD + DK \qquad BK := AB - AK \qquad BE := \sqrt{BK^2 + EK^2}$

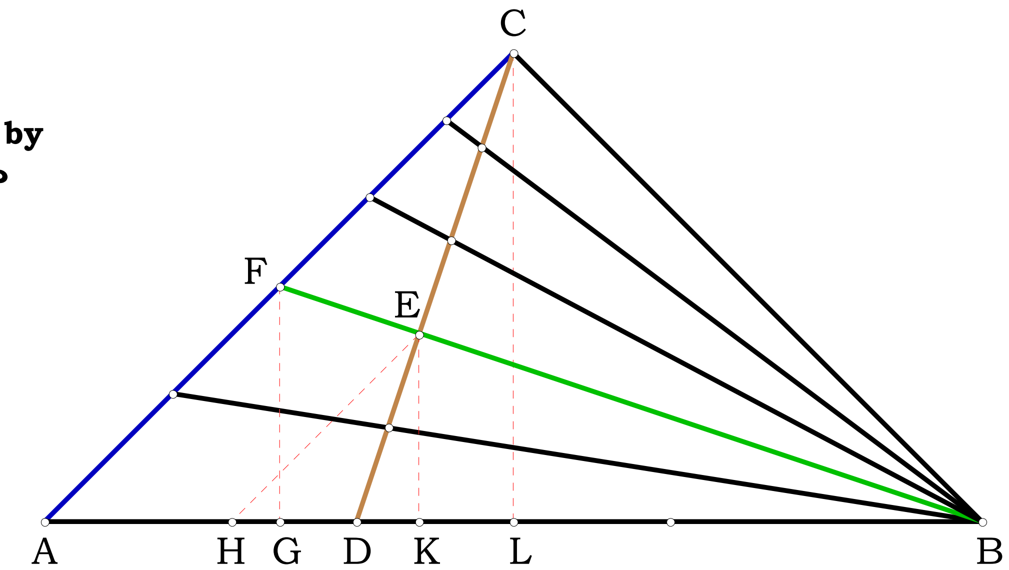
$HK := \frac{AL \cdot EK}{CL} \qquad BH := BK + HK \qquad EH := \frac{AC \cdot EK}{CL} \qquad AF := \frac{EH \cdot AB}{BH}$

$BF := \frac{BE \cdot AB}{BH} \qquad EF := BF - BE$

$\frac{BF}{EF} = 12.5 \qquad \frac{AC}{AF} = 1.533333 \qquad \frac{CD}{DE} = 1.666667$

Pyramid of Ratios I

Divide AB by N_1 then divide CD by N_2, what are BF/EF and AC/AF?



$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1} = 1.53333$

$\frac{X \cdot (W-Z)+Y \cdot Z}{Y \cdot W} = 1.53333$

$\frac{AC}{AF} = 1.53333$

$\frac{N_1 \cdot N_2}{N_2 - 1} = 12.50000$

$\frac{Y \cdot Z}{X \cdot (Z-W)} = 12.50000$

$\frac{BF}{EF} = 12.50000$

AC = 7.80151 cm

AF = 5.08794 cm

BF = 8.25996 cm

EF = 0.66080 cm

Unit = 1.00000

X = 4.00000

Y = 20.00000

$\frac{Y}{X} = 5.00000$

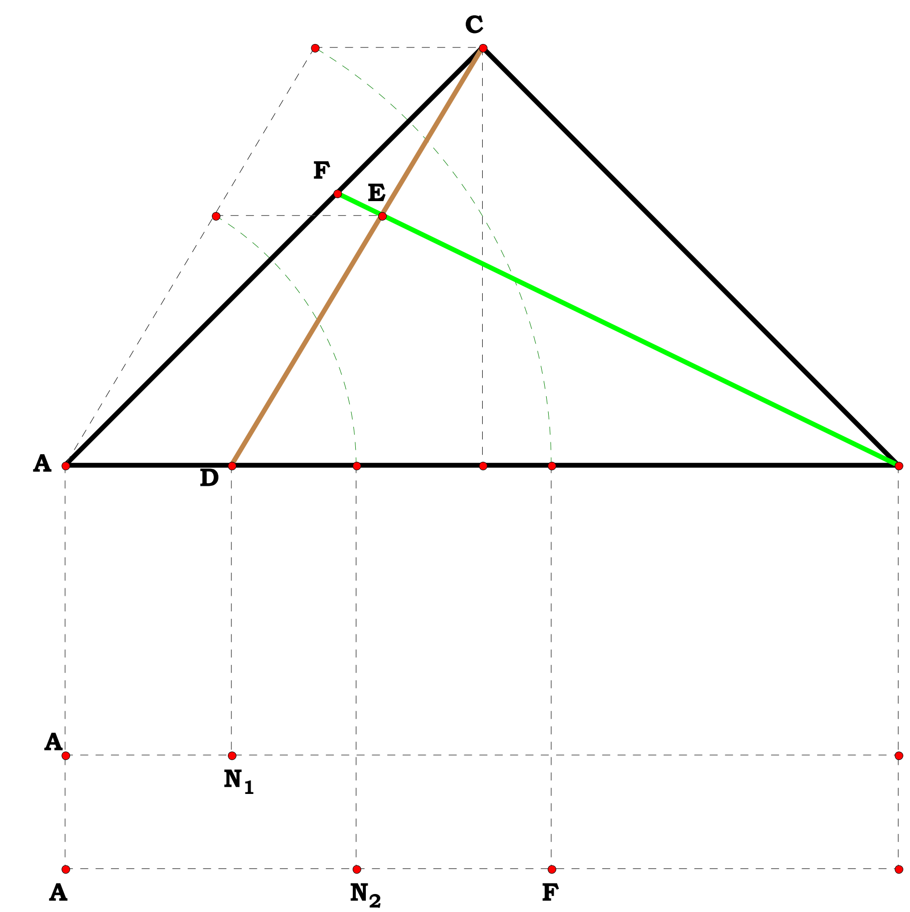
$N_1 = 5.00000$

W = 6.00000

Z = 10.00000

$\frac{Z}{W} = 1.66667$

$N_2 = 1.66667$



Definitions.

$$\frac{AC}{AF}-\frac{X\cdot(W-Z)+Y\cdot Z}{Y\cdot W}=0 \qquad \frac{BF}{EF}-\frac{Y\cdot Z}{X\cdot(Z-W)}=0$$

$$N_1-\frac{Y}{X}=0 \quad N_2-\frac{Z}{W}=0 \quad AD-\frac{X}{Y}=0 \quad AL-\frac{1}{2}=0 \quad AC-\frac{1}{\sqrt{2}}=0$$

$$CL-\frac{1}{2}=0 \quad DL-\frac{Y-2\cdot X}{2\cdot Y}=0 \quad CD-\frac{\sqrt{2\cdot X^2-2\cdot X\cdot Y+Y^2}}{\sqrt{2}\cdot Y}=0 \quad DE-\frac{\sqrt{2}\cdot W\cdot\sqrt{2\cdot X^2-2\cdot X\cdot Y+Y^2}}{2\cdot Y\cdot Z}=0$$

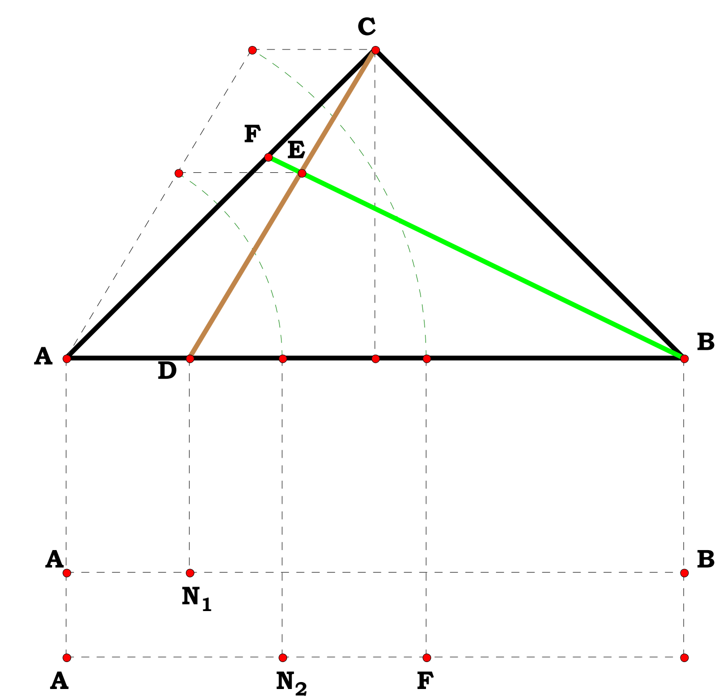
$$EK-\frac{W}{2\cdot Z}=0 \quad DK-\frac{W\cdot(Y-2\cdot X)}{2\cdot Y\cdot Z}=0 \quad AK-\frac{[W\cdot(Y-2\cdot X)+2\cdot X\cdot Z]}{2\cdot Y\cdot Z}=0 \quad BK-\frac{2\cdot(W\cdot X+Y\cdot Z)-(W\cdot Y+2\cdot X\cdot Z)}{2\cdot Y\cdot Z}=0$$

$$BE-\frac{\sqrt{2\cdot(X-Y)^2\cdot Z^2+[2\cdot W\cdot(Y-X)\cdot(2\cdot X-Y)]\cdot Z+W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)}}{\sqrt{2}\cdot Y\cdot Z}=0 \quad HK-\frac{W}{2\cdot Z}=0$$

$$BH-\frac{W\cdot X-X\cdot Z+Y\cdot Z}{Y\cdot Z}=0 \quad EH-\frac{\sqrt{2}\cdot W}{2\cdot Z}=0 \quad AF-\frac{\sqrt{2}\cdot W\cdot Y}{2\cdot(W\cdot X-X\cdot Z+Y\cdot Z)}=0$$

$$BF-\frac{\sqrt{2\cdot W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)+4\cdot Z^2\cdot(X-Y)^2+4\cdot W\cdot Z\cdot(X-Y)\cdot(Y-2\cdot X)}}{2\cdot(W\cdot X-X\cdot Z+Y\cdot Z)}=0$$

$$EF-\frac{X\cdot(W-Z)\cdot\sqrt{2\cdot W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)+4\cdot Z^2\cdot(X-Y)^2+4\cdot W\cdot Z\cdot(X-Y)\cdot(Y-2\cdot X)}}{2\cdot Y\cdot Z\cdot(X\cdot Z-W\cdot X-Y\cdot Z)}=0$$



Unit = 1.00000

X = 4.00000

Y = 20.00000

$\frac{Y}{X}$ = 5.00000

N_1 = 5.00000

W = 6.00000

Z = 10.00000

$\frac{Z}{W}$ = 1.66667

N_2 = 1.66667

$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1}$ = 1.53333

$\frac{X\cdot(W-Z)+Y\cdot Z}{Y\cdot W}$ = 1.53333

$\frac{AC}{AF}$ = 1.53333

$\frac{N_1\cdot N_2}{N_2-1}$ = 12.50000

$\frac{Y\cdot Z}{X\cdot(Z-W)}$ = 12.50000

$\frac{BF}{EF}$ = 12.50000

AC = 5.78095 cm

AF = 3.77019 cm

BF = 6.12067 cm

EF = 0.48965 cm

062793A

Given.

$N_1 := 3 \quad AB := N_1$

$N_2 := 6 \quad BC := N_2$

$N_3 := 4 \quad AC := N_3$

Describe A Circle About a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

Descriptions.

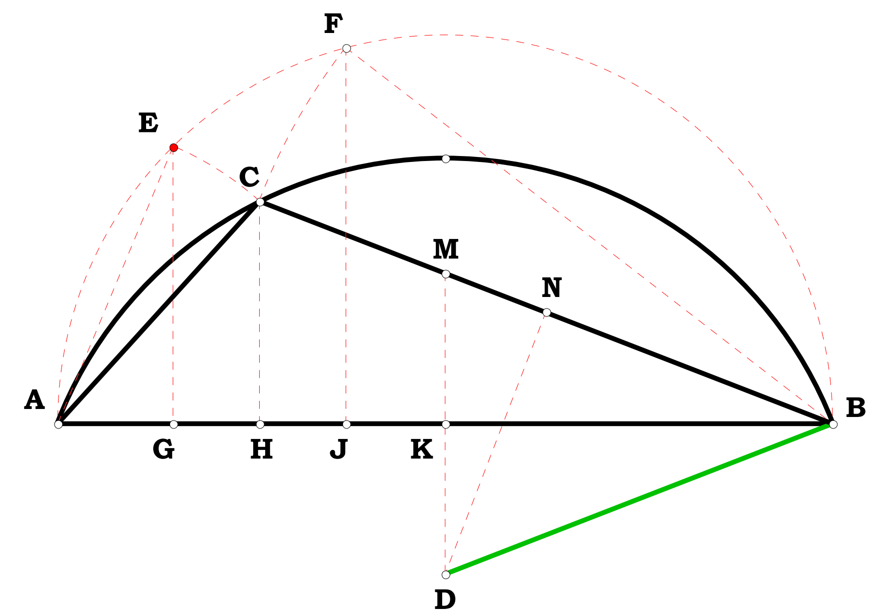
$\Delta := (AB + AC > BC)\cdot(AB + BC > AC)\cdot(BC + AC > AB) \qquad NOT(X) := X = \delta := 0 .. 2$

$BK := \frac{AB}{2} \quad AE := AC \quad BF := BC \quad AG := \frac{AE^2}{AB} \quad BJ := \frac{BF^2}{AB}$

$GJ := AB - (AG + BJ) \quad HJ := \frac{GJ}{2} \quad BH := BJ + HJ$

$CH := \sqrt{BC^2 - BH^2} \quad BN := \frac{BC}{2} \quad BM := \frac{BC \cdot BK}{BH}$

$MN := BM - BN \quad DN := \frac{BH \cdot MN}{CH} \quad BD := \sqrt{BN^2 + DN^2}$



Definitions.

$radius := if(\Delta, BD, 0) \qquad imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375412$

$imaginary\_radius = 0$

The construction is independent of the side one starts with.

$\Delta = 1$

$$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix} \quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix} \qquad R_\delta := \frac{S_{1_\delta} \cdot S_{2_\delta} \cdot S_{3_\delta}}{\sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}$$

The name of the Radius in terms of the givens.

$R^T = (3.375412 \quad 3.375412 \quad 3.375412)$ **The equation is a statement in regard to the relationship between each side of a triangle.**

$$BK-\frac{N_1}{2}=0 \qquad AE-N_3=0 \qquad BF-N_2=0 \qquad AG-\frac{N_3^{\,2}}{N_1}=0 \qquad BJ-\frac{N_2^{\,2}}{N_1}=0$$

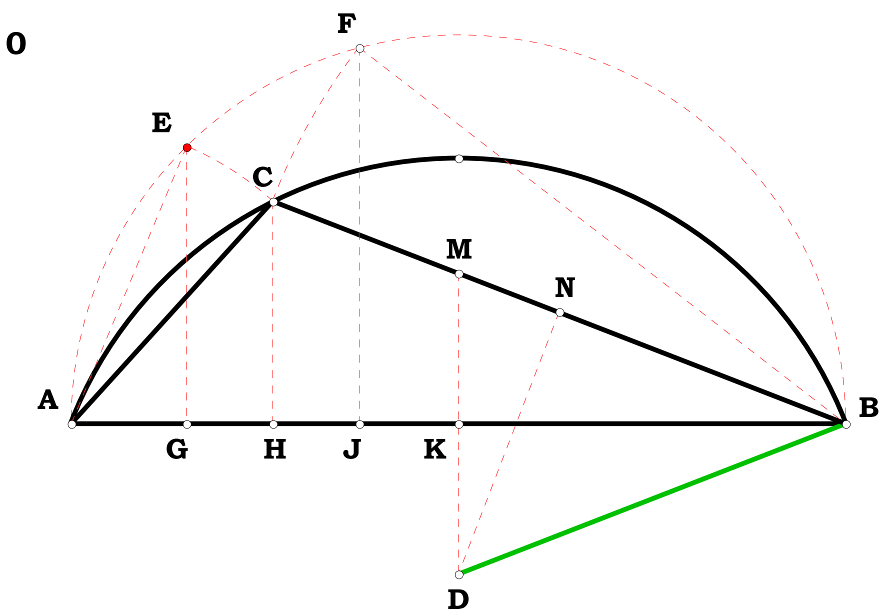
$$GJ-\frac{N_1^{\,2}-N_2^{\,2}-N_3^{\,2}}{N_1}=0 \qquad HJ-\frac{N_1^{\,2}-N_2^{\,2}-N_3^{\,2}}{2\cdot N_1}=0 \qquad BH-\frac{N_1^{\,2}+N_2^{\,2}-N_3^{\,2}}{2\cdot N_1}=0$$

$$CH-\frac{\sqrt{\left(N_1+N_2-N_3\right)\cdot\left(N_1-N_2+N_3\right)\cdot\left(N_2-N_1+N_3\right)\cdot\left(N_1+N_2+N_3\right)}}{2\cdot N_1}=0 \qquad BN-\frac{N_2}{2}=0$$

$$BM-\frac{N_1^{\,2}\cdot N_2}{N_1^{\,2}+N_2^{\,2}-N_3^{\,2}}=0 \qquad MN-\frac{N_2\cdot\left(N_1^{\,2}-N_2^{\,2}+N_3^{\,2}\right)}{2\cdot\left(N_1^{\,2}+N_2^{\,2}-N_3^{\,2}\right)}=0$$

$$DN-\frac{N_2\cdot\left(N_1^{\,2}-N_2^{\,2}+N_3^{\,2}\right)}{2\cdot\sqrt{\left(N_1+N_2-N_3\right)\cdot\left(N_1-N_2+N_3\right)\cdot\left(N_2-N_1+N_3\right)\cdot\left(N_1+N_2+N_3\right)}}=0$$

$$BD-\frac{N_1\cdot N_2\cdot N_3}{\sqrt{\left(N_1+N_2-N_3\right)\cdot\left(N_1-N_2+N_3\right)\cdot\left(N_2-N_1+N_3\right)\cdot\left(N_1+N_2+N_3\right)}}=0$$



AB = 4.03775 in. N_1 = 4.03775 in.
BC = 3.20102 in. N_2 = 3.20102 in.
AC = 1.55963 in. N_3 = 1.55963 in.
BD = 2.16667 in.

$$\frac{N_1\cdot N_2\cdot N_3}{\sqrt{(N_1+N_2+N_3)\cdot((N_1-N_2)+N_3)\cdot((N_2-N_1)+N_3)\cdot((N_1+N_2)-N_3)}}-BD = 0.00000\text{ in.}$$

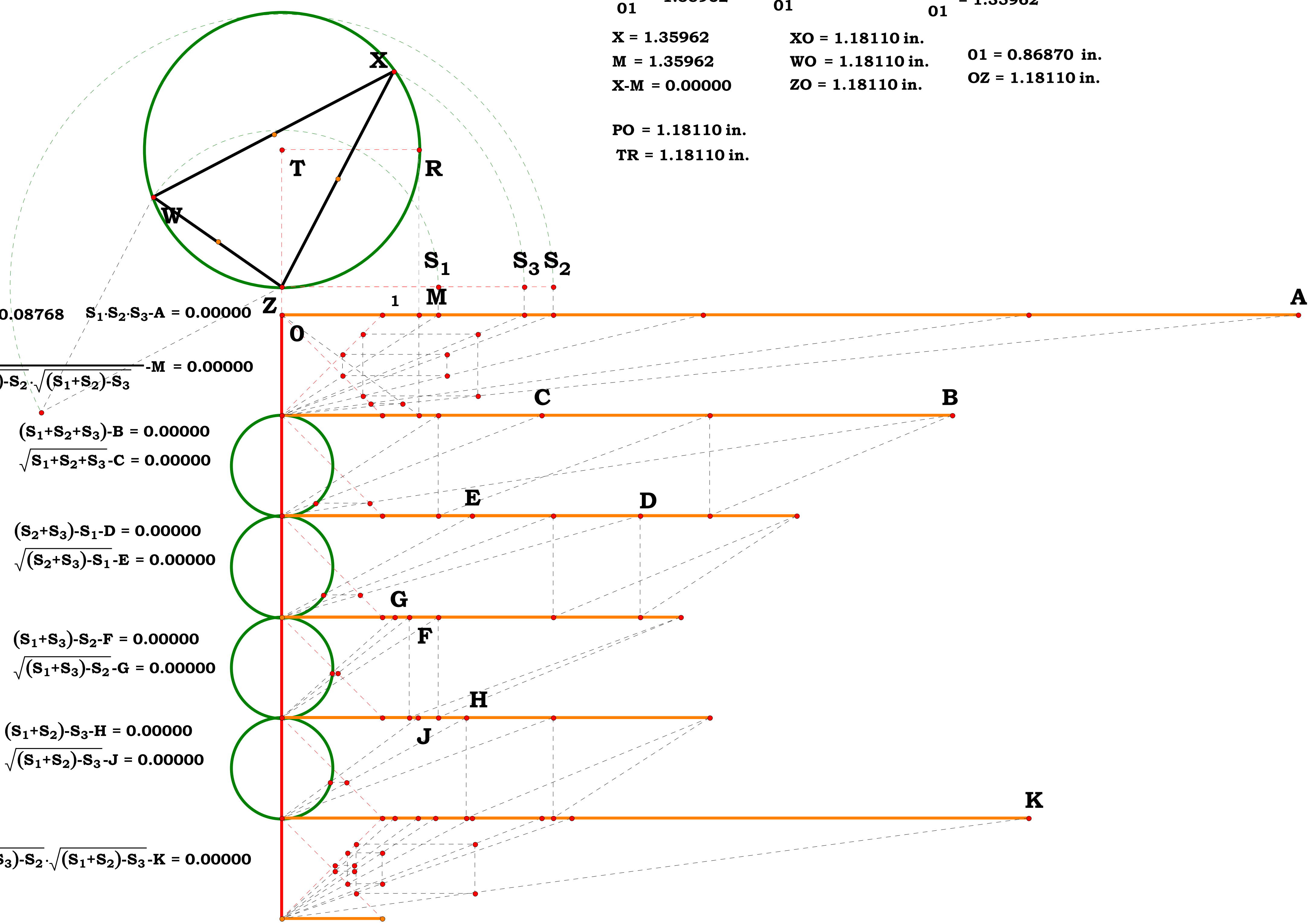
$\frac{XO}{01} = 1.35962$ $\frac{WO}{01} = 1.35962$ $\frac{ZO}{01} = 1.35962$

X = 1.35962
M = 1.35962
X-M = 0.00000

XO = 1.18110 in.
WO = 1.18110 in.
ZO = 1.18110 in.

01 = 0.86870 in.
OZ = 1.18110 in.

PO = 1.18110 in.
TR = 1.18110 in.



$S_1 = 1.55042$
$S_2 = 2.69740$
$S_3 = 2.41211$

A = 10.08768 $S_1 \cdot S_2 \cdot S_3 - A = 0.00000$

$$\frac{S_1 \cdot S_2 \cdot S_3}{\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3}} - M = 0.00000$$

B = 6.65993 $(S_1+S_2+S_3)-B = 0.00000$
C = 2.58068 $\sqrt{S_1+S_2+S_3}-C = 0.00000$

D = 3.55909 $(S_2+S_3)-S_1-D = 0.00000$
E = 1.88655 $\sqrt{(S_2+S_3)-S_1}-E = 0.00000$

F = 1.26512 $(S_1+S_3)-S_2-F = 0.00000$
G = 1.12478 $\sqrt{(S_1+S_3)-S_2}-G = 0.00000$

H = 1.83571 $(S_1+S_2)-S_3-H = 0.00000$
J = 1.35489 $\sqrt{(S_1+S_2)-S_3}-J = 0.00000$

K = 7.41948

$$\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3} - K = 0.00000$$

062793B

Unit.

$AB := 1$

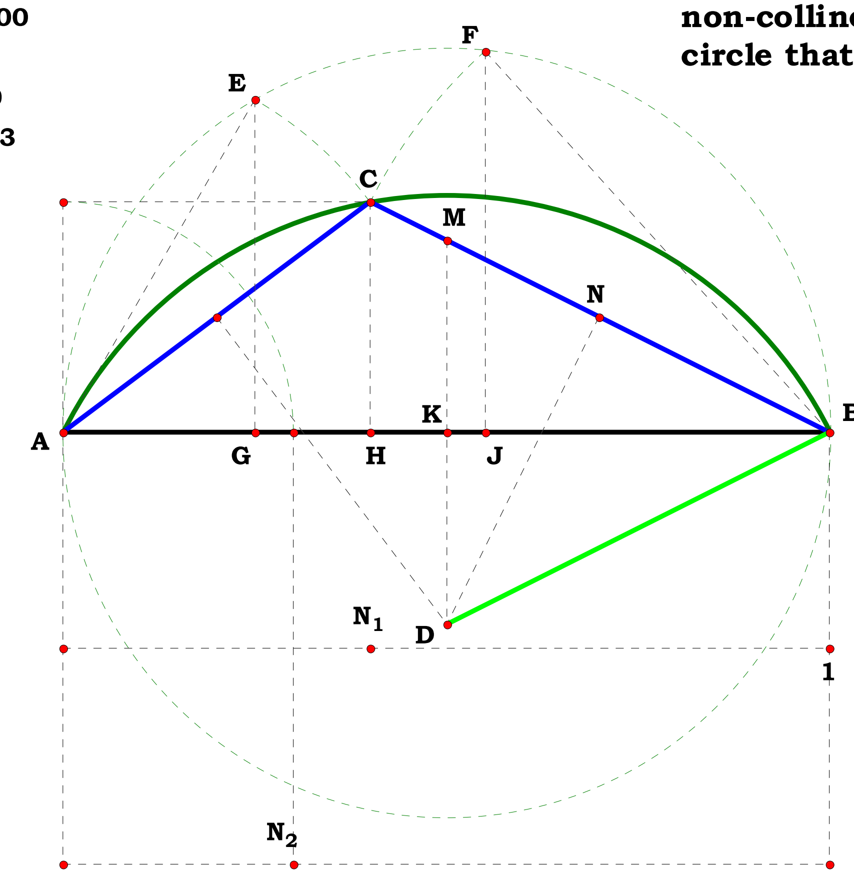
Given.

$W := 4 \quad Y := 3$

$X := 10 \quad Z := 10$

Describe A Circle About a Triangle

Unit = 1.00000
X/W = 2.50000
W = 4.00000
X = 10.00000
Z/Y = 3.33333
Y = 3.00000
Z = 10.00000
N_1 = 2.50000
N_2 = 3.33333



Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

Descriptions.

$$AH := \frac{W}{X} \quad CH := \frac{Y}{Z} \quad AC := \sqrt{AH^2 + CH^2} \quad BH := AB - AH$$

$$BC := \sqrt{BH^2 + CH^2} \quad BK := \frac{AB}{2} \quad AE := AC \quad BF := BC$$

$$AG := \frac{AE^2}{AB} \quad BJ := \frac{BF^2}{AB} \quad GJ := AB - (AG + BJ) \quad HJ := \frac{GJ}{2}$$

$$BN := \frac{BC}{2} \quad BM := \frac{BC \cdot BK}{BH} \quad MN := BM - BN \quad DN := \frac{BH \cdot MN}{CH}$$

$$BD := \sqrt{BN^2 + DN^2}$$

Definitions.

$$AH - \frac{W}{X} = 0 \quad CH - \frac{Y}{Z} = 0 \quad AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0 \quad BH - \frac{X - W}{X} = 0$$

$$BC - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0 \quad BK - \frac{1}{2} \quad AE - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0 \quad BF - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0$$

$$AG - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} = 0 \quad BJ - \frac{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}{X^2 \cdot Z^2} = 0 \quad GJ - \frac{2 \cdot \left(W \cdot X \cdot Z^2 - W^2 \cdot Z^2 - X^2 \cdot Y^2\right)}{X^2 \cdot Z^2} = 0 \quad HJ - \frac{\left(W \cdot X \cdot Z^2 - W^2 \cdot Z^2 - X^2 \cdot Y^2\right)}{X^2 \cdot Z^2} = 0$$

$$BN - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{2 \cdot X \cdot Z} = 0 \quad BM - \frac{\sqrt{X^2 \cdot \left(Y^2 + Z^2\right) + W \cdot Z^2 \cdot (W - 2 \cdot X)}}{2 \cdot Z \cdot (X - W)} = 0 \quad MN - \frac{W \cdot \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{2 \cdot X \cdot Z \cdot (X - W)} = 0$$

$$DN - \frac{W \cdot \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{2 \cdot X^2 \cdot Y} = 0 \quad BD - \frac{\sqrt{\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right) \cdot \left(W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2 + X^2 \cdot Y^2 + X^2 \cdot Z^2\right)}}{2 \cdot X^2 \cdot Y \cdot Z} = 0$$

Giving any side of a triangle as unity, then each of the remaing two sides are named as the algebraic names assigned to A and B like such:

$$A := \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z}$$

$$B := \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z}$$

$\delta := 0 .. 2$ The result is independent of the side one starts assignes to unity

$$S_1 := \begin{pmatrix} 1 \\ A \\ B \end{pmatrix} \qquad S_2 := \begin{pmatrix} A \\ B \\ 1 \end{pmatrix} \qquad S_3 := \begin{pmatrix} B \\ 1 \\ A \end{pmatrix}$$

$$R_\delta := \frac{S_{1_\delta} \cdot S_{2_\delta} \cdot S_{3_\delta}}{\sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}$$

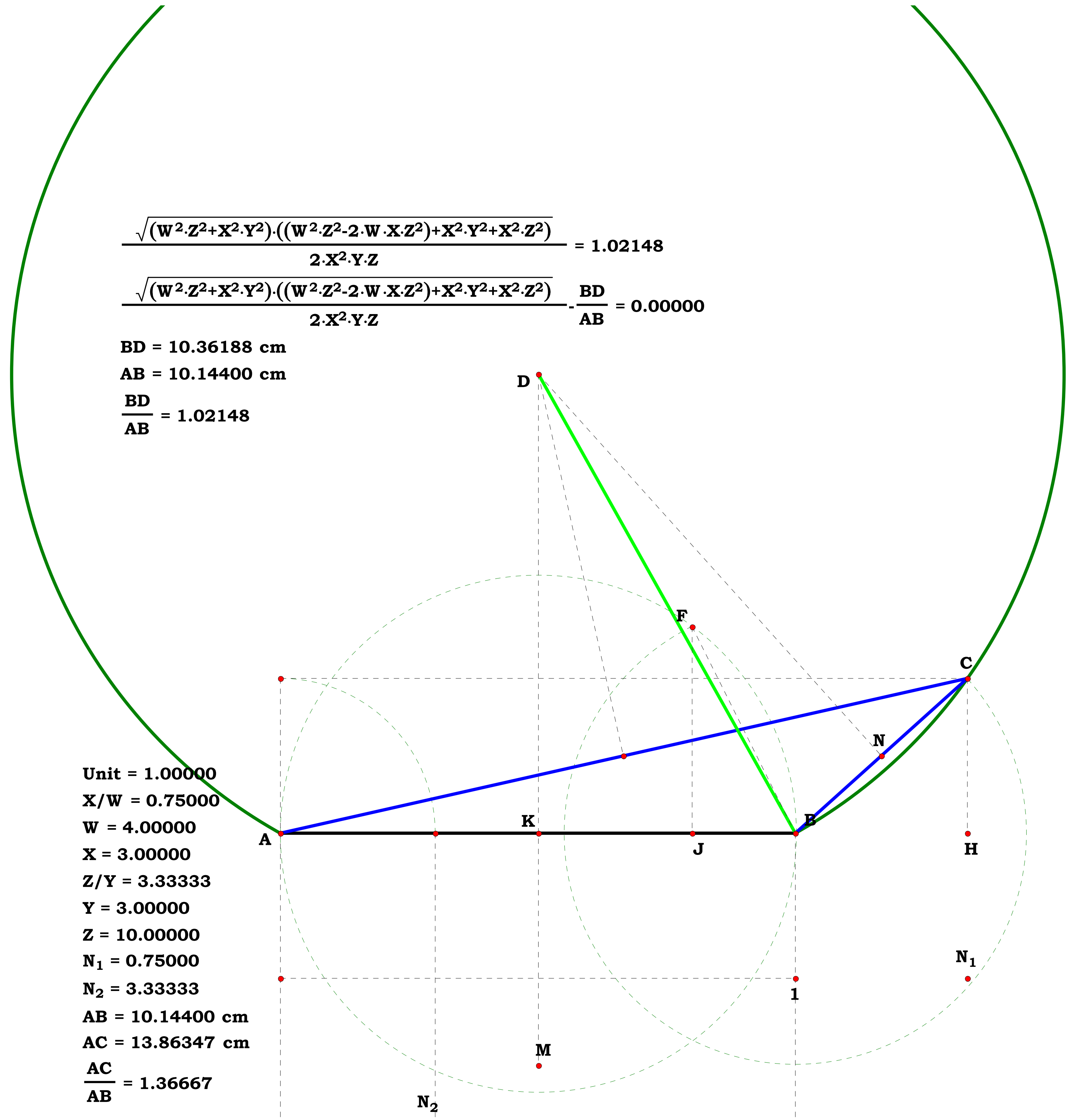
The Arithmetic name of the Radius in terms of the givens.

$R_\delta =$

0.559017
0.559017
0.559017

BD = 0.559017

The only time this process fails in in the Mythological Great Circle, which Shamen (sic) claim is a line. The real significance of the Great Circle is that in looking forward to one's future, these people actually end up where they started. I believe Dodson called it a Caucus Race of which he had plenty of experience. It is a whole lot more flattering to claim we have moved when all we did is stand still. the Couch Potato Philosopher.



071593A

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 3$

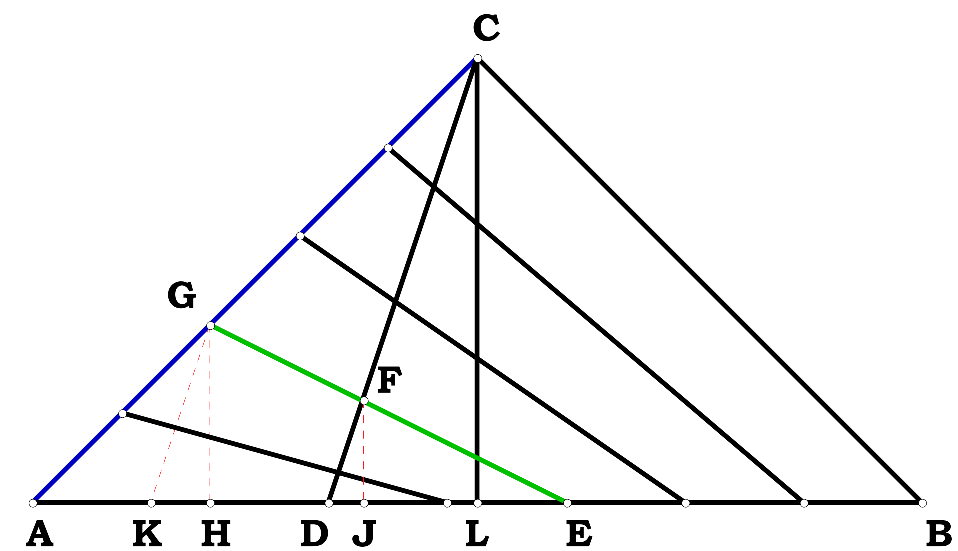
$N_2 := 5$

$\delta := 1 .. N_2$

Pyramid of Ratios II

Back in the day, I used to write shit up like this, but then I forgot just what in the hell I was doing, so, I had to invent other ways to write these up and to understand them.

AB is divided by N_1 and AC and BD is divided by N_2, what are EG/FG and CD/DF?



$AD := \frac{AB}{N_1}$ $\quad AC := \sqrt{\frac{AB^2}{2}}$ $\quad BD := AB - AD$ $\quad DE_\delta := \frac{BD \cdot \delta}{N_2}$ $\quad AG_\delta := \frac{AC \cdot \delta}{N_2}$

$AE_\delta := AD + DE_\delta$ $\quad AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}}$ $\quad GH_\delta := AH_\delta$ $\quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2}$ $\quad AL := \frac{AB}{2}$ $\quad DL := AL - AD$ $\quad CL := \sqrt{\frac{AC^2}{2}}$

$HK_\delta := \frac{DL \cdot GH_\delta}{CL}$ $\quad EK_\delta := EH_\delta + HK_\delta$ $\quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta}$ $\quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2}$ $\quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta}$ $\quad FG_\delta := EG_\delta - EF_\delta$

$CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right) =$

15
4.375
2.222222
1.40625
1

$\text{if}\left[\delta, N_2 \cdot \frac{\left[\left(N_2 + \delta \cdot N_1\right) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot \left(N_1 - 1\right)\right]}, 0\right] =$

15
4.375
2.222222
1.40625
1

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right) =$

1.5
2.333333
4
9
0

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot \left(N_1 - 2\right)}{N_2 - \delta}, 0\right] =$

1.5
2.333333
4
9
0

071593B

Unit.

$AB := 1$

Given.

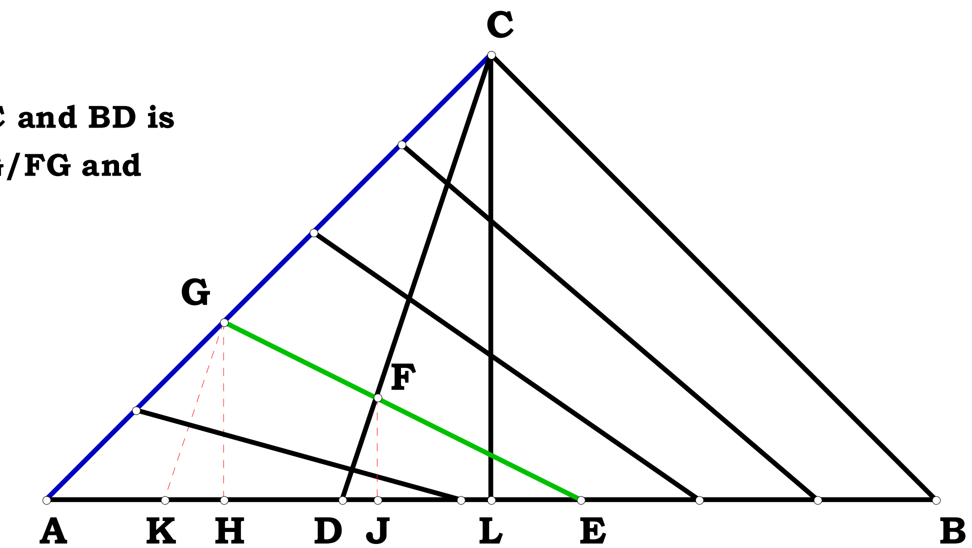
$N_1 := 3$

$N_2 := 5$

Pyramid of Ratios II

When I got to this point, I thought I had the bull gonads, but, I was not satisfied.

AB is divided by N_1 and AC and BD is divided by N_2, what are EG/FG and CD/DF?



Descriptions.

$AD := \frac{AB}{N_1}$ $\quad AC := \sqrt{\frac{AB^2}{2}}$ $\quad BD := AB - AD$ $\quad AL := \frac{AB}{2}$

$DE := \frac{BD}{N_2}$ $\quad AG := \frac{AC}{N_2}$ $\quad AE := AD + DE$ $\quad AH := \sqrt{\frac{AG^2}{2}}$

$GH := AH$ $\quad EH := AE - AH$ $\quad EG := \sqrt{EH^2 + GH^2}$ $\quad DL := AL - AD$

$CL := AL$ $\quad HK := \frac{DL \cdot AH}{AL}$ $\quad EK := EH + HK$

$DJ := \frac{HK \cdot DE}{EK}$ $\quad EF := \frac{EG \cdot DE}{EK}$ $\quad FG := EG - EF$

$FJ := \frac{GH \cdot DE}{EK}$ $\quad CD := \sqrt{CL^2 + DL^2}$ $\quad DF := \sqrt{DJ^2 + FJ^2}$

$\frac{EG}{FG} = 1.5$ $\quad \frac{CD}{DF} = 15$

Definitions.

$\frac{N_2 + N_1 - 2}{N_2 - 1} = 1.5$ $\quad \frac{N_2^{\,2} + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} = 15$

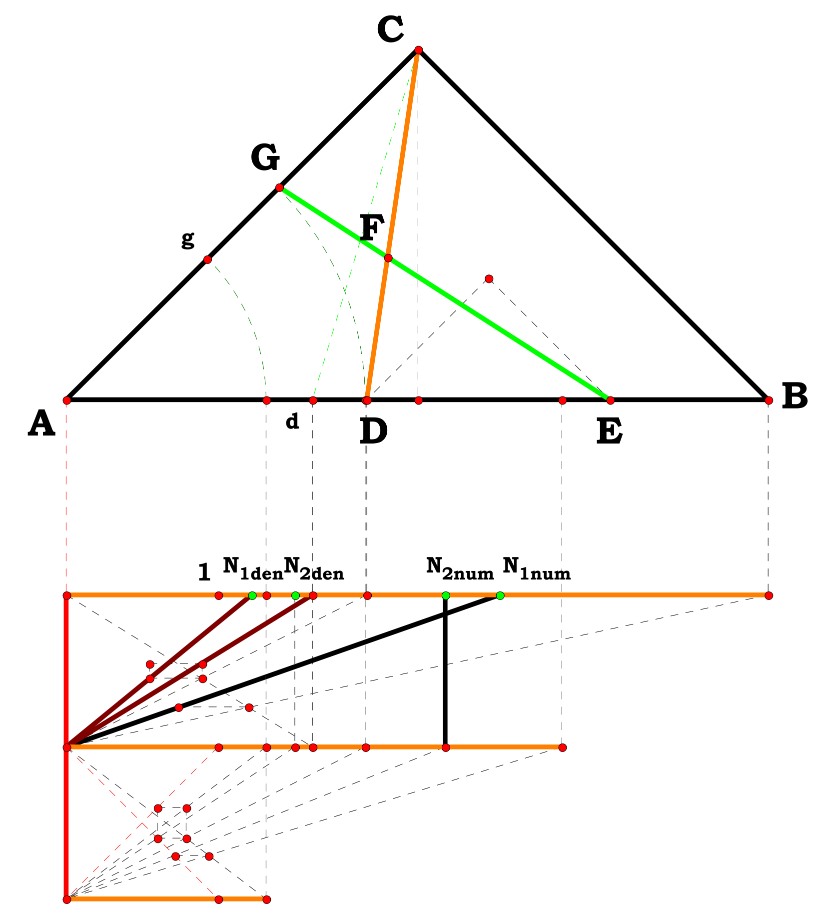
$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0$ $\quad \frac{CD}{DF} - \frac{N_2^{\,2} + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1}$

$N_1 = 2.32900$

$N_2 = 1.65606$

$\frac{N_{1\ num}}{N_{1\ den}} = 2.32900$

$\frac{N_{2\ num}}{N_{2\ dem}} = 1.65606$



$\frac{EG}{FG} = 3.02573$ $\quad \frac{(N_2+N_1)-2}{N_2-1} = 3.02573$ $\quad \frac{EG}{FG} - \frac{(N_2+N_1)-2}{N_2-1} = 0.00000$

$\frac{CD}{DF} = 2.47357$ $\quad \frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 2.47357$ $\quad \frac{CD}{DF} - \frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 0.00000$

071593C

Unit.

$AB := 1$

Given.

$W := 3$

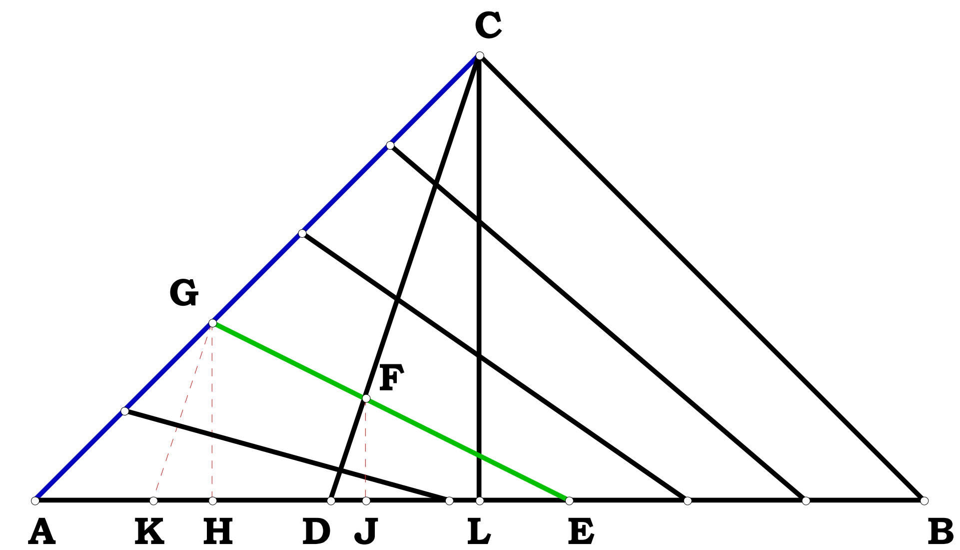
$X := 10$

$Y := 8$

$Z := 20$

Pyramid of Ratios II

What we have, is a way to understand how to write something, which shows a very intimate account of the ratios which, if we stop to think about it, we had set all along. We are learning the results of our own behavior applied to information.



Descriptions.

$$N_1 := \frac{X}{W} \quad N_2 := \frac{Z}{Y} \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$$

$$AL := \frac{AB}{2} \quad DE := \frac{BD}{N_2} \quad AG := \frac{AC}{N_2} \quad AE := AD + DE \quad AH := \sqrt{\frac{AG^2}{2}}$$

$$GH := AH \quad EH := AE - AH \quad EG := \sqrt{EH^2 + GH^2} \quad DL := AL - AD$$

$$CL := AL \quad HK := \frac{DL \cdot AH}{AL} \quad EK := EH + HK \quad DJ := \frac{HK \cdot DE}{EK}$$

$$EF := \frac{EG \cdot DE}{EK} \quad FG := EG - EF \quad FJ := \frac{GH \cdot DE}{EK}$$

$$CD := \sqrt{CL^2 + DL^2} \quad DF := \sqrt{DJ^2 + FJ^2}$$

$$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0 \qquad \frac{CD}{DF} - \left(\frac{N_2^{\,2} + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1}\right) = 0$$

Definitions.

$$N_1 - \frac{X}{W} = 0 \quad N_2 - \frac{Z}{Y} = 0 \quad AD - \frac{W}{X} = 0 \quad AC - \frac{1}{\sqrt{2}} = 0 \quad BD - \frac{(X - W)}{X} = 0$$

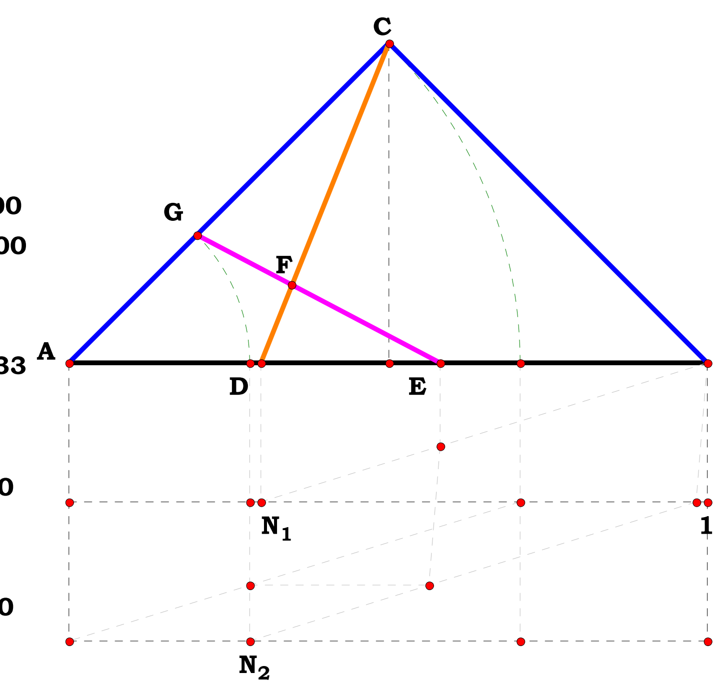
$$AL - \frac{1}{2} = 0 \quad DE - \frac{Y \cdot (X - W)}{X \cdot Z} = 0 \quad AG - \frac{\sqrt{2} \cdot Y}{2 \cdot Z} = 0 \quad AE - \frac{W \cdot (Z - Y) + X \cdot Y}{X \cdot Z} = 0 \quad AH - \frac{Y}{2 \cdot Z} = 0$$

$$GH - \frac{Y}{2 \cdot Z} = 0 \quad EH - \frac{2 \cdot W \cdot (Z - Y) + X \cdot Y}{2 \cdot X \cdot Z} = 0 \quad EG - \frac{\sqrt{Y^2 \cdot X^2 + [2 \cdot W \cdot Y \cdot (Z - Y)] \cdot X + 2 \cdot W^2 \cdot (Y - Z)^2}}{\sqrt{2} \cdot X \cdot Z} = 0$$

$$DL - \frac{(X - 2 \cdot W)}{2 \cdot X} = 0 \quad CL - \frac{1}{2} = 0 \quad HK - \frac{Y \cdot (X - 2 \cdot W)}{2 \cdot X \cdot Z} = 0 \quad EK - \frac{(W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)}{X \cdot Z} = 0$$

Unit = 1.00000
W/X = 0.30000
W = 3.00000
X = 10.00000
X/W = 3.33333
N_1 = 3.33333

Y/Z = 0.40000
Y = 8.00000
Z = 20.00000
Z/Y = 2.50000
N_2 = 2.50000



EG = 3.62887 cm
FG = 1.41999 cm

$\frac{EG}{FG}$ = 2.55556

$\frac{(N_1+N_2)-2}{N_2-1}$ = 2.55556

$\frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{W \cdot (Y-Z)}$ = 2.55556

CD = 4.55082 cm
DF = 1.10803 cm

$\frac{CD}{DF}$ = 4.10714

$\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1}$ = 4.10714

$\frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W-X)}$ = 4.10714

$$DJ - \frac{Y^2\cdot(W-X)\cdot(X-2\cdot W)}{2\cdot X\cdot Z\cdot(2\cdot W\cdot Y - W\cdot Z - X\cdot Y)} = 0$$

$$EF - \frac{Y\cdot(X-W)\cdot\sqrt{2\cdot X^2\cdot Y^2 + 4\cdot W^2\cdot(Y-Z)^2 - 4\cdot W\cdot X\cdot Y\cdot(Y-Z)}}{2\cdot X\cdot Z\cdot(W\cdot Z - 2\cdot W\cdot Y + X\cdot Y)} = 0$$

$$FG - \frac{W\cdot(Z-Y)\cdot\sqrt{2\cdot X^2\cdot Y^2 + 4\cdot W^2\cdot(Y-Z)^2 - 4\cdot W\cdot X\cdot Y\cdot(Y-Z)}}{2\cdot X\cdot Z\cdot(W\cdot Z - 2\cdot W\cdot Y + X\cdot Y)} = 0$$

$$FJ - \frac{Y^2\cdot(X-W)}{2\cdot Z\cdot(W\cdot Z - 2\cdot W\cdot Y + X\cdot Y)} = 0$$

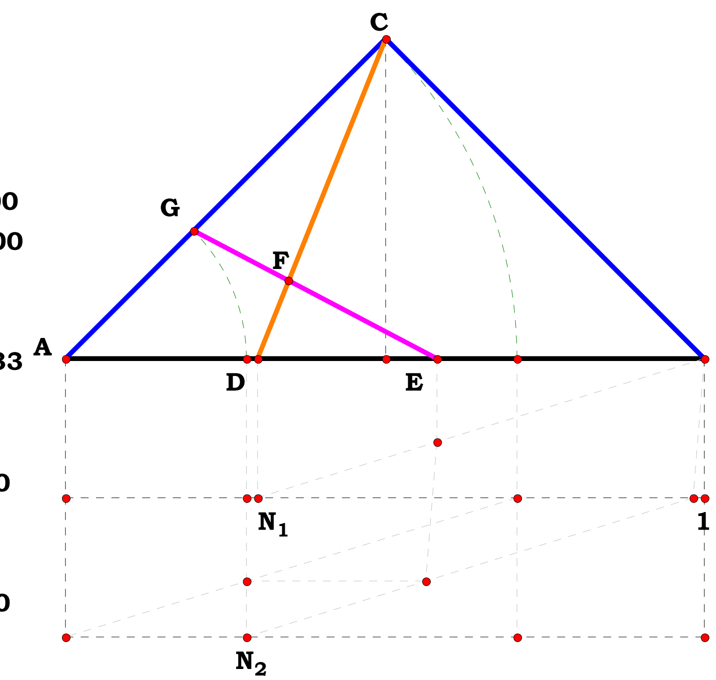
$$CD - \frac{\sqrt{2\cdot W^2 - 2\cdot W\cdot X + X^2}}{\sqrt{2}\cdot X} = 0$$

$$DF - \frac{Y^2\cdot(X-W)\cdot\sqrt{2\cdot W^2 - 2\cdot W\cdot X + X^2}}{\sqrt{2}\cdot X\cdot Z\cdot(W\cdot Z - 2\cdot W\cdot Y + X\cdot Y)} = 0$$

$$\frac{EG}{FG} - \frac{2\cdot W\cdot Y - W\cdot Z - X\cdot Y}{(Y-Z)\cdot W} = 0 \qquad \frac{CD}{DF} - \left[\frac{Z\cdot(2\cdot W\cdot Y - W\cdot Z - X\cdot Y)}{Y^2\cdot(W-X)}\right] = 0$$

Unit = 1.00000
W/X = 0.30000
W = 3.00000
X = 10.00000
X/W = 3.33333
N_1 = 3.33333

Y/Z = 0.40000
Y = 8.00000
Z = 20.00000
Z/Y = 2.50000
N_2 = 2.50000



EG = 3.62887 cm
FG = 1.41999 cm

$\frac{EG}{FG} = 2.55556$

$\frac{(N_1+N_2)-2}{N_2-1} = 2.55556$

$\frac{2\cdot W\cdot Y - W\cdot Z - X\cdot Y}{W\cdot(Y-Z)} = 2.55556$

CD = 4.55082 cm
DF = 1.10803 cm

$\frac{CD}{DF} = 4.10714$

$\frac{(N_2{}^2+N_1\cdot N_2)-2\cdot N_2}{N_1-1} = 4.10714$

$\frac{Z\cdot(2\cdot W\cdot Y - W\cdot Z - X\cdot Y)}{Y^2\cdot(W-X)} = 4.10714$

Unit.

$AB := 1$

Given.

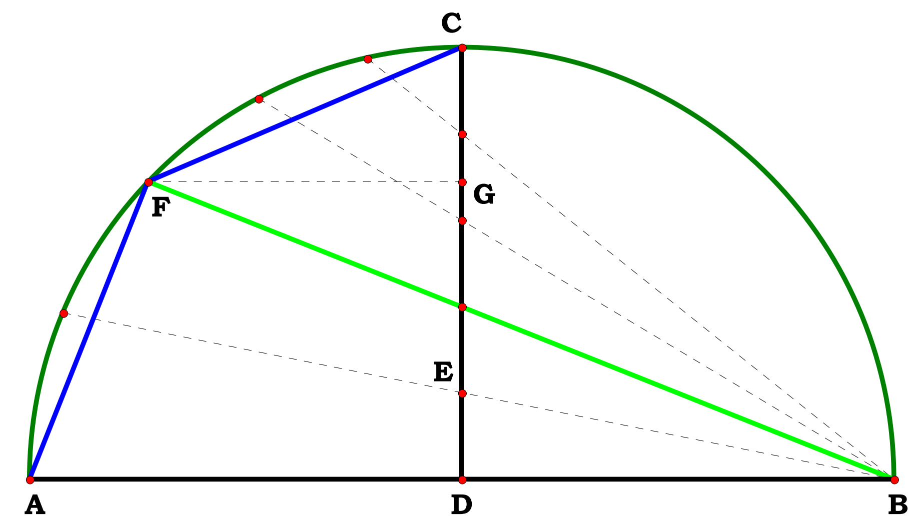
$N := 6$

$\delta := 1 .. N$

Pyramid of Ratios III

Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?

072593A



Descriptions.

$$AD := \frac{AB}{2} \quad BD := AD \quad CD := BD \quad BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N}$$

$$BE_\delta := \sqrt{BD^2 + (DE_\delta)^2} \quad BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$$

$$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2} \quad DG := DE + EG \quad CG_\delta := CD - DG_\delta$$

$$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$$

$\frac{DE_\delta}{DG_\delta} =$	$\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2} =$	$\frac{BE_\delta}{BF_\delta} =$	$\frac{BF_\delta}{BE_\delta} =$	$\frac{2}{\left[\left(\frac{\delta}{N}\right)^2 + 1\right]} =$	$\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right) =$	$\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right) =$
0.514	0.513889	0.513889	1.945946	1.945946	0.283	0.282843
0.556	0.555556	0.555556	1.8	1.8	0.707	0.707107
0.625	0.625	0.625	1.6	1.6	1.414	1.414214
0.722	0.722222	0.722222	1.384615	1.384615	2.828	2.828427
0.847	0.847222	0.847222	1.180328	1.180328	7.071	7.071068
1	1	1	1	1	0	0

072593B

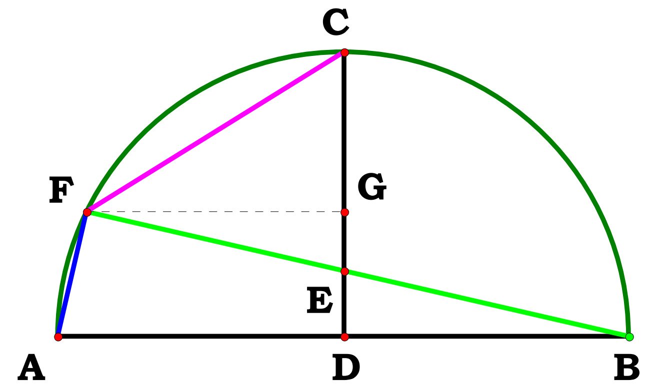
Unit.

$AB := 1$

Given.

$N := 5$

Pyramid of Ratios III



Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?

Descriptions.

$$CD := \frac{AB}{2} \qquad DE := \frac{CD}{N} \qquad BE := \sqrt{CD^2 + DE^2}$$

$$BF := \frac{CD \cdot AB}{BE} \qquad AF := \frac{DE \cdot BF}{CD} \qquad DG := \frac{DE \cdot BF}{BE}$$

$$CG := CD - DG \qquad FG := \frac{CD \cdot (DG - DE)}{DE} \qquad CF := \sqrt{FG^2 + CG^2}$$

Definitions.

$$\frac{BE}{BF} = 0.52 \qquad \frac{N^2 + 1}{2 \cdot N^2} = 0.52$$

$$\frac{AF}{CF} = 0.353553 \qquad \frac{\sqrt{2}}{N - 1} = 0.353553$$

$$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0 \qquad \frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$$

$$\frac{N_{um}}{N_{den}} = 2.99984$$

$N = 2.99984$

$BE = 1.75445$ in.

$BF = 3.15797$ in.

$$\frac{BE}{BF} = 0.55556$$

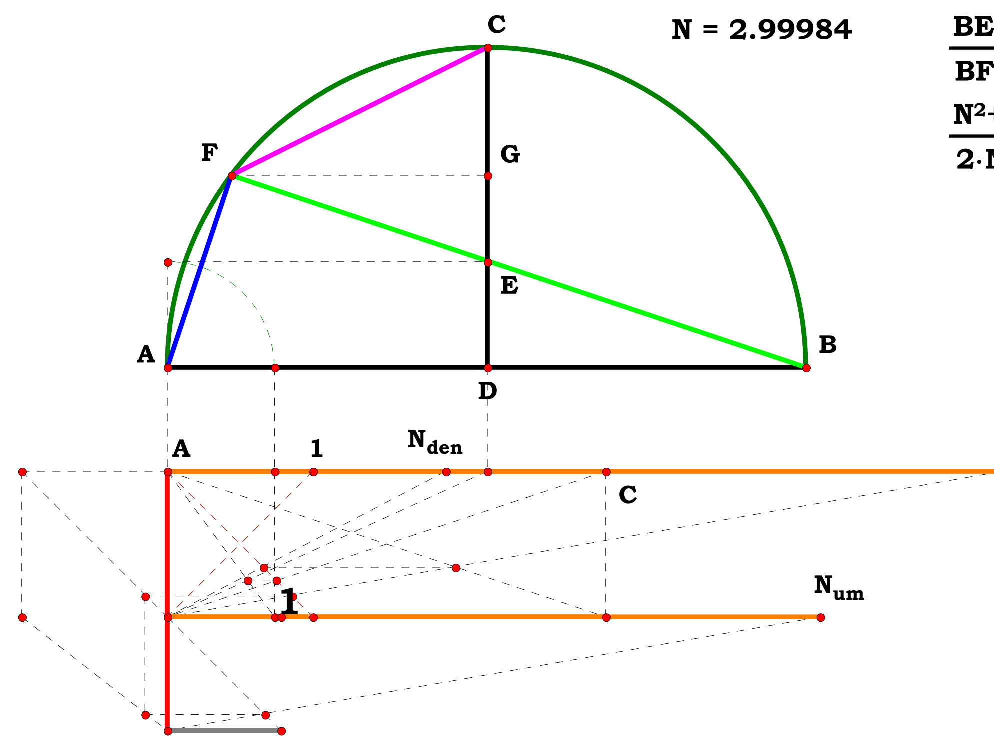
$$\frac{N^2+1}{2 \cdot N^2} - \frac{BE}{BF} = 0.00000$$

$AF = 1.05271$ in.

$CF = 1.48864$ in.

$$\frac{AF}{CF} = 0.70716$$

$$\frac{\sqrt{2}}{N-1} - \frac{AF}{CF} = 0.00000$$



072593C

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 4$

Descriptions.

$$N := \frac{Y}{X} \quad CD := \frac{AB}{2} \quad DE := \frac{CD}{N} \quad BE := \sqrt{CD^2 + DE^2}$$

$$BF := \frac{CD \cdot AB}{BE} \quad AF := \frac{DE \cdot BF}{CD} \quad DG := \frac{DE \cdot BF}{BE}$$

$$CG := CD - DG \quad FG := \frac{CD \cdot (DG - DE)}{DE} \quad CF := \sqrt{FG^2 + CG^2}$$

$$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0 \quad \frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$$

Definitions.

$$N - \frac{Y}{X} = 0 \quad CD - \frac{1}{2} \quad DE - \frac{X}{2 \cdot Y} = 0 \quad BE - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$$

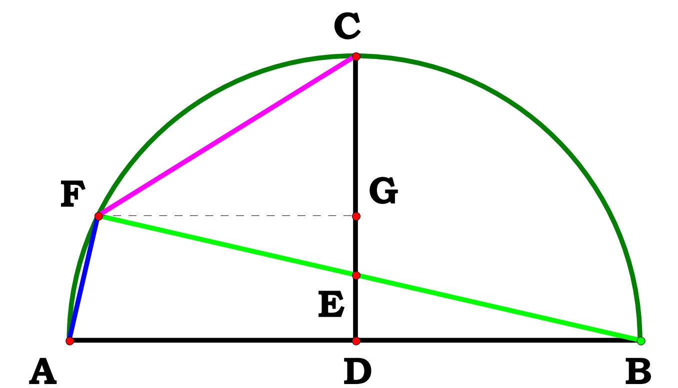
$$BF - \frac{Y}{\sqrt{X^2 + Y^2}} = 0 \quad AF - \frac{X}{\sqrt{X^2 + Y^2}} = 0 \quad DG - \frac{X \cdot Y}{X^2 + Y^2} = 0$$

$$CG - \frac{(X - Y)^2}{2 \cdot \left(X^2 + Y^2\right)} = 0 \quad FG - \frac{(Y - X) \cdot (X + Y)}{2 \cdot \left(X^2 + Y^2\right)} = 0$$

$$CF - \frac{(Y - X)}{\sqrt{2 \cdot \left(X^2 + Y^2\right)}} = 0$$

$$\frac{BE}{BF} - \frac{X^2 + Y^2}{2 \cdot Y^2} = 0 \quad \frac{AF}{CF} - \frac{\sqrt{2} \cdot X}{Y - X} = 0$$

Pyramid of Ratios III



Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?

Unit = 1.00000

X = 4.00000

Y = 20.00000

X/Y = 0.20000

Y/X = 5.00000

N = 5.00000

$\frac{N^2+1}{2 \cdot N^2} = 0.52000$

$\frac{\sqrt{2}}{N-1} = 0.35355$

BE = 4.78390 cm

BF = 9.19981 cm

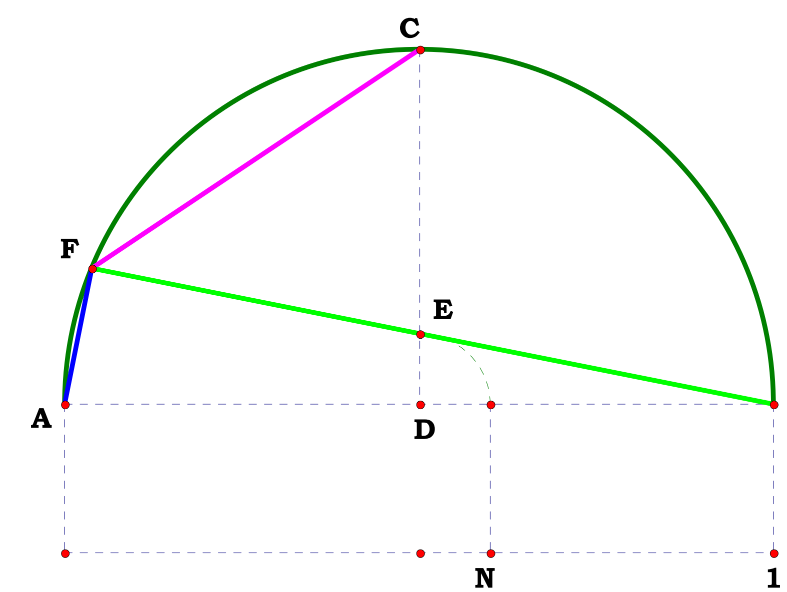
$\frac{BE}{BF} = 0.52000$

AF = 1.83996 cm

CF = 5.20420 cm

$\frac{X^2+Y^2}{2 \cdot Y^2} = 0.52000$

$\frac{\sqrt{2} \cdot X}{Y-X} = 0.35355$



110693A

Gruntwork I on the Delian Solution

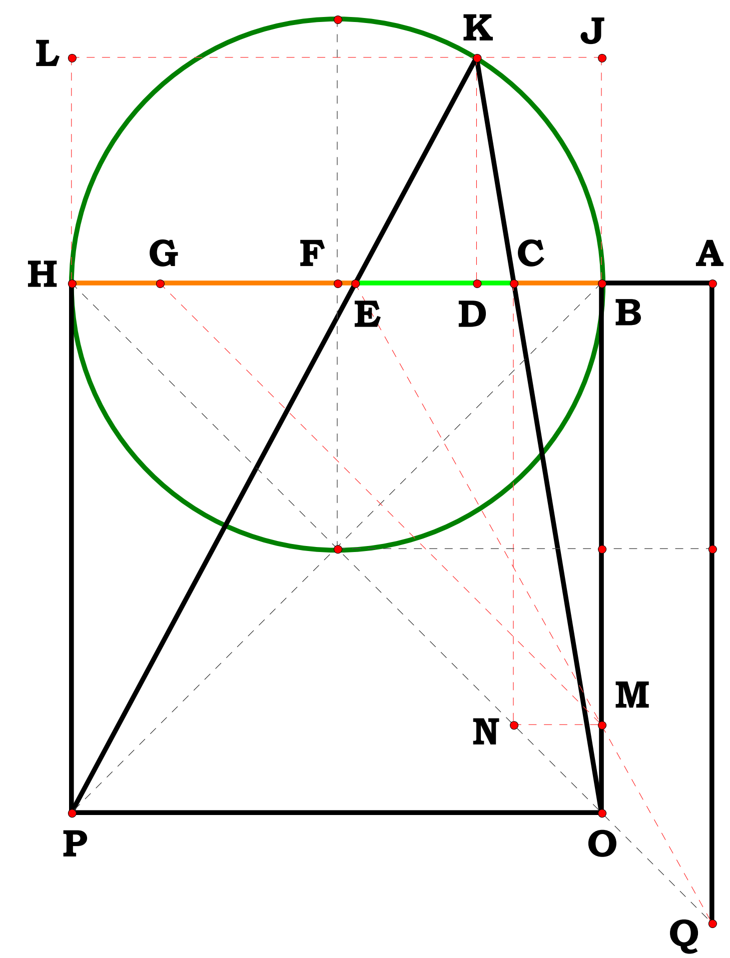
Unit.

$BH := 1$

Given.

$N := 5$

Does $(AB^2 \times AH)^{1/3} = AC$
and $(AB \times AH^2)^{1/3} = AE$?



Descriptions.

$BF := \frac{BH}{2}$ $\quad BD := \frac{BF}{N}$ $\quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $\quad JO := BH + DK$ $\quad BC := \frac{BD \cdot BH}{JO}$

$BG := BH - BC$ $\quad EH := \frac{DH \cdot BH}{JO}$ $\quad BE := BH - EH$ $\quad EG := EH - BC$

$GM := \sqrt{2 \cdot BG^2}$ $\quad HO := \sqrt{2 \cdot BH^2}$ $\quad HQ := \frac{GM \cdot EH}{EG}$ $\quad OQ := HQ - HO$

$AB := \frac{OQ}{\sqrt{2}}$ $\quad AC := AB + BC$ $\quad AE := AB + BE$ $\quad AH := AB + BH$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$$

Definitions.

$BF - \frac{1}{2} = 0$ $\quad BD - \frac{1}{2 \cdot N} = 0$ $\quad DH - \frac{(2 \cdot N - 1)}{2 \cdot N} = 0$ $\quad DK - \frac{\sqrt{2 \cdot N - 1}}{(2 \cdot N)} = 0$

$JO - \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)}{2 \cdot N} = 0$ $\quad BC - \frac{1}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$ $\quad BG - \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$

$EH - \frac{(2 \cdot N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$ $\quad BE - \frac{\left(\sqrt{2 \cdot N - 1} + 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$ $\quad EG - \frac{2 \cdot (N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$

$GM - \sqrt{2} \cdot \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)} = 0$ $\quad HO - \sqrt{2} = 0$

$$HQ - \frac{\sqrt{2}\cdot(2\cdot N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

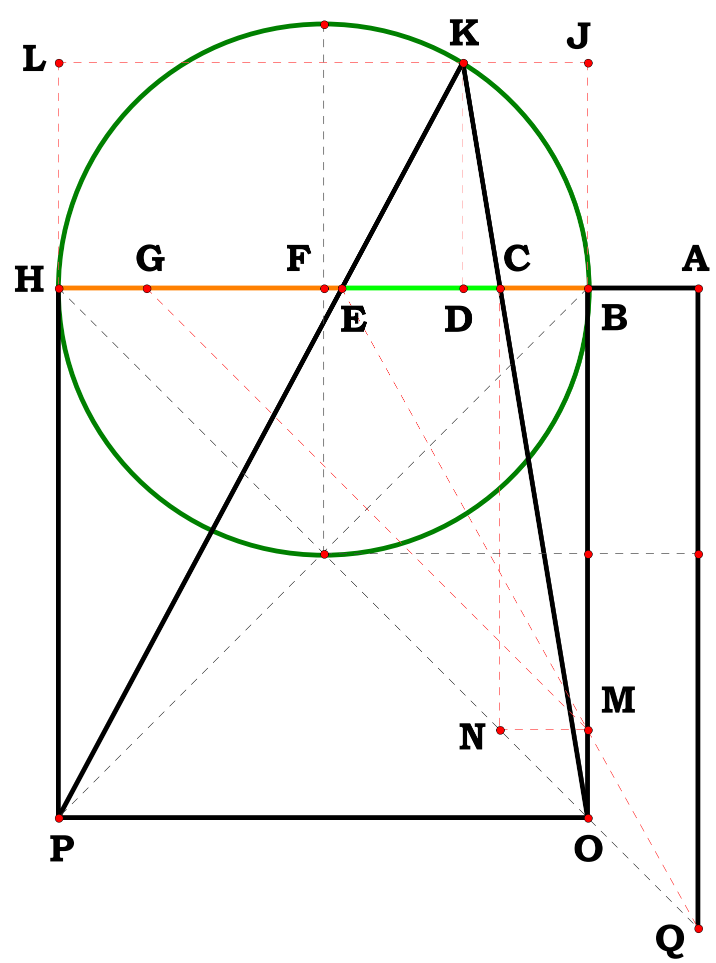
$$OQ - \frac{\sqrt{2}\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{\left(2\cdot N+\sqrt{2\cdot N-1}\right)\cdot(2\cdot N-2)} = 0$$

$$AB - \frac{\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AC - \frac{\left[2\cdot N+2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AE - \frac{\left[2\cdot N+(2\cdot N-1)^{\frac{3}{2}}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AH - \frac{\left[(2\cdot N-1)^{\frac{3}{2}}-4\cdot N+4\cdot N^2+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$



BF = 1.37917 in.

BD = 0.65310 in.

$\frac{BF}{BD} = 2.11173$

N = 2.11173

BH = 2.75833 in.

AE = 1.85725 in.

$$\frac{BH\cdot\left(\left(2\cdot N+(2\cdot N-1)^{\frac{3}{2}}\right)-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} - AE = 0.00000\text{ in.}$$

110693B

Unit.

$BH := 1$

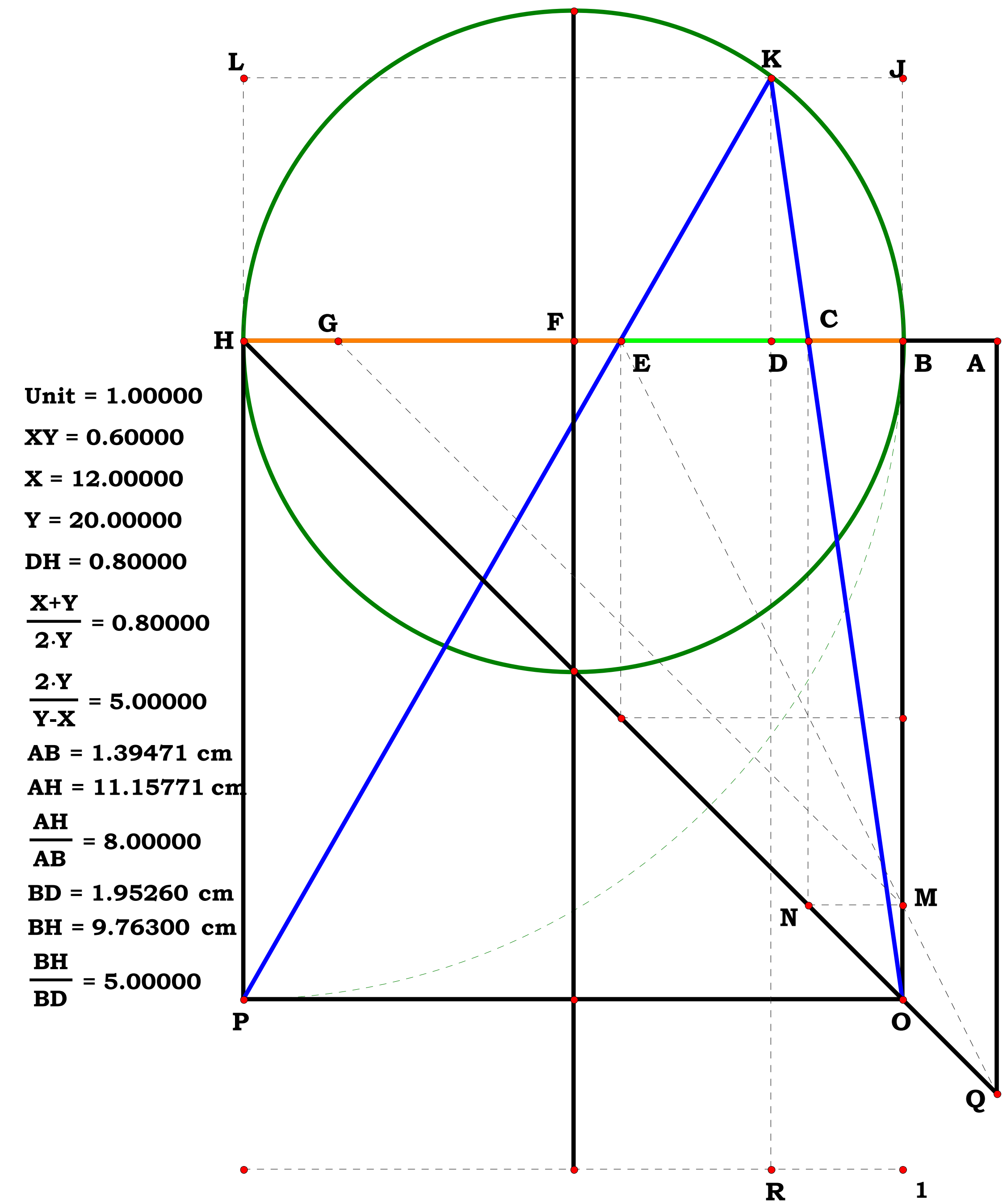
Given.

$Y := 20$

$X := 13$

Gruntwork I on the Delian Solution

Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/3} = AE$?



Descriptions.

$BF := \frac{BH}{2} \quad BD := \frac{Y-X}{2 \cdot Y} \quad DH := BH - BD \quad DK := \sqrt{BD \cdot DH} \quad JO := BH + DK$

$BC := \frac{BD \cdot BH}{JO} \quad BG := BH - BC \quad EH := \frac{DH \cdot BH}{JO} \quad BE := BH - EH \quad EG := EH - BC$

$GM := \sqrt{2 \cdot BG^2} \quad HO := \sqrt{2 \cdot BH^2} \quad HQ := \frac{GM \cdot EH}{EG} \quad OQ := HQ - HO$

$AB := \frac{OQ}{\sqrt{2}} \quad AC := AB + BC \quad AE := AB + BE \quad AH := AB + BH$

$\frac{BF}{BD} = 2.857143 \quad \frac{AH}{AB} = 10.235849 \quad \left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$

Definitions.

$BF - \frac{1}{2} = 0 \quad BD - \frac{Y-X}{2 \cdot Y} = 0 \quad DH - \frac{X+Y}{2 \cdot Y} = 0 \quad DK - \frac{\sqrt{Y^2 - X^2}}{2 \cdot Y} = 0$

$JO - \frac{2 \cdot Y + \sqrt{Y^2 - X^2}}{2 \cdot Y} = 0 \quad BC - \frac{Y-X}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0 \quad BG - \frac{X + Y + \sqrt{Y^2 - X^2}}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$

$EH - \frac{X+Y}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0 \quad BE - \frac{(X+Y) \cdot \sqrt{Y^2 - X^2} + (X-Y)^2}{X^2 + 3 \cdot Y^2} = 0 \quad EG - \frac{2 \cdot X}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$

$GM - \frac{\sqrt{2} \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{\left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad HO - \sqrt{2} = 0 \quad HQ - \frac{\sqrt{2} \cdot (X+Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad OQ - \frac{(X-Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right) \cdot \sqrt{2}}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$

$AB - \frac{(X-Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AC - \frac{(Y-X) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AE - \frac{(X+Y) \cdot \left(\sqrt{Y^2 - X^2} + Y - X\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AH - \frac{(X+Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$

Unit.

$BH := 1$

Given.

$N_1 := 5$

110993A

Descriptions.

$BG := \dfrac{BH}{2}$ $\quad CF := \dfrac{BH}{N_1}$ $\quad BL := CF$ $\quad GP := BG$

$BK := \dfrac{BL}{2}$ $\quad BD := BK$ $\quad NP := BD$ $\quad GN := GP - NP$ $\quad EN := BL$

$GE := \sqrt{GN^2 - EN^2}$ $\quad CE := BD$ $\quad BC := BG - (GE + CE)$

$GH := BG$ $\quad EF := BD$ $\quad FH := GH + GE - EF$ $\quad FQ := FH$

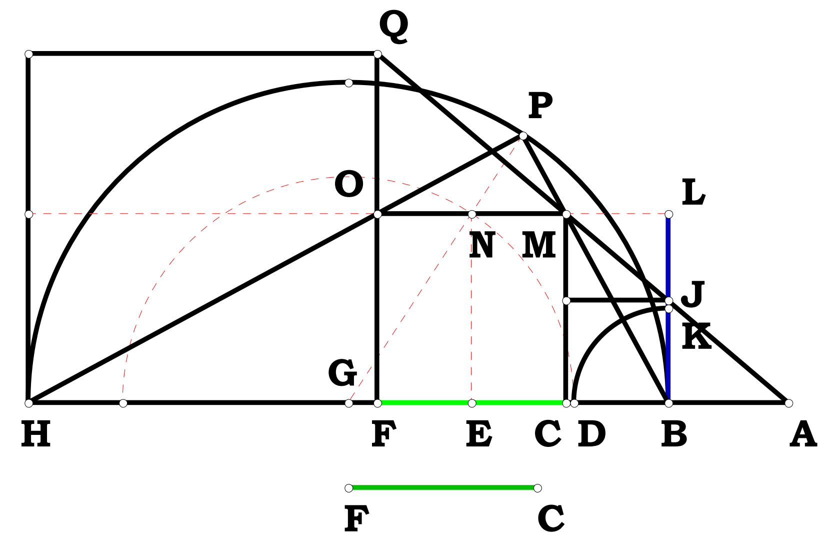
$FO := BL$ $\quad OQ := FQ - MO$ $\quad MO := CF$ $\quad AF := \dfrac{MO \cdot FQ}{OQ}$ $\quad AC := AF - CF$

$AH := AF + FH$ $\quad AB := AH - BH$

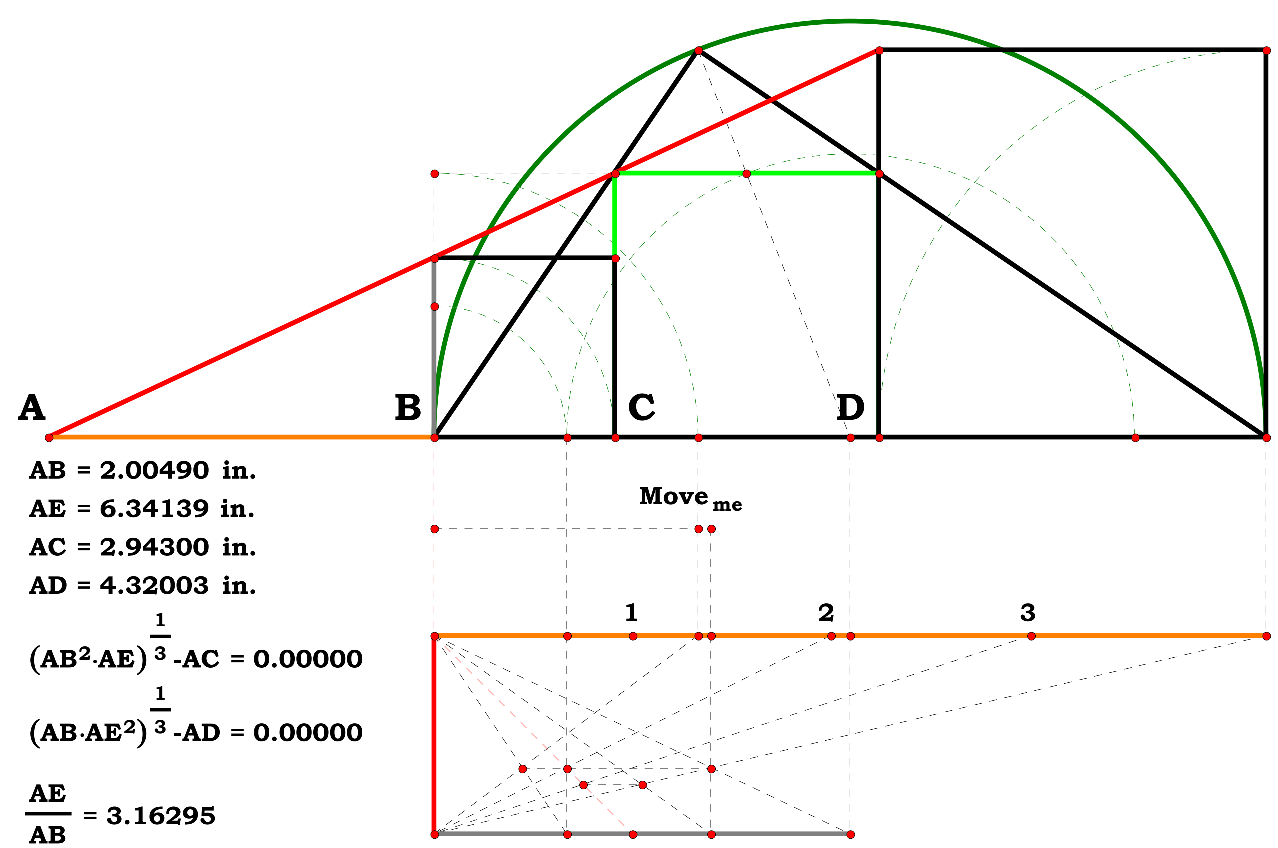
$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 51.980762113532$$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

Solve For Cube Root Placement



With straight edge and compass only, solve the given problem.
BH is the difference between the segments AH and AB.
CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



Definitions.

$$BH - 1 = 0 \qquad BG - \frac{1}{2} = 0 \qquad CF - \frac{1}{N_1} = 0 \qquad BK - \frac{1}{2 \cdot N_1} = 0 \qquad GN - \frac{N_1 - 1}{2 \cdot N_1} = 0$$

$$GE - \frac{\sqrt{(N_1+1)\cdot(N_1-3)}}{2 \cdot N_1} = 0 \qquad BC - \frac{N_1 - \sqrt{(N_1+1)\cdot(N_1-3)} - 1}{2 \cdot N_1} = 0$$

$$FH - \frac{N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 1}{2 \cdot N_1} = 0$$

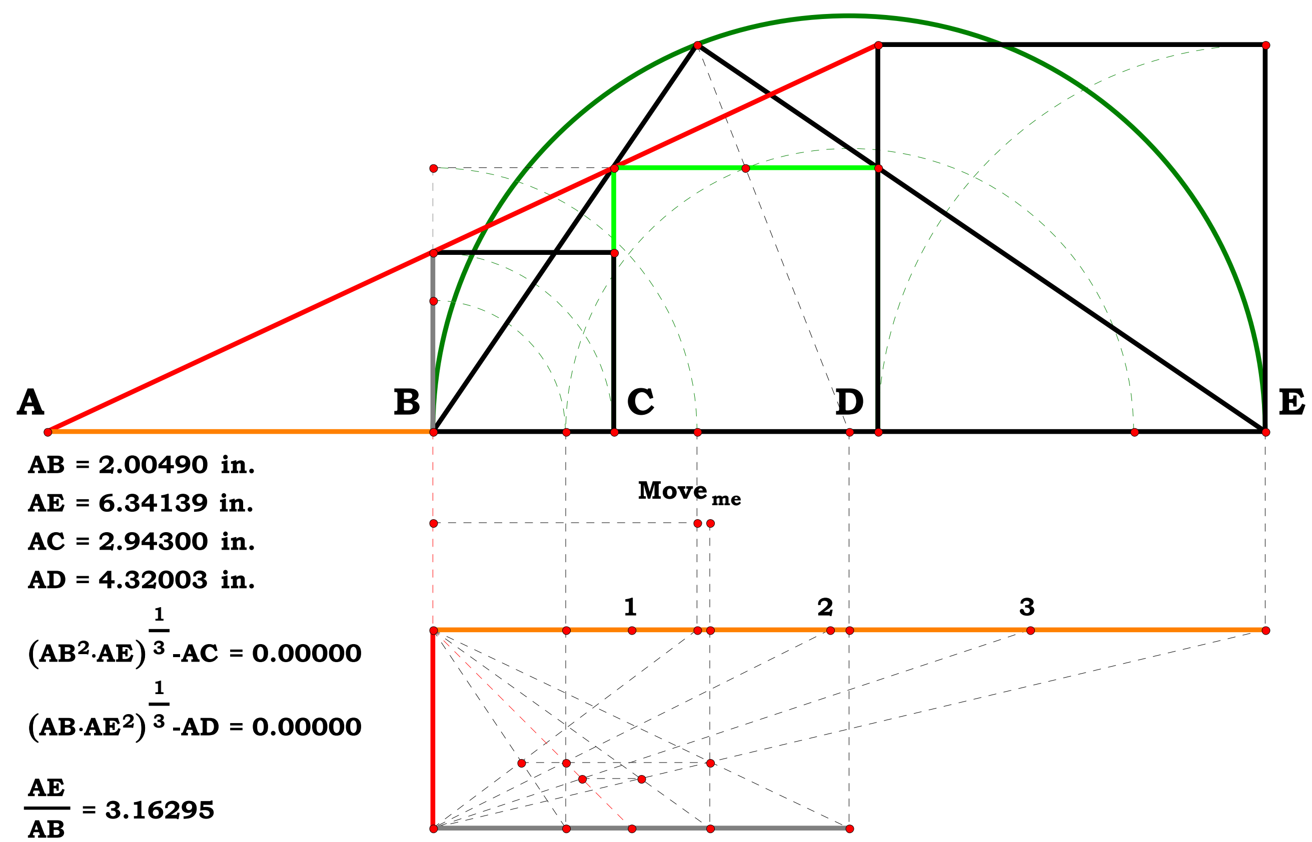
$$OQ - \frac{N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 3}{2 \cdot N_1} = 0$$

$$AF - \frac{N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 1}{N_1 \cdot \left[N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 3\right]} = 0$$

$$AC - \frac{2}{N_1 \cdot \left[N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 3\right]} = 0$$

$$AH - \frac{\left[N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 1\right]^2}{2 \cdot N_1 \cdot \left[N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 3\right]} = 0$$

$$AB - \frac{N_1 - \sqrt{(N_1+1)\cdot(N_1-3)} - 1}{N_1 \cdot \left[N_1 + \sqrt{(N_1+1)\cdot(N_1-3)} - 3\right]} = 0$$



Unit.

$BG := 1$

Given.

$Y := 20$

$X := 15$

110993B

Descriptions.

$BH := 2 \cdot BG \quad CF := \dfrac{2 \cdot X}{3 \cdot Y} \quad BL := CF \quad GP := BG$

$BK := \dfrac{BL}{2} \quad BD := BK \quad NP := BD \quad GN := GP - NP \quad EN := BL$

$GE := \sqrt{GN^2 - EN^2} \quad CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD \quad FH := GH + GE - EF \quad FQ := FH$

$FO := BL \quad OQ := FQ - \rbrack MO := CF \quad AF := \dfrac{MO \cdot FQ}{OQ} \quad AC := AF - CF$

$AH := AF + FH \quad AB := AH - BH$

$CH := CF + FH$

Arithmetic Names:

$CF = 0.5$

$FH = 1.309017$

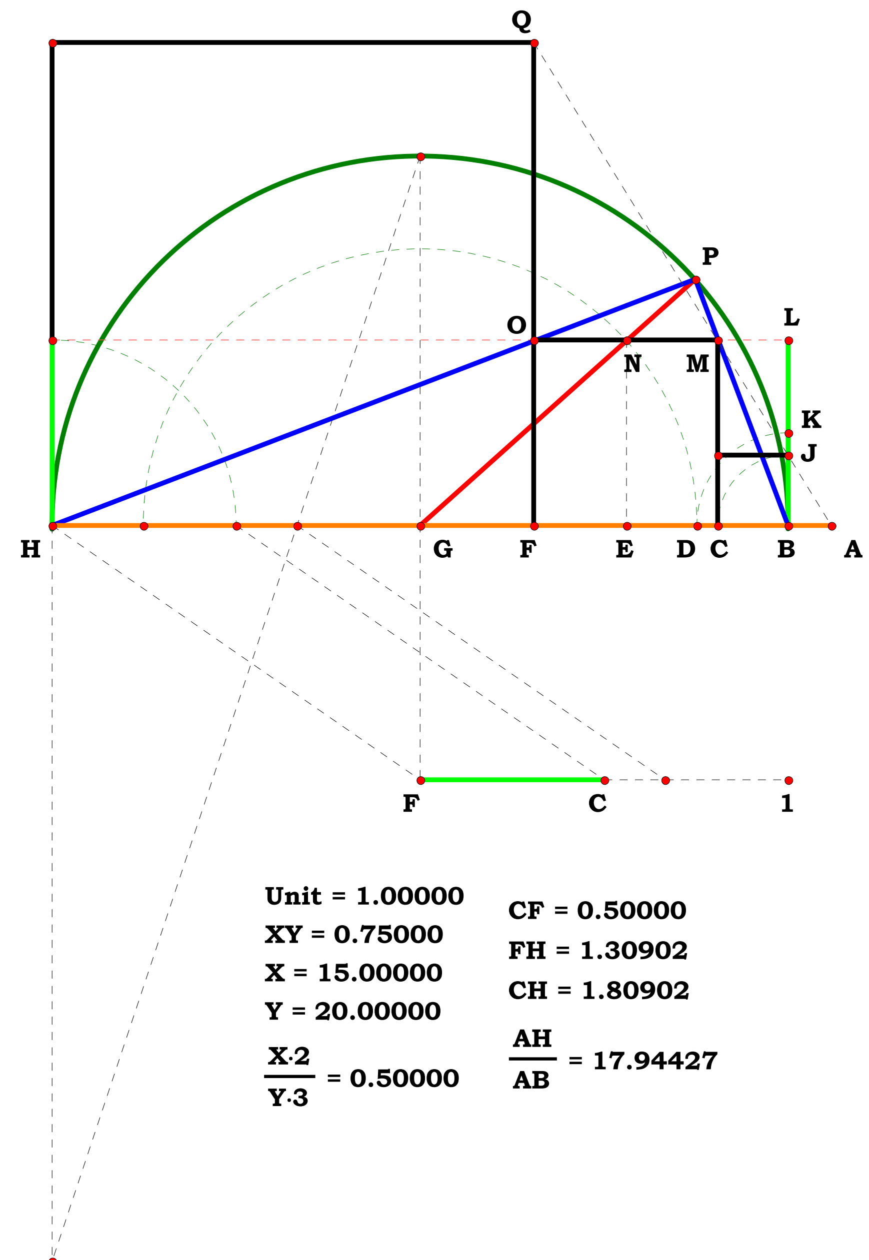
$CH = 1.809017$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \quad \dfrac{AH}{AB} = 17.944271909999$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

Solve For Cube Root Placement

With straight edge and compass only, solve the given problem. BH is the difference between the segments AH and AB. CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



Unit = 1.00000

XY = 0.75000

X = 15.00000

Y = 20.00000

$\dfrac{X \cdot 2}{Y \cdot 3} = 0.50000$

CF = 0.50000

FH = 1.30902

CH = 1.80902

$\dfrac{AH}{AB} = 17.94427$

Definitions.

$$BH-2=0 \quad CF-\frac{2\cdot X}{3\cdot Y}=0 \quad BL-\frac{2\cdot X}{3\cdot Y}=0 \quad GP-1=0$$

$$BK-\frac{X}{3\cdot Y}=0 \quad BD-\frac{X}{3\cdot Y}=0 \quad NP-\frac{X}{3\cdot Y}=0 \quad GN-\frac{3\cdot Y-X}{3\cdot Y}=0$$

$$EN-\frac{2\cdot X}{3\cdot Y}=0 \quad GE-\frac{\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}}{\sqrt{3}\cdot Y}=0 \quad CE-\frac{X}{3\cdot Y}=0$$

$$BC-\frac{3\cdot Y-X-\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}}{3\cdot Y}=0 \quad GH-1=0$$

$$EF-\frac{X}{3\cdot Y}=0 \quad FH-\frac{3\cdot Y-X+\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}}{3\cdot Y}=0$$

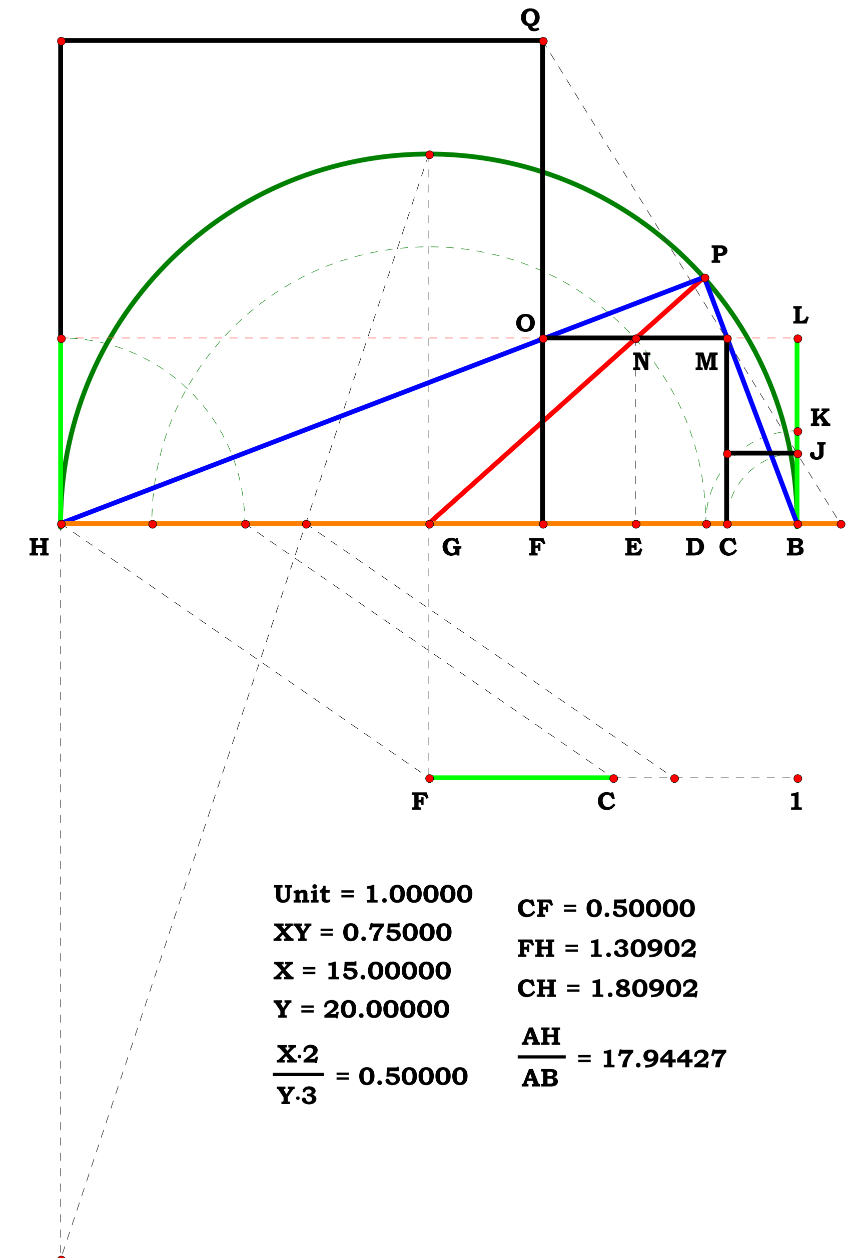
$$CH-\frac{X+3\cdot Y+\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}}{3\cdot Y}=0 \quad FQ-\frac{3\cdot Y-X+\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}}{3\cdot Y}=0$$

$$FO-\frac{2\cdot X}{3\cdot Y}=0 \quad OQ-\frac{3\cdot(Y-X)+\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}}{3\cdot Y}=0 \quad MO-\frac{2\cdot X}{3\cdot Y}=0$$

$$AF-\frac{2\cdot X\cdot\left(3\cdot Y-X+\sqrt{9\cdot Y^2-6\cdot X\cdot Y-3\cdot X^2}\right)}{3\cdot Y\cdot\left(3\cdot Y-3\cdot X+\sqrt{9\cdot Y^2-6\cdot X\cdot Y-3\cdot X^2}\right)}=0 \quad AC-\frac{4\cdot X^2}{3\cdot Y\cdot\left(3\cdot Y-3\cdot X+\sqrt{9\cdot Y^2-6\cdot X\cdot Y-3\cdot X^2}\right)}=0$$

$$AH-\frac{18\cdot Y^2-12\cdot X\cdot Y-2\cdot X^2-2\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}\cdot(X-3\cdot Y)\cdot\sqrt{3}}{3\cdot Y\cdot\left(3\cdot Y-3\cdot X+\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}\right)}=0 \quad AB-\frac{2\cdot X\cdot\left(3\cdot Y-X-\sqrt{3}\cdot\sqrt{3\cdot Y^2-2\cdot X\cdot Y-X^2}\right)}{3\cdot Y\cdot\left(3\cdot Y-3\cdot X+\sqrt{9\cdot Y^2-6\cdot X\cdot Y-3\cdot X^2}\right)}=0$$

$$\frac{AH}{AB}=17.944272$$



Unit = 1.00000
XY = 0.75000
X = 15.00000
Y = 20.00000
$\frac{X\cdot 2}{Y\cdot 3}$ = 0.50000

CF = 0.50000
FH = 1.30902
CH = 1.80902
$\frac{AH}{AB}$ = 17.94427

111093

Unit.

$AE := 1$

Given.

$N := 4$

Gruntwork II on the Delian Solution

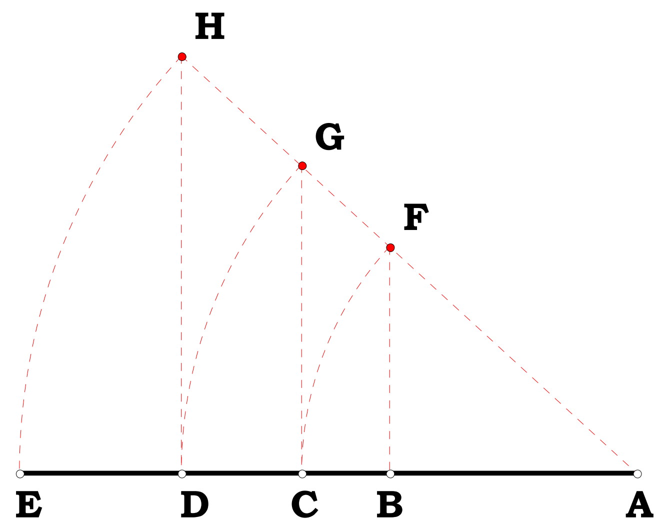
Descriptions.

$DE := \frac{AE}{N}$ $\quad AD := AE - DE$

$AH := AE$ $\quad AG := AD$

$AC := \frac{AD \cdot AD}{AE}$ $\quad AF := AC$

$AB := \frac{AC \cdot AC}{AD}$



Definitions.

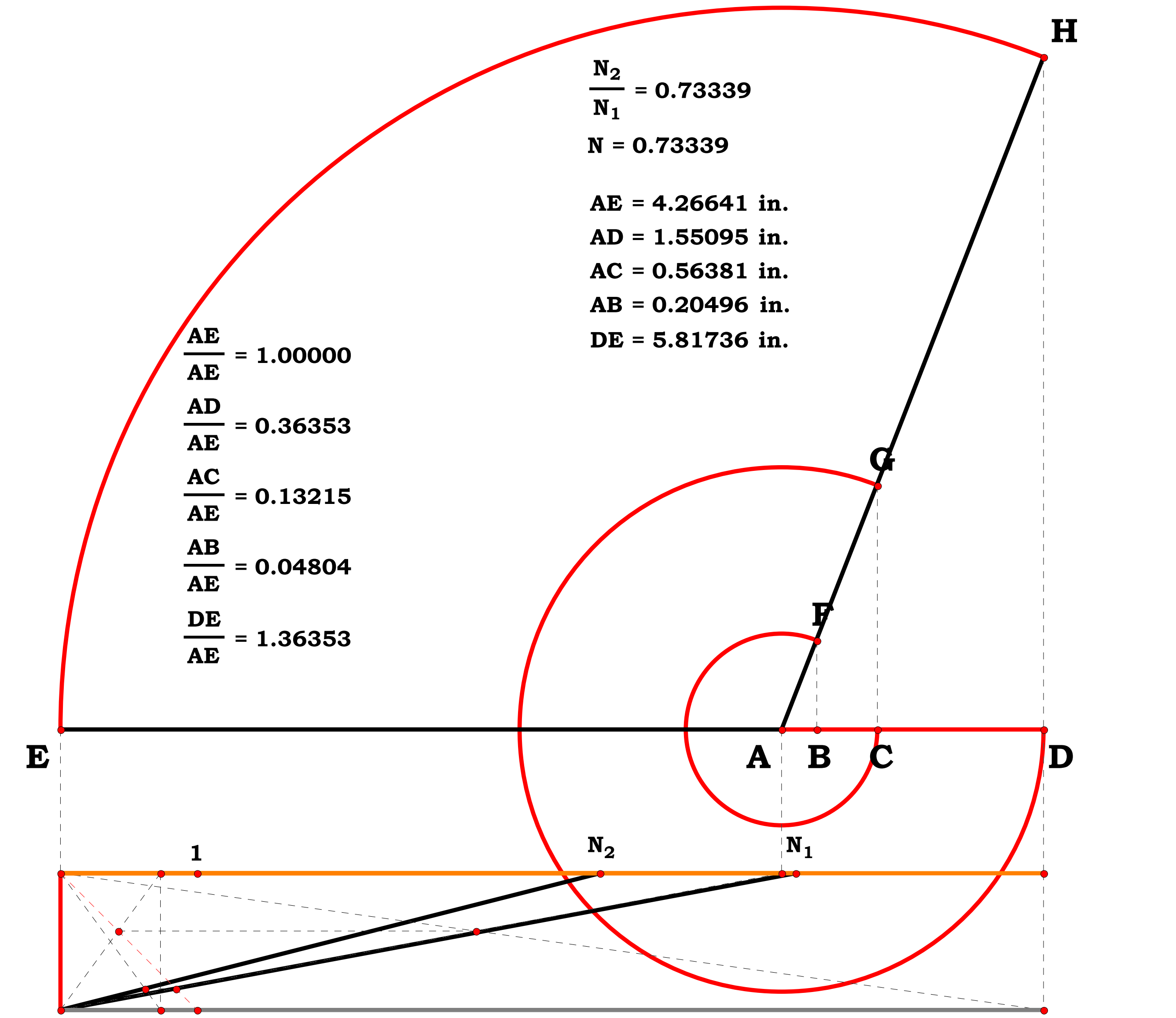
$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\qquad \left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2.37$ $\qquad \frac{AD}{AB} = 1.777778$ $\quad \frac{AC}{AB} = 1.333333$

Albebraic Names:

$\frac{1}{N} - DE = 0$ $\qquad 1 - \frac{1}{N} - AD = 0$ $\qquad \frac{(N-1)^2}{N^2} - AC = 0$

$\frac{(N-1)^3}{N^3} - AB = 0$



$\sqrt{\frac{1}{N}^2} = 1.36353$ $\qquad \sqrt{\left(1-\frac{1}{N}\right)^2} = 0.36353$ $\qquad \frac{(N-1)^2}{N^2} = 0.13215$ $\qquad \sqrt{\frac{(N-1)^{3\,2}}{N^3}} = 0.04804$

$\sqrt{\frac{1}{N}^2} - \frac{DE}{AE} = 0.00000$ $\qquad \sqrt{\left(1-\frac{1}{N}\right)^2} - \frac{AD}{AE} = 0.00000$ $\qquad \frac{(N-1)^2}{N^2} - \frac{AC}{AE} = 0.00000$ $\qquad \sqrt{\frac{(N-1)^{3\,2}}{N^3}} - \frac{AB}{AE} = 0.00000$

Unit.

$AB := 1$

Given.

$N := 3$

111193A

Descriptions.

The Archimedean Paper Trisector

When I looked up the Archimedean Paper Trisector, which is all I found. I did not find where anyone had bothered to complete the figure, for it was obvious to me that the figure was simply not complete. The first task then in writing up the figure is to simply complete the figure.

Once one understands that the angle on the center is twice the angle from the circumference one can then start to work filling in the figure to include the APT. One can see, not only here, but in other figures that trisection is involved with the right triangle and square roots.

$AD := \frac{AB}{N}$ $\quad BD := AB - AD$ $\quad CD := \sqrt{AD \cdot BD}$ $\quad BC := \sqrt{CD^2 + BD^2}$

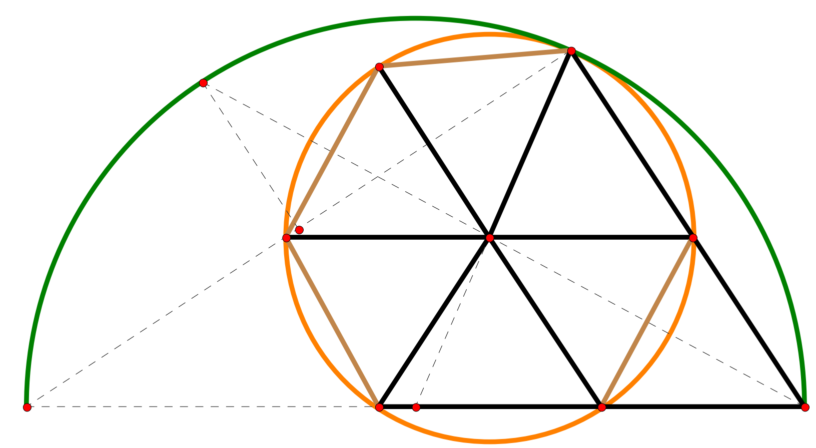
$AC := \sqrt{AD^2 + CD^2}$ $\quad BH := \frac{BC}{2}$ $\quad AO := \frac{AB}{2}$ $\quad HO := \frac{AC}{2}$ $\quad HL := \frac{CD}{2}$ $\quad LO := \frac{AD}{2}$

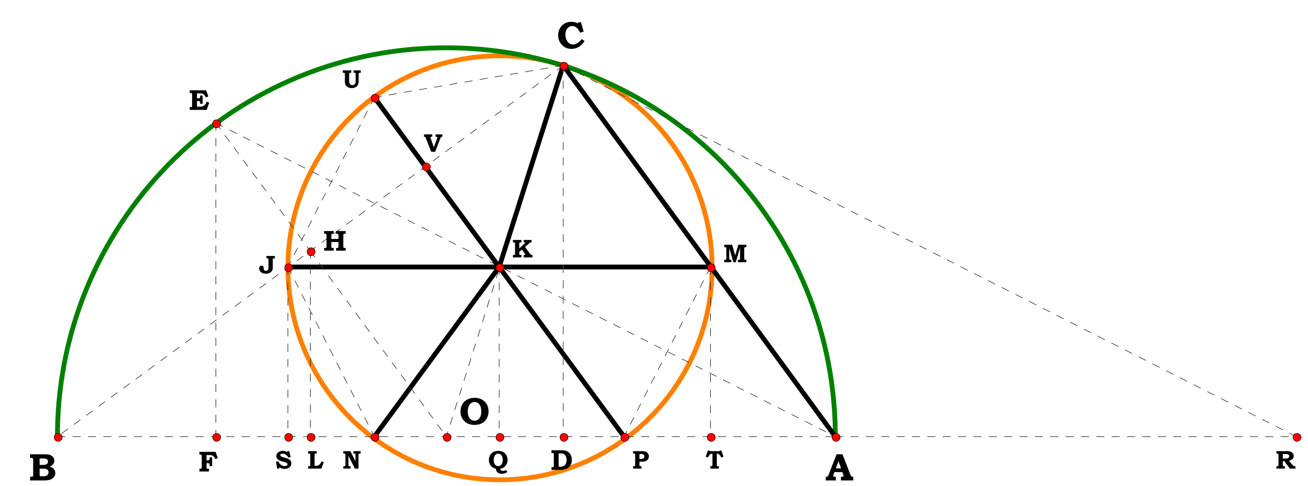
$OF := \frac{LO \cdot AO}{HO}$ $\quad AF := AO + OF$ $\quad BF := AB - AF$ $\quad EF := \sqrt{BF \cdot AF}$ $\quad DR := \frac{AF \cdot CD}{EF}$

$DO := AO - AD$ $\quad OR := DR + DO$ $\quad KQ := \frac{CD \cdot AO}{OR}$ $\quad OK := \frac{AO \cdot KQ}{CD}$

$CK := AO - OK$ $\quad QP := \sqrt{CK^2 - KQ^2}$ $\quad OQ := \frac{DO \cdot KQ}{CD}$ $\quad EH := AO - HO$

$AP := AO - (OQ + QP)$ $\quad AP - CK = 0$





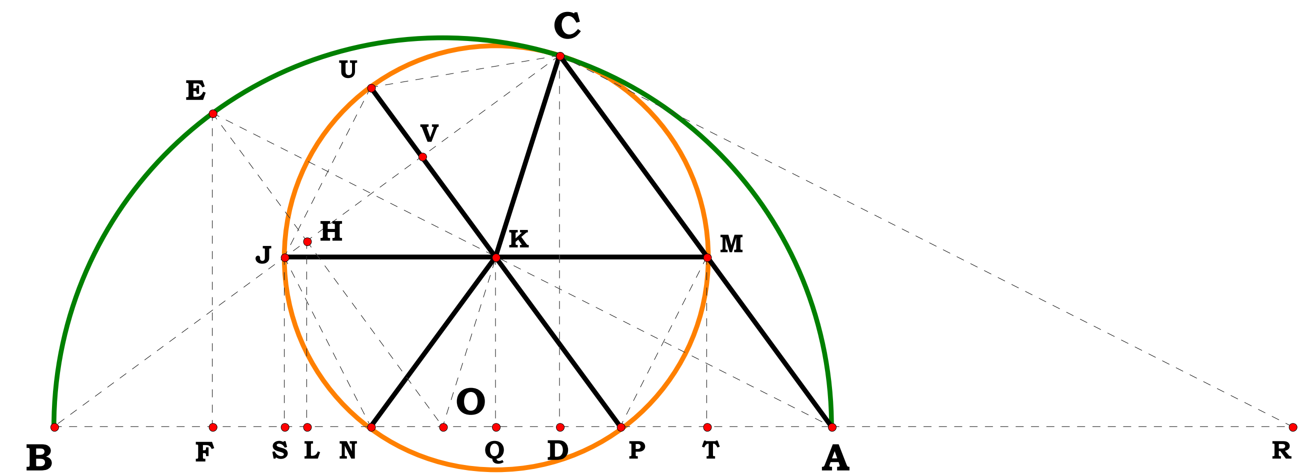
$$CJ := \frac{BC \cdot CK}{AO} \qquad AQ := AO - OQ \qquad SO := CK - OQ \qquad BS := AO - SO$$

$$BJ := BC - CJ \qquad JS := \sqrt{BJ^2 - BS^2} \qquad JS - KQ = 0 \qquad SN := SO + OQ - QP$$

$$JN := \sqrt{JS^2 + SN^2} \qquad UV := \frac{EH \cdot CJ}{BC} \qquad JV := \frac{CJ}{2} \qquad JU := \sqrt{JV^2 + UV^2} \qquad CU := JU$$

$$AT := \frac{AD \cdot CK}{AC} \qquad PT := AP - AT \qquad MT := \frac{CD \cdot AP}{AC} \qquad MP := \sqrt{PT^2 + MT^2}$$

$$MP - JN = 0 \qquad MP - JU = 0 \qquad MP - CU = 0$$



Definitions.

$$AD - \frac{1}{N} = 0 \qquad BD - \frac{N-1}{N} = 0 \qquad CD - \frac{\sqrt{N-1}}{N} = 0 \qquad BC - \frac{\sqrt{N-1}}{\sqrt{N}} = 0 \qquad AC - \frac{1}{\sqrt{N}} = 0 \qquad BH - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0$$

$$AO - \frac{1}{2} = 0 \qquad HO - \frac{1}{2 \cdot \sqrt{N}} = 0 \qquad HL - \frac{\sqrt{N-1}}{2 \cdot N} = 0 \qquad LO - \frac{1}{2 \cdot N} = 0 \qquad OF - \frac{1}{2 \cdot \sqrt{N}} = 0 \qquad AF - \frac{\sqrt{N}+1}{2 \cdot \sqrt{N}} = 0 \qquad BF - \frac{\sqrt{N}-1}{2 \cdot \sqrt{N}} = 0$$

$$EF - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0 \qquad DR - \frac{\sqrt{N-1} \cdot \left(\sqrt{N}+1\right)}{N \cdot \sqrt{\left(\sqrt{N}-1\right) \cdot \left(\sqrt{N}+1\right)}} = 0 \qquad DO - \frac{\sqrt{(N-2)^2}}{2 \cdot N} = 0 \qquad OR - \frac{\sqrt{N}+2}{2 \cdot \sqrt{N}} = 0 \qquad KQ - \frac{\sqrt{N-1}}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0$$

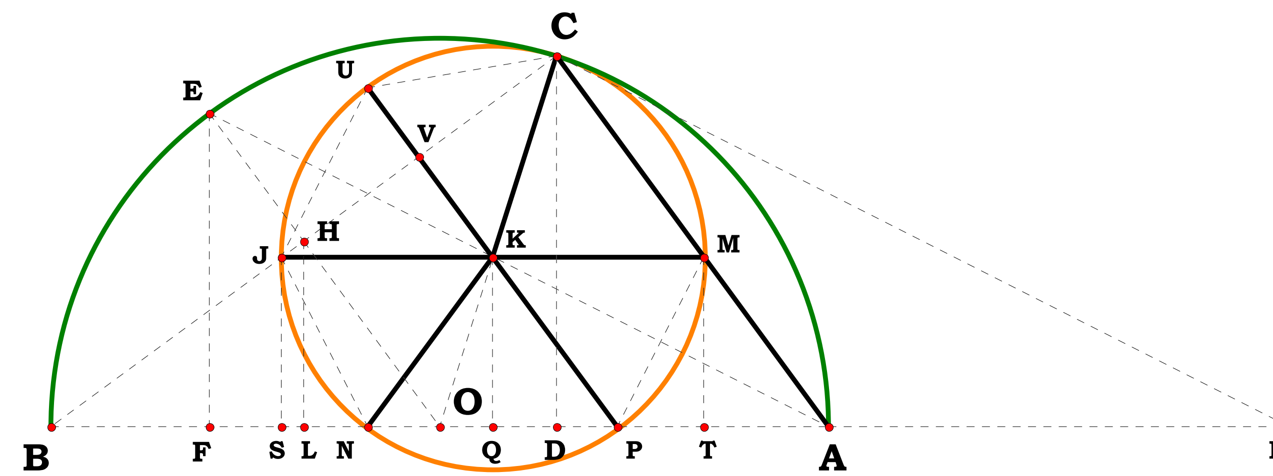
$$OK - \frac{\sqrt{N}}{2 \cdot \left(\sqrt{N}+2\right)} = 0 \qquad QP - \frac{1}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad OQ - \frac{\sqrt{(N-2)^2}}{2 \cdot \sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad EH - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0$$

$$AP - \frac{1}{\sqrt{N}+2} = 0 \qquad CK - \frac{1}{\sqrt{N}+2} = 0$$

In logic, things which have the same name are equal. CK equals AP.

$$CJ - \frac{2\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AQ - \frac{\sqrt{N}+1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SO - -\frac{N-2\cdot\sqrt{N}-2}{2\cdot\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$BS - \frac{1\cdot(N-1)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad BJ - \frac{1\cdot\sqrt{N-1}}{\sqrt{N+2}} = 0 \qquad JS - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$



$$JS - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SN - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JN - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad UV - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad JV - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JU - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad CU - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AT - \frac{1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$PT - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MT - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0$$

111193B1

Descriptions.

Unit $:= 1$

Given.

$N := 4$

$A := N - Unit \qquad AB := \frac{A}{2} \qquad AN := N - A$

$NK := \sqrt{N \cdot AN} \qquad BK := N - (NK + AB)$

$DK := \sqrt{AB^2 + BK^2} \qquad KM := \frac{BK \cdot NK}{DK}$

$CE := \frac{N - AB}{2} \qquad MP := \frac{CE}{2} \qquad CK := CE - BK$

$KP := \frac{CK}{2} \qquad NP := NK - KP \qquad CF := \frac{MP \cdot CE}{NP}$

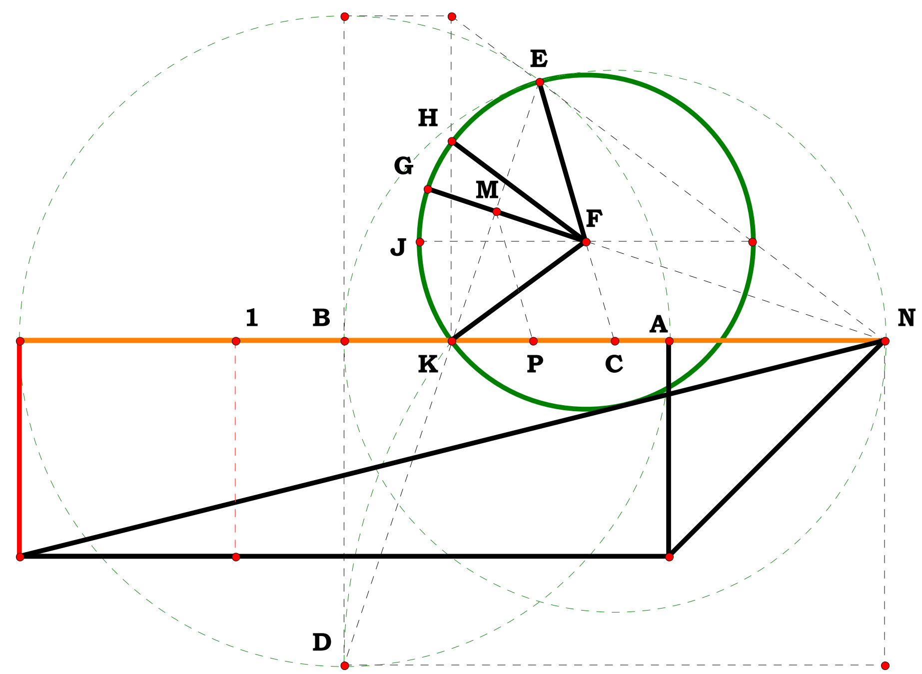
$EF := CE - CF \qquad EM := KM$

$EF = 0.769231 \qquad KM = 0.632456$

Definitions.

$$EF - \frac{\sqrt{N}\cdot(N+1)}{N + 4\cdot\sqrt{N} + 1} = 0 \qquad EM - \frac{\sqrt{2}\cdot\sqrt{N}\cdot\left(\sqrt{N}-1\right)^2}{2\cdot\sqrt{(N+1)\cdot\left(\sqrt{N}-1\right)^2}} = 0$$

$$\frac{EF}{EM} - \frac{(N+1)\cdot\sqrt{(2\cdot N+2)\cdot\left(\sqrt{N}-1\right)^2}}{\left(\sqrt{N}-1\right)^2\cdot\left(N+4\cdot\sqrt{N}+1\right)} = 0$$

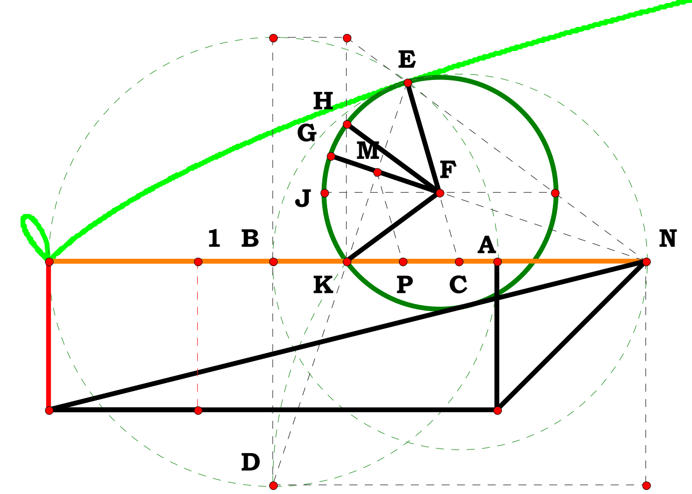


N = 4.00000
EM = 1.81051 cm
GM = 0.94862 cm
m∠EFG = 55.30485°
m∠HFG = 18.43495°
m∠EFK = 110.60969°
m∠HFJ = 36.86990°

EF = 0.76923
EM = 0.63246

$\frac{m\angle EFG}{m\angle HFG} = 3.00000 \qquad \frac{\sqrt{N}\cdot(N+1)}{N+4\cdot\sqrt{N}+1}-EF = 0.00000$

$\frac{m\angle EFK}{m\angle HFJ} = 3.00000 \qquad \frac{\sqrt{2\cdot N}\cdot(\sqrt{N}-1)^2}{2\cdot\sqrt{(N+1)\cdot(\sqrt{N}-1)^2}}-EM = 0.00000$



N = 4.00000
EM = 1.24826 cm
GM = 0.65403 cm
m∠EFG = 55.30485°
m∠HFG = 18.43495°
m∠EFK = 110.60969°
m∠HFJ = 36.86990°

EF = 0.76923
EM = 0.63246

$\frac{m\angle EFG}{m\angle HFG} = 3.00000 \qquad \frac{\sqrt{N}\cdot(N+1)}{N+4\cdot\sqrt{N}+1}-EF = 0.00000$

$\frac{m\angle EFK}{m\angle HFJ} = 3.00000 \qquad \frac{\sqrt{2\cdot N}\cdot(\sqrt{N}-1)^2}{2\cdot\sqrt{(N+1)\cdot(\sqrt{N}-1)^2}}-EM = 0.00000$

Unit := 1

Given.

N := 3

111193B2

Descriptions.

$H := \frac{N}{2}$ $\quad J := N - Unit \quad F := N - \sqrt{(N + J)}$

$FK := \sqrt{F^2 + J^2} \quad FN := N - F$

$EF := \frac{F \cdot FN}{FK} \quad CF := 2 \cdot EF \quad EG := \frac{H}{2}$

$FH := H - F \quad FG := \frac{FH}{2} \quad GN := FN - FG$

$BH := \frac{EG \cdot H}{GN} \quad BC := H - BH \quad BC = 0.897763$

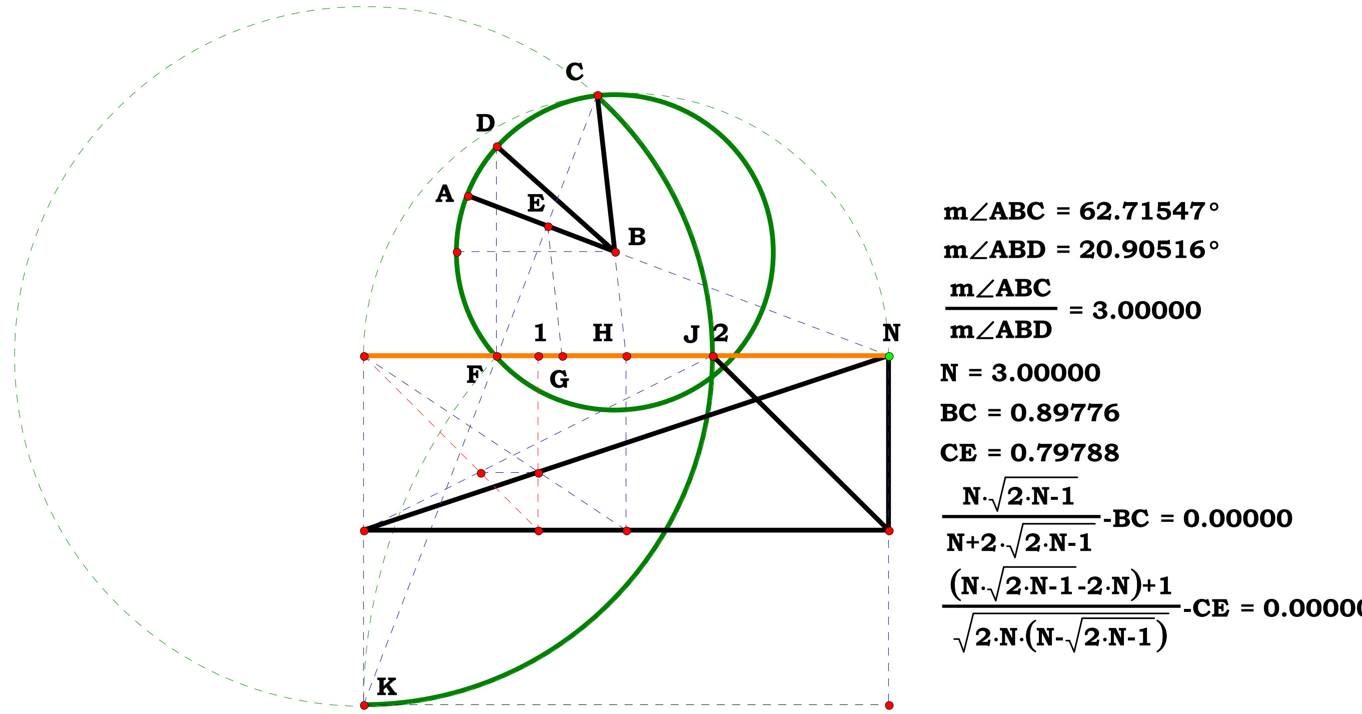
$CE := EF \quad CE = 0.797878$

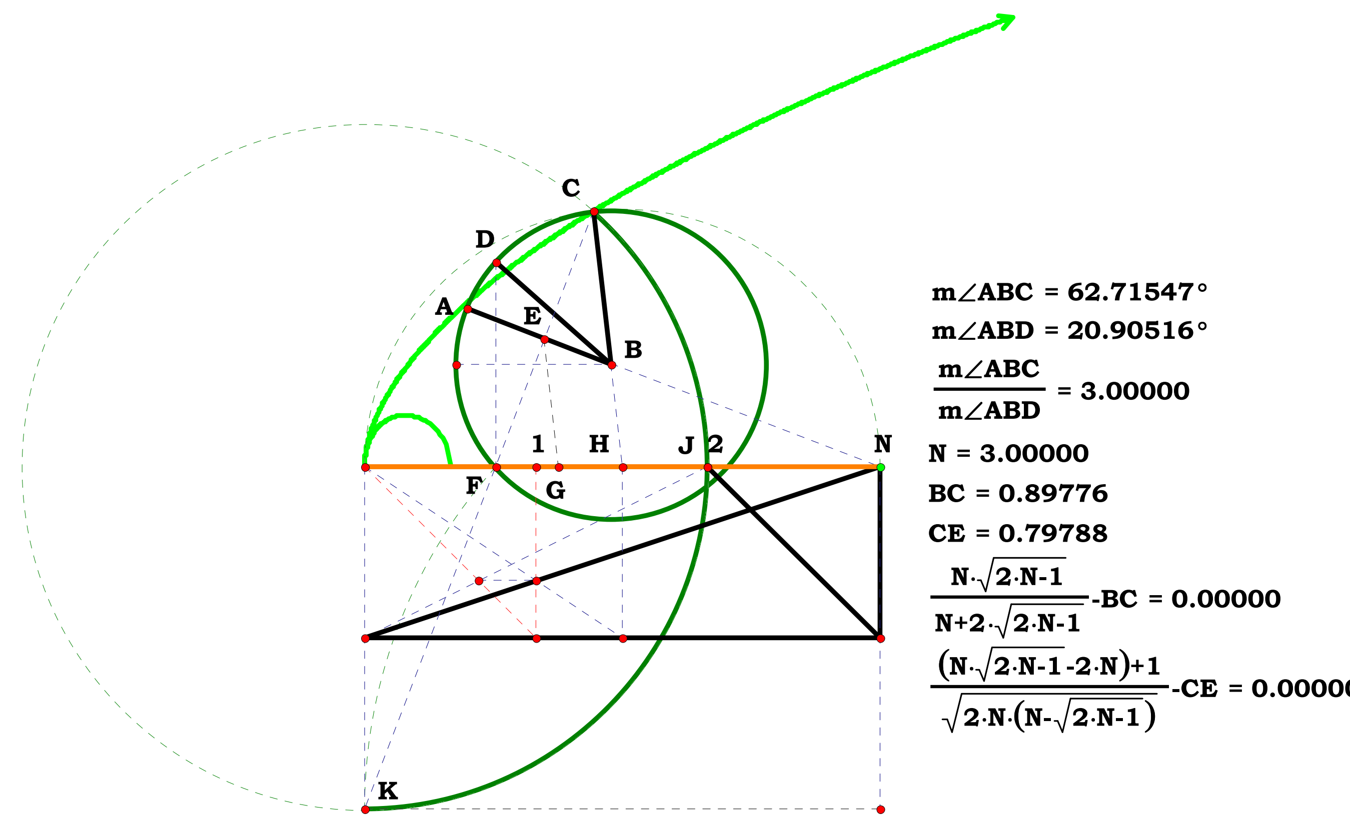
Definitions.

$$BC - \frac{N \cdot \sqrt{2 \cdot N - 1}}{N + 2 \cdot \sqrt{2 \cdot N - 1}} = 0$$

$$CE - \frac{\left(N \cdot \sqrt{2 \cdot N - 1} - 2 \cdot N + 1\right)}{\sqrt{2 \cdot N \cdot \left(N - \sqrt{2 \cdot N - 1}\right)}} = 0$$

$$\frac{BC}{CE} - \frac{\sqrt{2} \cdot \sqrt{2 \cdot N - 1} \cdot N \cdot \sqrt{N^2 - \sqrt{2 \cdot N - 1} \cdot N}}{\sqrt{2 \cdot N - 1} \cdot \left(N^2 - 4 \cdot N + 2\right) - N + 2 \cdot N^2} = 0$$





Unit := 1

Given.

N := 5

111193B3

Descriptions.

$P := N + Unit \qquad F := P - \sqrt{P} \qquad FP := P - F$

$O := \frac{N}{2} \qquad FO := F - O \quad OP := P - O$

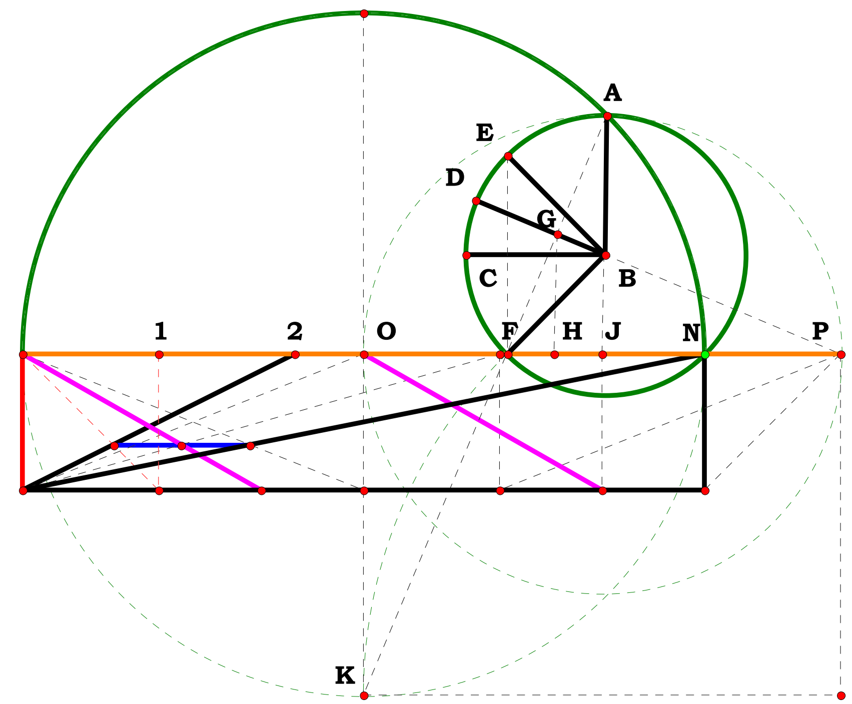
$AJ := \frac{OP}{2} \qquad FK := \sqrt{FO^2 + O^2} \qquad FG := \frac{FO \cdot FP}{FK}$

$GH := \frac{AJ}{2} \qquad FJ := AJ - FO$

$FH := \frac{FJ}{2} \qquad PH := FP - FH \qquad BJ := \frac{GH \cdot AJ}{PH}$

$AB := AJ - BJ \qquad AG := FG$

$AB = 1.020745 \qquad AG = 0.948914$



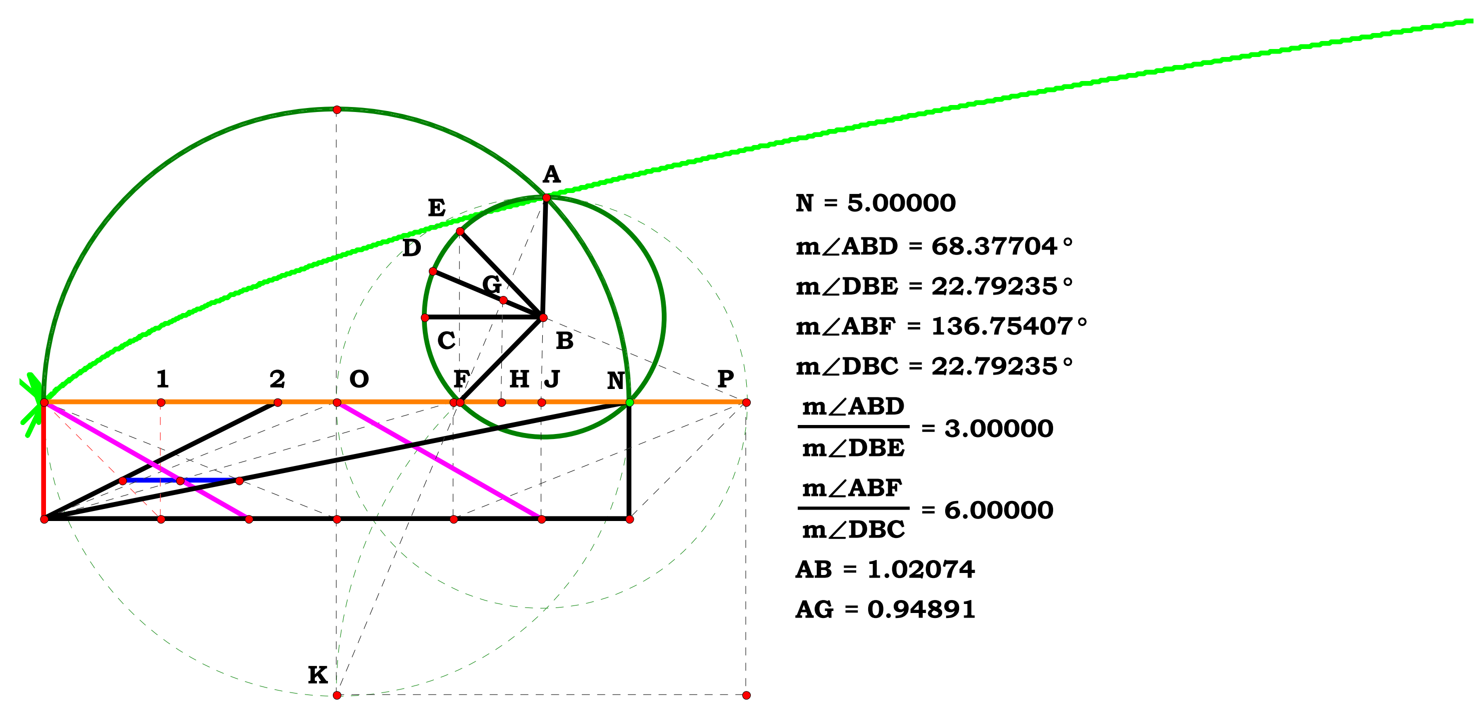
N = 5.00000
m∠ABD = 68.37704°
m∠DBE = 22.79235°
m∠ABF = 136.75407°
m∠DBC = 22.79235°
$\frac{m\angle ABD}{m\angle DBE} = 3.00000$
$\frac{m\angle ABF}{m\angle DBC} = 6.00000$
AB = 1.02074
AG = 0.94891

Definitions.

$$AB - \frac{\sqrt{N+1} \cdot (N+2)}{N + 4 \cdot \sqrt{N+1} + 2} = 0$$

$$AG - \frac{2 \cdot \sqrt{2} \cdot \left(\sqrt{N+1}\right)^3 - 2 \cdot \sqrt{2} \cdot N - 2 \cdot \sqrt{2} - \sqrt{2} \cdot N \cdot \sqrt{N+1}}{2 \cdot \sqrt{4 \cdot N + N^2 + 2 \cdot N \cdot \sqrt{N+1} - 4 \cdot \left(\sqrt{N+1}\right)^3 + 4}} = 0$$

$$\frac{AB}{AG} - \frac{\sqrt{2} \cdot \sqrt{N+1} \cdot (N+2) \cdot \sqrt{4 \cdot N + N^2 + 2 \cdot N \cdot \sqrt{N+1} - 4 \cdot \left(\sqrt{N+1}\right)^3 + 4}}{\left(N + 4 \cdot \sqrt{N+1} + 2\right) \cdot \left[2 \cdot \left(\sqrt{N+1}\right)^3 - N \cdot \sqrt{N+1} - 2 \cdot N - 2\right]} = 0$$



N = 5.00000
m∠ABD = 68.37704°
m∠DBE = 22.79235°
m∠ABF = 136.75407°
m∠DBC = 22.79235°
$\frac{m\angle ABD}{m\angle DBE} = 3.00000$
$\frac{m\angle ABF}{m\angle DBC} = 6.00000$
AB = 1.02074
AG = 0.94891

111193B4

Unit.

$AC := 1$

Given.

$N := 5$

Descriptions.

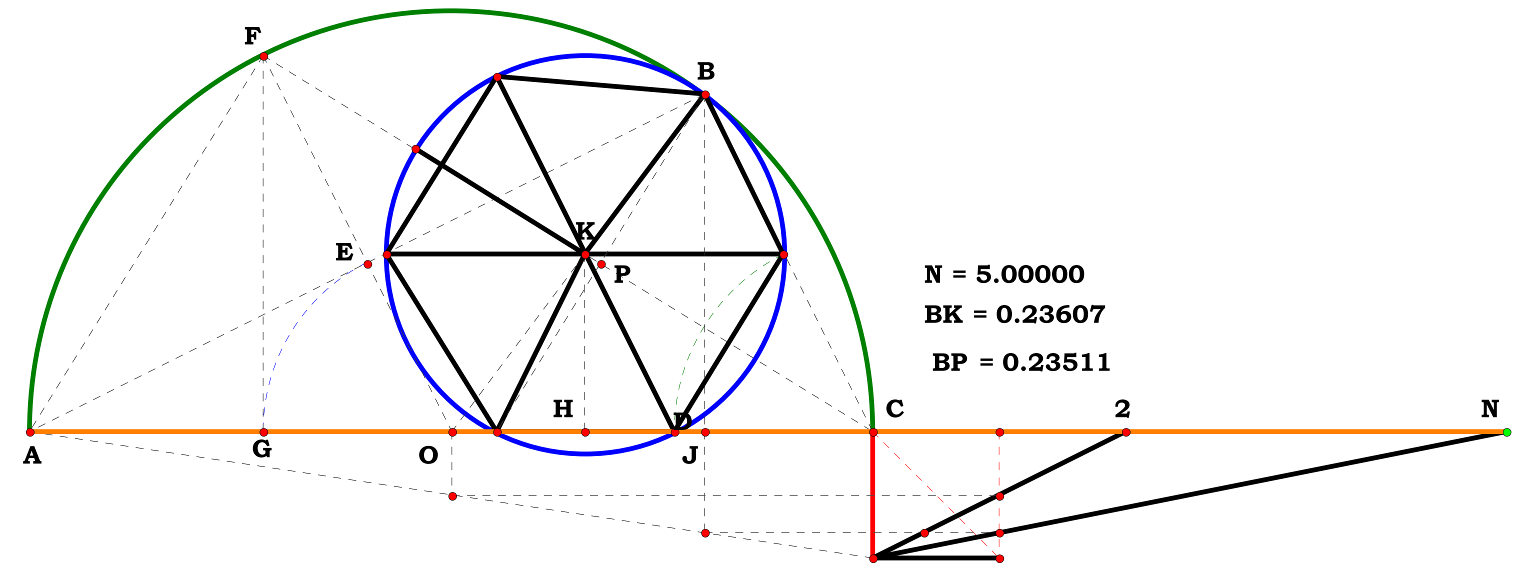
$$AO := \frac{AC}{2} \quad CO := AO \quad CD := \frac{AC}{N} \quad AD := AC - CD \quad BD := \sqrt{AD \cdot CD}$$

$$AB := \sqrt{AD^2 + BD^2} \quad BE := \frac{AB}{2} \quad FG := BE \quad EO := \sqrt{AO^2 - BE^2}$$

$$AG := AO - EO \quad CG := AC - AG \quad DO := AD - AO \quad GM := \frac{DO \cdot FG}{BD}$$

$$CM := CG + GM \quad FM := \sqrt{GM^2 + FG^2} \quad KO := \frac{FM \cdot AO}{CM}$$

$$BK := AO - KO \quad BK = 0.236068$$



Definitions.

$$BK - \frac{1}{\sqrt{N}+2} = 0 \quad N - \left(\frac{1}{BK} - 2\right)^2 = 0 \quad BC := \sqrt{CD^2 + BD^2} \quad CF := \sqrt{FG^2 + CG^2}$$

$$CP := \frac{CF \cdot BC}{AC} \quad BP := \sqrt{BC^2 - CP^2} \quad KP := \sqrt{BK^2 - BP^2}$$

$$KP - \frac{\sqrt{2} \cdot \sqrt{N^3 + 4 \cdot N^{\frac{3}{2}} - 3 \cdot N^{\frac{5}{2}}}}{2 \cdot \sqrt{4 \cdot N^3 + N^4 + 4 \cdot N^{\frac{7}{2}}}} = 0 \quad KP - \frac{\sqrt{(\sqrt{N}+1) \cdot (\sqrt{N}-2)^2}}{\sqrt{2 \cdot N^{\frac{3}{2}} \cdot (\sqrt{N}+2)^2}} = 0$$

$$\frac{BK}{KP} = 11.135164 \quad \frac{\sqrt{2} \cdot \sqrt{\left[(\sqrt{N})^3 \cdot (\sqrt{N}+2)^2\right]}}{(\sqrt{N}+2) \cdot \sqrt{(\sqrt{N}+1) \cdot (\sqrt{N}-2)^2}} = 11.135164 \quad \frac{BK}{KP} - \frac{\sqrt{2} \cdot \sqrt{\left[(\sqrt{N})^3 \cdot (\sqrt{N}+2)^2\right]}}{(\sqrt{N}+2) \cdot \sqrt{(\sqrt{N}+1) \cdot (\sqrt{N}-2)^2}} = 0$$

Unit.

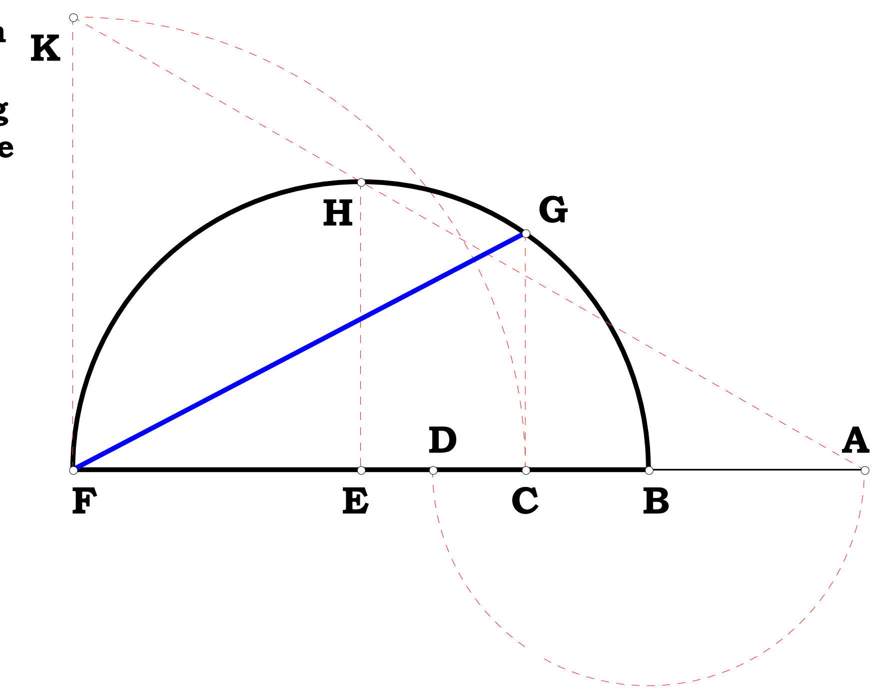
$BF := 1$

To Square A Circle Off The Base Of A Right Triangle.

Using the approximation, π = 22/7, square the circle off the base of a right triangle.

111293

Sometime in 1992, I remembered reading that some man spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost the figure, so I set out to find it - or something that could pass for it. It took a couple hours so I wonder what he did with the rest of his time?



Descriptions.

$BE := \frac{BF}{2} \quad EH := BE \quad BD := \frac{3}{4} \cdot BE \quad AB := BD$

$AE := AB + BE \quad AF := AB + BF \quad FK := \frac{EH \cdot AF}{AE} \quad CF := FK$

$BC := BF - CF \quad CG := \sqrt{BC \cdot CF} \quad FG := \sqrt{CF^2 + CG^2}$

$\pi\_A := \frac{FG^2}{BE^2}$

Definitions.

$\pi\_A - \frac{22}{7} = 0$

$\pi = 3.14159265359$

$\pi\_A = 3.142857142857$

$\frac{\pi}{\pi\_A} = 0.999597662505843$

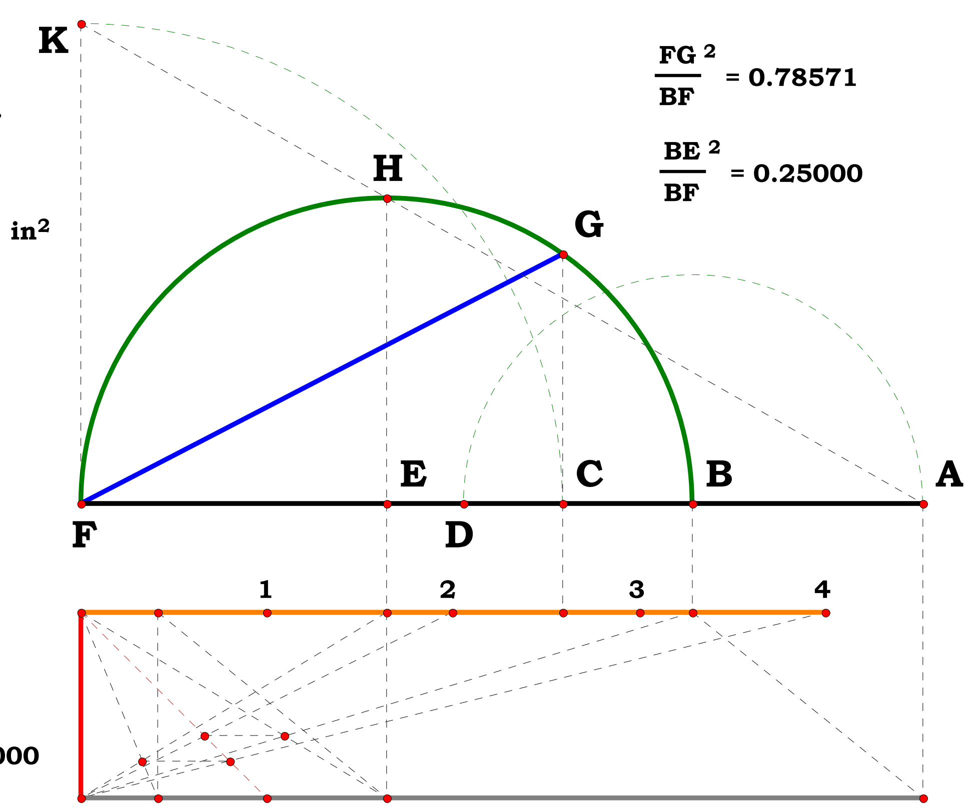
DE = 0.39839 in.
BF = 3.18715 in.
AB = 1.19518 in.
$AB \cdot BF$ = 3.80922 in^2
FG = 2.82511 in.
BE = 1.59357 in.
BF = 3.18715 in.

$\frac{22}{\frac{FG^2}{BF}} = 28.00000$

$\frac{7}{\frac{BE^2}{BF}} = 28.00000$

FK = 2.50419 in.

$\left(\frac{FK}{BF}\right) \cdot 28 = 22.00000$



$\frac{FG^2}{BF} = 0.78571$

$\frac{BE^2}{BF} = 0.25000$

Unit.

$BJ := 1$

Given.

$N_1 := 2$

Exploring Cube Roots Plate A

111893A

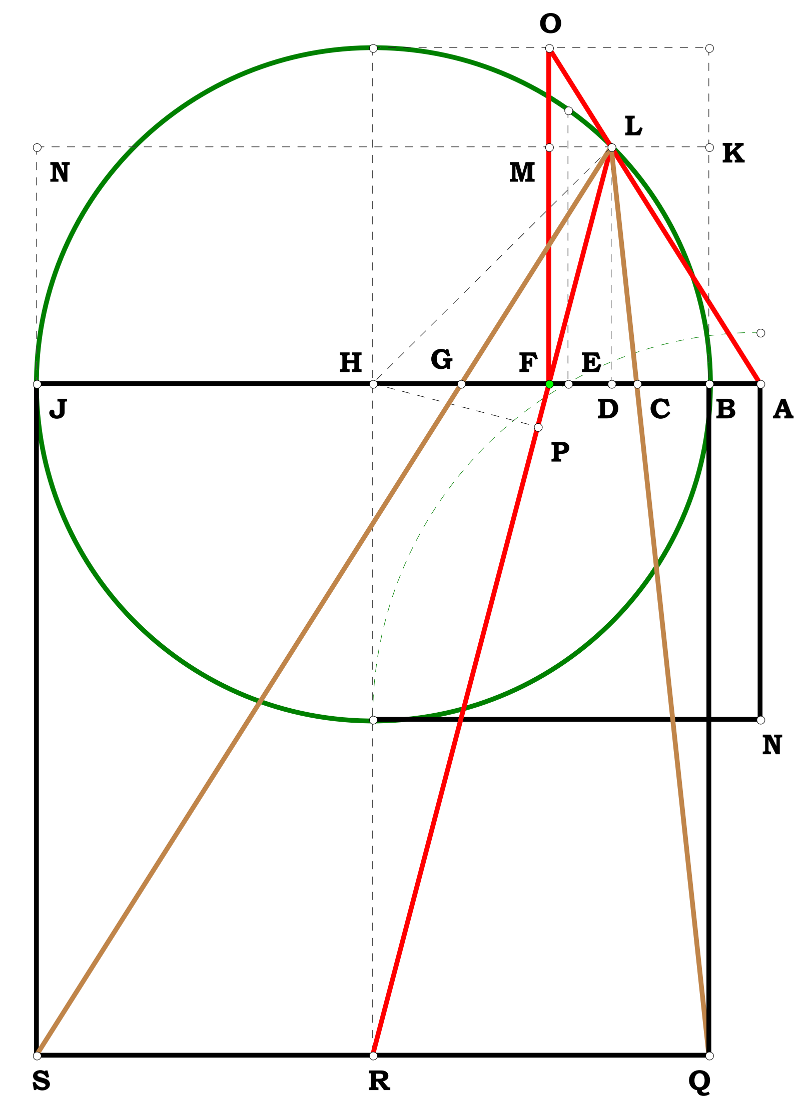
Descriptions. **describe AB.**

$BH := \frac{BJ}{2}$ $\quad BD := \frac{BH}{N_1}$ $\quad HJ := BH$

$DH := BH - BD$ $\quad HR := BJ$ $\quad DJ := DH + HJ$

$DL := \sqrt{BD \cdot DJ}$ $\quad DF := \frac{DH \cdot DL}{DL + HR}$ $\quad FO := BH$ $\quad BF := BD + DF$

$MO := FO - DL$ $\quad LM := DF$ $\quad AF := \frac{LM \cdot FO}{MO}$ $\quad AB := AF - BF$



Definitions.

$BJ - 1 = 0$ $\quad BH - \frac{1}{2} = 0$ $\quad BD - \frac{1}{2 \cdot N_1} = 0$ $\quad HJ - \frac{1}{2} = 0$ $\quad DH - \frac{N_1 - 1}{2 \cdot N_1} = 0$

$HR - 1 = 0$ $\quad DJ - \frac{2 \cdot N_1 - 1}{2 \cdot N_1} = 0$ $\quad DL - \frac{\sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$

$DF - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right)} = 0$ $\quad FO - \frac{1}{2} = 0$ $\quad BF - \frac{\sqrt{2 \cdot N_1 - 1} + 2}{2 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right)} = 0$

$MO - \frac{N_1 - \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$ $\quad LM - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right)} = 0$

$AF - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right) \cdot \left(N_1 - \sqrt{2 \cdot N_1 - 1}\right)} = 0$ $\quad AB - \frac{\sqrt{2 \cdot N_1 - 1} - 1}{2 \cdot \left(2 \cdot N_1^{\,2} - 2 \cdot N_1 - N_1 \cdot \sqrt{2 \cdot N_1 - 1} + 1\right)} = 0$

$$BH-\frac{N_1}{2} = 0.00000 \text{ in.}$$

$$BD-\frac{N_1}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$HJ-\frac{N_1}{2} = 0.00000 \text{ in.}$$

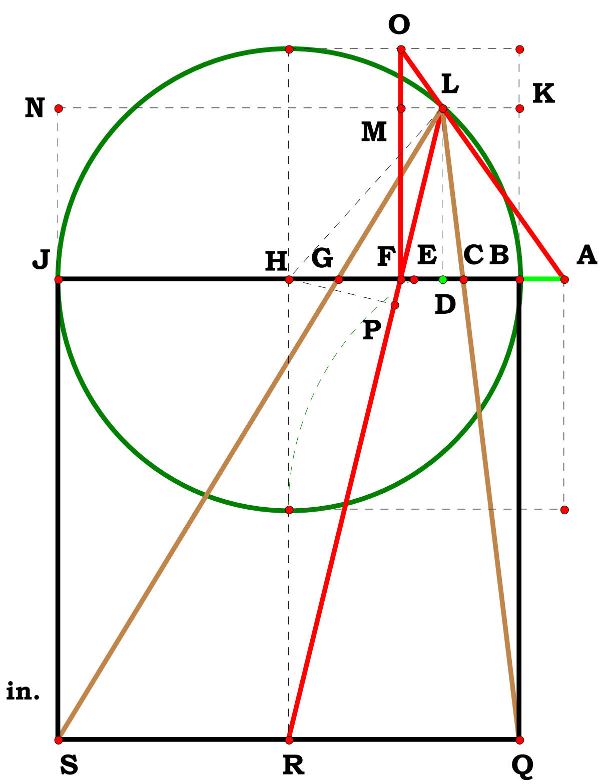
$$DH-\frac{N_1\cdot(N_2-1)}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$HR-N_1 = 0.00000 \text{ in.}$$

$$DJ-\frac{N_1\cdot(2\cdot N_2-1)}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$DL-\frac{N_1\cdot\sqrt{2\cdot N_2-1}}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$DF-\frac{N_1\cdot(N_2-1)\cdot\sqrt{2\cdot N_2-1}}{2\cdot N_2\cdot\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$



$$FO-\frac{N_1}{2} = 0.00000 \text{ in.}$$

$$BF-\frac{N_1\cdot\left(\sqrt{2\cdot N_2-1}+2\right)}{2\cdot\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$

$$MO-\frac{N_1\cdot\left(N_2-\sqrt{2\cdot N_2-1}\right)}{2\cdot N_2} = 0.00000 \text{ in.}$$

$$LM-\frac{N_1\cdot(N_2-1)\cdot\sqrt{2\cdot N_2-1}}{2\cdot N_2\cdot\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$

$$AF-\frac{N_1\cdot(N_2-1)\cdot\sqrt{2\cdot N_2-1}}{\left(2\cdot N_2+\sqrt{2\cdot N_2-1}\right)\cdot\left(2\cdot N_2-2\cdot\sqrt{2\cdot N_2-1}\right)} = 0.00000 \text{ in.}$$

$$AB-\frac{N_1\cdot\left(\sqrt{2\cdot N_2-1}-1\right)}{2\cdot\left(\left(2\cdot N_2^2-2\cdot N_2-N_2\cdot\sqrt{2\cdot N_2-1}\right)+1\right)} = 0.00000 \text{ in.}$$

$N_1 = 2.40000$ in.

$N_2 = 3.01550$

BJ = 2.40000 in.

BH = 1.20000 in.

BD = 0.39794 in.

HJ = 1.20000 in.

$$\frac{BH}{BD} = 3.01550$$

DH = 0.80206 in.

HR = 2.40000 in.

DJ = 2.00206 in.

DL = 0.89258 in.

DF = 0.21743 in.

FO = 1.20000 in.

BF = 0.61537 in.

MO = 0.30742 in.

LM = 0.21743 in.

AF = 0.84873 in.

AB = 0.23336 in.

Given.

$BJ := 1$

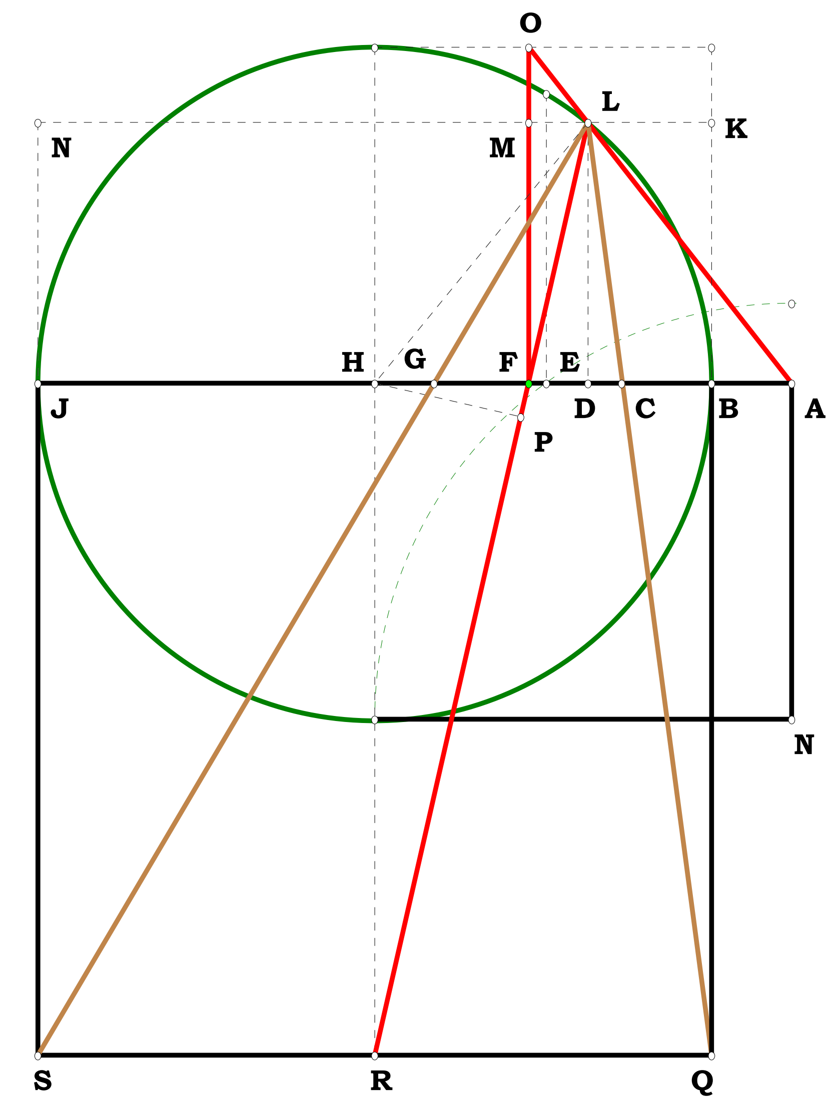
Given.

$N_1 := 3.01550$

Exploring Cube Roots Plate B

Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root. Notice in this write-up I chose the wrong point to proportion. I get the right answers, but the equations are more complicated. Compare features to plate A.

111893B



Descriptions.

$BH := \frac{BJ}{2}$ $\quad HL := BH$ $\quad BF := \frac{BH}{N_1}$ $\quad FH := BH - BF$ $\quad HR := BJ$ $\quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR}$ $\quad PH := \frac{HR \cdot FP}{FH}$ $\quad LP := \sqrt{HL^2 - PH^2}$ $\quad FL := LP - FP$ $\quad DF := \frac{FH \cdot FL}{FR}$

$DL := \frac{HR \cdot FL}{FR}$ $\quad FO := BH$ $\quad FM := DL$ $\quad MO := FO - FM$ $\quad LM := DF$ $\quad AF := \frac{LM \cdot FO}{MO}$

$AB := AF - BF$ $\quad BQ := BJ$ $\quad BK := DL$ $\quad BD := BF - DF$ $\quad KQ := BQ + BK$ $\quad KL := BD$

$BC := \frac{KL \cdot BQ}{KQ}$ $\quad DJ := BJ - BD$ $\quad LN := DJ$ $\quad JS := BJ$ $\quad JN := DL$ $\quad NS := JS + JN$

$GJ := \frac{LN \cdot JS}{NS}$ $\quad BG := BJ - GJ$ $\quad AC := AB + BC$ $\quad AG := AB + BG$ $\quad AJ := AB + BJ$

Definitions.

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0$ $\quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

$BJ - 1 = 0$ $\quad BH - \frac{1}{2} = 0$ $\quad HL - \frac{1}{2} = 0$ $\quad BF - \frac{1}{2 \cdot N_1} = 0$ $\quad FH - \frac{N_1 - 1}{2 \cdot N_1} = 0$ $\quad HR - 1 = 0$

$FR - \frac{\sqrt{5 \cdot {N_1}^2 - 2 \cdot N_1 + 1}}{2 \cdot N_1} = 0$ $\quad FP - \frac{\left(N_1 - 1\right)^2}{2 \cdot N_1 \cdot \sqrt{5 \cdot {N_1}^2 - 2 \cdot N_1 + 1}} = 0$ $\quad PH - \frac{N_1 - 1}{\sqrt{5 \cdot {N_1}^2 - 2 \cdot N_1 + 1}} = 0$

$$LP-\frac{\sqrt{N_1^2+6\cdot N_1-3}}{2\cdot\sqrt{5\cdot N_1^2-2\cdot N_1+1}}=0 \qquad FL-\frac{-\left(N_1^2-2\cdot N_1-N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+1\right)}{2\cdot N_1\cdot\sqrt{5\cdot N_1^2-2\cdot N_1+1}}=0$$

$$DF-\frac{-\left(N_1-1\right)\cdot\left(N_1^2-2\cdot N_1-N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+1\right)}{2\cdot N_1\cdot\left(5\cdot N_1^2-2\cdot N_1+1\right)}=0 \qquad DL-\frac{2\cdot N_1-N_1^2+N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}-1}{5\cdot N_1^2-2\cdot N_1+1}=0$$

$$FO-\frac{1}{2}=0 \qquad FM-\frac{2\cdot N_1-N_1^2+N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}-1}{5\cdot N_1^2-2\cdot N_1+1}=0 \qquad MO-\frac{7\cdot N_1^2-6\cdot N_1-2\cdot N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+3}{2\cdot\left(5\cdot N_1^2-2\cdot N_1+1\right)}=0$$

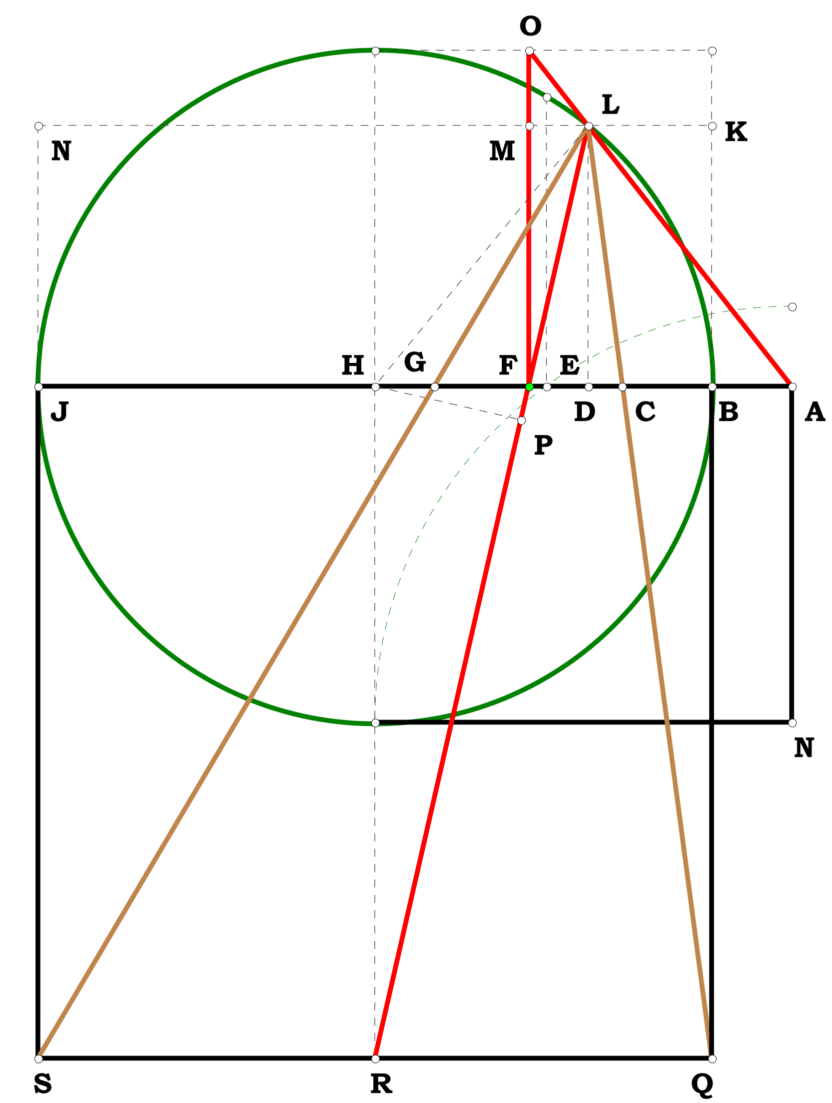
$$LM-\frac{-\left(N_1-1\right)\cdot\left(N_1^2-2\cdot N_1-N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+1\right)}{2\cdot N_1\cdot\left(5\cdot N_1^2-2\cdot N_1+1\right)}=0 \qquad AF-\frac{\left(1-N_1\right)\cdot\left(N_1^2-2\cdot N_1-N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+1\right)}{2\cdot N_1\cdot\left(7\cdot N_1^2-6\cdot N_1-2\cdot N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+3\right)}=0$$

$$AB-\frac{\left(N_1^2+N_1\right)\cdot\sqrt{N_1^2+6\cdot N_1-3}+3\cdot N_1-4\cdot N_1^2-N_1^3-2}{2\cdot N_1\cdot\left(7\cdot N_1^2-6\cdot N_1-2\cdot N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}+3\right)}=0 \qquad BQ-1=0$$

$$BK-\frac{2\cdot N_1-N_1^2+N_1\cdot\sqrt{N_1^2+6\cdot N_1-3}-1}{5\cdot N_1^2-2\cdot N_1+1}=0 \qquad BD-\frac{\left(1-N_1\right)\cdot\sqrt{N_1^2+6\cdot N_1-3}+N_1^2+2\cdot N_1+1}{2\cdot\left(5\cdot N_1^2-2\cdot N_1+1\right)}=0$$

$$KQ-\frac{N_1\cdot\left(4\cdot N_1+\sqrt{N_1^2+6\cdot N_1-3}\right)}{5\cdot N_1^2-2\cdot N_1+1}=0 \qquad KL-\frac{2\cdot N_1+N_1^2+\left(1-N_1\right)\cdot\sqrt{N_1^2+6\cdot N_1-3}+1}{2\cdot\left(5\cdot N_1^2-2\cdot N_1+1\right)}=0$$

$$BC-\frac{2\cdot N_1+N_1^2-\sqrt{N_1^2+6\cdot N_1-3}\cdot\left(N_1-1\right)+1}{2\cdot N_1\cdot\left(4\cdot N_1+\sqrt{N_1^2+6\cdot N_1-3}\right)}=0 \qquad DJ-\frac{\left(N_1-1\right)\cdot\sqrt{N_1^2+6\cdot N_1-3}+9\cdot N_1^2-6\cdot N_1+1}{2\cdot\left(5\cdot N_1^2-2\cdot N_1+1\right)}=0$$



$$LN-\frac{\left(N_1-1\right)\cdot\sqrt{N_1{}^2+6\cdot N_1-3}+9\cdot N_1{}^2-6\cdot N_1+1}{2\cdot\left(5\cdot N_1{}^2-2\cdot N_1+1\right)}=0$$

$$JS-1=0$$

$$JN-\frac{2\cdot N_1-N_1{}^2+N_1\cdot\sqrt{N_1{}^2+6\cdot N_1-3}-1}{5\cdot N_1{}^2-2\cdot N_1+1}=0$$

$$NS-\frac{N_1\cdot\left(4\cdot N_1+\sqrt{N_1{}^2+6\cdot N_1-3}\right)}{5\cdot N_1{}^2-2\cdot N_1+1}=0$$

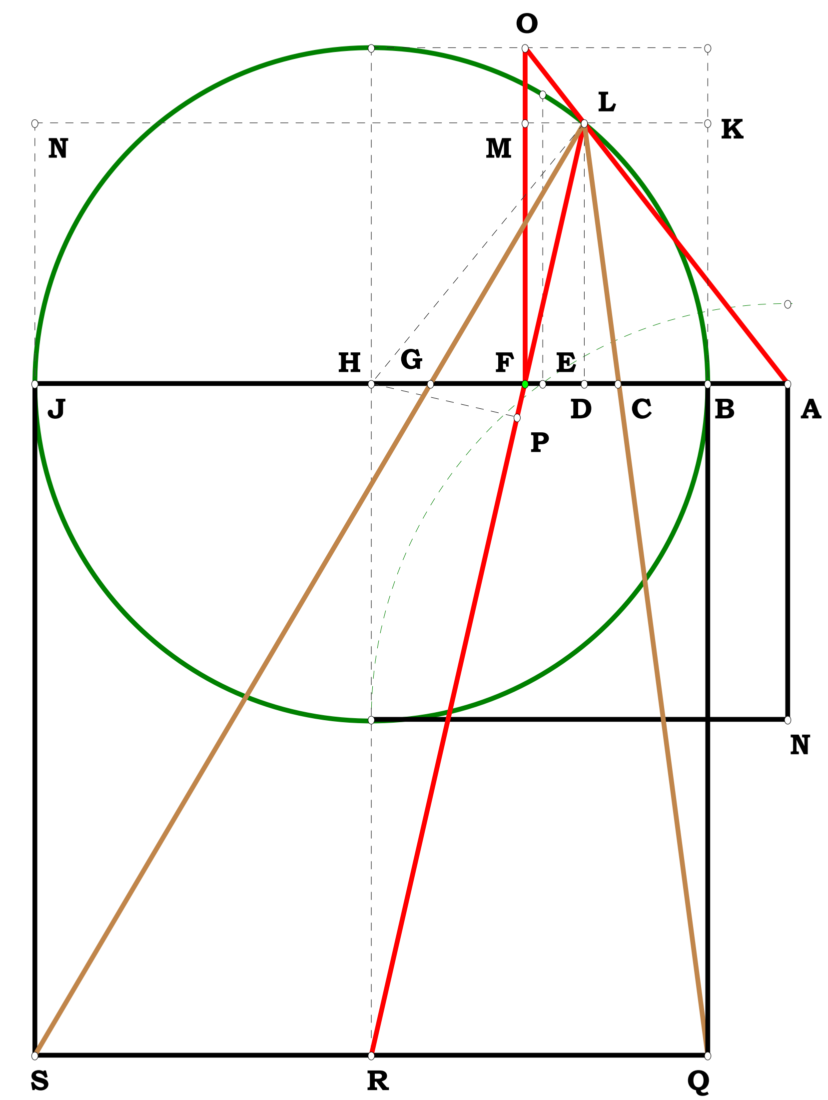
$$GJ-\frac{\left(N_1-1\right)\cdot\sqrt{N_1{}^2+6\cdot N_1-3}+9\cdot N_1{}^2-6\cdot N_1+1}{2\cdot N_1\cdot\left(4\cdot N_1+\sqrt{N_1{}^2+6\cdot N_1-3}\right)}=0$$

$$BG-\frac{3-N_1+\sqrt{N_1{}^2+6\cdot N_1-3}}{6\cdot N_1}=0$$

$$AC-\frac{\left(9\cdot N_1{}^2-6\cdot N_1{}^3-8\cdot N_1+1\right)\cdot\sqrt{N_1{}^2+6\cdot N_1-3}+\left(N_1+1\right)\cdot\left(6\cdot N_1{}^3+3\cdot N_1{}^2-8\cdot N_1+3\right)}{2\cdot N_1\cdot\left(26\cdot N_1{}^3-36\cdot N_1{}^2+18\cdot N_1\right)-2\cdot N_1\cdot\left(N_1{}^2+6\cdot N_1-3\right)\cdot\sqrt{N_1{}^2+6\cdot N_1-3}}=0$$

$$AG-\frac{N_1{}^2-4\cdot N_1{}^3+1+\sqrt{N_1{}^2+6\cdot N_1-3}\cdot\left(4\cdot 1\cdot N_1{}^2-3\cdot 1\cdot N_1+1\right)-2\cdot N_1}{6\cdot N_1-12\cdot N_1{}^2+14\cdot N_1{}^3-4\cdot\sqrt{N_1{}^2+6\cdot N_1-3}\cdot N_1{}^2}=0$$

$$AJ-\frac{13\cdot N_1{}^3-16\cdot N_1{}^2-2+\sqrt{N_1{}^2+6\cdot N_1-3}\cdot\left(1\cdot N_1-3\cdot 1\cdot N_1{}^2\right)+9\cdot N_1}{6\cdot N_1-12\cdot N_1{}^2+14\cdot N_1{}^3-4\cdot\sqrt{N_1{}^2+6\cdot N_1-3}\cdot N_1{}^2}=0$$



Etc.

$$BF\text{-}\frac{N_1}{2\cdot N_2} = 0.00000\text{ in.}$$

$$AF\text{-}\frac{N_1\cdot(N_2-1)\cdot\left(\left(N_2^2-2\cdot N_2-N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)+1\right)}{2\cdot N_2\cdot\left(\left(\left(6\cdot N_2-7\cdot N_2^2\right)+2\cdot N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-3\right)} = 0.00000\text{ in.}$$

$$AB\text{-}\frac{N_1\cdot\left(\left(\left(\left(3\cdot N_2+N_2^2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-4\cdot N_2^2-N_2^3\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-2\right)}{2\cdot\left(\left(3\cdot N_2-2\cdot N_2^2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}-6\cdot N_2^2\right)+7\cdot N_2^3\right)} = 0.00000\text{ in}$$

Animate Point

$N_1 = 2.40000$ in.

$N_2 = 1.86107$

$$FH\text{-}\frac{N_1\cdot(N_2-1)}{2\cdot N_2} = 0.00000\text{ in.}$$

$$FR\text{-}\frac{N_1\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}}{2\cdot N_2} = 0.00000\text{ in.}$$

$$FP\text{-}\frac{N_1\cdot(N_2-1)^2}{2\cdot N_2\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\text{ in.}$$

$$PH\text{-}\frac{N_1\cdot(N_2-1)}{\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\text{ in.}$$

$$LP\text{-}\frac{N_1\cdot\sqrt{(N_2^2+6\cdot N_2)-3}}{2\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\text{ in.}$$

$$FL\text{-}\frac{N_1\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{2\cdot N_2\cdot\sqrt{(5\cdot N_2^2-2\cdot N_2)+1}} = 0.00000\text{ in.}$$

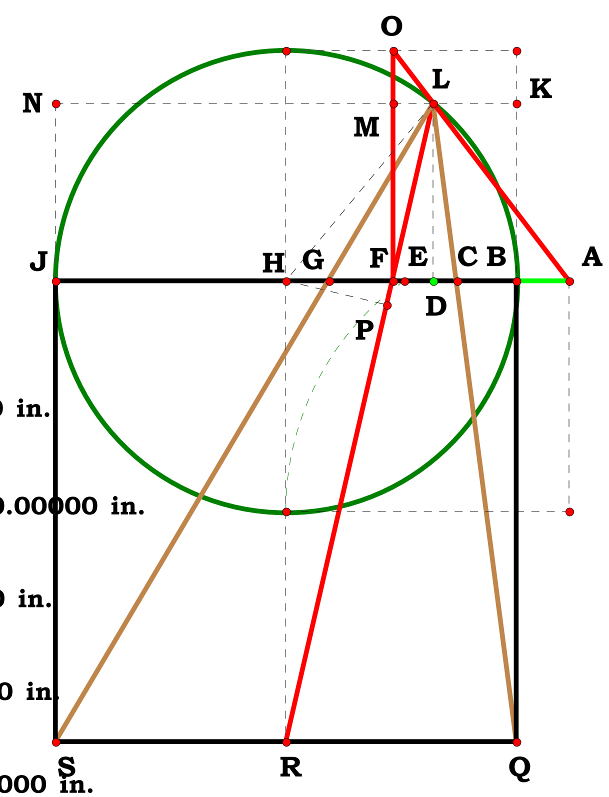
$$DF\text{-}\frac{N_1\cdot(N_2-1)\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{2\cdot N_2\cdot\left(\left(5\cdot N_2^2-2\cdot N_2\right)+1\right)} = 0.00000\text{ in.}$$

$$DL\text{-}\frac{N_1\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{(5\cdot N_2^2-2\cdot N_2)+1} = 0.00000\text{ in.}$$

$$FM\text{-}\frac{N_1\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{(5\cdot N_2^2-2\cdot N_2)+1} = 0.00000\text{ in.}$$

$$MO\text{-}\frac{N_1\cdot\left(\left(7\cdot N_2^2-6\cdot N_2-2\cdot N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)+3\right)}{2\cdot\left(\left(5\cdot N_2^2-2\cdot N_2\right)+1\right)} = 0.00000\text{ in.}$$

$$LM\text{-}\frac{N_1\cdot(N_2-1)\cdot\left(\left(\left(2\cdot N_2-N_2^2\right)+N_2\cdot\sqrt{(N_2^2+6\cdot N_2)-3}\right)-1\right)}{2\cdot N_2\cdot\left(\left(5\cdot N_2^2-2\cdot N_2\right)+1\right)} = 0.00000\text{ in.}$$



BJ = 2.40000 in.

BH = 1.20000 in.

BF = 0.64479 in.

$\frac{BH}{BF} = 1.86107$

FH = 0.55521 in.

FR = 2.46338 in.

FP = 0.12514 in.

PH = 0.54092 in.

LP = 1.07117 in.

FL = 0.94603 in.

DF = 0.21322 in.

DL = 0.92169 in.

FM = 0.92169 in.

MO = 0.27831 in.

LM = 0.21322 in.

AF = 0.91936 in.

AB = 0.27457 in.

Unit.

$BK := 1$

Given.

$N_1 := 4$

111893C

Descriptions.

Exploring Cube Roots Plate C

If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.

$BH := \dfrac{BK}{2}$ $\quad BD := \dfrac{BH}{N_1}$ $\quad DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $\quad BQ := BK$ $\quad KS := BK$ $\quad HR := BK$

$BC := \dfrac{BD \cdot BQ}{BQ + DN}$ $\quad GK := \dfrac{DK \cdot KS}{KS + DN}$ $\quad BG := BK - GK$

$DH := BH - BD$ $\quad FH := \dfrac{DH \cdot HR}{HR + DN}$ $\quad BF := BH - FH$

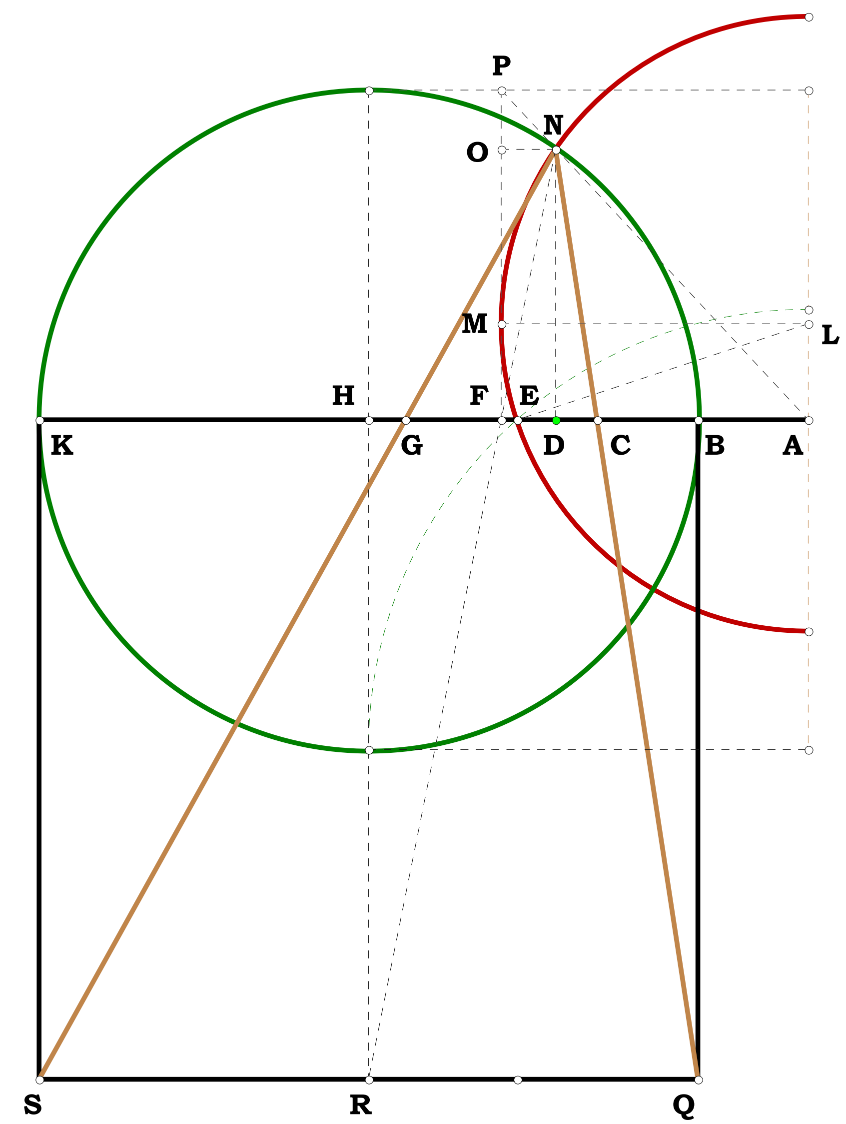
$CF := BF - BC$ $\quad AL := CF$ $\quad DF := BF - BD$

$NO := DF$ $\quad FP := BH$ $\quad PO := FP - DN$

$AD := \dfrac{NO \cdot DN}{PO}$ $\quad AB := AD - BD$

$AF := AB + BF$ $\quad LM := AF$ $\quad EL := AF$ $\quad AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $\quad AE_2 := \sqrt{AB \cdot AK}$ $\quad AE_1 - AE_2 = 0$



Definitions.

$BH - \dfrac{1}{2} = 0$ $\quad BD - \dfrac{1}{2 \cdot N_1} = 0$ $\quad DK - \dfrac{(2 \cdot N_1 - 1)}{2 \cdot N_1} = 0$

$DN - \dfrac{\sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$ $\quad BQ - 1 = 0$ $\quad KS - 1 = 0$

$$HR-1=0 \qquad BC-\frac{1}{2\cdot N_1+\sqrt{2\cdot N_1-1}}=0 \qquad GK-\frac{2\cdot N_1-1}{2\cdot N_1+\sqrt{2\cdot N_1-1}}=0$$

$$BG-\frac{\sqrt{2\cdot N_1-1}+1}{2\cdot N_1+\sqrt{2\cdot N_1-1}}=0 \qquad DH-\frac{N_1-1}{2\cdot N_1}=0 \qquad FH-\frac{N_1-1}{2\cdot N_1+\sqrt{2\cdot N_1-1}}=0$$

$$BF-\frac{\sqrt{2\cdot N_1-1}+2}{2\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)}=0 \qquad CF-\frac{\sqrt{2\cdot N_1-1}}{2\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)}=0 \qquad AL-\frac{\sqrt{2\cdot N_1-1}}{2\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)}=0$$

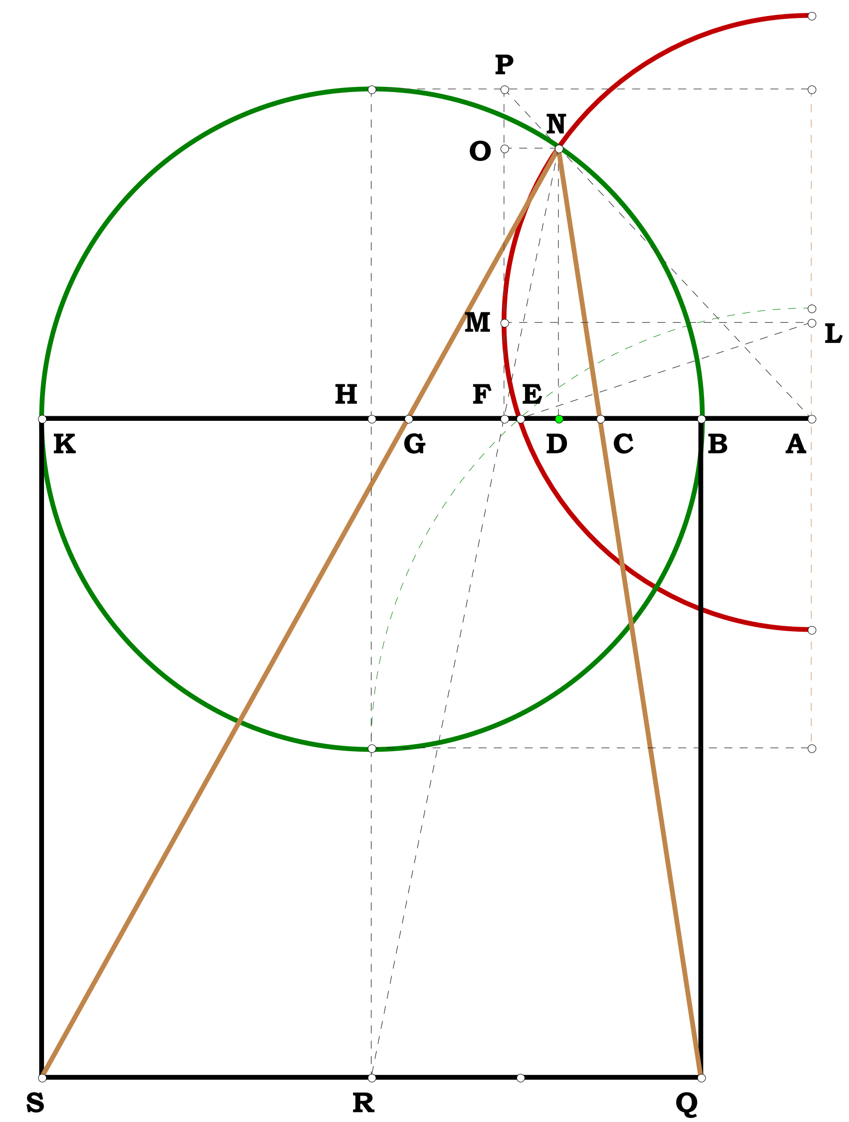
$$DF-\frac{\left(N_1-1\right)\cdot\sqrt{2\cdot N_1-1}}{2\cdot N_1\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)}=0 \qquad NO-\frac{\left(N_1-1\right)\cdot\sqrt{2\cdot N_1-1}}{2\cdot N_1\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)}=0$$

$$FP-\frac{1}{2}=0 \qquad PO-\frac{N_1-\sqrt{2\cdot N_1-1}}{2\cdot N_1}=0 \qquad AD-\frac{\left(N_1-1\right)\cdot\left(\sqrt{2\cdot N_1-1}\right)^2}{2\cdot N_1\cdot\left(N_1-\sqrt{2\cdot N_1-1}\right)\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)}=0$$

$$AB-\frac{\left(\sqrt{2\cdot N_1-1}-1\right)}{2\cdot\left(2\cdot N_1^2-2\cdot N_1-N_1\cdot\sqrt{2\cdot N_1-1}+1\right)}=0 \qquad AF-\frac{\left(N_1-1\right)\cdot\left(2\cdot N_1+2\cdot N_1\cdot\sqrt{2\cdot N_1-1}-1\right)}{2\cdot\left(3\cdot N_1+\sqrt{2\cdot N_1-1}-6\cdot N_1^2+4\cdot N_1^3-2\cdot N_1\cdot\sqrt{2\cdot N_1-1}\right)}=0 \qquad EL-AF=0 \qquad LM-AF=0$$

$$AK-\frac{\left[4\cdot N_1^2-\left(2\cdot N_1-1\right)^{\frac{3}{2}}-4\cdot N_1+1\right]}{2\cdot\left(2\cdot N_1^2-2\cdot N_1-N_1\cdot\sqrt{2\cdot N_1-1}+1\right)}=0 \qquad AE_1-\sqrt{\left[\frac{\left(N_1-1\right)^2\cdot\left(2\cdot N_1+2\cdot N_1\cdot\sqrt{2\cdot N_1-1}-1\right)^2}{4\cdot\left(3\cdot N_1+\sqrt{2\cdot N_1-1}-6\cdot N_1^2+4\cdot N_1^3-2\cdot N_1\cdot\sqrt{2\cdot N_1-1}\right)^2}-\frac{\left(2\cdot N_1-1\right)}{4\cdot\left(2\cdot N_1+\sqrt{2\cdot N_1-1}\right)^2}\right]}=0$$

$$AE_2-\sqrt{\left[\frac{\left(\sqrt{2\cdot N_1-1}-1\right)\cdot\left[4\cdot N_1^2-\left(2\cdot N_1-1\right)^{\frac{3}{2}}-4\cdot N_1+1\right]}{4\cdot\left(2\cdot N_1-2\cdot N_1^2+N_1\cdot\sqrt{2\cdot N_1-1}-1\right)^2}\right]}=0 \qquad AE_1-AE_2=0$$



111893D

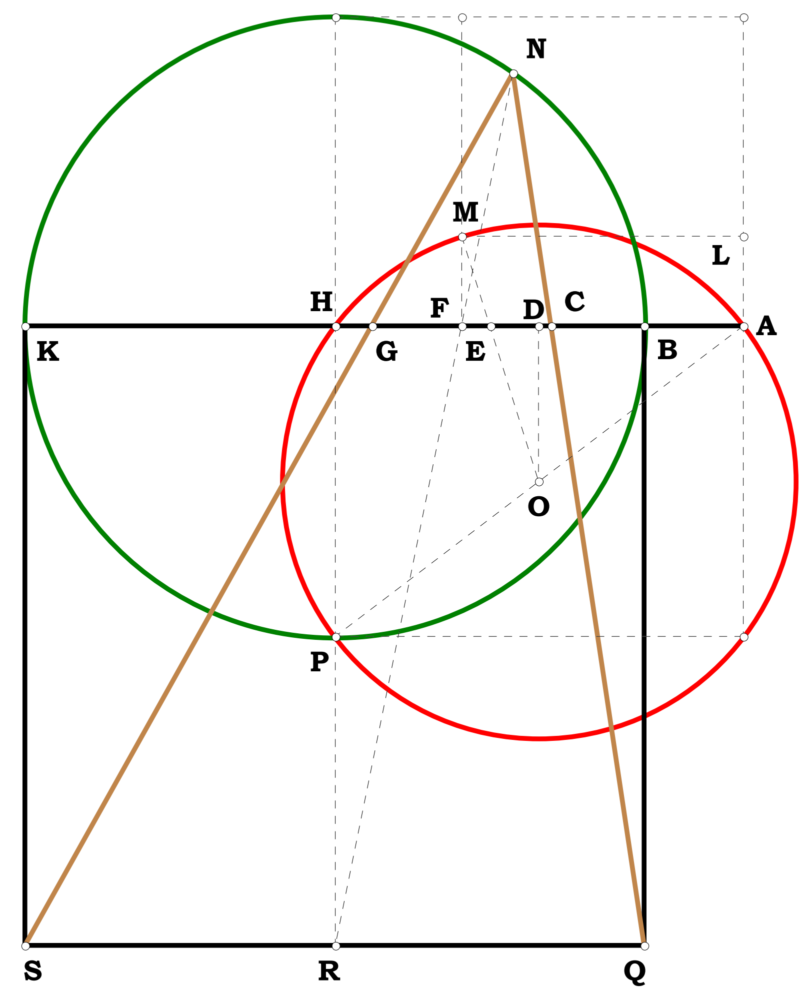
$AB := 1$

Given.

$N_1 := 3$

Exploring Cube Roots Plate D

The circle AO passes through point M. FM equals half of CG.



Descriptions.

$BK := N_1 \quad AK := BK + AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \qquad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

$CG := AG - AC \qquad CF := \frac{CG}{2} \qquad BH := \frac{BK}{2}$

$AH := AB + BH \qquad HP := BH \qquad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \qquad DO := \frac{HP}{2} \qquad AF := AC + CF \qquad AD := \frac{AH}{2}$

$DF := AF - AD \qquad FM := CF \qquad MO := AO$

Definitions.

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0 \qquad AK - \left(1 + N_1\right) = 0$

$AC - \left(N_1 + 1\right)^{\frac{1}{3}} = 0 \qquad AG - \left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} = 0 \qquad CG - \left[\left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - \left(N_1 + 1\right)^{\frac{1}{3}}\right] = 0$

$CF - \frac{\left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - \left(N_1 + 1\right)^{\frac{1}{3}}}{2} = 0 \qquad BH - \frac{N_1}{2} = 0 \qquad AH - \frac{2 + N_1}{2} = 0 \qquad HP - \frac{N_1}{2} = 0 \qquad AP - \frac{\sqrt{N_1^2 + 2 \cdot N_1 + 2}}{\sqrt{2}} = 0$

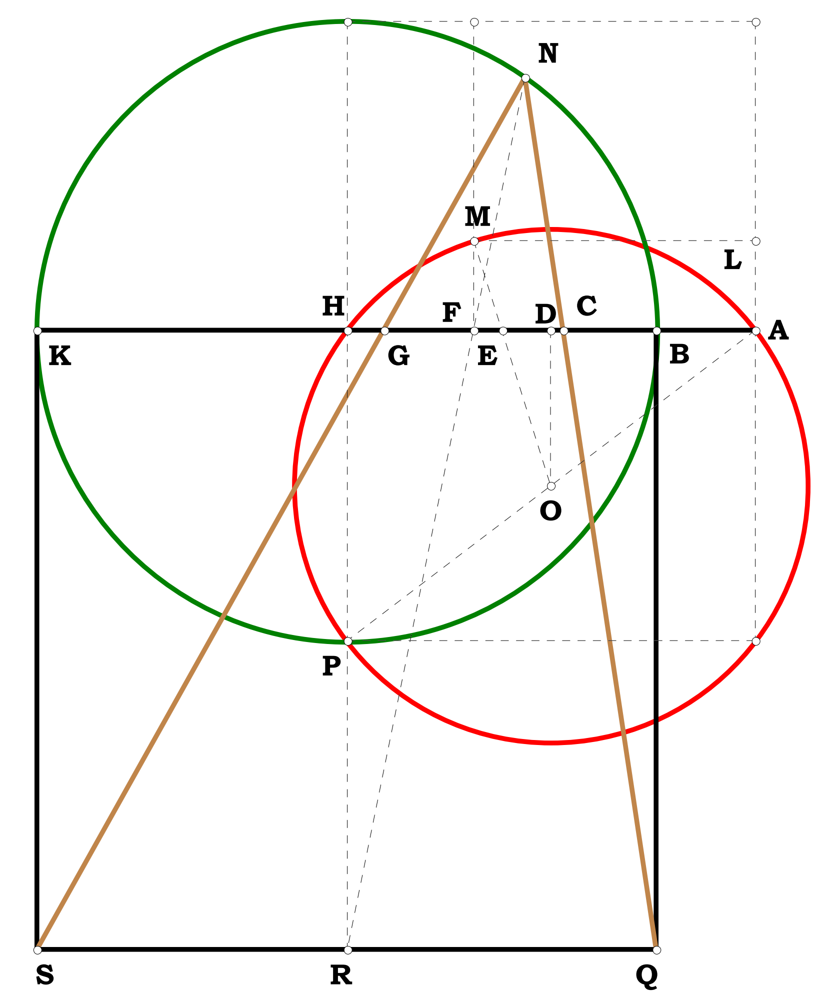
$$AO - \frac{\sqrt{2}}{4}\cdot\sqrt{\left(N_1{}^2 + 2\cdot N_1 + 2\right)} = 0 \qquad DO - \frac{N_1}{4} = 0 \qquad AF - \frac{\left[\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} + \left(N_1+1\right)^{\frac{1}{3}}\right]}{2} = 0$$

$$AD - \frac{2+N_1}{4} = 0 \qquad DF - \frac{\left[2\cdot\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} - N_1 - 2 + 2\cdot\left(N_1+1\right)^{\frac{1}{3}}\right]}{4} = 0$$

$$FM - \frac{\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} - \left(N_1+1\right)^{\frac{1}{3}}}{2} = 0 \qquad MO - \frac{\sqrt{2}}{4}\cdot\sqrt{2\cdot 1^2 + 2\cdot 1\cdot N_1 + N_1{}^2} = 0$$

$$MO^2 - \frac{\left[\begin{aligned}&2\cdot N_1 + N_1{}^2 - 4\cdot N_1\cdot\left(N_1+1\right)^{\frac{1}{3}} - 4\cdot\left(N_1+1\right)^{\frac{1}{3}} \ \ldots \\ &+ 4\cdot\left(N_1+1\right)^{\frac{2}{3}} - 4\cdot\left(N_1{}^2+2\cdot N_1+1\right)^{\frac{1}{3}} + 4\cdot\left(N_1{}^2+2\cdot N_1+1\right)^{\frac{2}{3}} + 2\end{aligned}\right]}{8} = 0$$

$$\frac{\left[\left(N_1+1\right)^2\right]^{\frac{2}{3}}}{2} + \frac{\left(N_1+1\right)^{\frac{2}{3}}}{2} - \frac{\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}}}{2} - \frac{\left(N_1+1\right)^{\frac{1}{3}}}{2} - \frac{N_1\cdot\left(N_1+1\right)^{\frac{1}{3}}}{2} = 0$$



Unit. $AB := 1$

Given. $AE := 2$

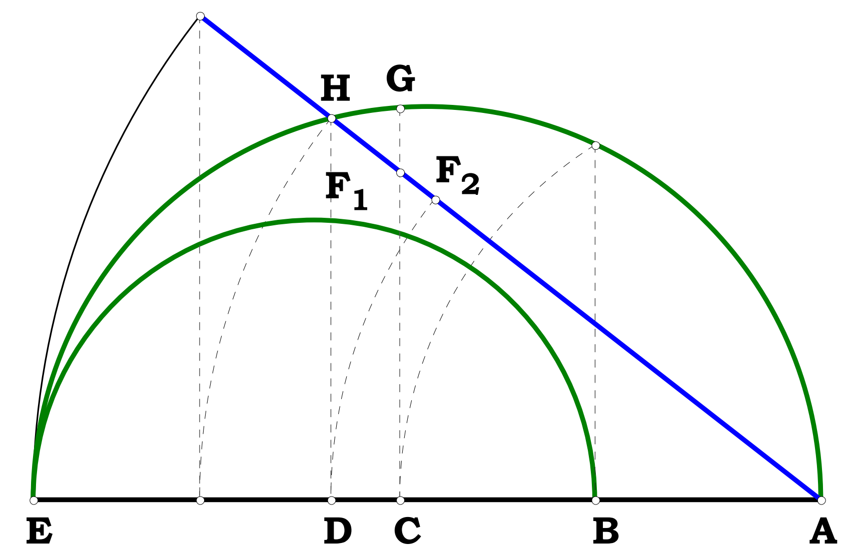
$\delta := 0 .. \Delta$

112293 Cube by Iteration

Descriptions.

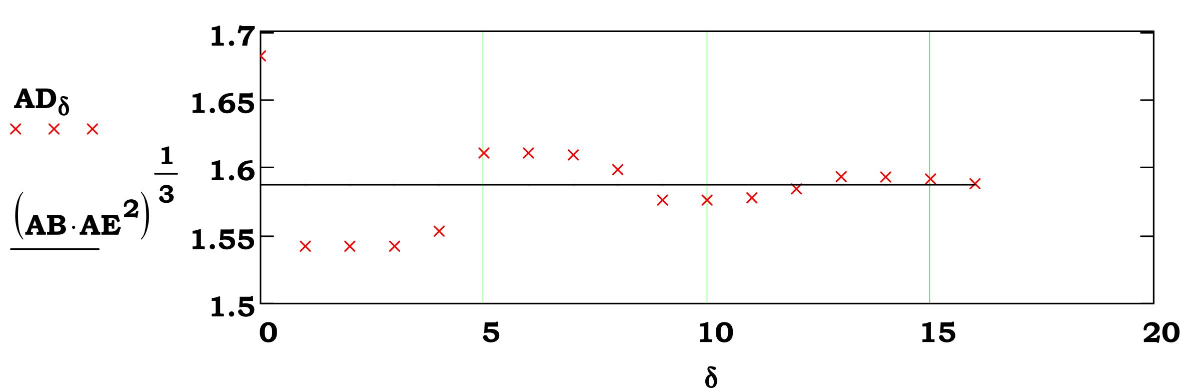
When F_1 and F_2 are the same point on C, then a sixth root series has been constructed. Use iteration to place F_2 on F_1.

$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$



$$\begin{pmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{pmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[\dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG}\right]^2} \end{bmatrix} \qquad \begin{pmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{pmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix}$$

$\Delta \equiv 16$



$\left(AB^2 \cdot AE^4\right)^{\frac{1}{6}} - AD_\Delta = -6.761441 \times 10^{-4}$ $\qquad$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD_\Delta = -6.761441 \times 10^{-4}$

Not a very promising prospect!

112493

Unit.

$AE := 1$

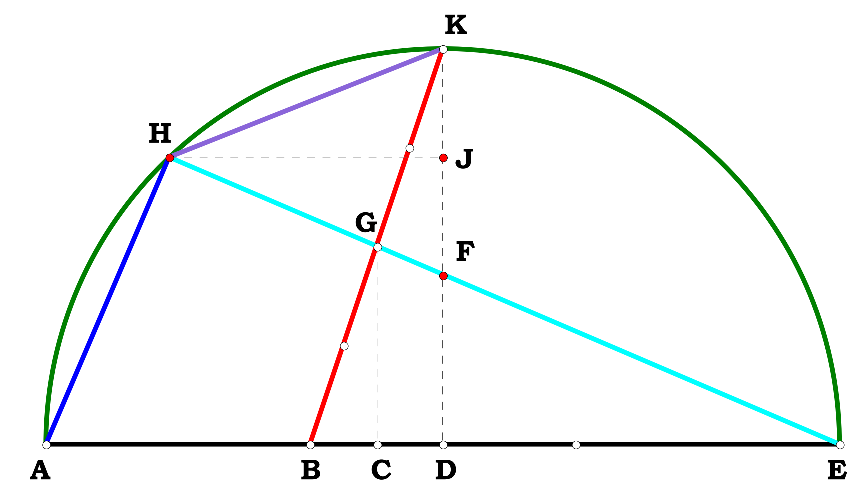
Given.

$N_1 := 3 \qquad \alpha := 1 .. N_1 - 1$

$N_2 := 5 \qquad \beta := 1 .. N_2 - 1$

POR Series IV

Generalize the work of 07/25/93 for dividing the base AE with K constant.



Descriptions.

$AB := \frac{AE}{N_1} \qquad AD := \frac{AE}{2} \qquad DK := AD \qquad DE := AD$

$BD := AD - AB \qquad BK := \sqrt{BD^2 + DK^2} \qquad BG := \frac{BK}{N_2} \qquad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \qquad CE := AE - AC \qquad DF := \frac{CG \cdot DE}{CE} \qquad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \qquad AH := \frac{DF \cdot AE}{EF} \qquad EH := \frac{DE \cdot AE}{EF} \qquad GH := EH - EG \qquad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \qquad HJ := \frac{DE \cdot FH}{EF} \qquad DJ := DF + FJ \quad JK := DK - DJ \qquad HK := \sqrt{HJ^2 + JK^2}$

Definitions.

$\frac{AH}{HK} = 0.265165 \qquad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265165$

$\frac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot \left(2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1{}^2 - 2 \cdot N_1 + 2\right)} = 2.85$

$SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

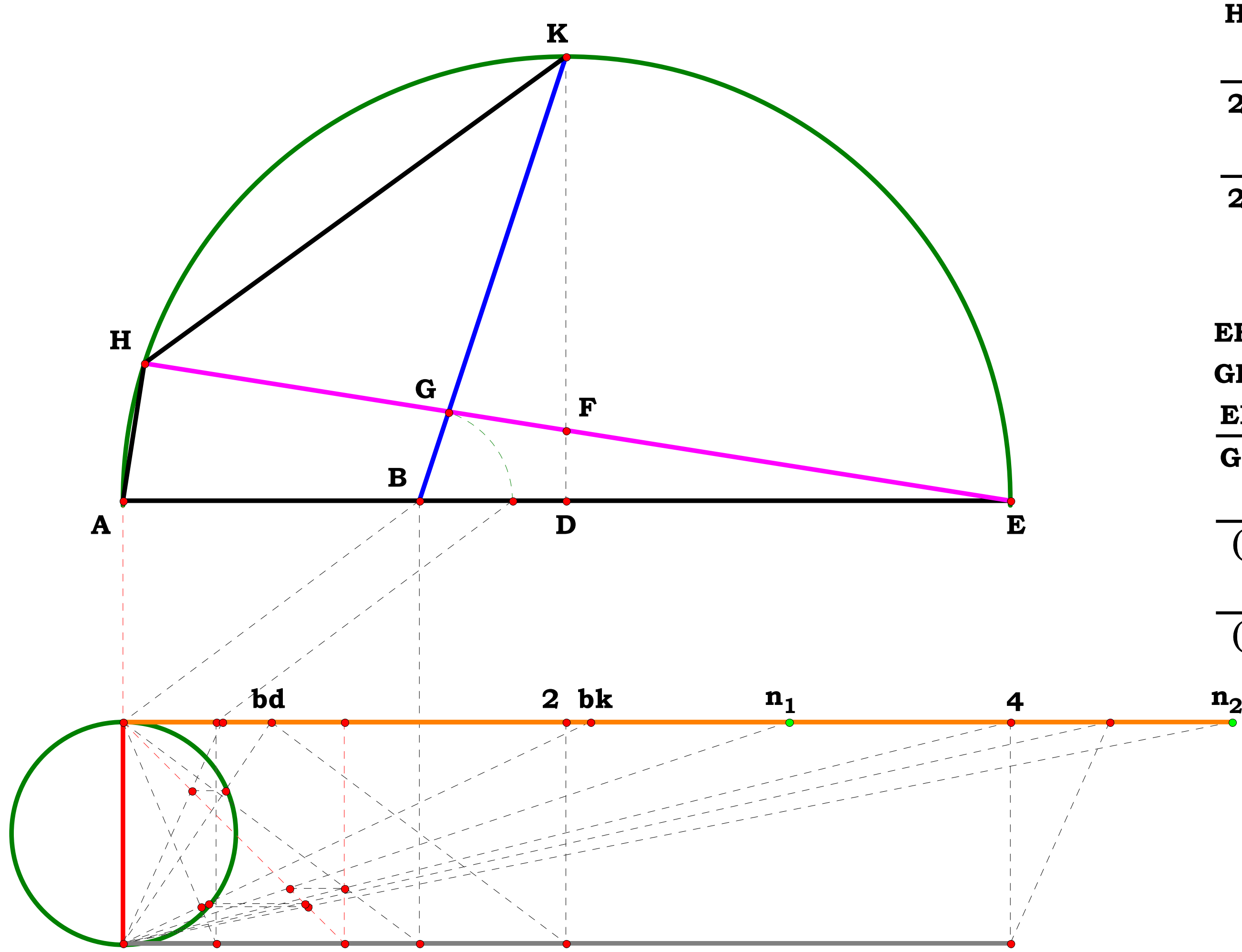
$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot \left(2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1{}^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta\right)}$

SeriesAH =

	1	2	3	4
1	0.265165	0.707107	1.59099	4.242641
2	0.53033	1.414214	3.181981	8.485281

SeriesEH =

	1	2	3	4
1	2.85	3	3.643	6
2	1.65	2	2.786	5.25



$$HK = 2.35269$$

$$\frac{AH}{HK} = 0.26517$$

$$\frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1-1) \cdot (N_2-1)} = 0.26517$$

$$\frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1-1) \cdot (N_2-1)} - \frac{AH}{HK} = 0.00000$$

$$EH = 3.95105$$

$$GH = 1.38633$$

$$\frac{EH}{GH} = 2.85000$$

$$\frac{N_1 \cdot N_2 \cdot ((2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1) + 2)}{(N_2-1) \cdot ((((2 \cdot N_1 \cdot N_2 - 2 \cdot N_2) + N_1^2) - 2 \cdot N_1) + 2)} = 2.85000$$

$$\frac{N_1 \cdot N_2 \cdot ((2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1) + 2)}{(N_2-1) \cdot ((((2 \cdot N_1 \cdot N_2 - 2 \cdot N_2) + N_1^2) - 2 \cdot N_1) + 2)} - \frac{EH}{GH} = 0.00000$$

120293

Unit.

$AH := 1$

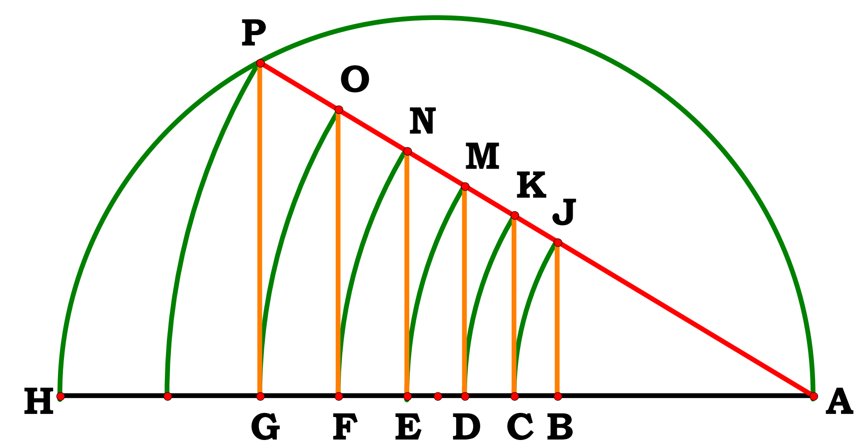
Given.

$\delta := 1 .. 7$

$\Delta := 1 .. 6$

POR Roots and Powers (Pyramid of Ratio Series V)

Is the progression noticed in 112993 a continuous phenomenon?



Descriptions.

$AP_\delta := \frac{AH}{\delta}$ $\quad AG_\delta := \frac{(AP_\delta)^2}{AH}$ $\quad AO_\delta := AG_\delta$

$AF_\delta := \frac{(AG_\delta)^2}{AP_\delta}$ $\quad AE_\delta := \frac{(AF_\delta)^2}{AO_\delta}$ $\quad AN_\delta := AF_\delta$

$AD_\delta := \frac{(AE_\delta)^2}{AN_\delta}$ $\quad AM_\delta := AE_\delta$

$AC_\delta := \frac{(AD_\delta)^2}{AM_\delta}$ $\quad AK_\delta := AD_\delta$ $\quad AB_\delta := \frac{(AC_\delta)^2}{AK_\delta}$

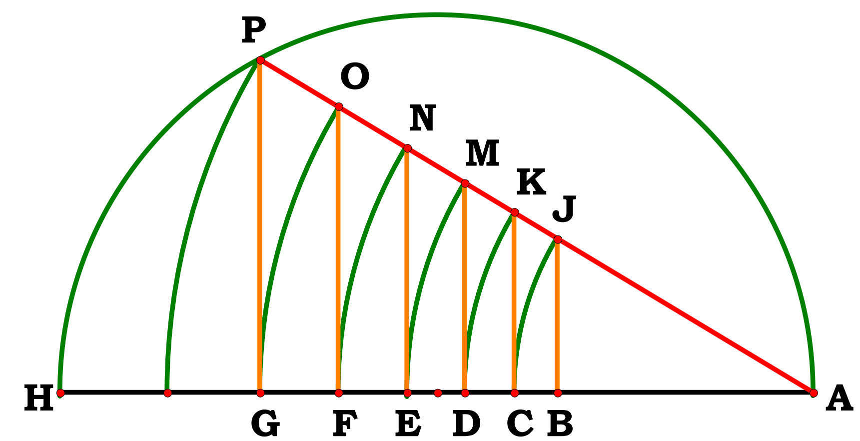
Definitions.

$$\frac{AH}{AP_\Delta} = \quad \left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}} = \quad \left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}} = \quad \left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}} = \quad \left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}} = \quad \left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}} = \quad \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} =$$

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6

$$\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}} = \frac{AC_\Delta}{AB_\Delta} =$$

1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6



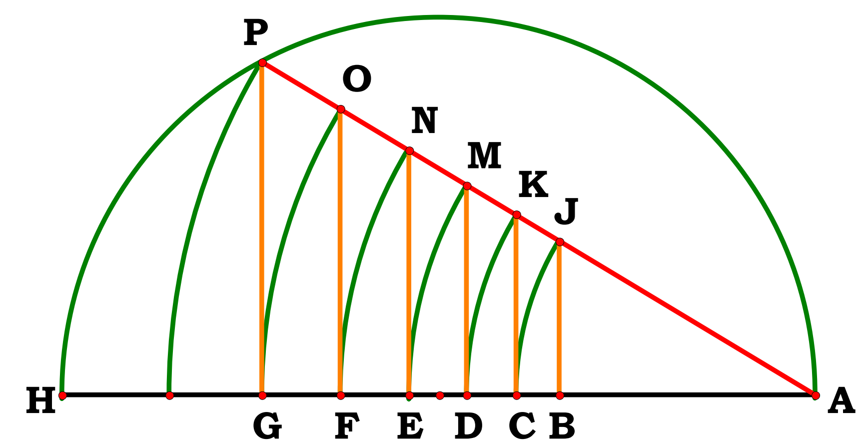
$$\begin{pmatrix} \frac{AH}{1}, \frac{AH}{1^2}, \frac{AH}{1^3}, \frac{AH}{1^4}, \text{etc} \\ \frac{AH}{2}, \frac{AH}{2^2}, \frac{AH}{2^3}, \frac{AH}{2^4}, \text{etc} \\ \frac{AH}{3}, \frac{AH}{3^2}, \frac{AH}{3^3}, \frac{AH}{3^4}, \text{etc} \\ \frac{AH}{4}, \frac{AH}{4^2}, \frac{AH}{4^3}, \frac{AH}{4^4}, \text{etc} \\ \frac{AH}{5}, \frac{AH}{5^2}, \frac{AH}{5^3}, \frac{AH}{5^4}, \text{etc} \end{pmatrix}$$

$AP_\Delta =$	$AG_\Delta =$	$AF_\Delta =$	$AE_\Delta =$	$AD_\Delta =$	$AC_\Delta =$	$AB_\Delta =$
1	1	1	1	1	1	1
0.5	0.25	0.125	0.0625	0.03125	0.015625	$7.8125\cdot10^{-3}$
0.333333	0.111111	0.037037	0.012346	$4.115226\cdot10^{-3}$	$1.371742\cdot10^{-3}$	$4.572474\cdot10^{-4}$
0.25	0.0625	0.015625	$3.90625\cdot10^{-3}$	$9.765625\cdot10^{-4}$	$2.441406\cdot10^{-4}$	$6.103516\cdot10^{-5}$
0.2	0.04	$8\cdot10^{-3}$	$1.6\cdot10^{-3}$	$3.2\cdot10^{-4}$	$6.4\cdot10^{-5}$	$1.28\cdot10^{-5}$
0.166667	0.027778	$4.62963\cdot10^{-3}$	$7.716049\cdot10^{-4}$	$1.286008\cdot10^{-4}$	$2.143347\cdot10^{-5}$	$3.572245\cdot10^{-6}$

$AP_\Delta =$	$AG_\Delta\cdot\frac{AH}{AP_\Delta} =$	$AF_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^2 =$	$AE_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^3 =$	$AD_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^4 =$	$AC_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^5 =$	$AB_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^6 =$
1	1	1	1	1	1	1
0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.333333	0.333333	0.333333	0.333333	0.333333	0.333333	0.333333
0.25	0.25	0.25	0.25	0.25	0.25	0.25
0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.166667	0.166667	0.166667	0.166667	0.166667	0.166667	0.166667

$$\frac{AH^2}{AP_\Delta} = \left(\frac{AH^3}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH^4}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH^5}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH^6}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH^7}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH^8}{AB_\Delta}\right)^{\frac{1}{7}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} \cdot AH =$$

1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6



$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{3}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{3}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{3}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{3}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{3}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{3}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{3} =$$

1	1	1	1	1	1	1
8	8	8	8	8	8	8
27	27	27	27	27	27	27
64	64	64	64	64	64	64
125	125	125	125	125	125	125
216	216	216	216	216	216	216

$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{7}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{7}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{7}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{7}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{7}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{7}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{\frac{7}{1}} =$$

1	1	1	1	1	1	1
128	128	128	128	128	128	128
$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$	$2.187\cdot10^3$
$1.6384\cdot10^4$	$1.6384\cdot10^4$	$1.6384\cdot10^4$	$1.6384\cdot10^4$	$1.6384\cdot10^4$	$1.6384\cdot10^4$	$1.6384\cdot10^4$
$7.8125\cdot10^4$	$7.8125\cdot10^4$	$7.8125\cdot10^4$	$7.8125\cdot10^4$	$7.8125\cdot10^4$	$7.8125\cdot10^4$	$7.8125\cdot10^4$
$2.79936\cdot10^5$	$2.79936\cdot10^5$	$2.79936\cdot10^5$	$2.79936\cdot10^5$	$2.79936\cdot10^5$	$2.79936\cdot10^5$	$2.79936\cdot10^5$

$$GH_\delta := AH - AG_\delta \qquad GP_\delta := \sqrt{AG_\delta \cdot GH_\delta} \qquad FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta} \qquad EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta}$$

$$DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta} \qquad CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta} \qquad BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$$

$\dfrac{GP_\Delta}{\Delta^5} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{FO_\Delta}{\Delta^4} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{EN_\Delta}{\Delta^3} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{DM_\Delta}{\Delta^2} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{CK_\Delta}{\Delta} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$BJ_\Delta =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$GP_\Delta =$

0
0.433013
0.31427
0.242061
0.195959
0.164336

$FO_\Delta =$

0
0.216506
0.104757
0.060515
0.039192
0.027389

$EN_\Delta =$

0
0.108253
0.034919
0.015129
$7.838367 \cdot 10^{-3}$
$4.564876 \cdot 10^{-3}$

$DM_\Delta =$

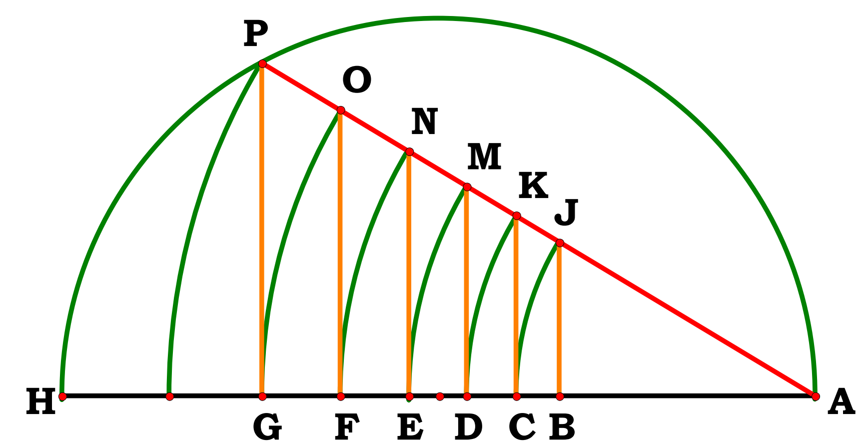
0
0.054127
0.01164
$3.78221 \cdot 10^{-3}$
$1.567673 \cdot 10^{-3}$
$7.608127 \cdot 10^{-4}$

$CK_\Delta =$

0
0.027063
$3.879873 \cdot 10^{-3}$
$9.455526 \cdot 10^{-4}$
$3.135347 \cdot 10^{-4}$
$1.268021 \cdot 10^{-4}$

$BJ_\Delta =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$



$\dfrac{GP_\Delta}{\left(\dfrac{AH}{AP_\Delta}\right)^5} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{FO_\Delta}{\left(\dfrac{AH}{AP_\Delta}\right)^4} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{EN_\Delta}{\left(\dfrac{AH}{AP_\Delta}\right)^3} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\dfrac{DM_\Delta}{\left(\dfrac{AH}{AP_\Delta}\right)^2} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

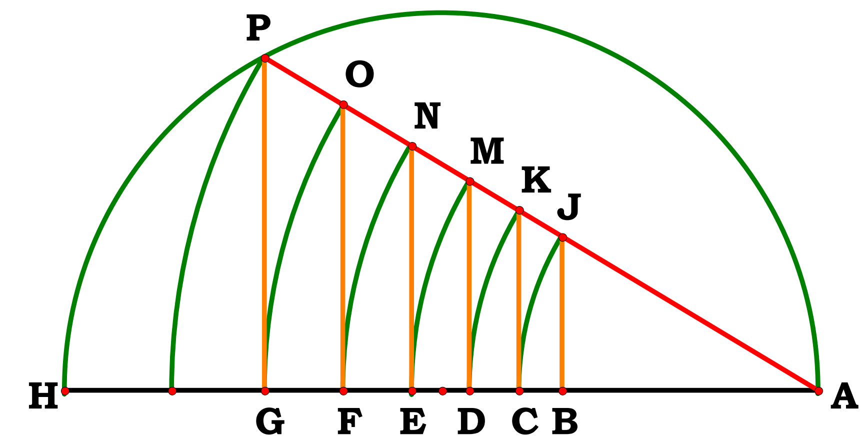
$\dfrac{CK_\Delta}{\dfrac{AH}{AP_\Delta}} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$BJ_\Delta =$

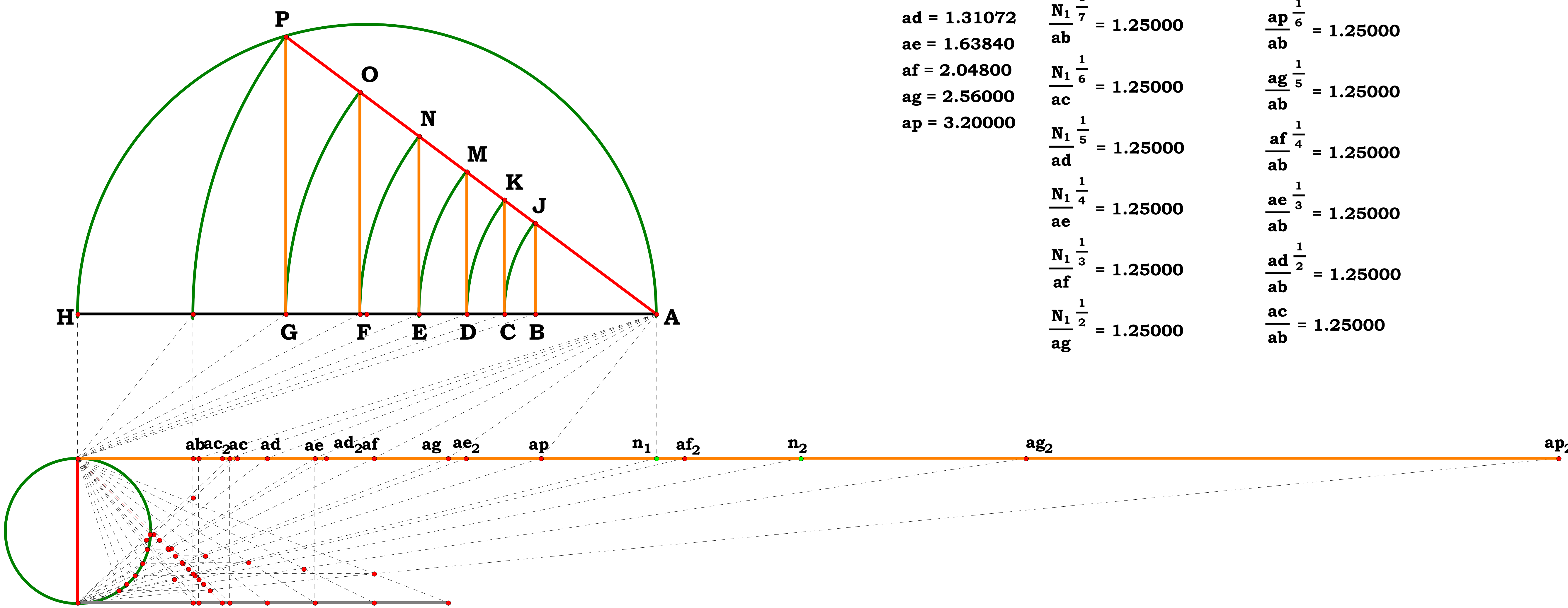
0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

GP_{Δ} =	FO_{Δ} =	EN_{Δ} =	DM_{Δ} =	CK_{Δ} =	BJ_{Δ} =
0	0	0	0	0	0
0.433013	0.216506	0.108253	0.054127	0.027063	0.013532
0.31427	0.104757	0.034919	0.01164	$3.879873 \cdot 10^{-3}$	$1.293291 \cdot 10^{-3}$
0.242061	0.060515	0.015129	$3.78221 \cdot 10^{-3}$	$9.455526 \cdot 10^{-4}$	$2.363881 \cdot 10^{-4}$
0.195959	0.039192	$7.838367 \cdot 10^{-3}$	$1.567673 \cdot 10^{-3}$	$3.135347 \cdot 10^{-4}$	$6.270694 \cdot 10^{-5}$
0.164336	0.027389	$4.564876 \cdot 10^{-3}$	$7.608127 \cdot 10^{-4}$	$1.268021 \cdot 10^{-4}$	$2.113369 \cdot 10^{-5}$



$$\frac{\sqrt{AG_2 \cdot GH_2}}{2} = \mathbf{0.216506} \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^2} = \mathbf{0.108253} \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^3} = \mathbf{0.054127}$$

$$\frac{\sqrt{AG_3 \cdot GH_3}}{3} = \mathbf{0.104757} \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^2} = \mathbf{0.034919} \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^3} = \mathbf{0.01164}$$



$ab = 0.83886$

$ac = 1.04858$

$ad = 1.31072$

$ae = 1.63840$

$af = 2.04800$

$ag = 2.56000$

$ap = 3.20000$

$$\frac{N_1}{ap} = 1.25000$$

$${\frac{N_1}{ab}}^{\frac{1}{7}} = 1.25000$$

$${\frac{N_1}{ac}}^{\frac{1}{6}} = 1.25000$$

$${\frac{N_1}{ad}}^{\frac{1}{5}} = 1.25000$$

$${\frac{N_1}{ae}}^{\frac{1}{4}} = 1.25000$$

$${\frac{N_1}{af}}^{\frac{1}{3}} = 1.25000$$

$${\frac{N_1}{ag}}^{\frac{1}{2}} = 1.25000$$

$${\frac{ap}{ab}}^{\frac{1}{6}} = 1.25000$$

$${\frac{ag}{ab}}^{\frac{1}{5}} = 1.25000$$

$${\frac{af}{ab}}^{\frac{1}{4}} = 1.25000$$

$${\frac{ae}{ab}}^{\frac{1}{3}} = 1.25000$$

$${\frac{ad}{ab}}^{\frac{1}{2}} = 1.25000$$

$$\frac{ac}{ab} = 1.25000$$

120493

Unit.

$AB := 1$

Given.

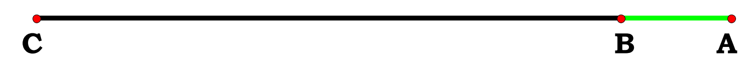
$N_1 := 5 \qquad AC := N_1$

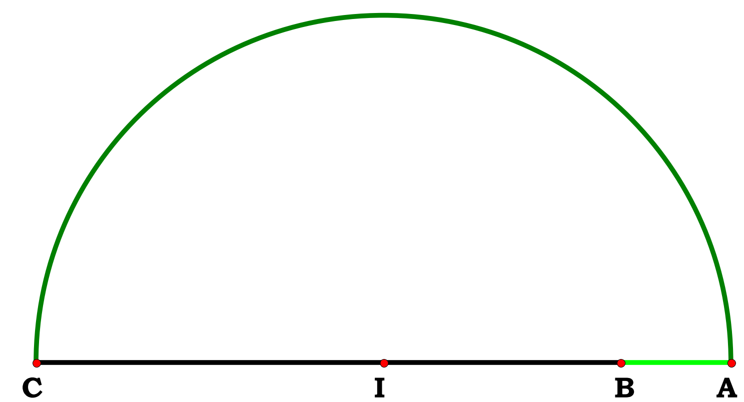
Exponential Series M^(1/2^N)

Descriptions.

To use the digital indexing system to apply names, let AC be the thing with which we seek to name an exponential series on. AB is our unit. As a number is a ratio, numbers are two dimensional.

A number is no more than a digital name used with the stipulation that the indexing system is further qualified by using as standard difference. In other words that the concept of ratio will employ a name called a number. Ratio, however, is independent of the naming convention. Given any ration, say, M, describe a two prime exponential series, M^1/2^N, where N is any whole ratio.



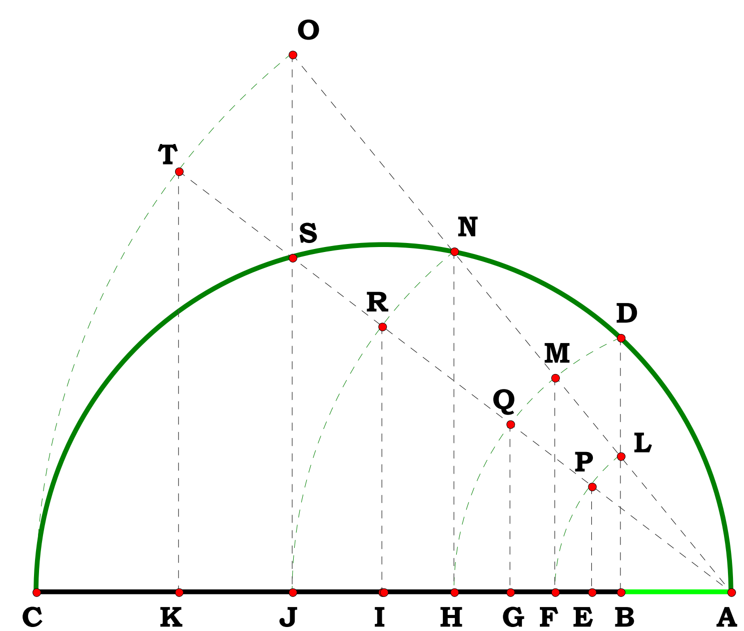


The circle is a two dimensional object which is capable of producing every ratio between two differences.

$BC := AC - AB \qquad BD := \sqrt{BC \cdot AB} \qquad AH := \sqrt{AB^2 + BD^2} \qquad CH := AC - AH$

$HN := \sqrt{CH \cdot AH} \qquad AJ := \sqrt{AH^2 + HN^2} \qquad CJ := AC - AJ \qquad JS := \sqrt{CJ \cdot AJ}$

$AK := \sqrt{JS^2 + AJ^2} \qquad AI := \dfrac{AJ^2}{AK} \qquad AG := \dfrac{AH^2}{AI} \qquad AF := \dfrac{AH^2}{AJ} \qquad AE := \dfrac{AF^2}{AG}$



Definitions.

$AC^0 - AB = 0$

$AC^{\frac{1}{8}} - AE = 0 \qquad AC^{\frac{2}{8}} - AF = 0 \qquad AC^{\frac{3}{8}} - AG = 0 \qquad AC^{\frac{4}{8}} - AH = 0$

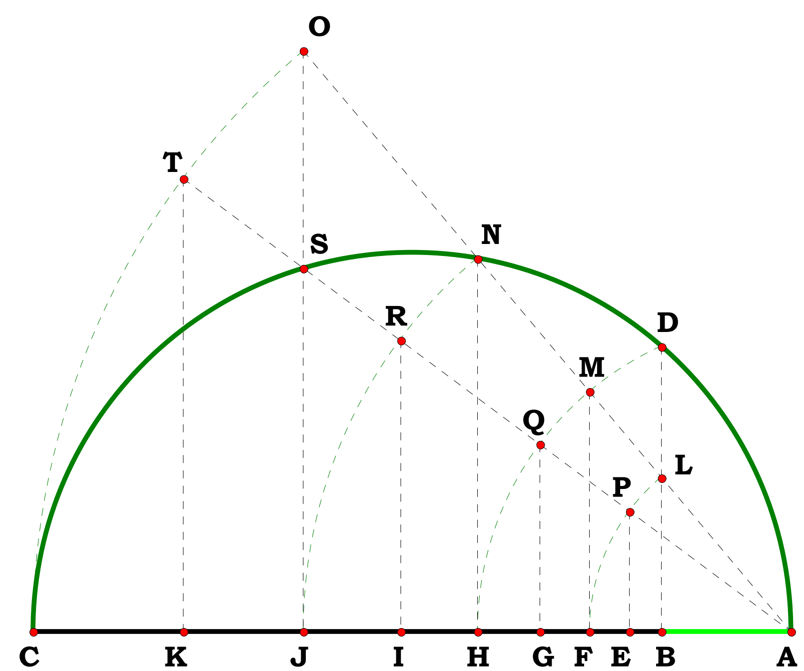
$AC^{\frac{5}{8}} - AI = 0 \qquad AC^{\frac{6}{8}} - AJ = 0 \qquad AC^{\frac{7}{8}} - AK = 0 \qquad AC^{\frac{8}{8}} - AC = 0$

$AC^{\frac{1}{8}} - AE = 0$ $\quad$ $AC^{\frac{2}{8}} - AF = 0$

$AC^{\frac{3}{8}} - AG = 0$ $\quad$ $AC^{\frac{4}{8}} - AH = 0$

$AC^{\frac{5}{8}} - AI = 0$ $\quad$ $AC^{\frac{6}{8}} - AJ = 0$

$AC^{\frac{7}{8}} - AK = 0$ $\quad$ $AC^{\frac{8}{8}} - AC = 0$



$AC^0 = 1.00000$	$N_1 = 1.00000$	$\frac{AB}{AB} = 1.00000$
$AC^{\frac{1}{8}} = 1.24752$	$AE = 1.24752$	$\frac{AE}{AB} = 1.24752$
$AC^{\frac{2}{8}} = 1.55630$	$AF = 1.55630$	$\frac{AF}{AB} = 1.55630$
$AC^{\frac{3}{8}} = 1.94151$	$AG = 1.94151$	$\frac{AG}{AB} = 1.94151$
$AC^{\frac{4}{8}} = 2.42207$	$AH = 2.42207$	$\frac{AH}{AB} = 2.42207$
$AC^{\frac{5}{8}} = 3.02157$	$AI = 3.02157$	$\frac{AI}{AB} = 3.02157$
$AC^{\frac{6}{8}} = 3.76946$	$AJ = 3.76946$	$\frac{AJ}{AB} = 3.76946$
$AC^{\frac{7}{8}} = 4.70247$	$AK = 4.70247$	$\frac{AK}{AB} = 4.70247$
$AC^{\frac{8}{8}} = 5.86641$	$AC = 5.86641$	$\frac{AC}{AB} = 5.86641$

AC = 3.95000 in.
AB = 0.67333 in.
AE = 0.83998 in.
AF = 1.04790 in.
AG = 1.30727 in.
AH = 1.63084 in.
AI = 2.03450 in.
AJ = 2.53807 in.
AK = 3.16629 in.

$AC^0 - N_1 = 0.00000$
$AC^{\frac{1}{8}} - AE = 0.00000$
$AC^{\frac{2}{8}} - AF = 0.00000$
$AC^{\frac{3}{8}} - AG = 0.00000$
$AC^{\frac{4}{8}} - AH = 0.00000$
$AC^{\frac{5}{8}} - AI = 0.00000$
$AC^{\frac{6}{8}} - AJ = 0.00000$
$AC^{\frac{7}{8}} - AK = 0.00000$
$AC^{\frac{8}{8}} - AC = 0.00000$

Alternate method of creating an exponential series.

$N_1 = 2.97175$

$N_1^{\frac{8}{8}} = 2.97175$

$N_1^{\frac{7}{8}} = 2.59350$

$N_1^{\frac{6}{8}} = 2.26339$

$N_1^{\frac{5}{8}} = 1.97530$

$N_1^{\frac{4}{8}} = 1.72388$

$N_1^{\frac{3}{8}} = 1.50446$

$N_1^{\frac{2}{8}} = 1.31296$

$N_1^{\frac{1}{8}} = 1.14585$

$N_1^{\frac{9}{8}} = 3.40517$

$N_1^{\frac{10}{8}} = 3.90181$

$N_1^{\frac{11}{8}} = 4.47087$

$N_1^{\frac{-1}{8}} = 0.87272$

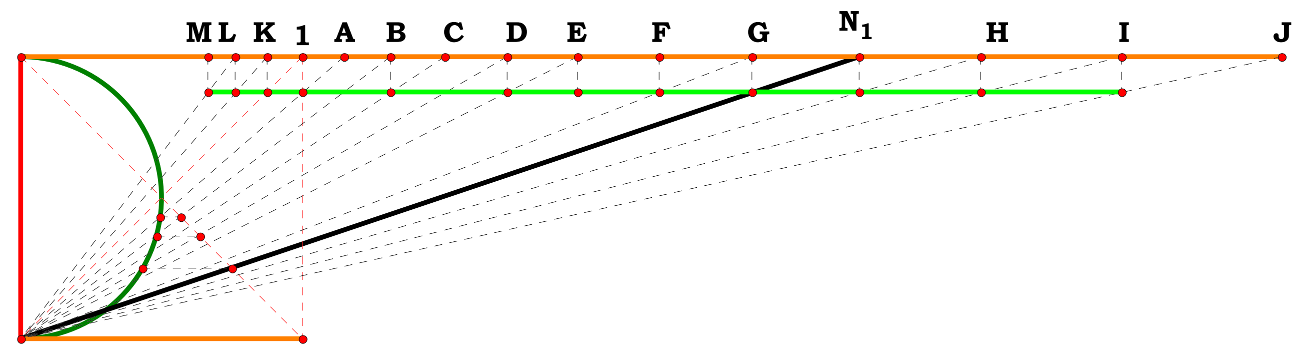
$N_1^{\frac{-2}{8}} = 0.76163$

$N_1^{\frac{-3}{8}} = 0.66469$

A = 1.14585
B = 1.31296
C = 1.50446
D = 1.72388
E = 1.97530
F = 2.26339
G = 2.59350
H = 3.40517
I = 3.90181
J = 4.47087
K = 0.87272
L = 0.76163
M = 0.66469

$N_1^0 = 1.00000$

$N_1^{\frac{8}{8}} - N_1 = 0.00000$



$N_1^{\frac{-1}{8}} - K = 0.00000$

$N_1^{\frac{-2}{8}} - L = 0.00000$

$N_1^{\frac{-3}{8}} - M = 0.00000$

$N_1^{\frac{1}{8}} - A = 0.00000$

$N_1^{\frac{2}{8}} - B = 0.00000$

$N_1^{\frac{3}{8}} - C = 0.00000$

$N_1^{\frac{4}{8}} - D = 0.00000$

$N_1^{\frac{5}{8}} - E = 0.00000$

$N_1^{\frac{6}{8}} - F = 0.00000$

$N_1^{\frac{7}{8}} - G = 0.00000$

$N_1^{\frac{9}{8}} - H = 0.00000$

$N_1^{\frac{10}{8}} - I = 0.00000$

$N_1^{\frac{11}{8}} - J = 0.00000$

Unit.

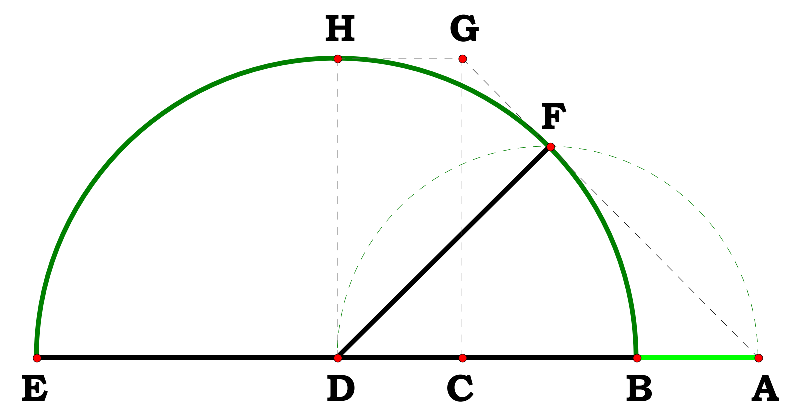
$BE := 1$

Given.

$N_1 := 6$

120693A

Alternate Method: Square Root
Common Segment Common Endpoint



Descriptions.

$AB := N_1$ $\quad AE := AB + BE$ $\quad BD := \dfrac{BE}{2}$

$AE - (N_1 + BE) = 0$ $\quad BD - \dfrac{BE}{2} = 0$ $\quad DF := BD$

$AD := BD + AB$ $\quad DF - \dfrac{BE}{2} = 0$ $\quad AD - \dfrac{2 \cdot N_1 + BE}{2} = 0$

$AF := \sqrt{AD^2 - DF^2}$ $\quad AC := AF$

Definitions.

$\sqrt{AB \cdot AE} - AC = 0$

$AF - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

$AC - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

120693B

Unit.

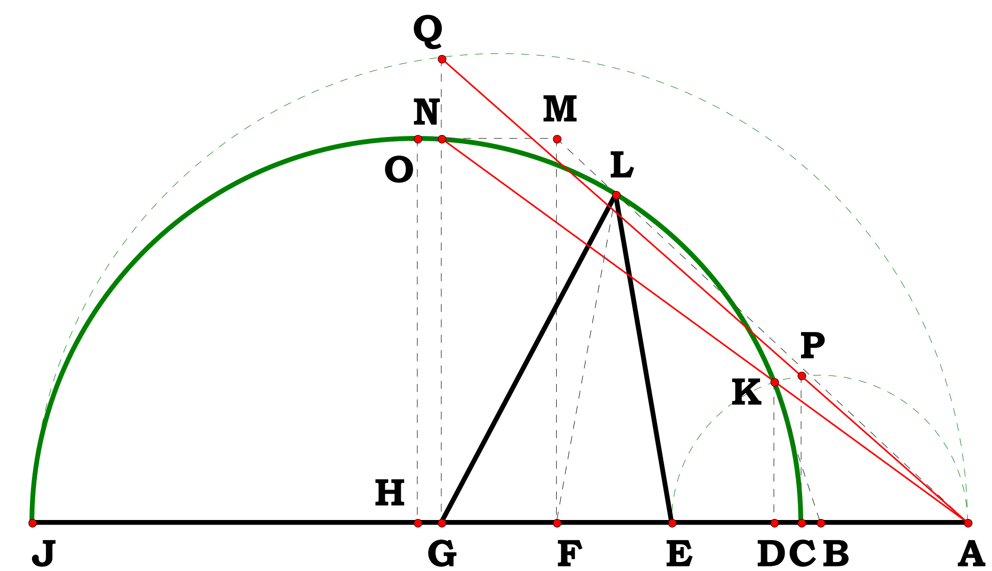
$CJ := 1$

Given.

$N_1 := 5$

Are A, P and Q collinear? Are A, K and N collinear?

Gruntwork IV on the Delian Solution.



Descriptions.

$AC := N_1$ $\quad AJ := AC + CJ \quad AE := \left(AC^2 \cdot AJ\right)^{\frac{1}{3}} \quad AG := \left(AC \cdot AJ^2\right)^{\frac{1}{3}}$

$CG := AG - AC \quad GJ := CJ - CG \quad GN := \sqrt{CG \cdot GJ} \quad AB := \dfrac{AE}{2} \quad CE := AE - AC$

$CH := \dfrac{CJ}{2} \quad BK := AB \quad HK := CH \quad HJ := CH \quad AH := AJ - HJ \quad BH := AH - AB$

$BD := \dfrac{BK^2 + BH^2 - HK^2}{2 \cdot BH} \quad AD := AB + BD \quad DE := AE - AD \quad DK := \sqrt{AD \cdot DE}$

$GQ := \sqrt{AG \cdot GJ} \quad CP := \sqrt{AC \cdot CE} \quad \dfrac{AG}{GN} - \dfrac{AD}{DK} = -8.526 \quad \dfrac{AG}{GQ} - \dfrac{AC}{CP} = 0$

Definitions.

$AJ - \left(N_1 + 1\right) = 0 \quad AE - \left(N_1^{\,3} + N_1^{\,2}\right)^{\frac{1}{3}} = 0 \quad AG - \left(N_1^{\,3} + 2 \cdot N_1^{\,2} + N_1\right)^{\frac{1}{3}} = 0$

$CG - \left[\left[N_1 \cdot \left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - N_1\right] = 0 \quad GJ - \left[N_1 - \left(N_1^{\,3} + 2 \cdot N_1^{\,2} + N_1\right)^{\frac{1}{3}} + 1\right] = 0$

$GN - \sqrt{\left(2 \cdot N_1 + 1\right) \cdot \left(N_1^{\,3} + 2 \cdot N_1^{\,2} + N_1\right)^{\frac{1}{3}} - \left[N_1 + N_1^{\,2} + \left(N_1^{\,3} + 2 \cdot N_1^{\,2} + N_1\right)^{\frac{2}{3}}\right]} = 0$

121193A

Unit.

$AB := 1$

Given.

$N := 6$

Descriptions.

$AL := AB \cdot N \qquad BL := AL - AB \qquad BK := \dfrac{BL}{2} \qquad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ \qquad EJ := AJ - AE \qquad FJ := \dfrac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ \qquad BF := BL - FL \qquad FP := \sqrt{BF \cdot FL} \qquad KR := BK \qquad KL := BK$

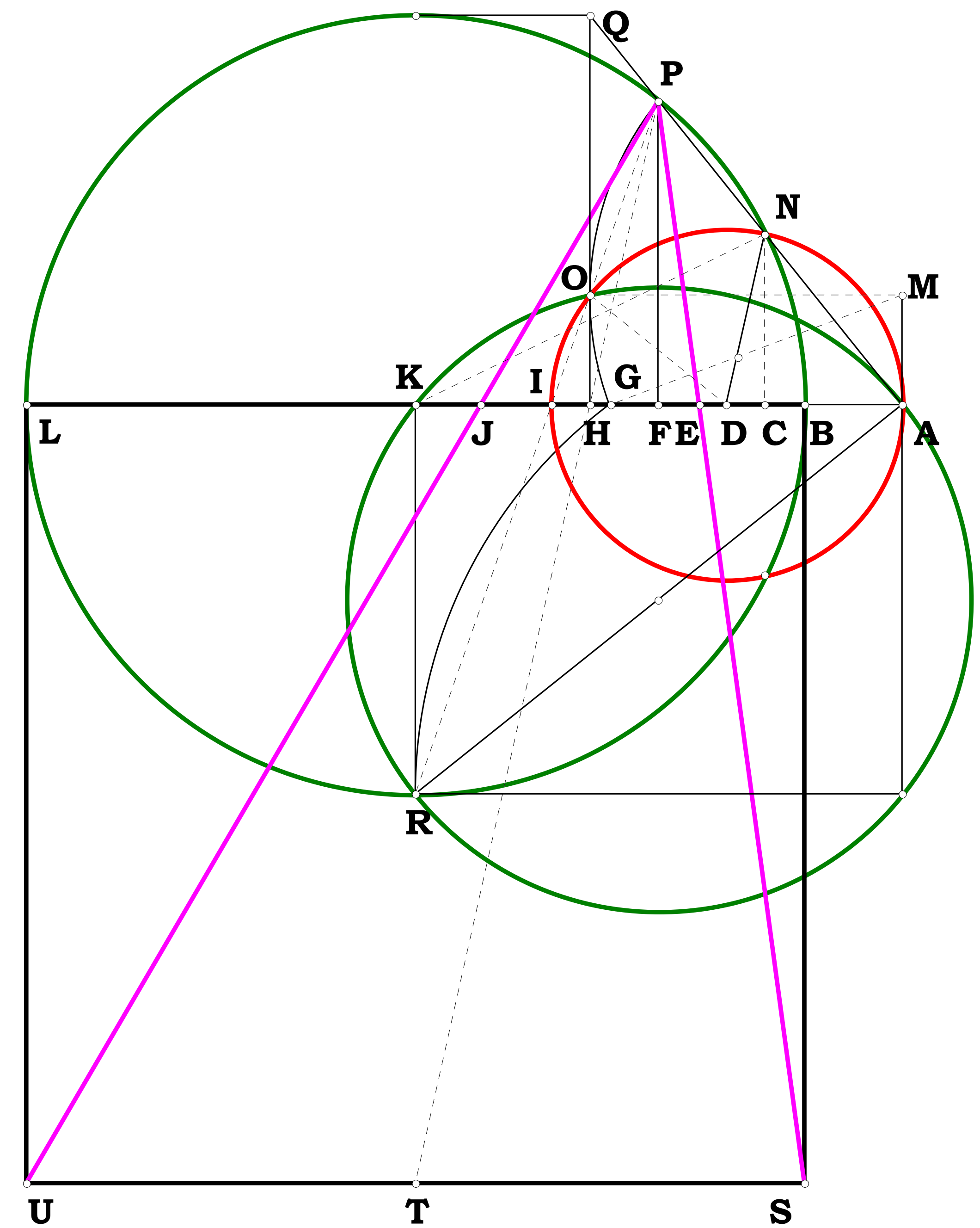
$FK := FL - KL \qquad IK := \dfrac{FK \cdot KR}{KR + FP} \qquad AK := BK + AB \qquad AI := AK - IK \qquad AD := \dfrac{AI}{2}$

$KT := BL \qquad FH := \dfrac{FK \cdot FP}{KT + FP} \qquad AF := BF + AB \qquad AH := AF + FH \qquad HI := AI - AH$

$HO := \sqrt{AH \cdot HI} \qquad DN := AD \qquad KN := BK \qquad DK := AK - AD \qquad CK := \dfrac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK \qquad CI := AI - AC \qquad CN := \sqrt{AC \cdot CI} \qquad \dfrac{KR}{IK} - \dfrac{HO}{HI} = 0 \qquad \dfrac{AF}{FP} - \dfrac{AC}{CN} = 0$

The structure in red appears to be a constant.



Definitions.

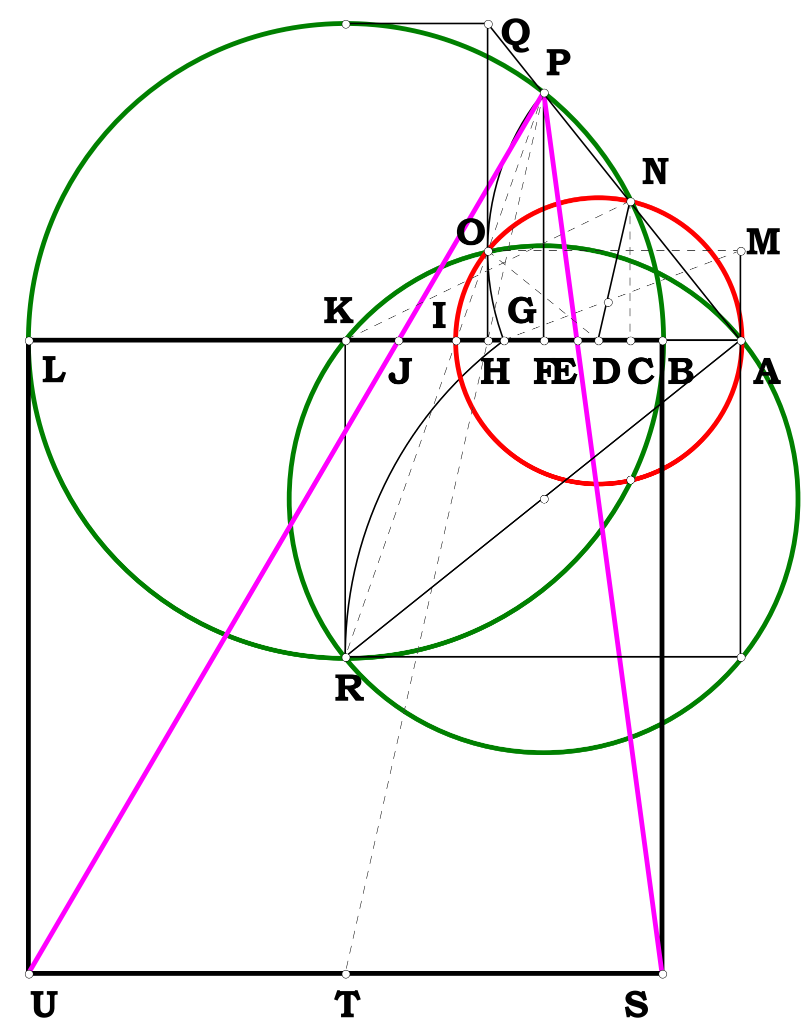
$AL - N = 0 \qquad BL - (N-1) = 0 \qquad BK - \frac{N-1}{2} = 0 \qquad AE - N^{\frac{1}{3}} = 0 \qquad AJ - N^{\frac{2}{3}} = 0$

$BE - \left(N^{\frac{1}{3}} - 1\right) = 0 \qquad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \qquad JL - \left(N - N^{\frac{2}{3}}\right) = 0 \qquad EJ - N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) = 0$

$FJ - \frac{N \cdot \left(N^{\frac{1}{3}} - 1\right)}{N^{\frac{2}{3}} + 1} = 0 \qquad FL - \frac{N^{\frac{2}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0 \qquad BF - \frac{N-1}{N^{\frac{2}{3}} + 1} = 0 \qquad FP - \frac{N^{\frac{1}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0$

$FK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \qquad IK - \frac{\left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$

$AK - \frac{N+1}{2} = 0 \qquad AI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{2}{3}} + 1\right)}{N^{\frac{1}{3}} + 1} = 0 \qquad AD - \frac{N + N^{\frac{1}{3}}}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad FH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \qquad AF - \frac{\left(N^{\frac{1}{3}}\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{2}{3}} + 1} = 0$

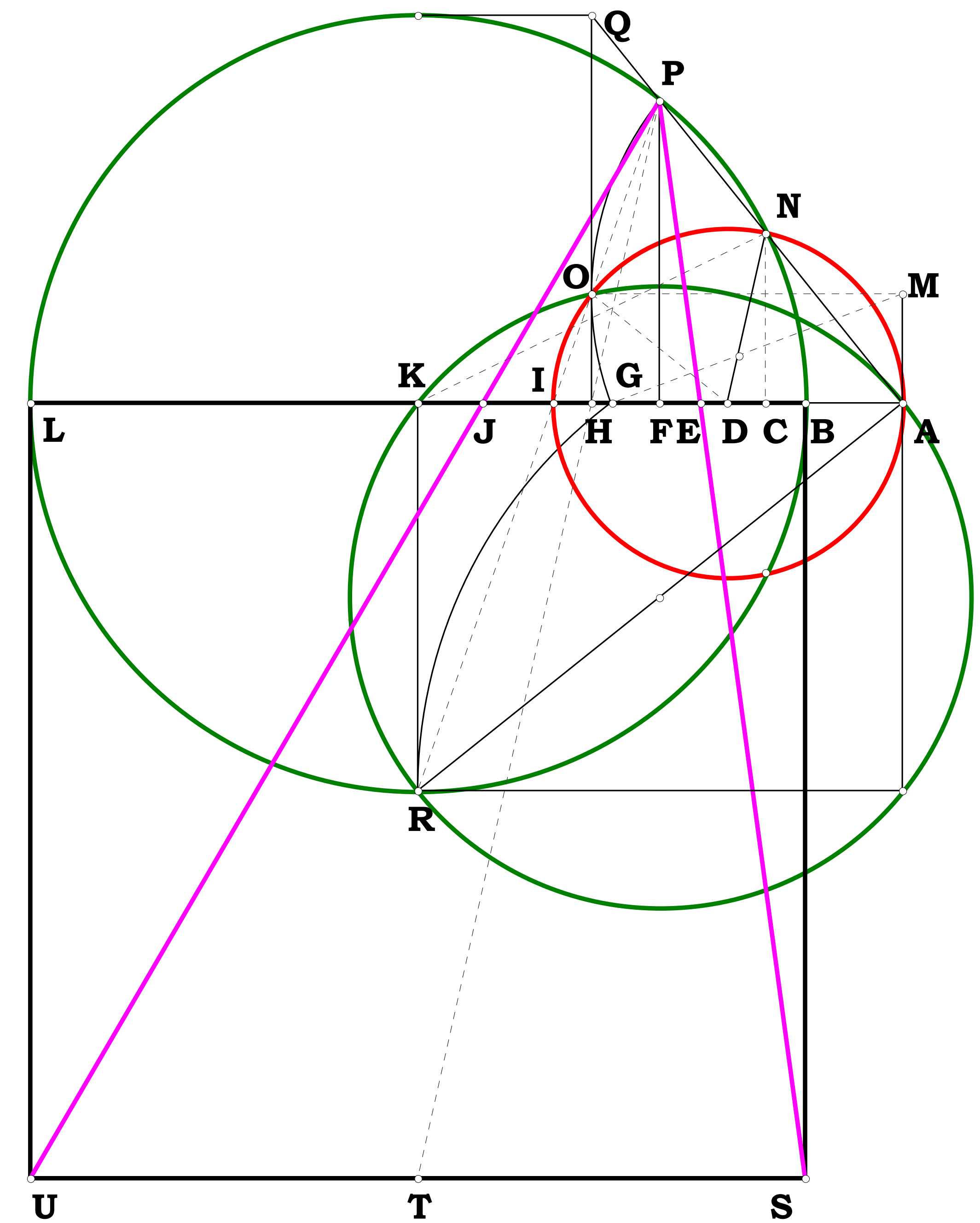


$$AH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{2} = 0 \qquad HI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad HO - \frac{\sqrt{N^{\frac{2}{3}} - 2 \cdot N + N^{\frac{4}{3}}}}{2} = 0$$

$$DK - \frac{N^{\frac{4}{3}} + 1}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad CK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$AC - \frac{\left(N^{\frac{1}{3}}\right)^3 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{4}{3}} + 1} = 0 \qquad CI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)^2}{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$CN - \frac{N^{\frac{2}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)}{\left(N^{\frac{4}{3}} + 1\right)} = 0 \qquad \frac{KR}{IK} - \frac{N^{\frac{1}{3}} + 1}{N^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{N - 1} = 0$$



121193B

Unit.

$AB := 1$

Given.

$Y := 4$

$X := 20$

Descriptions.

$AL := \frac{X}{Y}$ $\quad BL := AL - AB$ $\quad BK := \frac{BL}{2}$ $\quad AE := (AL)^{\frac{1}{3}}$ $\quad AJ := AL^{\frac{2}{3}}$

$BE := AE - AB$ $\quad BJ := AJ - AB$ $\quad JL := BL - BJ$ $\quad EJ := AJ - AE$ $\quad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $\quad BF := BL - FL$ $\quad FP := \sqrt{BF \cdot FL}$ $\quad KR := BK$ $\quad KL := BK$

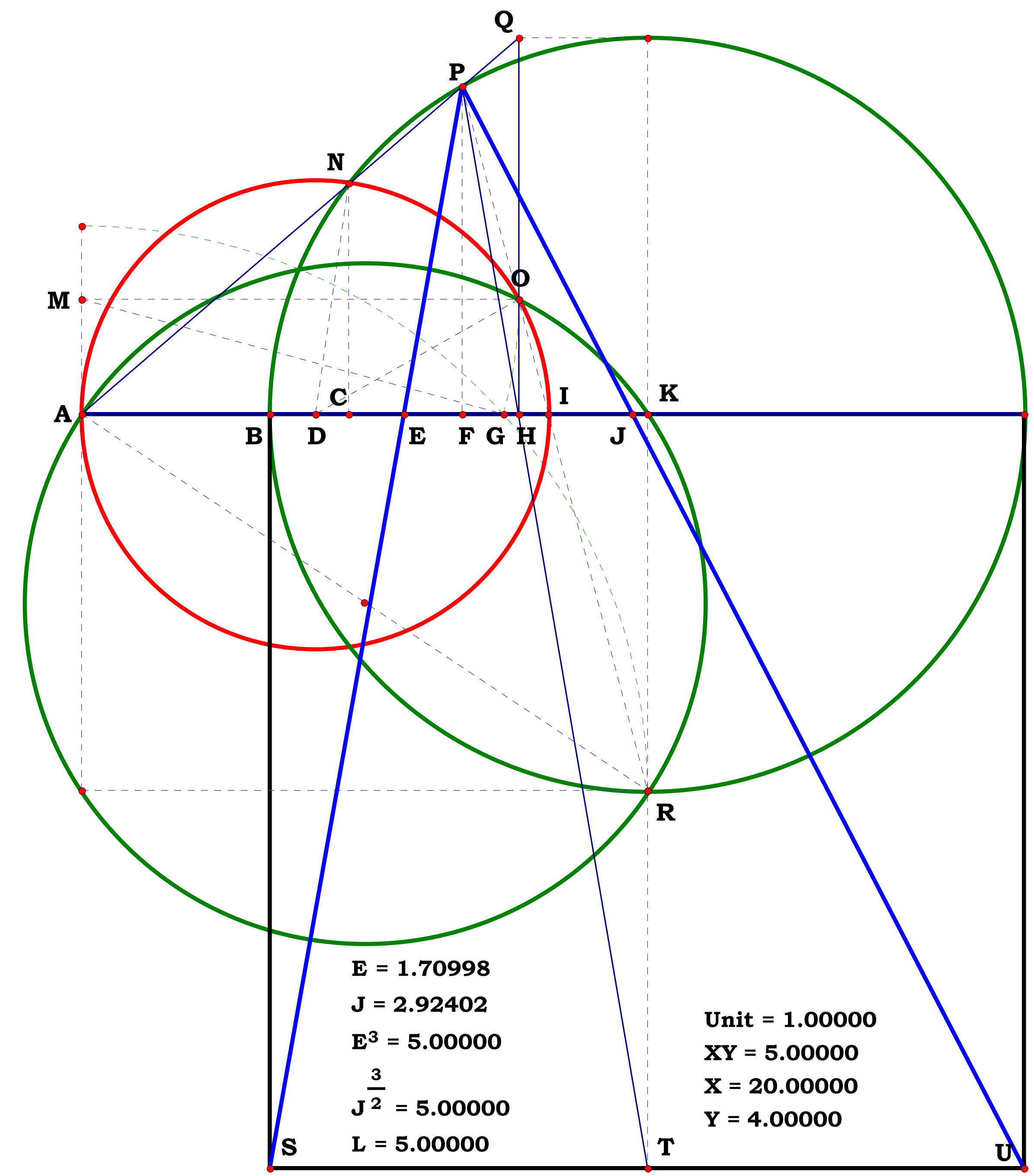
$FK := FL - KL$ $\quad IK := \frac{FK \cdot KR}{KR + FP}$ $\quad AK := BK + AB$ $\quad AI := AK - IK$ $\quad AD := \frac{AI}{2}$

$KT := BL$ $\quad FH := \frac{FK \cdot FP}{KT + FP}$ $\quad AF := BF + AB$ $\quad AH := AF + FH$ $\quad HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $\quad DN := AD$ $\quad KN := BK$ $\quad DK := AK - AD$ $\quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $\quad CI := AI - AC$ $\quad CN := \sqrt{AC \cdot CI}$ $\quad \frac{KR}{IK} - \frac{HO}{HI} = 0$ $\quad \frac{AF}{FP} - \frac{AC}{CN} = 0$

The structure in red appears to be a constant.



Definitions.

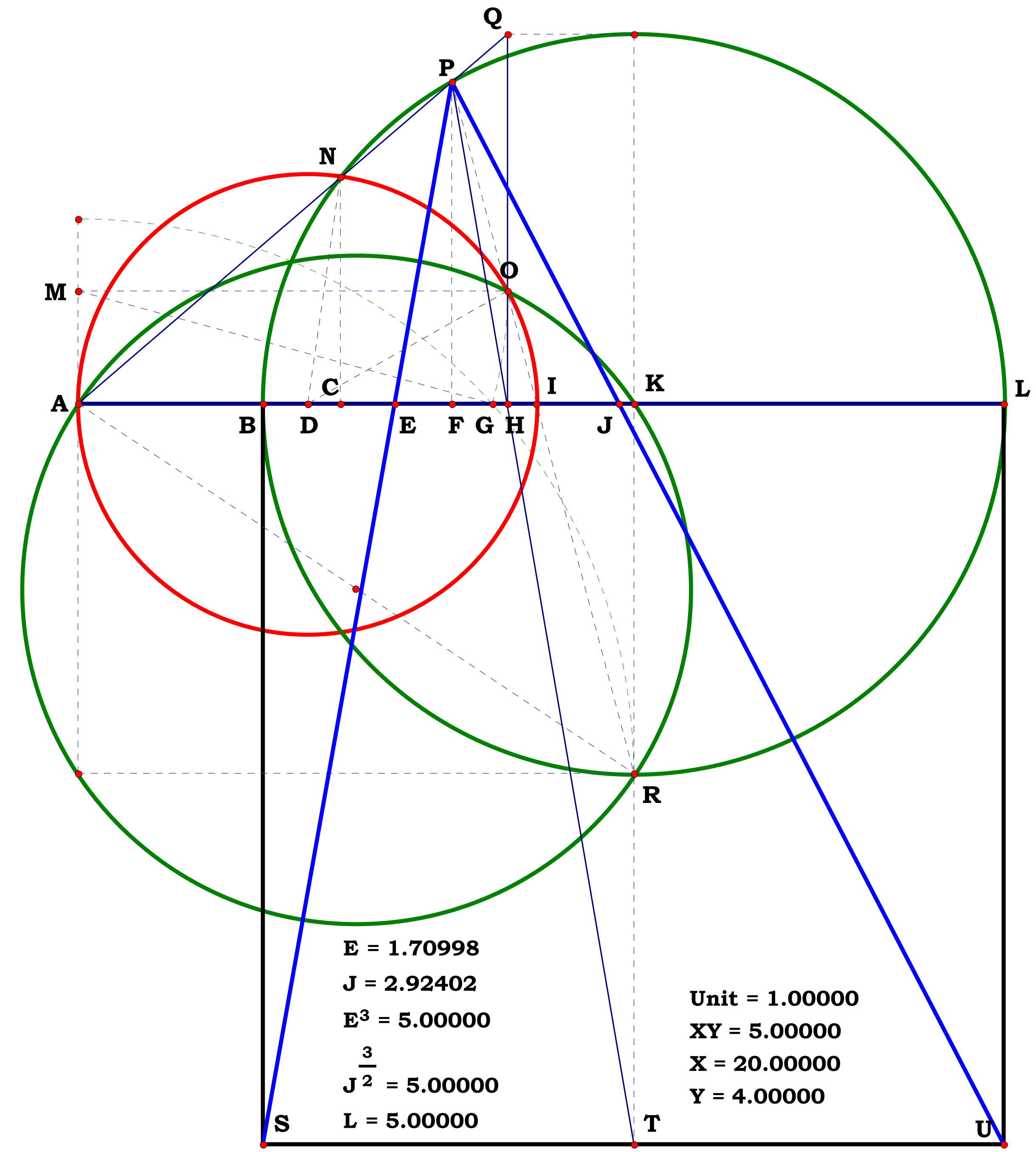
$$AL - \frac{X}{Y} = 0 \qquad BL - \frac{X-Y}{Y} = 0 \qquad BK - \frac{X-Y}{2\cdot Y} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{1}{3}} = 0 \qquad AJ - \left(\frac{X}{Y}\right)^{\frac{2}{3}} = 0$$

$$BE - \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0 \qquad BJ - \left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} - 1\right] = 0 \qquad JL - \left[\frac{X}{Y} - \left(\frac{X}{Y}\right)^{\frac{2}{3}}\right] = 0 \qquad EJ - \left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0$$

$$FJ - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FL - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad BF - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = FP - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

$$FK - \left[\frac{X-Y}{2\cdot Y} - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}\right] = 0 \qquad IK - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]\cdot(X-Y)}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AK - \frac{X+Y}{2\cdot Y} = 0$$

$$AI - \frac{X + Y\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AD - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}{2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad FH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 2} = 0 \qquad AF - \frac{X + Y\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

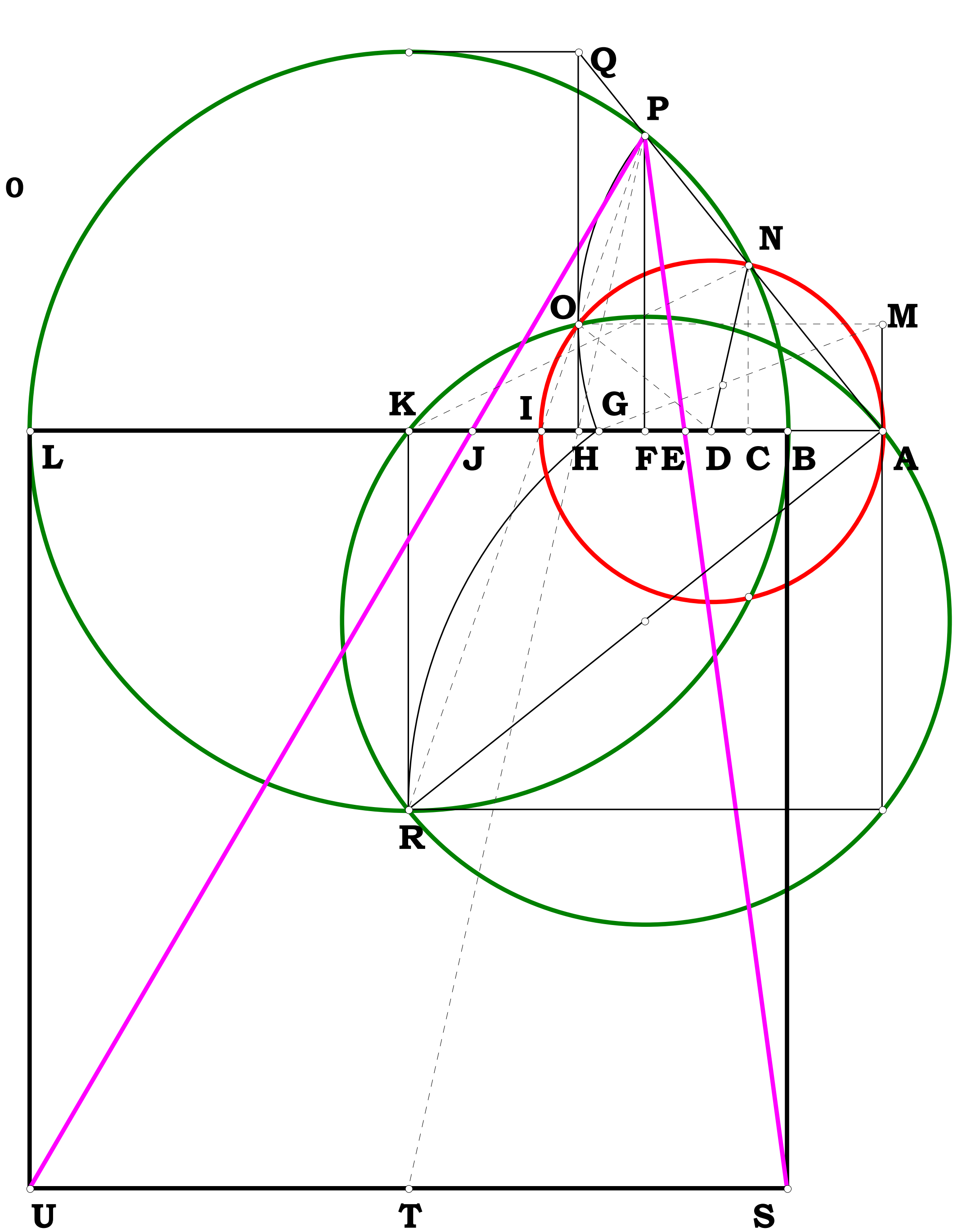


$$AH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2} = 0 \qquad HI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 2} = 0 \qquad HO - \frac{\sqrt{\left(\frac{X}{Y}\right)^{\frac{2}{3}} - \frac{2\cdot X}{Y} + \frac{X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}}}{2} = 0$$

$$DK - \frac{Y + X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad CK - -\frac{(X - Y)\cdot\left[Y - X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}\right]}{2\cdot Y^2\cdot\left[\frac{X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y} + 1\right]} = 0 \qquad AC - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + \left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]^2}{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CN - \frac{(X - Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y + X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}} = 0 \qquad \frac{KR}{IK} - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1}{\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{X - Y} = 0$$



121193C

Unit.

$BL := 1$

Given.

$Y := 5$

$X := 10$

Descriptions.

$AB := 1 \qquad AL := \frac{X}{Y} \qquad BK := \frac{BL}{2} \qquad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ \qquad EJ := AJ - AE \qquad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ \qquad BF := BL - FL \qquad FP := \sqrt{BF \cdot FL} \qquad KR := BK \qquad KL := BK$

$FK := FL - KL \qquad IK := \frac{FK \cdot KR}{KR + FP} \qquad AK := BK + AB \qquad AI := AK - IK \qquad AD := \frac{AI}{2}$

$KT := BL \qquad FH := \frac{FK \cdot FP}{KT + FP} \qquad AF := BF + AB \qquad AH := AF + FH \qquad HI := AI - AH$

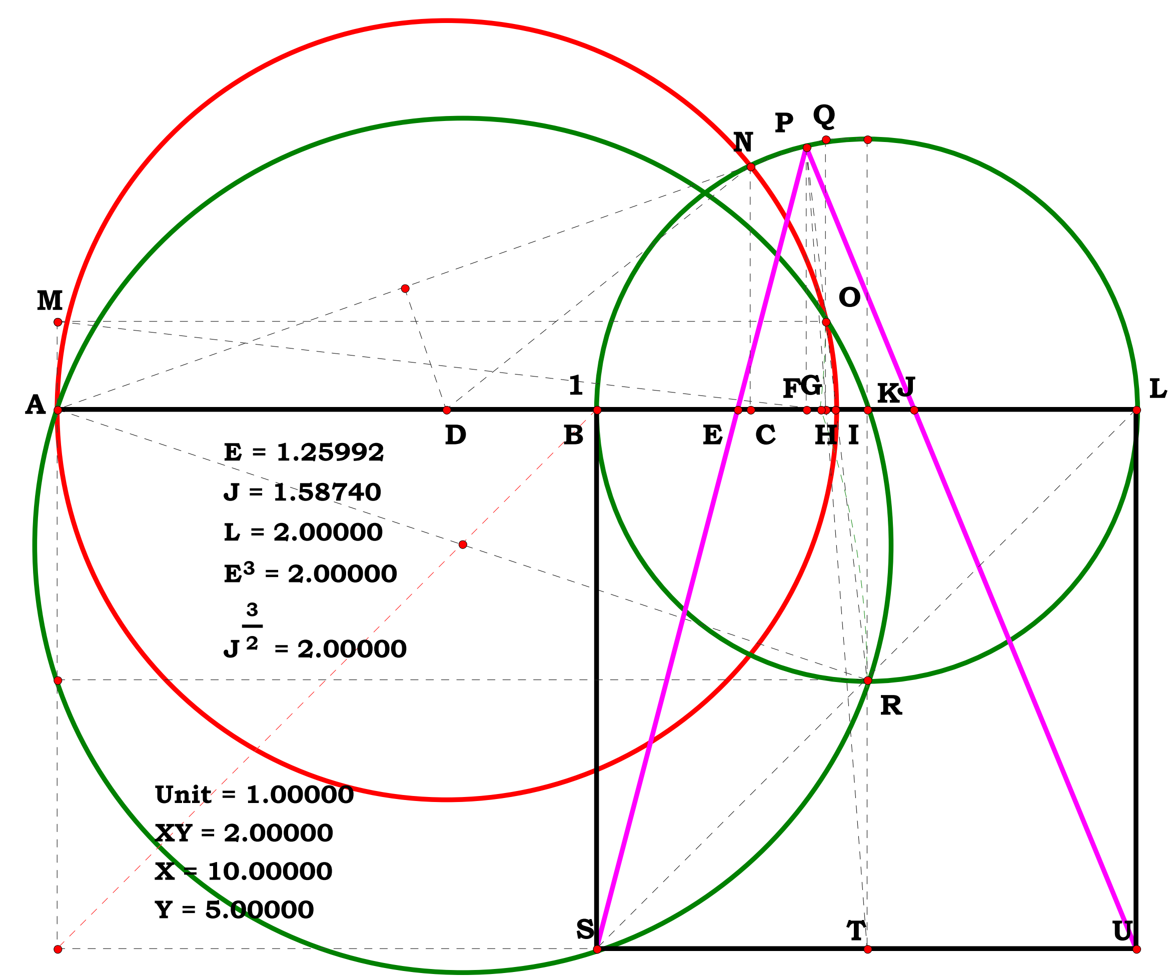
$HO := \sqrt{AH \cdot HI} \qquad DN := AD \qquad KN := BK \qquad DK := AK - AD \qquad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK \qquad CI := AI - AC \qquad CN := \sqrt{AC \cdot CI} \qquad \frac{KR}{IK} - \frac{HO}{HI} = 0 \qquad \frac{AF}{FP} - \frac{AC}{CN} = 0$

$AE = 1.259921 \qquad AE^3 = 2$

$AJ = 1.587401 \qquad AJ^{\frac{3}{2}} = 2$

The structure in red appears to be a constant.



Definitions.

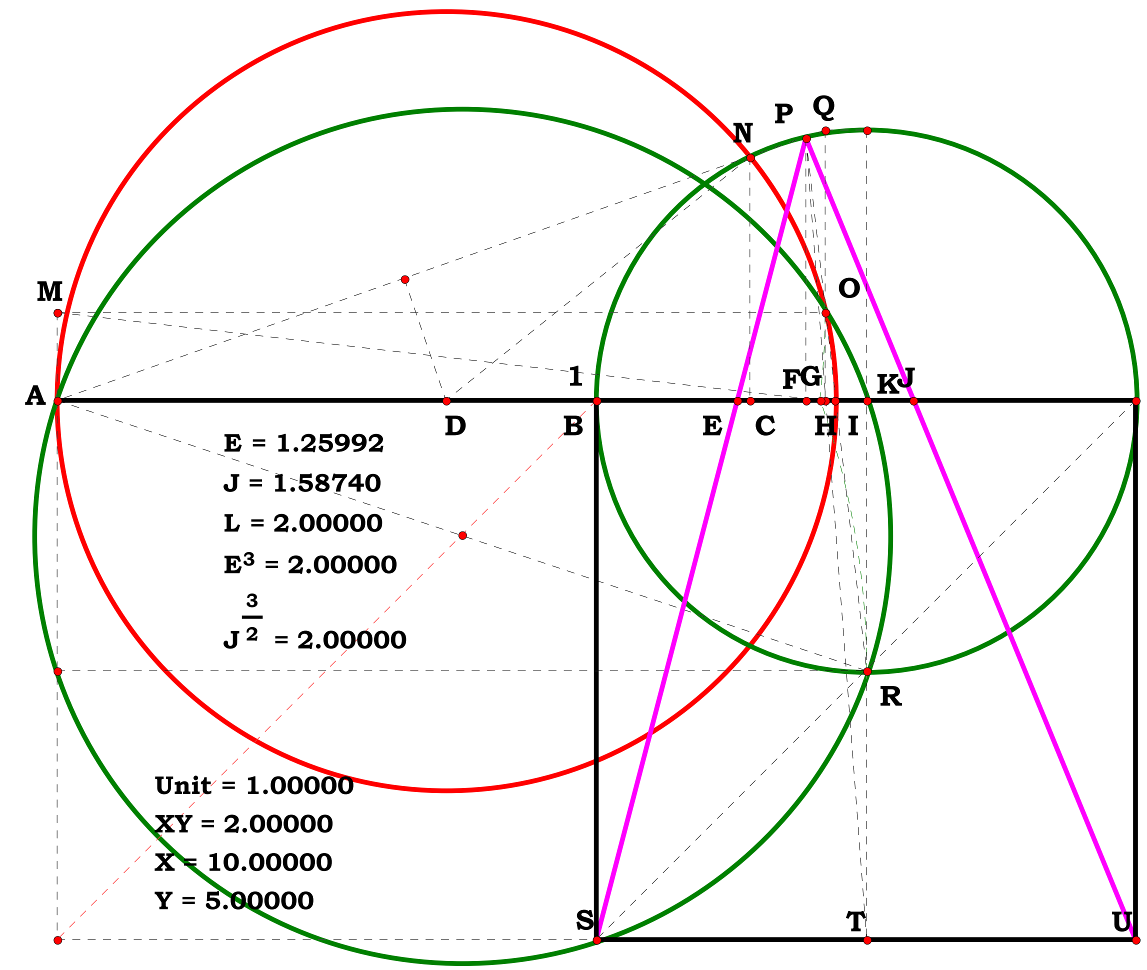
$$AL - \frac{X}{Y} = 0 \qquad BL - \frac{X-Y}{Y} = 0 \qquad BK - \frac{X-Y}{2\cdot Y} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{1}{3}} = 0 \qquad AJ - \left(\frac{X}{Y}\right)^{\frac{2}{3}} = 0$$

$$BE - \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0 \qquad BJ - \left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} - 1\right] = 0 \qquad JL - \left[\frac{X}{Y} - \left(\frac{X}{Y}\right)^{\frac{2}{3}}\right] = 0 \qquad EJ - \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0$$

$$FJ - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FL - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad BF - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = FP - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

$$FK - \left[\frac{X-Y}{2\cdot Y} - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}\right] = 0 \qquad IK - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]\cdot(X-Y)}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AK - \frac{X+Y}{2\cdot Y} = 0$$

$$AI - \frac{X + Y\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AD - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}{2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad FH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 2} = 0 \qquad AF - \frac{X + Y\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

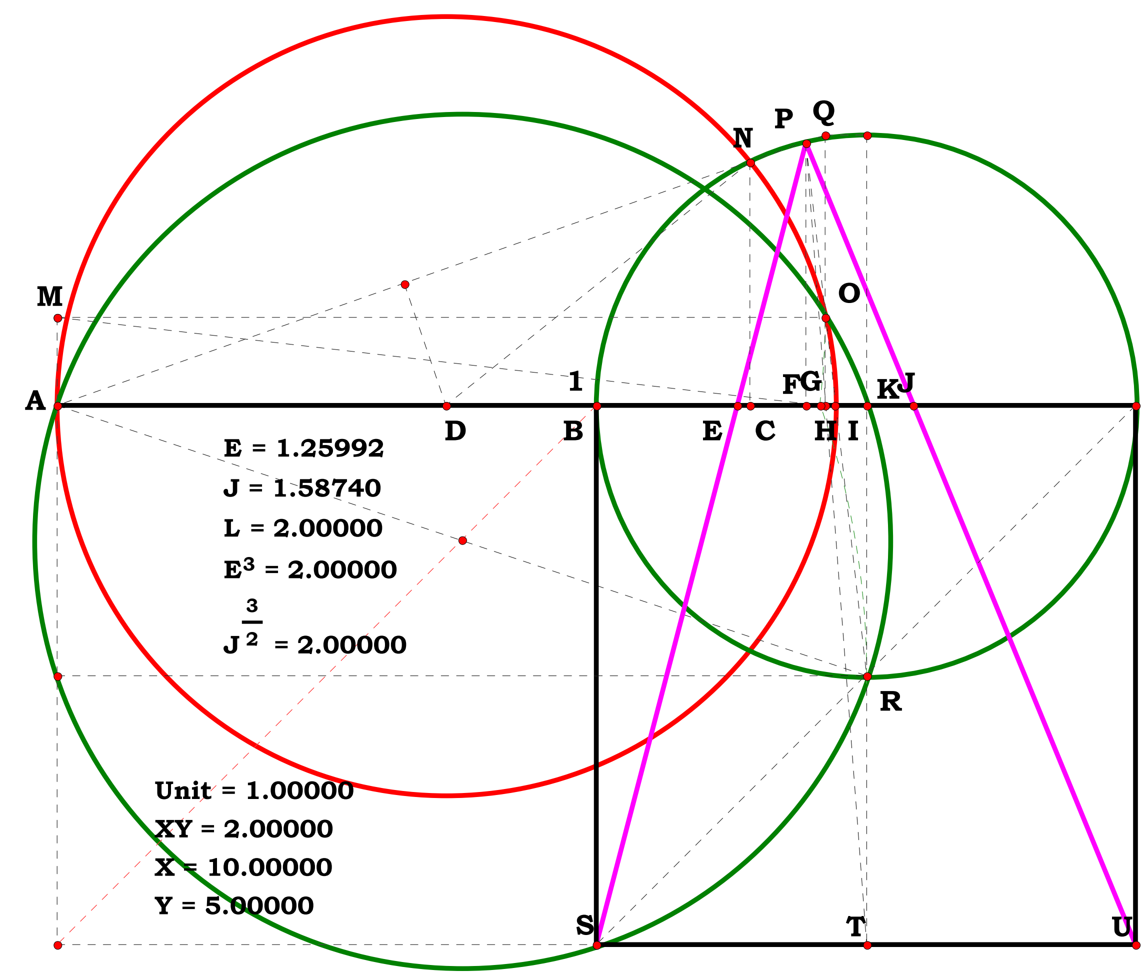


$$AH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}+1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2} = 0 \qquad HI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}-1\right]^2\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}+2} = 0 \qquad HO - \frac{\sqrt{\left(\frac{X}{Y}\right)^{\frac{2}{3}} - \frac{2\cdot X}{Y} + \frac{X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}}}{2} = 0$$

$$DK - \frac{Y + X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}+1\right]} = 0 \qquad CK - -\frac{(X-Y)\cdot\left[Y - X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}\right]}{2\cdot Y^2\cdot\left[\frac{X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}+1\right]} = 0 \qquad AC - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}+1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{4}{3}}+1\right]} = 0$$

$$CI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}-1\right]^2\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}+\left(\frac{X}{Y}\right)^{\frac{2}{3}}+1\right]^2}{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}+1\right]\cdot\left[\left(\frac{X}{Y}\right)^{\frac{4}{3}}+1\right]} = 0$$

$$CN - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y + X\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}} = 0 \qquad \frac{KR}{IK} - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}+1}{\left(\frac{X}{Y}\right)^{\frac{1}{3}}-1} = 0 \qquad \frac{AF}{FP} - \frac{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}}+1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{X-Y} = 0$$



121293A1

Given.

$N_1 := 5$

$N_2 := 3$

Common Segment Common Midpoint

Square root by common segment common midpoint.
Given AFand BE is GH their root?

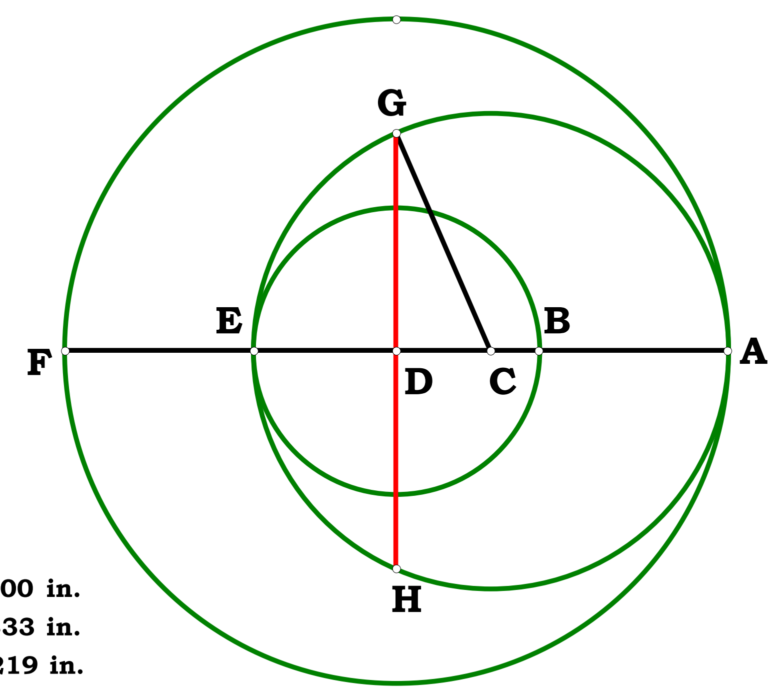
Descriptions.

$AF := N_1 \qquad BE := N_2$

$AD := \dfrac{AF}{2} \qquad BD := \dfrac{BE}{2} \qquad AB := AD - BD$

$AE := BE + AB \qquad AC := \dfrac{AE}{2} \qquad CG := AC$

$CD := AD - AC \qquad GH := 2\cdot\sqrt{CG^2 - CD^2}$



AF = 3.45000 in.
BE = 1.48333 in.
GH = 2.26219 in.
$\sqrt{AF\cdot BE}$-GH = 0.00000 in.

Definitions.

$$GH - \sqrt{AF\cdot BE} = 0$$

$$GH - \sqrt{N_1\cdot N_2} = 0$$

121293A2

Unit.

$AF := 1$

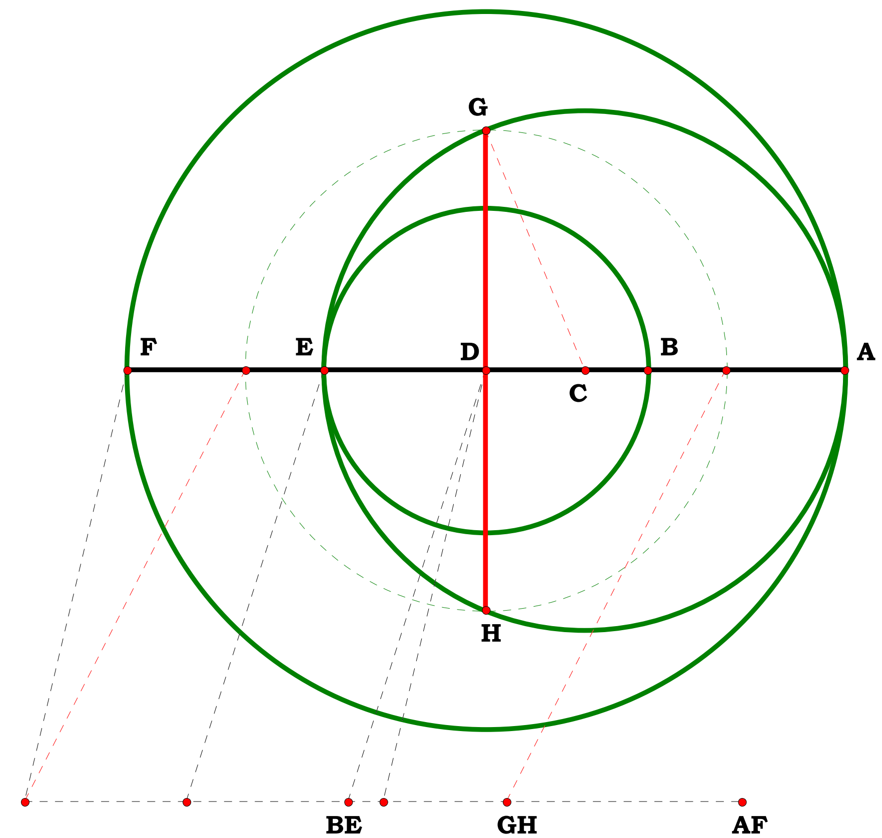
Given.

$Y := 20$

$X := 9$

Common Segment Common Midpoint

Square root by common segment common midpoint.
Given AFand BE is GH their root?



Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000

AF = 1.00000
BE = 0.45000
GH = 0.67082
$\sqrt{BE}$ = 0.67082

Descriptions.

$BE := \frac{X}{Y}$ $\quad AD := \frac{AF}{2}$ $\quad BD := \frac{BE}{2}$

$AB := AD - BD$ $\quad AE := BE + AB$

$AC := \frac{AE}{2}$ $\quad CG := AC$ $\quad CD := AD - AC$

$GH := 2 \cdot \sqrt{CG^2 - CD^2}$ $\quad GH - \sqrt{AF \cdot BE} = 0$

Definitions.

$BE - \frac{X}{Y} = 0$ $\quad AD - \frac{1}{2} = 0$ $\quad BD - \frac{X}{2 \cdot Y} = 0$

$AB - \left(\frac{Y - X}{2 \cdot Y}\right) = 0$ $\quad AE - \left(\frac{X + Y}{2 \cdot Y}\right) = 0$

$AC - \frac{X + Y}{4 \cdot Y} = 0$ $\quad CG - \frac{X + Y}{4 \cdot Y} = 0$

$CD - \frac{Y - X}{4 \cdot Y} = 0$ $\quad GH - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$GH - \sqrt{\frac{X}{Y}} = 0$

121293B1

Given.

$N_1 := 1$

$N_2 := 4$

$N_3 := 3$

Descriptions.

Generalize The Previous Square Root Figure

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

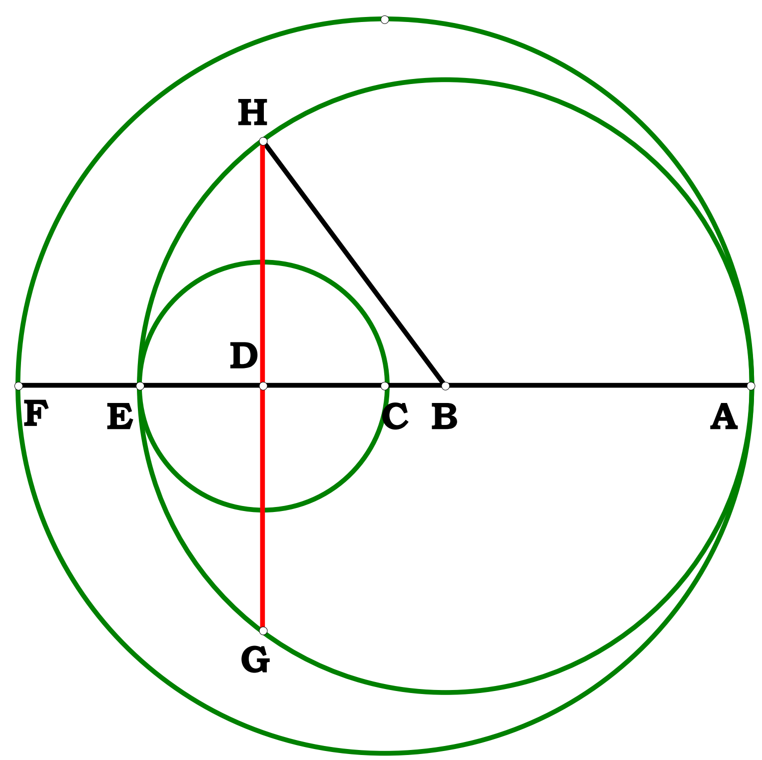
$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

Definitions.

$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$



AF = 3.81667 in.

DF = 1.27449 in.

DE = 0.64116 in.

$\frac{AF}{DF} = 2.99466$

$\frac{DF}{DE} = 1.98779$

$$\frac{2 \cdot AF \cdot \sqrt{\frac{AF}{DF} - 1}}{\frac{AF}{DF} \cdot \sqrt{\frac{DF}{DE}}} = 2.55338 \text{ in.}$$

HG = 2.55338 in.

$$\frac{2 \cdot AF \cdot \sqrt{\frac{AF}{DF} - 1}}{\frac{AF}{DF} \cdot \sqrt{\frac{DF}{DE}}} - HG = 0.00000 \text{ in.}$$

121293B2

Descriptions.

$AF := 1$

Given.

$Y := 20$

$X := 8$

$W := 9$

$V := 3$

Generalize The Previous Square Root Figure

$$DF := \frac{X}{Y} \qquad AD := AF - DF$$

$$DE := DF - \frac{DF \cdot V}{W} \qquad AE := AD + DE$$

$$AB := \frac{AE}{2} \qquad BD := AD - AB \qquad BH := AB$$

$$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2} \qquad GH = 0.8$$

Definitions.

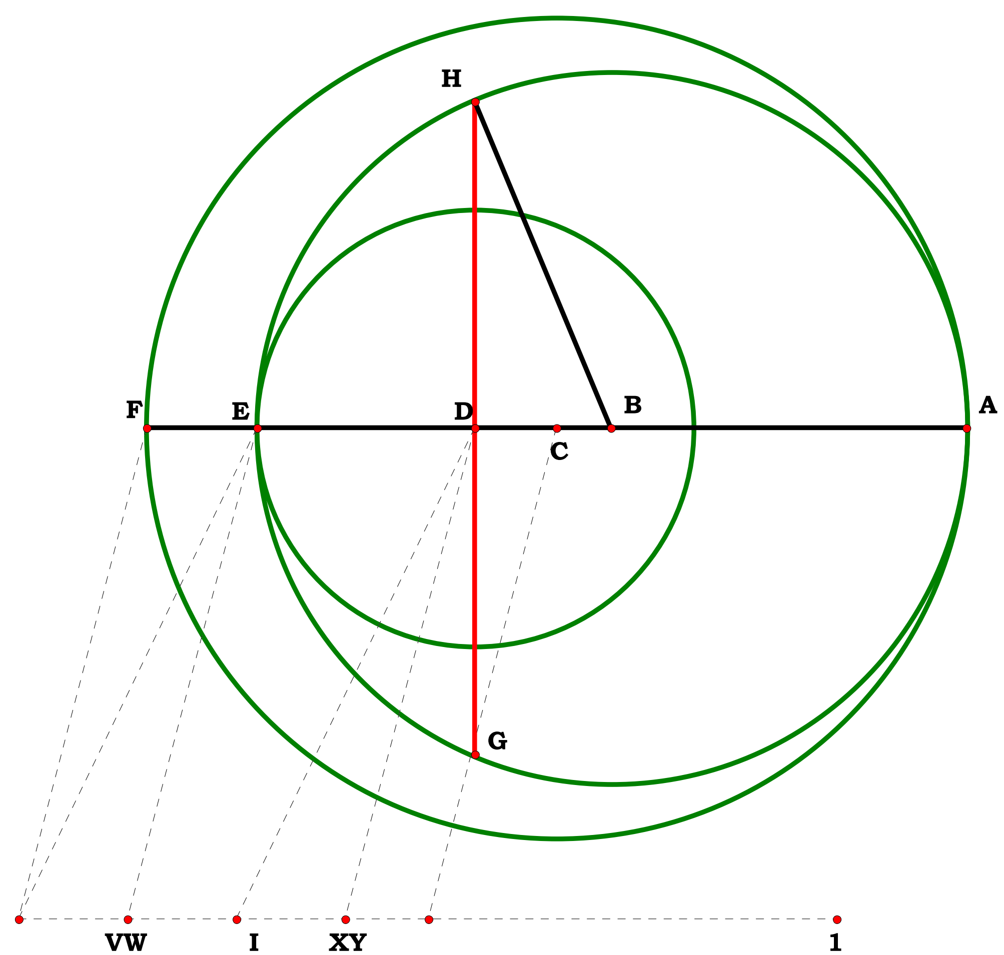
$$DF - \frac{X}{Y} = 0 \quad AD - \frac{Y - X}{Y} = 0 \quad DE - \frac{X \cdot (W - V)}{W \cdot Y}$$

$$AE - \frac{W \cdot Y - V \cdot X}{W \cdot Y} = 0 \qquad AB - \frac{W \cdot Y - V \cdot X}{2 \cdot W \cdot Y} = 0$$

$$BD - \frac{V \cdot X - 2 \cdot W \cdot X + W \cdot Y}{2 \cdot W \cdot Y} = 0$$

$$BH - \frac{W \cdot Y - V \cdot X}{2 \cdot W \cdot Y} = 0 \qquad GH = 0.8$$

$$GH - 2 \cdot \frac{\sqrt{X \cdot (V - W) \cdot (X - Y)}}{\sqrt{W} \cdot Y} = 0$$



Unit = 1.00000
XY = 0.40000
X = 8.00000
Y = 20.00000

VW = 0.33333
V = 3.00000
W = 9.00000

$\frac{Y}{X} = 2.50000$

$\frac{W}{V} = 3.00000$

$\frac{GH}{AF} = 0.80000$

$\frac{2 \cdot \sqrt{X \cdot (V-W) \cdot (X-Y)}}{\sqrt{W} \cdot Y} - \frac{GH}{AF} = 0.00000$

Unit.

$AR := 1$

Using 120493

Given.

$\Delta := 5 \qquad \delta := 2 .. \Delta + 1$

121693A

Descriptions.

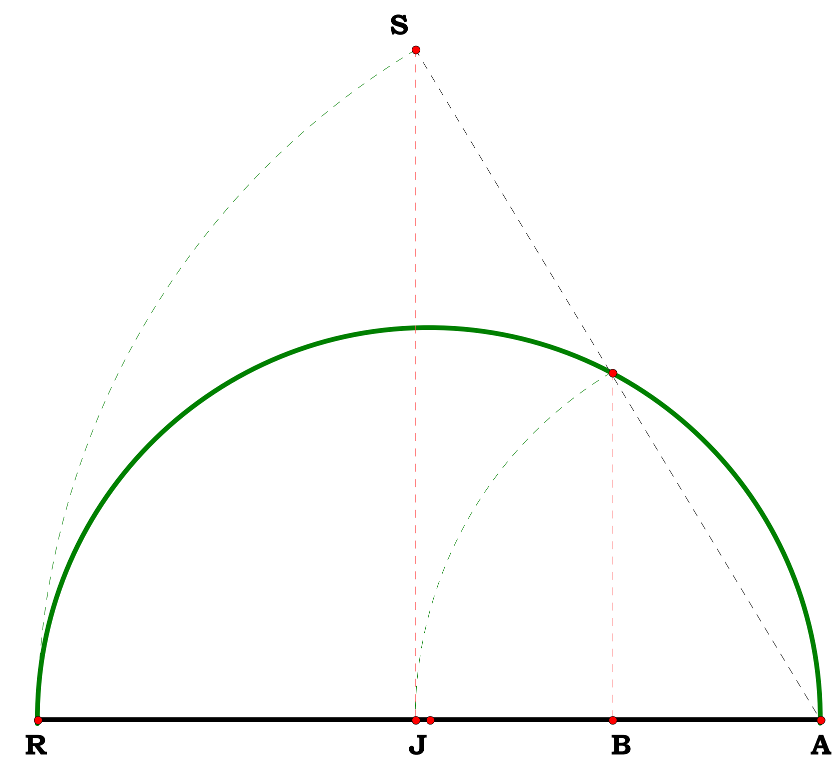
$$AB_\delta := \frac{AR}{\delta}$$

$$AJ_\delta := \sqrt{AB_\delta \cdot AR} \qquad JR_\delta := AR - AJ_\delta$$

$$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \qquad AW_\delta := \sqrt{\left(AJ_\delta\right)^2 + \left(JW_\delta\right)^2}$$

$AB_\delta =$	$AJ_\delta =$
0.5	0.707107
0.333333	0.57735
0.25	0.5
0.2	0.447214
0.166667	0.408248

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.



Euclidean Exponential Series

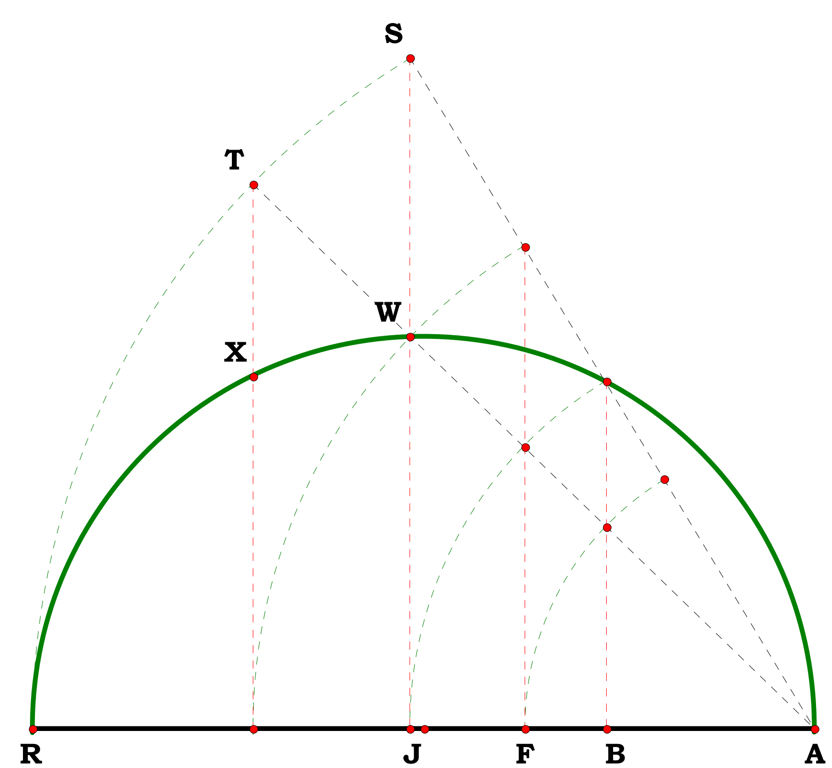
$$AT := AR \qquad AN_\delta := AW_\delta \qquad AF_\delta := \frac{\left(AJ_\delta\right)^2}{AW_\delta}$$

$$NR_\delta := AR - AN_\delta \qquad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$$

$$AX_\delta := \sqrt{\left(AN_\delta\right)^2 + \left(NX_\delta\right)^2}$$

Definitions.

$AB_\delta =$	$AF_\delta =$	$AJ_\delta =$	$AN_\delta =$
0.5	0.594604	0.707107	0.840896
0.333333	0.438691	0.57735	0.759836
0.25	0.353553	0.5	0.707107
0.2	0.29907	0.447214	0.66874
0.166667	0.260847	0.408248	0.638943

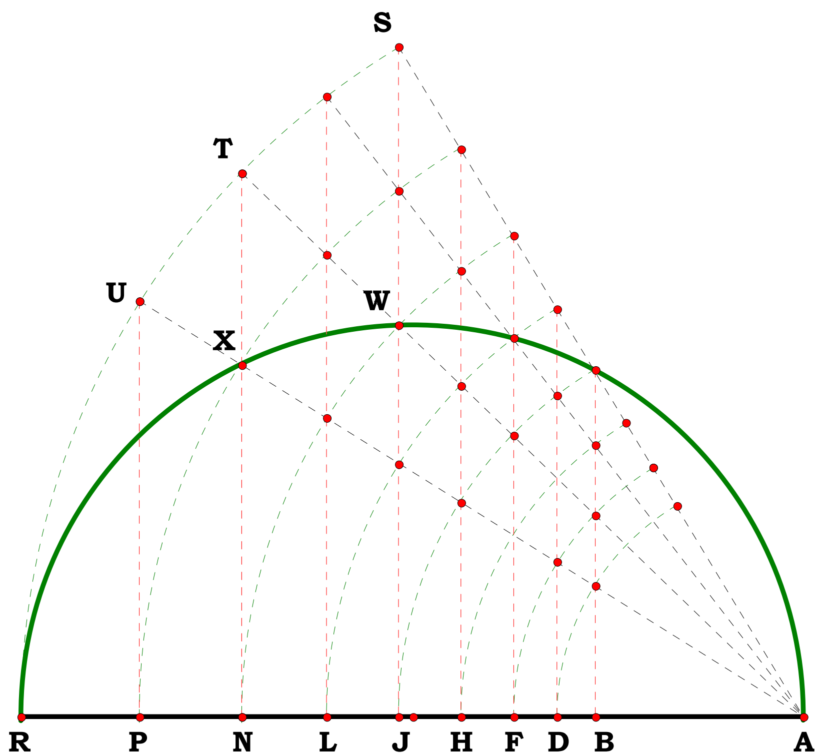


What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.

$$\mathbf{AU} := \mathbf{AR} \qquad \mathbf{AP}_\delta := \mathbf{AX}_\delta \qquad \mathbf{AL}_\delta := \frac{(\mathbf{AN}_\delta)^2}{\mathbf{AX}_\delta} \qquad \mathbf{AH}_\delta := \frac{(\mathbf{AJ}_\delta)^2}{\mathbf{AL}_\delta} \qquad \mathbf{AD}_\delta := \frac{(\mathbf{AF}_\delta)^2}{\mathbf{AH}_\delta}$$

$$\mathbf{PR}_\delta := \mathbf{AR} - \mathbf{AP}_\delta \qquad \mathbf{PY}_\delta := \sqrt{\mathbf{AP}_\delta \cdot \mathbf{PR}_\delta} \qquad \mathbf{AY}_\delta := \sqrt{(\mathbf{AP}_\delta)^2 + (\mathbf{PY}_\delta)^2}$$

$\mathbf{AB}_\delta =$	$\mathbf{AD}_\delta =$	$\mathbf{AF}_\delta =$	$\mathbf{AH}_\delta =$	$\mathbf{AJ}_\delta =$	$\mathbf{AL}_\delta =$	$\mathbf{AN}_\delta =$	$\mathbf{AP}_\delta =$
0.5	0.545254	0.594604	0.64842	0.707107	0.771105	0.840896	0.917004
0.333333	0.382401	0.438691	0.503268	0.57735	0.662338	0.759836	0.871686
0.25	0.297302	0.353553	0.420448	0.5	0.594604	0.707107	0.840896
0.2	0.244569	0.29907	0.365716	0.447214	0.546873	0.66874	0.817765
0.166667	0.208506	0.260847	0.326329	0.408248	0.510732	0.638943	0.799339

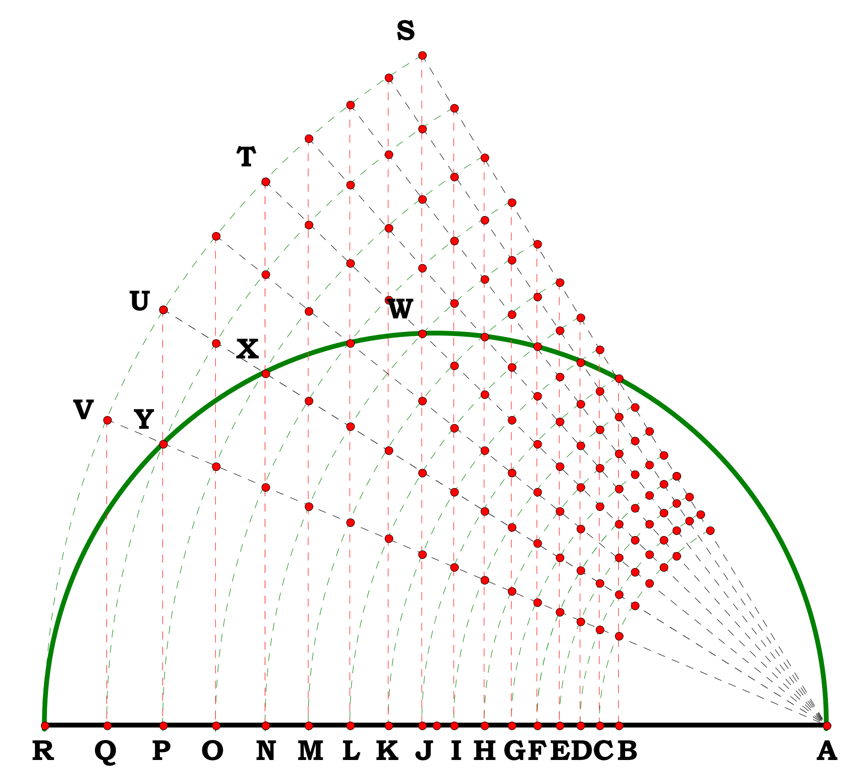


$$\mathbf{AV} := \mathbf{AR} \qquad \mathbf{AQ}_\delta := \mathbf{AY}_\delta \qquad \mathbf{AO}_\delta := \frac{(\mathbf{AP}_\delta)^2}{\mathbf{AY}_\delta} \qquad \mathbf{AM}_\delta := \frac{(\mathbf{AN}_\delta)^2}{\mathbf{AO}_\delta}$$

$$\mathbf{AK}_\delta := \frac{(\mathbf{AL}_\delta)^2}{\mathbf{AM}_\delta} \qquad \mathbf{AI}_\delta := \frac{(\mathbf{AJ}_\delta)^2}{\mathbf{AK}_\delta} \qquad \mathbf{AG}_\delta := \frac{(\mathbf{AH}_\delta)^2}{\mathbf{AI}_\delta}$$

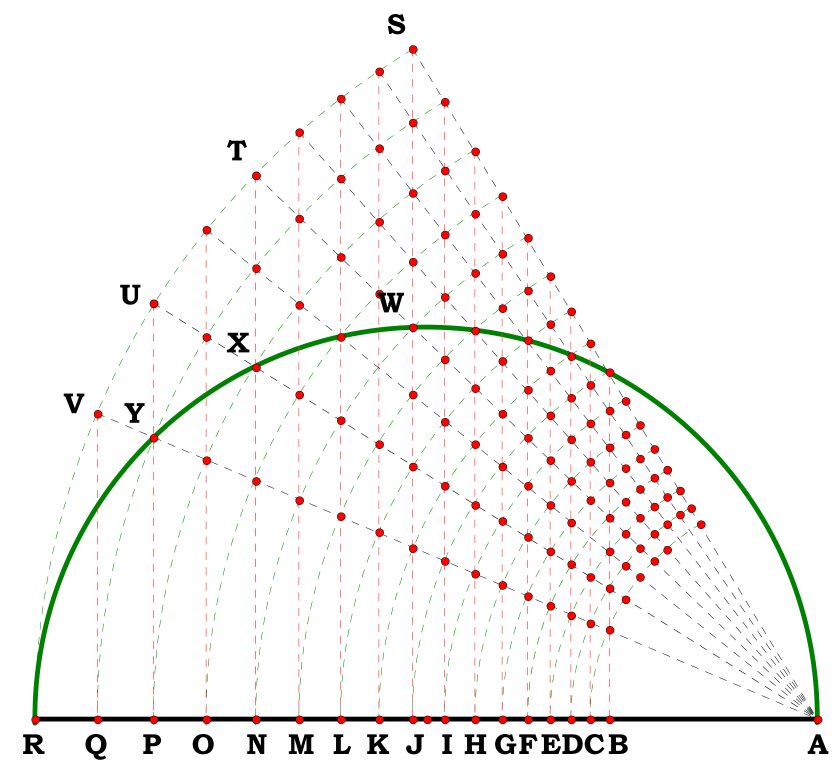
$$\mathbf{AE}_\delta := \frac{(\mathbf{AF}_\delta)^2}{\mathbf{AG}_\delta} \qquad \mathbf{AC}_\delta := \frac{(\mathbf{AD}_\delta)^2}{\mathbf{AE}_\delta}$$

$\mathbf{AB}_\delta =$	$\mathbf{AC}_\delta =$	$\mathbf{AD}_\delta =$	$\mathbf{AE}_\delta =$	$\mathbf{AF}_\delta =$
0.5	0.522137	0.545254	0.569394	0.594604
0.333333	0.357025	0.382401	0.40958	0.438691
0.25	0.272627	0.297302	0.32421	0.353553
0.2	0.221165	0.244569	0.27045	0.29907
0.166667	0.186416	0.208506	0.233213	0.260847



$AG_\delta =$	$AH_\delta =$	$AI_\delta =$	$AJ_\delta =$	$AK_\delta =$	$AL_\delta =$
0.620929	0.64842	0.677128	0.707107	0.738413	0.771105
0.469872	0.503268	0.539038	0.57735	0.618386	0.662338
0.385553	0.420448	0.458502	0.5	0.545254	0.594604
0.330718	0.365716	0.404417	0.447214	0.494539	0.546873
0.291757	0.326329	0.364998	0.408248	0.456624	0.510732

$AM_\delta =$	$AN_\delta =$	$AO_\delta =$	$AP_\delta =$	$AQ_\delta =$
0.805245	0.840896	0.878126	0.917004	0.957603
0.709414	0.759836	0.813841	0.871686	0.933641
0.64842	0.707107	0.771105	0.840896	0.917004
0.604744	0.66874	0.739508	0.817765	0.904304
0.571252	0.638943	0.714655	0.799339	0.894058

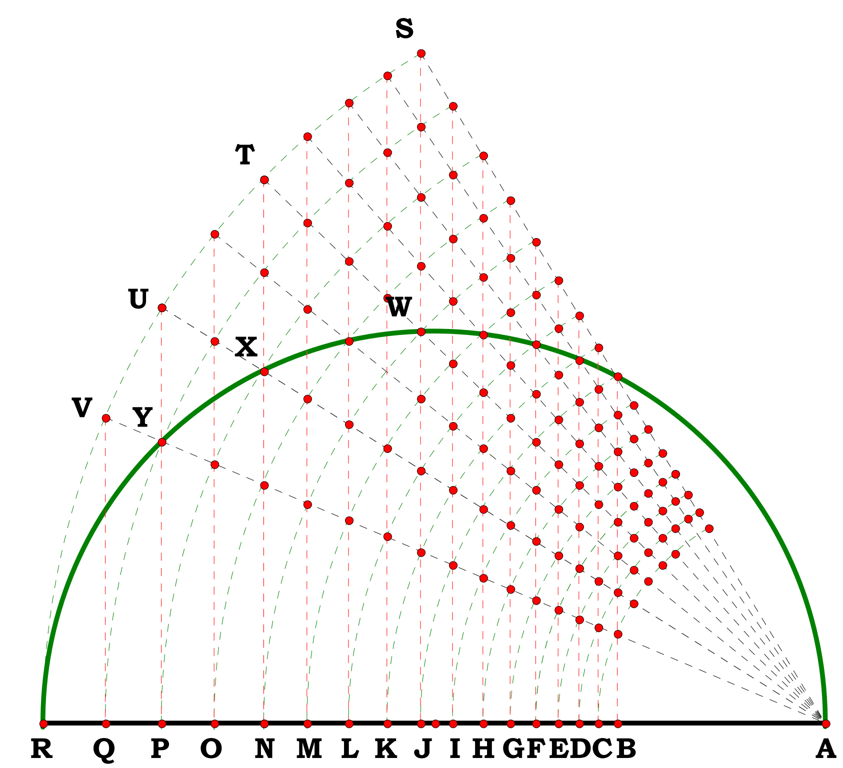


Values found by the investigator of 12_14_93

$AQ_\delta =$	$(AB_\delta)^{\frac{1}{16}} =$	$AP_\delta =$	$(AB_\delta)^{\frac{1}{8}} =$	$AN_\delta =$	$(AB_\delta)^{\frac{1}{4}} =$	$AL_\delta =$	$(AB_\delta)^{\frac{3}{8}} =$	$AI_\delta =$	$(AB_\delta)^{\frac{9}{16}} =$
0.957603	0.957603	0.917004	0.917004	0.840896	0.840896	0.771105	0.771105	0.677128	0.677128
0.933641	0.933641	0.871686	0.871686	0.759836	0.759836	0.662338	0.662338	0.539038	0.539038
0.917004	0.917004	0.840896	0.840896	0.707107	0.707107	0.594604	0.594604	0.458502	0.458502
0.904304	0.904304	0.817765	0.817765	0.66874	0.66874	0.546873	0.546873	0.404417	0.404417
0.894058	0.894058	0.799339	0.799339	0.638943	0.638943	0.510732	0.510732	0.364998	0.364998

$AH_\delta =$	$(AB_\delta)^{\frac{5}{8}} =$	$AG_\delta =$	$(AB_\delta)^{\frac{11}{16}} =$	$AF_\delta =$	$(AB_\delta)^{\frac{3}{4}} =$
0.64842	0.64842	0.620929	0.620929	0.594604	0.594604
0.503268	0.503268	0.469872	0.469872	0.438691	0.438691
0.420448	0.420448	0.385553	0.385553	0.353553	0.353553
0.365716	0.365716	0.330718	0.330718	0.29907	0.29907
0.326329	0.326329	0.291757	0.291757	0.260847	0.260847

$(AB_\delta)^{\frac{13}{16}}$



$AE_\delta = \quad (AB_\delta)^{\frac{13}{16}} =$

0.569394	0.569394
0.40958	0.40958
0.32421	0.32421
0.27045	0.27045
0.233213	0.233213

$AD_\delta = \quad (AB_\delta)^{\frac{7}{8}} =$

0.545254	0.545254
0.382401	0.382401
0.297302	0.297302
0.244569	0.244569
0.208506	0.208506

$AC_\delta = \quad (AB_\delta)^{\frac{15}{16}} =$

0.522137	0.522137
0.357025	0.357025
0.272627	0.272627
0.221165	0.221165
0.186416	0.186416

$AQ_\delta = \quad (AB_\delta)^{\frac{1}{16}} =$

0.957603	0.957603
0.933641	0.933641
0.917004	0.917004
0.904304	0.904304
0.894058	0.894058

$AP_\delta = \quad (AB_\delta)^{\frac{2}{16}} =$

0.917004	0.917004
0.871686	0.871686
0.840896	0.840896
0.817765	0.817765
0.799339	0.799339

$AO_\delta = \quad (AB_\delta)^{\frac{3}{16}} =$

0.878126	0.878126
0.813841	0.813841
0.771105	0.771105
0.739508	0.739508
0.714655	0.714655

$AN_\delta = \quad (AB_\delta)^{\frac{4}{16}} =$

0.840896	0.840896
0.759836	0.759836
0.707107	0.707107
0.66874	0.66874
0.638943	0.638943

$AM_\delta = \quad (AB_\delta)^{\frac{5}{16}} =$

0.805245	0.805245
0.709414	0.709414
0.64842	0.64842
0.604744	0.604744
0.571252	0.571252

$AL_\delta = \quad (AB_\delta)^{\frac{6}{16}} =$

0.771105	0.771105
0.662338	0.662338
0.594604	0.594604
0.546873	0.546873
0.510732	0.510732

$AK_\delta = \quad (AB_\delta)^{\frac{7}{16}} =$

0.738413	0.738413
0.618386	0.618386
0.545254	0.545254
0.494539	0.494539
0.456624	0.456624

$AJ_\delta = \quad (AB_\delta)^{\frac{8}{16}} =$

0.707107	0.707107
0.57735	0.57735
0.5	0.5
0.447214	0.447214
0.408248	0.408248

$AI_\delta = \quad (AB_\delta)^{\frac{9}{16}} =$

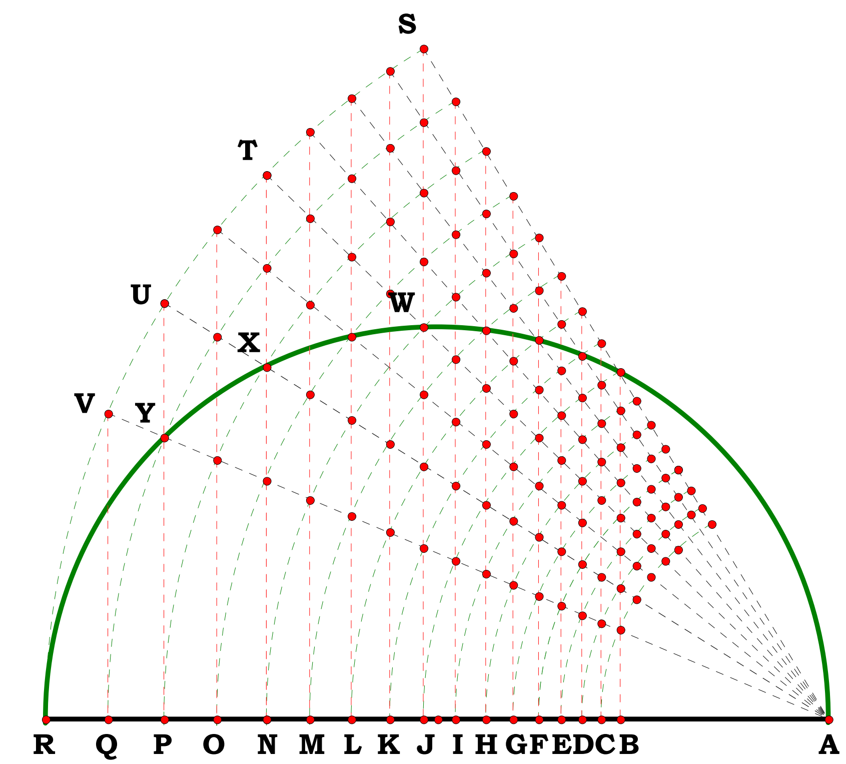
0.677128	0.677128
0.539038	0.539038
0.458502	0.458502
0.404417	0.404417
0.364998	0.364998

$AH_\delta = \quad (AB_\delta)^{\frac{10}{16}} =$

0.64842	0.64842
0.503268	0.503268
0.420448	0.420448
0.365716	0.365716
0.326329	0.326329

$AG_{\delta} =$	$\left(AB_{\delta}\right)^{\frac{11}{16}} =$
0.620929	0.620929
0.469872	0.469872
0.385553	0.385553
0.330718	0.330718
0.291757	0.291757

$AF_{\delta} =$	$\left(AB_{\delta}\right)^{\frac{12}{16}} =$
0.594604	0.594604
0.438691	0.438691
0.353553	0.353553
0.29907	0.29907
0.260847	0.260847



$AE_{\delta} =$	$\left(AB_{\delta}\right)^{\frac{13}{16}} =$
0.569394	0.569394
0.40958	0.40958
0.32421	0.32421
0.27045	0.27045
0.233213	0.233213

$AD_{\delta} =$	$\left(AB_{\delta}\right)^{\frac{14}{16}} =$
0.545254	0.545254
0.382401	0.382401
0.297302	0.297302
0.244569	0.244569
0.208506	0.208506

Resultant Equation

$$\left(A^{\delta}\cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

or

$$\left(A^{DIV-\delta}\cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

depending on direction of transcription.

$AC_{\delta} =$	$\left[\left(AB_{\delta}\right)^{\frac{15}{16}}\right] =$
0.522137	0.522137
0.357025	0.357025
0.272627	0.272627
0.221165	0.221165
0.186416	0.186416

$AB_{\delta} =$	$\left[\left(AB_{\delta}\right)^{\frac{16}{16}}\right] =$
0.5	0.5
0.333333	0.333333
0.25	0.25
0.2	0.2
0.166667	0.166667

Given.

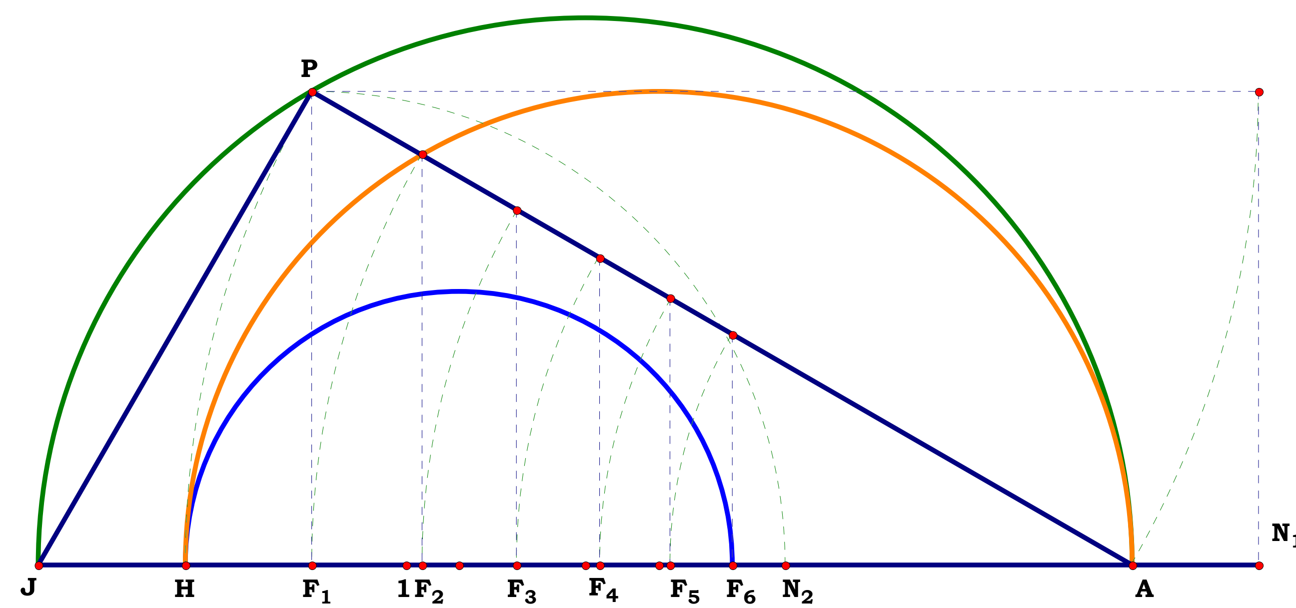
$DIV \equiv 7$

$\Delta := DIV$

$\delta := 1 .. \Delta$

121693B

Exponential Series



Descriptions.

$AP := 10 \qquad AH := AP \qquad PF_1 := 5 \qquad AF_1 := \sqrt{AP^2 - (PF_1)^2}$

$AF_2 := \dfrac{AF_1 \cdot AF_1}{AP} \qquad AF_{\delta+1} := \dfrac{AF_1 \cdot AF_\delta}{AP}$

$\dfrac{PF_1{}^2}{AF_2} = 3.333333 \qquad \dfrac{PF_1{}^2}{\left(\dfrac{10}{3}\right)} = 7.5 \qquad \dfrac{AP}{PF_1} = 2$

$\sqrt{AF_\Delta \cdot AH} = 6.044456$

$AF_\Delta = 3.653545 \qquad \dfrac{AH}{AF_\Delta} = 2.737068$

Definitions.

$\left[(AF_\Delta)^{\Delta-\delta} \cdot (AH)^{\delta}\right]^{\frac{1}{\Delta}} = \qquad \text{if}(\Delta - \delta, AF_{\Delta-\delta}, 0) = \qquad \Delta - \delta =$

4.21875	4.21875	6
4.871393	4.871393	5
5.625	5.625	4
6.495191	6.495191	3
7.5	7.5	2
8.660254	8.660254	1
10	0	0

$F_1 1 = 1.25400$ cm

$AP = 12.54000$ cm

$AH = 12.54000$ cm

$AF_1 = 10.85996$ cm

$AF_2 = 9.40500$ cm

$AF_3 = 8.14497$ cm

$AF_4 = 7.05375$ cm

$AF_5 = 6.10873$ cm

$AF_6 = 5.29031$ cm

$\dfrac{AH}{F_1 1} = 10.00000$

$\dfrac{AP}{F_1 1} = 10.00000$

$\dfrac{AF_1}{F_1 1} = 8.66025$

$\dfrac{AF_2}{F_1 1} = 7.50000$

$\dfrac{AF_3}{F_1 1} = 6.49519$

$\dfrac{AF_4}{F_1 1} = 5.62500$

$\dfrac{AF_5}{F_1 1} = 4.87139$

$\dfrac{AF_6}{F_1 1} = 4.21875$

Unit.

$L := 1$

Given.

$Y := 20$

$X := 14$

$DIV := 10$

$\Delta := DIV$

$\delta := 1 .. \Delta$

121693C

Descriptions.

Every number is expressible as a ratio.

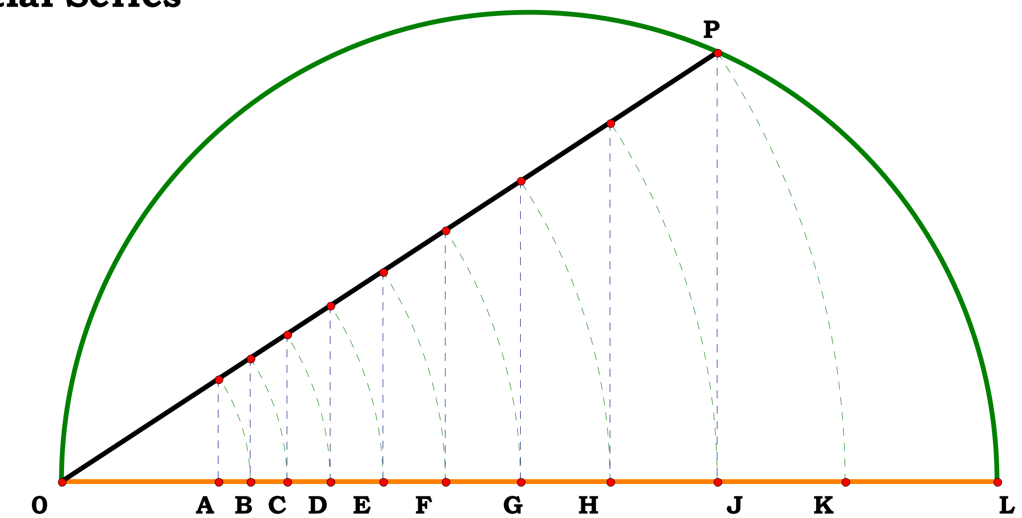
$OJ := \frac{X}{Y} \quad OK := \sqrt{OJ} \quad OJ_1 := OK^2$

$OJ_2 := \frac{OJ^2}{OK} \quad OJ_{\delta+1} := \frac{OJ_1 \cdot OJ_\delta}{OK}$

$\left[(OJ_\Delta)^{\Delta-\delta} \cdot (OK)^\delta\right]^{\frac{1}{\Delta}} =$	$\text{if}(\Delta-\delta, OJ_{\Delta-\delta}, 0) =$	$\Delta - \delta =$	$\text{if}\left[\Delta-\delta, \left[(OJ)_{\Delta-\delta}\right]^{\frac{1}{[\Delta-(\delta-1)]}}, 0\right] =$
0.16807	0.16807	9	0.83666
0.200882	0.200882	8	0.83666
0.2401	0.2401	7	0.83666
0.286974	0.286974	6	0.83666
0.343	0.343	5	0.83666
0.409963	0.409963	4	0.83666
0.49	0.49	3	0.83666
0.585662	0.585662	2	0.83666
0.7	0.7	1	0.83666
0.83666	0	0	0

Definitions.

Exponential Series



$A = 0.16807$
$B = 0.20088$
$C = 0.24010$
$D = 0.28697$
$E = 0.34300$
$F = 0.40996$
$G = 0.49000$
$H = 0.58566$
$J = 0.70000$
$K = 0.83666$
$L = 1.00000$
$Unit = 1.00000$
$XY = 0.70000$
$X = 14.00000$
$Y = 20.00000$

$A^{\frac{1}{10}} - K = 0.00000$
$A^{\frac{2}{10}} - J = 0.00000$
$A^{\frac{3}{10}} - H = 0.00000$
$A^{\frac{4}{10}} - G = 0.00000$
$A^{\frac{5}{10}} - F = 0.00000$
$A^{\frac{6}{10}} - E = 0.00000$
$A^{\frac{7}{10}} - D = 0.00000$
$A^{\frac{8}{10}} - C = 0.00000$
$A^{\frac{9}{10}} - B = 0.00000$

$\frac{X}{Y} = 0.70000$
$A^{\frac{1}{5}} = 0.70000$
$B^{\frac{2}{9}} = 0.70000$
$C^{\frac{3}{12}} = 0.70000$
$D^{\frac{4}{14}} = 0.70000$
$F^{\frac{6}{15}} = 0.70000$
$G^{\frac{7}{14}} = 0.70000$
$H^{\frac{8}{12}} = 0.70000$
$K^2 = 0.70000$

$\sqrt{\frac{X}{Y}} = 0.83666$
$J^{\frac{1}{2}} - K = 0.00000$
$H^{\frac{1}{3}} - K = 0.00000$
$G^{\frac{1}{4}} - K = 0.00000$
$F^{\frac{1}{5}} - K = 0.00000$
$E^{\frac{1}{6}} - K = 0.00000$
$D^{\frac{1}{7}} = 0.83666$
$C^{\frac{1}{8}} - K = 0.00000$
$B^{\frac{1}{9}} - K = 0.00000$
$A^{\frac{1}{10}} - K = 0.00000$

$K^2 - J = 0.00000$
$K^3 - H = 0.00000$
$K^4 - G = 0.00000$
$K^5 - F = 0.00000$
$K^6 - E = 0.00000$
$K^7 - D = 0.00000$
$K^8 - C = 0.00000$
$K^9 - B = 0.00000$
$K^{10} - A = 0.00000$

121693D

Unit.

$L := 1$

Given.

$Y := 20$

$X := 14$

$DIV := 10$

$\Delta := DIV$

$\delta := 1 .. \Delta$

Exponential Series

Descriptions.

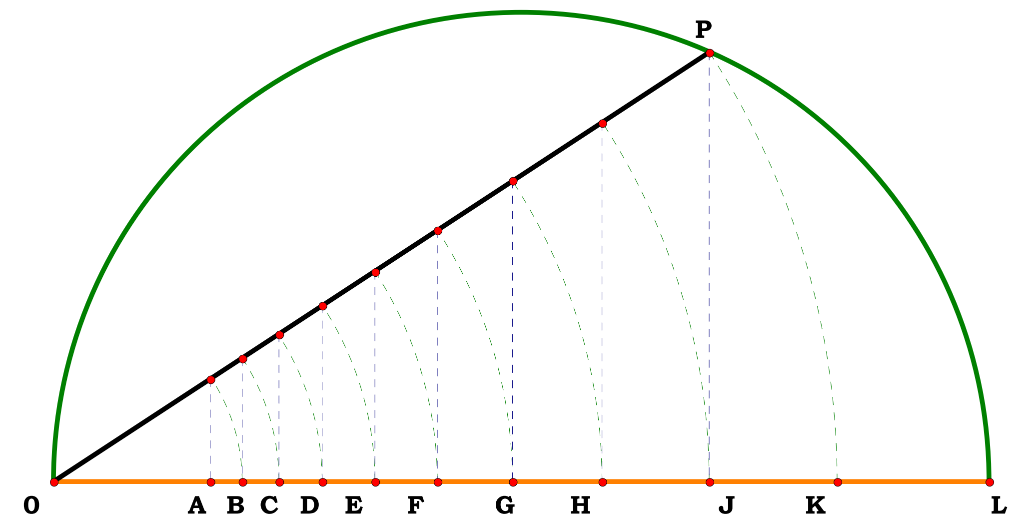
The first was X/Y, and this is Y/X.

$OJ := \frac{Y}{X}$ $\quad OK := \sqrt{OJ}$ $\quad OJ_1 := OK^2$

$OJ_2 := \frac{OJ^2}{OK}$ $\quad OJ_{\delta+1} := \frac{OJ_1 \cdot OJ_\delta}{OK}$ $\quad \sqrt{\frac{Y}{X}} = 1.195229$

$\left[(OJ_\Delta)^{\Delta-\delta} \cdot (OK)^\delta\right]^{\frac{1}{\Delta}} =$	$\text{if}(\Delta-\delta, OJ_{\Delta-\delta}, 0) =$	$\Delta-\delta+1 =$	$\left(\sqrt{\frac{Y}{X}}\right)^{\Delta-\delta+1} =$	$\delta =$	$\left(\sqrt{\frac{Y}{X}}\right)^{\delta} =$
5.949902	5.949902	10	5.949902	1	1.195229
4.978045	4.978045	9	4.978045	2	1.428571
4.164931	4.164931	8	4.164931	3	1.707469
3.484632	3.484632	7	3.484632	4	2.040816
2.915452	2.915452	6	2.915452	5	2.439242
2.439242	2.439242	5	2.439242	6	2.915452
2.040816	2.040816	4	2.040816	7	3.484632
1.707469	1.707469	3	1.707469	8	4.164931
1.428571	1.428571	2	1.428571	9	4.978045
1.195229	0	1	1.195229	10	5.949902

Definitions.



A = 0.16807
B = 0.20088
C = 0.24010
D = 0.28697
E = 0.34300
F = 0.40996
G = 0.49000
H = 0.58566
J = 0.70000
K = 0.83666
L = 1.00000
Unit = 1.00000
XY = 0.70000
X = 14.00000
Y = 20.00000

$A^{\frac{1}{10}} - K = 0.00000$
$A^{\frac{2}{10}} - J = 0.00000$
$A^{\frac{3}{10}} - H = 0.00000$
$A^{\frac{4}{10}} - G = 0.00000$
$A^{\frac{5}{10}} - F = 0.00000$
$A^{\frac{6}{10}} - E = 0.00000$
$A^{\frac{7}{10}} - D = 0.00000$
$A^{\frac{8}{10}} - C = 0.00000$
$A^{\frac{9}{10}} - B = 0.00000$

$\frac{X}{Y} = 0.70000$
$A^{\frac{1}{5}} = 0.70000$
$B^{\frac{2}{9}} = 0.70000$
$C^{\frac{3}{12}} = 0.70000$
$D^{\frac{4}{14}} = 0.70000$
$F^{\frac{6}{15}} = 0.70000$
$G^{\frac{7}{14}} = 0.70000$
$H^{\frac{8}{12}} = 0.70000$
$K^2 = 0.70000$

$\sqrt{\frac{X}{Y}} = 0.83666$
$J^{\frac{1}{2}} - K = 0.00000$
$H^{\frac{1}{3}} - K = 0.00000$
$G^{\frac{1}{4}} - K = 0.00000$
$F^{\frac{1}{5}} - K = 0.00000$
$E^{\frac{1}{6}} - K = 0.00000$
$D^{\frac{1}{7}} = 0.83666$
$C^{\frac{1}{8}} - K = 0.00000$
$B^{\frac{1}{9}} - K = 0.00000$
$A^{\frac{1}{10}} - K = 0.00000$

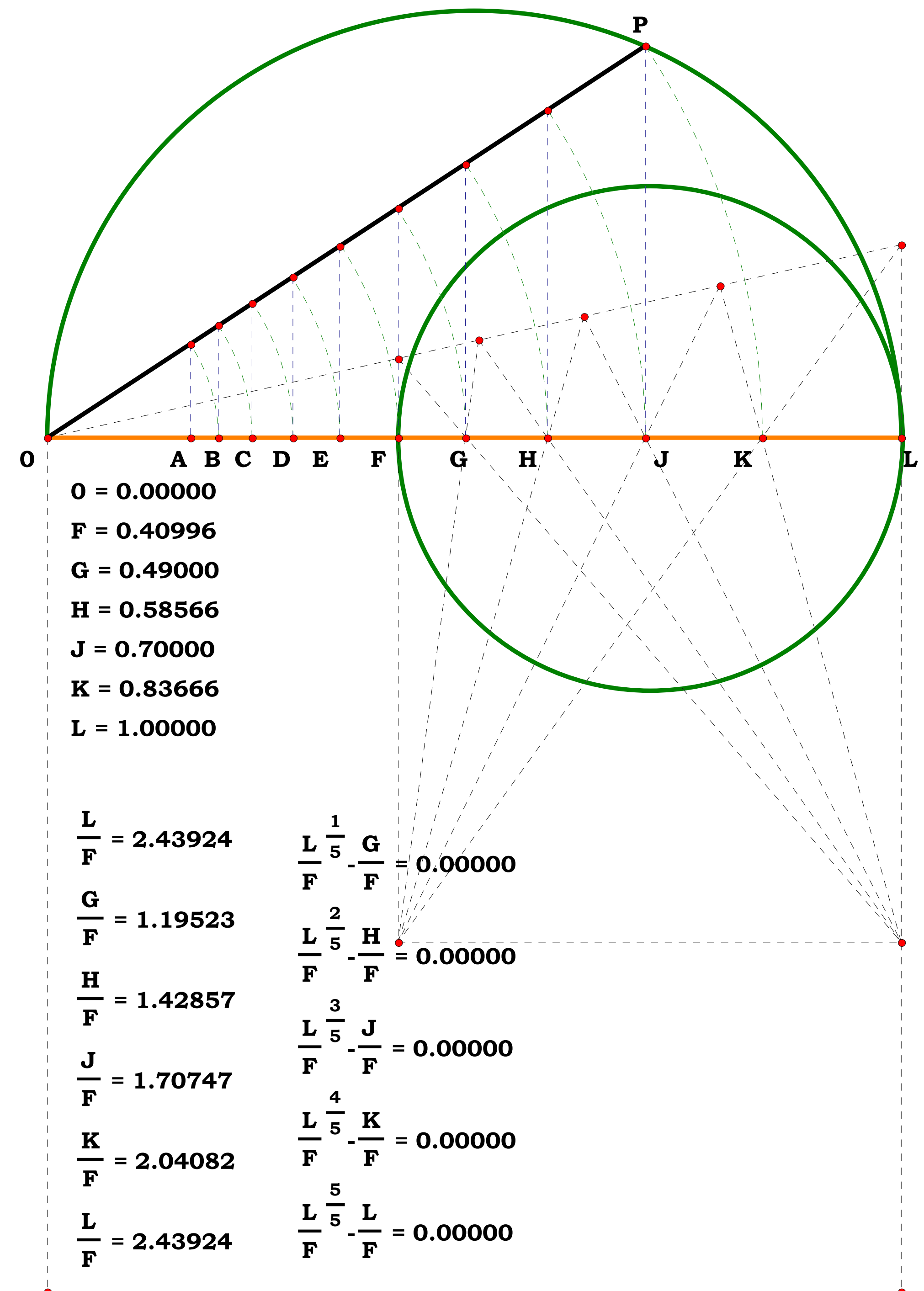
$K^2 - J = 0.00000$
$K^3 - H = 0.00000$
$K^4 - G = 0.00000$
$K^5 - F = 0.00000$
$K^6 - E = 0.00000$
$K^7 - D = 0.00000$
$K^8 - C = 0.00000$
$K^9 - B = 0.00000$
$K^{10} - A = 0.00000$
$\frac{L}{K} = 1.19523$

And if given any member of a series, save the square root, we have from 042906

Let us plead the 5th.

So, if one is clever enough, which I am not, one can actually do any root seriesl from the fact that any number is expressible as a ratio, and triangles just love ratios.

At one time I was seriously considering changing the name of the Delian Quest to One Circle, One Square, and One Line, but it did not stick.



$0 = 0.00000$

$F = 0.40996$

$G = 0.49000$

$H = 0.58566$

$J = 0.70000$

$K = 0.83666$

$L = 1.00000$

$\sqrt{\frac{X}{Y}} = 0.83666$

$J^{\frac{1}{2}} - K = 0.00000$

$H^{\frac{1}{3}} - K = 0.00000$

$G^{\frac{1}{4}} - K = 0.00000$

$F^{\frac{1}{5}} - K = 0.00000$

$E^{\frac{1}{6}} - K = 0.00000$

$D^{\frac{1}{7}} = 0.83666$

$C^{\frac{1}{8}} - K = 0.00000$

$B^{\frac{1}{9}} - K = 0.00000$

$A^{\frac{1}{10}} - K = 0.00000$

$K^2 - J = 0.00000$

$K^3 - H = 0.00000$

$K^4 - G = 0.00000$

$K^5 - F = 0.00000$

$K^6 - E = 0.00000$

$K^7 - D = 0.00000$

$K^8 - C = 0.00000$

$K^9 - B = 0.00000$

$K^{10} - A = 0.00000$

$\frac{L}{K} = 1.19523$

$\frac{L}{F} = 2.43924$

$\frac{G}{F} = 1.19523$

$\frac{H}{F} = 1.42857$

$\frac{J}{F} = 1.70747$

$\frac{K}{F} = 2.04082$

$\frac{L}{F} = 2.43924$

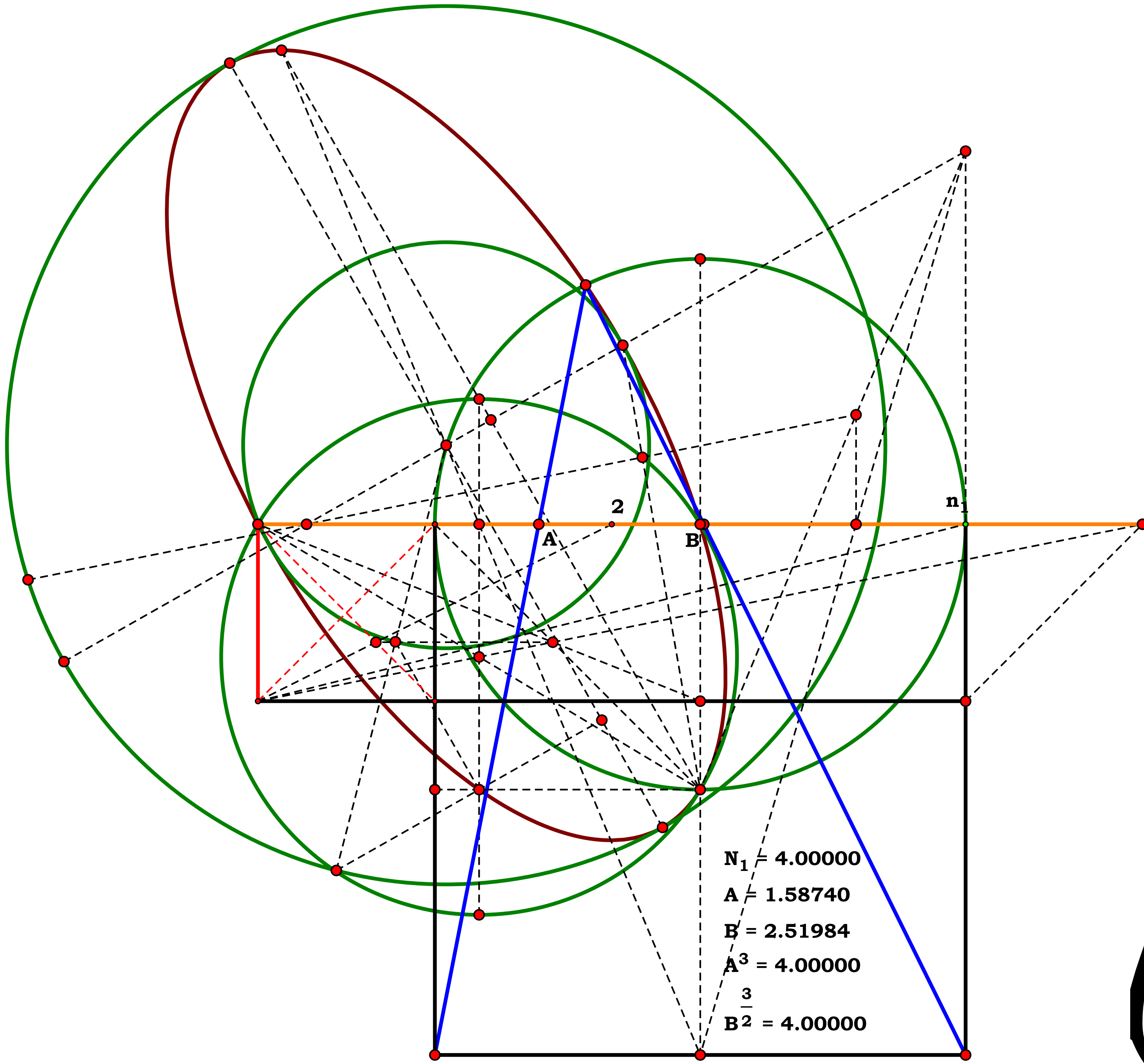
$\frac{L}{F}^{\frac{1}{5}} - \frac{G}{F} = 0.00000$

$\frac{L}{F}^{\frac{2}{5}} - \frac{H}{F} = 0.00000$

$\frac{L}{F}^{\frac{3}{5}} - \frac{J}{F} = 0.00000$

$\frac{L}{F}^{\frac{4}{5}} - \frac{K}{F} = 0.00000$

$\frac{L}{F}^{\frac{5}{5}} - \frac{L}{F} = 0.00000$



The Delian Quest 1994

John Clark

040694A

Descriptions.

Unit.

Given.

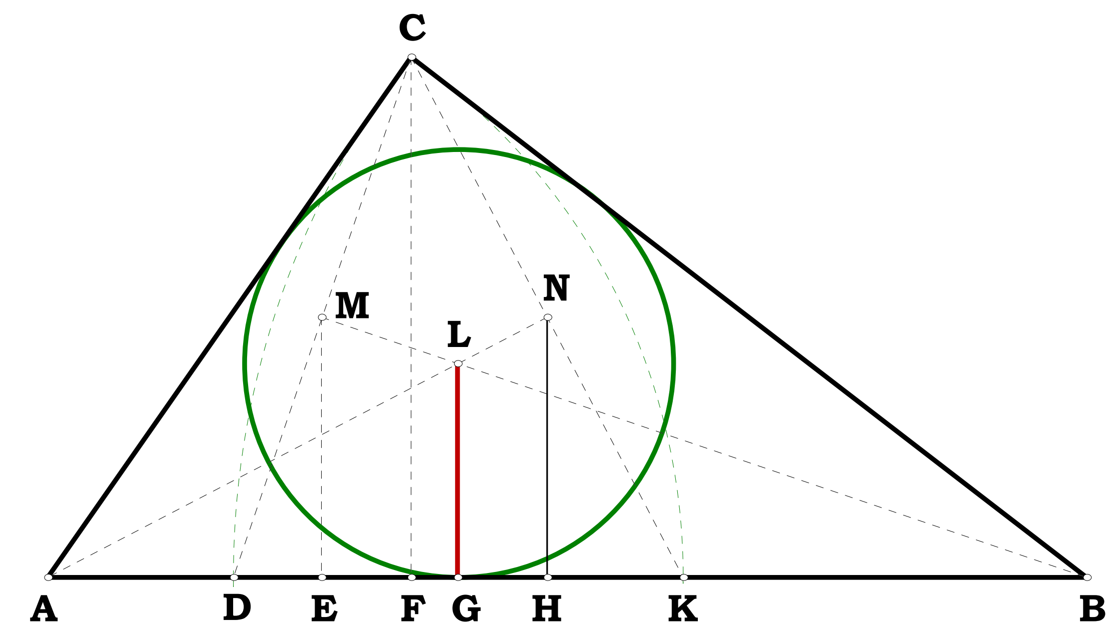
$N_1 := 3 \quad AB := N_1$

$N_2 := 4 \quad BC := N_2$

$N_3 := 5 \quad AC := N_3$

Inscribing A Circle In A Given Triangle

Given three sides of a triangle, what is the length of the inscribed radius?



$AK := AC \quad BD := BC \quad AF := \dfrac{AC^2 + AB^2 - BC^2}{2 \cdot AB} \quad FK := AK - AF$

$CF := \sqrt{AC^2 - AF^2} \quad AH := AF + \dfrac{FK}{2} \quad HN := \dfrac{CF}{2} \quad BF := AB - AF$

$DF := BD - BF \quad CD := \sqrt{CF^2 + DF^2} \quad BM := \dfrac{CF \cdot BD}{CD} \quad BE := \dfrac{CF \cdot BM}{CD}$

$GL := \dfrac{HN \cdot AB}{AH + BE}$

Definitions. $S_1 := \begin{pmatrix} AB \\ BC \\ AC \end{pmatrix} \quad S_2 := \begin{pmatrix} BC \\ AC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} AC \\ AB \\ BC \end{pmatrix} \quad \delta := 0 .. 2$

$$Radius_\delta := \frac{\sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}{2 \cdot \sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}}} \qquad Radius = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$AK - N_3 = 0 \quad BD - N_2 = 0 \quad AF - \dfrac{N_1{}^2 - N_2{}^2 + N_3{}^2}{2 \cdot N_1} = 0 \quad FK - \dfrac{(N_2 - N_1 + N_3) \cdot (N_1 + N_2 - N_3)}{2 \cdot N_1} = 0$

$CF - \dfrac{\sqrt{(N_1 + N_2 + N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_1 + N_2 - N_3) \cdot (N_2 - N_1 + N_3)}}{2 \cdot N_1} = 0 \quad AH - \dfrac{(N_1 - N_2 + N_3) \cdot (N_1 + N_2 + N_3)}{4 \cdot N_1} = 0$

$HN - \dfrac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{4 \cdot N_1} = 0 \quad BF - \dfrac{N_1{}^2 + N_2{}^2 - N_3{}^2}{2 \cdot N_1} = 0 \quad DF - \dfrac{(N_2 - N_1 + N_3) \cdot (N_1 - N_2 + N_3)}{2 \cdot N_1} = 0$

$CD - \dfrac{\sqrt{N_2 \cdot (N_2 - N_1 + N_3) \cdot (N_1 - N_2 + N_3)}}{\sqrt{N_1}} = 0 \quad BM - \dfrac{N_2 \cdot \sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot \sqrt{N_1} \cdot \sqrt{N_2 \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3)}} = 0 \quad BE - \dfrac{(N_1 + N_2 - N_3) \cdot (N_1 + N_2 + N_3)}{4 \cdot N_1} = 0$

$GL - \dfrac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot (N_1 + N_2 + N_3)} = 0$

Imagine that, another equation for the radius along with every other definition!

DE = 0.58830

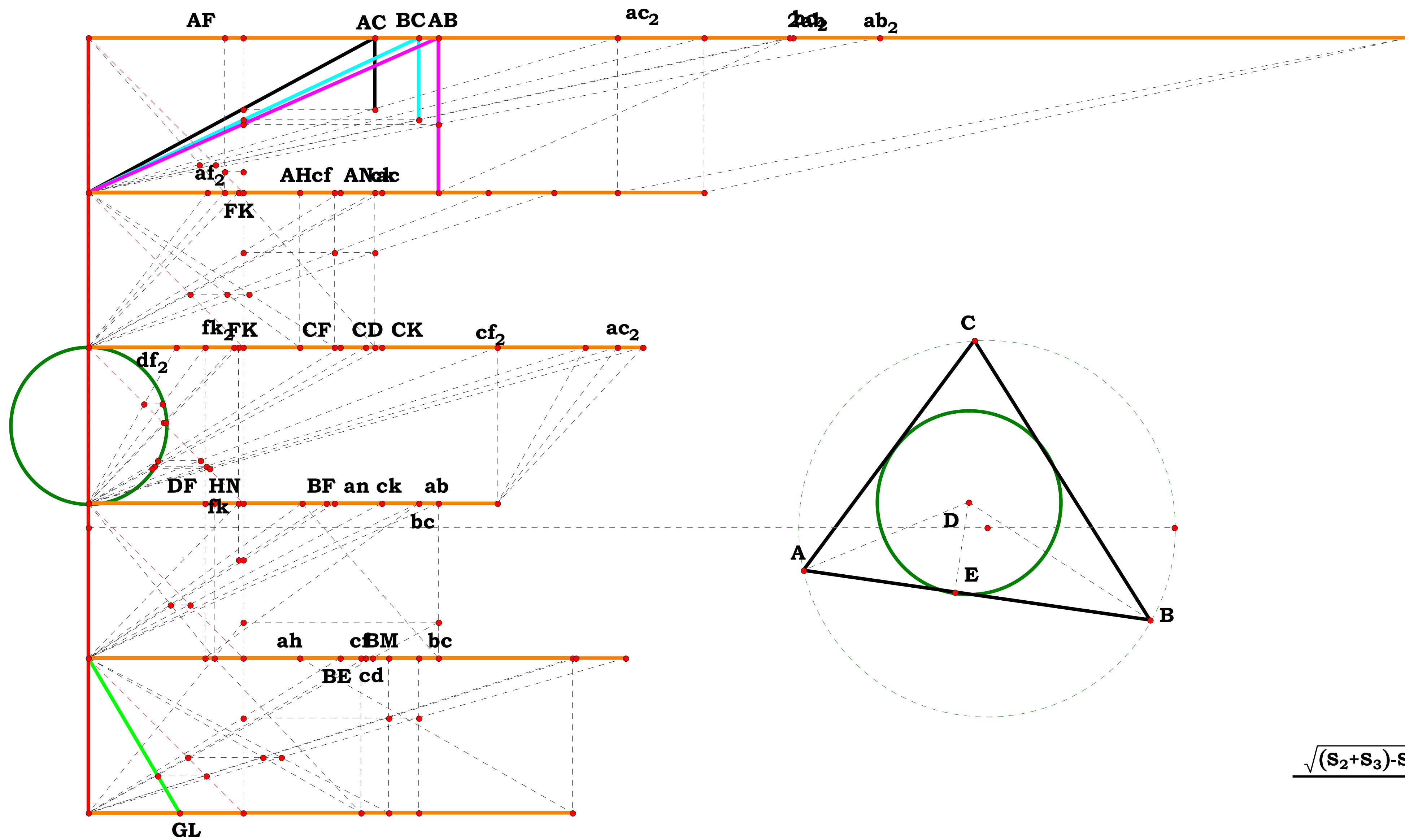
GL = 0.58830

DE-GL = 0.00000

AB = 2.25959

BC = 2.13235

AC = 1.84623



S_1 = 2.25959

S_2 = 2.13235

S_3 = 1.84623

$$\frac{\sqrt{(S_2+S_3)-S_1}\cdot\sqrt{(S_1+S_3)-S_2}\cdot\sqrt{(S_1+S_2)-S_3}}{2\cdot\sqrt{S_1+S_2+S_3}}-GL = 0.00000$$

040694A

Unit.

$AB := 1$

Given.

$W := 8 \quad Y := 11$

$X := 20 \quad Z := 20$

AB is the working side or base.

Inscribing A Circle In A Given Triangle

Given three sides of a triangle, what is the length of the inscribed radius?

Descriptions.

$AF := \frac{W}{X} \quad AF = 0.4 \quad AP := \frac{Y}{Z} \quad AP = 0.55 \quad CF := AP$

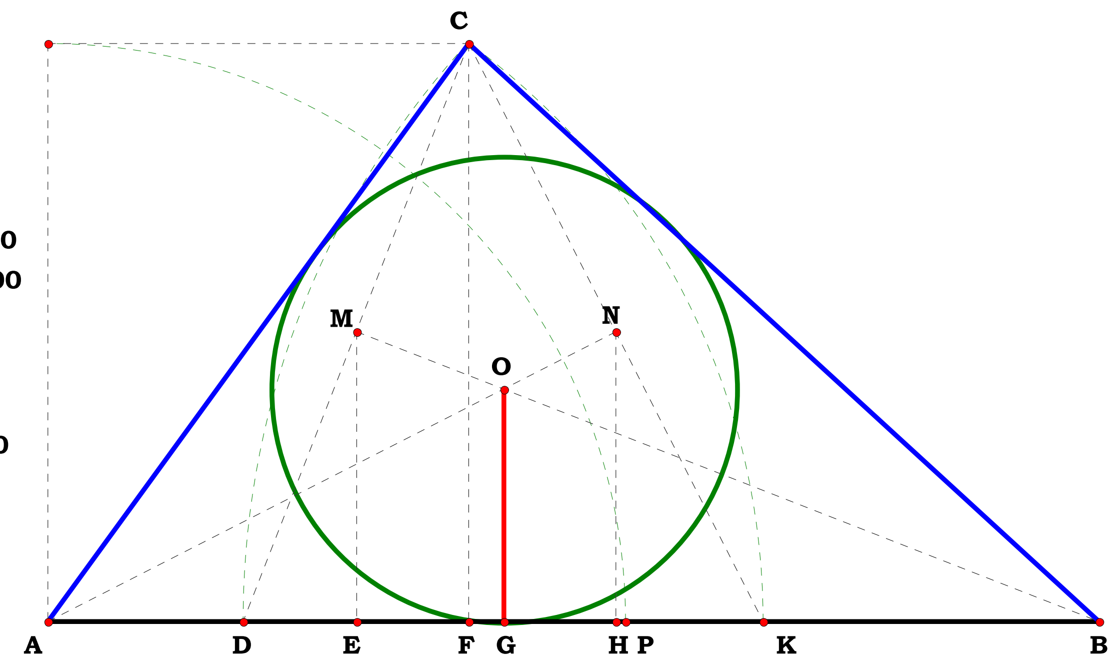
$AC := \sqrt{AF^2 + CF^2} \quad BF := AB - AF \quad BC := \sqrt{BF^2 + CF^2}$

$AK := AC \quad BD := BC \quad FK := AK - AF \quad HN := \frac{CF}{2}$

$AH := AF + \frac{FK}{2} \quad DF := BD - BF \quad BE := BF + \frac{DF}{2} \quad EM := HN$

$GO := \frac{EM \cdot AB}{BE + AH} \qquad GO = 0.220528$

Unit = 1.00000
W/X = 0.40000
W = 8.00000
X = 20.00000
AF = 0.40000
Y/Z = 0.55000
Y = 11.00000
Z = 20.00000
AP = 0.55000
GO = 0.22053



$$\frac{X \cdot Y}{X \cdot Z + \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}} - GO = 0.00000$$

Definitions.

$AF - \frac{W}{X} = 0 \quad AP - \frac{Y}{Z} = 0 \quad CF - \frac{Y}{Z} = 0$

$AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0 \quad BF - \frac{X - W}{X} = 0$

$BC - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0 \quad AK - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0 \quad BD - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0$

$FK - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - W \cdot Z}{X \cdot Z} = 0 \quad HN - \frac{Y}{2 \cdot Z} = 0 \quad AH - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W \cdot Z}{2 \cdot X \cdot Z} = 0 \quad DF - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)} + Z \cdot (W - X)}{X \cdot Z} = 0$

$BE - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)} + Z \cdot (X - W)}{2 \cdot X \cdot Z} = 0 \quad EM - \frac{Y}{2 \cdot Z} = 0 \quad GO - \frac{X \cdot Y}{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)} + \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + X \cdot Z} = 0$

Unit.

$AB := 1$

Given.

$N := 5$

042194A

The Cradle

Are A, M, N colinear?

Descriptions.

$AL := AB \cdot N \qquad AF := \sqrt{AB \cdot AL}$

$AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \qquad BP := BL \qquad LR := BL \qquad FL := AL - AF$

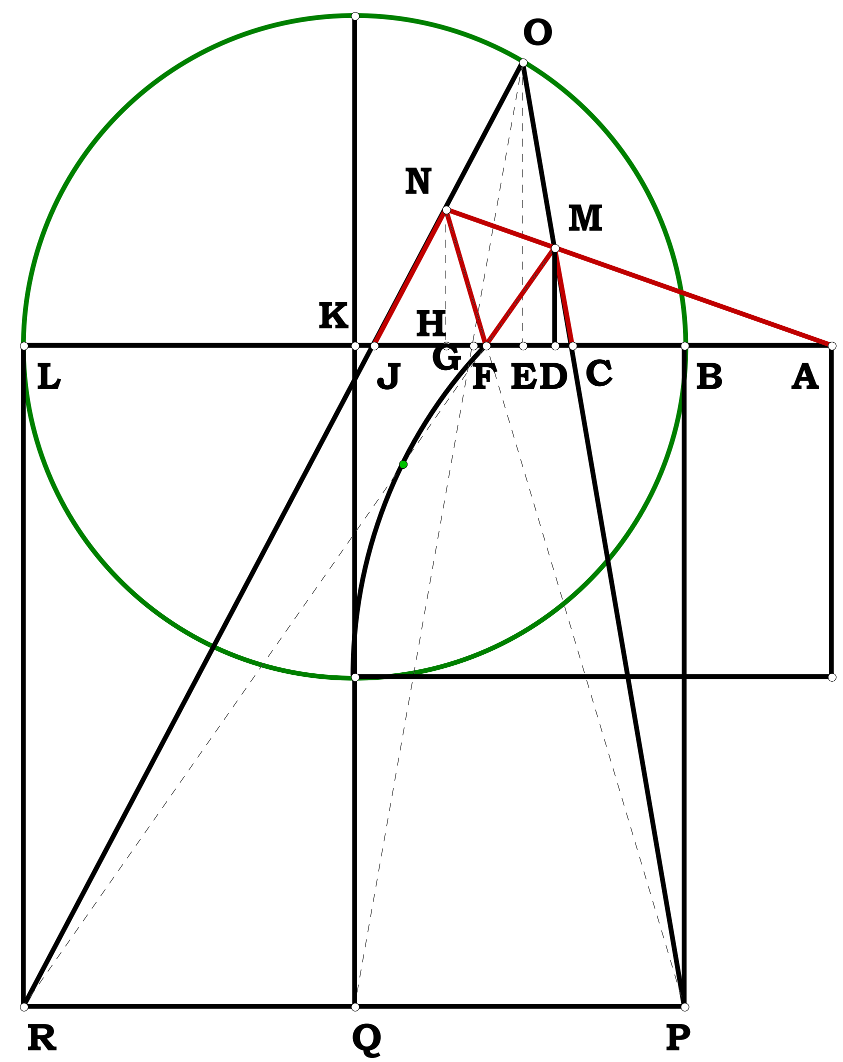
$BC := AC - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ$

$BF := AF - AB \qquad FJ := AJ - AF \qquad CF := AF - AC$

$$FG := \frac{BF \cdot FJ}{BF + JL} \qquad GN := \frac{BP \cdot FG}{BF} \qquad CD := \frac{BC \cdot CF}{BC + FL}$$

$$DM := \frac{BP \cdot CD}{BC} \qquad AD := AC + CD \qquad AG := AF + FG$$

$$\frac{AG}{GN} - \frac{AD}{DM} = 0$$



Definitions.

$AL - N = 0 \qquad AF - \sqrt{N} = 0 \qquad AC - N^{\frac{1}{3}} = 0$

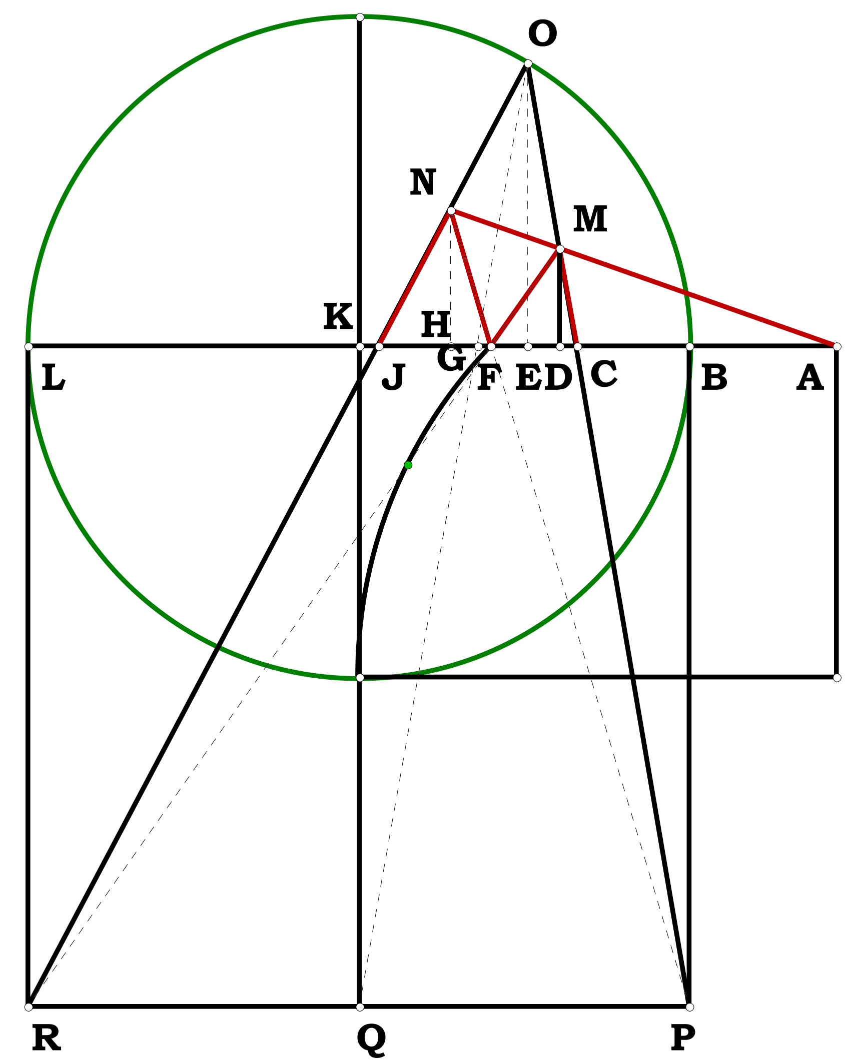
$AJ - N^{\frac{2}{3}} = 0 \qquad BL - (N - 1) = 0 \qquad FL - \left(N - \sqrt{N}\right) = 0$

$BC - \left(N^{\frac{1}{3}} - 1\right) = 0 \qquad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \qquad JL - \left(N - N^{\frac{2}{3}}\right) = 0 \qquad BF - \left(\sqrt{N} - 1\right) = 0$

$FJ - \left(N^{\frac{2}{3}} - \sqrt{N}\right) = 0 \qquad CF - \left[\sqrt{N} - (N)^{\frac{1}{3}}\right] = 0 \qquad FG - \dfrac{\sqrt{N}\cdot\left(\sqrt{N} - 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$

$GN - \dfrac{\sqrt{N}\cdot(N - 1)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad CD - \dfrac{N^{\frac{2}{3}} - N^{\frac{1}{3}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$

$DM - \dfrac{N^{\frac{4}{3}} - N^{\frac{1}{3}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad AD - \dfrac{N + \sqrt{N} + N^{\frac{2}{3}} + N^{\frac{5}{6}} + N^{\frac{7}{6}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$



$AG - \dfrac{N + N^{\frac{2}{3}} + N^{\frac{4}{3}} + N^{\frac{5}{6}} + N^{\frac{7}{6}}}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad \dfrac{AG}{GN} - \dfrac{\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}}}{N - 1} = 0 \qquad \dfrac{AD}{DM} - \dfrac{\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}}}{N - 1} = 0$

042194B

Unit.

$AB := 1$

Given.

$X := 9$

$Y := 20$

The Cradle

Are A, M, N colinear?

Descriptions.

$AD := \frac{X}{Y}$ $\quad AE := \sqrt{AD}$ $\quad AE = 0.67082$ $\quad AC := AD^{\frac{3}{2}}$ $\quad AC = 0.301869$

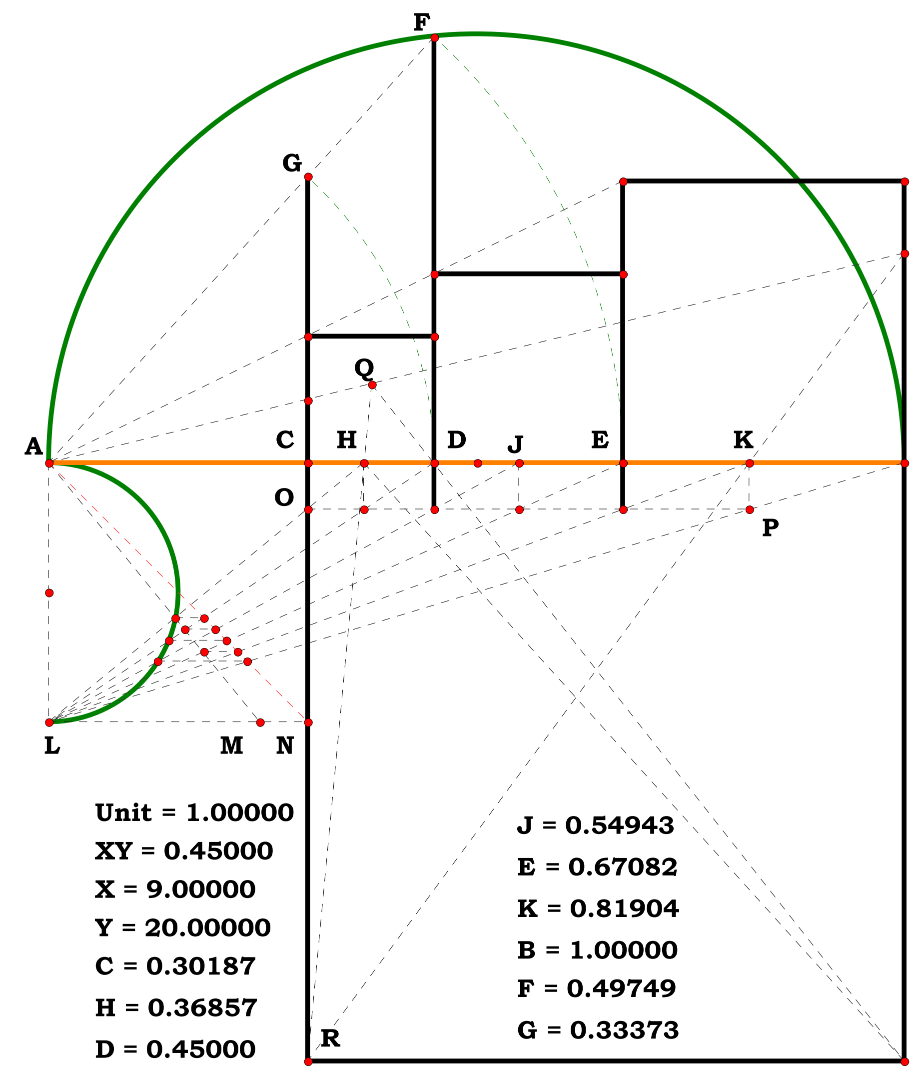
$\frac{AB}{AC} = 3.312693$ $\quad AH := AC^{\frac{5}{6}}$ $\quad AH = 0.368566$ $\quad AD - AC^{\frac{4}{6}} = 0$

$AJ := AC^{\frac{3}{6}}$ $\quad AJ = 0.549426$ $\quad AE := AC^{\frac{2}{6}}$ $\quad AE = 0.67082$

$AK := AC^{\frac{1}{6}}$ $\quad AK = 0.819036$ $\quad AB - AC^{\frac{0}{6}} = 0$

In this plate one can see some of the various ways to project a root series. We have one method down in the little unit box, L, M, and N. We have the parallel line method shown by O and P. We have the peak method R, Q, and S. and we have the method I started wtih. So, eventually one can show, many ways that each of them are part of a root series. So, here I will simply cash out.

When every possible root series is constructed in exactly the same way, it is very odd to say that one cannot do this or that root series. We have to learn how to start and stop them, and how many geometric recursions we want between those two limits. Learning the different ways to produce those series and learning how they are all interrelated can do nothing more than help us in that regard.



Definitions.

$AD - \frac{X}{Y} = 0$ $\quad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$AC := \left(\frac{X}{Y}\right)^{\frac{3}{2}}$ $\quad AH - \left[\left(\frac{X}{Y}\right)^{\frac{3}{2}}\right]^{\frac{5}{6}} = 0$

It is perfectly silly, but Mathcad 15 will not reduce these expressions. You have to do it manually.

$AH - \left(\frac{X}{Y}\right)^{\frac{5}{4}} = 0$ $\quad AJ - \left(\frac{X}{Y}\right)^{\frac{3}{4}} = 0$

$AE - \left(\frac{X}{Y}\right)^{\frac{1}{2}} = 0$ $\quad AK - \left(\frac{X}{Y}\right)^{\frac{1}{4}} = 0$ $\quad AB - \left(\frac{X}{Y}\right)^{\frac{0}{12}} = 0$

Given.

Tangents and Similarity Points.

042694A

What is the Algebraic names of the similarity points C and F in relation to the radius of each circle and the difference between their centers?

I believe that I was almost laughing when I drew this up originally. I made it an acronym side show. I actually get annoyed with acronyms. It scared me so much I never did it again. But, for the last version of DQ. I should at least put a dress on the graphics and remove the acorns. For some reason, I always liked this write-up.

Descriptions.

I will work with point C first.

Given AD = large radius
BE = small radius
AB = difference between centers

$AD := 4 \quad BE := 2 \quad AB := 7 \qquad DE := AD - BE$

$$AC := \frac{AB \cdot AD}{DE} \qquad AC = 14$$

AC "External similarity point Origin to center of Radius Large"

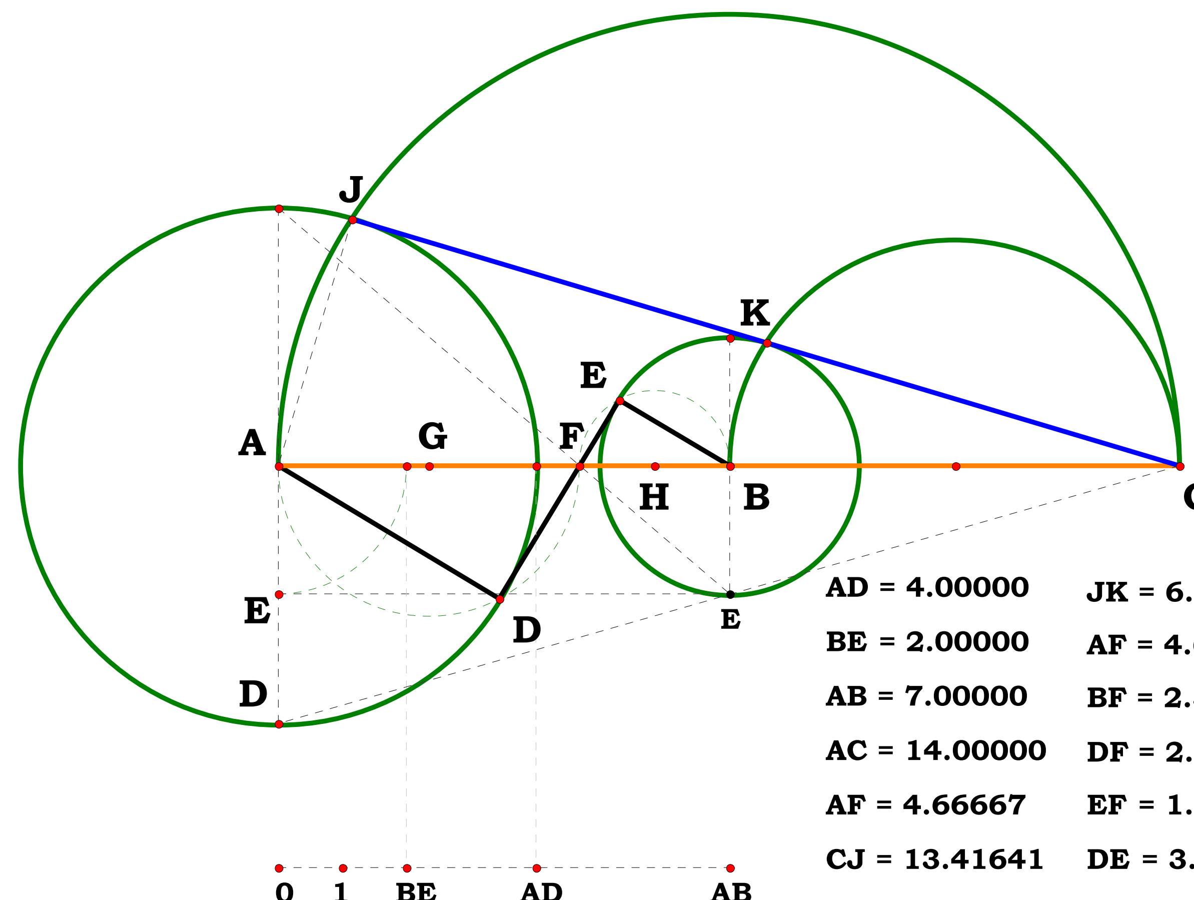
$$AC := \text{if}\left(AD \neq BE, \text{if}\left(BE > AD, 0, \frac{AB \cdot AD}{AD - BE}\right), \infty\right) \qquad AC = 14$$

What is the length of line JC tangent to both circles?

$$CJ := \sqrt{AC^2 - AD^2} \qquad CJ = 13.416408$$

And what is the formula?
JC " External similarity point Origin to Tangent (Large Radius)"

$$CJ - AD \cdot \frac{\sqrt{(AD - BE + AB) \cdot (-AD + BE + AB)}}{AD - BE} = 0$$



AD = 4.00000
BE = 2.00000
AB = 7.00000
AC = 14.00000
AF = 4.66667
CJ = 13.41641
CK = 6.70820

JK = 6.70820
AF = 4.66667
BF = 2.33333
DF = 2.40370
EF = 1.20185
DE = 3.60555

What is the length of the line tangent to the least circle (CK)?

$BC := AC - AB \qquad BC = 7$

$CK := \sqrt{BC^2 - BE^2}$

$CK = 6.708204$

And what is the formula?

CK " External similarity point Origin to Tangent (Small Radius)"

$$CK - BE \cdot \frac{\sqrt{-(AD - BE + AB) \cdot (AD - BE - AB)}}{AD - BE} = 0$$

Lastly what is the length of line from tangent to tangent of these circles?

$JK := CJ - CK$

$JK = 6.708204$

And what is the formula for, JK, Tangent to Tangent"?

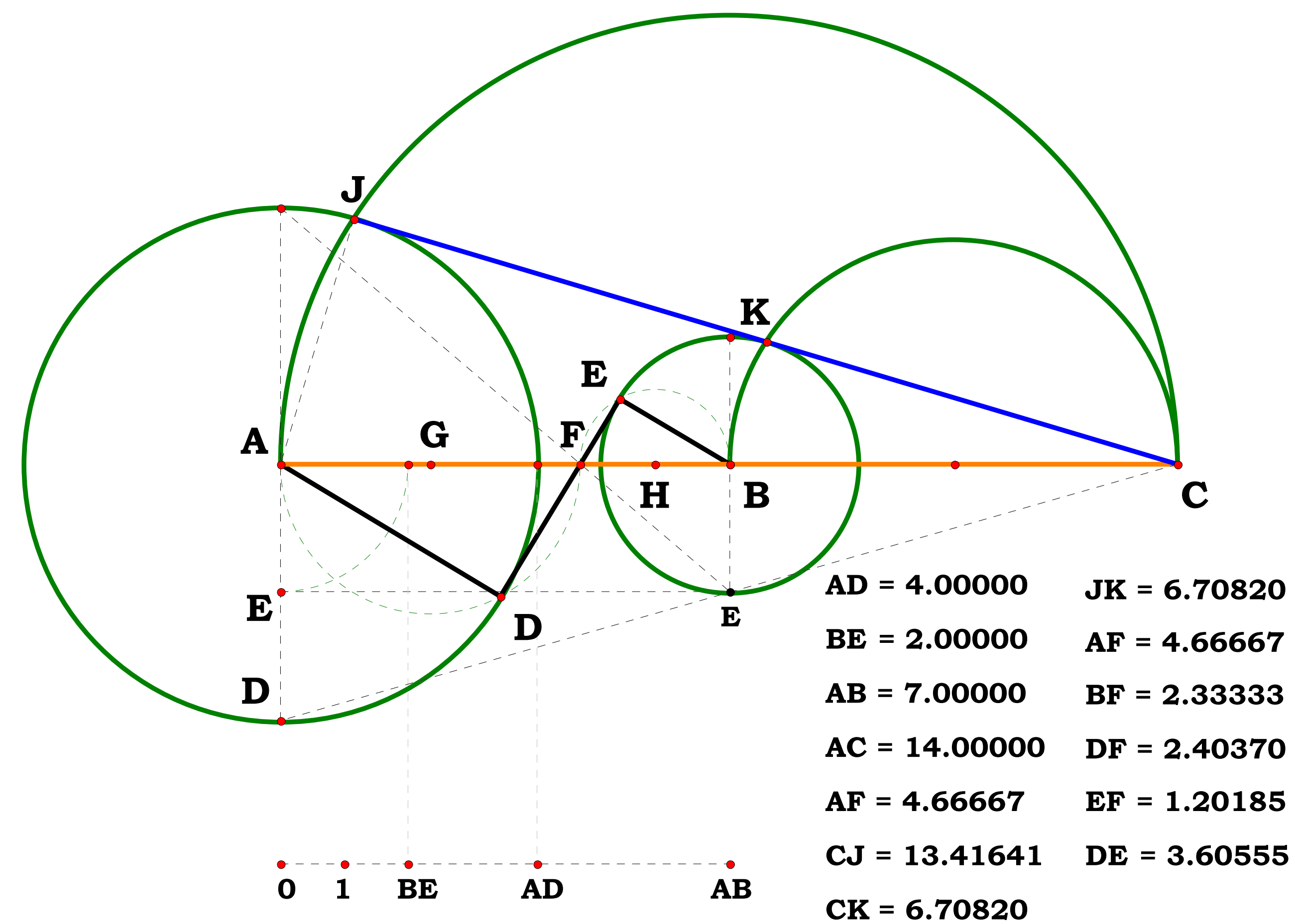
$$JK - \sqrt{-(AD - BE + AB) \cdot (AD - BE - AB)} = 0$$

I will now turn my attention to the point F the internal similarity point.

$AF := \dfrac{AB \cdot AD}{AD + BE} \qquad AF = 4.666667$

AF "Internal similarity point to center of Radius Large"

$$AF - AB \cdot \frac{AD}{AD + BE} = 0$$



$\mathbf{BF} := \mathbf{AB} - \mathbf{AF} \qquad \mathbf{BF} = 2.333333$

BF "Internal similarity point to center of Radius Small"

$$\mathbf{BF} - \mathbf{AB} \cdot \frac{\mathbf{BE}}{\mathbf{AD} + \mathbf{BE}} = 0$$

$\mathbf{DF} := \sqrt{\mathbf{AF}^2 - \mathbf{AD}^2} \qquad \mathbf{DF} = 2.403701$

DF "Internal similarity point Origin to Tangent (Large Radius)"

$$\mathbf{DF} - \mathbf{AD} \cdot \frac{\sqrt{-(\mathbf{AD} + \mathbf{BE} - \mathbf{AB}) \cdot (\mathbf{AD} + \mathbf{BE} + \mathbf{AB})}}{(\mathbf{AD} + \mathbf{BE})} = 0$$

$\mathbf{DF} = 2.403701 \qquad \mathbf{EF} := \sqrt{\mathbf{BF}^2 - \mathbf{BE}^2} \qquad \mathbf{EF} = 1.20185$

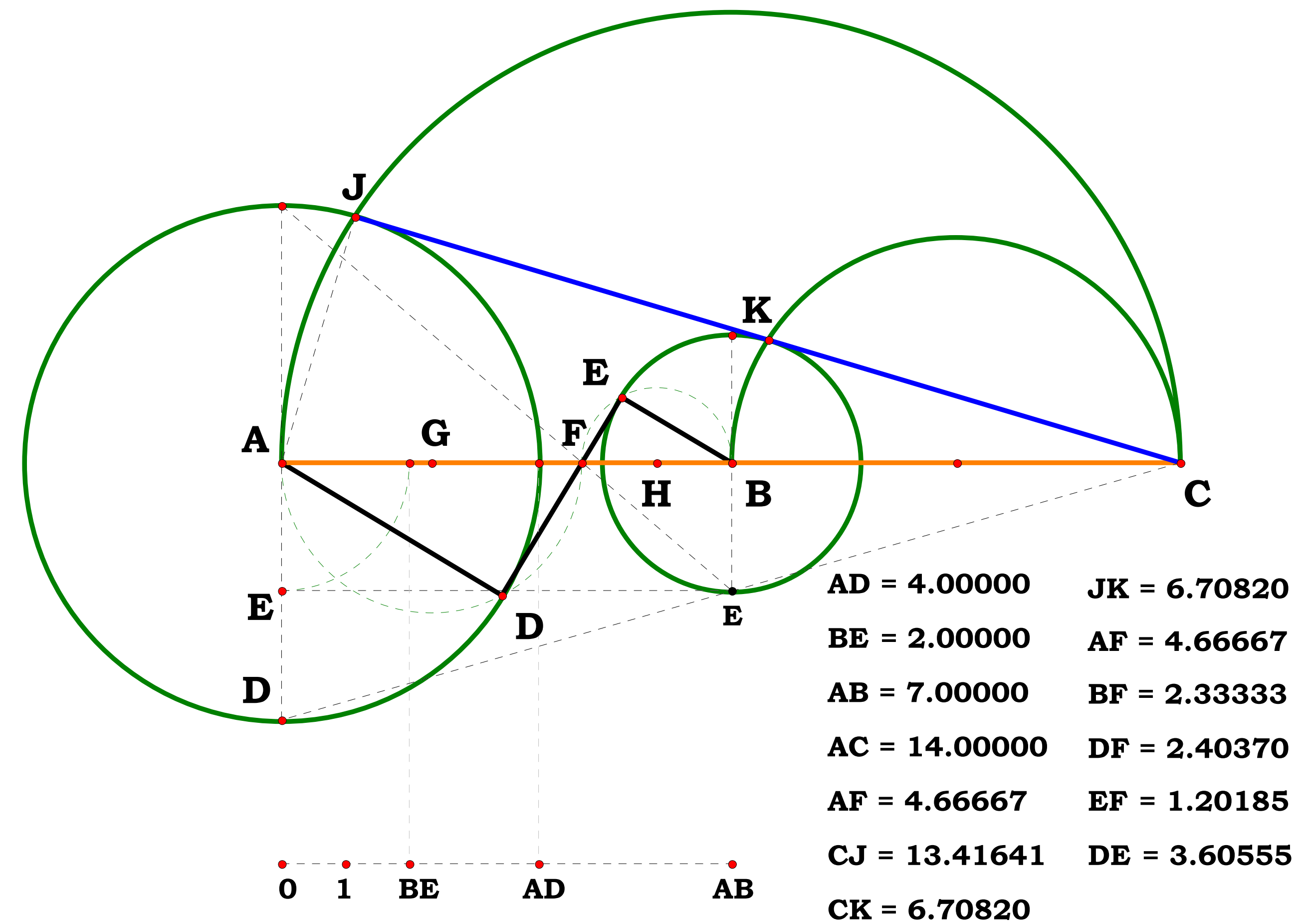
EF "Internal similarity point Origin to Tangent (Small Radius)"

$$\mathbf{EF} - \mathbf{BE} \cdot \frac{\sqrt{-(\mathbf{AD} + \mathbf{BE} - \mathbf{AB}) \cdot (\mathbf{AD} + \mathbf{BE} + \mathbf{AB})}}{\mathbf{AD} + \mathbf{BE}} = 0$$

$\mathbf{DE} := \mathbf{DF} + \mathbf{EF} \qquad \mathbf{DE} = 3.605551$

DE "Internal similarity point Tangent to Tangent"

$\mathbf{DE} - \sqrt{-(\mathbf{AD} + \mathbf{BE} - \mathbf{AB}) \cdot (\mathbf{AD} + \mathbf{BE} + \mathbf{AB})} = 0 \qquad \mathbf{DE} = 3.605551$



042694B

Given.

$N_1 := 3$

$N_2 := 2$

$N_3 := 7$

Tangents and Similarity Points.

Descriptions.

$R_1 := \sqrt{N_1{}^2}$ $\qquad R_2 := \sqrt{N_2{}^2}$

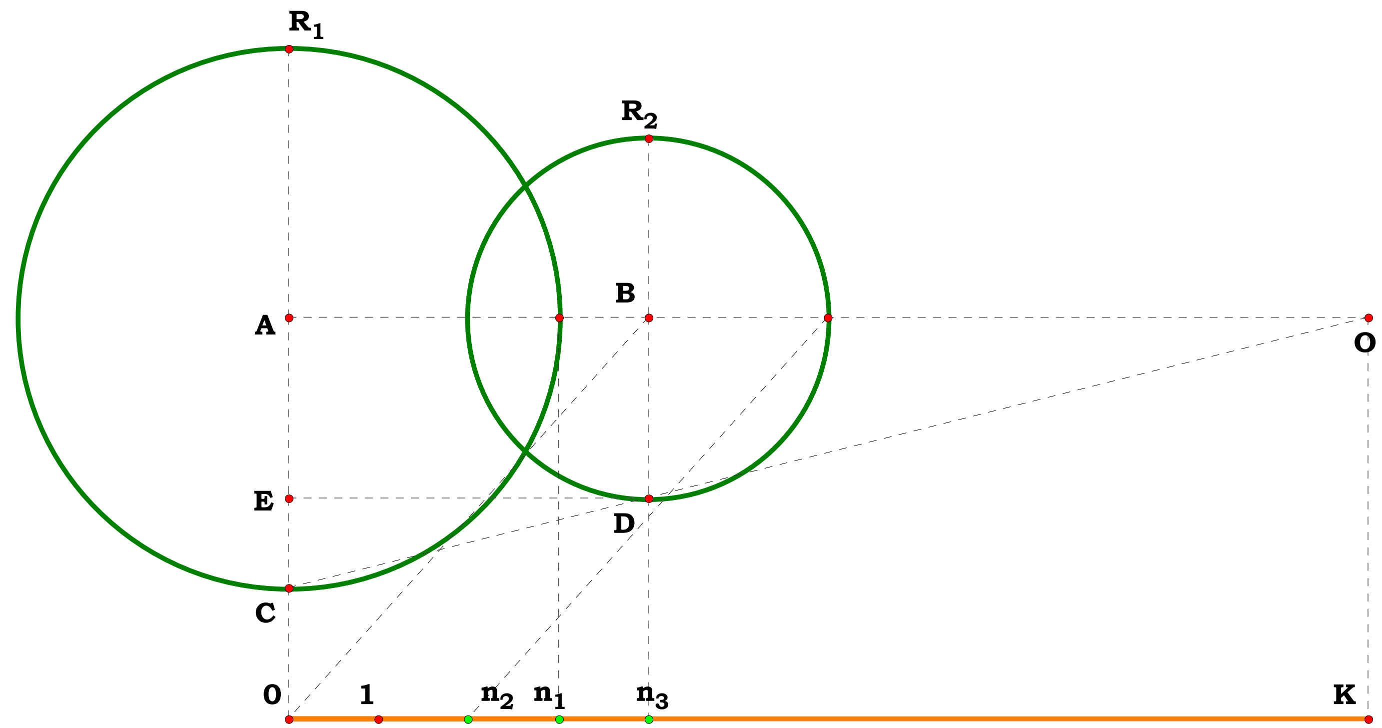
What is the length of the AO, O being the similarity point?

$AC := R_1 \qquad BD := R_2 \qquad AB := N_3$

$DE := AB \qquad AE := BD \qquad CE := AC - AE$

$AO := \dfrac{DE \cdot AC}{CE} \qquad AO = 21$

$\dfrac{N_3 \cdot R_1}{R_1 - R_2} = 21 \qquad AO - \dfrac{N_3 \cdot R_1}{R_1 - R_2} = 0$



What is the length of the tangent GO?

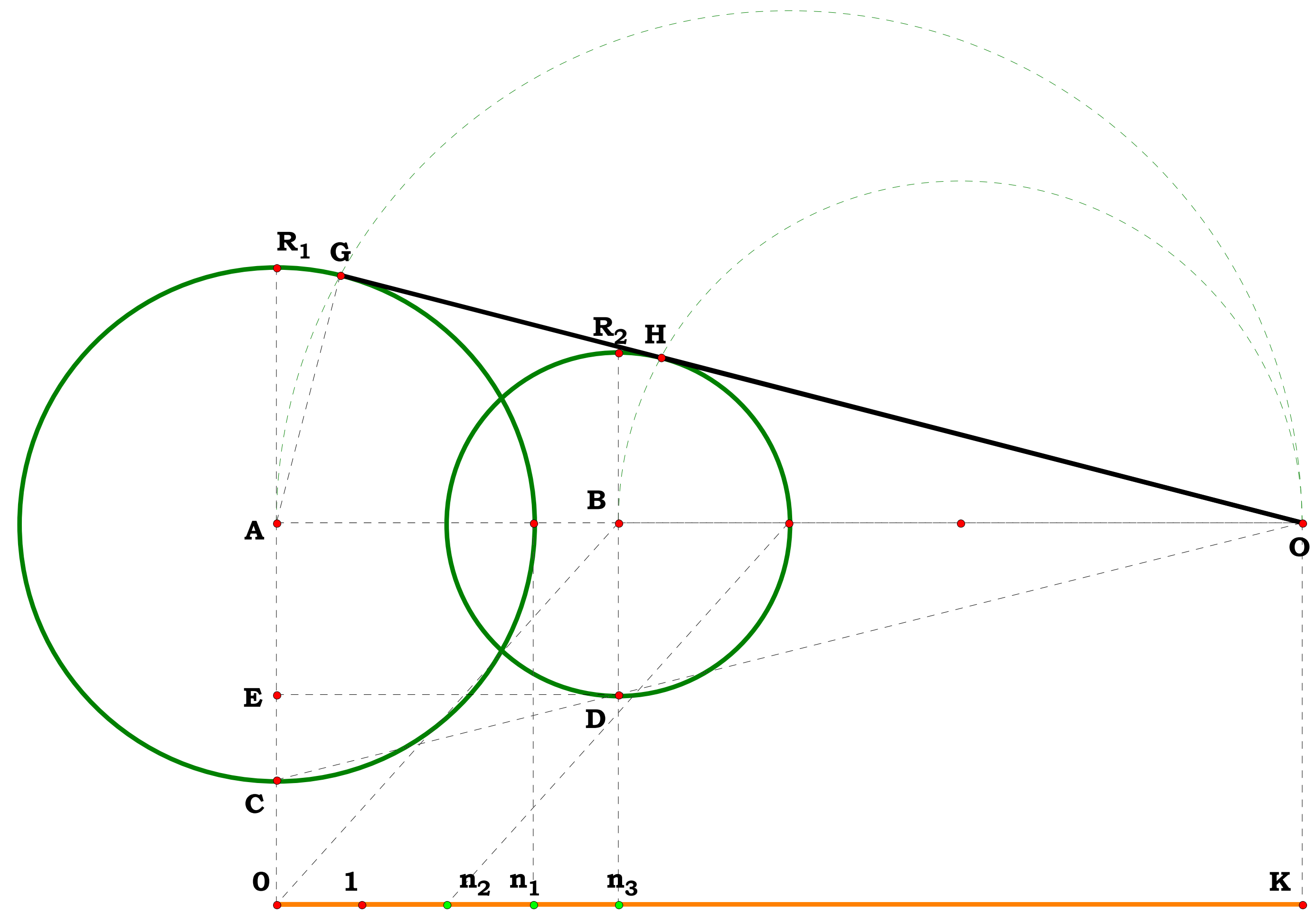
$$AG := R_1 \qquad GO := \sqrt{AO^2 - AG^2}$$

$$GO - \frac{R_1 \cdot \sqrt{N_3{}^2 - R_1{}^2 + 2 \cdot R_1 \cdot R_2 - R_2{}^2}}{\sqrt{\left(R_1 - R_2\right)^2}} = 0$$

What is the length of the tangent HO?

$$BH := R_2 \qquad HO := \sqrt{\left(\frac{N_3 \cdot R_1}{R_1 - R_2} - N_3\right)^2 - R_2{}^2}$$

$$HO - \frac{R_2 \cdot \sqrt{N_3{}^2 - R_1{}^2 + 2 \cdot R_1 \cdot R_2 - R_2{}^2}}{\sqrt{\left(R_1 - R_2\right)^2}} = 0$$



What is the length of line tangent to tangent of these circles?

$$GH := \frac{GO \cdot AB}{AO} \qquad GH - \sqrt{N_3{}^2 - R_1{}^2 + 2 \cdot R_1 \cdot R_2 - R_2{}^2} = 0$$

What are the names of the tangents AP and BP to the similarity point P?

$$AP := \frac{N_3 \cdot R_1}{R_1 + R_2} \qquad BP := N_3 - \frac{N_3 \cdot R_1}{R_1 + R_2}$$

What is JP?

$$JP := \sqrt{AP^2 - R_1{}^2} \qquad JP - \frac{R_1 \cdot \sqrt{N_3{}^2 - R_1{}^2 - 2 \cdot R_1 \cdot R_2 - R_2{}^2}}{R_1 + R_2} = 0$$

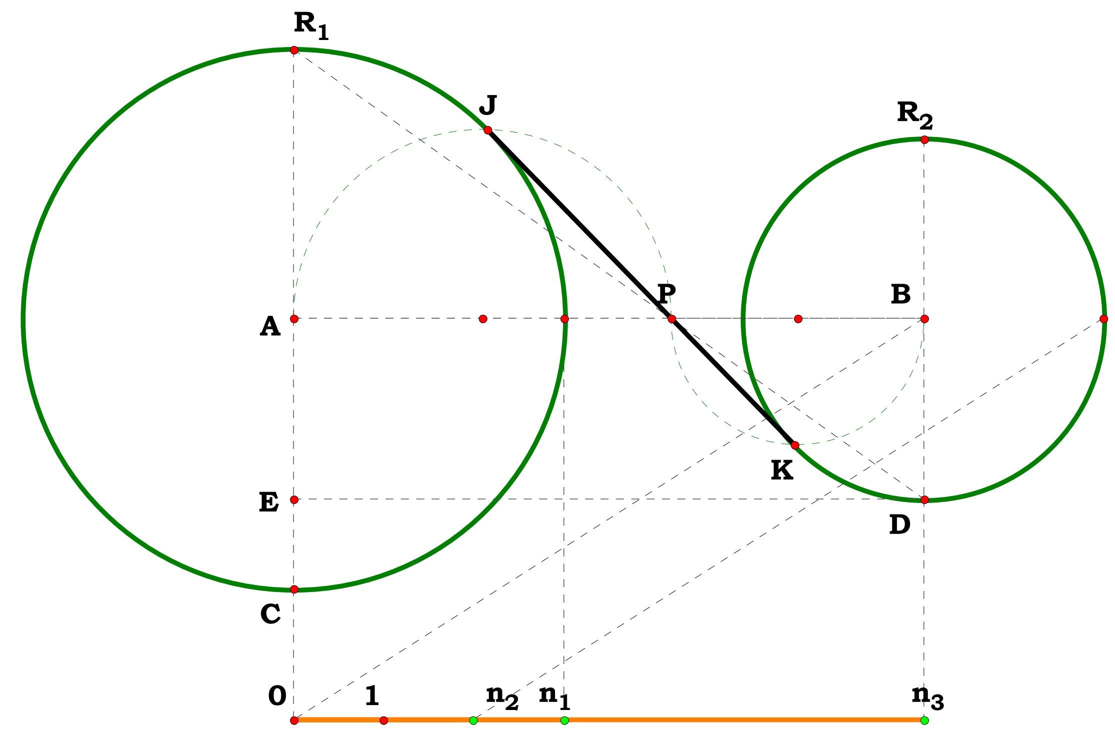
What is KP?

$$BP - \frac{N_3 \cdot R_2}{R_1 + R_2} = 0$$

$$KP := \frac{JP \cdot BP}{AP} \qquad KP - \frac{R_2 \cdot \sqrt{N_3{}^2 - R_1{}^2 - 2 \cdot R_1 \cdot R_2 - R_2{}^2}}{R_1 + R_2} = 0$$

What is JK?

$$JK := \frac{JP \cdot AB}{AP} \qquad JK - \sqrt{N_3{}^2 - R_1{}^2 - 2 \cdot R_1 \cdot R_2 - R_2{}^2} = 0$$



042694C

Descriptions.

Unit.

$AB := 1$

Given.

$W := 8 \quad Y := 4$

$X := 20 \quad Z := 20$

What is the Algebraic names of the similarity points C and F in relation to the radius of each circle and the difference between their centers?

I believe that I was almost laughing when I drew this up originally. I made it an acronym side show. I actually get annoyed with acronyms. It scared me so much I never did it again. But, for the last version of DQ. I should at least put a dress on the graphics and remove the acorns. For some reason, I always liked this write-up.

I will work with point C first.

Given AD = large radius
BE = small radius
AB = difference between centers

$$AD := \frac{W}{X} \qquad BE := \frac{Y}{Z} \qquad AB := 1$$

$$DE := AD - BE \qquad AC := \frac{AB \cdot AD}{DE} \qquad AC = 2$$

AC "External similarity point Origin to center of Radius Large"

$$AC := \text{if}\left(AD \neq BE\,,\,\text{if}\left(BE > AD\,,\,0\,,\,\frac{AB \cdot AD}{AD - BE}\right),\infty\right) \qquad AC = 2$$

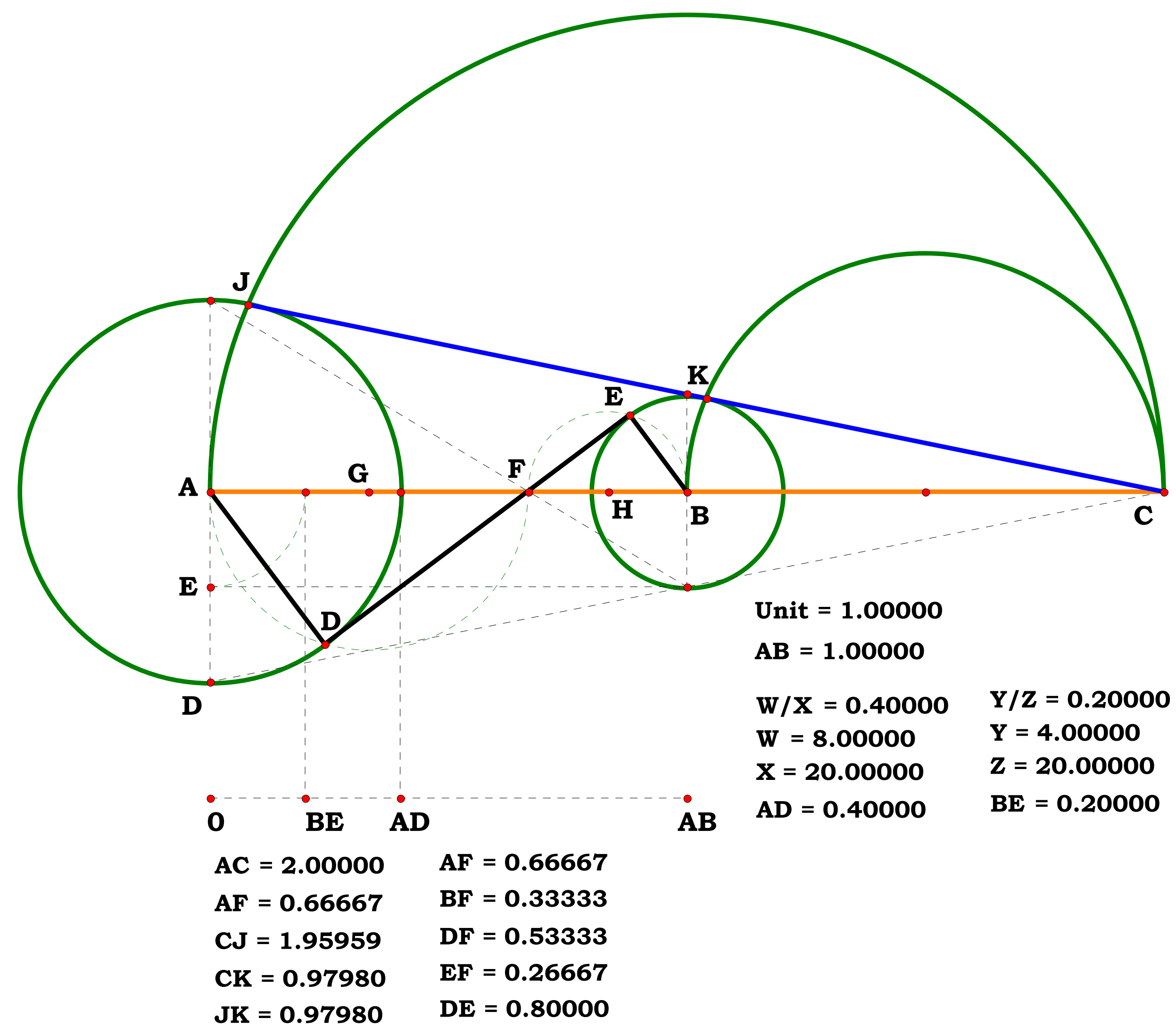
What is the length of line JC tangent to both circles?

$$CJ := \sqrt{AC^2 - AD^2} \qquad CJ = 1.959592$$

And what is the formula?
JC " External similarity point Origin to Tangent (Large Radius)"

$$CJ - \frac{W \cdot \sqrt{(W \cdot Z - X \cdot Y + X \cdot Z) \cdot (X \cdot Y - W \cdot Z + X \cdot Z)}}{X \cdot (W \cdot Z - X \cdot Y)} = 0$$

Tangents and Similarity Points.



What is the length of the line tangent to the least circle (CK)?

$$BC := AC - AB \qquad BC = 1$$

$$CK := \sqrt{BC^2 - BE^2}$$

$$CK = 0.979796$$

And what is the formula?

CK " External similarity point Origin to Tangent (Small Radius)"

$$CK - \frac{Y \cdot \sqrt{(W \cdot Z - X \cdot Y + X \cdot Z) \cdot (X \cdot Y - W \cdot Z + X \cdot Z)}}{Z \cdot (W \cdot Z - X \cdot Y)} = 0$$

Lastly what is the length of line from tangent to tangent of these circles?

$$JK := CJ - CK$$

$$JK = 0.979796$$

And what is the formula for, JK, Tangent to Tangent"?

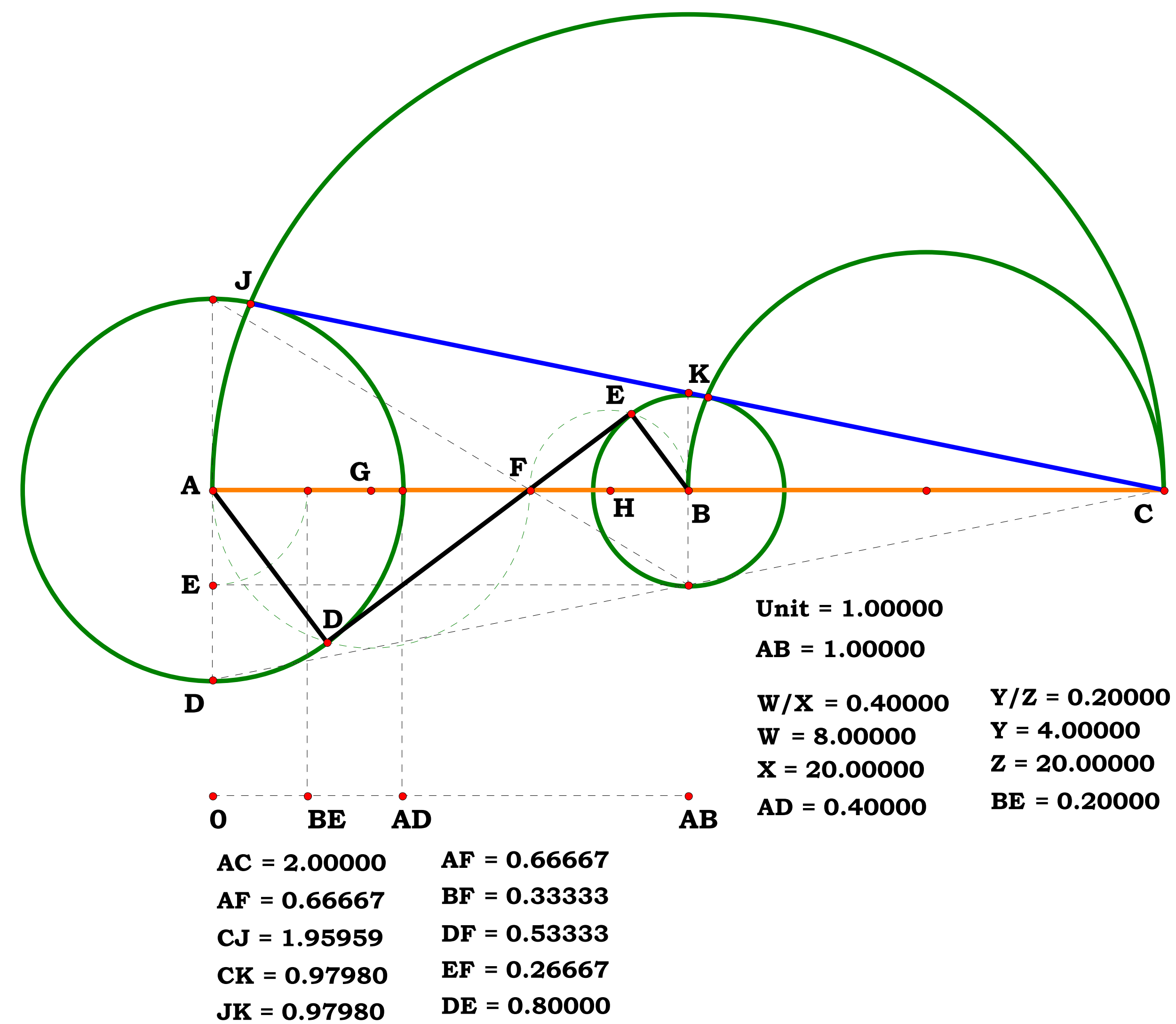
$$JK - \frac{\sqrt{(W \cdot Z - X \cdot Y + X \cdot Z) \cdot (X \cdot Y - W \cdot Z + X \cdot Z)}}{X \cdot Z} = 0$$

I will now turn my attention to the point F the internal similarity point.

$$AF := \frac{AB \cdot AD}{AD + BE} \qquad AF = 0.666667$$

AF "Internal similarity point to center of Radius Large"

$$AF - \frac{W \cdot Z}{W \cdot Z + X \cdot Y} = 0$$



$$BF := AB - AF \qquad BF = 0.333333$$

BF "Internal similarity point to center of Radius Small"

$$BF - \frac{Y \cdot X}{W \cdot Z + X \cdot Y} = 0$$

$$DF := \sqrt{AF^2 - AD^2} \qquad DF = 0.533333$$

DF "Internal similarity point Origin to Tangent (Large Radius)"

$$DF - AD \cdot \frac{\sqrt{-(AD + BE - AB) \cdot (AD + BE + AB)}}{(AD + BE)} = 0$$

$$DF = 0.533333 \qquad EF := \sqrt{BF^2 - BE^2} \qquad EF = 0.266667$$

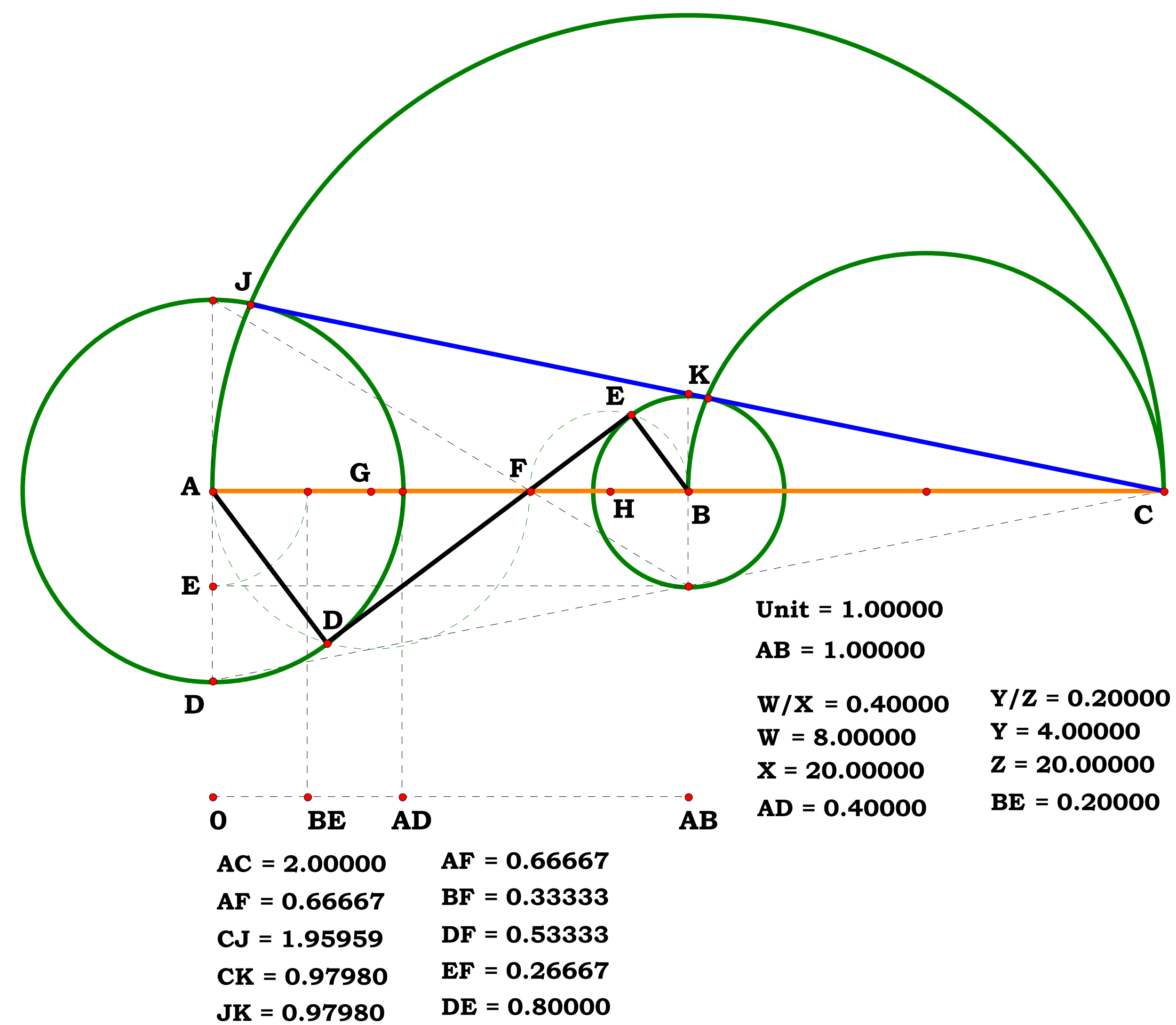
EF "Internal similarity point Origin to Tangent (Small Radius)"

$$EF - \frac{Y \cdot \sqrt{(W \cdot Z + X \cdot Y + X \cdot Z) \cdot (X \cdot Z - X \cdot Y - W \cdot Z)}}{Z \cdot (W \cdot Z + X \cdot Y)} = 0$$

$$DE := DF + EF \qquad DE = 0.8$$

DE "Internal similarity point Tangent to Tangent"

$$DE - \frac{\sqrt{(X \cdot Z - X \cdot Y - W \cdot Z) \cdot (W \cdot Z + X \cdot Y + X \cdot Z)}}{X \cdot Z} = 0$$



042794A

Descriptions.

Given.

$N_1 := 3$

$N_2 := 2$

$N_3 := 6$

$R_1 := \sqrt{N_1^{\,2}} \qquad R_2 := \sqrt{N_2^{\,2}} \qquad AB := N_3$

$AC := \dfrac{R_1^{\,2}}{AB} \qquad BD := \dfrac{R_2^{\,2}}{AB} \qquad CD := AB - (AC + BD)$

$CE := \dfrac{CD}{2} \qquad AE := AC + CE \qquad BE := AB - AE$

$$AE - \frac{N_3^{\,2} + R_1^{\,2} - R_2^{\,2}}{2 \cdot N_3} = 0$$

$$BE - \frac{N_3^{\,2} - R_1^{\,2} + R_2^{\,2}}{2 \cdot N_3} = 0$$

Definitions.

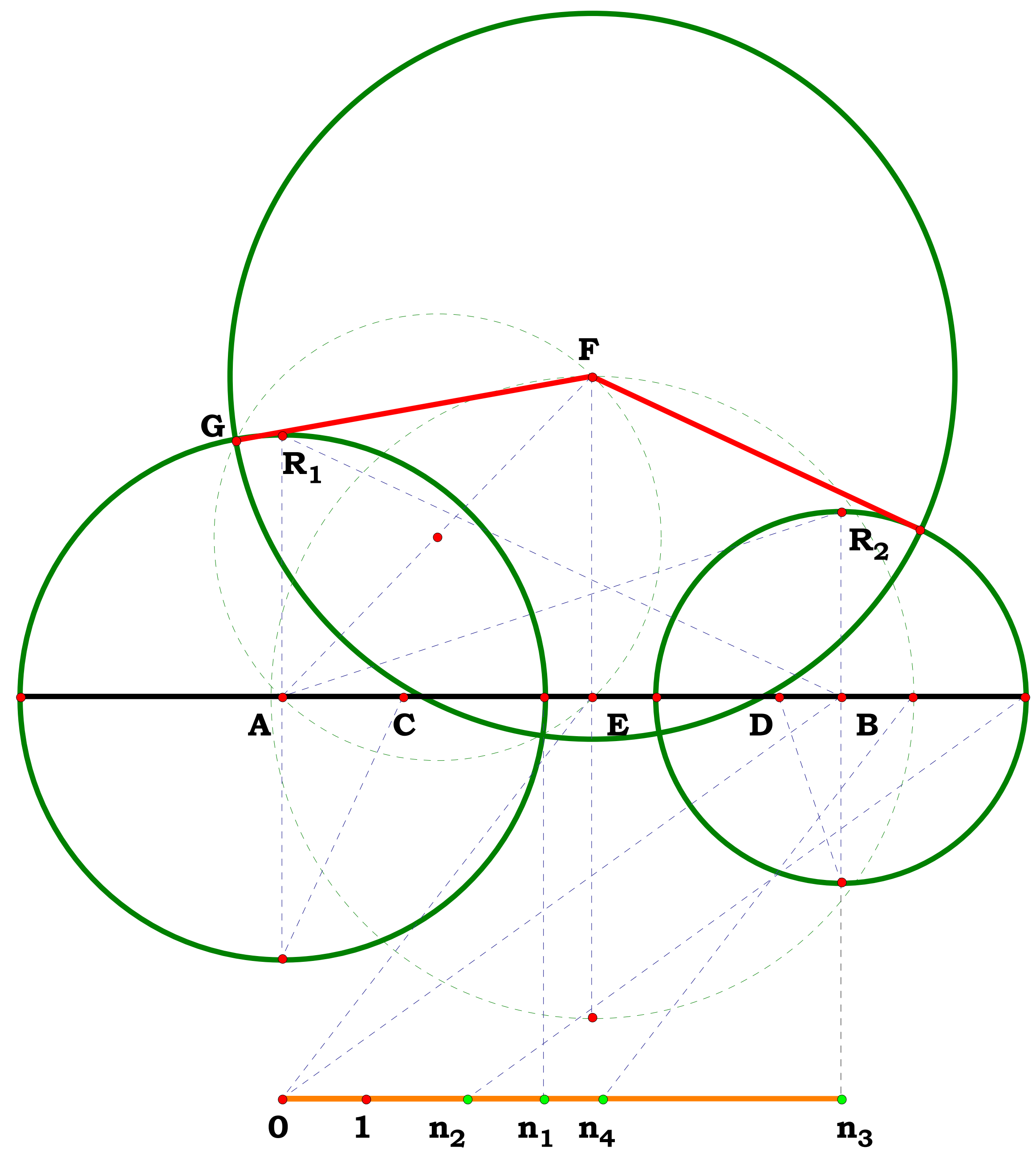
If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.

$N_4 := 3 \qquad AF := \sqrt{AE^2 + N_4^{\,2}} \qquad FG := \sqrt{AF^2 - R_1^{\,2}}$

$$FG - \frac{\sqrt{N_3^{\,2} \cdot \left(N_3^{\,2} + 4 \cdot N_4^{\,2} - 2 \cdot R_1^{\,2} - 2 \cdot R_2^{\,2}\right) + \left(R_1 - R_2\right)^2 \cdot \left(R_1 + R_2\right)^2}}{2 \cdot \sqrt{N_3^{\,2}}} = 0$$

A drawomg solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by **Heinrich Dörrie. It does not lend itself to formal geometry, so I developed my own method, this is actually one of the methods I developed and it is quite simple. I was actually surprised to see how undeveloped it was.**
One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.

The Chordal or Power Line of two Circles



Unit.

$AB := 1$

Given.

$U := 8 \quad W := 3 \quad Y := 12$

042794B

$V := 15 \quad X := 8 \quad Z := 16$

The Chordal or Power Line of two Circles

Descriptions.

A drawomg solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by **Heinrich Dörrie. It does not lend itself to formal geometry, so I developed my own method, this is actually one of the methods I developed and it is quite simple. I was actually surprised to see how undeveloped it was.**
One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.

$$AH := \frac{U}{V} \qquad BJ := \frac{W}{X} \qquad EF := \frac{Y}{Z}$$

$$AC := \frac{AH^2}{AB} \qquad BD := \frac{BJ^2}{AB} \qquad CD := AB - (AC + BD)$$

$$CE := \frac{CD}{2} \qquad AE := AC + CE \qquad BE := AB - AE$$

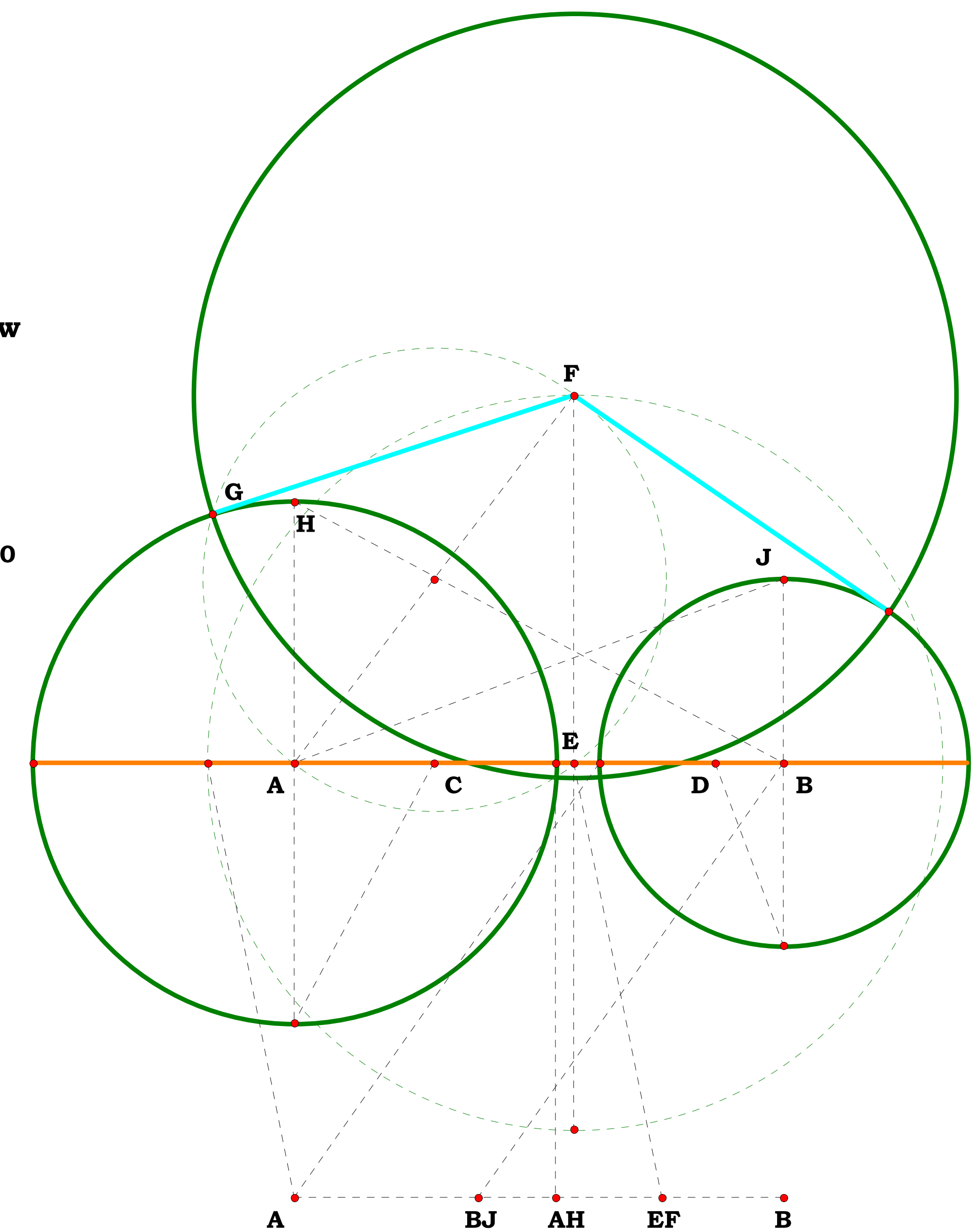
$$AE - \frac{U^2 \cdot X^2 - V^2 \cdot W^2 + V^2 \cdot X^2}{2 \cdot V^2 \cdot X^2} = 0 \qquad AE = 0.57191$$

$$BE - -\frac{U^2 \cdot X^2 - V^2 \cdot W^2 - V^2 \cdot X^2}{2 \cdot V^2 \cdot X^2} = 0 \qquad BE = 0.42809$$

Unit = 1.00000
XY = 0.53333
U = 8.00000
V = 15.00000
AH = 0.53333

XY = 0.37500
W = 3.00000
X = 8.00000
BJ = 0.37500

XY = 0.75000
Y = 12.00000
Z = 16.00000
AE = 0.57191
EF = 0.75000
BE = 0.42809
FG = 0.77791
AF = 0.94318



Definitions.

$$AF := \frac{\sqrt{\left(U^2 \cdot X^2 - V^2 \cdot W^2 + V^2 \cdot X^2\right)^2 \cdot Z^2 + 4 \cdot V^4 \cdot X^4 \cdot Y^2}}{2 \cdot V^2 \cdot X^2 \cdot Z}$$

$AF = 0.943176$

$$FG := \frac{\sqrt{\left[\left(U^4 - 2 \cdot U^2 \cdot V^2 + V^4\right) \cdot X^4 - 2 \cdot V^2 \cdot W^2 \cdot \left(U^2 + V^2\right) \cdot X^2 + V^4 \cdot W^4\right] \cdot Z^2 + 4 \cdot V^4 \cdot X^4 \cdot Y^2}}{2 \cdot V^2 \cdot X^2 \cdot Z}$$

$FG = 0.777905$

042894

Descriptions.

Given.

$N_1 := 3$ $\quad D_1 := 7.81447$

$N_2 := 1$ $\quad D_2 := 6.96686$

$N_3 := 2$ $\quad D_3 := 5.33279$

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate an Algebraic name for the power point and the length of the resultant tangent.

$R_1 := \sqrt{N_1^2} \qquad R_2 := \sqrt{N_2^2} \qquad R_3 := \sqrt{N_3^2} \qquad AC := D_1 \qquad AE := D_2 \qquad CE := D_3$

$$AG := \frac{\sqrt{\left(R_1^2 + D_1^2 - R_2^2\right)^2}}{2 \cdot D_1} \qquad AH := \frac{\sqrt{\left(R_1^2 + D_2^2 - R_3^2\right)^2}}{2 \cdot D_2} \qquad AM := \frac{\sqrt{\left(D_2^2 + D_1^2 - D_3^2\right)^2}}{2 \cdot D_1}$$

$$EM := \sqrt{AE^2 - AM^2} \qquad AK := \frac{AE \cdot AH}{AM} \qquad GK := AK - AG \qquad GJ := \frac{AM \cdot GK}{EM}$$

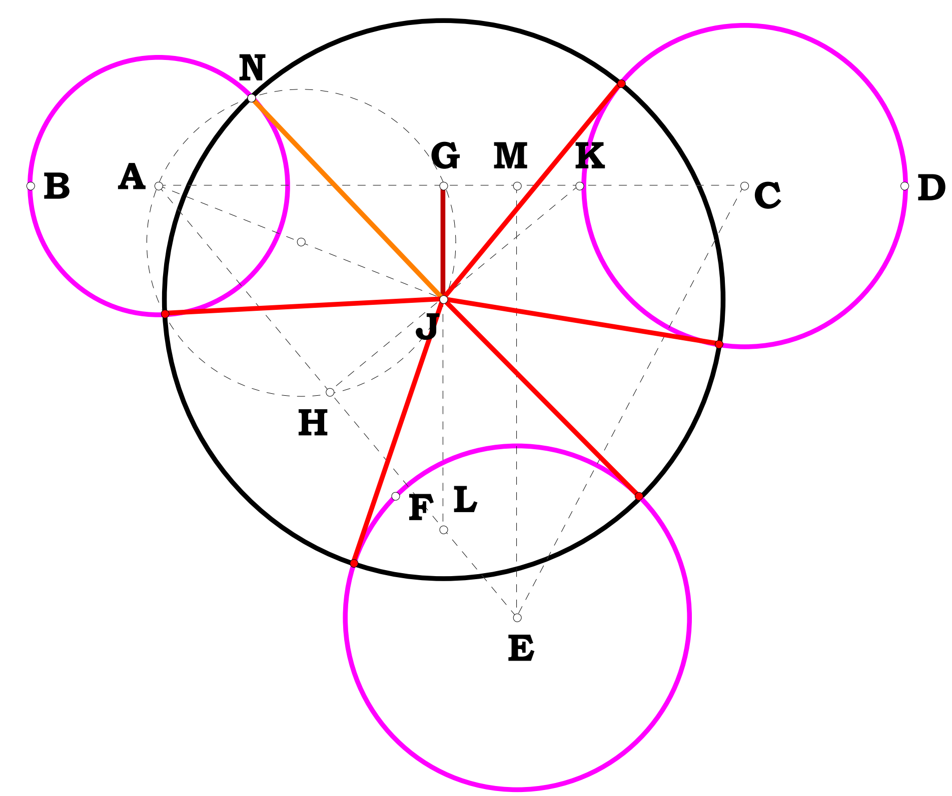
$$AJ := \sqrt{AG^2 + GJ^2} \qquad AN := R_1 \qquad JN := \sqrt{AJ^2 - AN^2}$$

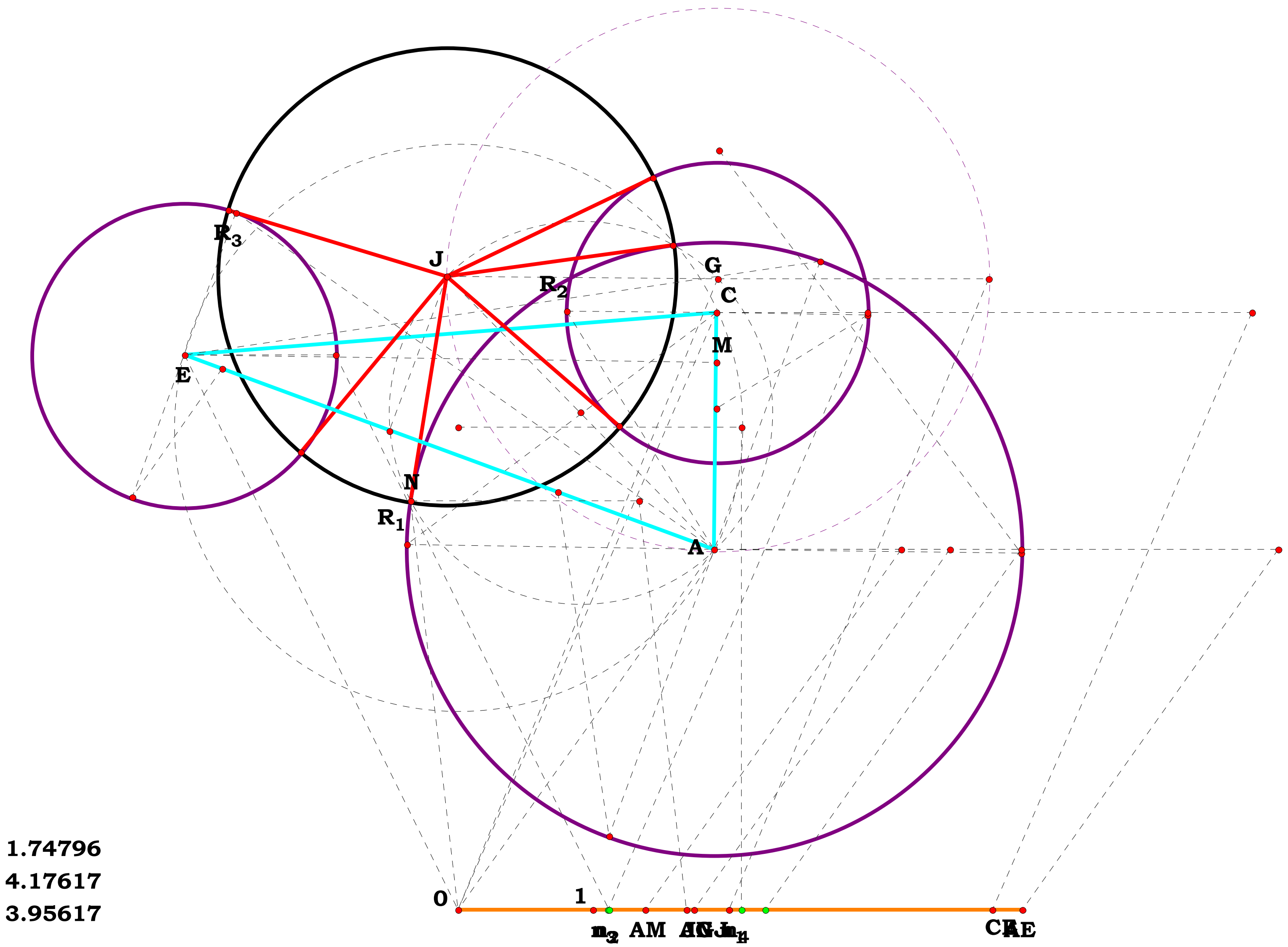
Definitions.

$$GJ - \frac{2 \cdot D_1^2 \cdot \sqrt{\left(D_2^2 + R_1^2 - R_3^2\right)^2} - \sqrt{\left(D_1^2 + D_2^2 - D_3^2\right)^2} \cdot \sqrt{\left(D_1^2 + R_1^2 - R_2^2\right)^2}}{2 \cdot \sqrt{D_1^2} \cdot \sqrt{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_2 - D_1 + D_3\right) \cdot \left(D_1 + D_2 + D_3\right)}} = 0$$

$$JN - \sqrt{\frac{\begin{array}{l} D_1^2 \cdot R_3^2 \cdot \left(R_1^2 + R_2^2 - R_3^2 - D_1^2\right) + D_1^2 \cdot D_2^2 \cdot \left(R_2^2 + R_3^2 - D_3^2\right) \ldots \\ + D_2^2 \cdot R_1^2 \cdot \left(D_3^2 + R_2^2 - R_3^2\right) + D_1^2 \cdot D_3^2 \cdot \left(R_1^2 + R_3^2\right) + D_3^2 \cdot R_1^2 \cdot \left(R_2^2 + R_3^2 - D_3^2 - R_1^2\right) \ldots \\ + D_2^2 \cdot R_2^2 \cdot \left(R_3^2 - D_2^2 - R_2^2\right) + D_3^2 \cdot R_2^2 \cdot \left(D_2 - R_3\right) \cdot \left(D_2 + R_3\right) - D_1^2 \cdot R_1^2 \cdot R_2^2 \end{array}}{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 + D_2 + D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_1 - D_2 - D_3\right)}} = 0$$

Power Point





N_1 = 2.27017
N_2 = 1.11001
N_3 = 1.12235
N_4 = 2.09689

R_1 = 2.27017
R_2 = 1.11001
R_3 = 1.12235

AC = 1.74796
AE = 4.17617
CE = 3.95617
AM = 1.38576
GJ = 2.00571
JN = 1.68883

D_1 = 1.74796
D_2 = 4.17617
D_3 = 3.95617

Animate Points

$$\frac{\sqrt{((D_1^2+D_2^2)-D_3^2)^2}}{2\cdot D_1}\text{-AM} = 0.00000$$

$$\frac{2\cdot D_1^2\cdot\sqrt{((D_2^2+R_1^2)-R_3^2)^2}-\sqrt{((D_1^2+D_2^2)-D_3^2)^2}\cdot\sqrt{((D_1^2+R_1^2)-R_2^2)^2}}{2\cdot\sqrt{D_1^2}\cdot\sqrt{((D_1+D_2)-D_3)\cdot((D_1-D_2)+D_3)\cdot((D_2-D_1)+D_3)\cdot(D_1+D_2+D_3)}}\text{-GJ} = 0.00000$$

$$\sqrt{\frac{(D_1^2\cdot R_3^2\cdot((R_1^2+R_2^2)-R_3^2-D_1^2)+D_1^2\cdot D_2^2\cdot((R_2^2+R_3^2)-D_3^2))+(D_2^2\cdot R_1^2\cdot((D_3^2+R_2^2)-R_3^2)+D_1^2\cdot D_3^2\cdot(R_1^2+R_3^2)+D_3^2\cdot R_1^2\cdot((R_2^2+R_3^2)-D_3^2-R_1^2))+((D_2^2\cdot R_2^2\cdot(R_3^2-D_2^2-R_2^2)+D_3^2\cdot R_2^2\cdot(D_2-R_3)\cdot(D_2+R_3))-D_1^2\cdot R_1^2\cdot R_2^2)}{(((D_1+D_2)-D_3)\cdot(D_1+D_2+D_3)\cdot((D_1-D_2)+D_3)\cdot(D_1-D_2-D_3))}}\text{-JN} = 0.00000$$

Given.

$AF := 2.652$

$BE := 1.390$

$AB := 2.992$

042994A

Teeter-Totter

Given the rectangle ABDE, and some point F, collinear with AD, what are CE, CF, EF, BC?

Descriptions.

$AD := BE \qquad DE := AB$

$DF := AF - AD \qquad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$CE := \dfrac{EF \cdot BE}{DF} \qquad CF := EF + CE \qquad BC := \dfrac{AB \cdot CE}{EF}$

Definitions.

See the end note on the next plate for those who want to make AD greater than AF.

$EF = 3.247262$

$$EF - \sqrt{AF^2 - 2 \cdot AF \cdot BE + BE^2 + AB^2} = 0$$

$CE = 3.576619$

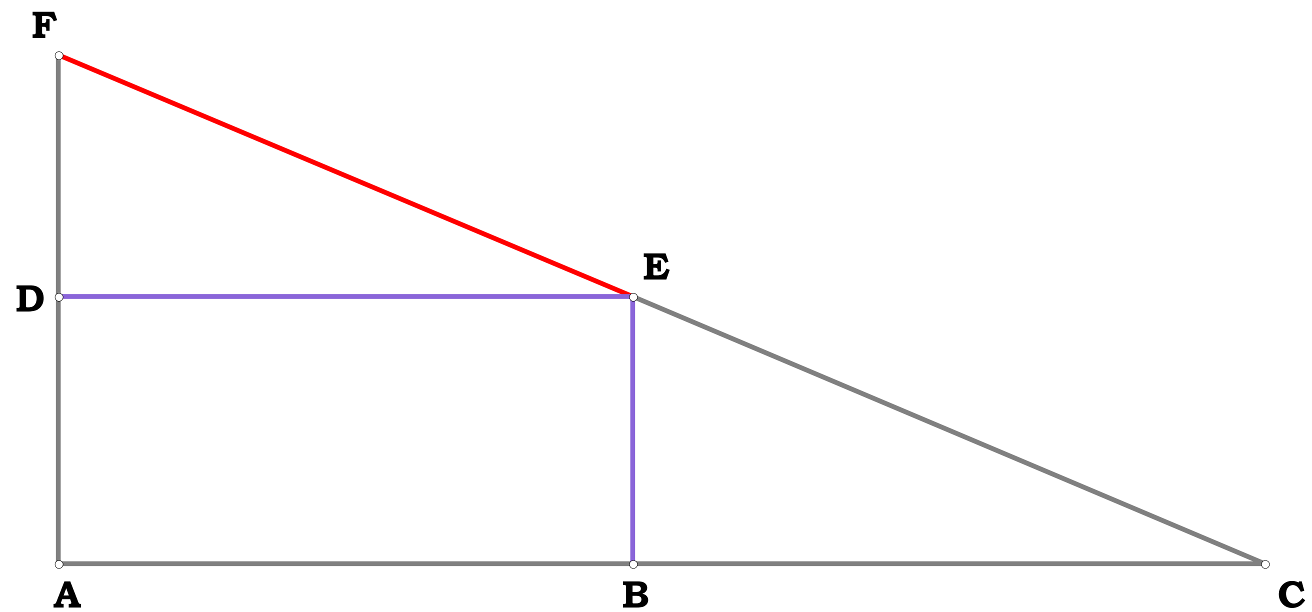
$$CE - \frac{\sqrt{AF^2 - 2 \cdot AF \cdot BE + BE^2 + AB^2} \cdot BE}{AF - BE} = 0$$

$CF = 6.823881$

$$CF - \frac{AF \cdot \sqrt{AF^2 - 2 \cdot AF \cdot BE + BE^2 + AB^2}}{AF - BE} = 0$$

$BC = 3.295468$

$$BC - \frac{BE \cdot AB}{AF - BE} = 0$$



AB = 2.992 in.
BE = 1.390 in.
AF = 2.652 in.
EF = 3.247 in.
CE = 3.574 in.
CF = 6.821 in.
BC = 3.292 in.

$\sqrt{(AF^2 \text{-} 2 \cdot AF \cdot BE) + BE^2 + AB^2} = 3.247$ in.

$EF - \sqrt{(AF^2 \text{-} 2 \cdot AF \cdot BE) + BE^2 + AB^2} = 0.000$ in.

$CE - \dfrac{BE \cdot \sqrt{(AF^2 \text{-} 2 \cdot AF \cdot BE) + BE^2 + AB^2}}{AF - BE} = 0.000$ in.

$CF - \dfrac{AF \cdot \sqrt{(AF^2 \text{-} 2 \cdot AF \cdot BE) + BE^2 + AB^2}}{AF - BE} = 0.000$ in.

$BC - \dfrac{BE \cdot AB}{AF - BE} = 0.000$ in.

042994B

Unit.

$AB := 1$

Given.

$W := 6 \quad Y := 15$

$X := 20 \quad Z := 18$

Teeter-Totter

Descriptions.

$BE := \frac{W}{X} \quad AF := \frac{Y}{Z} \quad AD := BE \quad DE := AB$

$BE = 0.3 \quad AF = 0.833333$

$DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$CE := \frac{EF \cdot BE}{DF} \quad CF := EF + CE \quad BC := \frac{AB \cdot CE}{EF}$

Definitions.

$EF = 1.133333$

$$EF - \frac{\sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0$$

$CE = 0.6375$

$$CE - \frac{W \cdot \sqrt{X^2 \cdot \left(Y^2 + Z^2\right) + W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y)}}{X \cdot (X \cdot Y - W \cdot Z)} = 0$$

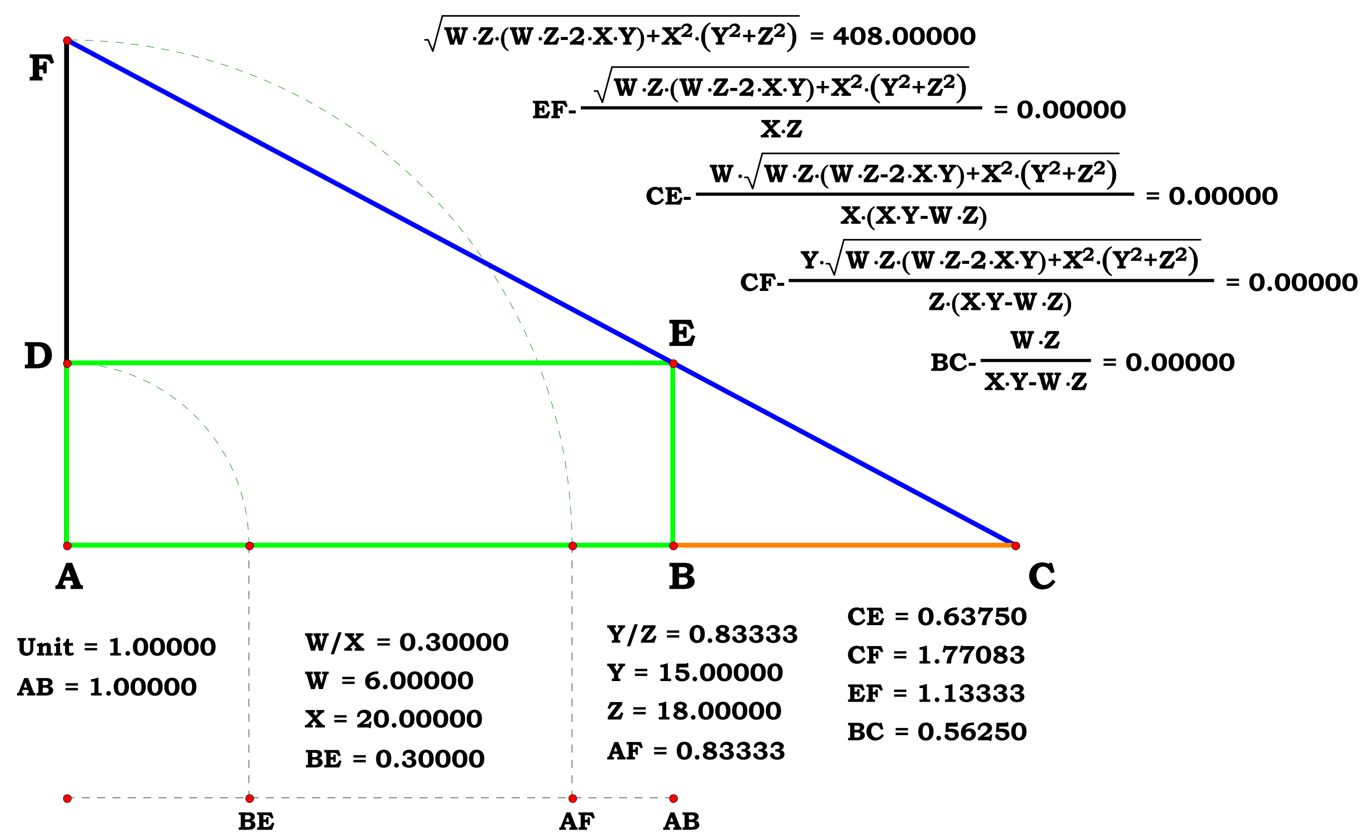
$CF = 1.770833$

$$CF - \frac{Y \cdot \sqrt{X^2 \cdot \left(Y^2 + Z^2\right) + W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y)}}{Z \cdot (X \cdot Y - W \cdot Z)} = 0$$

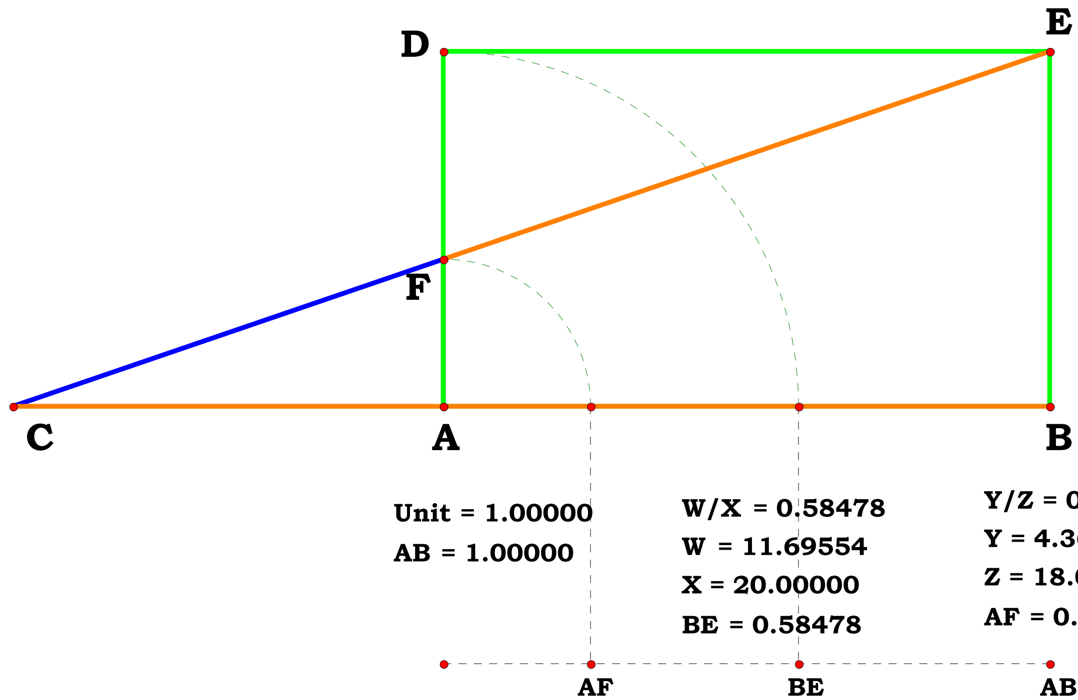
$BC = 0.5625$

$$BC - \frac{W \cdot Z}{X \cdot Y - W \cdot Z} = 0$$

Given the rectangle ABDE, and some point F, collinear with AD, what are CE, CF, EF, BC?



On both of these plates, if you are going to invert D and F, then you have to make a small change to some of the equations, making sure the the result is always positive.



Unit = 1.00000
AB = 1.00000

W/X = 0.58478
W = 11.69554
X = 20.00000
BE = 0.58478

Y/Z = 0.24268
Y = 4.36832
Z = 18.00000
AF = 0.24268

CE = 1.80667
CF = 0.74978
EF = 1.05690
BC = 1.70941

$$\sqrt{W\cdot Z\cdot(W\cdot Z-2\cdot X\cdot Y)+X^2\cdot(Y^2+Z^2)} = 380.48222$$

$$EF-\frac{\sqrt{W\cdot Z\cdot(W\cdot Z-2\cdot X\cdot Y)+X^2\cdot(Y^2+Z^2)}}{X\cdot Z} = 0.00000$$

$$CE-\frac{W\cdot\sqrt{W\cdot Z\cdot(W\cdot Z-2\cdot X\cdot Y)+X^2\cdot(Y^2+Z^2)}}{\sqrt{(X\cdot(X\cdot Y-W\cdot Z))^2}} = 0.00000$$

$$CF-\frac{Y\cdot\sqrt{W\cdot Z\cdot(W\cdot Z-2\cdot X\cdot Y)+X^2\cdot(Y^2+Z^2)}}{\sqrt{(Z\cdot(X\cdot Y-W\cdot Z))^2}} = 0.00000$$

$$BC-\frac{W\cdot Z}{\sqrt{(X\cdot Y-W\cdot Z)^2}} = 0.00000$$

043094A

Descriptions.

$$AC := \sqrt{AB^2 + BC^2}$$

$$CD := \frac{BC \cdot AC}{AB}$$

$$BD := \sqrt{CD^2 - BC^2}$$

Definitions.

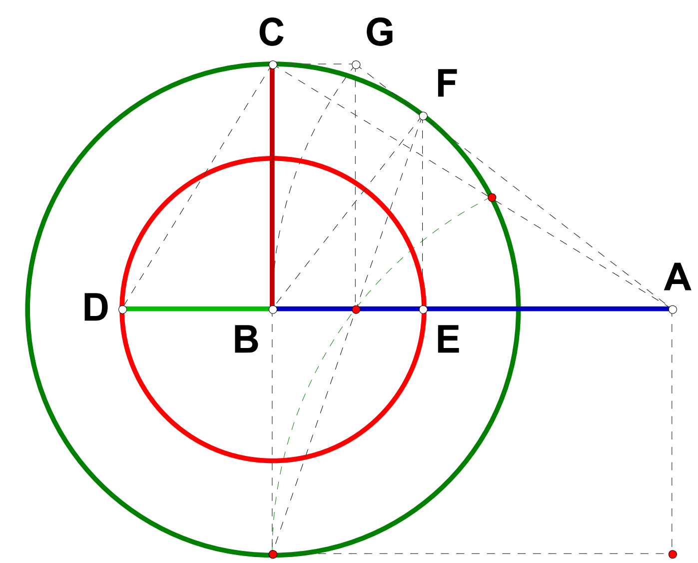
$$\frac{BC^2}{AB} - BD = 0$$

Given.

$AB := 5$

$BC := 2$

Division N^2



043094B

Descriptions.

$$BC := \frac{X}{Y}$$

$$AC := \sqrt{AB^2 + BC^2}$$

$$CD := \frac{BC \cdot AC}{AB}$$

$$BD := \sqrt{CD^2 - BC^2}$$

Definitions.

$$\left(\frac{X}{Y}\right)^2 - BD = 0$$

Unit.

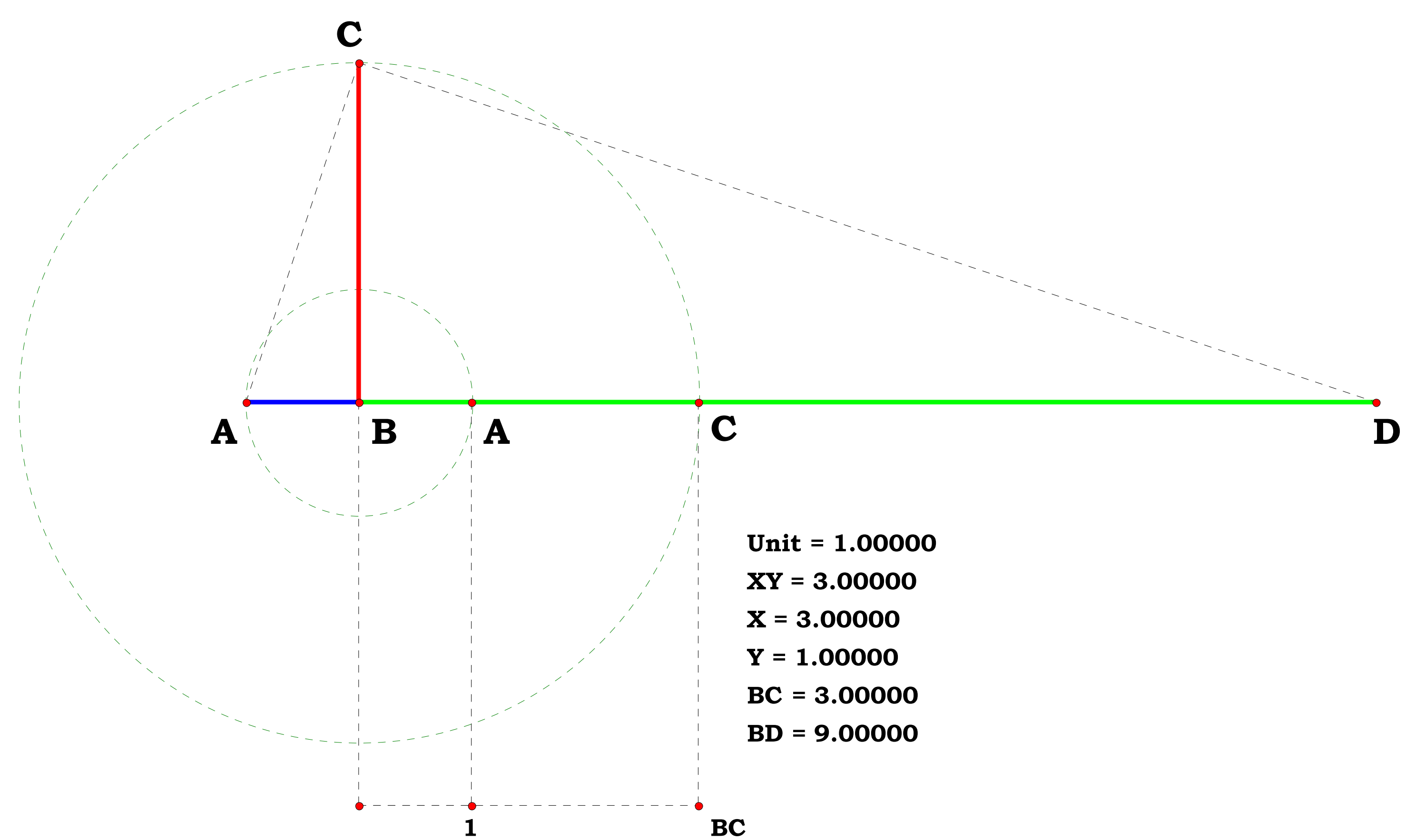
$AB := 1$

Given.

$X := 3$

$Y := 1$

Division N^2



Unit = 1.00000

XY = 3.00000

X = 3.00000

Y = 1.00000

BC = 3.00000

BD = 9.00000

050194A

Given.

$R_1 := 3 \quad DE := R_1$

$R_2 := 2 \quad BC := R_2$

Two Circles And A Parallel

Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.

Descriptions.

$CN := BC \quad EQ := DE \quad CD := BC \quad CE := CD + DE$

$ES := CN \quad NS := CE \quad SQ := EQ - ES \quad AE := \dfrac{NS \cdot EQ}{SQ}$

$AD := AE - DE \quad EP := DE \quad AP := \sqrt{AE^2 - EP^2}$

$DO := \dfrac{EP \cdot AD}{AP} \quad DL := \dfrac{DO \cdot DE}{CD} \quad AC := AD - CD \quad CM := BC$

$AM := \dfrac{AP \cdot AC}{AE} \quad AO := \dfrac{AE \cdot AD}{AP} \quad MO := AO - AM \quad MR := \dfrac{AD \cdot MO}{AO}$

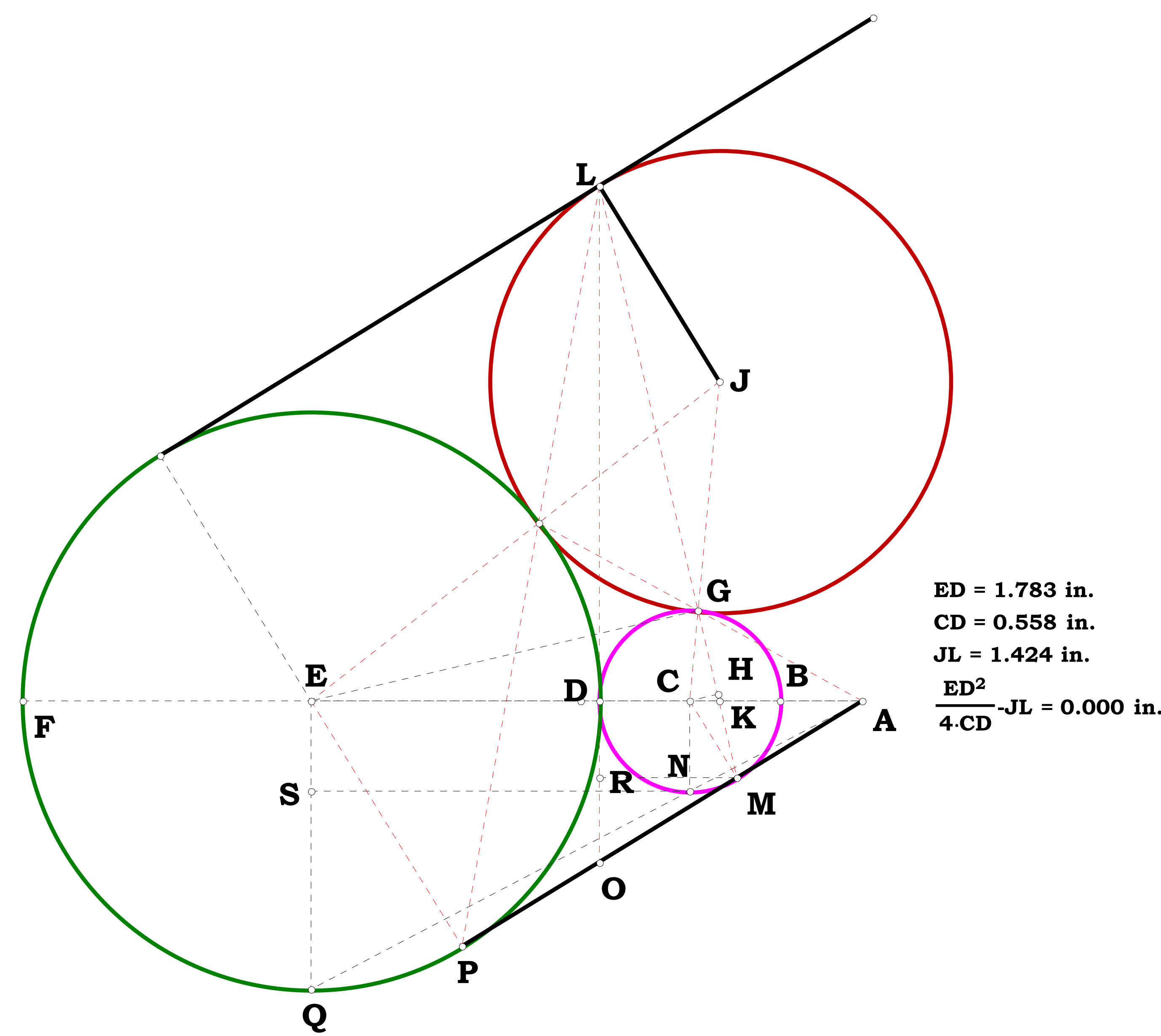
$RO := \dfrac{DO \cdot MR}{AD} \quad DR := DO - RO \quad LR := DR + DL \quad ML := \sqrt{MR^2 + LR^2}$

$DK := \dfrac{MR \cdot DL}{LR} \quad CK := DK - CD \quad CH := \dfrac{LR \cdot CK}{ML}$

$MH := \sqrt{CM^2 - CH^2} \quad MG := 2 \cdot MH \quad GL := ML - MG \quad GJ := \dfrac{CM \cdot GL}{MG}$

Definitions.

$R_3 := \dfrac{{R_1}^2}{4 \cdot R_2} \quad GJ - R_3 = 0$



050194B

Unit.

$mn := 1$

Given.

$W := 13$ $\quad Y := 4$

$X := 20$ $\quad Z := 15$

Two Circles And A Parallel

Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite F which is tangent to the larger circle.

Descriptions.

The results of Plate A tells us the equation for the remaining radius, and knowing that it is tangent to both circles the construction becomes obvious.

$AD := \dfrac{W}{X} \qquad BD := \dfrac{Y}{Z}$

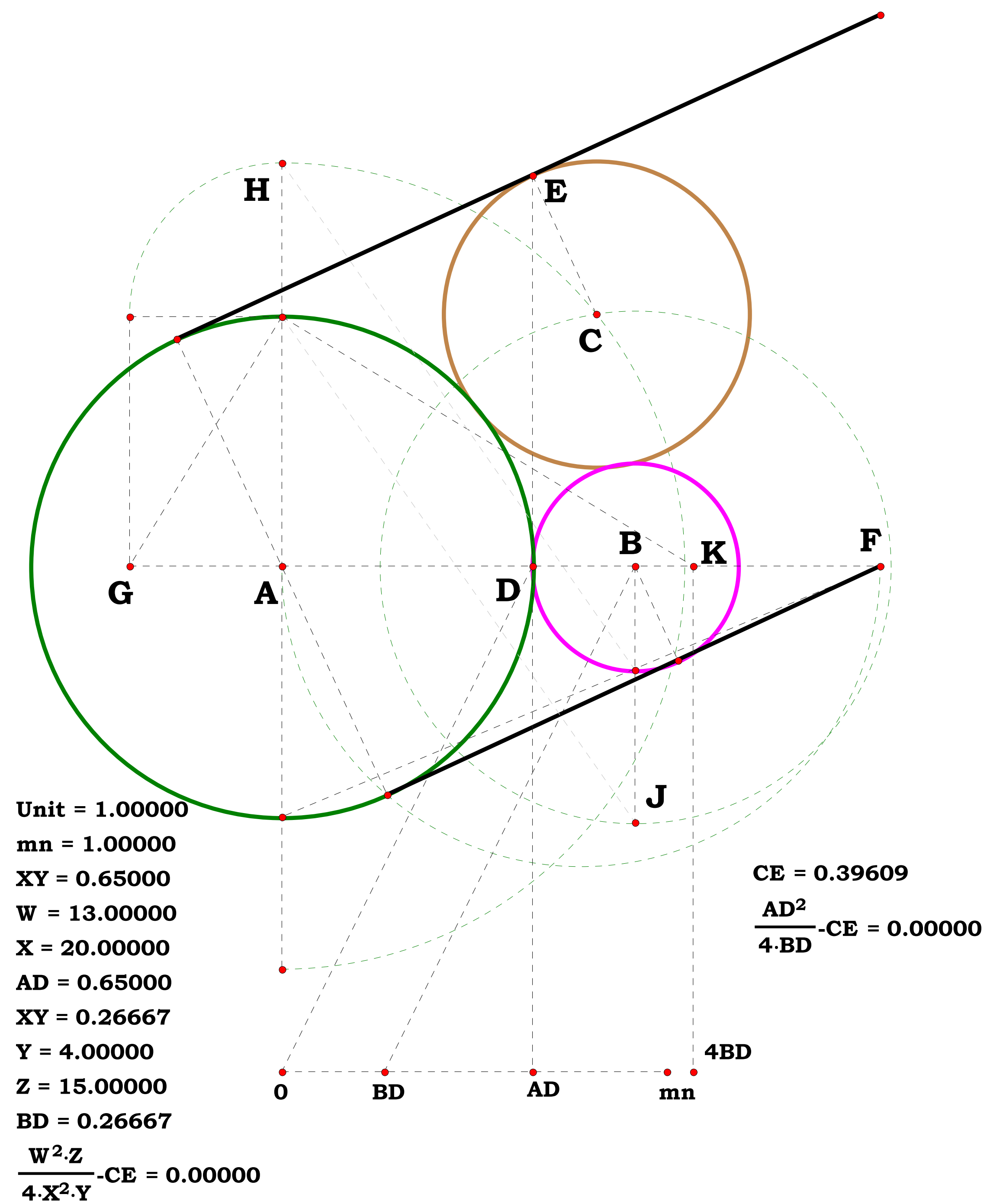
$AK := 4 \cdot BD \qquad AG := \dfrac{AD^2}{AK}$

$AH := AD + AG \qquad BJ := BD + AG$

$CE := \dfrac{AD^2}{4 \cdot BD} \qquad CE = 0.396094$

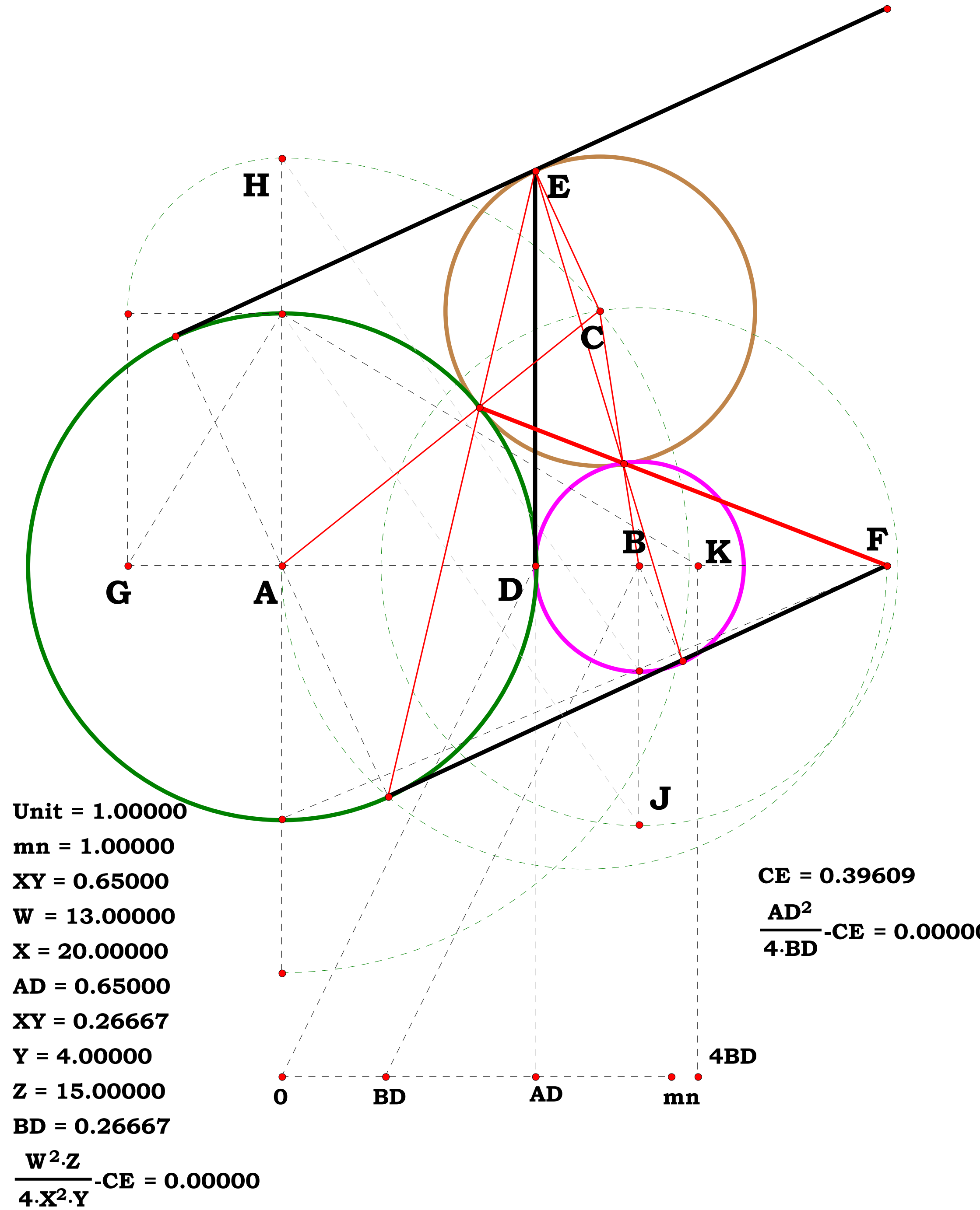
Definitions.

$$CE - \frac{W^2 \cdot Z}{4 \cdot X^2 \cdot Y} = 0$$



If one compare this plate, B, with A, they find something missing, some understanding as to why it is so. Although one can construct figures from equations, the finished construction may need to be aughmented with structures which are implicit in the equation.

Just knowing ones projection is dependent on the powerline, a lot less construction is needed than any of my plates on this show.



050494A

Given.

$R_1 := 1.49167$ $\quad FK := R_1$

$R_2 := .70833$ $\quad BC := R_2$

$D := 2.65$ $\quad CK := D$

$N := 15.18411$

Two Circles And A Tangent

Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.

Descriptions.

$$FL := 2 \cdot FK \qquad FG := \frac{FL}{N} \qquad AK := \frac{D \cdot R_1}{R_1 - R_2}$$

$$EK := \frac{R_1{}^2 + D^2 - R_2{}^2}{2 \cdot D} \qquad AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$$

$$GL := FL - FG \qquad GM := \sqrt{FG \cdot GL} \qquad AJ := \frac{AQ \cdot AQ}{AK} \qquad AF := AK - FK$$

$$FJ := AJ - AF \qquad JL := FL - FJ \qquad JQ := \sqrt{FJ \cdot JL} \qquad GJ := FJ - FG$$

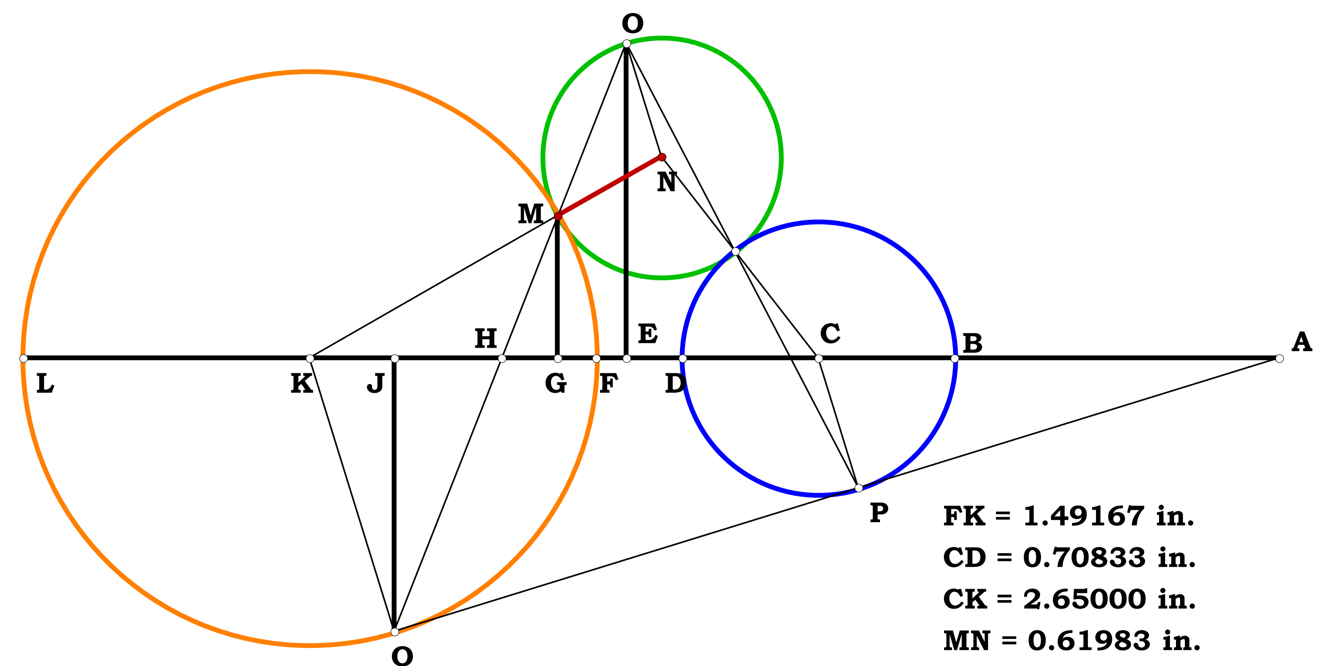
$$QM := \sqrt{(JQ + GM)^2 + GJ^2} \qquad GH := \frac{GJ \cdot GM}{JQ + GM} \qquad HM := \frac{QM \cdot GM}{JQ + GM}$$

$$EF := EK - FK \qquad EH := EF + FG + GH \qquad HO := \frac{HM \cdot EH}{GH} \qquad MO := HO - HM$$

$$KM := FK \qquad MN := \frac{KM \cdot MO}{QM} \qquad MN = 0.619833$$

Definitions.

$$MN - \frac{(4 \cdot R_1 \cdot D) - N \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2N \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$



FK = 1.49167 in.

CD = 0.70833 in.

CK = 2.65000 in.

MN = 0.61983 in.

FG = 0.19648 in.

$\frac{2 \cdot FK}{FG} = 15.18411$

050494B

Unit.

$CK := 1$

Given.

$U := 9 \quad W := 4 \quad Y := 16$

$V := 14 \quad X := 17 \quad Z := 20$

Two Circles And A Tangent

Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.

Descriptions.

$$FK := \frac{U}{V} \qquad BC := \frac{W}{X} \qquad GK := \frac{FK \cdot Y}{Z} \qquad FG := FK - GK$$

$$FL := 2 \cdot FK \qquad AK := \frac{CK \cdot FK}{FK - BC} \qquad EK := \frac{FK^2 + CK^2 - BC^2}{2 \cdot CK}$$

$$AQ := FK \cdot \frac{\sqrt{(FK - BC + CK) \cdot (-FK + BC + CK)}}{FK - BC}$$

$$GL := FL - FG \qquad GM := \sqrt{FG \cdot GL} \qquad AJ := \frac{AQ \cdot AQ}{AK} \qquad AF := AK - FK$$

$$FJ := AJ - AF \qquad JL := FL - FJ \qquad JQ := \sqrt{FJ \cdot JL} \qquad GJ := FJ - FG$$

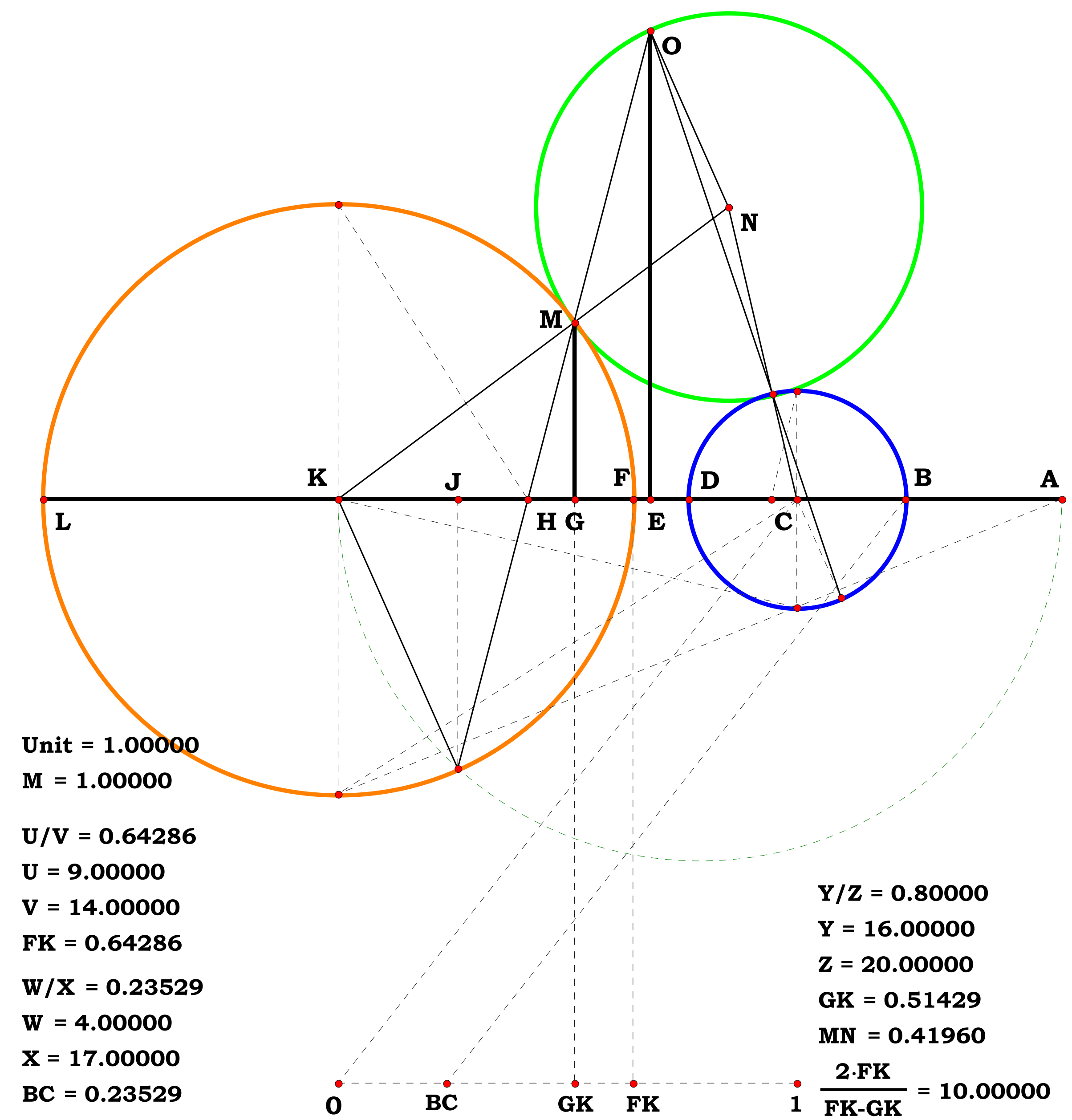
$$QM := \sqrt{(JQ + GM)^2 + GJ^2} \qquad GH := \frac{GJ \cdot GM}{JQ + GM} \qquad HM := \frac{QM \cdot GM}{JQ + GM}$$

$$EF := EK - FK \qquad EH := EF + FG + GH \qquad HO := \frac{HM \cdot EH}{GH} \qquad MO := HO - HM$$

$$KM := FK \qquad MN := \frac{KM \cdot MO}{QM} \qquad MN = 0.419597$$

Definitions.

$$MN - \frac{X^2 \cdot \left[Z \cdot \left(U^2 + V^2\right) - 2 \cdot U \cdot V \cdot Y\right] - V^2 \cdot W^2 \cdot Z}{2 \cdot V \cdot X \cdot (V \cdot W \cdot Z - U \cdot X \cdot Z + V \cdot X \cdot Y)} = 0$$



050694

Unit

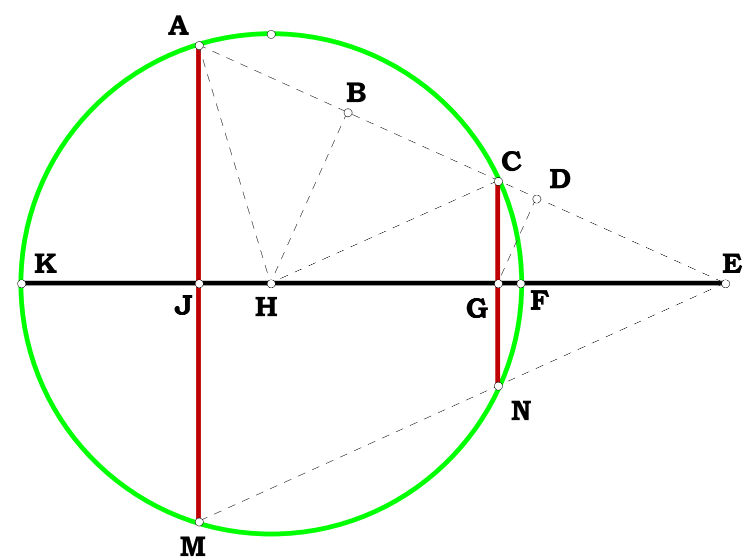
FH := 1

Given.

N := 2.423

A Ratio In Trisection

What is AJ to CG?



Descriptions.

$$CE := FH \qquad CG := \frac{FH}{N} \qquad EG := \sqrt{CE^2 - CG^2} \qquad CD := \frac{CG^2}{CE}$$

$$DG := \sqrt{CG^2 - CD^2} \qquad EH := 2 \cdot EG \qquad BH := \frac{DG \cdot EH}{EG} \qquad CH := FH$$

$$BC := \sqrt{CH^2 - BH^2} \qquad AC := 2 \cdot BC \qquad AE := AC + CE \qquad AJ := \frac{CG \cdot AE}{CE}$$

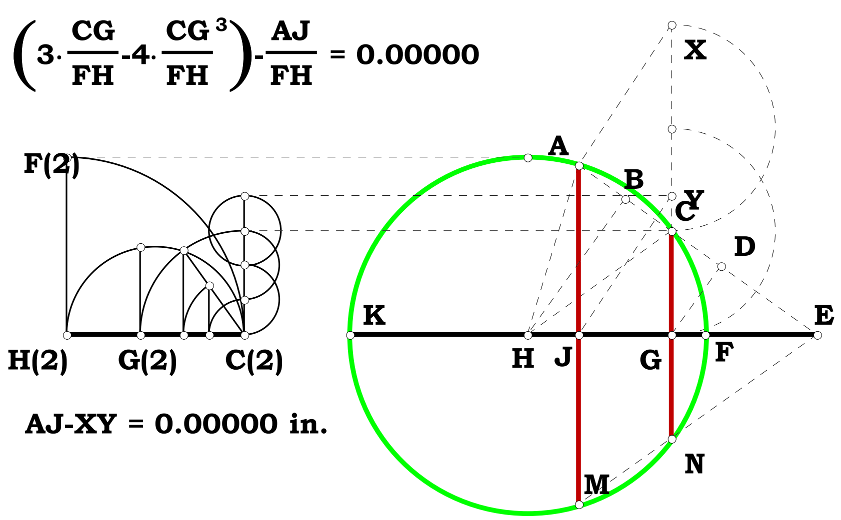
$$3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0 \qquad 3 \cdot CG - 4 \cdot CG^3 - AJ = 0 \qquad AJ - \frac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$$

Definitions.

$$AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$$

The resultant equation suggests this construction.

$$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG}{FH}^3\right) - \frac{AJ}{FH} = 0.00000$$



AJ-XY = 0.00000 in.

Unit.

$AE := 1$

Given.

$N := 5$

A Trisection Ratio with the Paper Trisector

The figure works on the fact that trisection takes place as point K moves between .5 and 1 of half the radius. Thus one can examine it by a simple fact. Division in this method will take place, for the Paper Trisector, over 3/4 of the semi-circle.

In trisection, what is the ratio of FG/EK?

050794A

Descriptions.

$$EH := \frac{AE}{2} \qquad HK := \frac{EH}{N} \qquad AK := AE + EH + HK$$

$$EJ := AE \qquad EK := EH + HK \qquad AD := \frac{EJ \cdot AK}{EK} \qquad CD := AE$$

$$AC := AD - CD \qquad BC := \frac{AC}{2} \qquad CE := AE \qquad BE := \sqrt{CE^2 - BC^2}$$

$$BD := CD + BC \qquad DE := \sqrt{BD^2 + BE^2} \qquad DF := \frac{BD \cdot AD}{DE}$$

$$EG := AE \qquad DG := DE + EG \qquad FG := DG - DF$$

$$HK = 0.1 \qquad FG = 0.361715$$

$$EK = 0.6 \qquad AK = 1.6$$

Definitions.

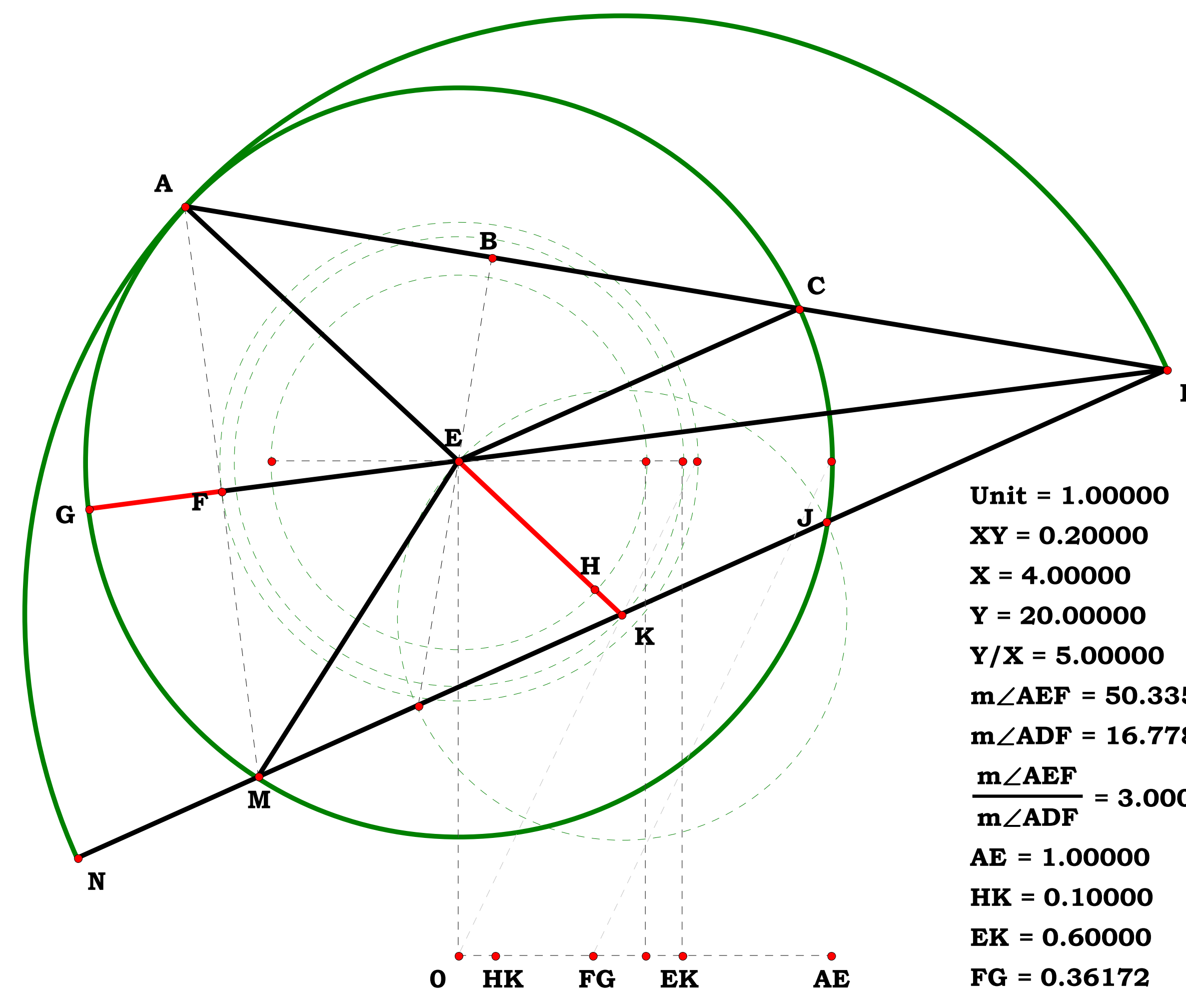
$$EH - \frac{1}{2} = 0 \qquad HK - \frac{1}{2 \cdot N} = 0 \qquad AK - \frac{3 \cdot N + 1}{2 \cdot N} = 0 \qquad EJ - 1 = 0 \qquad EK - \frac{N+1}{2 \cdot N} = 0$$

$$AD - \frac{3 \cdot N + 1}{N + 1} = 0 \qquad CD - 1 = 0 \qquad AC - \frac{2 \cdot N}{N+1} = 0 \qquad BC - \frac{N}{N+1} = 0 \qquad CE - 1 = 0$$

$$BE - \frac{\sqrt{2 \cdot N + 1}}{(N+1)} = 0 \qquad BD - \frac{2 \cdot N + 1}{N + 1} = 0 \qquad DE - \frac{\sqrt{2 \cdot (2 \cdot N + 1)}}{\sqrt{N+1}} = 0 \qquad EG - 1 = 0$$

$$DF - \frac{(3 \cdot N + 1) \cdot \sqrt{4 \cdot N + 2}}{2 \cdot (N+1)^{\frac{3}{2}}} = 0 \qquad DG - \frac{\sqrt{2} \cdot \sqrt{2 \cdot N + 1} + \sqrt{N+1}}{\sqrt{N+1}} = 0$$

$$FG - \frac{\left(\sqrt{2} - \sqrt{2} \cdot N\right) \cdot \sqrt{2 \cdot N + 1} + 2 \cdot (N+1)^{\frac{3}{2}}}{2 \cdot (N+1)^{\frac{3}{2}}} = 0 \qquad \frac{FG}{EK} - \frac{N \cdot \left[(1 - N) \cdot \sqrt{2} \cdot \sqrt{2 \cdot N + 1} + 2 \cdot (N+1)^{\frac{3}{2}}\right]}{(N+1)^{\frac{5}{2}}} = 0$$



Unit = 1.00000

XY = 0.20000

X = 4.00000

Y = 20.00000

Y/X = 5.00000

m∠AEF = 50.33596°

m∠ADF = 16.77865°

$\frac{m\angle AEF}{m\angle ADF} = 3.00000$

AE = 1.00000

HK = 0.10000

EK = 0.60000

FG = 0.36172

AK = 1.60000

N = 5.00000

$$\frac{N \cdot \left((1-N) \cdot \sqrt{2} \cdot \sqrt{2 \cdot N+1} + 2 \cdot (N+1)^{\frac{3}{2}}\right)}{(N+1)^{\frac{5}{2}}} - \frac{FG}{EK} = 0.00000$$

$$\frac{\left(\sqrt{2} - \sqrt{2} \cdot N\right) \cdot \sqrt{2 \cdot N+1} + 2 \cdot (N+1)^{\frac{3}{2}}}{2 \cdot (N+1)^{\frac{3}{2}}} - FG = 0.00000$$

Unit.

$AE := 1$

Given.

$X := 5 \quad Y := 17$

050794B

A Trisection Ratio with the Paper Trisector

The figure works on the fact that trisection takes place as point K moves between .5 and 1 of half the radius. Thus one can examine it by a simple fact. Division in this method will take place, for the Paper Trisector, over 3/4 of the semi-circle.

Descriptions.

$EH := \frac{AE}{2} \quad N := \frac{Y}{X} \quad HK := \frac{EH}{N} \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

$HK = 0.147059 \quad FG = 0.486472 \quad N = 3.4$

$EK = 0.647059 \quad AK = 1.647059$

Definitions.

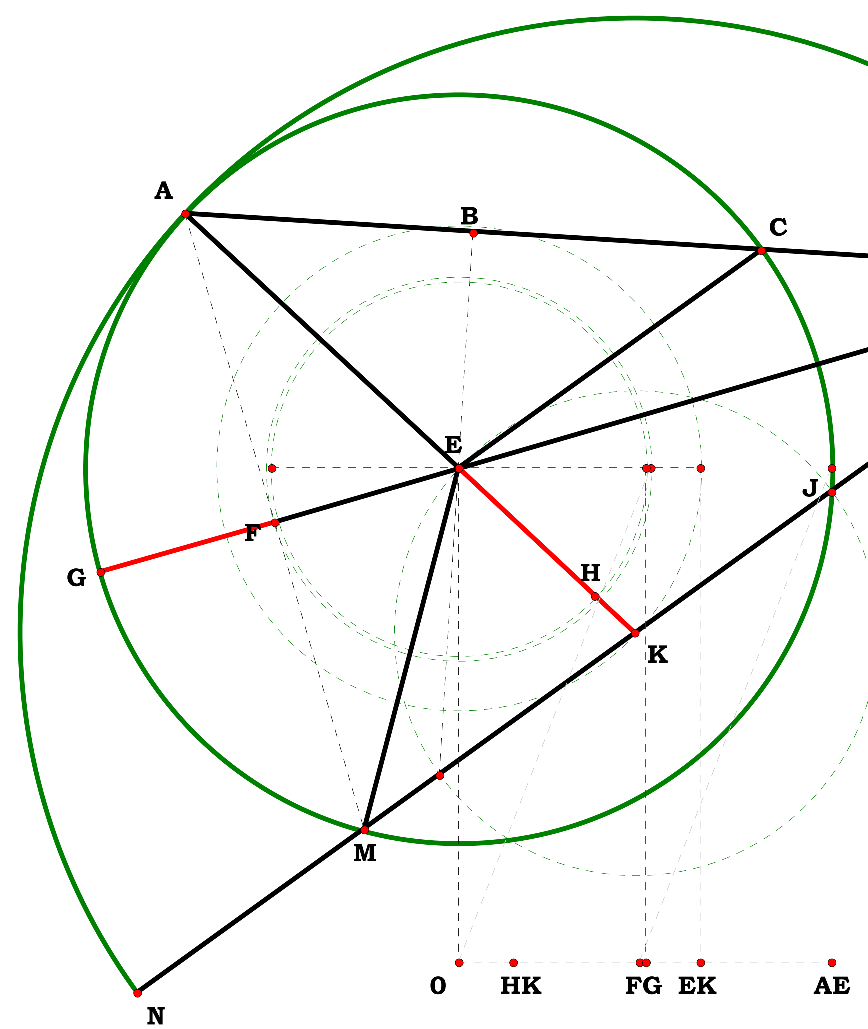
$HK - \frac{X}{2 \cdot Y} = 0 \quad EK - \frac{X+Y}{2 \cdot Y} = 0$

$$FG - \frac{\sqrt{X + 2 \cdot Y} \cdot (X - Y) \cdot \sqrt{2} + 2 \cdot (X+Y) \cdot \sqrt{X+Y}}{2 \cdot (X+Y)^{\frac{3}{2}}} = 0$$

$$AK - \frac{X + 3 \cdot Y}{2 \cdot Y} = 0$$

$$\frac{FG}{EK} - \frac{Y \cdot \left[\sqrt{X + 2 \cdot Y} \cdot \left(X^2 - Y^2\right) \cdot \sqrt{2} + 2 \cdot (X+Y)^{\frac{5}{2}}\right]}{(X+Y)^{\frac{7}{2}}} = 0$$

In trisection, what is the ratio of FG/EK?



Unit = 1.00000
XY = 0.29412
X = 5.00000
Y = 17.00000
Y/X = 3.40000
m∠AEF = 59.10085°
m∠ADF = 19.70028°

$\frac{m\angle AEF}{m\angle ADF} = 3.00000$

AE = 1.00000
HK = 0.14706
EK = 0.64706
FG = 0.48647
AK = 1.64706
N = 3.40000

$$\frac{\sqrt{X+2\cdot Y}\cdot(X-Y)\cdot\sqrt{2}+2\cdot(X+Y)\cdot\sqrt{(X+Y)}}{2\cdot(X+Y)^{\frac{3}{2}}}-FG = 0.00000$$

$$\frac{N\cdot\left((1-N)\cdot\sqrt{2}\cdot\sqrt{2\cdot N+1}+2\cdot(N+1)^{\frac{3}{2}}\right)}{(N+1)^{\frac{5}{2}}}-\frac{FG}{EK} = 0.00000$$

$$\frac{Y\cdot\left(\sqrt{X+2\cdot Y}\cdot(X^2-Y^2)\cdot\sqrt{2}+2\cdot(X+Y)^{\frac{5}{2}}\right)}{(X+Y)^{\frac{7}{2}}}-\frac{FG}{EK} = 0.00000$$

$$\frac{(\sqrt{2}-\sqrt{2}\cdot N)\cdot\sqrt{2\cdot N+1}+2\cdot(N+1)^{\frac{3}{2}}}{2\cdot(N+1)^{\frac{3}{2}}}-FG = 0.00000$$

051694A

Unit

$CE := 1$

Given.

$Y := 17 \quad N_1 := 2$

$X := 6$

Descriptions.

Choose a point along CF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.
The number of marbles one can stack this way is equal to the number of E/J, or N_1.

$$DE := \frac{X}{Y} \quad EJ := CE \cdot N_1 \quad DJ := \sqrt{DE^2 + EJ^2} \quad JG := \frac{EJ^2}{DJ}$$

$$BE := CE \quad EG := \sqrt{EJ^2 - JG^2} \quad BG := \sqrt{BE^2 - EG^2}$$

$$BJ := BG + JG \quad JK := CE \quad BD := BJ - DJ \quad DH := \frac{JK \cdot BD}{BJ}$$

$$DH = 0.301428$$

Definitions.

$$DE - \frac{X}{Y} = 0 \quad EJ - N_1 = 0 \quad DJ - \frac{\sqrt{N_1^2 \cdot Y^2 + X^2}}{Y} = 0$$

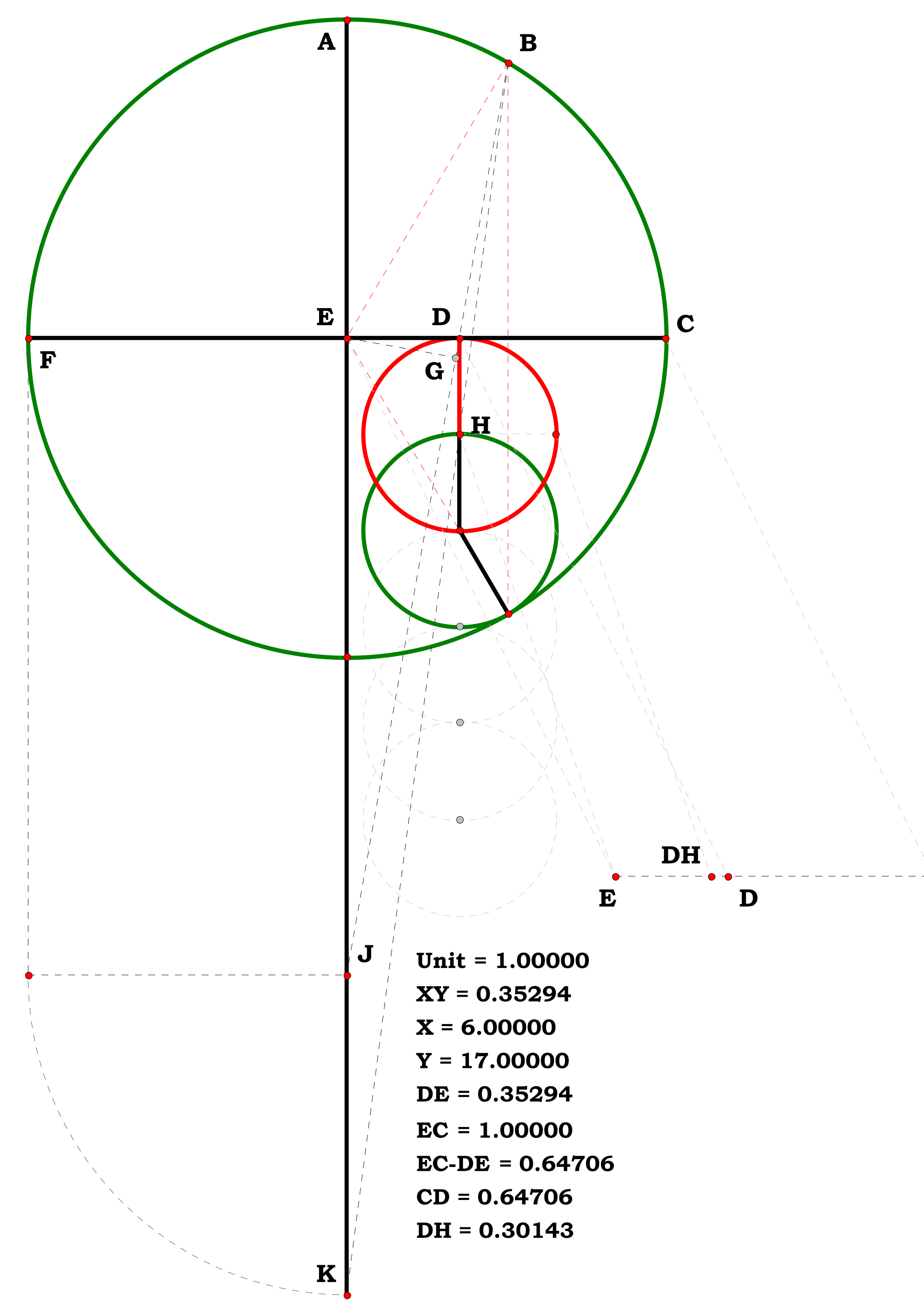
$$JG - \frac{N_1^2 \cdot Y}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad BE - 1 = 0 \quad EG - \frac{N_1 \cdot X}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$BG - \frac{\sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad BJ - \frac{N_1^2 \cdot Y + \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$JK - 1 = 0 \quad BD - \frac{Y \cdot \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2} - X^2}{Y \cdot \sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$DH - \frac{Y \cdot \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2} - X^2}{Y \cdot \left[N_1^2 \cdot Y + \sqrt{X^2 - N_1^2 \cdot \left(X^2 - Y^2\right)}\right]} = 0$$

Tangent Diameter and Circles



051694B

Unit.

$AF := 1$

Given.

$X := 5 \quad N_1 := 3$

$Y := 13$

Descriptions.

Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.
The number of marbles one can stack this way is equal to the number of F/J, or N_1.

$$DF := AF \quad FJ := N_1 \quad HJ := AF \quad EF := \frac{X}{Y} \quad EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$$

$$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG \quad BJ := BE + EJ \quad EK := \frac{HJ \cdot BE}{BJ}$$

$$AJ := FJ + AF \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot EK\right]^2} \quad BC - EK = 0 \quad EK = 0.224477$$

Definitions.

$$DF - 1 = 0 \quad FJ - N_1 = 0 \quad HJ - 1 = 0 \quad EF - \frac{X}{Y} = 0 \quad EJ - \frac{\sqrt{N_1^2 \cdot Y^2 + X^2}}{Y} = 0$$

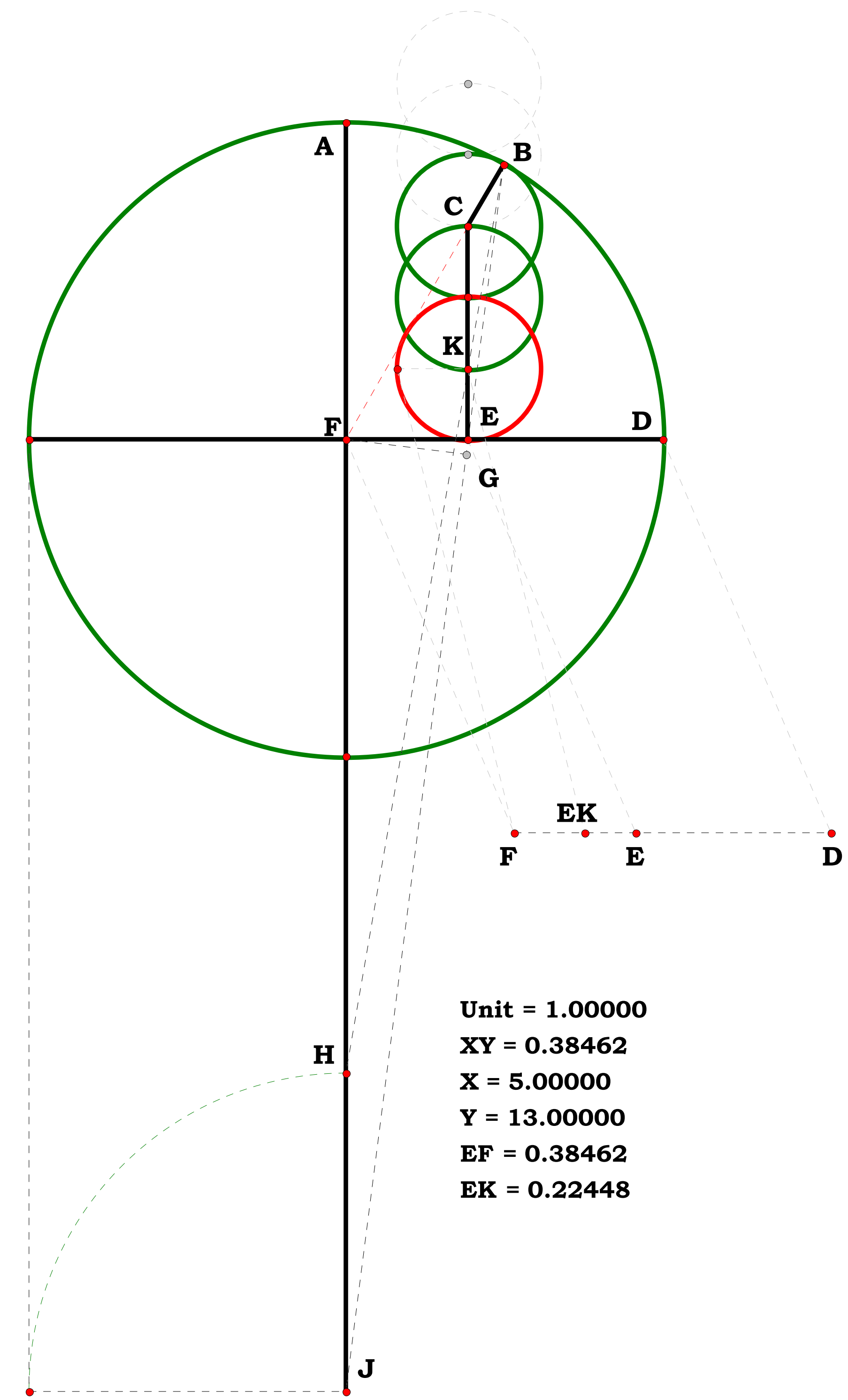
$$EG - \frac{X^2}{Y \cdot \sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad BF - 1 = 0 \quad FG - \frac{N_1 \cdot X}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad AJ - \left(N_1 + 1\right) = 0$$

$$BG - \frac{\sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad BE - \frac{Y \cdot \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2} - X^2}{Y \cdot \sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$BJ - \frac{N_1^2 \cdot Y + \sqrt{N_1^2 \cdot Y^2 - N_1^2 \cdot X^2 + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad EK - \frac{Y \cdot \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2} - X^2}{Y \cdot \left[N_1^2 \cdot Y + \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2}\right]} = 0$$

$$BC - \frac{Y \cdot \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2} - X^2}{Y \cdot \left[N_1^2 \cdot Y + \sqrt{N_1^2 \cdot \left(Y^2 - X^2\right) + X^2}\right]} = 0$$

Tangent Diameter and Circles



102794A

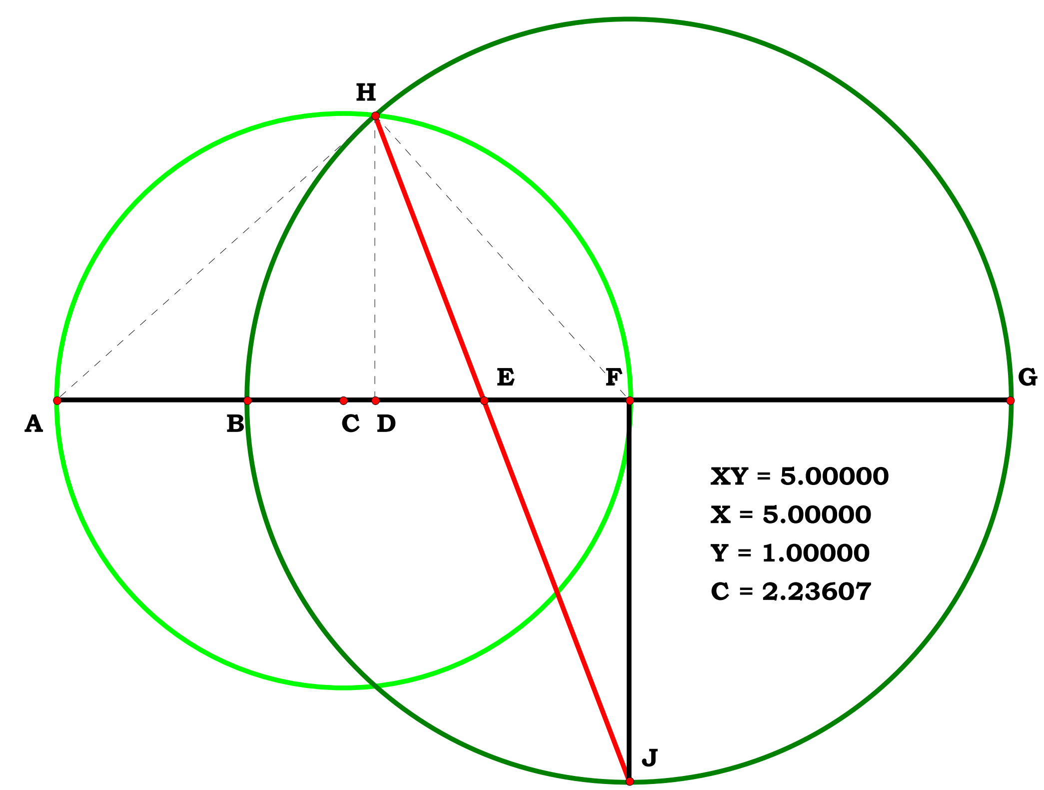
Unit.

$Y := 1 \qquad AB := Y$

Given.

$X := 5 \qquad N := X$

Trivial Method Square Root



Descriptions.

$AG := N \qquad BG := AG - AB \qquad BF := \frac{BG}{2}$

$AF := AB + BF \qquad FH := BF \qquad DF := \frac{FH^2}{AF} \qquad AD := AF - DF$

$BD := AD - AB \qquad DG := BG - BD \qquad FJ := BF$

$DH := \sqrt{BD \cdot DG} \qquad DE := \frac{DF \cdot DH}{DH + FJ} \qquad AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0 \qquad AE = 2.236068$

Definitions.

The following is as far as Mathcad 15 will get you, the rest you have to do by hand. As I noted elsware, Mathcad is not the sharpest tool in the shed, I hope!

$$AE - \frac{\sqrt{(N+1)^2}\cdot\left[\sqrt{N\cdot(N-1)^2}\cdot\sqrt{(N+1)^2} - 2\cdot N + 2\cdot N^2\right]}{(N+1)\cdot\left[N\cdot\sqrt{(N+1)^2} - \sqrt{(N+1)^2} + 2\cdot\sqrt{N\cdot(N-1)^2}\right]} = 0$$

$$AE - \sqrt{N} = 0$$

AE is the square root of AB x AG.

Now, if you are wondering why I say trivial, it is because I foundf that word in a dictionary and thought it would look good standing there.

102794B

Unit.

$Y := 1 \qquad AB := Y$

Given.

$X := 4 \qquad N := X$

Descriptions.

$AG := N \quad AC := \frac{AB}{2} \quad CG := AG - AC \quad CF := \frac{CG}{2}$

$AF := AC + CF \quad FH := CF \quad CD := \frac{AC^2}{CG} \quad AD := AC + CD$

$DG := CG - CD \quad CJ := AC \quad BD := AB - AD$

$DH := \sqrt{BD \cdot AD} \quad CE := \frac{CD \cdot AC}{DH + AC} \quad EG := CG - CE$

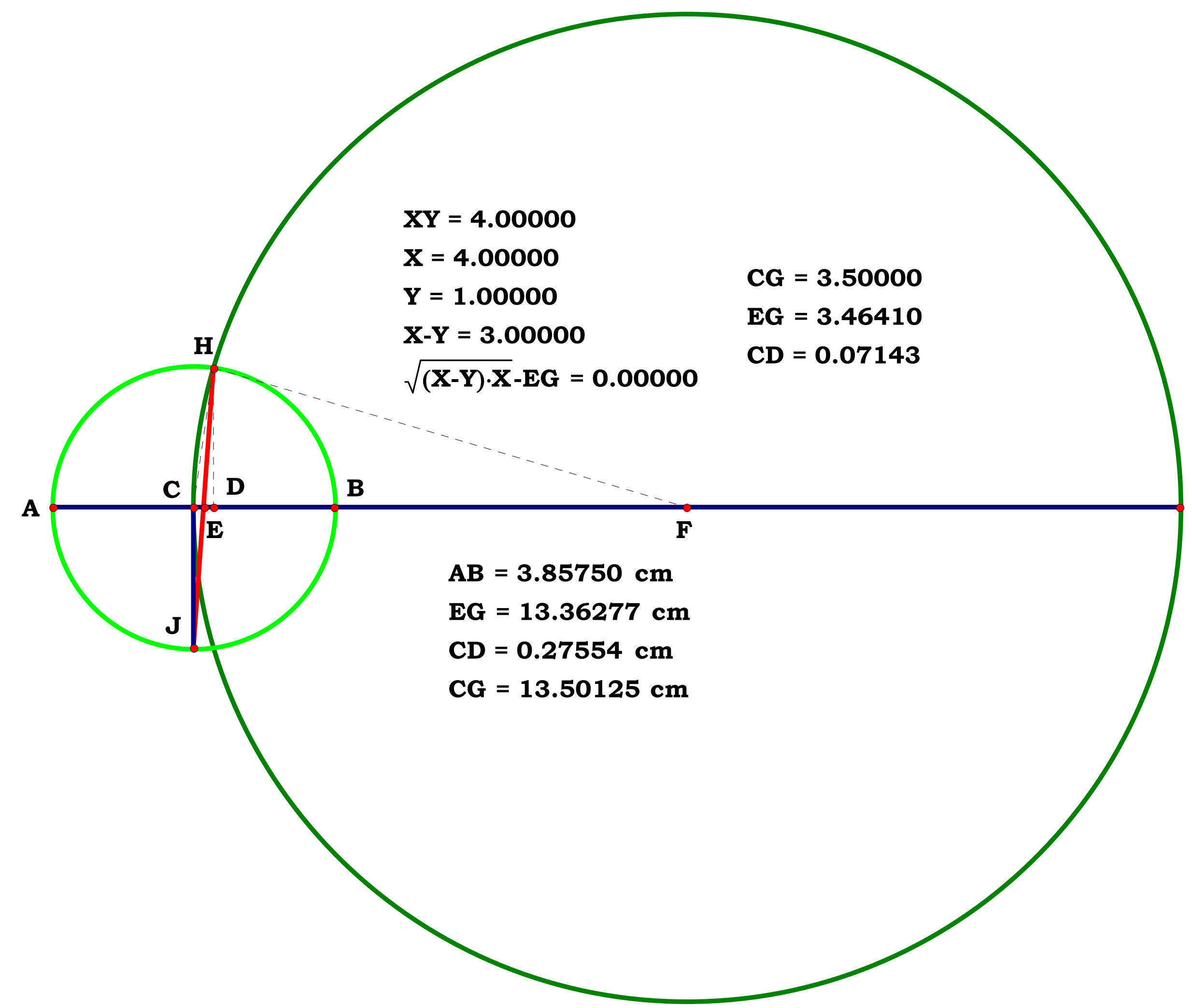
$\sqrt{N \cdot (N - AB)} - EG = 0$

$CG = 3.5 \quad EG = 3.464102 \quad CD = 0.071429$

Definitions.

$$EG - \frac{\left(N^2 - N + 1\right) \cdot \sqrt{N \cdot (N-1)} + 2 \cdot N^2 - 2 \cdot N}{N^2 - N + 2 \cdot \sqrt{N \cdot (N-1)} + 1} = 0$$

Trivial Method Square Root



AE is the square root of BG x AG.

102794C

Given.

$X := 6 \quad Y := 20$

Unit.

$AG := 1$

Trivial Method Square Root

Descriptions.

In plates A and B, we took one or the other things involved in computatrion and called it unity. Here, both are grouped as a unit and one can see the whole of all the possible interactions between the two in a simple figure.

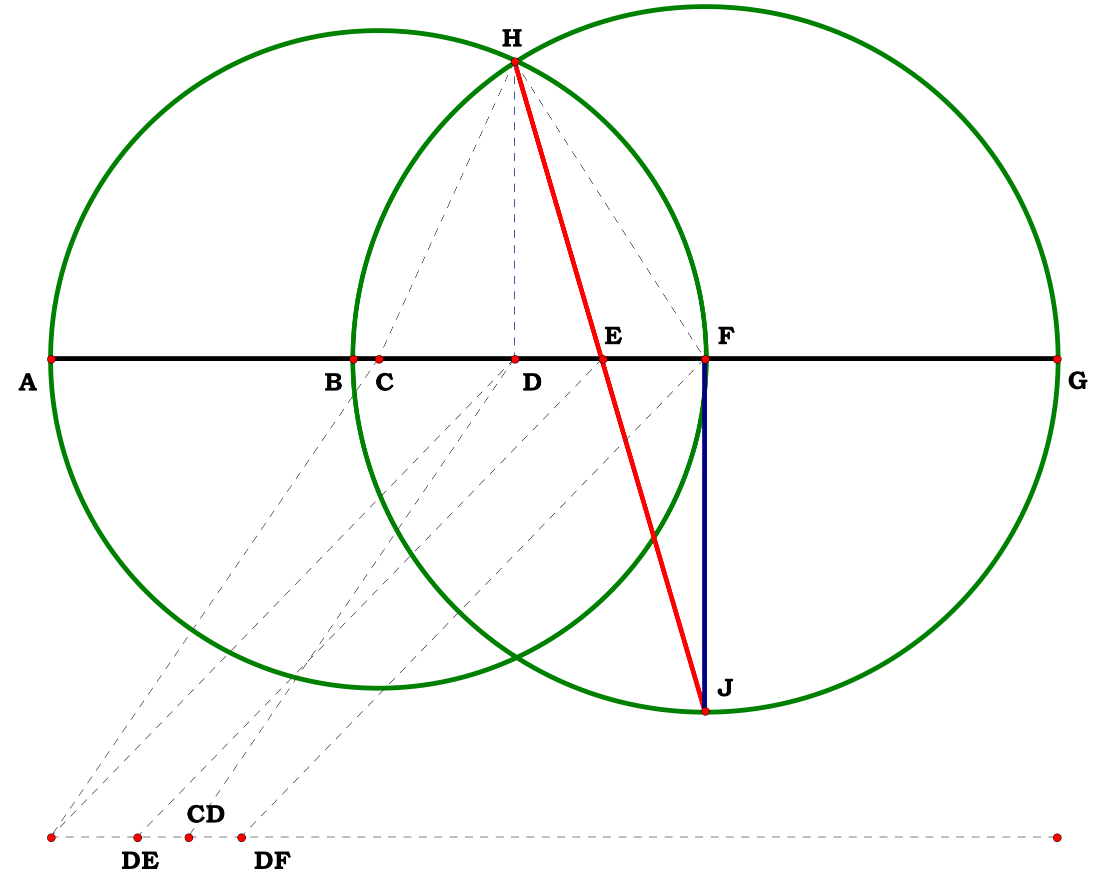
$$AB := \frac{X}{Y} \quad BG := \frac{Y-X}{Y} \quad FH := \frac{BG}{2} \quad AF := AB + FH$$

$$CH := \frac{AF}{2} \quad CD := \frac{2 \cdot CH^2 - FH^2}{2 \cdot CH} \quad DF := CH - CD$$

$$DH := \sqrt{(CH + CD) \cdot DF} \quad DE := \frac{DF \cdot DH}{DH + FH}$$

$$AE := CH + CD + DE \quad AE = 0.547723$$

$$\sqrt{\frac{X}{Y}} - AE = 0$$



Definitions.

$$AB - \frac{X}{Y} = 0 \quad BG - \frac{Y-X}{Y} = 0 \quad FH - \frac{Y-X}{2 \cdot Y} = 0$$

$$AF - \frac{X+Y}{2 \cdot Y} = 0 \quad CH - \frac{X+Y}{4 \cdot Y} = 0$$

$$CD - \frac{\left(6 \cdot X \cdot Y - X^2 - Y^2\right)}{4 \cdot Y \cdot (X+Y)} = 0 \quad DF - \frac{(X-Y)^2}{2 \cdot Y \cdot (X+Y)} = 0$$

$$DH - \frac{(Y-X) \cdot \sqrt{X}}{\sqrt{Y} \cdot (X+Y)} = 0 \quad DE - \frac{\sqrt{X} \cdot \left(\sqrt{X} - \sqrt{Y}\right)^2}{\sqrt{Y} \cdot (X+Y)} = 0$$

$$AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$$

Unit = 1.00000

XY = 0.30000
X = 6.00000
Y = 20.00000
B = 0.30000
$\sqrt{B}$ = 0.54772
C = 0.54772

A = 0.00000
B = 0.30000
C = 0.32500
D = 0.46154
E = 0.54772
F = 0.65000

G = 1.00000
DE = 0.08618
DF = 0.18846
CD = 0.13654

AE is the square root of AB x AG, which is always 1. One should come to understand that considering two things, and the relation between them, they are grouped as one thing and are proportional to that whole. Thus, a simple 1, or unit, produces every possible solution there ever can be in terms between 0 and 1. So, slide B from A to G and see every possible root that can exist.

102894A

Descriptions.

Unit.

$AB := 1$

Given.

$N := 5$

$AH := AB \cdot N \quad BH := AH - AB$

$BG := \dfrac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \dfrac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

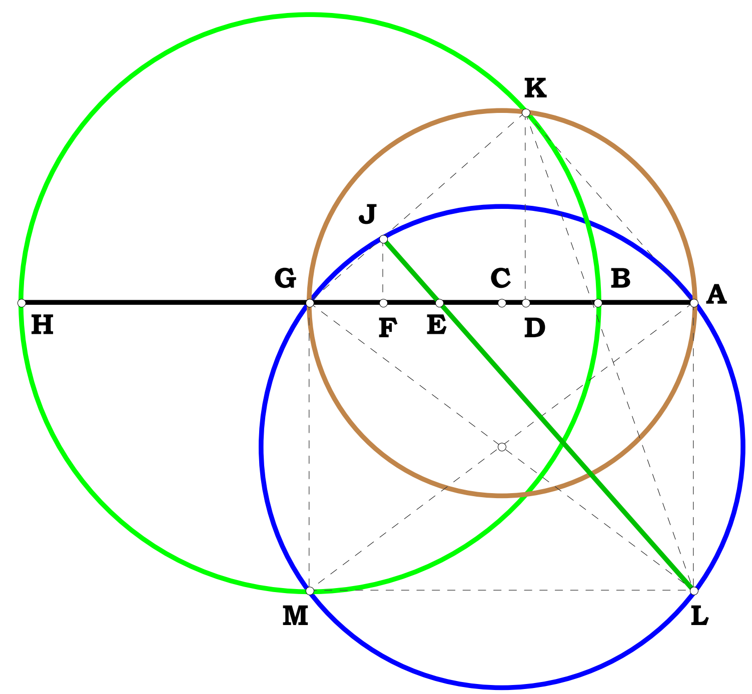
$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

$GJ := \dfrac{GL^2 + GK^2 - KL^2}{2 \cdot GK} \quad FG := \dfrac{DG \cdot GJ}{GK}$

$AF := AG - FG \quad FJ := \dfrac{DK \cdot GJ}{GK} \quad EF := \dfrac{AF \cdot FJ}{FJ + AL}$

$AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0 \quad AE = 2.236068$

Trivial Method Square Root



AE is the square root of AB x AH.

$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$$GJ - \frac{S_2{}^2 + S_1{}^2 - S_3{}^2}{2 \cdot S_1} = 0$$

Definitions.

Another fine example of Mathcad's inability to function rationally. Even to get the reduction this far required too much manual labor, but if one wants to continue, then one will get to the simple result.

$$AE - \frac{(N-1)^2 \cdot (N+1) \cdot \sqrt{N \cdot (N-1)^2} + 2 \cdot N \cdot (N-1) \cdot (N+1) \cdot \sqrt{(N+1)^2}}{\left(N^3 - 3 \cdot N^2 + 3 \cdot N - 1\right) \cdot \sqrt{(N+1)^2} + \sqrt{N \cdot (N-1)^2} \cdot \left(2 \cdot N^2 + 4 \cdot N + 2\right)} = 0$$

$$AE - \sqrt{N} = 0$$

102894B

Given.

$X := 6 \quad Y := 20$

Unit.

$AH := 1$

Descriptions.

$AB := \frac{X}{Y} \quad BH := \frac{Y - X}{Y} \quad BH := AH - AB \quad BG := \frac{BH}{2}$

$GK := BG \quad AG := AB + BG \quad DG := \frac{GK^2}{AG} \quad AD := AG - DG$

$AL := BG \quad GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG$

$DH := BH - BD \quad DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

$GJ := \frac{GL^2 + GK^2 - KL^2}{2 \cdot GK} \quad FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG$

$FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF$

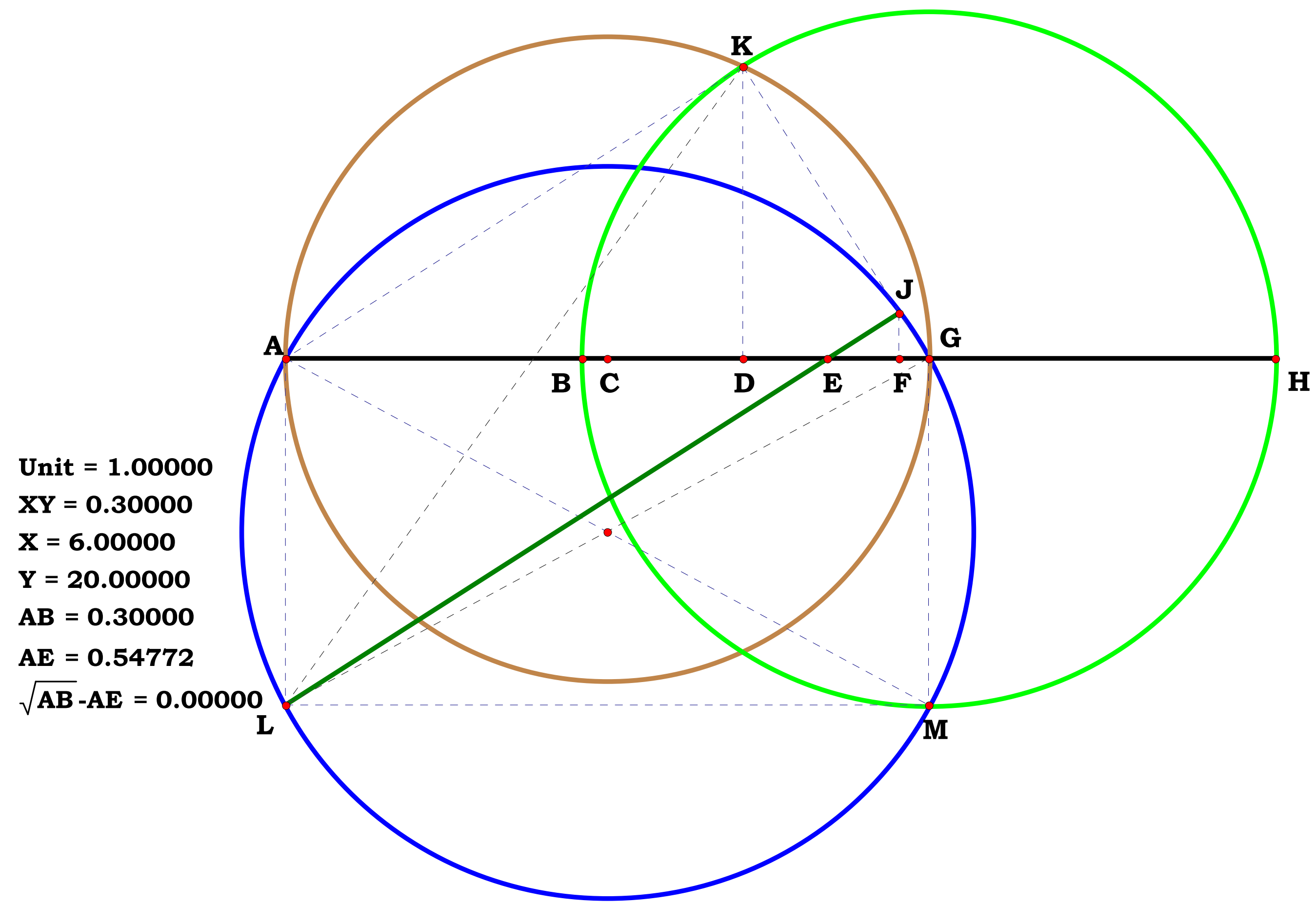
$\sqrt{AB \cdot AH} - AE = 0 \quad AE = 0.547723$

Definitions.

And again, this method judges both parties equally.

$AE - \sqrt{\frac{X}{Y}} = 0 \quad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

Trivial Method Square Root



AE is the square root of AB x AH which is always unity.

Unit.

$AB := 1$

Given.

$N := 22$

103194A

Descriptions.

$AF := N \quad BF := AF - AB \quad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \quad BC := \frac{AB \cdot (BG + GH)}{BJ} \quad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \quad CE := \frac{CF \cdot CH}{(CH + FK)}$

$EF := CF - CE \quad BE := BC + CE \quad DF := BF - BE \quad CD := AD - AC$

Definitions.

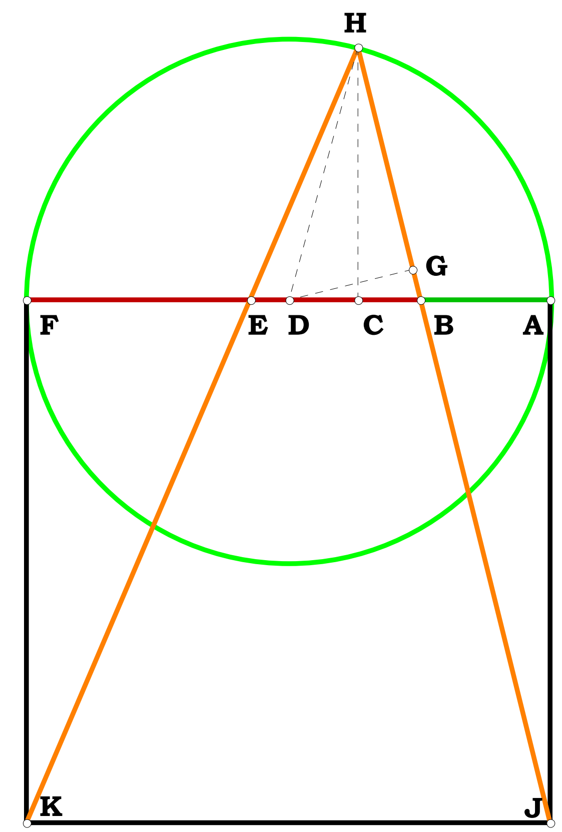
I cannot get Mathcad to transform one into the other.

$BE^2 - EF = 0 \quad \sqrt{EF} - BE = 0$

$$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$$

$$EF - \frac{2 \cdot N^2 - \sqrt{4 \cdot N - 3} - 2 \cdot N + 1}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} = 0$$

Square Root of a Segment



Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square.

110194A

Given.

$N_1 := 4.55192$

$N_2 := 3.86362$

Duplicate Ratios

Given BG and BC find AB, AG, such that $(AB^2 \cdot AG)^{1/3} = BC$. For obvious reasons, BC between BG. It seems this is the first time I drew the figure in Sketchpad, all my other graphics came from TommyCad in the early 90's,

I am going to be presenting two identical write-ups of the figure, the only difference will be in N_2 in order to show a problem with so called mathematicians today.

Descriptions.

$BG := N_1 \quad BC := N_1 - N_2 \quad BN := BG \quad BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN} \quad FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN} \quad GM := DL \quad BD := BC + CD$

$DG := BG - BD \quad LM := DG \quad GO := BG \quad MO := GO + GM \quad EG := \frac{LM \cdot GO}{MO}$

$CG := BG - BC \quad CE := CG - EG \quad CJ := BC \quad CK := CE \quad IJ := BC$

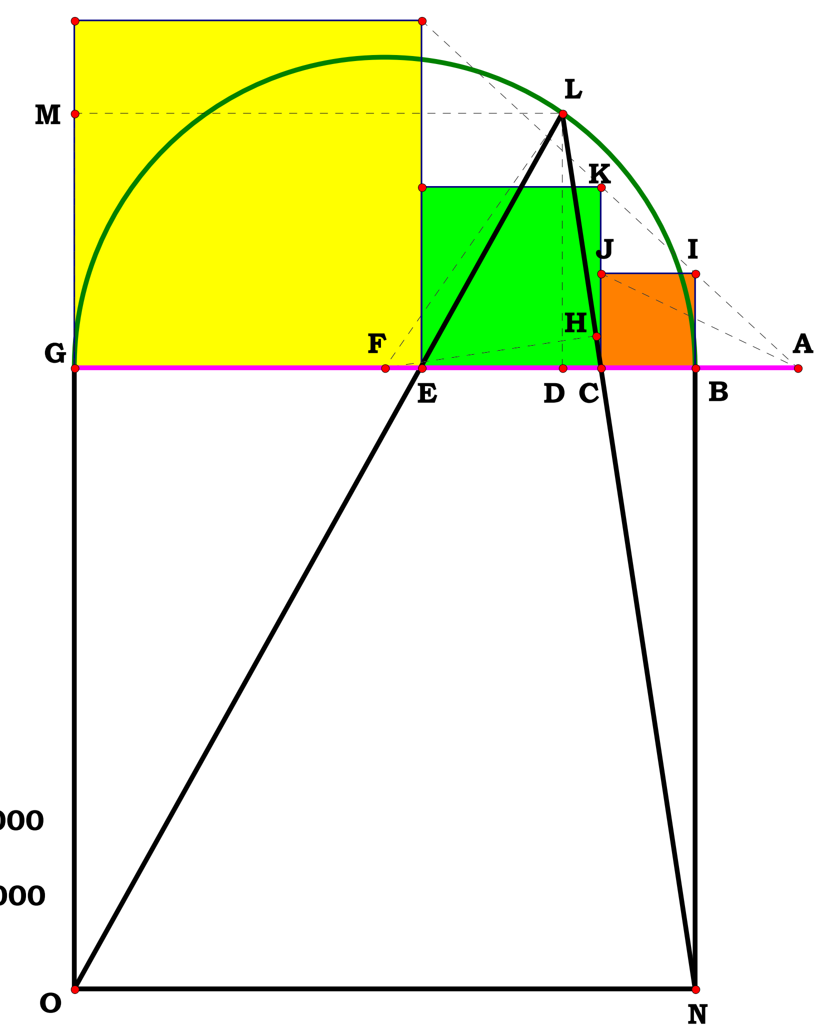
$JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2} \quad BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE \quad AC := AB + BC \quad AB = 0.746998 \quad AG = 5.298918$

$AE = 2.757812$

Definitions.

$$\left(AB^2 \cdot AG\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$$



$N_1 = 4.55192$

$N_2 = 3.86362$

$AB = 0.74699$

$BF = 2.27596$

$AG = 5.29891$

$AC = 1.43528$

$AE = 2.75779$

$(AB^2 \cdot AG)^{\frac{1}{3}} = 1.43528$

$(AB \cdot AG^2)^{\frac{1}{3}} = 2.75779$

$(AB^2 \cdot AG)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AG^2)^{\frac{1}{3}} - AE = 0.00000$

Here, everything looks as it should and we might get comfortable, but let us just move C.

Given any point on a segment, cut the segment into duplicate ratios with that point.

$EG = 2.541106 \quad CE = 1.322514 \quad BC = 0.6883 \quad \frac{EG}{CE} - \frac{CE}{BC} = 0$

110194B

Given.

$N_1 := 4.55192$

$N_2 := 2.63187$

Given AG - AB = BG and $(AB^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BC between BG. It seems this is the first time I drew the figure in Sketchpad, all my other graphics came from TommyCad in the early 90's,

I am going to be presenting two identical write-ups of the figure, the only difference will be in N_2 in order to show a problem with so called mathematicians today.

Duplicate Ratios

Descriptions.

$BG := N_1 \quad BC := N_1 - N_2 \quad BN := BG \quad BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN} \quad FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN} \quad GM := DL \quad BD := BC + CD$

$DG := BG - BD \quad LM := DG \quad GO := BG \quad MO := GO + GM \quad EG := \frac{LM \cdot GO}{MO}$

$CG := BG - BC \quad CE := CG - EG \quad CJ := BC \quad CK := CE \quad IJ := BC$

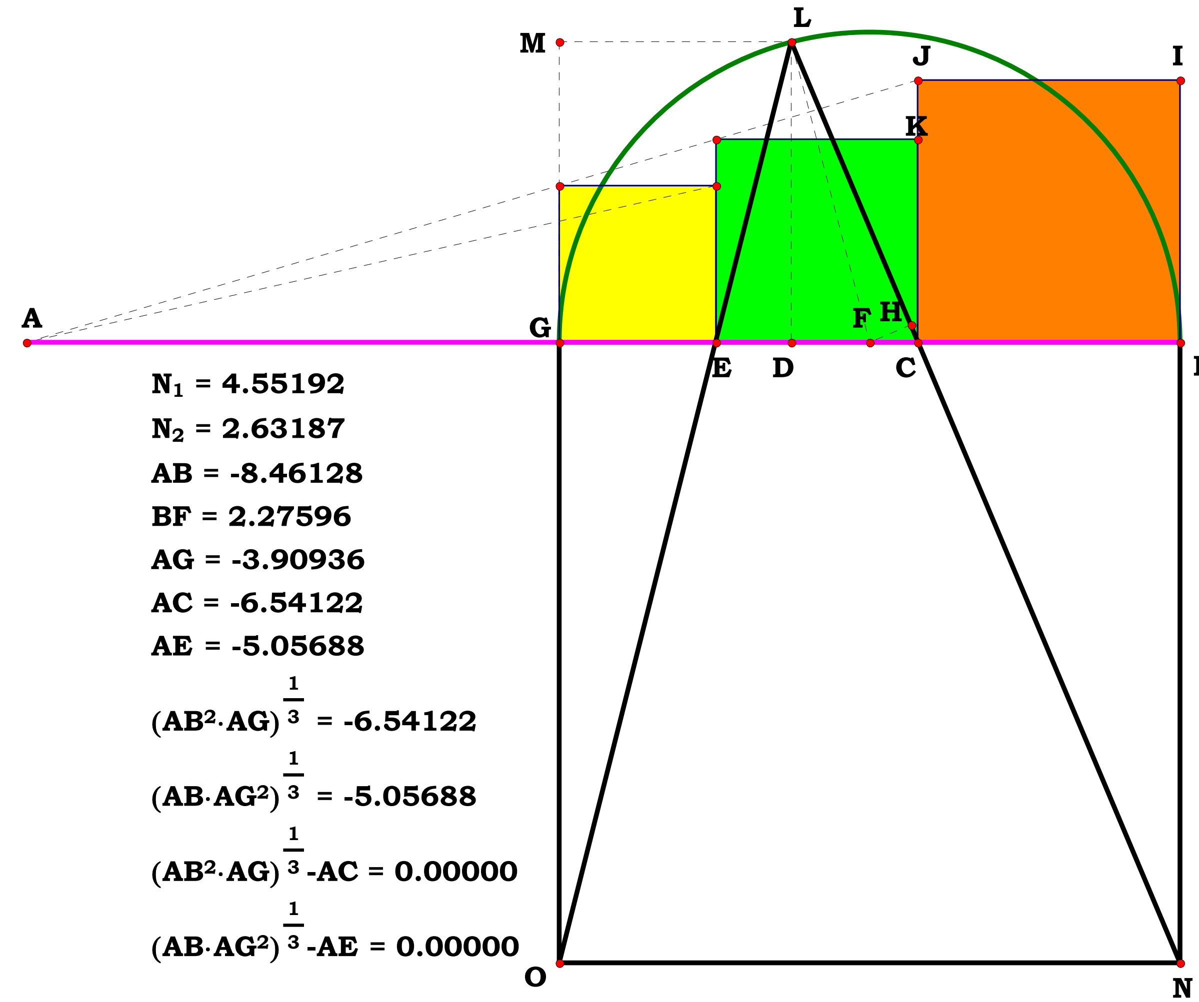
$JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2} \quad BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE \quad AC := AB + BC \quad AB = -8.461312 \quad AG = -3.909392$

$AE = -5.056911$

Definitions.

$(AB^2 \cdot AG)^{\frac{1}{3}} = 3.270631 + 5.664899i \qquad (AB \cdot AG^2)^{\frac{1}{3}} - AE = 7.585367 + 4.379414i$



$N_1 = 4.55192$
$N_2 = 2.63187$
AB = -8.46128
BF = 2.27596
AG = -3.90936
AC = -6.54122
AE = -5.05688
$(AB^2 \cdot AG)^{\frac{1}{3}}$ = -6.54122
$(AB \cdot AG^2)^{\frac{1}{3}}$ = -5.05688
$(AB^2 \cdot AG)^{\frac{1}{3}}$-AC = 0.00000
$(AB \cdot AG^2)^{\frac{1}{3}}$-AE = 0.00000

For those who believe that math is perfect, that mathematicians are literate, here we have an example of simple geometry and a child's drawing program showing the illiteracy of one of the top math programs world-wide. How is it possible for two identical equations with identical inputs produce two different resullts? How is it possible not to know what your own grammar means? Do the equation in Windows Calculator, it will even get it right. Why can't a so called high end math program?

110194C

Given.

$N_1 := 4.55192$

$N_2 := 2.63187$

Duplicate Ratios

Procrastinated write-up?

Descriptions.

Maybe I am remembering badly, but there was something about trying to find a way to construct duplicate ratios? How hard can it be? one can find duplicate ratios on any segment given any point, two of them in fact.

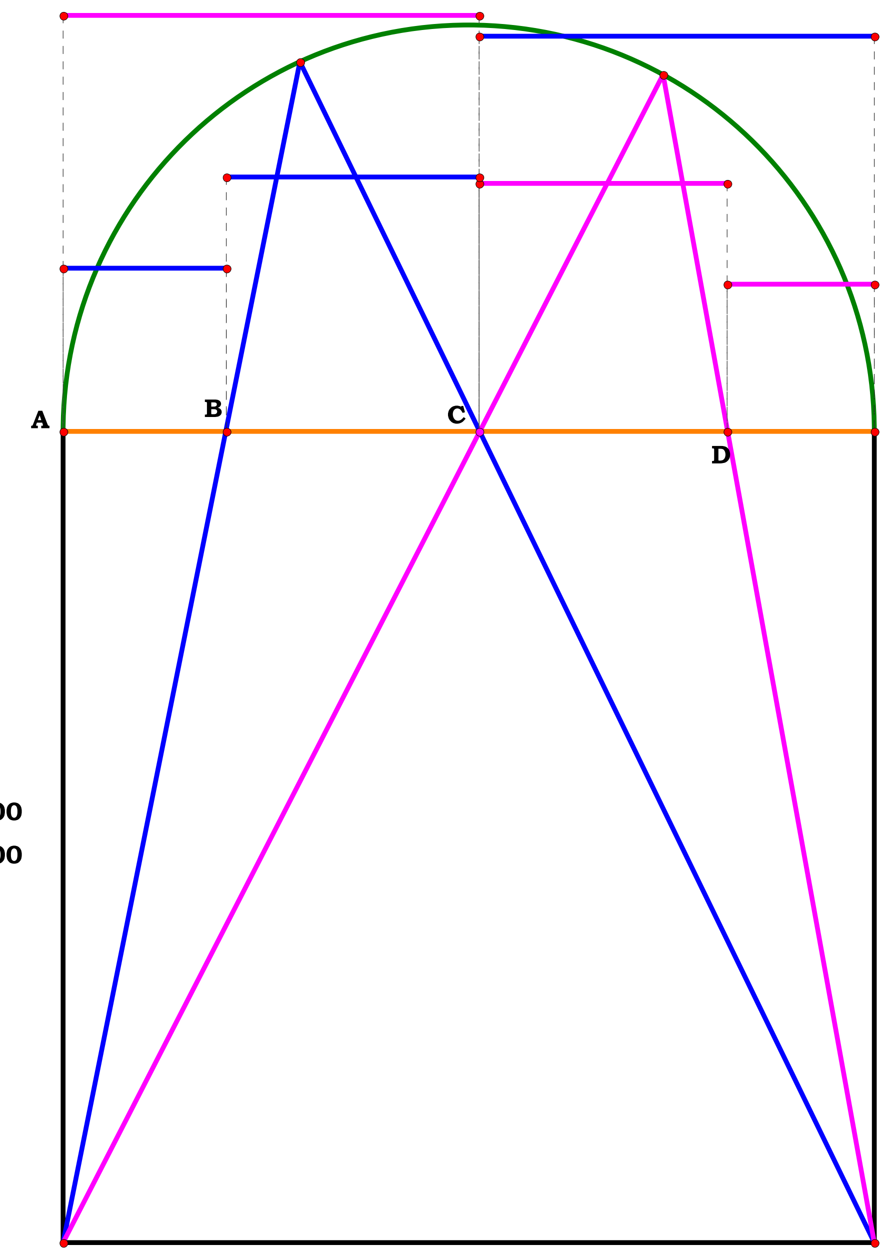
I do not think I need to keep writing the figure up, but maybe later.

A = 0.00000
AB = 0.20070
AC = 0.51326
AD = 0.81848
AE = 1.00000

BC = 0.31255
CE = 0.48674
CD = 0.30523
DE = 0.18152

$\sqrt{AB \cdot CE} - BC = 0.00000$

$\sqrt{AC \cdot DE} - CD = 0.00000$



Definitions.

122494A

Unit.

$AB := 1$

Given.

$N := 5$

Descriptions.

$AJ := AB \cdot N$ $\quad AF := \sqrt{AB \cdot AJ}$ $\quad BJ := AJ - AB$ $\quad BG := \dfrac{BJ}{2}$

$AG := AB + BG$ $\quad GS := BG$ $\quad DG := \dfrac{GS^2}{AG}$ $\quad FG := AG - AF$

$BD := BG - DG$ $\quad DJ := BJ - BD$ $\quad DS := \sqrt{BD \cdot DJ}$

$FK := \dfrac{DS \cdot FG}{DG}$ $\quad BF := AF - AB$ $\quad BK := \sqrt{BF^2 + FK^2}$

$FI := \dfrac{DJ \cdot FK}{DS}$ $\quad BI := FI + BF$ $\quad BP := \dfrac{BK \cdot BJ}{BI}$

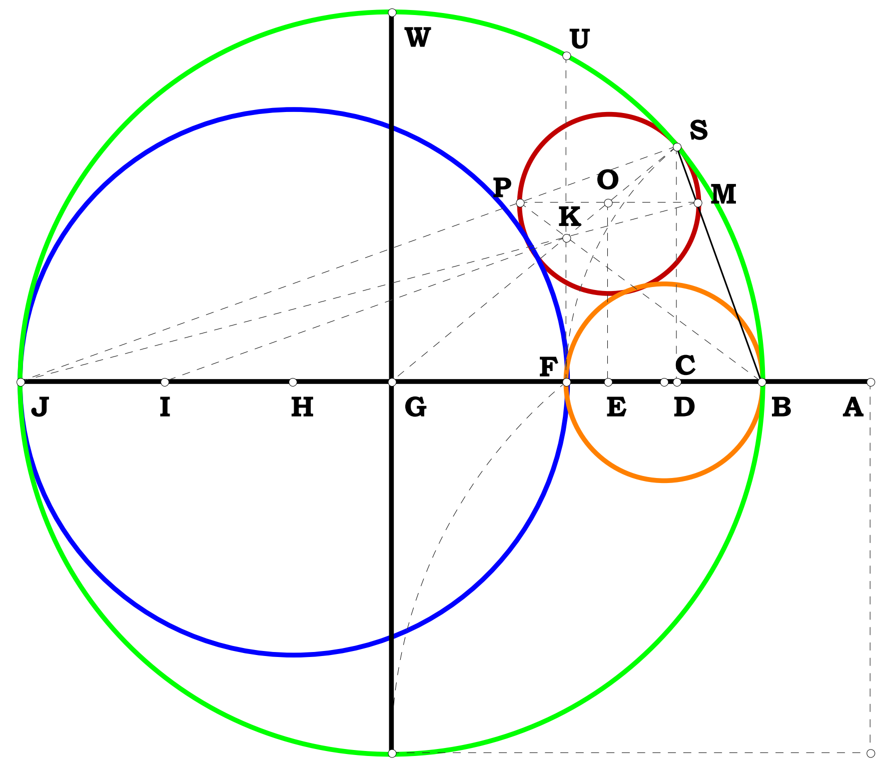
$KP := BP - BK$ $\quad MP := \dfrac{BJ \cdot KP}{BK}$ $\quad OS := \dfrac{MP}{2}$

Definitions.

$$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0 \qquad OS - \frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(N + \sqrt{N} + 1\right)} = 0$$

Power Line At Square Root

In this square root figure, what is the Algebraic definition of the tangent circle OS?



AB = 1.43933 cm

AJ = 11.23950 cm

OS = 1.18009 cm

$\dfrac{AB}{AB} = 1.00000$ $\quad AB = 1.00000$

$\dfrac{AJ}{AB} = 7.80882$ $\quad N = 7.80882$

$\dfrac{OS}{AB} = 0.81989$ $\quad OS = 0.81989$

$\dfrac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(\sqrt{N}+N+1\right)} - OS = 0.00000$

Power Line At Square Root

In this square root figure, what is the Algebraic definition of the tangent circle OS Given just point D?

Given.

$Y := 20 \quad X := 16$

Unit.

$AH := \frac{Y}{Y}$

122494B

Descriptions.

$AD := \frac{X}{Y} \quad AC := \frac{AH}{2} \quad DO := \sqrt{AD \cdot (AH - AD)} \quad CD := \sqrt{AC^2 - DO^2}$

$CK := \frac{AC^2}{CD} \quad AK := CK + AC \quad DK := AK - AD \quad DJ := \frac{DK}{2}$

$AJ := AD + DJ \quad CJ := AJ - AC \quad CG := \frac{AC^2}{CJ} \quad AG := CG + AC$

$GV := \sqrt{AG \cdot (AH - AG)} \quad DP := \frac{DJ \cdot CD}{AC} \quad DW := \frac{AG \cdot DP}{GV}$

$DH := AH - AD \quad HP := \sqrt{DH^2 + DP^2} \quad AW := AD - DW$

$HR := \frac{HP \cdot AH}{AH - AW} \quad PR := HR - HP \quad RT := \frac{AH \cdot PR}{HP} \quad ST := \frac{RT}{2}$

$ST = 0.095238$

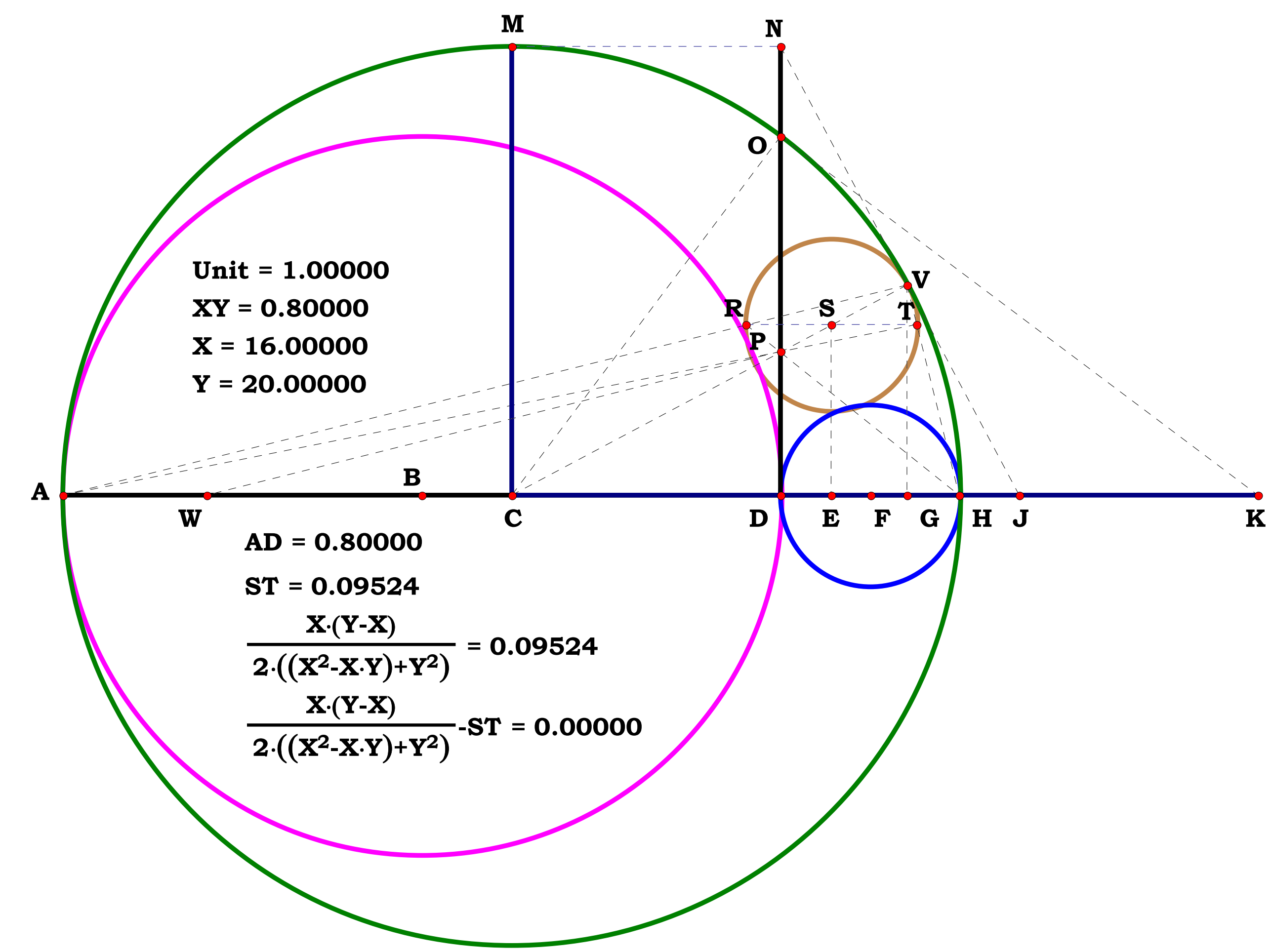
Definitions.

$AD - \frac{X}{Y} = 0 \quad AC - \frac{1}{2} = 0 \quad DO - \frac{\sqrt{X \cdot (Y - X)}}{Y} = 0 \quad CD - \frac{(2 \cdot X - Y)}{2 \cdot Y} = 0 \quad CK - \frac{Y}{2 \cdot (2 \cdot X - Y)} = 0$

$AK - \frac{X}{2 \cdot X - Y} = 0 \quad DK - \frac{2 \cdot X \cdot (Y - X)}{Y \cdot (2 \cdot X - Y)} = 0 \quad DJ - \frac{2 \cdot X \cdot (Y - X)}{2 \cdot Y \cdot (2 \cdot X - Y)} = 0 \quad AJ - \frac{X^2}{Y \cdot (2 \cdot X - Y)} = 0 \quad CJ - \frac{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2}{2 \cdot Y \cdot (2 \cdot X - Y)} = 0 \quad CG - \frac{Y \cdot (2 \cdot X - Y)}{2 \cdot \left(2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2\right)} = 0$

$AG - \frac{X^2}{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2} = 0 \quad GV - \frac{X \cdot (Y - X)}{\left(2 \cdot X^2 + Y^2 - 2 \cdot X \cdot Y\right)} = 0 \quad DP - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad DW - \frac{X^2}{Y^2} = 0 \quad DH - \frac{Y - X}{Y} = 0 \quad HP - \frac{\sqrt{X^2 + Y^2} \cdot (Y - X)}{Y^2} = 0$

$AW - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad HR - \frac{(Y - X) \cdot \sqrt{X^2 + Y^2}}{X^2 - X \cdot Y + Y^2} = 0 \quad PR - \frac{\sqrt{X^2 + Y^2} \cdot X \cdot (X - Y)^2}{Y^2 \cdot \left(X^2 - X \cdot Y + Y^2\right)} = 0 \quad RT - \frac{X \cdot (Y - X)}{X^2 - X \cdot Y + Y^2} = 0 \quad ST - \frac{X \cdot (Y - X)}{2 \cdot \left(X^2 - X \cdot Y + Y^2\right)} = 0$



122595A

Unit.

$AB := 1$

Given.

$N := 5$

$\Delta := 5$

$\delta := 1 .. \Delta$

Two Prime Exponential Series Developed Through The Powerline Progression

Descriptions.

$AO := AB \cdot N \quad AG := \sqrt{AB \cdot AO} \quad BO := AO - AB \quad BJ := \frac{BO}{2} \quad JZ := BJ \quad JV := BJ \quad JO := BJ$

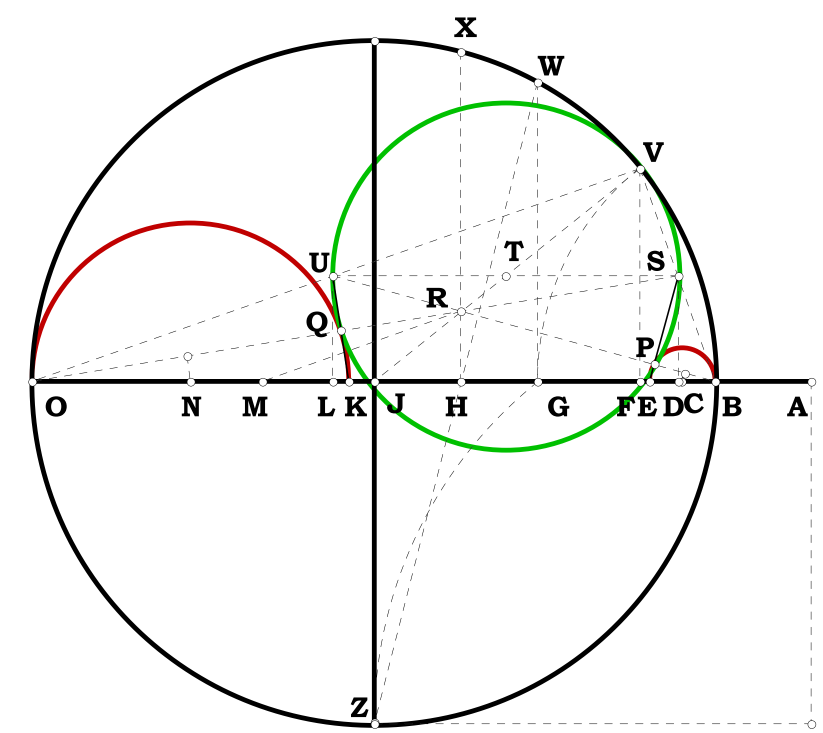
$BG_1 := AG - AB \quad GO_1 := BO - BG_1 \quad GW_1 := \sqrt{BG_1 \cdot GO_1} \quad GJ_1 := BJ - BG_1 \quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$

$$\begin{pmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{pmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - (BG_\delta + GH_\delta) \\ \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]} \\ BJ - (BG_\delta + GH_\delta) \\ \dfrac{[BJ - (BG_\delta + GH_\delta)] \cdot \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}}{JZ + \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}} \end{bmatrix}$$

$HJ := BJ - BG_\Delta \quad FJ := \frac{(N-1)^2}{2 \cdot (N+1)} \quad BF := BJ - FJ \quad FO := FJ + JO \quad FV := \sqrt{BF \cdot FO} \quad HR := \frac{FV \cdot HJ}{FJ}$

$BH := BJ - HJ \quad BR := \sqrt{HR^2 + BH^2} \quad HM := \frac{FO \cdot HR}{FV} \quad BU := \frac{BR \cdot BO}{BH + HM} \quad RU := BU - BR \quad SU := \frac{BO \cdot RU}{BR}$

$TV := \frac{SU}{2} \quad PU := \frac{BH \cdot SU}{BR} \quad BP := BU - PU \quad BE := \frac{BR \cdot BP}{BH} \quad AE := AB + BE$



Definitions.

$N^{\frac{1}{2^\Delta}} - AE = 0$

$\delta =$	$N^{\frac{1}{2^\delta}} =$
1	2.236068
2	1.495349
3	1.222845
4	1.105823
5	1.051581

Descriptions.

$$LU := \frac{HR \cdot BU}{BR} \quad BL := \frac{BH \cdot BU}{BR} \quad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \quad DS := LU \quad OS := \frac{OR \cdot DS}{HR}$$

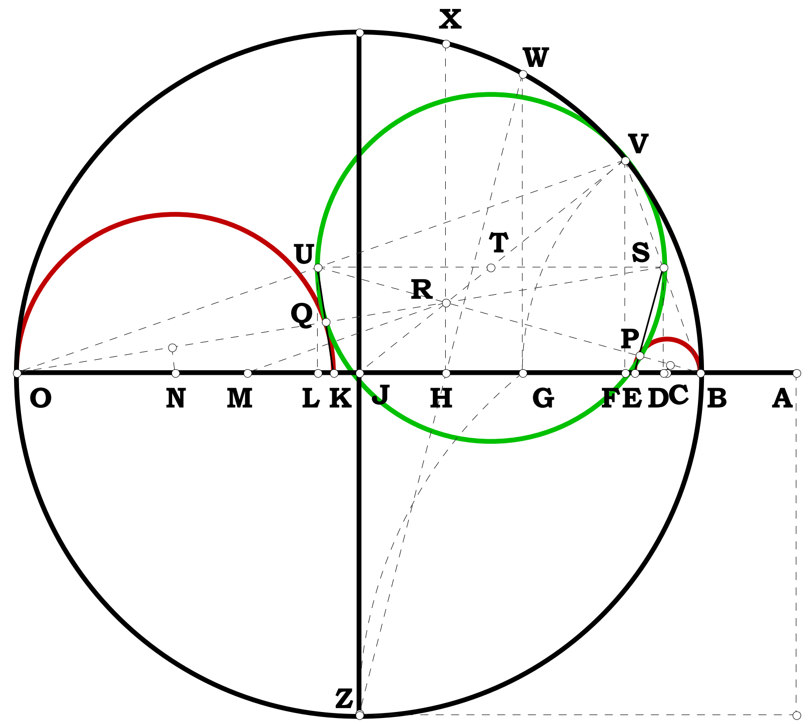
$$DO := \frac{HO \cdot DS}{HR} \quad QS := \frac{DO \cdot SU}{OS} \quad OQ := OS - QS$$

$$KO := \frac{OS \cdot OQ}{DO} \quad AK := AO - KO$$

Definitions.

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$

$\delta =$	$N^{\frac{2^{\delta}-1}{2^{\delta}}} =$
1	2.236068
2	3.343702
3	4.088827
4	4.521519
5	4.754745



122595B

Given.

$X := 6 \quad Y := 20$

Unit.

$\frac{Y}{Y} = 1$

Two Prime Exponential Series Developed Through The Powerline Progression

Descriptions.

A series is actually the recursion of a process. In this case, you will find that M through C and then again through ever G, we have produced an exponential series with a simple square root figure. One can see, there are many places one can start this figure and determine the rest of it.

$DH := \frac{Y}{Y} \quad BD := 2 \cdot DH \quad CH := \frac{X}{Y} \quad HM := DH$

$CM := \sqrt{CH^2 + HM^2} \quad LM := \frac{HM \cdot BD}{CM} \quad LM = 1.915653$

$JL := CH \cdot \frac{BD}{CM} \quad JL = 0.574696 \quad FH := \frac{HM^2}{CH}$

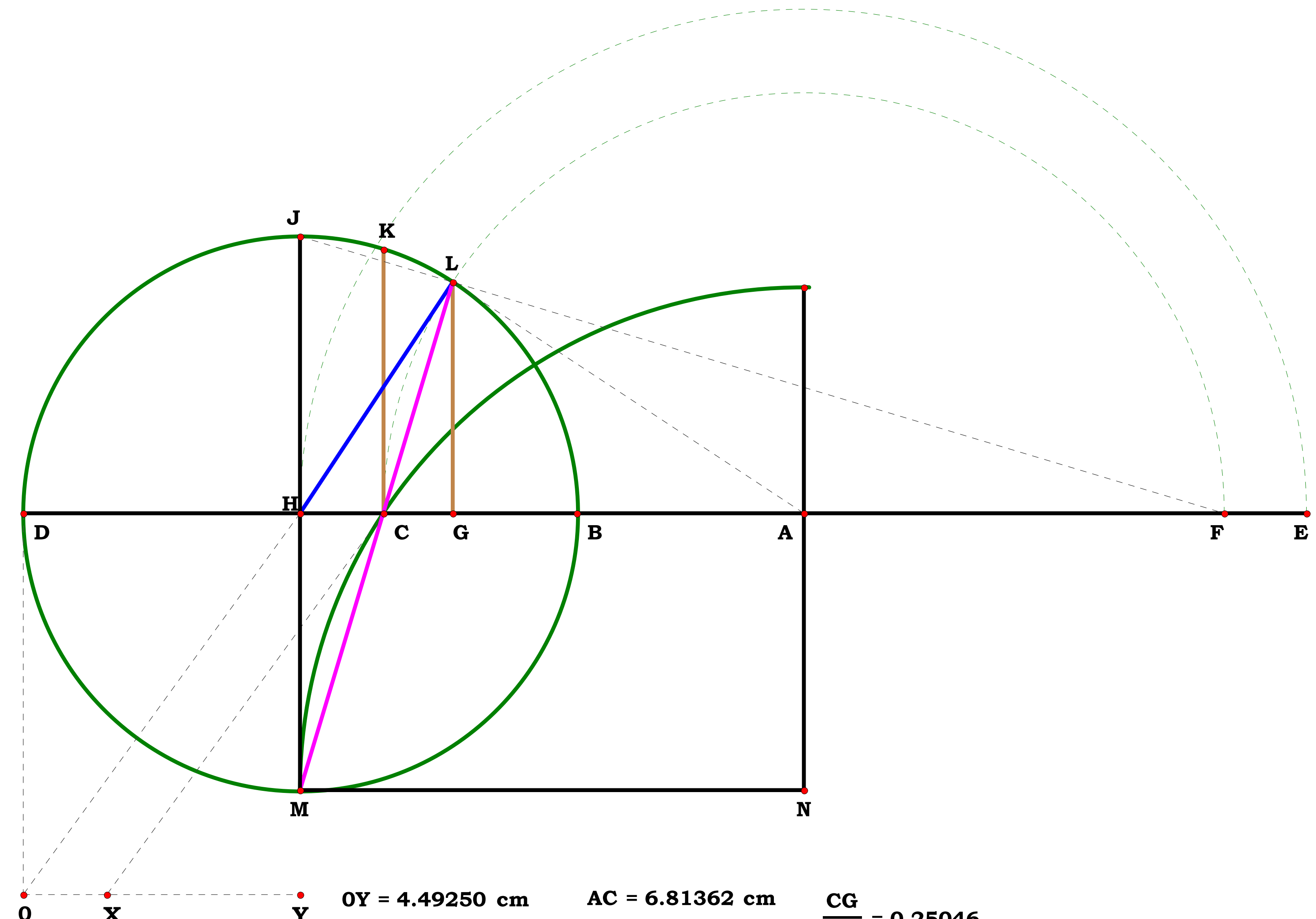
$DF := FH + DH \quad DF = 4.333333 \quad CF := FH - CH$

$CD := CH + DH \quad AC := \frac{CF}{2} \quad AD := AC + CD$

$AB := AD - BD \quad \sqrt{AB \cdot AD} - AC = 0$

$CG := \frac{CH \cdot (LM - CM)}{CM} \quad BG := DH - (CH + CG)$

$GL := \sqrt{BG \cdot (BD - BG)} \quad GL = 0.834862$



Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000
x_F = 4.33333

0Y = 4.49250 cm
LM = 8.60607 cm
$\frac{LM}{0Y}$ = 1.91565
LJ = 2.58182 cm
$\frac{LJ}{0Y}$ = 0.57470

AC = 6.81362 cm
$\frac{AC}{0Y}$ = 1.51667
AH = 8.16137 cm
$\frac{AH}{0Y}$ = 1.81667
CG = 1.12519 cm

$\frac{CG}{0Y}$ = 0.25046
GL = 3.75062 cm
$\frac{GL}{0Y}$ = 0.83486

Definitions.

$$DH-1=0 \qquad BD := 2 \qquad CH-\frac{X}{Y}=0 \qquad HM-1=0 \qquad CM-\frac{\sqrt{X^2+Y^2}}{Y}=0$$

$$LM-\frac{2\cdot Y}{\sqrt{X^2+Y^2}}=0 \qquad JL-\frac{2\cdot X}{\sqrt{X^2+Y^2}}=0 \qquad FH-\frac{Y}{X}=0 \qquad DF-\frac{X+Y}{X}=0$$

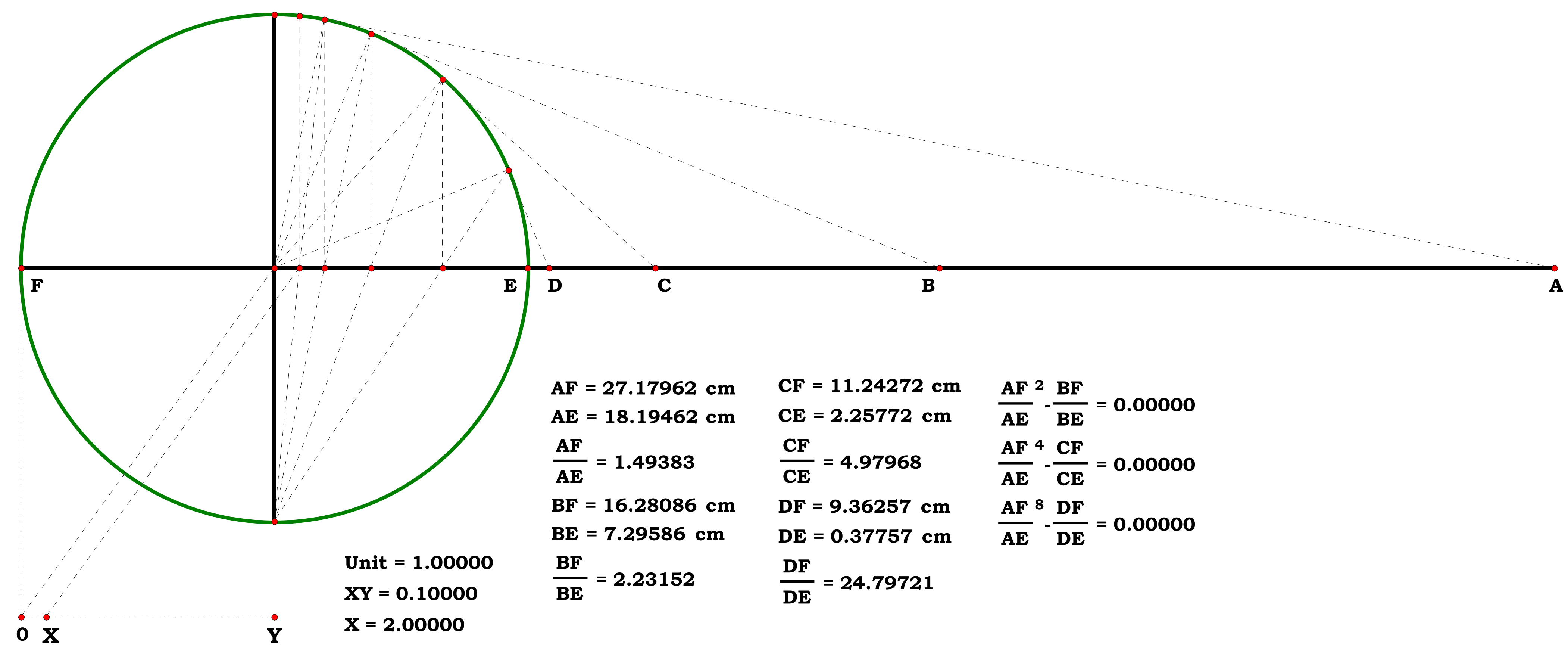
$$CF-\frac{Y^2-X^2}{X\cdot Y}=0 \qquad CD-\frac{X+Y}{Y}=0$$

$$AC-\frac{Y^2-X^2}{2\cdot X\cdot Y}=0 \qquad AD-\frac{(X+Y)^2}{2\cdot X\cdot Y}$$

$$AB-\frac{(X-Y)^2}{2\cdot X\cdot Y}=0 \quad AC-\frac{\left(Y^2-X^2\right)}{2\cdot X\cdot Y}=0$$

$$CG-\frac{X\cdot\left(Y^2-X^2\right)}{\left(X^2+Y^2\right)\cdot Y}=0 \qquad BG-\frac{(X-Y)^2}{X^2+Y^2}=0$$

$$GL-\frac{Y^2-X^2}{X^2+Y^2}=0$$



Unit = 1.00000
XY = 0.10000
X = 2.00000
Y = 20.00000

AF = 27.17962 cm
AE = 18.19462 cm
$\frac{AF}{AE}$ = 1.49383
BF = 16.28086 cm
BE = 7.29586 cm
$\frac{BF}{BE}$ = 2.23152

CF = 11.24272 cm
CE = 2.25772 cm
$\frac{CF}{CE}$ = 4.97968
DF = 9.36257 cm
DE = 0.37757 cm
$\frac{DF}{DE}$ = 24.79721

$\frac{AF}{AE}^{2}-\frac{BF}{BE}$ = 0.00000
$\frac{AF}{AE}^{4}-\frac{CF}{CE}$ = 0.00000
$\frac{AF}{AE}^{8}-\frac{DF}{DE}$ = 0.00000

Unit.

$AB := 1$

Given.

$N := 5$

122694A

Descriptions.

$AF := AB \cdot N \quad BF := AF - AB$

$BE := \frac{BF}{2} \quad EF := BE \quad EK := BE$

$AD := \sqrt{AB \cdot AF} \quad CE := \frac{(N-1)^2}{2 \cdot (N+1)} \quad CF := CE + EF$

$DF := AF - AD \quad BC := BF - CF \quad CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF} \qquad DG_1 = 1.236068$

$BD := AD - AB \quad DG_2 := \frac{EK \cdot BD}{BE} \quad DG_1 - DG_2 = 0$

Definitions.

This might give you something to think about.

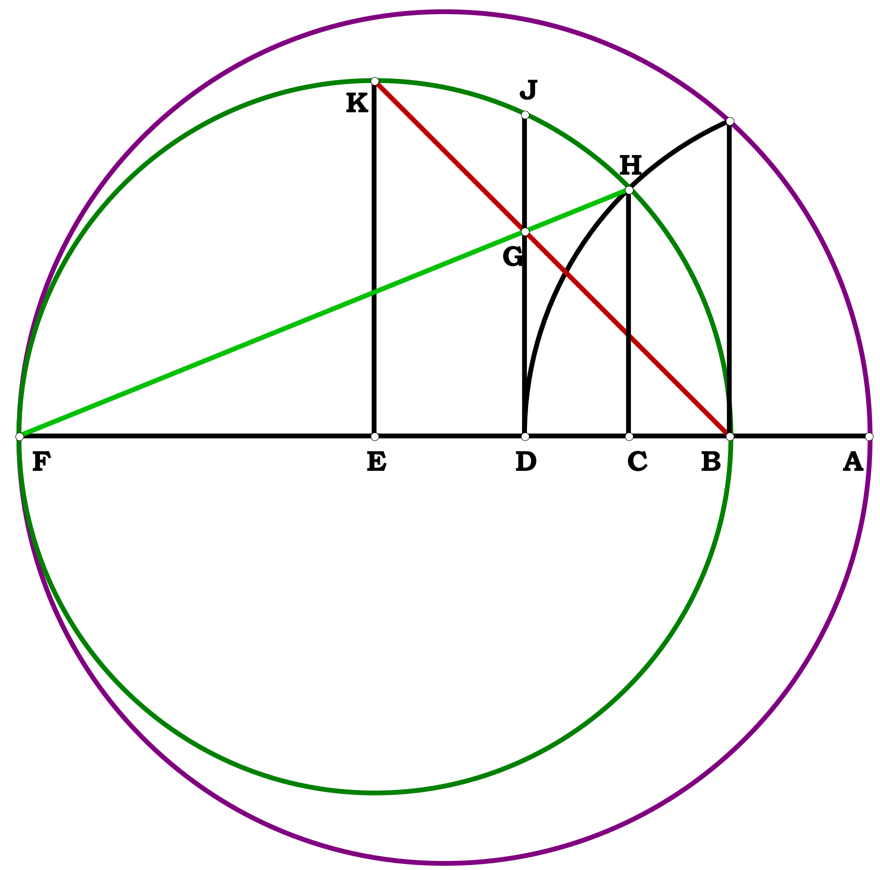
$$DG_1 - \frac{(N+1) \cdot \sqrt{N \cdot (N-1)^2} \cdot \left(N - \sqrt{N}\right)}{N \cdot (N-1) \cdot \sqrt{(N+1)^2}} = 0$$

$$DG_2 - \left(\sqrt{N} - 1\right) = 0$$

Exponential Series

Is Point G on DJ?

Is G, the intersection of FH and BK, on DJ?



122694B

Descriptions.

Given.

$X := 7$

$Y := 20$

Unit.

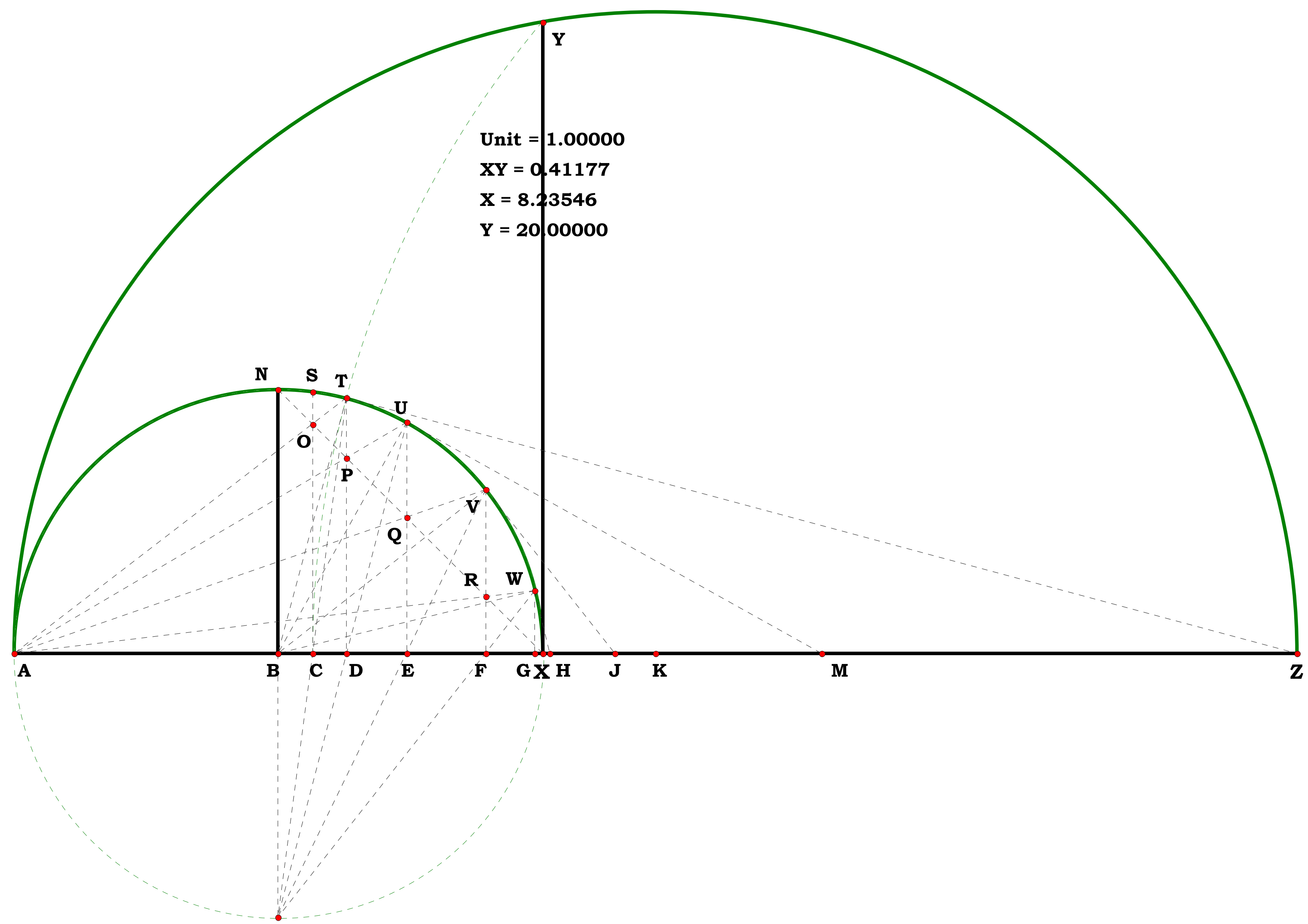
$\frac{Y}{Y} = 1$

Procrastinated Write-up

Exponential Series

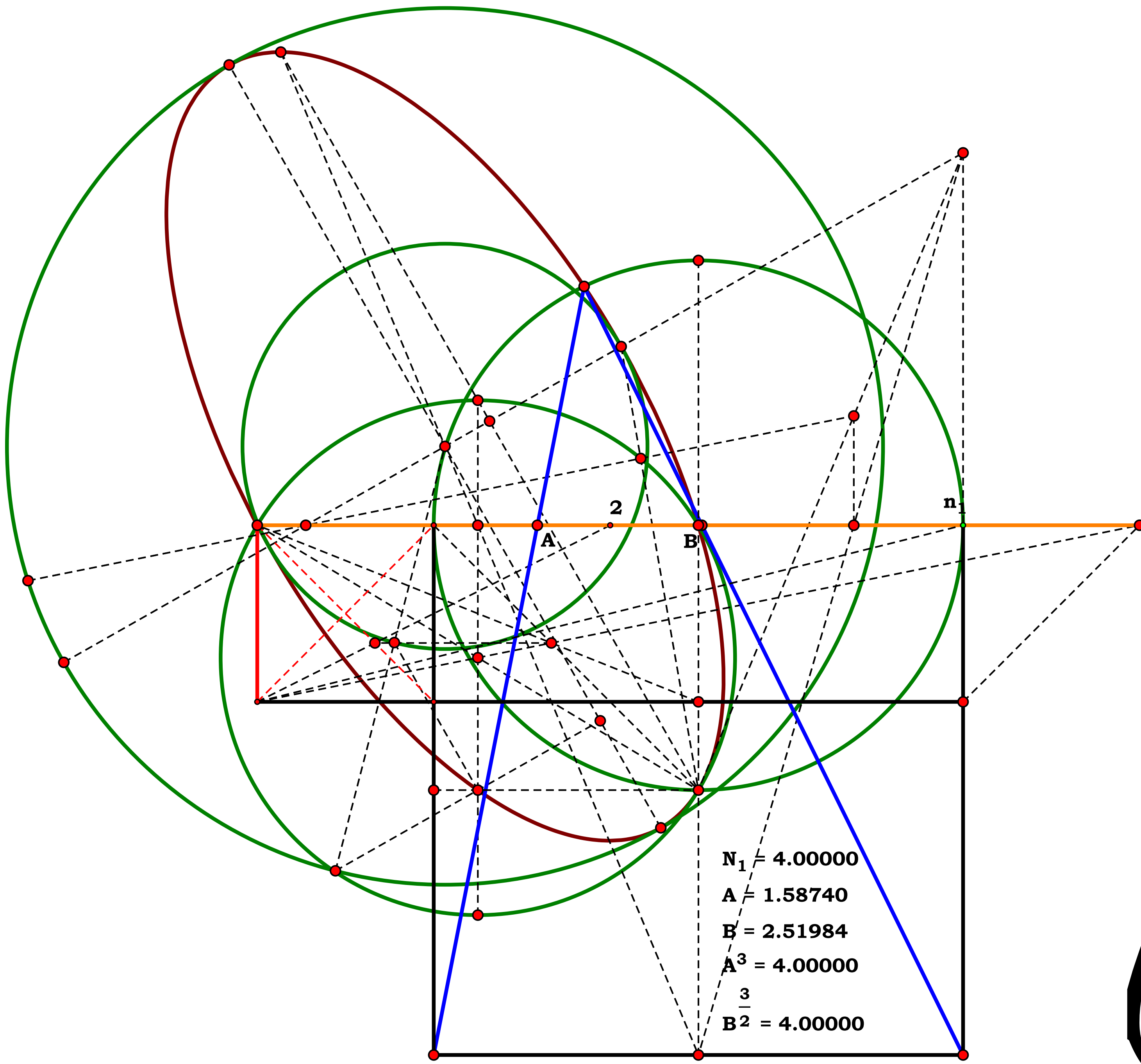
Leaving love and me out of the equation.

Definitions.



The Delian Quest 1995

John Clark



Unit.

$AC := 1$

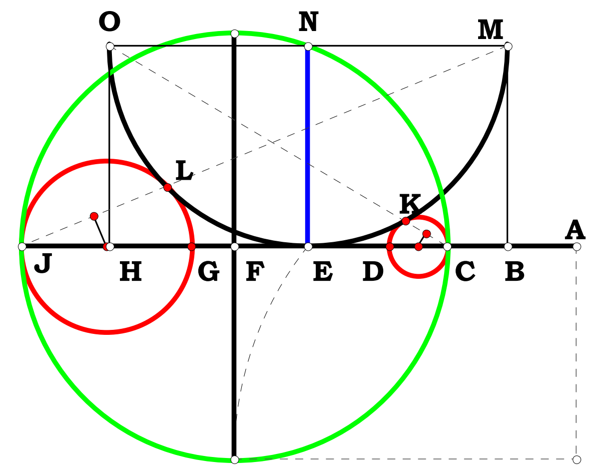
Given.

$N := 5$

010695A

Descriptions.

Alternate Method Quad Roots



$AJ := AC \cdot N \quad CJ := AJ - AC \quad AE := \sqrt{AC \cdot AJ} \quad CE := AE - AC$

$EJ := CJ - CE \quad EN := \sqrt{CE \cdot EJ} \quad BM := EN \quad HO := EN$

$MN := EN \quad NO := \,]BE := EN \quad EH := EN \quad BJ := BE + EJ$

$MJ := \sqrt{BJ^2 + BM^2} \quad MO := MN + NO \quad ML := \dfrac{BJ \cdot MO}{MJ}$

$JL := MJ - ML \quad GJ := \dfrac{MJ \cdot JL}{BJ} \quad AG := AJ - GJ \quad \left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

$CH := CE + EH \quad CO := \sqrt{CH^2 + HO^2} \quad KO := \dfrac{CH \cdot MO}{CO} \quad CK := CO - KO \quad CD := \dfrac{CO \cdot CK}{CH}$

$AD := AC + CD \quad \left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0$

Definitions.

$N^{\frac{3}{4}} - AG = 0 \quad N^{\frac{1}{4}} - AD = 0$

100695B

Descriptions.

Given.

$X := 6$

$Y := 20$

Unit.

$FG := \frac{Y}{Y}$

Finding the Unit for a Figure.

Supplement to 100695

$BF := 2 \cdot FG \qquad DG := \frac{X}{Y} \qquad DF := FG + DG \qquad BD := FG - DG$

$DH := \sqrt{DF \cdot BD} \qquad JK := 2 \cdot DH \qquad FK := \sqrt{(DF + DH)^2 + DH^2}$

$KU := \frac{(DF + DH) \cdot JK}{FK} \qquad FU := FK - KU \qquad EF := \frac{JK \cdot FU}{KU}$

$BJ := \sqrt{(DH + BD)^2 + DH^2} \qquad JV := \frac{(DH + BD) \cdot JK}{BJ}$

$BV := BJ - JV \qquad BC := \frac{JK \cdot BV}{JV}$

Use 041694, Tangents and Similarity Points for the next equation.

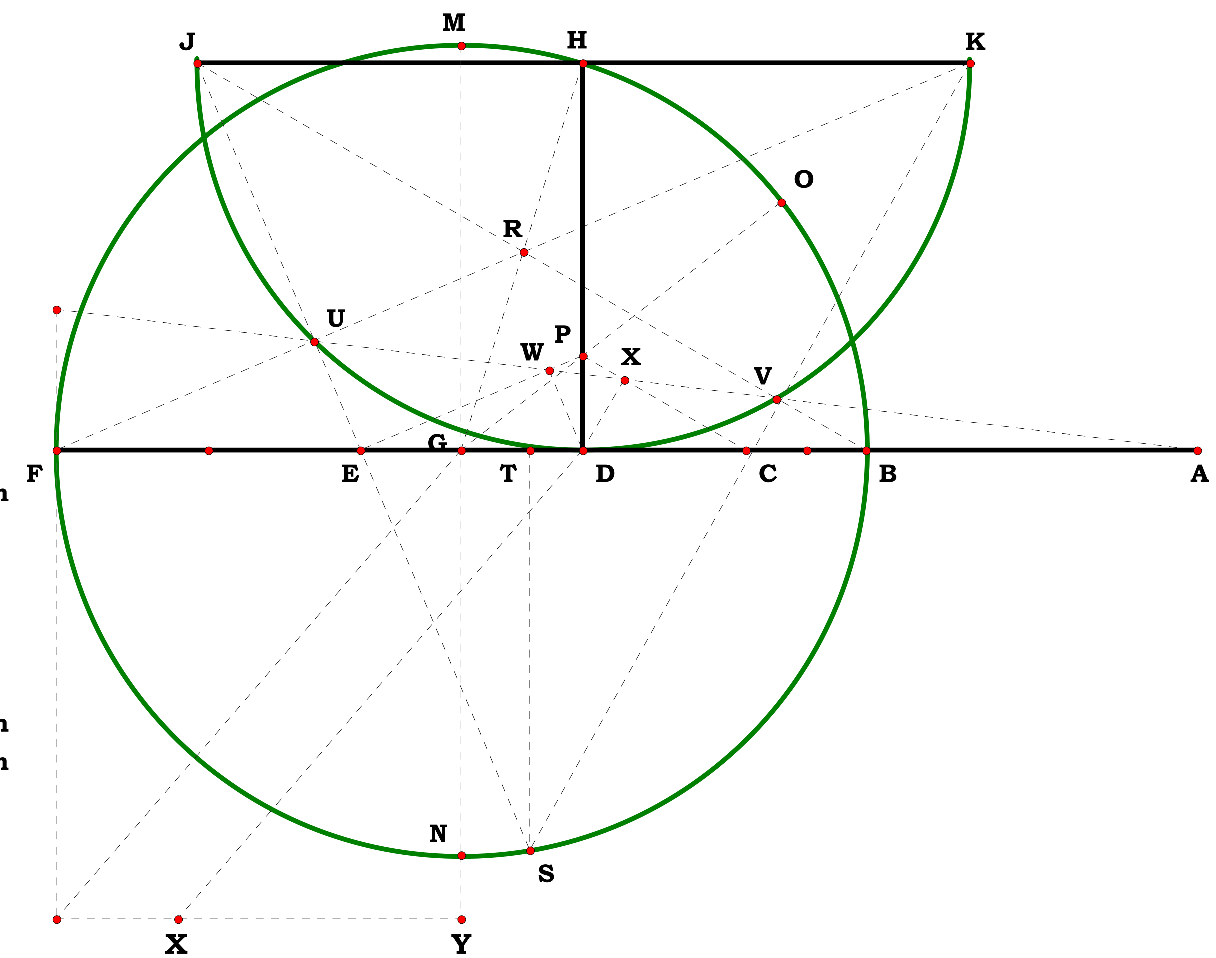
$AF := \frac{EF \cdot (BC - BF)}{BC - EF} \qquad AB := AF - BF \qquad AC := AB + BC$

$AD := AB + BD \qquad AE := AF - EF$

$\left(\frac{AF}{AB}\right)^{\frac{1}{4}} - \frac{AC}{AB} = 0 \qquad \left(\frac{AF}{AB}\right)^{\frac{2}{4}} - \frac{AD}{AB} = 0 \qquad \left(\frac{AF}{AB}\right)^{\frac{3}{4}} - \frac{AE}{AB} = 0$

What the above tells us is that the figure has to be redrawn such that AB is the unit if we want to express the figure as a quad root series.
The following plate will do just that.

Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000
BF = 10.72067 cm
AB = 4.37761 cm
BC = 1.58807 cm
CD = 2.16417 cm
DE = 2.94926 cm
EF = 4.01917 cm
AF = 15.09827 cm
BF = 10.72067 cm
CF = 9.13260 cm
DF = 6.96843 cm
TF = 6.25594 cm
FG = 5.36033 cm



Definitions.

$$BF - 2 = 0 \qquad DG - \frac{X}{Y} = 0 \qquad DF - \frac{X+Y}{Y} = 0 \qquad BD - \frac{Y-X}{Y} = 0 \qquad DH - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$$

$$JK - \frac{2 \cdot \sqrt{Y^2 - X^2}}{Y} = 0 \qquad FK - \frac{\sqrt{(X+Y) \cdot \left(3 \cdot Y - X + 2 \cdot \sqrt{Y^2 - X^2}\right)}}{Y} = 0$$

$$KU - \frac{2 \cdot \sqrt{Y^2 - X^2} \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{\sqrt{(X+Y) \cdot \left(3 \cdot Y - X + 2 \cdot \sqrt{Y^2 - X^2}\right)} \cdot Y} = 0$$

$$FU - \frac{(X+Y)^2}{Y \cdot \sqrt{2 \cdot X \cdot \sqrt{Y^2 - X^2} + 2 \cdot Y \cdot \sqrt{Y^2 - X^2} - X^2 + 3 \cdot Y^2 + 2 \cdot X \cdot Y}} = 0$$

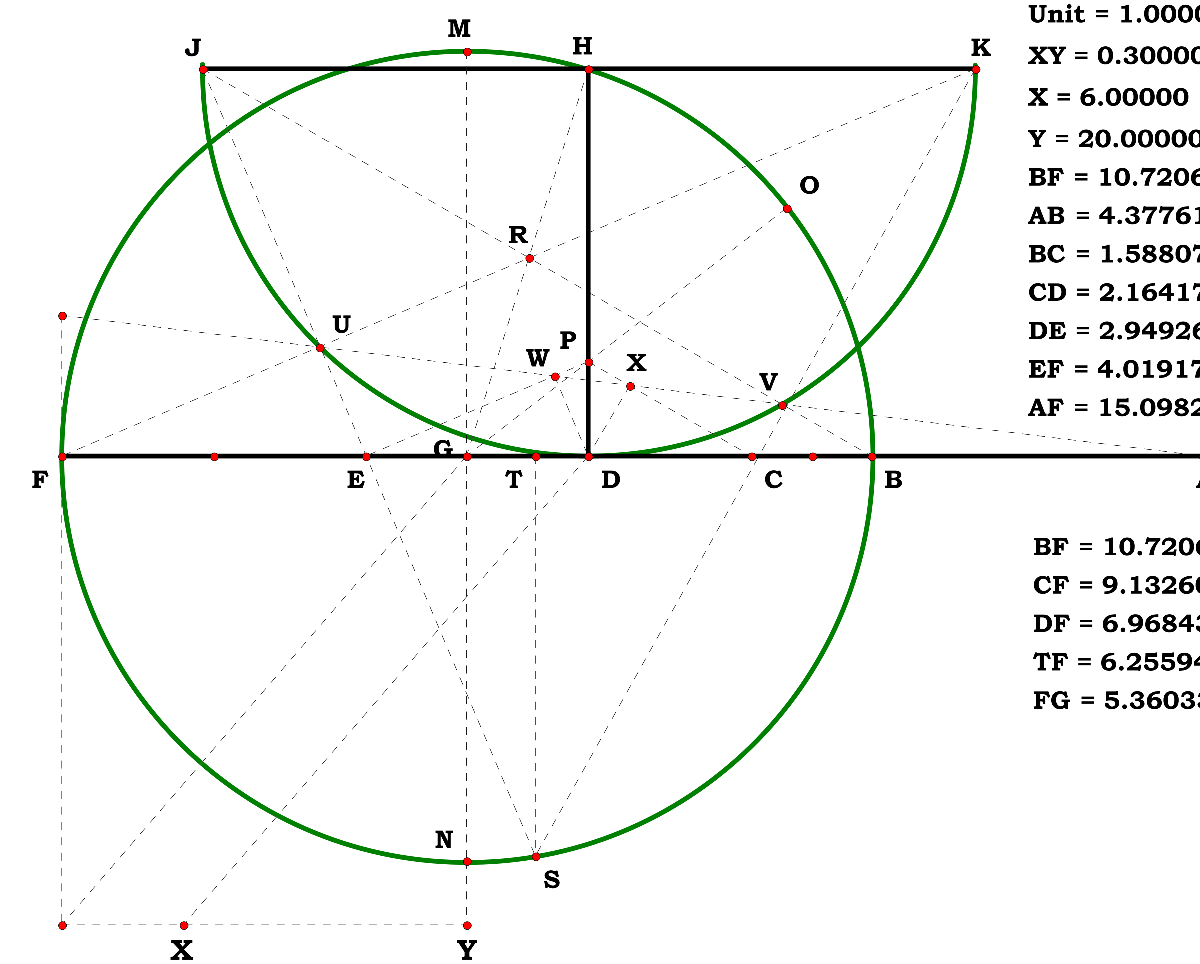
$$EF - \frac{\sqrt{(X+Y) \cdot \left(3 \cdot Y - X + 2 \cdot \sqrt{Y^2 - X^2}\right)} \cdot (X+Y)^2}{Y \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right) \cdot \sqrt{2 \cdot X \cdot \sqrt{Y^2 - X^2} + 2 \cdot Y \cdot \sqrt{Y^2 - X^2} - X^2 + 3 \cdot Y^2 + 2 \cdot X \cdot Y}} = 0$$

$$BJ - \frac{\sqrt{(Y-X) \cdot \left(X + 3 \cdot Y + 2 \cdot \sqrt{Y^2 - X^2}\right)}}{Y} = 0 \qquad JV - \frac{2 \cdot \left(Y - X + \sqrt{Y^2 - X^2}\right) \cdot \sqrt{Y^2 - X^2}}{Y \cdot \sqrt{(Y-X) \cdot \left(X + 3 \cdot Y + 2 \cdot \sqrt{Y^2 - X^2}\right)}} = 0$$

$$BV := \frac{(X-Y)^2}{Y \cdot \sqrt{2 \cdot Y \cdot \sqrt{Y^2 - X^2} - 2 \cdot X \cdot \sqrt{Y^2 - X^2} - X^2 + 3 \cdot Y^2 - 2 \cdot X \cdot Y}}$$

$$BC - \frac{(X-Y)^2 \cdot \sqrt{(Y-X) \cdot \left(X + 3 \cdot Y + 2 \cdot \sqrt{Y^2 - X^2}\right)}}{Y \cdot \left(Y - X + \sqrt{Y^2 - X^2}\right) \cdot \sqrt{2 \cdot Y \cdot \sqrt{Y^2 - X^2} - 2 \cdot X \cdot \sqrt{Y^2 - X^2} - X^2 + 3 \cdot Y^2 - 2 \cdot X \cdot Y}} = 0$$

$$AF - \frac{(X+Y)^2}{2 \cdot X \cdot Y} = 0 \qquad AB - \frac{(X-Y)^2}{2 \cdot X \cdot Y} = 0 \qquad AC - \frac{(X-Y)^2 \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot Y \cdot \left(Y - X + \sqrt{Y^2 - X^2}\right)} = 0$$



Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000
BF = 10.72067 cm
AB = 4.37761 cm
BC = 1.58807 cm
CD = 2.16417 cm
DE = 2.94926 cm
EF = 4.01917 cm
AF = 15.09827 cm

BF = 10.72067 cm
CF = 9.13260 cm
DF = 6.96843 cm
TF = 6.25594 cm
FG = 5.36033 cm

100695C

Given.

$X := 9$

$Y := 2$

Unit.

$AB := \frac{Y}{Y}$

Finding the Unit for a Figure.

Supplement to 100695

Mathcad 15 is not able to reduce these equations to the final powers of $\frac{X}{Y}$, therefore one is going to have to do that manually.

Descriptions.

$AF := \frac{X}{Y}$ $\quad BF := AF - AB$ $\quad BO := \frac{BF}{2}$ $\quad AD := \sqrt{AF}$

$BD := AD - AB$ $\quad DF := BF - BD$ $\quad DG := \sqrt{BD \cdot DF}$

$HJ := 2 \cdot DG$ $\quad BJ := \sqrt{(BD + DG)^2 + DG^2}$

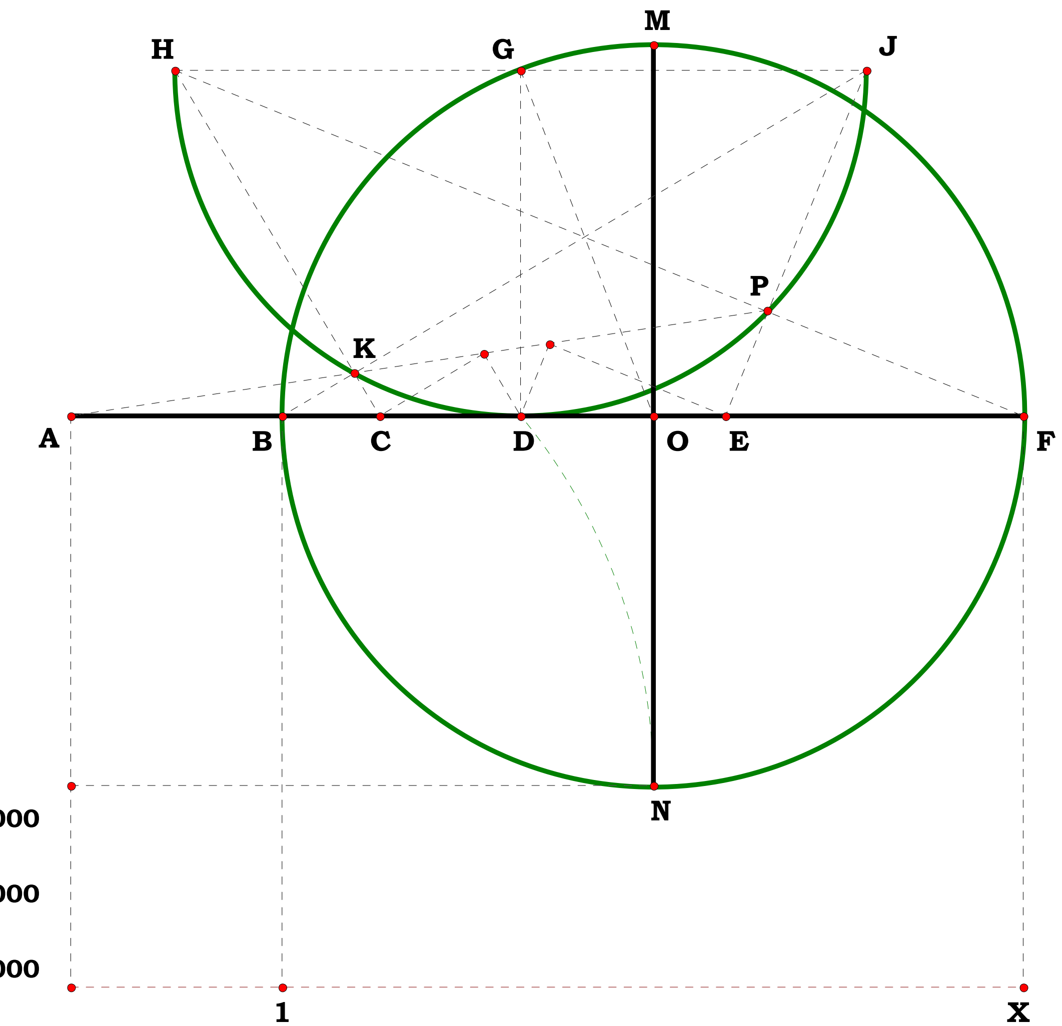
$JK := \frac{(BD + DG) \cdot HJ}{BJ}$ $\quad BK := BJ - JK$ $\quad BC := \frac{HJ \cdot BK}{JK}$

$AC := AB + BC$ $\quad FH := \sqrt{(DF + DG)^2 + DG^2}$

$HP := \frac{(DF + DG) \cdot HJ}{FH}$ $\quad FP := FH - HP$ $\quad EF := \frac{HJ \cdot FP}{HP}$

$AE := AF - EF$ $\quad AF^{\frac{1}{4}} - AC = 0$ $\quad AF^{\frac{2}{4}} - AD = 0$ $\quad AF^{\frac{3}{4}} - AE = 0$

Unit = 1.00000
XY = 4.50000
X = 9.00000
Y = 2.00000
AB = 1.00000
AC = 1.45648
AD = 2.12132
AE = 3.08965
AF = 4.50000
AC^4 = 4.50000
$AD^{\frac{4}{2}}$ = 4.50000
$AE^{\frac{4}{3}}$ = 4.50000
$AF^{\frac{1}{4}}$-AC = 0.00000
$AF^{\frac{2}{4}}$-AD = 0.00000
$AF^{\frac{3}{4}}$-AE = 0.00000



Definitions.

$AF - \frac{X}{Y} = 0$ $\quad BF - \frac{X - Y}{Y} = 0$ $\quad BO - \frac{X - Y}{2 \cdot Y} = 0$ $\quad AD - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$BD - \frac{\sqrt{X} - \sqrt{Y}}{\sqrt{Y}} = 0$ $\quad DF - \frac{\sqrt{X} \cdot (\sqrt{X} - \sqrt{Y})}{Y} = 0$ $\quad DG - \frac{X^{\frac{1}{4}} \cdot (\sqrt{X} - \sqrt{Y})}{(Y^3)^{\frac{1}{4}}} = 0$ $\quad HJ - \frac{2 \cdot X^{\frac{1}{4}} \cdot (\sqrt{X} - \sqrt{Y})}{(Y^3)^{\frac{1}{4}}} = 0$

$$BJ - \frac{\sqrt{(\sqrt{X} - \sqrt{Y})^2 \cdot \left[2 \cdot \sqrt{X} \cdot Y + \sqrt{Y^3} + 2 \cdot X^{\frac{1}{4}} \cdot \sqrt{Y} \cdot (Y^3)^{\frac{1}{4}}\right]}}{\sqrt{Y \cdot \sqrt{Y^3}}} = 0$$

$$JK - \frac{2 \cdot X^{\frac{1}{4}} \cdot (\sqrt{X} - \sqrt{Y}) \cdot \sqrt{Y \cdot \sqrt{Y^3}} \cdot (\sqrt{X} - \sqrt{Y}) \cdot \left[X^{\frac{1}{4}} \cdot \sqrt{Y} + (Y^3)^{\frac{1}{4}}\right]}{\sqrt{Y^3} \cdot \sqrt{(\sqrt{X} - \sqrt{Y})^2 \cdot \left[2 \cdot \sqrt{X} \cdot Y + \sqrt{Y^3} + 2 \cdot X^{\frac{1}{4}} \cdot \sqrt{Y} \cdot (Y^3)^{\frac{1}{4}}\right]} \cdot \sqrt{Y}} = 0$$

$$BK - \frac{Y^3\cdot\left(\sqrt{X}-\sqrt{Y}\right)^2}{\sqrt{Y^3}\cdot\sqrt{Y\cdot\sqrt{Y^3}}\cdot\sqrt{\left(\sqrt{X}-\sqrt{Y}\right)^2\cdot\left[2\cdot\sqrt{X}\cdot Y+\sqrt{Y^3}+2\cdot X^{\frac{1}{4}}\cdot\sqrt{Y}\cdot\left(Y^3\right)^{\frac{1}{4}}\right]}} = 0 \qquad BC - \frac{Y^{\frac{5}{2}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{\left(Y^3\right)^{\frac{3}{4}}\cdot\left[X^{\frac{1}{4}}\cdot\sqrt{Y}+\left(Y^3\right)^{\frac{1}{4}}\right]} = 0$$

$$AC - \frac{X^{\frac{1}{4}}\cdot\sqrt{Y}\cdot\left[X^{\frac{1}{4}}\cdot Y^2+\left(Y^3\right)^{\frac{3}{4}}\right]}{\left(Y^3\right)^{\frac{3}{4}}\cdot\left[X^{\frac{1}{4}}\cdot\sqrt{Y}+\left(Y^3\right)^{\frac{1}{4}}\right]} = 0 \qquad AC - \left(\frac{X}{Y}\right)^{\frac{1}{4}} = 0$$

$$FH - \frac{X^{\frac{1}{4}}\cdot\sqrt{X^{\frac{3}{2}}\cdot\sqrt{Y^3}+2\cdot X\cdot Y^2+2\cdot Y^3-4\cdot\sqrt{X}\cdot Y^{\frac{5}{2}}-2\cdot X\cdot\sqrt{Y}\cdot\sqrt{Y^3}+\sqrt{X}\cdot Y\cdot\sqrt{Y^3}+2\cdot X^{\frac{5}{4}}\cdot Y\cdot\left(Y^3\right)^{\frac{1}{4}}+2\cdot X^{\frac{1}{4}}\cdot Y^2\cdot\left(Y^3\right)^{\frac{1}{4}}-4\cdot X^{\frac{3}{4}}\cdot Y^{\frac{3}{2}}\cdot\left(Y^3\right)^{\frac{1}{4}}}}{Y\cdot\left(Y^3\right)^{\frac{1}{4}}} = 0$$

$$HP - \frac{2\cdot X^{\frac{1}{4}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)^2\cdot\left[Y+X^{\frac{1}{4}}\cdot\left(Y^3\right)^{\frac{1}{4}}\right]}{\left(Y^3\right)^{\frac{1}{4}}\cdot\sqrt{X^{\frac{3}{2}}\cdot\sqrt{Y^3}+2\cdot X\cdot Y^2+2\cdot Y^3-4\cdot\sqrt{X}\cdot Y^{\frac{5}{2}}-2\cdot X\cdot\sqrt{Y}\cdot\sqrt{Y^3}+\sqrt{X}\cdot Y\cdot\sqrt{Y^3}+2\cdot X^{\frac{5}{4}}\cdot Y\cdot\left(Y^3\right)^{\frac{1}{4}}+2\cdot X^{\frac{1}{4}}\cdot Y^2\cdot\left(Y^3\right)^{\frac{1}{4}}-4\cdot X^{\frac{3}{4}}\cdot Y^{\frac{3}{2}}\cdot\left(Y^3\right)^{\frac{1}{4}}}} = 0$$

$$FP - \frac{X^{\frac{3}{4}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)^2\cdot\left(Y^3\right)^{\frac{1}{4}}}{Y\cdot\sqrt{X^{\frac{3}{2}}\cdot\sqrt{Y^3}+2\cdot X\cdot Y^2+2\cdot Y^3-4\cdot\sqrt{X}\cdot Y^{\frac{5}{2}}-2\cdot X\cdot\sqrt{Y}\cdot\sqrt{Y^3}+\sqrt{X}\cdot Y\cdot\sqrt{Y^3}+2\cdot X^{\frac{5}{4}}\cdot Y\cdot\left(Y^3\right)^{\frac{1}{4}}+2\cdot X^{\frac{1}{4}}\cdot Y^2\cdot\left(Y^3\right)^{\frac{1}{4}}-4\cdot X^{\frac{3}{4}}\cdot Y^{\frac{3}{2}}\cdot\left(Y^3\right)^{\frac{1}{4}}}} = 0 \qquad EF - \frac{X^{\frac{3}{4}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)\cdot\left(Y^3\right)^{\frac{1}{4}}}{Y\cdot\left[Y+X^{\frac{1}{4}}\cdot\left(Y^3\right)^{\frac{1}{4}}\right]} = 0$$

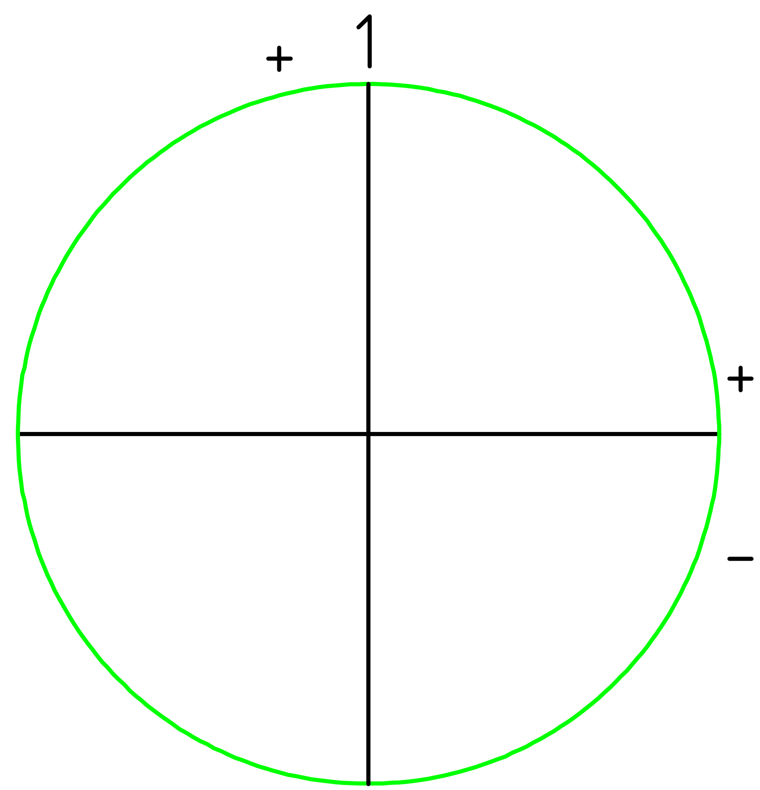
$$AE - \frac{\left(X^{\frac{1}{4}}\right)^3\cdot\left[X^{\frac{1}{4}}\cdot\sqrt{Y}+\left(Y^3\right)^{\frac{1}{4}}\right]}{\sqrt{Y}\cdot\left[Y+X^{\frac{1}{4}}\cdot\left(Y^3\right)^{\frac{1}{4}}\right]} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{3}{4}} = 0$$

011295A

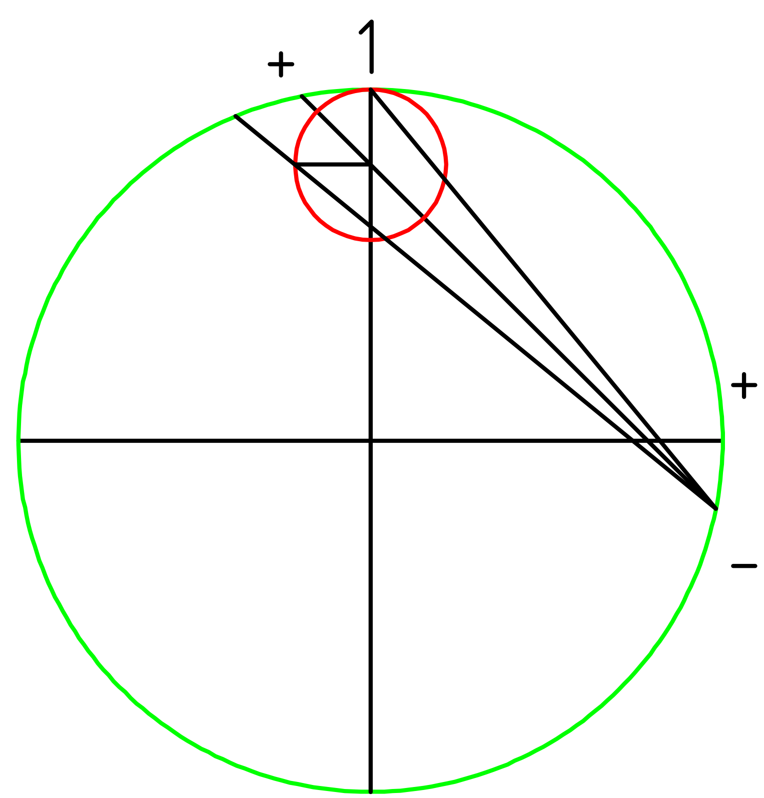
Archimedean **Trisection Revisited.**

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.



The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have $\frac{90}{8} = 11.25$ **my starting angle of the s will be 90 degrees.**



$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$

$B := 1 + \frac{1}{8} - \frac{1}{8}$ $\qquad B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\qquad \frac{B}{4} \cdot 90 = 22.5$

$8 + 1 - 1 = 8$ $\qquad 8 \cdot 11.25 = 90$

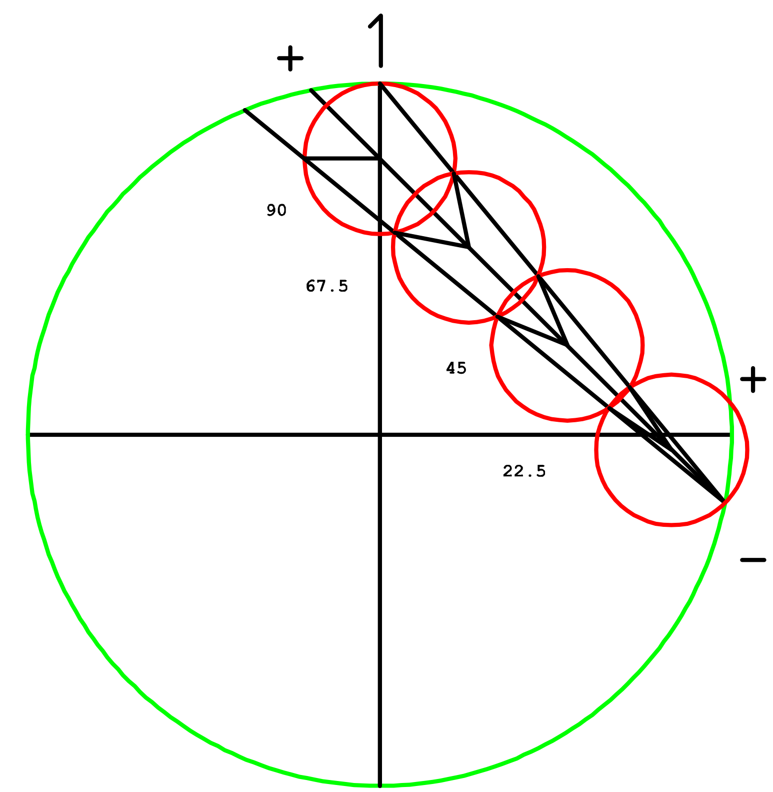
$8 + 1 - 1 - 2 = 6$ $\qquad 6 \cdot 11.25 = 67.5$

$8 + 1 - 1 - 2 - 2 = 4$ $\qquad 4 \cdot 11.25 = 45$

$8 + 1 - 1 - 2 - 2 - 2 = 2$ $\qquad 2 \cdot 11.25 = 22.5$

$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$

$\mathrm{mod}(8 + 1 - 1, 2) = 0$



I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $\qquad B = 1.125$ $\qquad \frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

$8 + 1 + 1 - 1 = 9$ $\qquad 9 \cdot 11.25 = 101.25$

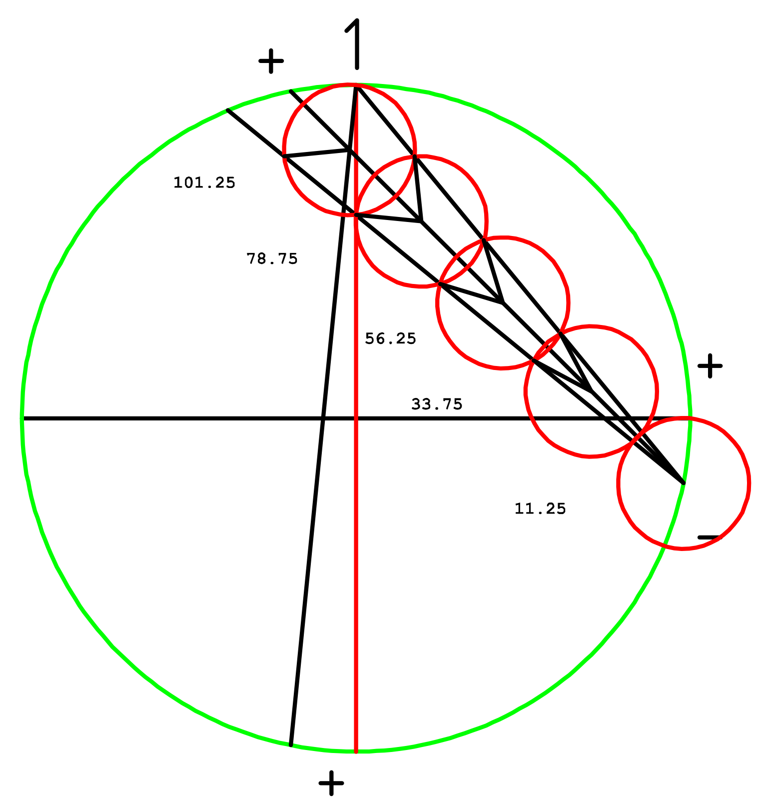
$8 + 1 + 1 - 1 - 2 = 7$ $\qquad 7 \cdot 11.25 = 78.75$

$8 + 1 + 1 - 1 - 2 - 2 = 5$ $\qquad 5 \cdot 11.25 = 56.25$

$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ $\qquad 3 \cdot 11.25 = 33.75$

$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ $\qquad 1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$



$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $\qquad B = 1.125 \qquad \frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

$8 + 1 = 9 \qquad 9 \cdot 11.25 = 101.25$

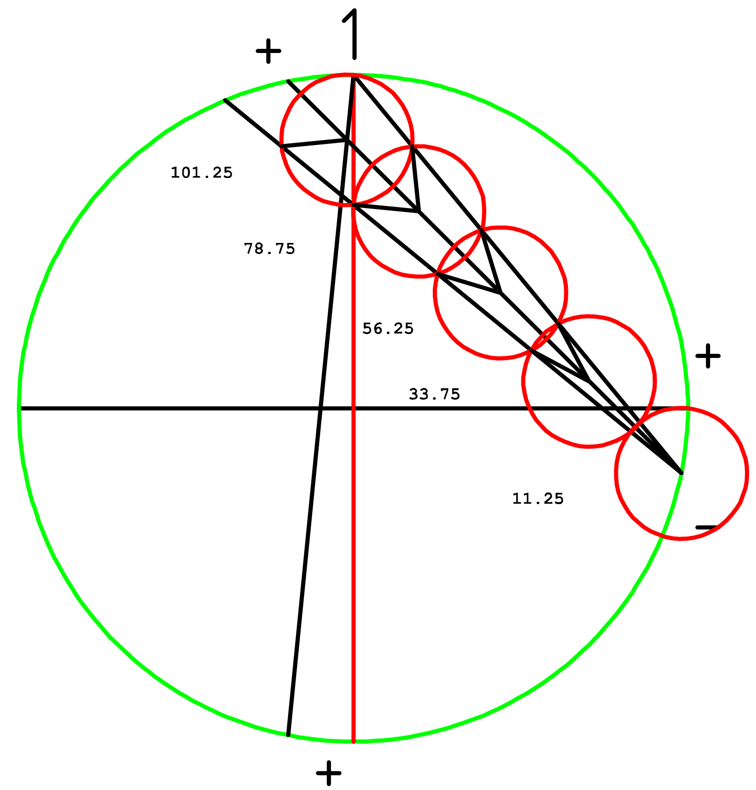
$8 + 1 - (1 \cdot 2) = 7 \qquad 7 \cdot 11.25 = 78.75$

$8 + 1 - (2 \cdot 2) = 5 \qquad 5 \cdot 11.25 = 56.25$

$8 + 1 - (3 \cdot 2) = 3 \qquad 3 \cdot 11.25 = 33.75$

$8 + 1 - (4 \cdot 2) = 1 \qquad 1 \cdot 11.25 = 11.25$

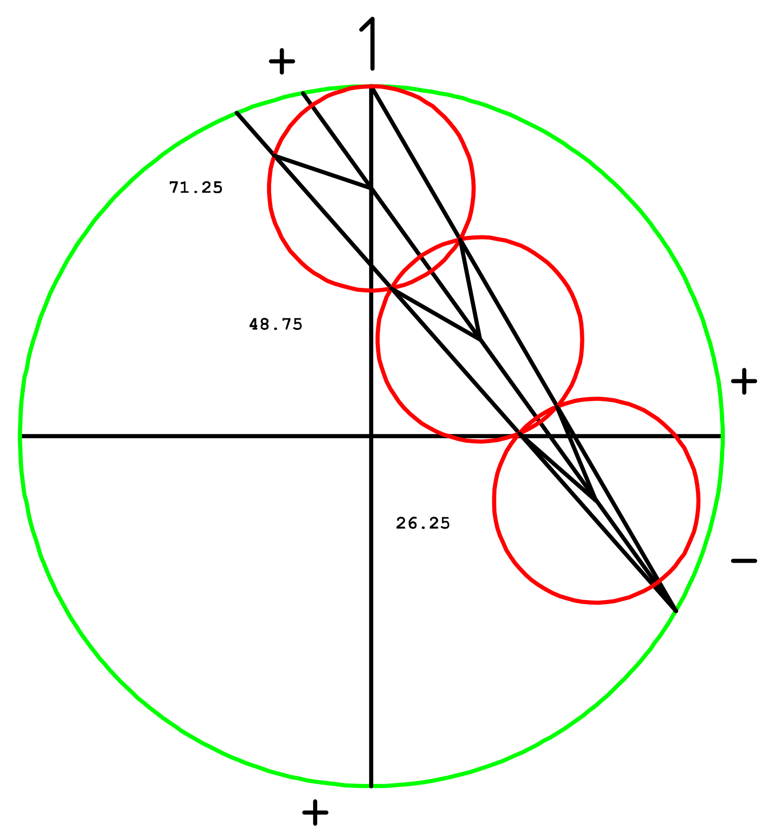
$\mathrm{mod}(8 + 1, 2) = 1$



$B := 1 + \frac{3}{24} - \frac{8}{24}$ $\quad B = 0.791667 \qquad \frac{19}{24} = 0.791667$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25 \qquad \frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.749526$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.249905 \qquad \frac{B \cdot .166666}{3.166666} \cdot 90 = 3.749986$



$(24 + 3) - 8 = 19 \qquad 19 \cdot 3.75 = 71.25$

$(24 + 3) - 8 - (1 \cdot 6) = 13 \qquad 13 \cdot 3.75 = 48.75$

$(24 + 3) - 8 - (2 \cdot 6) = 7 \qquad 7 \cdot 3.75 = 26.25$

$(24 + 3) - 8 - (3 \cdot 6) = 1 \qquad 1 \cdot 3.75 = 3.75$

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $\quad B = 2.25 \quad \frac{18}{8} = 2.25$

$\frac{B\cdot 9}{9}\cdot 90 = 202.5 \qquad \frac{B\cdot 8}{9}\cdot 90 = 180$

$\frac{B\cdot 7}{9}\cdot 90 = 157.5 \qquad \frac{B\cdot .6}{9}\cdot 90 = 13.5$

$8 + 1 - 1 + 10 = 18 \qquad 18\cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2\cdot 1) = 16 \quad 16\cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2\cdot 2) = 14 \quad 14\cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2\cdot 3) = 12 \quad 12\cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2\cdot 4) = 10 \quad 10\cdot 11.25 = 112.5$

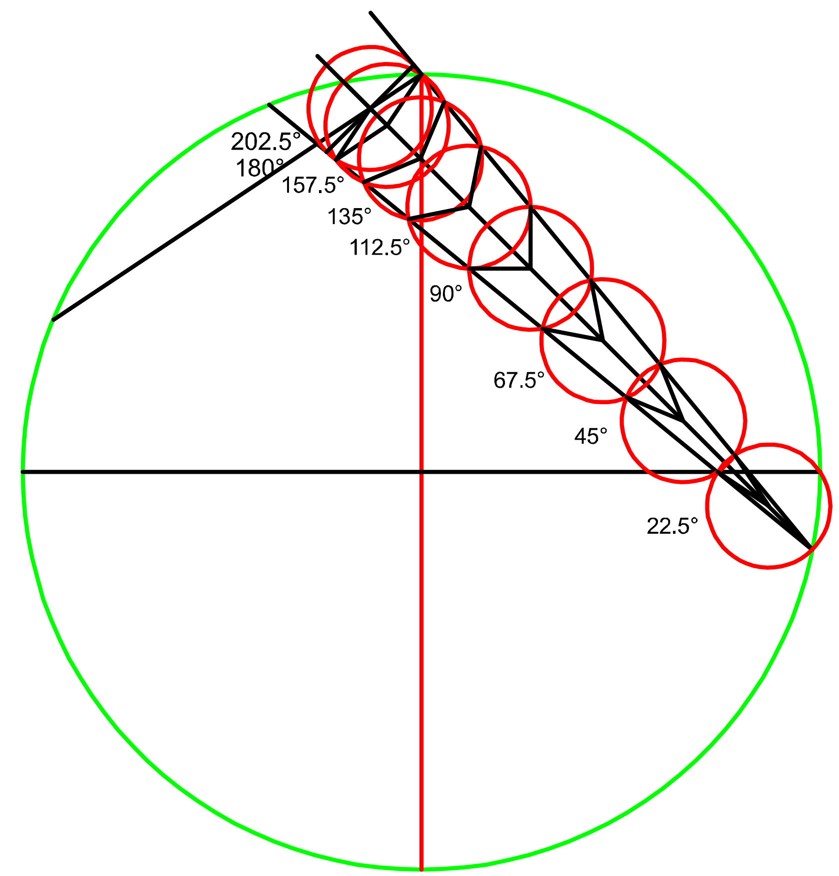
$8 + 1 - 1 + 10 - (2\cdot 5) = 8 \quad 8\cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2\cdot 6) = 6 \quad 6\cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2\cdot 7) = 4 \quad 4\cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2\cdot 8) = 2 \qquad 2\cdot 11.25 = 22.5$

$\mathrm{mod}[(8 + 1 - 1) + 10, 2] = 0$



$B := 1 + \frac{1}{7} - \frac{2}{7} \qquad B = 0.857143 \qquad \frac{6}{7} = 0.857143$

$\frac{B\cdot 6}{6}\cdot 90 = 77.142857 \qquad \frac{B\cdot 4}{6}\cdot 90 = 51.428571$

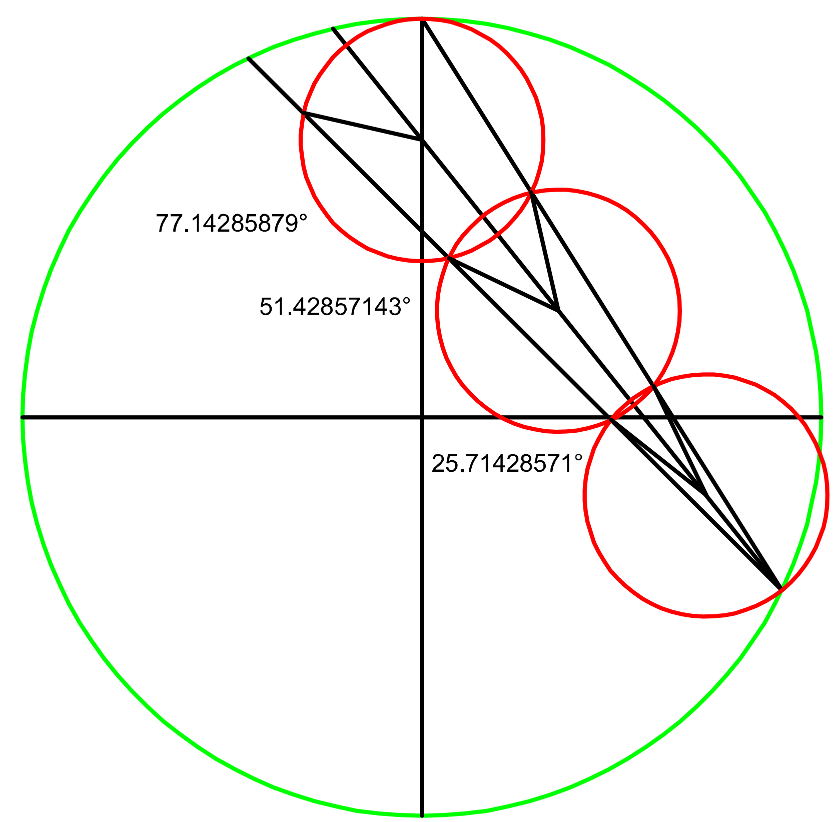
$\frac{B\cdot 2}{6}\cdot 90 = 25.714286 \qquad c := \frac{90}{7}$

$7 + 1 - (1\cdot 2) = 6 \qquad 6\cdot c = 77.142857$

$7 + 1 - (2\cdot 2) = 4 \qquad 4\cdot c = 51.428571$

$7 + 1 - (3\cdot 2) = 2 \qquad 2\cdot c = 25.714286$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$



$B := 1 + \frac{1}{7} - \frac{1}{7}$ $\quad B = 1 \quad \frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90 \qquad \frac{B \cdot 5}{7} \cdot 90 = 64.285714$

$\frac{B \cdot 3}{7} \cdot 90 = 38.57142 \qquad \frac{B \cdot 1}{7} \cdot 90 = 12.857143$

$7 + 1 - 1 = 7 \qquad 7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5 \qquad 5 \cdot c = 64.285714$

$7 + 1 - 1 - (2 \cdot 2) = 3 \qquad 3 \cdot c = 38.571429$

$7 + 1 - 1 - (3 \cdot 2) = 1 \qquad 1 \cdot c = 12.857143$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$

$B := 1 + \frac{8}{56} - \frac{7}{56} \qquad B = 1.017857$

$\frac{B \cdot 57}{57} \cdot 90 = 91.607143 \qquad \frac{B \cdot 41}{57} \cdot 90 = 65.892857$

$\frac{B \cdot 25}{57} \cdot 90 = 40.178571 \qquad c := \frac{90}{56}$

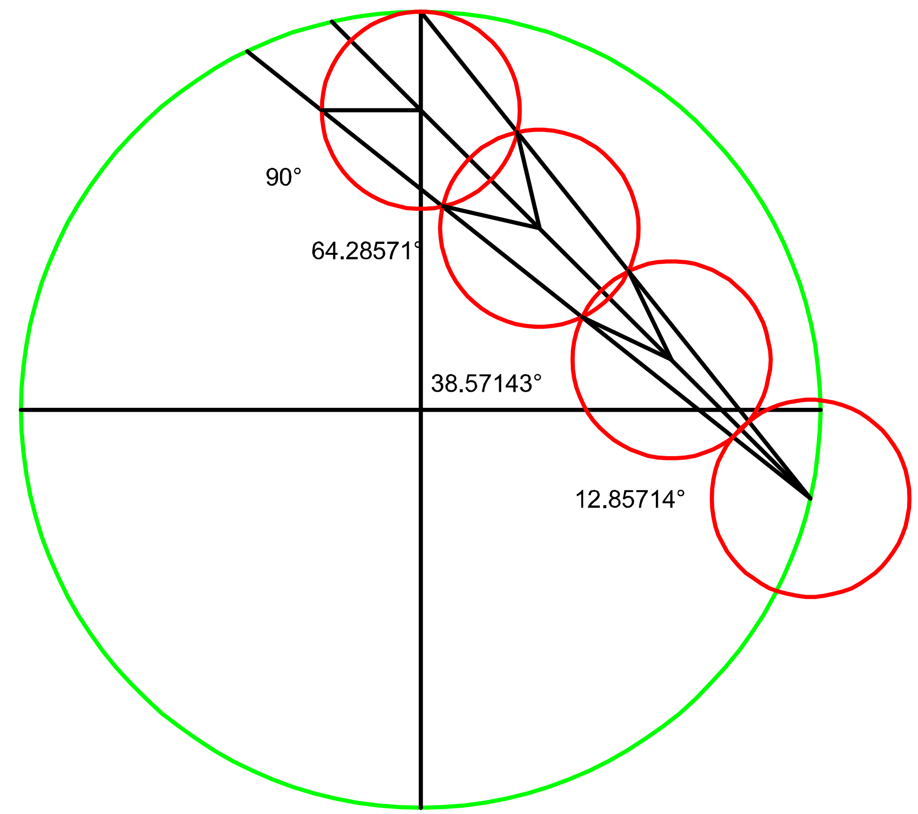
$56 + 8 - 7 = 57 \qquad 57 \cdot c = 91.607143$

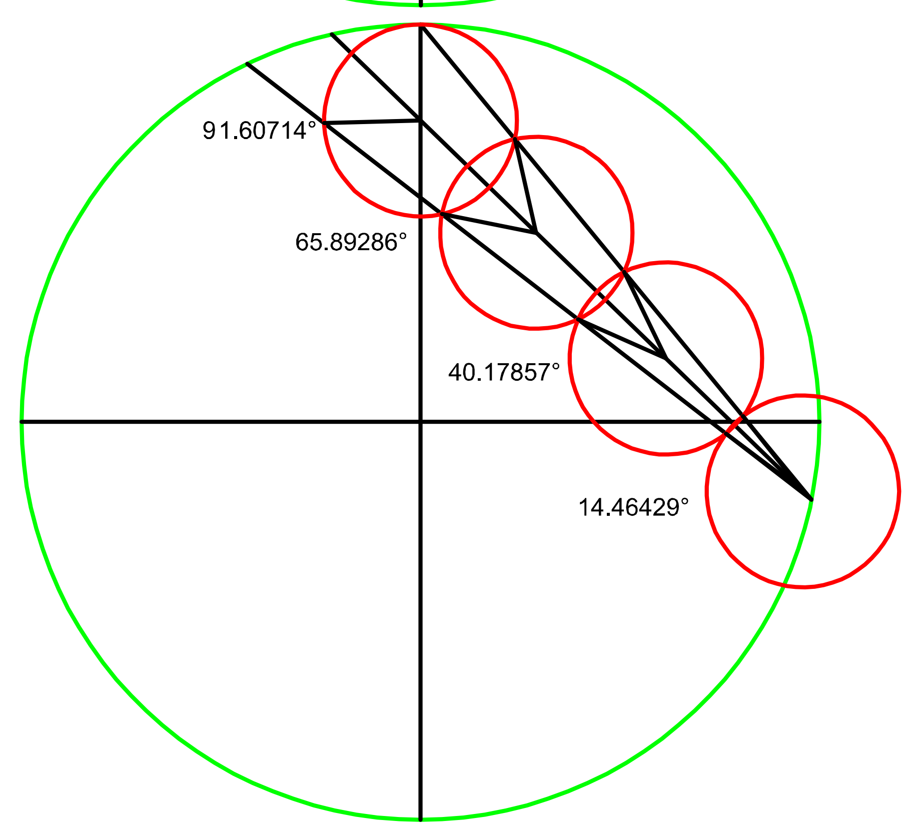
$56 + 8 - 7 - (1 \cdot 16) = 41 \qquad 41 \cdot c = 65.892857$

$56 + 8 - 7 - (2 \cdot 16) = 25 \qquad 25 \cdot c = 40.178571$

$56 + 8 - 7 - (3 \cdot 16) = 9 \qquad 9 \cdot c = 14.464286$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$





$\mathbf{B} := 1 + \frac{1}{7} - \frac{1}{7}$ $\qquad \mathbf{B} = 1$

$\frac{\mathbf{B} \cdot 7}{7} \cdot 90 = 90$ $\qquad \frac{\mathbf{B} \cdot 5}{7} \cdot 90 = 64.285714$

$\frac{\mathbf{B} \cdot 3}{7} \cdot 90 = 38.571429$ $\qquad \frac{\mathbf{B} \cdot 1}{7} \cdot 90 = 12.857143$

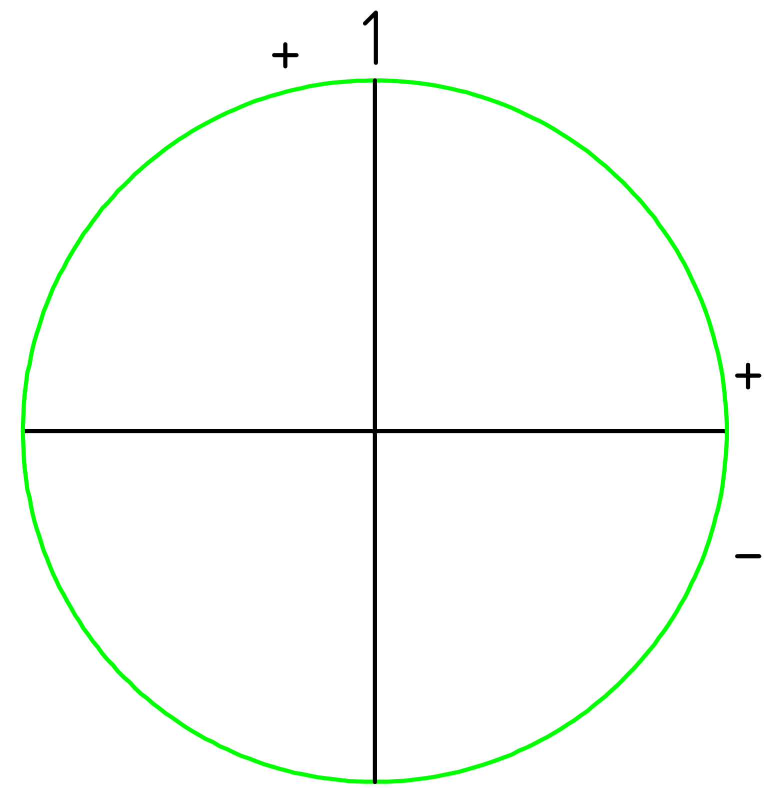
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

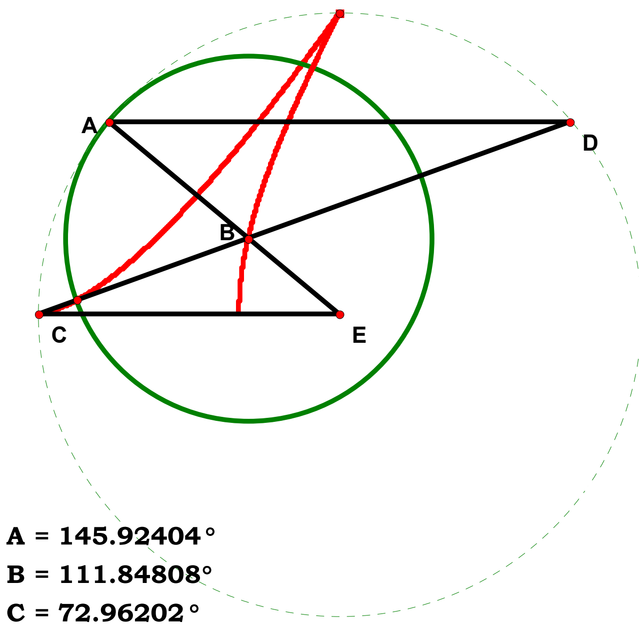
$\mathbf{mod}(7 + 1 - 1 , 2) = 1$



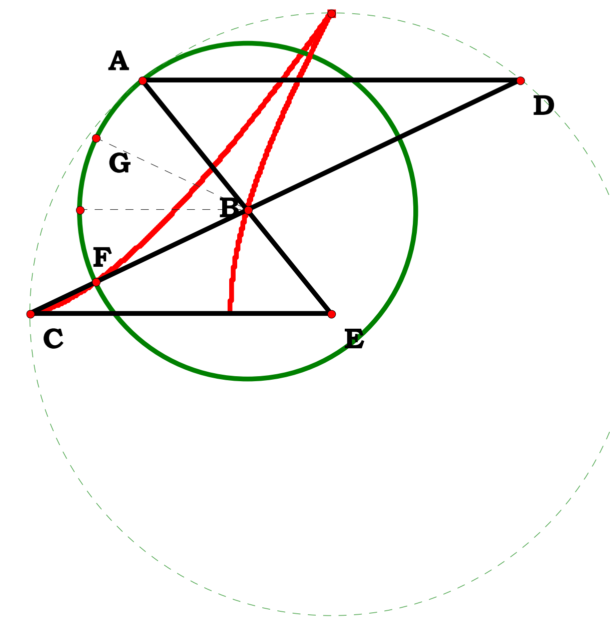
011295B

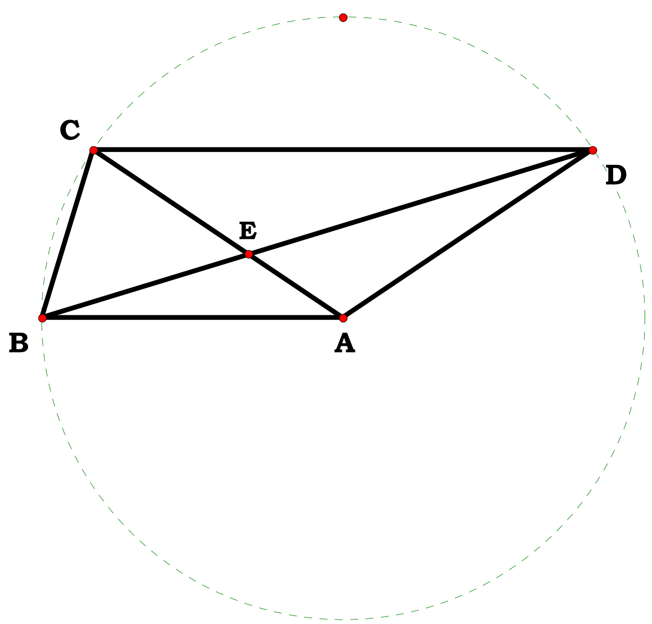
Archimedean Trisection Revisited.

$m\angle ABC = 59.71415°$
$m\angle ADC = 19.90472°$
$\frac{m\angle ABC}{m\angle ADC} = 3.00000$
$m\angle BCE = 19.90472°$



$m\angle ABC = 76.48479°$
$m\angle ADC = 25.49493°$
$\frac{m\angle ABC}{m\angle ADC} = 3.00000$
$m\angle BCE = 25.49493°$
$m\angle ABF = 76.48479°$
$m\angle ABG = 25.49493°$
$\frac{m\angle ABF}{m\angle ABG} = 3.00000$





$m\angle BAD = 145.92404°$ A = 145.92404°
$m\angle CAD = 111.84808°$ B = 111.84808°
$m\angle ABC = 72.96202°$ C = 72.96202°
$m\angle ABD = 17.03798°$ D = 17.03798°
$m\angle BCA = 72.96202°$
$m\angle BCD = 107.03798°$ F = 107.03798° E = 72.96202°
$m\angle ADB = 17.03798°$ G = 17.03798°
$m\angle ADC = 34.07596°$ H = 34.07596°

$m\angle AEB = 128.88606°$ J = 128.88606°
$m\angle BEC = 51.11394°$ K = 51.11394°
$m\angle CED = 128.88606°$ M = 128.88606°
$m\angle DEA = 51.11394°$ N = 51.11394°

$\frac{A}{B} = 1.30466$	$\frac{B}{A} = 0.76648$	$\frac{C}{A} = 0.50000$	$\frac{D}{A} = 0.11676$	$\frac{F}{A} = 0.73352$	$\frac{H}{A} = 0.23352$	$\frac{M}{A} = 0.88324$	$\frac{N}{A} = 0.35028$
$\frac{A}{C} = 2.00000$	$\frac{B}{C} = 1.53296$	$\frac{C}{B} = 0.65233$	$\frac{D}{B} = 0.15233$	$\frac{F}{B} = 0.95699$	$\frac{H}{B} = 0.30466$	$\frac{M}{B} = 1.15233$	$\frac{N}{B} = 0.45699$
$\frac{A}{D} = 8.56463$	$\frac{B}{D} = 6.56463$	$\frac{C}{D} = 4.28232$	$\frac{D}{C} = 0.23352$	$\frac{F}{C} = 1.46704$	$\frac{H}{C} = 0.46704$	$\frac{M}{C} = 1.76648$	$\frac{N}{C} = 0.70056$
$\frac{A}{F} = 1.36329$	$\frac{B}{F} = 1.04494$	$\frac{C}{F} = 0.68165$	$\frac{D}{F} = 0.15918$	$\frac{F}{D} = 6.28232$	$\frac{H}{D} = 2.00000$	$\frac{M}{D} = 7.56463$	$\frac{N}{D} = 3.00000$
$\frac{A}{H} = 4.28232$	$\frac{B}{H} = 3.28232$	$\frac{C}{H} = 2.14116$	$\frac{D}{H} = 0.50000$	$\frac{F}{H} = 3.14116$	$\frac{H}{F} = 0.31835$	$\frac{M}{F} = 1.20412$	$\frac{N}{F} = 0.47753$
$\frac{A}{M} = 1.13219$	$\frac{B}{M} = 0.86781$	$\frac{C}{M} = 0.56610$	$\frac{D}{M} = 0.13219$	$\frac{F}{M} = 0.83049$	$\frac{H}{M} = 0.26439$	$\frac{M}{H} = 3.78232$	$\frac{N}{H} = 1.50000$
$\frac{A}{N} = 2.85488$	$\frac{B}{N} = 2.18821$	$\frac{C}{N} = 1.42744$	$\frac{D}{N} = 0.33333$	$\frac{F}{N} = 2.09411$	$\frac{H}{N} = 0.66667$	$\frac{M}{N} = 2.52154$	$\frac{N}{M} = 0.39658$

040195A

Descriptions.

Given.

$N_1 := 5$

$N_2 := 12$

$\delta := 0 .. 2$

Exponential Series-Roots and Powers

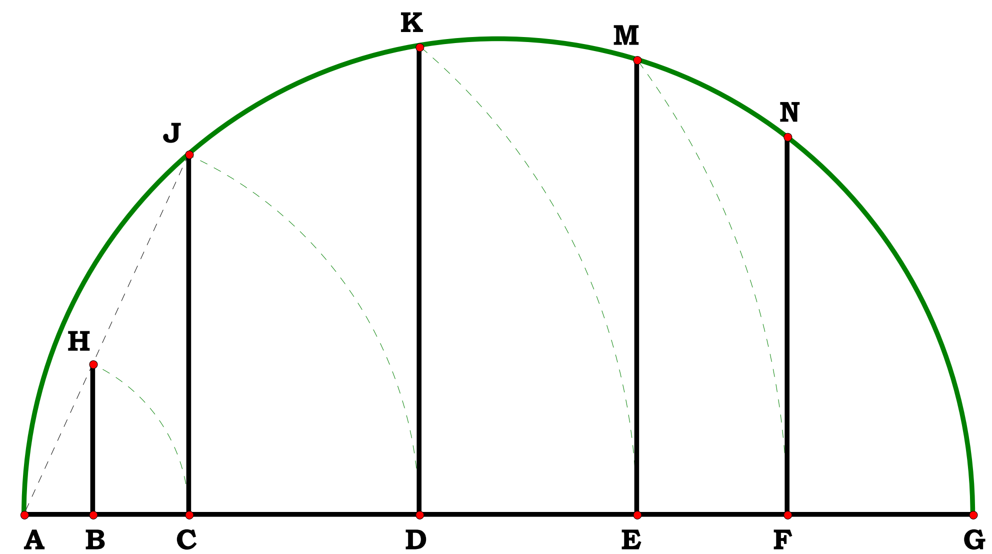
I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.

$AG := N_2 \quad AD_0 := N_1 \quad AN := AD_0$

$AJ_1 := AD_0 \quad DF_0 := AG - AD_0$

$DO_0 := \sqrt{AD_0 \cdot DF_0} \quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AG - AO_\delta \\ \sqrt{AO_\delta \cdot (AG - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AG - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$



Unit = 1.00000

XY = 0.41667

X = 5.00000

Y = 12.00000

AB = 0.07234

AC = 0.17361

AD = 0.41667

AE = 0.64550

AF = 0.80343

$\frac{Unit}{AB} = 13.82400$

$\frac{Unit}{AC} = 5.76000$

$\frac{Unit}{AD} = 2.40000$

$\frac{Unit}{AE} = 1.54919$

$\frac{Unit}{AF} = 1.24467$

Definitions.

$$\sum_\delta \left[\frac{AG}{AD_\delta} - \left(\frac{N_2}{N_1}\right)^{\frac{1}{2^\delta}} \right] = 0$$

$\left(\frac{N_2}{N_1}\right)^{\delta+1} =$

2.4
5.76
13.824

$\left(\frac{N_2}{N_1}\right)^{\frac{1}{2^\delta}} =$

2.4
1.549193
1.244666

$\frac{AG}{AD_\delta} =$

2.4
1.549193
1.244666

$\left(\frac{N_2}{N_1}\right)^{\frac{1}{2^\delta}} =$

2.4
1.549193
1.244666

040195B

Descriptions.

Given.

$X := 5$

$Y := 12$

$\delta := 0 .. 2$

Exponential Series-Roots and Powers

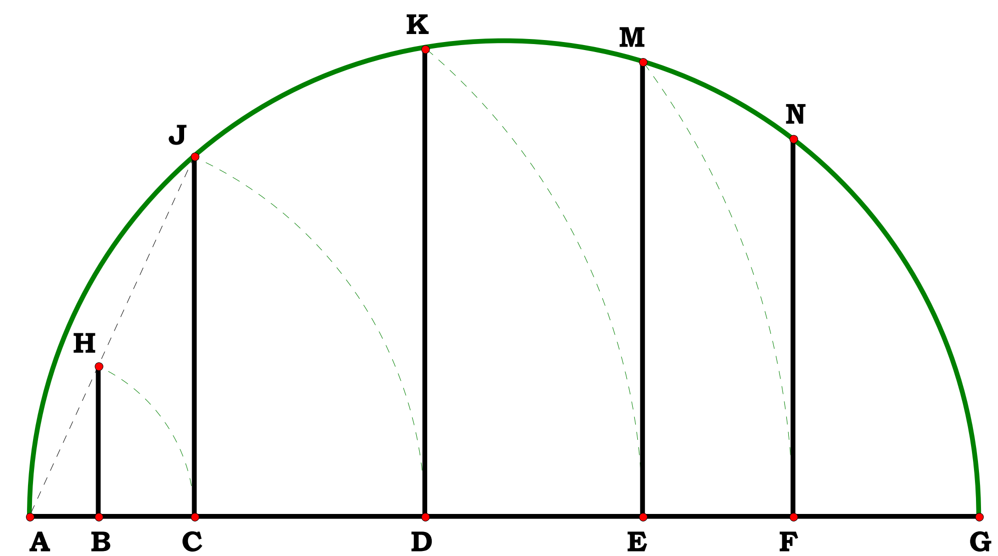
I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.

$AG := \frac{Y}{Y}$ $\quad AD_0 := \frac{Y}{X}$ $\quad AN := AD_0$

$AJ_1 := AD_0$ $\quad DF_0 := AG - AD_0$

$DO_0 := \sqrt{AD_0 \cdot DF_0}$ $\quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AG - AO_\delta \\ \sqrt{AO_\delta \cdot (AG - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AG - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$



Unit = 1.00000
XY = 0.41667
X = 5.00000
Y = 12.00000

AB = 0.07234
AC = 0.17361
AD = 0.41667
AE = 0.64550
AF = 0.80343

$\frac{Unit}{AB} = 13.82400$

$\frac{Unit}{AC} = 5.76000$

$\frac{Unit}{AD} = 2.40000$

$\frac{Unit}{AE} = 1.54919$

$\frac{Unit}{AF} = 1.24467$

Definitions.

$$\sum_{\delta} \left[AD_\delta - \left(\frac{Y}{X}\right)^{\frac{1}{2^\delta}} \right] = 0$$

$\left(\frac{Y}{X}\right)^{\delta+1} =$

2.4
5.76
13.824

$\left(\frac{Y}{X}\right)^{\frac{1}{2^\delta}} =$

2.4
1.549193
1.244666

$AD_\delta =$

2.4
1.549193
1.244666

$\left(\frac{Y}{X}\right)^{\frac{1}{2^\delta}} =$

2.4
1.549193
1.244666

$AE := 5.20700$

$Unit := AE$

Given.

$AB := 1.11300 \quad N_1 := AB$

$AC := 2.02712 \quad N_2 := AC$

091395A1

Given AE, AB, AC what is GH?

Descriptions.

$$BC := (AC - AB) \qquad GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2}$$

$$GK = 4.27657 \qquad GH = 2.138285$$

Definitions.

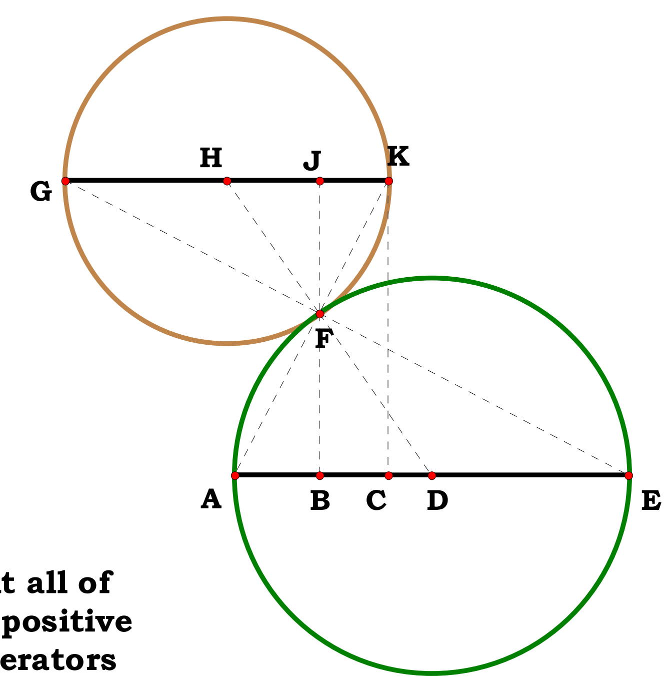
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_2 - N_1}{\sqrt{(N_2 - N_1)^2}} = 1$$

$$GH - \left[\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot \frac{N_2 - N_1}{\sqrt{(N_2 - N_1)^2}}\right] = 0$$

Combinaing we get: $GH - \dfrac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} = 0$

A Study In Placement



AE = 5.20700 cm
AB = 1.11300 cm
AC = 2.02712 cm
CK = 3.88781 cm
EF = 4.61708 cm
GK = 4.27653 cm
GH = 2.13827 cm
BC = 0.91412 cm

Unit = 5.20700 cm
N_1 = 1.11300 cm
N_2 = 2.02712 cm
AC-AB = 0.91412 cm

$$\frac{AE}{AB}-\frac{GK}{BC} = 0.00000$$

$$\frac{AE \cdot BC}{AB}\text{-}GK = 0.00000 \text{ cm}$$

$$\frac{(AC\text{-}AB)}{\sqrt{(AC\text{-}AB)^2}} = 1.00000$$

Is GK outside side AE? = 1.00000

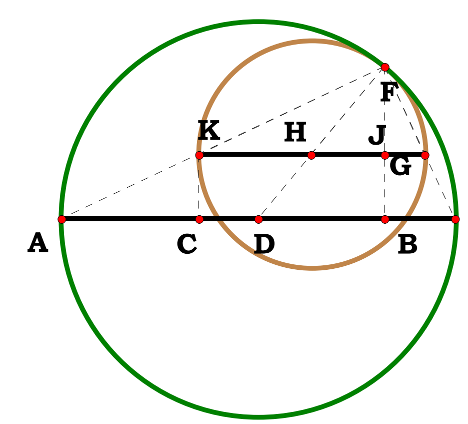
X = 1.00000

$$\frac{Unit \cdot (N_2\text{-}N_1)}{2 \cdot N_1} = 2.13827 \text{ cm}$$

$$\frac{Unit \cdot (N_2\text{-}N_1)}{2 \cdot N_1} \cdot X\text{-}GH = 0.00000 \text{ cm}$$

$$\frac{Unit \cdot \sqrt{(N_1\text{-}N_2)^2}}{2 \cdot N_1}\text{-}GH = 0.00000 \text{ cm}$$

$$\frac{Unit \cdot (N_2\text{-}N_1)}{2 \cdot N_1} \cdot \frac{(AC\text{-}AB)}{\sqrt{(AC\text{-}AB)^2}}\text{-}GH = 0.00000 \text{ cm}$$



$$\frac{Unit \cdot \sqrt{(N_1\text{-}N_2)^2}}{2 \cdot N_1}\text{-}GH = 0.00000 \text{ cm}$$

Given.

$W := 4 \qquad Y := 6$

$X := 20 \qquad Z := 17$

A Study In Placement

091395A2

Given AE, AB, AC what is GH?

Descriptions.

$$AB := \frac{W}{X} \qquad AC := \frac{Y}{Z}$$

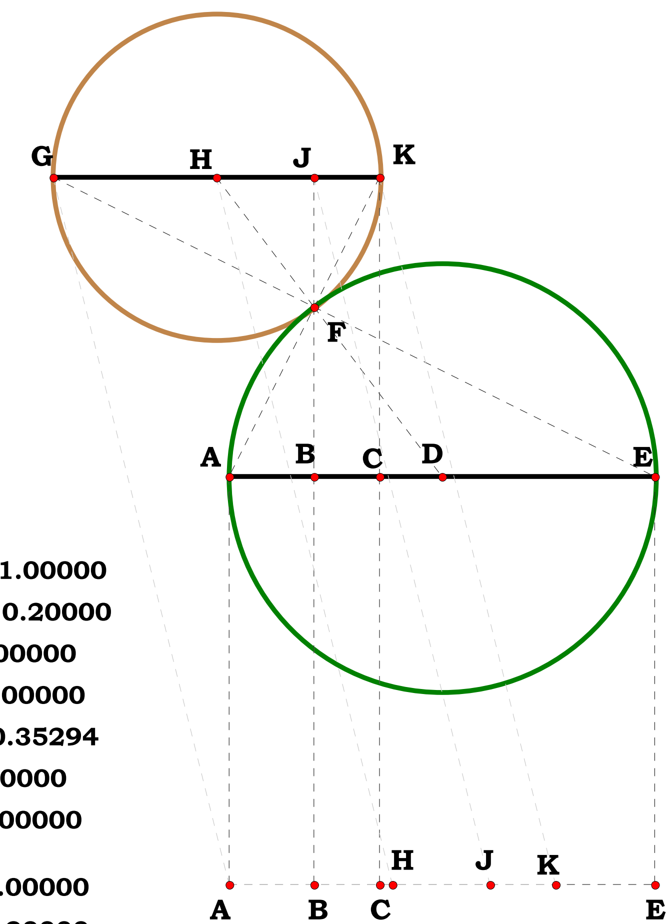
$$BC := (AC - AB) \qquad GK := \frac{BC}{AB} \qquad GH := \frac{GK}{2}$$

$GK = 0.764706 \quad GH = 0.382353$

Definitions.

$$AB - \frac{W}{X} = 0 \qquad AC - \frac{Y}{Z} = 0 \qquad BC - \frac{X \cdot Y - W \cdot Z}{X \cdot Z} = 0$$

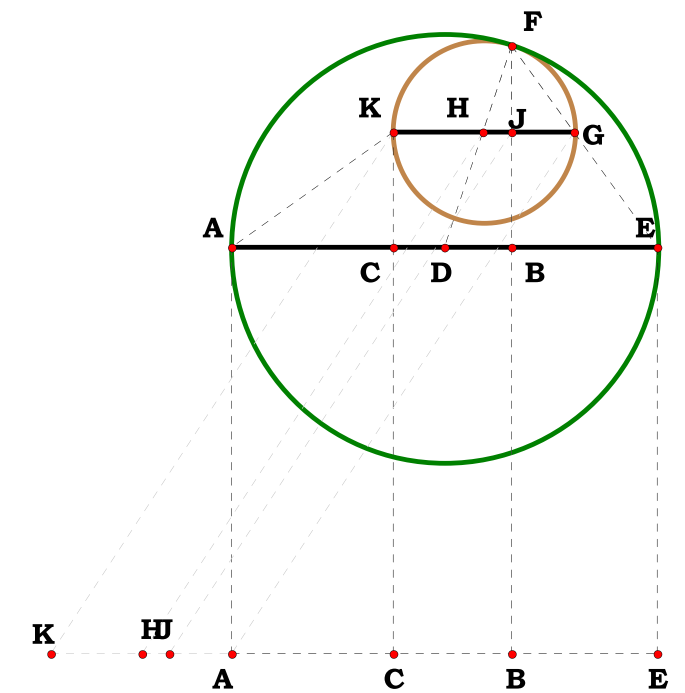
$$GK - \frac{X \cdot Y - W \cdot Z}{W \cdot Z} = 0 \qquad GH - \frac{X \cdot Y - W \cdot Z}{2 \cdot W \cdot Z} = 0$$



Unit = 1.00000
W/X = 0.20000
W = 4.00000
X = 20.00000
Y/Z = 0.35294
Y = 6.00000
Z = 17.00000

AA = 0.00000
AB = 0.20000
AC = 0.35294
GH = 0.38235
GJ = 0.61176
GK = 0.76471
AE = 1.00000

$\frac{X \cdot Y - W \cdot Z}{2 \cdot W \cdot Z}$-GH = 0.00000



AA = 0.00000
AB = 0.65823
AC = 0.37923
GH = -0.21193
GJ = -0.14486
GK = -0.42386
AE = 1.00000

$\frac{X \cdot Y - W \cdot Z}{2 \cdot W \cdot Z}$-GH = 0.00000

Unit = 1.00000
W/X = 0.65823
W = 13.16452
X = 20.00000

Y/Z = 0.37923
Y = 6.44696
Z = 17.00000

$AE := 5.20700$

$Unit := AE$

Given.

$EF := 4.58113 \quad N_1 := EF$

$AC := 1.96362 \quad N_2 := AC$

091395B1

Given AE, EF AC what is GH?

Descriptions.

$$AB := AE - \frac{EF^2}{AE} \qquad BC := (AC - AB) \qquad GK := \frac{AE \cdot BC}{AB}$$

$$GH := \frac{GK}{2} \qquad GK = 3.48358 \qquad GH = 1.74179$$

Definitions.

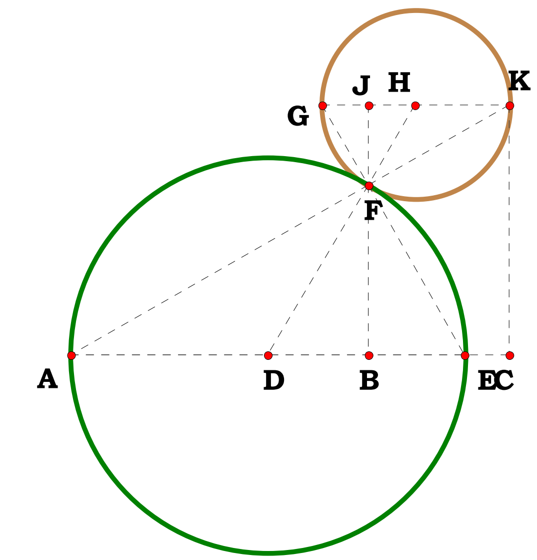
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_1^2 - Unit^2 + N_2 \cdot Unit}{\sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}} = 1$$

$$GH - \left[\frac{\left(N_2 \cdot Unit^2 - Unit^3 + Unit \cdot N_1^2\right)}{2 \cdot Unit^2 - 2 \cdot N_1^2} \cdot \frac{N_1^2 - Unit^2 + N_2 \cdot Unit}{\sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}}\right] = 0$$

$$GH - \frac{Unit \cdot \sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}}{2 \cdot \left(Unit^2 - N_1^2\right)} = 0$$

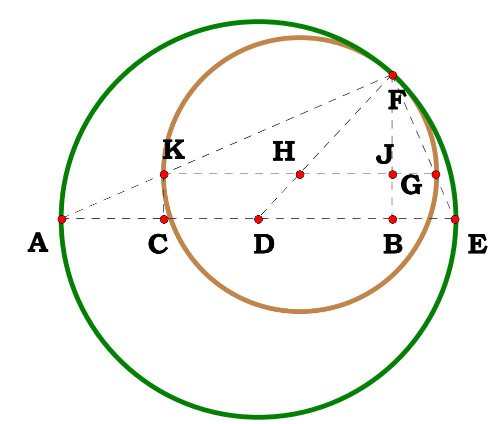
A Study In Placement



$AE = 5.20700$ cm
$AB = 3.92817$ cm
$AC = 5.79479$ cm
$CK = 3.30635$ cm
$EF = 2.58048$ cm
$GK = 2.47430$ cm
$GH = 1.23715$ cm
$BC = 1.86662$ cm

$Unit = 5.20700$ cm
$N_1 = 2.58048$ cm
$N_2 = 5.79479$ cm
$\frac{AE}{AB} - \frac{GK}{BC} = 0.00000$
$\frac{AE \cdot BC}{AB} - GK = 0.00000$ cm
$AC-AB = 1.86662$ cm
$\frac{(AC-AB)}{\sqrt{(AC-AB)^2}} = 1.00000$
Is GK outside AE? = 1.00000
$X = 1.00000$

$$\frac{Unit \cdot \sqrt{((N_1^2-Unit^2)+N_2 \cdot Unit)^2}}{2 \cdot (Unit^2-N_1^2)} - GH = 0.00000 \text{ cm}$$



$$\frac{Unit \cdot \sqrt{((N_1^2-Unit^2)+N_2 \cdot Unit)^2}}{2 \cdot (Unit^2-N_1^2)} - GH = 0.00000 \text{ cm}$$

091395B2

Given AE, EF AC what is GH?

Descriptions.

Given.

$W := 6 \quad Y := 14$

$X := 20 \quad Z := 17$

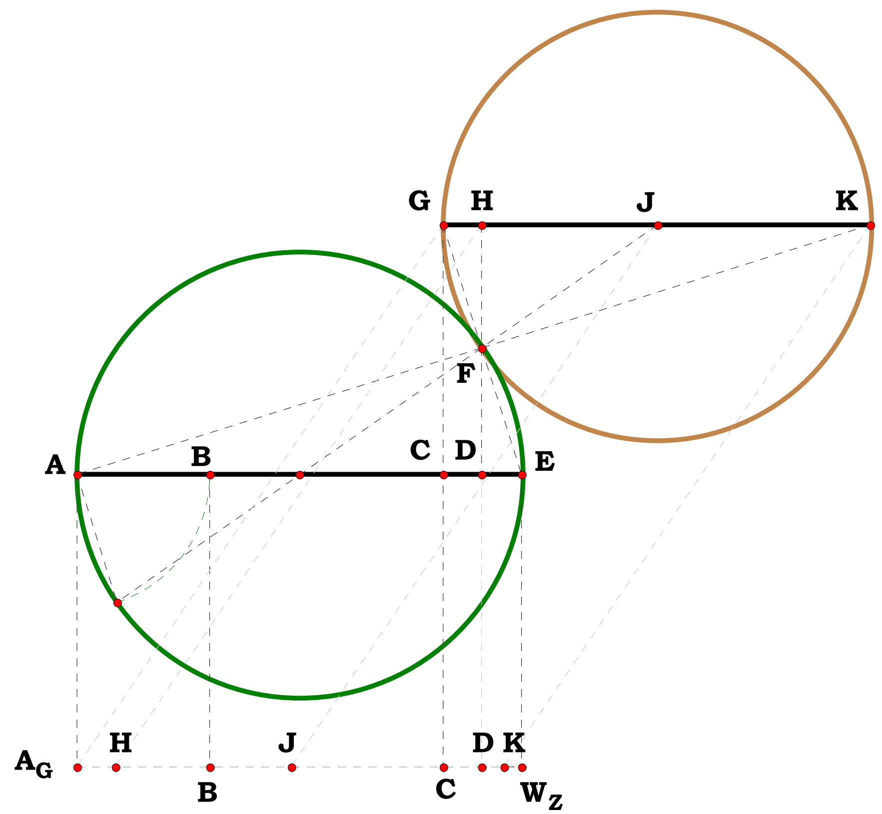
$AE := 1$

$$AB := \frac{W}{X} \qquad EF := AB \qquad AD := AE - AB^2 \qquad AC := \frac{Y}{Z}$$

$$CD := AD - AC \qquad DE := AE - AD \qquad GK := \frac{AE \cdot CD}{DE}$$

$$GJ := \frac{GK}{2} \qquad GJ = 0.480392$$

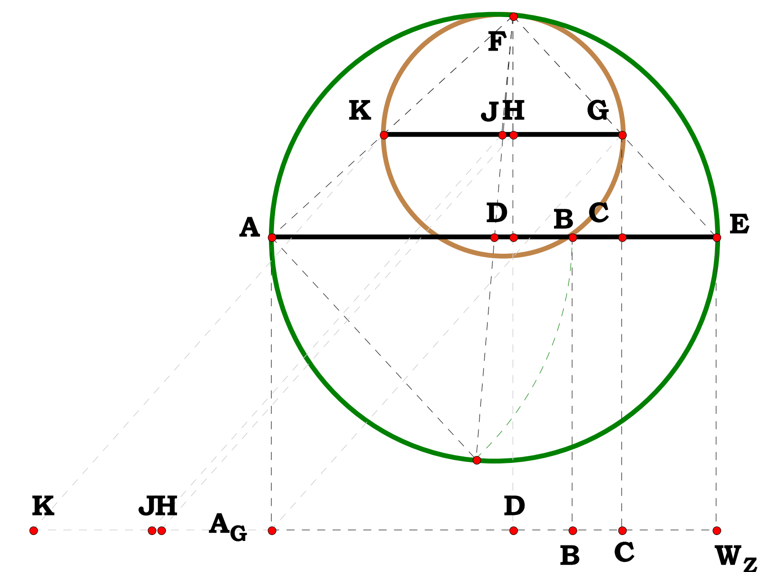
A Study In Placement



Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.82353
Y = 14.00000
Z = 17.00000

A_G = 0.00000
GH = 0.08647
AB = 0.30000
GJ = 0.48039
AC = 0.82353
GK = 0.96078
AD = 0.91000
AW_Z = 1.00000

$$GJ - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{2 \cdot W^2 \cdot Z} = 0.00000$$



Unit = 1.00000
W/X = 0.67737
W = 13.54733
X = 20.00000
Y/Z = 0.78759
Y = 13.38903
Z = 17.00000

A_G = 0.00000
GH = -0.24642
AB = 0.67737
GJ = -0.26853
AC = 0.78759
GK = -0.53706
AD = 0.54117
AW_Z = 1.00000

$$GJ - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{2 \cdot W^2 \cdot Z} = 0.00000$$

Definitions.

$$AB - \frac{W}{X} = 0 \qquad EF - \frac{W}{X} = 0 \qquad AD := \frac{X^2 - W^2}{X^2}$$

$$AC - \frac{Y}{Z} = 0 \qquad CD - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{X^2 \cdot Z} = 0$$

$$DE - \frac{W^2}{X^2} = 0 \qquad GK - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{W^2 \cdot Z} = 0$$

$$GJ - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{2 \cdot W^2 \cdot Z} = 0$$

$AE := 5.20700$

$Unit := AE$

Given.

$CK := 2.82631 \quad N_1 := CK$

$AC := 4.41895 \quad N_2 := AC$

091395C1

Given AE, CK, AC what is GH?

Descriptions.

$$CM := \frac{CK^2}{AC} \qquad AB := \frac{AC \cdot AE}{AC + CM} \qquad BC := AC - AB$$

$$GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2} \qquad GK = 1.019626 \qquad GH = 0.509813$$

Definitions.

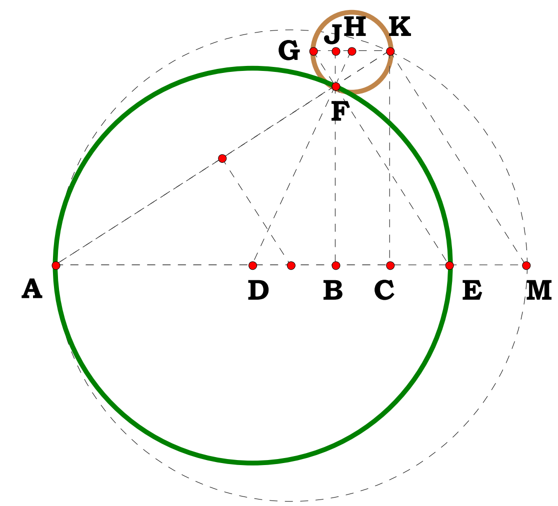
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: *Is the Brown Circle outside the Green one?* If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2 \cdot \left(N_1^2 + N_2^2\right)}} = 1$$

$$GH - \left[\frac{N_1^2 + N_2^2 - Unit \cdot N_2}{2 \cdot N_2} \cdot \frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2 \cdot \left(N_1^2 + N_2^2\right)}}\right] = 0$$

$$GH - \frac{\left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2}{2 \cdot \sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2}} = 0$$

A Study In Placement



$AE = 5.20700$ cm
$AB = 3.69534$ cm
$AC = 4.41895$ cm
$CK = 2.82631$ cm
$EF = 2.80557$ cm
$GK = 1.01963$ cm
$GH = 0.50981$ cm
$BC = 0.72362$ cm
$CM = 1.80767$ cm

$Unit = 5.20700$ cm
$N_1 = 2.82631$ cm
$N_2 = 4.41895$ cm
$AC\text{-}AB = 0.72362$ cm

$$\frac{AE}{AB} - \frac{GK}{BC} = 0.00000$$

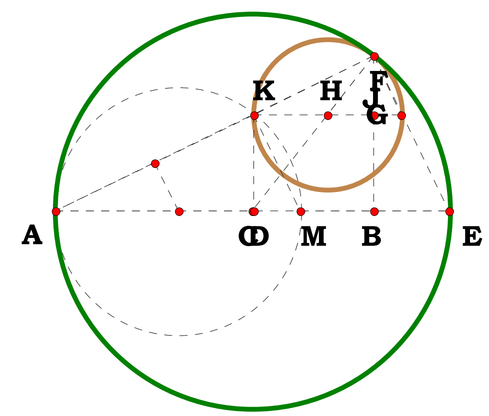
$$\frac{AE \cdot BC}{AB} - GK = 0.00000 \text{ cm}$$

$$\frac{(AC-AB)}{\sqrt{(AC-AB)^2}} = 1.00000$$

Is GK outside AE? = 1.00000

$X = 1.00000$

$$\frac{((N_1^2+N_2^2)-Unit \cdot N_2)^2}{2 \cdot \sqrt{N_2^2 \cdot ((N_1^2+N_2^2)-Unit \cdot N_2)^2}} - GH = 0.00000 \text{ cm}$$



$$\frac{((N_1^2+N_2^2)-Unit \cdot N_2)^2}{2 \cdot \sqrt{N_2^2 \cdot ((N_1^2+N_2^2)-Unit \cdot N_2)^2}} - GH = 0.00000 \text{ cm}$$

Combinaing we get:

$$\frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2 \cdot \left(N_1^2 + N_2^2\right)}}$$

And since MC cannot figure this out, we finish it by hand.

Given.

$W := 12 \quad Y := 6$

$X := 10 \quad Z := 8$

Unit.

$AE := 1$

091395C2

Given AE, CK, AC what is GH?

Descriptions.

$$CK := \frac{Y}{Z} \qquad AC := \frac{W}{X} \qquad CM := \frac{CK^2}{AC}$$

$$AB := \frac{AC \cdot AE}{AC + CM} \qquad BC := AC - AB$$

$$GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2}$$

$AB = 0.719101$

$GK = 0.66875 \qquad GH = 0.334375$

Definitions.

$$CK - \frac{Y}{Z} = 0 \qquad AC - \frac{W}{X} = 0$$

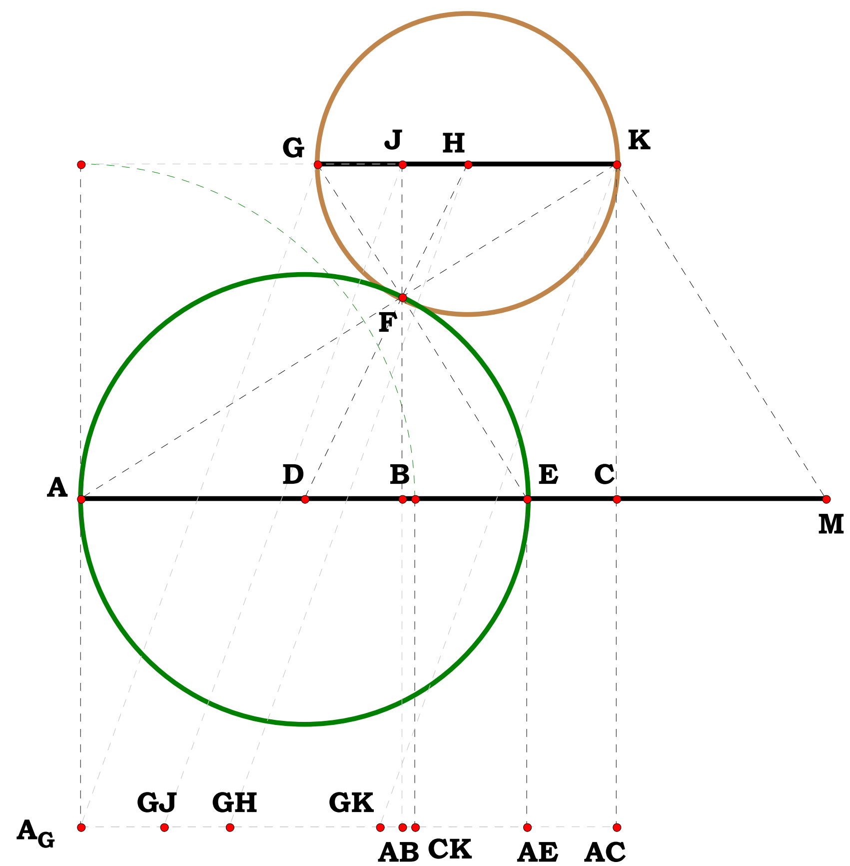
$$CM - \frac{X \cdot Y^2}{W \cdot Z^2} = 0 \qquad AB - \frac{W^2 \cdot Z^2}{W^2 \cdot Z^2 + X^2 \cdot Y^2} = 0$$

$$BC := \frac{W \cdot \left(W^2 \cdot Z^2 - W \cdot X \cdot Z^2 + X^2 \cdot Y^2\right)}{X \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)}$$

$$GK - \frac{W^2 \cdot Z^2 - W \cdot X \cdot Z^2 + X^2 \cdot Y^2}{W \cdot X \cdot Z^2} = 0$$

$$GH - \frac{W^2 \cdot Z^2 - W \cdot X \cdot Z^2 + X^2 \cdot Y^2}{2 \cdot W \cdot X \cdot Z^2} = 0$$

A Study In Placement



Unit = 1.00000

W/X = 1.20000

W = 12.00000

X = 10.00000

Y/Z = 0.75000

Y = 6.00000

Z = 8.00000

A_G = 0.00000

GJ = 0.18785

GH = 0.33438

GK = 0.66875

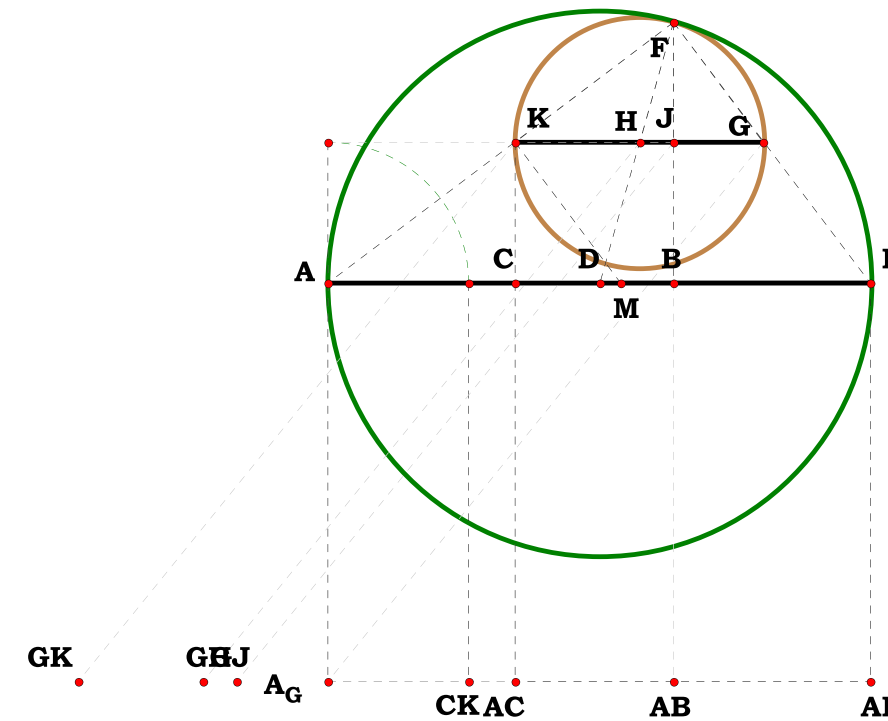
AB = 0.71910

CK = 0.75000

AE = 1.00000

AC = 1.20000

$$\frac{(W^2 \cdot Z^2 - W \cdot X \cdot Z^2) + X^2 \cdot Y^2}{2 \cdot W \cdot X \cdot Z^2} - GH = 0.00000$$



Unit = 1.00000

W/X = 0.34412

W = 3.44118

X = 10.00000

Y/Z = 0.26022

Y = 2.08175

Z = 8.00000

A_G = 0.00000

GJ = -0.16702

GH = -0.22955

GK = -0.45911

AB = 0.63620

CK = 0.26022

AE = 1.00000

AC = 0.34412

$$\frac{(W^2 \cdot Z^2 - W \cdot X \cdot Z^2) + X^2 \cdot Y^2}{2 \cdot W \cdot X \cdot Z^2} - GH = 0.00000$$

101495A

Descriptions.

$AF := N_1 \quad AK := N_2 \quad BF := AF - AB$

$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$DJ := BO \quad AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

$\sqrt{AB \cdot AF} - AC = 0$

Definitions.

$AC - \sqrt{N_1} = 0$

Unit.

$AB := 1$

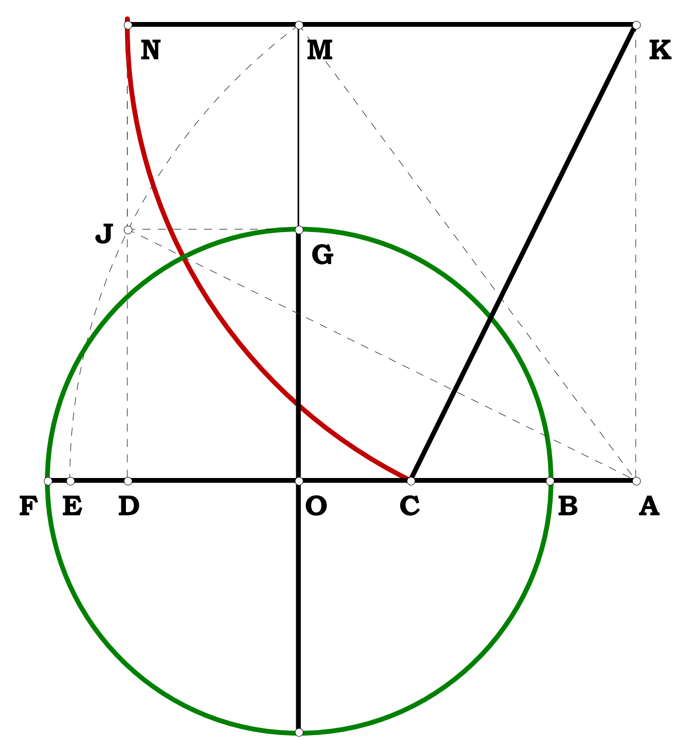
Given.

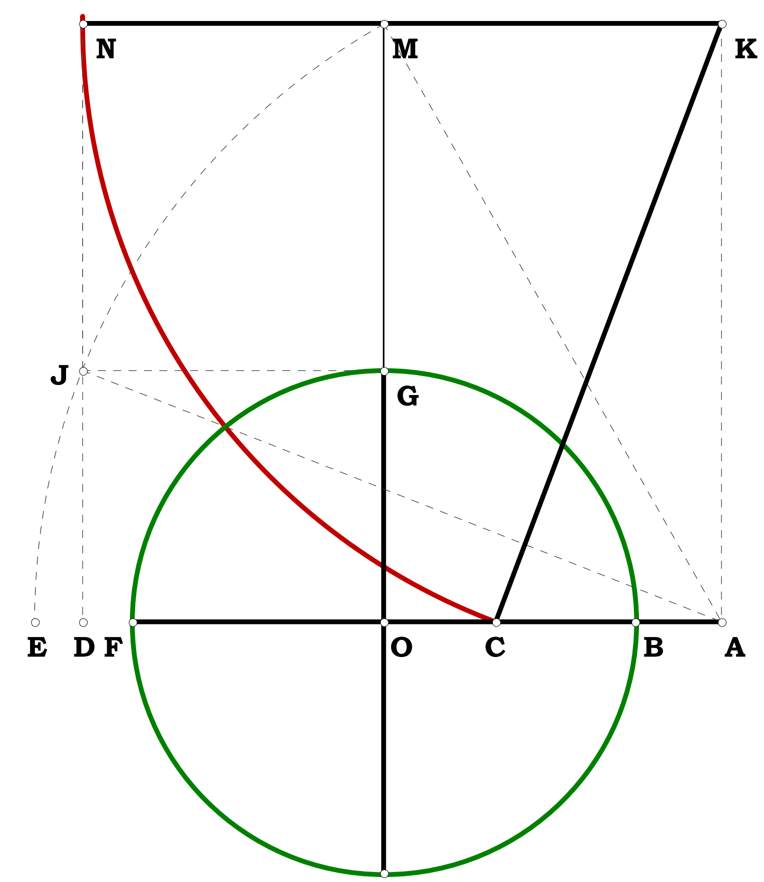
$N_1 := 5$

$N_2 := 4$

Alternate Method Square Root

For any AK is AC the root of AB x AF?





101495B

Given.

$W := 5$ $\quad Y := 6$

$X := 20$ $\quad Z := 6$

Unit.

$BF := 1$

Descriptions.

$AB := \frac{W}{X} \qquad AF := AB + 1 \qquad AK := \frac{Y}{Z}$

$BO := \frac{BF}{2} \qquad AO := AB + BO$

$KM := AO \qquad AM := \sqrt{AK^2 + KM^2}$

$DJ := BO \qquad AJ := AM$

$AD := \sqrt{AJ^2 - DJ^2} \qquad CK := AD$

$AC := \sqrt{CK^2 - AK^2} \qquad \sqrt{AB \cdot AF} - AC = 0$

Definitions.

$AB - \frac{W}{X} = 0 \qquad AF - \frac{W + X}{X} = 0 \qquad AK - \frac{Y}{Z} = 0$

$BO - \frac{1}{2} = 0 \qquad AO - \frac{2 \cdot W + X}{2 \cdot X} = 0 \qquad KM - \frac{2 \cdot W + X}{2 \cdot X} = 0$

$$AM - \frac{\sqrt{4 \cdot W \cdot Z^2 \cdot (W + X) + X^2 \cdot \left(4 \cdot Y^2 + Z^2\right)}}{2 \cdot X \cdot Z} = 0$$

$$DJ - \frac{1}{2} = 0 \qquad AJ - \frac{\sqrt{4 \cdot W \cdot Z^2 \cdot (W + X) + X^2 \cdot \left(4 \cdot Y^2 + Z^2\right)}}{2 \cdot X \cdot Z} = 0$$

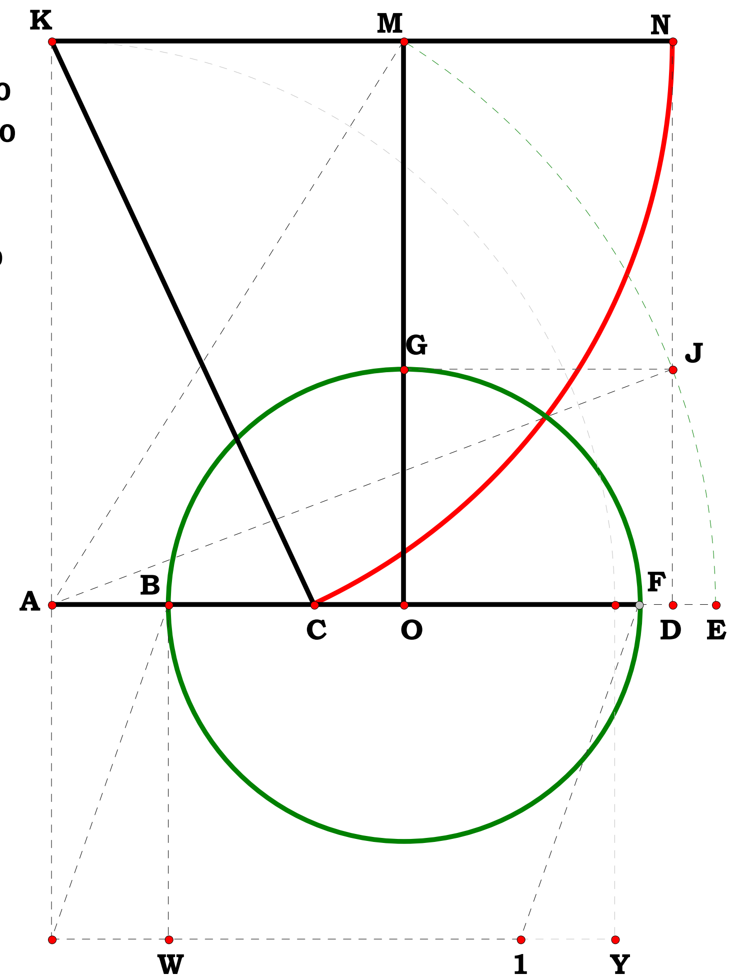
$$AD - \frac{\sqrt{W \cdot Z^2 \cdot (W + X) + X^2 \cdot Y^2}}{X \cdot Z} = 0 \qquad CK - \frac{\sqrt{W \cdot Z^2 \cdot (W + X) + X^2 \cdot Y^2}}{X \cdot Z} = 0$$

$$AC := \frac{\sqrt{W \cdot (W + X)}}{X} \qquad \frac{W \cdot (W + X)}{X} = 6.25$$

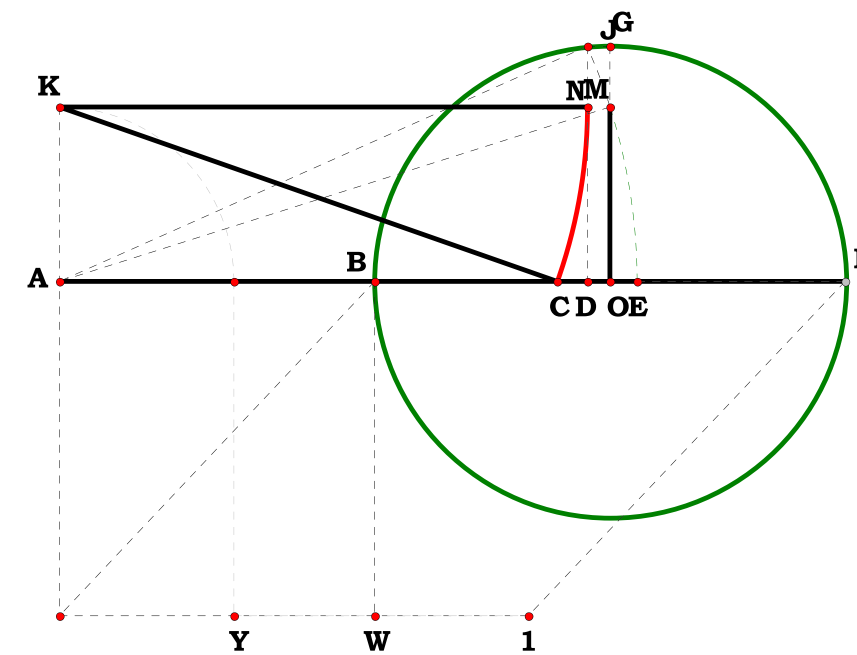
Alternate Method Square Root

For any AK is AC the root of AB x AF?

Unit = 1.00000
W/X = 0.25000
W = 5.00000
X = 20.00000
Y/Z = 1.20000
Y = 6.00000
Z = 5.00000
A = 0.00000
AB = 0.25000
AC = 0.55902
AO = 0.75000
AF = 1.25000
AD = 1.32382
AE = 1.41510



Unit = 1.00000
W/X = 0.67286
W = 13.45712
X = 20.00000
Y/Z = 0.37134
Y = 1.85669
Z = 5.00000
A = 0.00000
AB = 0.67286
AC = 1.06094
AO = 1.17286
AF = 1.67286
AD = 1.12405
AE = 1.23024



$\frac{\sqrt{W \cdot (W+X)}}{X}$-AC = 0.00000

Y and Z have wholly disappeared out of the results, therefore, it does not make any difference what value given, as long as some value is given. It is structures such as this, equations such as this, which proves that the perceptible is not guaranteed by the grammatical result of a single grammar, in short, it does not creat a thing. One must pair a logic with an analogic.

Unit.

$AE := 1$

Given.

$N_1 := 2$

102095A

Descriptions.

$$AB := \frac{AE}{N_1} \qquad BE := AE - AB \qquad BF := \sqrt{AB \cdot BE} \qquad AF := \sqrt{AB^2 + BF^2}$$

$$AC := AF \qquad CE := AE - AC \qquad EG := CE \qquad DE := \frac{EG^2}{AE}$$

$$BD := AE - (AB + DE) \qquad \frac{BD^2}{4 \cdot (AB \cdot DE)} = 1 \qquad AD := AE - DE \qquad DG := \sqrt{AD \cdot DE}$$

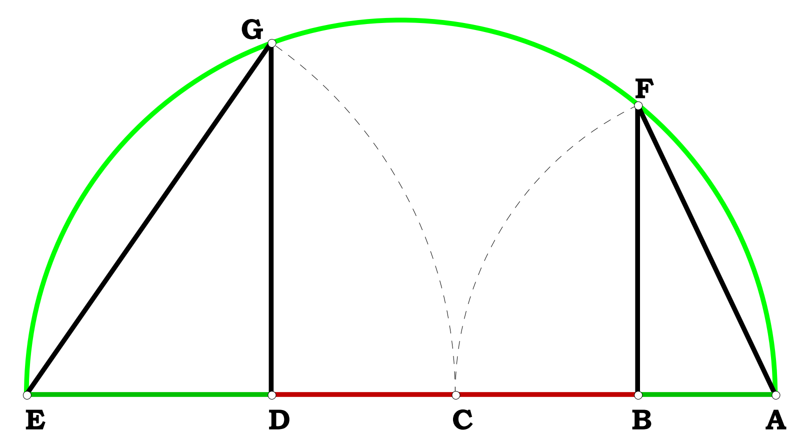
Definitions.

$$\frac{1}{N_1} - AB = 0 \qquad 1 - \frac{1}{N_1} - BE = 0 \qquad \sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0 \qquad \sqrt{\frac{N_1}{N_1^{\,2}}} - AF = 0$$

$$1 - \sqrt{\frac{N_1}{N_1^{\,2}}} - CE = 0 \qquad 1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0 \qquad 2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$$

$$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0 \qquad \frac{\sqrt{4 \cdot \sqrt{N_1} - 5 \cdot N_1 + 2 \cdot N_1^{\frac{3}{2}} - 1}}{N_1} - DG = 0$$

Four Times The Square



Given.

$X := 4$

$Y := 20$

Unit.

$AE := 1$

102095B

Descriptions.

$AB := \frac{X}{Y} \qquad BE := AE - AB \qquad BF := \sqrt{AB \cdot BE} \qquad AF := \sqrt{AB^2 + BF^2}$

$AC := AF \qquad CE := AE - AC \qquad EG := CE \qquad DE := \frac{EG^2}{AE}$

$BD := AE - (AB + DE) \qquad AD := AE - DE \qquad DG := \sqrt{AD \cdot DE}$

$\frac{BD^2}{AB \cdot DE} = 4 \qquad \frac{BD^2}{4 \cdot (AB \cdot DE)} = 1$

$AF = 0.447214 \qquad AD = 0.694427$

Definitions.

$AB - \frac{X}{Y} = 0 \qquad BE - \frac{Y - X}{Y} = 0 \qquad BF - \frac{\sqrt{X \cdot (Y - X)}}{Y} = 0$

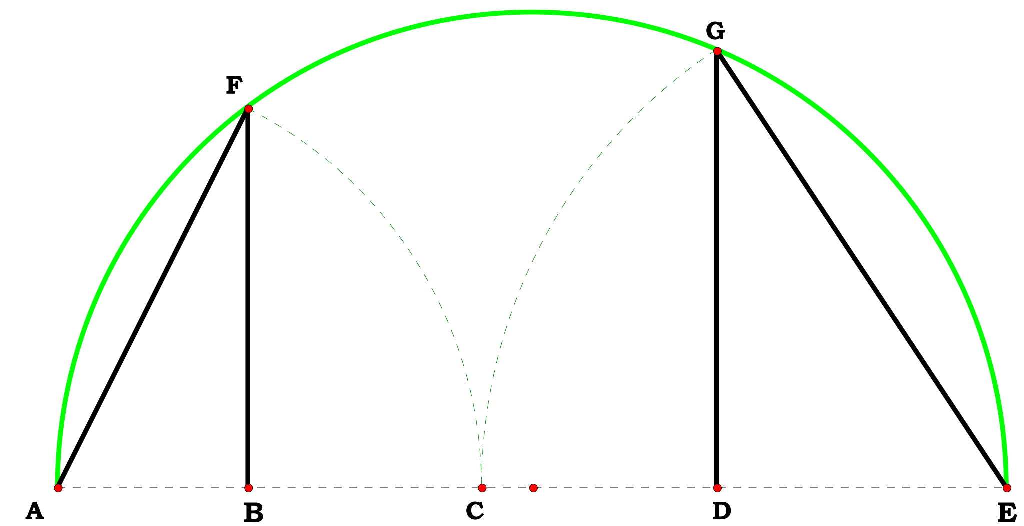
$AF - \frac{\sqrt{X}}{\sqrt{Y}} = 0 \qquad AC - \frac{\sqrt{X}}{\sqrt{Y}} = 0 \qquad CE - \frac{\sqrt{Y} - \sqrt{X}}{\sqrt{Y}} = 0$

$EG - \frac{\sqrt{Y} - \sqrt{X}}{\sqrt{Y}} = 0 \qquad DE - \frac{\left(\sqrt{Y} - \sqrt{X}\right)^2}{Y} = 0$

$BD - \frac{2 \cdot \sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y} = 0 \qquad AD - \frac{\sqrt{X} \cdot \left(2 \cdot \sqrt{Y} - \sqrt{X}\right)}{Y} = 0$

$DG - \frac{X^{\frac{1}{4}} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \sqrt{2 \cdot \sqrt{Y} - \sqrt{X}}}{Y} = 0$

Four Times The Square



Unit = 1.00000 AB = 0.20000
XY = 0.20000 AC = 0.44721
X = 4.00000 AD = 0.69443
Y = 20.00000

Unit.

$AG := 1$

Given.

$N_1 := 5$

$N_2 := 2$

110195A

Descriptions.

$AE := \frac{AG}{2}$ $\quad EG := AE$ $\quad EF := \frac{AG}{2 \cdot N_1}$ $\quad AF := AE + EF$

$FG := EG - EF$ $\quad FN := \sqrt{AF \cdot FG}$ $\quad GN := \sqrt{FN^2 + FG^2}$ $\quad GK := GN$

$EK := \sqrt{GK^2 - EG^2}$ $\quad EO := \frac{EG \cdot EF}{EK}$ $\quad OK := EO + EK$ $\quad DE := \frac{AE}{N_2}$

$DO := \sqrt{DE^2 + EO^2}$ $\quad DJ := OK - DO$ $\quad CD := \frac{DE \cdot DJ}{DO}$ $\quad CE := CD + DE$

$CJ := \frac{EO \cdot DJ}{DO}$ $\quad AC := AE - CE$ $\quad AJ := \sqrt{AC^2 + CJ^2}$ $\quad AL := AJ$ $\quad AB := \frac{AL^2}{AG}$

$CG := AG - AC$ $\quad GJ := \sqrt{CG^2 + CJ^2}$ $\quad GM := GJ$ $\quad DG := \frac{GM^2}{AG}$

$BD := AG - (AB + DG)$

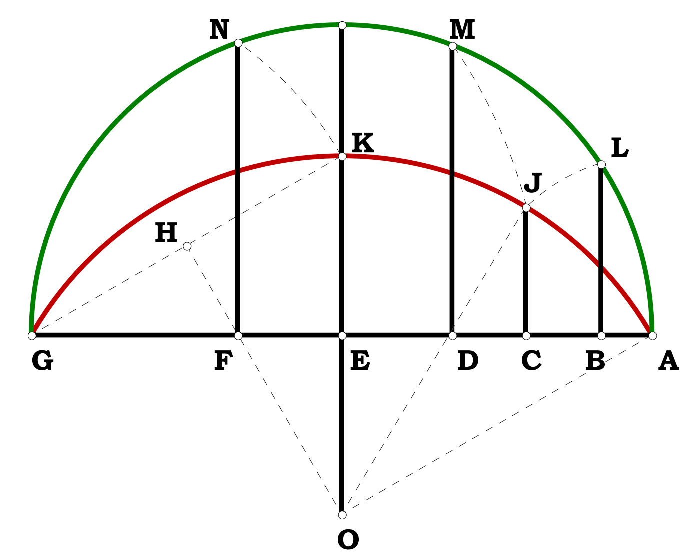
Definitions.

$$\frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$$

A Modification Of The Square Root Figure, Gemini Roots

On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.

Although the final equation is wholly correct, the write-up, itself, is pure garbage. I remember doing this plate right once, but I no longer seem to have that write-up, so I did it again in the plate B. This errant write-up goes back at least as far as 2001. When I started to examine it, the faults it contains shortly after a few equations puzzles me. Maybe that is what happens when you work 12 hours a day, seven days a week for a long time. Well, I often get equations before I write them up, but rarely do so badly on the write-up as this is. Or, the write-up could be the result of a complete lapse in sanity by yours truly. At any rate, it stops being correct after the definition of EO. After EO, it makes no sense at all. Not even on the graphic for this is D a mobile point!. The working point is C, not D.



110195B

Descriptions.

Given.

$W := 15 \quad Y := 14$

$X := 20 \quad Z := 20$

Unit.

$AG := 1$

$$AE := \frac{AG}{2} \quad EF := \frac{Y}{4 \cdot Z} \quad AF := AE - EF \quad AB := \frac{W}{X}$$

$$FG := AE + EF \quad FN := \sqrt{AF \cdot FG} \quad AN := \sqrt{FN^2 + AF^2}$$

$$AK := AN \quad EK := \sqrt{AK^2 - AE^2} \quad EO := \frac{AE \cdot EF}{EK}$$

$$OK := EO + EK \quad BG := AG - AB$$

$$PQ := 2 \cdot OK \quad SQ := \frac{PQ - AG}{2} \quad RJ := \sqrt{(AB + SQ) \cdot [PQ - (AB + SQ)]}$$

$$BJ := RJ - EO \quad AJ := \sqrt{AB^2 + BJ^2} \quad AM := AJ \quad AC := \frac{AJ^2}{AG}$$

$$BC := AB - AC \quad AD := AB + BC \quad CD := 2 \cdot BC \quad DG := AG - AD$$

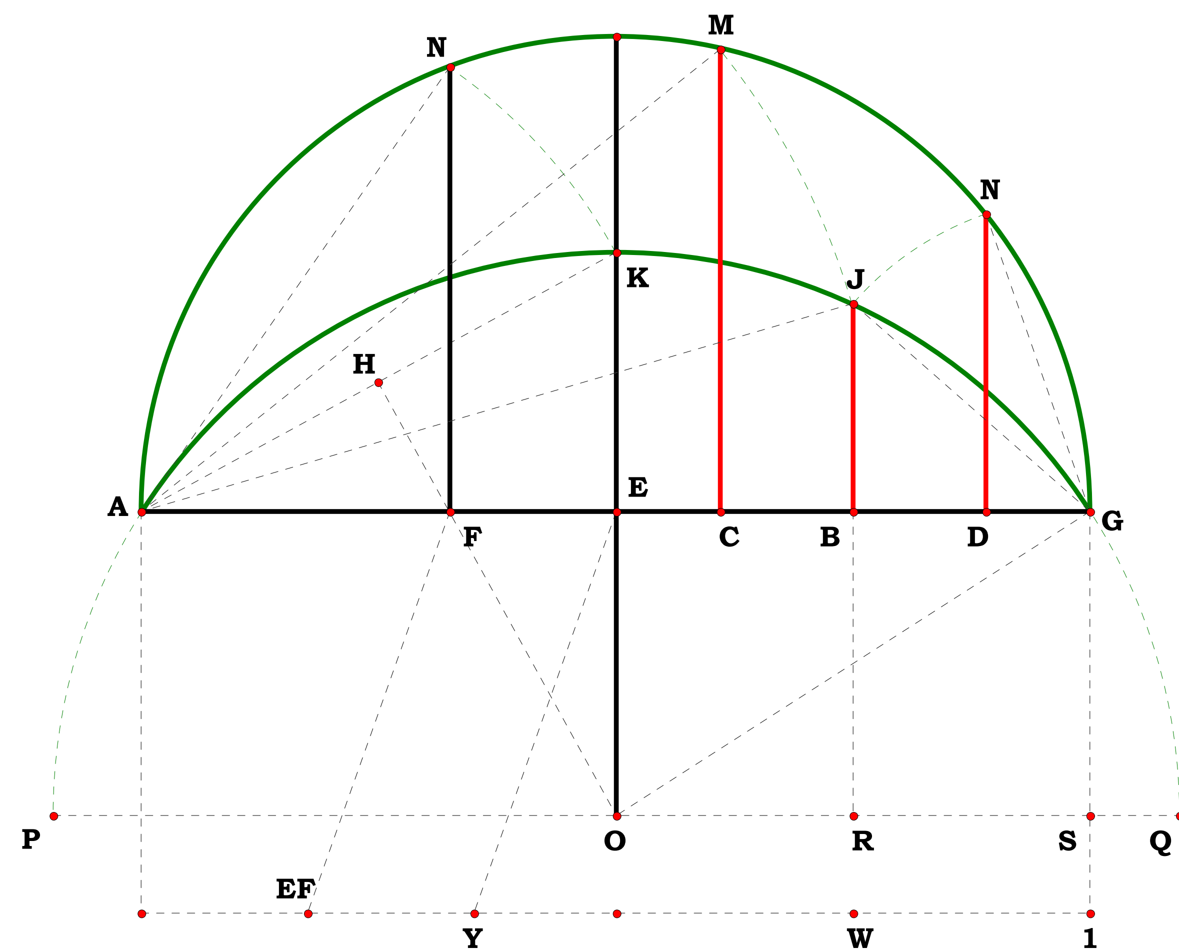
$$N := \frac{AG}{2 \cdot EF} \quad N = 2.857143 \quad \frac{\sqrt{AC \cdot DG}}{CD} = 0.928571$$

$$\frac{N-1}{2} = 0.928571 \quad \frac{N-1}{2} - \frac{\sqrt{AC \cdot DG}}{CD} = 0$$

$$\frac{1 - 2 \cdot EF}{4 \cdot EF} - \frac{\sqrt{AC \cdot DG}}{CD} = 0$$

A Modification Of The Square Root Figure, Gemini Roots

On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.



Unit = 1.00000	A = 0.00000
W/X = 0.75000	AF = 0.32500
W = 15.00000	AE = 0.50000
X = 20.00000	AG = 1.00000
Y/Z = 0.70000	EF = 0.17500
Y = 14.00000	AC = 0.61030
Z = 20.00000	AB = 0.75000
	AD = 0.88970

$$\frac{1\text{-}2 \cdot EF}{4 \cdot EF} - \frac{\sqrt{AC \cdot (AG\text{-}AD)}}{AD\text{-}AC} = 0.00000$$

Definitions.

$$AE-\frac{1}{2}=0 \quad EF-\frac{Y}{4\cdot Z}=0 \quad AF-\frac{(2\cdot Z-Y)}{4\cdot Z}=0 \quad AB-\frac{W}{X}=0 \quad FG-\frac{Y+2\cdot Z}{4\cdot Z}=0 \quad FN-\frac{\sqrt{4\cdot Z^2-Y^2}}{4\cdot Z}=0 \quad AN-\frac{\sqrt{(2\cdot Z-Y)}}{2\cdot\sqrt{Z}}=0 \quad AK-\frac{\sqrt{(2\cdot Z-Y)}}{2\cdot\sqrt{Z}}=0 \quad EK-\frac{\sqrt{(Z-Y)}}{\sqrt{4\cdot Z}}=0$$

$$EO-\frac{Y}{4\cdot\sqrt{Z}\cdot\sqrt{Z-Y}}=0 \quad OK-\frac{2\cdot Z-Y}{4\cdot\sqrt{Z}\cdot\sqrt{Z-Y}}=0 \quad BG-\frac{X-W}{X}=0 \quad PQ-\frac{2\cdot Z-Y}{2\cdot\sqrt{Z}\cdot\sqrt{Z-Y}}=0 \quad SQ-\frac{2\cdot Z-Y-2\cdot\sqrt{Z}\cdot\sqrt{Z-Y}}{4\cdot\sqrt{Z}\cdot\sqrt{Z-Y}}=0$$

$$RJ-\frac{\sqrt{\left\lfloor\left\lfloor 2\cdot\sqrt{Z}\cdot(X-2\cdot W)\cdot\sqrt{Z-Y}+X\cdot(Y-2\cdot Z)\right\rfloor\cdot\left\lfloor 2\cdot\sqrt{Z}\cdot(2\cdot W-X)\cdot\sqrt{Z-Y}+X\cdot(Y-2\cdot Z)\right\rfloor\right\rfloor}}{4\cdot X\cdot\sqrt{(Z-Y)\cdot Z}}=0 \quad BJ-\frac{\sqrt{Z}\cdot\sqrt{Z-Y}\cdot\sqrt{16\cdot W^2\cdot Z\cdot(Y-Z)-16\cdot W\cdot X\cdot Z\cdot(Y-Z)+X^2\cdot Y^2}-X\cdot Y\cdot\sqrt{Z^2-Y\cdot Z}}{4\cdot X\cdot\sqrt{Z}\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}}=0$$

$$AJ-\frac{\sqrt{\sqrt{Z}\cdot(Y-Z)\cdot\left(8\cdot W\cdot Y\cdot Z-X\cdot Y^2-8\cdot W\cdot Z^2\right)-Y\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W^2\cdot Z\cdot(Y-Z)-16\cdot W\cdot X\cdot Z\cdot(Y-Z)+X^2\cdot Y^2}}}{\sqrt{8\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2}}=0$$

$$AM-\frac{\sqrt{\sqrt{Z}\cdot(Y-Z)\cdot\left(8\cdot W\cdot Y\cdot Z-X\cdot Y^2-8\cdot W\cdot Z^2\right)-Y\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W^2\cdot Z\cdot(Y-Z)-16\cdot W\cdot X\cdot Z\cdot(Y-Z)+X^2\cdot Y^2}}}{\sqrt{8\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2}}=0$$

$$AC-\frac{8\cdot\left(\sqrt{Z}\right)^3\cdot W\cdot(Y-Z)^2-\sqrt{Z}\cdot X\cdot Y^2\cdot(Y-Z)-Y\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{X^2\cdot Y^2+16\cdot W\cdot Z\cdot(Y-Z)\cdot(W-X)}}{8\cdot X\cdot Z^{\frac{3}{2}}\cdot(Y-Z)^2}=0$$

$$BC-\frac{Y\cdot\left(\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W^2\cdot Y\cdot Z-16\cdot W^2\cdot Z^2-16\cdot W\cdot X\cdot Y\cdot Z+16\cdot W\cdot X\cdot Z^2+X^2\cdot Y^2}-X\cdot Y\cdot Z^{\frac{3}{2}}+X\cdot Y^2\cdot\sqrt{Z}\right)}{8\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2}=0$$

$$AD-\frac{\sqrt{Z}\cdot(Y-Z)\cdot\left(X\cdot Y^2+8\cdot W\cdot Y\cdot Z-8\cdot W\cdot Z^2\right)+Y\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W^2\cdot Z\cdot(Y-Z)-16\cdot W\cdot X\cdot Z\cdot(Y-Z)+X^2\cdot Y^2}}{8\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2}=0$$

$$CD - \frac{Y\cdot\left(\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W^2\cdot Y\cdot Z-16\cdot W^2\cdot Z^2-16\cdot W\cdot X\cdot Y\cdot Z+16\cdot W\cdot X\cdot Z^2+X^2\cdot Y^2}-X\cdot Y\cdot Z^{\frac{3}{2}}+X\cdot Y^2\cdot\sqrt{Z}\right)}{4\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2} = 0$$

$$DG - \frac{\sqrt{Z}\cdot(Y-Z)\cdot\left[8\cdot\left(W\cdot Z^2+X\cdot Y\cdot Z\right)-\left[X\cdot Y^2+8\cdot Z\cdot(W\cdot Y+X\cdot Z)\right]\right]-Y\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W\cdot Z\cdot(Y-Z)\cdot(W-X)+X^2\cdot Y^2}}{8\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2} = 0$$

This is all a bit out of hand, maybe?

$$N := \frac{AG}{2\cdot EF} \qquad N = 2.857143$$

$$\frac{N-1}{2} = 0.928571 \qquad \frac{N-1}{2} - \frac{\sqrt{AC\cdot DG}}{CD} = 0 \qquad \frac{\sqrt{AC\cdot DG}}{CD} = 0.928571$$

110595A

Descriptions.

Unit.

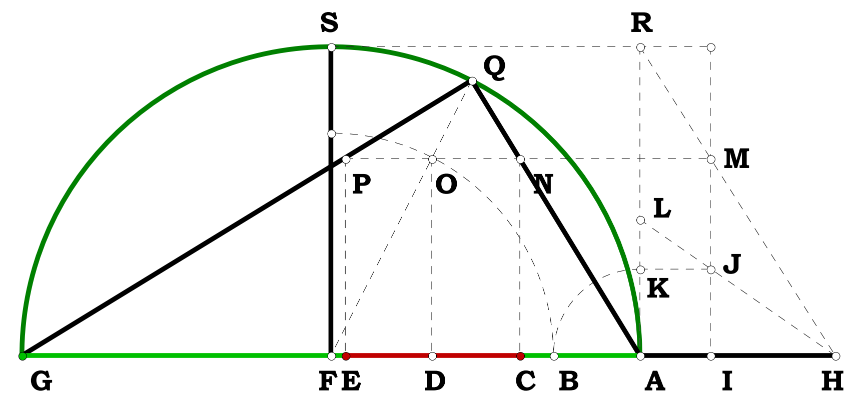
$AG := 1$

Given.

$N_1 := 3$

$N_2 := 5$

Alternate Method Gemini Roots



$AF := \frac{AG}{2}$ $\quad AR := AF$ $\quad FQ := AF$ $\quad FG := AF$

$AL := \frac{AR}{N_1}$ $\quad IM := \frac{AR}{N_2}$ $\quad AK := \frac{AL \cdot IM}{AR}$ $\quad DO := IM$

$AB := AK$ $\quad BF := AF - AB$ $\quad FO := BF$ $\quad OQ := FQ - FO$

$NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO}$ $\quad NP - 2 \cdot AK = 0$ $\quad CD := AK$

$DE := AK$ $\quad DF := \sqrt{FO^2 - DO^2}$ $\quad AD := AF - DF$

$AC := AD - CD$ $\quad EG := FG + DF - DE$ $\quad CE := NP$

$$\frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$$

Definitions.

$AF - \frac{1}{2} = 0$ $\quad AR - \frac{1}{2} = 0$ $\quad FQ - \frac{1}{2} = 0$ $\quad FG - \frac{1}{2} = 0$ $\quad AL - \frac{1}{2 \cdot N_1} = 0$ $\quad IM - \frac{1}{2 \cdot N_2} = 0$ $\quad DO - \frac{1}{2 \cdot N_2} = 0$

$AK - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad AB - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad CD - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad DE - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$

$BF - \frac{N_1 \cdot N_2 - 1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad FO - \frac{N_1 \cdot N_2 - 1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad OQ - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad NP - \frac{1}{N_1 \cdot N_2} = 0$ $\quad CE - \frac{1}{N_1 \cdot N_2} = 0$

$DF - \frac{\sqrt{(N_1 \cdot N_2 - N_1 - 1) \cdot (N_1 + N_1 \cdot N_2 - 1)}}{2 \cdot N_1 \cdot N_2} = 0$ $\quad AD - \frac{N_1 \cdot N_2 - \sqrt{N_1^2 \cdot N_2^2 - N_1^2 - 2 \cdot N_1 \cdot N_2 + 1}}{2 \cdot N_1 \cdot N_2} = 0$ $\quad AC - \frac{N_1 \cdot N_2 - \sqrt{N_1^2 \cdot N_2^2 - N_1^2 - 2 \cdot N_1 \cdot N_2 + 1} - 1}{2 \cdot N_1 \cdot N_2} = 0$

$EG - \frac{N_1 \cdot N_2 + \sqrt{N_1^2 \cdot N_2^2 - N_1^2 - 2 \cdot N_1 \cdot N_2 + 1} - 1}{2 \cdot N_1 \cdot N_2} = 0$ $\quad \frac{N_1}{2} = 1.5$ $\quad \frac{\sqrt{AC \cdot EG}}{CE} = 1.5$

110595B

Unit.

$AB := 1$

Given.

$W := 8 \qquad Y := 13$

$X := 20 \qquad Z := 20$

Descriptions.

$BC := AB \quad AP := AB \quad RT := AB \quad BM := AB$

$AC := 2 \cdot AB \quad AN := \frac{W}{X} \quad OR := \frac{Y}{Z} \quad RS := \frac{AN \cdot OR}{RT}$

$EH := OR \quad FJ := OR \quad GK := OR$

$BJ := AB - RS \quad JM := RS$

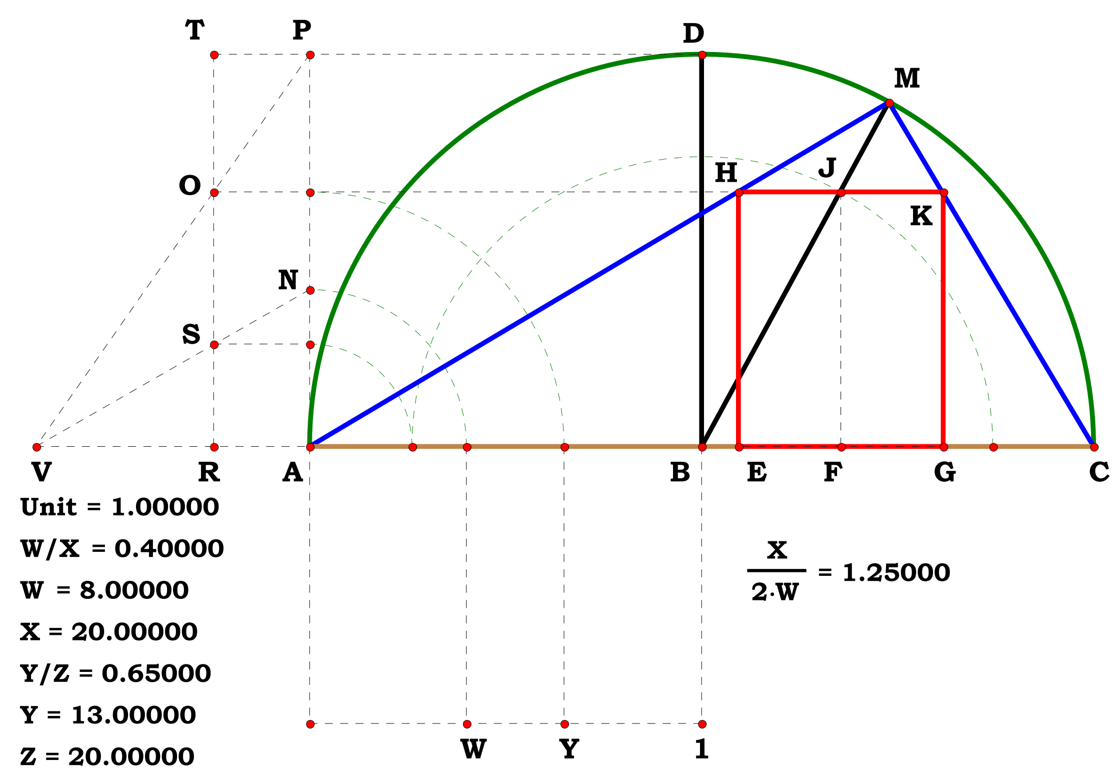
$HK := 2 \cdot JM \quad EG := HK$

$BF := \sqrt{BJ^2 - FJ^2} \quad BG := BF + JM$

$CG := BC - BG \quad AG := AC - CG \quad AE := AG - EG$

$$\frac{AB}{2 \cdot AN} - \frac{\sqrt{AE \cdot CG}}{EG} = 0$$

Alternate Method Gemini Roots



Definitions.

$BC - 1 = 0 \quad AP - 1 = 0 \quad RT - 1 = 0 \quad BM - 1 = 0$

$AC - 2 = 0 \quad AN - \frac{W}{X} = 0 \quad OR - \frac{Y}{Z} = 0 \quad RS - \frac{W \cdot Y}{X \cdot Z} = 0$

$EH - \frac{Y}{Z} = 0 \quad FJ - \frac{Y}{Z} = 0 \quad GK - \frac{Y}{Z} = 0 \quad BJ - \frac{X \cdot Z - W \cdot Y}{X \cdot Z} = 0 \quad JM - \frac{W \cdot Y}{X \cdot Z} = 0 \quad HK - \frac{2 \cdot W \cdot Y}{X \cdot Z} = 0 \quad EG - \frac{2 \cdot W \cdot Y}{X \cdot Z} = 0$

$$BF - \frac{\sqrt{(W \cdot Y - X \cdot Y - X \cdot Z) \cdot (W \cdot Y + X \cdot Y - X \cdot Z)}}{X \cdot Z} = 0$$

$$BG - \frac{\sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2} + W \cdot Y}{X \cdot Z} = 0$$

$$CG - \frac{X \cdot Z - W \cdot Y - \sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2}}{X \cdot Z} = 0$$

$$AG - \frac{\sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2} + W \cdot Y + X \cdot Z}{X \cdot Z} = 0$$

$$AE - \frac{\sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2} - W \cdot Y + X \cdot Z}{X \cdot Z} = 0$$

$$\frac{\sqrt{AE \cdot CG}}{EG} - \frac{X}{2 \cdot W} = 0 \qquad \frac{AB}{2 \cdot AN} - \frac{X}{2 \cdot W} = 0$$

Unit.

$AH := 1$

Given.

$N_1 := 5$

120195A

Descriptions.

$$AE := \frac{AH}{2} \qquad EH := AE \quad EP := AE \qquad AP := \sqrt{2 \cdot AE^2}$$

$$AB := \frac{AE}{N_1} \qquad CE := AB \quad CH := EH + CE \qquad CL := \sqrt{2 \cdot CE^2}$$

$$AM := \frac{CL \cdot AH}{CH} \qquad MP := AP - AM \qquad NP := \frac{EP \cdot MP}{AP}$$

Definitions.

$$\frac{1}{2} \cdot \frac{(N_1 - 1)}{(N_1 + 1)} - NP = 0 \qquad \frac{(N_1 - 1)}{(N_1 + 1)} - 2 \cdot NP = 0$$

$$AE - \frac{1}{2} = 0 \quad EH - \frac{1}{2} = 0 \quad EP - \frac{1}{2} = 0$$

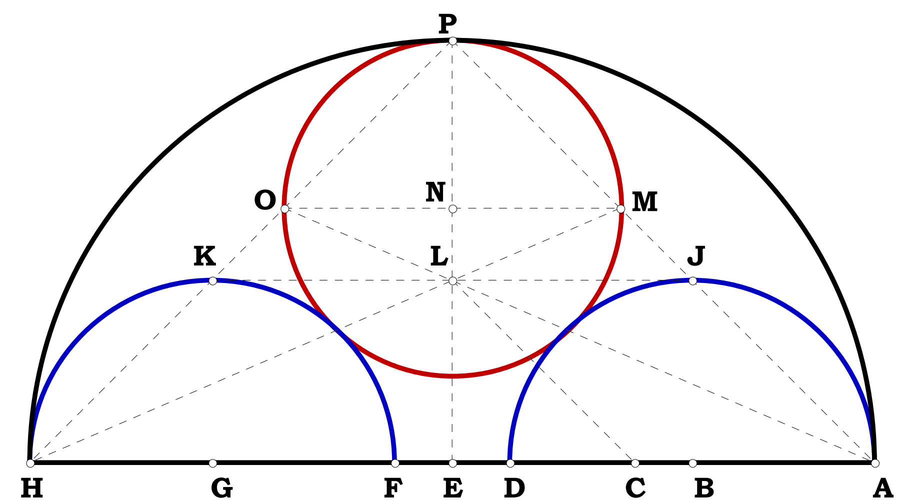
$$AP - \frac{\sqrt{2}}{2} = 0 \qquad AB - \frac{1}{2 \cdot N_1} = 0 \qquad CE - \frac{1}{2 \cdot N_1} = 0$$

$$CH - \frac{N_1 + 1}{2 \cdot N_1} = 0 \qquad CL - \frac{1}{\sqrt{2} \cdot N_1} = 0 \qquad AM - \frac{\sqrt{2}}{N_1 + 1} = 0$$

$$MP - \frac{\sqrt{2} \cdot (N_1 - 1)}{2 \cdot (N_1 + 1)} = 0 \qquad NP - \frac{N_1 - 1}{2 \cdot (N_1 + 1)} = 0$$

Method For Equals

Given AB find NP.



120195B

Given.

$X := 8$

$Y := 20$

Unit.

$AE := \frac{Y}{Y}$

Descriptions.

$AH := 2 \cdot AE \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$

$AB := \frac{X}{Y} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$

$AM := \frac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \frac{EP \cdot MP}{AP}$

Definitions.

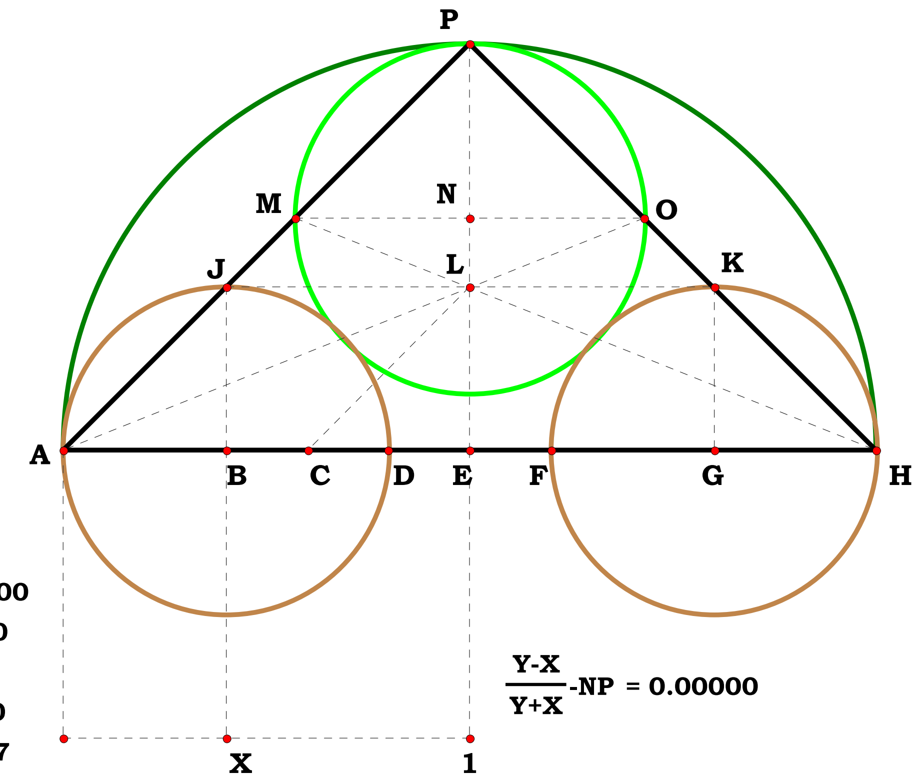
$EH - 1 = 0 \quad EP - 1 = 0 \quad AH - 2 = 0 \quad AP - \sqrt{2} = 0$

$AB - \frac{X}{Y} = 0 \quad CE - \frac{X}{Y} = 0 \quad CH - \frac{X+Y}{Y} = 0 \quad CL - \frac{\sqrt{2} \cdot X}{Y} = 0$

$AM - \frac{2 \cdot \sqrt{2} \cdot X}{X+Y} = 0 \quad MP - \frac{(Y-X) \cdot \sqrt{2}}{X+Y} = 0 \quad NP - \frac{Y-X}{X+Y} = 0$

Method For Equals

Given AB find NP.



Unit = 1.00000
XY = 0.40000
X = 8.00000
Y = 20.00000
NP = 0.42857

$\frac{Y-X}{Y+X}$-NP = 0.00000

27 **Euler's Straight Line**

In all triangles the center of the circumscribed circle, the point of intersection of the medians, and the point of intersection of the altitudes are situated in this order in a straight line—the Euler line—and are spaced in such a manner that the altitude intersection is twice as far from the median intersection as the center of the circumscribed circle is.

Leonhard Euler (1707–1783) was one of the greatest and most fertile mathematicians of all time. His writings comprise 45 volumes and over 700 papers, most of them long ones, published in periodicals.

The above theorem is among the results of the paper "Solutio facilis problematum quorundam geometricorum difficillimorum," which appeared in the journal *Novi commentarii Academiae Petropolitanae* (*ad annum 1765*).

The following proof of the Euler theorem is distinguished by its great simplicity.

In the triangle ABC let M be the midpoint of side AB, S the median intersection, which lies on CM, so that

$$SC = 2 \cdot SM, \tag{1}$$

and U the center of the circle of circumscription, lying on the perpendicular bisector of AB.

We extend US by SO so that

$$SO = 2 \cdot SU, \tag{2}$$

and join O to C.

According to (1) and (2) the triangles MUS and COS are similar. Consequently, $CO \parallel MU$, i.e., $CO \perp AB$, or expressed verbally, the line connecting the point O with a vertex of the triangle is perpendicular to the side of the triangle opposite the vertex; consequently, the connecting line is an altitude of the triangle.

The three altitudes consequently pass through point 0. This is, therefore, the altitude intersection, and Euler's theorem is proved.

NOTE. Our proof contains at the same time the solution to the interesting

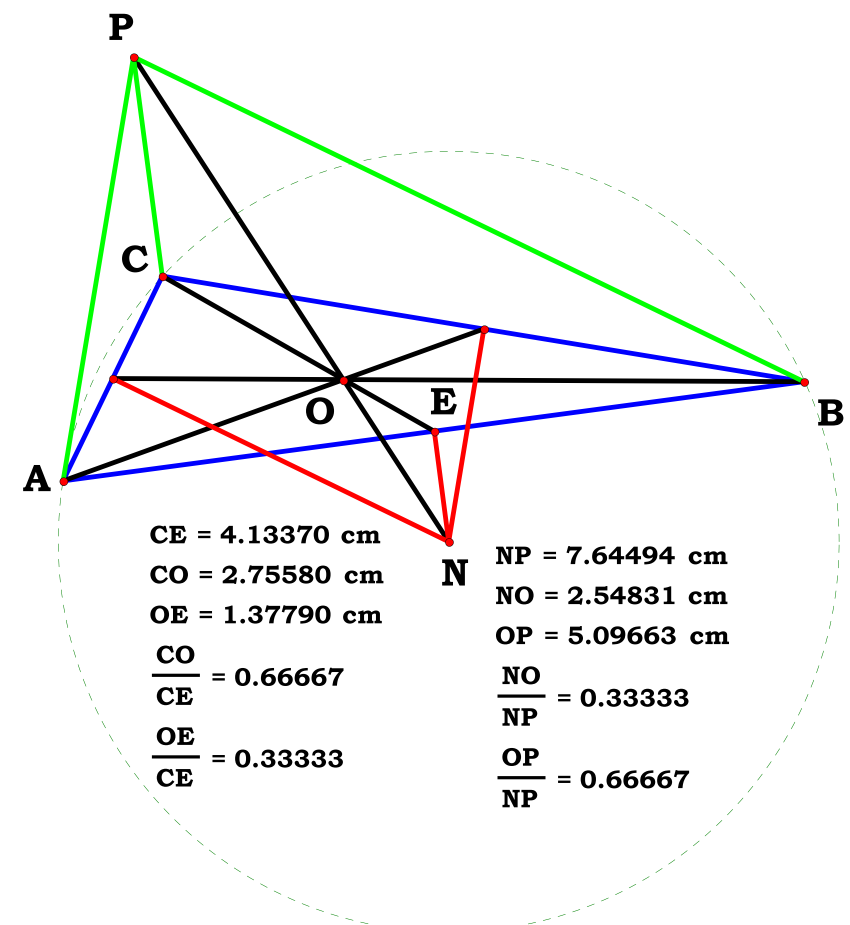
Euler's Straight Line

120795A

Dover republished this book in 1965. The only time I ever drew it up I was still using TommyCad. The graphic, on the following page, is not even appropriate for the figure. The proof is not valid at all. In a proof, one has to have their Lets, only be the given three lines of a triangle.

Euler's Straight Line
In all triangles the center of the circumscribed circle, the point of intersection of the medians, and the point of intersection of the altitudes are situated in this order in a straight line-the Euler line-and are spaced in such a manner that the altitude intersection is twice as far from the median intersection as the center of the circumscribed circle is.

So, what is the so called proposition, which is not labled as such, try to say? What does the above statement mean?



Let us take any triangle whatsoever and construct three points with it. One is the center of the circle which circumscribes it, point N, the second is perpendicular of each point to the segment opposite to it. These three will also converge at what is being called an altitude, point P. Next let us take the midpoint of each segment and form a segment with the opposite vertex point. These three will also construct one point, O. These three points will be collinear, and if we take NP for the unit, ON will be 1/3rd of it, while OP the other 2/3rds.

This plate in my releases has never been drawn up by me in Sketchpad. It is still the origional grapic I did in TommyCad. So, in this revision I will correct that issue.

The real problem that I see, is that every median is also cut in the same ratio, no different than the so called Euler's line as if Euler had anything what so ever to do than connent the dots. which is hardly a great feat for any so called mathematician, or anyone playing dots to begin with. The most obsurd thing which ego does for someone is incite them to simply rename a name in a particular grammar.
If this is Euler's straight line, are the the rest of his lines crooked?

120795AR

Descriptions.

Unit.

Given.

$\delta := 0 .. 2$

$Side\_1 := 2$

$Side\_2 := 3$

$Side\_3 := 4$

Scholia: ***100 Great Problems of Elementary Mathematics*** **H. Dorrie Problem 27 Euler's Line**

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$$AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$$

$$TRIANGLE := (Side\_1 + Side\_2 > Side\_3) \cdot (Side\_1 + Side\_3 > Side\_2) \cdot (Side\_2 + Side\_3 > Side\_1) \quad TRIANGLE = 1$$

$$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta \quad Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta}$$

$$Ah_\delta := AB_\delta - Bh_\delta \quad hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2} \quad Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2}$$

$$BE_\delta := AE_\delta \quad Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta} \quad fg_\delta := Bf_\delta - Bg_\delta$$

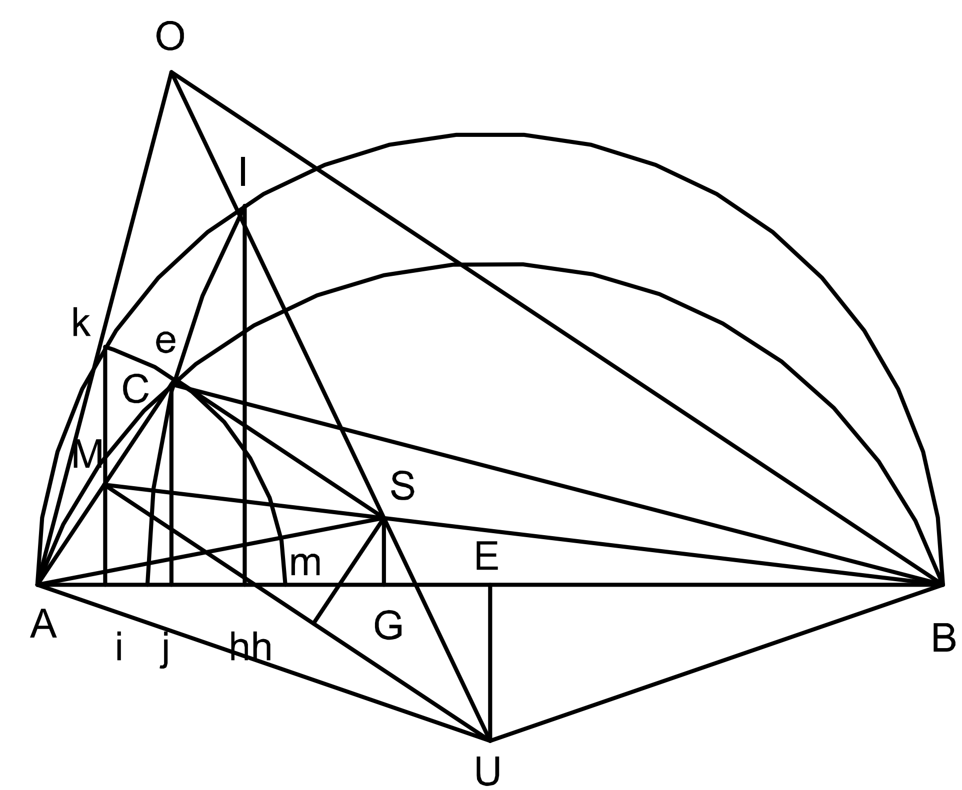
$$Ug_\delta := \text{if}\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right) \quad BU_\delta := \text{if}\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right] \quad AM_\delta := \frac{AC_\delta}{2}$$

$$AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta \quad GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2}$$

$$BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2} \quad BS_\delta := \frac{2 \cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$$

$$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2} \quad MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta$$

$$MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2} \cdot \frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2} \cdot AM_\delta - \frac{1}{2} \cdot \frac{(MS_\delta)^2}{AM_\delta}$$



The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$\mathbf{eM_\delta := Ae_\delta - AM_\delta}$ $\quad \mathbf{Sm_\delta := eM_\delta}$ $\quad \mathbf{Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2}}$ $\quad \mathbf{Mm_\delta := Se_\delta}$

$\mathbf{Um_\delta := if\left[AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta\right]}$ $\quad \mathbf{SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2}}$ $\quad \mathbf{UO_\delta := 3 \cdot SU_\delta}$

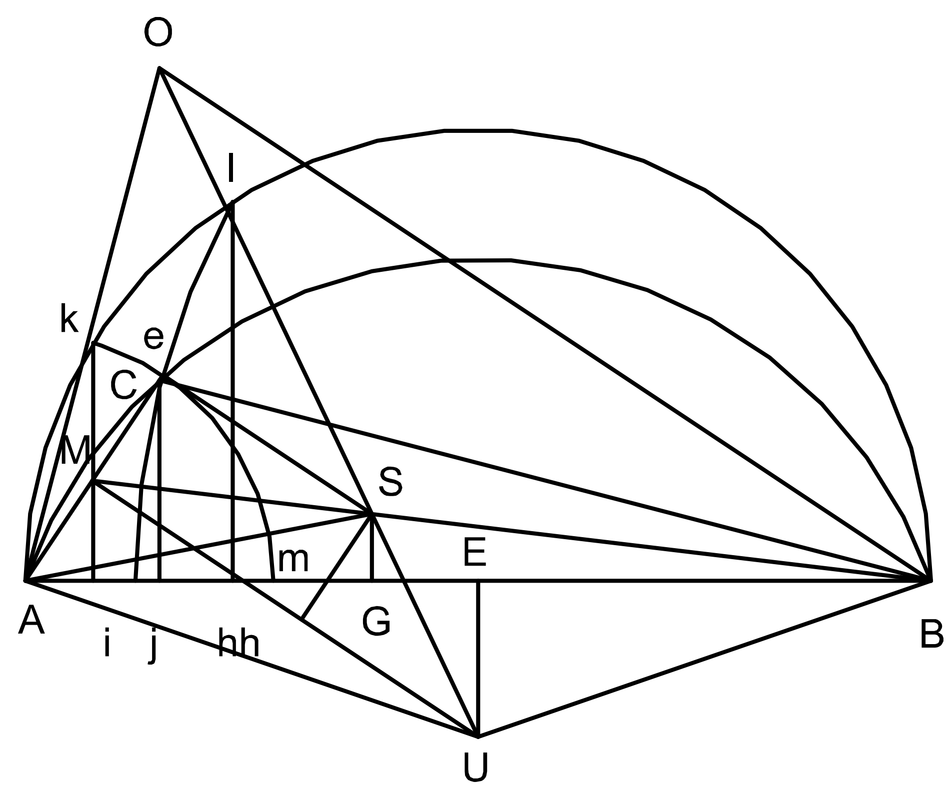
Due to the way in which certain lines lay, the above switch was needed.

Definitions.

Is this a TRIANGLE = 1 ? Now if you are wondering why thumbs up means that things are a go, or okay, think about it.

$\mathbf{SU_\delta} =$	$\mathbf{UO_\delta} =$	$\mathbf{AU_\delta} =$
1.021981	3.065942	2.065591
1.021981	3.065942	2.065591
1.021981	3.065942	2.065591

$\mathbf{\dfrac{SU_\delta}{UO_\delta}} =$
0.333333
0.333333
0.333333



120795B

Unit.

Given.

$AB := 10.49867$

$BC := 9.21723$

$AC := 3.47398$

Euler's Straight Line

Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

Given three sides of a triangle, determine the length of the Euler line.
Work the drawing from each of the sides.

In this write up, I will use equations from previous versions of the DQ. 062793, 010893.

Descriptions.

$$AD := \frac{AB}{2} \qquad AO := \frac{AB \cdot AC \cdot BC}{\sqrt{AB+AC+BC}\cdot\sqrt{AC+BC-AB}\cdot\sqrt{AB-AC+BC}\cdot\sqrt{AB+AC-BC}}$$

$$AJ := \frac{AB^2+AC^2-BC^2}{2\cdot AB} \qquad CD := \frac{\sqrt{2\cdot AC^2-AB^2+2\cdot BC^2}}{2} \qquad AE := \frac{\sqrt{2\cdot AB^2-BC^2+2\cdot AC^2}}{2}$$

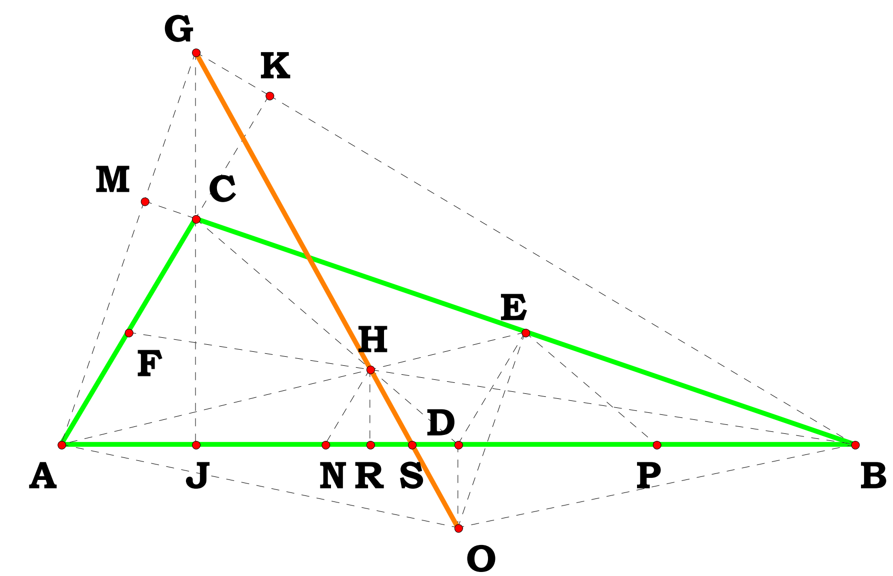
$$BF := \frac{\sqrt{2\cdot BC^2-AC^2+2\cdot AB^2}}{2} \qquad DE := \frac{AC}{2} \qquad DP := \frac{AD}{2} \qquad EP := \frac{CD}{2} \qquad AP := AD+DP$$

$$DH := \frac{EP\cdot AD}{AP} \qquad \frac{CD}{DH}=3 \qquad DN := \frac{AD\cdot DH}{CD} \qquad NR := \frac{AJ\cdot DH}{CD} \qquad BJ := AB-AJ$$

$$CJ := \sqrt{BC^2-BJ^2} \qquad HR := \frac{CJ\cdot DH}{CD} \qquad DR := DN-NR \qquad DO := \sqrt{AO^2-AD^2}$$

$$HO := \sqrt{(HR+DO)^2+DR^2} \qquad AR := AD-DR \qquad JR := AR-AJ \qquad RS := \frac{DR\cdot HR}{DO+HR}$$

$$HS := \frac{HO\cdot HR}{DO+HR} \qquad JS := JR+RS \qquad GS := \frac{HS\cdot JS}{RS} \qquad OS := HO-HS \qquad GO := GS+OS \qquad \frac{GO}{HO}=3$$



AB = 10.49867 cm	EP = 2.28896 cm
AC = 3.47398 cm	DH = 1.52597 cm
BC = 9.21723 cm	CJ = 2.98450 cm
AO = 5.36446 cm	DN = 1.74978 cm
AJ = 1.77800 cm	NR = 0.59267 cm
CD = 4.57792 cm	RS = 0.54810 cm
AE = 6.31712 cm	HS = 1.13583 cm
BF = 9.72484 cm	JS = 2.86232 cm
DE = 1.73699 cm	GS = 5.93160 cm
DP = 2.62467 cm	OS = 1.26206 cm
HR = 0.99483 cm	GO = 7.19366 cm

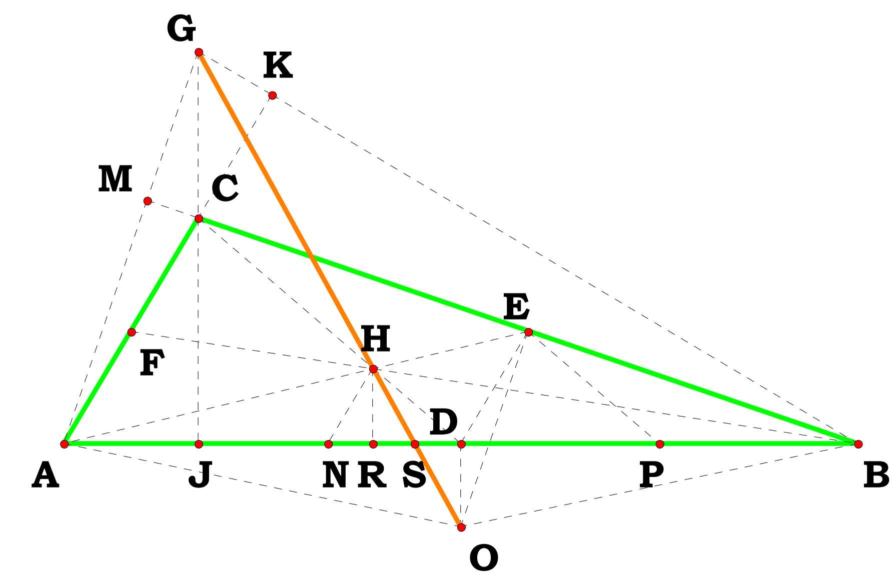
$AO = 5.364458$	$BF = 9.724843$	$EP = 2.288963$	$NR = 0.592667$	$GS = 5.931605$
$AJ = 1.778$	$DE = 1.73699$	$DH = 1.525976$	$RS = 0.548099$	$OS = 1.262059$
$CD = 4.577927$	$DP = 2.624668$	$CJ = 2.984502$	$HS = 1.135829$	$GO = 7.193663$
$AE = 6.317117$	$HR = 0.994834$	$DN = 1.749778$	$JS = 2.862322$	

Definitions.

$$AD-\frac{AB}{2}=0 \qquad AO-\frac{AB\cdot AC\cdot BC}{\sqrt{AB+AC+BC}\cdot\sqrt{AC+BC-AB}\cdot\sqrt{AB-AC+BC}\cdot\sqrt{AB+AC-BC}}=0 \qquad AJ-\frac{AB^2+AC^2-BC^2}{2\cdot AB}=0$$

$$CD-\frac{\sqrt{2\cdot AC^2-AB^2+2\cdot BC^2}}{2}=0 \qquad AE-\frac{\sqrt{2\cdot AB^2-BC^2+2\cdot AC^2}}{2}=0 \qquad BF-\frac{\sqrt{2\cdot BC^2-AC^2+2\cdot AB^2}}{2}=0 \qquad DE-\frac{AC}{2}=0$$

$$DP-\frac{AB}{4}=0 \quad EP-\frac{\sqrt{2\cdot AC^2-AB^2+2\cdot BC^2}}{4}=0 \quad AP-\frac{3\cdot AB}{4}=0 \quad DH-\frac{\sqrt{2\cdot AC^2-AB^2+2\cdot BC^2}}{6}=0$$



$$NR-\frac{AB^2+AC^2-BC^2}{6\cdot AB}=0 \quad BJ-\frac{AB^2-AC^2+BC^2}{2\cdot AB}=0 \quad DN-\frac{AB}{6}=0 \quad DR-\frac{(BC-AC)\cdot(AC+BC)}{6\cdot AB}=0$$

$$CJ-\frac{\sqrt{(AB+AC-BC)\cdot(AB-AC+BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)}}{2\cdot AB}=0 \quad AR-\frac{3\cdot AB^2+AC^2-BC^2}{6\cdot AB}=0$$

$$HR-\frac{\sqrt{(AB+AC-BC)\cdot(AB-AC+BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)}}{6\cdot AB}=0$$

$$HO-\frac{\sqrt{AB^6-AB^4\cdot AC^2-AB^4\cdot BC^2-AB^2\cdot AC^4+3\cdot AB^2\cdot AC^2\cdot BC^2-AB^2\cdot BC^4+AC^6-AC^4\cdot BC^2-AC^2\cdot BC^4+BC^6}}{\sqrt{(AB+AC-BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\cdot(9\cdot AB-9\cdot AC+9\cdot BC)}}=0$$

$$JR-\frac{(BC-AC)\cdot(AC+BC)}{3\cdot AB}=0 \quad RS-\frac{(AB+AC-BC)\cdot(AB-AC+BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\cdot(AC+BC)\cdot(BC-AC)}{6\cdot AB\cdot\left[3\cdot AB^4-3\cdot AB^2\cdot AC^2-3\cdot AB^2\cdot BC^2+(AB+AC-BC)\cdot(AB-AC+BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\right]}=0$$

$$HS-\frac{(AB+AC-BC)\cdot(AB-AC+BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\cdot\sqrt{AB^6-AB^4\cdot AC^2-AB^4\cdot BC^2-AB^2\cdot AC^4+3\cdot AB^2\cdot AC^2\cdot BC^2-AB^2\cdot BC^4+AC^6-AC^4\cdot BC^2-AC^2\cdot BC^4+BC^6}}{\left[3\cdot AB^4-3\cdot AB^2\cdot AC^2-3\cdot AB^2\cdot BC^2+(AB+AC-BC)\cdot(AB-AC+BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\right]\cdot\sqrt{(AB+AC-BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\cdot(9\cdot AB-9\cdot AC+9\cdot BC)}}=0$$

$$JS-\frac{(BC-AC)\cdot(AC+BC)\cdot\left(AB^2-AC^2+BC^2\right)\cdot\left(AB^2+AC^2-BC^2\right)}{2\cdot AB\cdot\left(2\cdot AB^4-AB^2\cdot AC^2-AB^2\cdot BC^2-AC^4+2\cdot AC^2\cdot BC^2-BC^4\right)}=0 \quad DO-\frac{AB\cdot\left(AB^2-AC^2-BC^2\right)}{2\cdot\sqrt{(AB+AC+BC)\cdot(AB-AC+BC)\cdot(AB+AC-BC)\cdot(AC-AB+BC)}}=0$$

$$GS-\frac{3\cdot\left(AB^2+AC^2-BC^2\right)\cdot\left(AB^2-AC^2+BC^2\right)\cdot\sqrt{AB^6-AB^4\cdot AC^2-AB^4\cdot BC^2-AB^2\cdot AC^4+3\cdot AB^2\cdot AC^2\cdot BC^2-AB^2\cdot BC^4+AC^6-AC^4\cdot BC^2-AC^2\cdot BC^4+BC^6}}{\sqrt{(AB+AC-BC)\cdot(AC-AB+BC)\cdot(AB+AC+BC)\cdot(9\cdot AB-9\cdot AC+9\cdot BC)}\cdot\left(2\cdot AB^4-AB^2\cdot AC^2-AB^2\cdot BC^2-AC^4+2\cdot AC^2\cdot BC^2-BC^4\right)}=0$$

$$OS-\frac{3\cdot\sqrt{AB^6-AB^4\cdot AC^2-AB^4\cdot BC^2-AB^2\cdot AC^4+3\cdot AB^2\cdot AC^2\cdot BC^2-AB^2\cdot BC^4+AC^6-AC^4\cdot BC^2-AC^2\cdot BC^4+BC^6}\cdot AB^2\cdot\left(AB^2-AC^2-BC^2\right)}{\sqrt{18\cdot AB^2\cdot AC^2-9\cdot AB^4+18\cdot AB^2\cdot BC^2-9\cdot AC^4+18\cdot AC^2\cdot BC^2-9\cdot BC^4}\cdot\left(2\cdot AB^4-AB^2\cdot AC^2-AB^2\cdot BC^2-AC^4+2\cdot AC^2\cdot BC^2-BC^4\right)}=0$$

$$GO-\frac{\sqrt{AB^6-AB^4\cdot AC^2-AB^4\cdot BC^2-AB^2\cdot AC^4+3\cdot AB^2\cdot AC^2\cdot BC^2-AB^2\cdot BC^4+AC^6-AC^4\cdot BC^2-AC^2\cdot BC^4+BC^6}}{\sqrt{2\cdot AB^2\cdot AC^2-AB^4+2\cdot AB^2\cdot BC^2-AC^4+2\cdot AC^2\cdot BC^2-BC^4}}=0$$

120795C

Descriptions.

Given.

$W := 6 \quad Y := 3$

$X := 20 \quad Z := 13$

Unit.

$AB := \frac{Y}{Y}$

Euler's Straight Line

Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

In this write up, I will use equations from previous versions of the DQ. 062793, 010893.

$AM := \frac{Y}{Z} \quad BM := AB - AM \quad CM := \frac{W}{X} \quad AC := \sqrt{AM^2 + CM^2} \quad BC := \sqrt{BM^2 + CM^2}$

$AD := \frac{AB}{2} \quad AO := \frac{AB \cdot AC \cdot BC}{\sqrt{AB + AC + BC} \cdot \sqrt{AC + BC - AB} \cdot \sqrt{AB - AC + BC} \cdot \sqrt{AB + AC - BC}}$

$AM := \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} \quad CD := \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} \quad AE := \frac{\sqrt{2 \cdot AB^2 - BC^2 + 2 \cdot AC^2}}{2}$

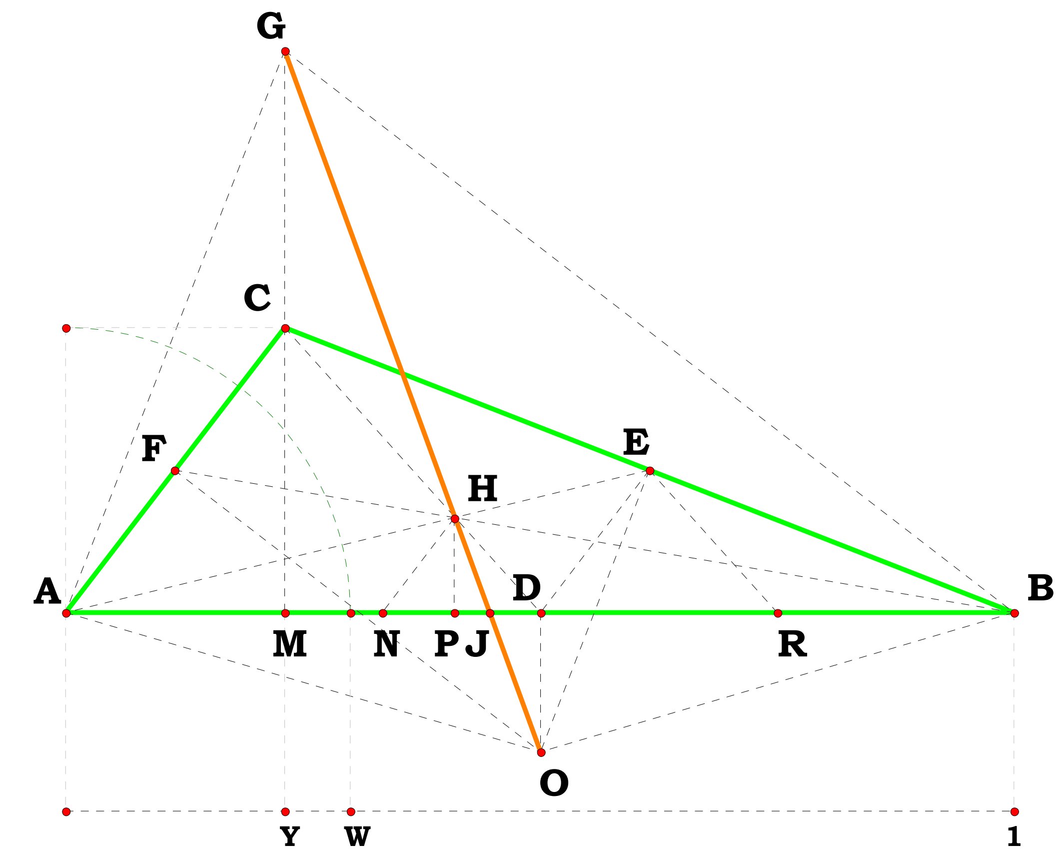
$BF := \frac{\sqrt{2 \cdot BC^2 - AC^2 + 2 \cdot AB^2}}{2} \quad DE := \frac{AC}{2} \quad DR := \frac{AD}{2} \quad ER := \frac{CD}{2} \quad AR := AD + DR$

$DH := \frac{ER \cdot AD}{AR} \quad \frac{CD}{DH} = 3 \quad DN := \frac{AD \cdot DH}{CD} \quad NP := \frac{AM \cdot DH}{CD} \quad BM := AB - AM$

$HP := \frac{CM \cdot DH}{CD} \quad DP := DN - NP \quad DO := \sqrt{AO^2 - AD^2}$

$HO := \sqrt{(HP + DO)^2 + DP^2} \quad AP := AD - DP \quad JP := AP - AM \quad PJ := \frac{DP \cdot HP}{DO + HP}$

$HJ := \frac{HO \cdot HP}{DO + HP} \quad MJ := JP + PJ \quad GJ := \frac{HJ \cdot MJ}{PJ} \quad OJ := HO - HJ \quad GO := GJ + OJ \quad \frac{GO}{HO} = 3$



Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.23077
Y = 3.00000
Z = 13.00000

AC = 0.37849
BC = 0.82566
AO = 0.52084
GO = 0.78518

Definitions.

$AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0 \quad BC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{X \cdot Z} = 0 \quad AD - \frac{1}{2} = 0$

$$AO - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{\sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} + \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + X \cdot Z} \cdot \sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} + \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - X \cdot Z} \cdot \sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} + -\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + X \cdot Z} \cdot \sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} + X \cdot Z}} = 0$$

$AM - \frac{Y}{Z} = 0$ $\quad CD - \frac{\sqrt{4\cdot W^2\cdot Z^2 + 4\cdot X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{2\cdot X\cdot Z} = 0$ $\quad AE - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2 + 2\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{2\cdot X\cdot Z} = 0$ $\quad BF - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + 4\cdot X^2\cdot Z^2}}{2\cdot X\cdot Z} = 0$

$DE - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}}{2\cdot X\cdot Z} = 0$ $\quad ER - \frac{\sqrt{4\cdot W^2\cdot Z^2 + 4\cdot X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{4\cdot X\cdot Z} = 0$ $\quad AR - \frac{3}{4} = 0$ $\quad DH - \frac{\sqrt{4\cdot W^2\cdot Z^2 + 4\cdot X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{6\cdot X\cdot Z} = 0$

$DR - \frac{1}{4} = 0$ $\quad NP - \frac{Y}{3\cdot Z} = 0$ $\quad BM - \frac{Z - Y}{Z} = 0$ $\quad DN - \frac{1}{6} = 0$ $\quad DP - -\frac{\left(\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2 - 2\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2} + \sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}\right)\cdot\left(\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - BC\cdot X\cdot Z\right)}{6\cdot X^2\cdot Z^2} = 0$

$CM - \frac{W}{X} = 0$ $\quad AP - \frac{Y + Z}{3\cdot Z} = 0$ $\quad HP - \frac{W}{3\cdot X} = 0$

$$HO - \frac{\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{6\cdot W\cdot X\cdot Z^2} = 0$$

$JP - \frac{Z - 2\cdot Y}{3\cdot Z} = 0$ $\quad PJ - -\frac{W^2\cdot Z\cdot(Z - 2\cdot Y)}{3\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$

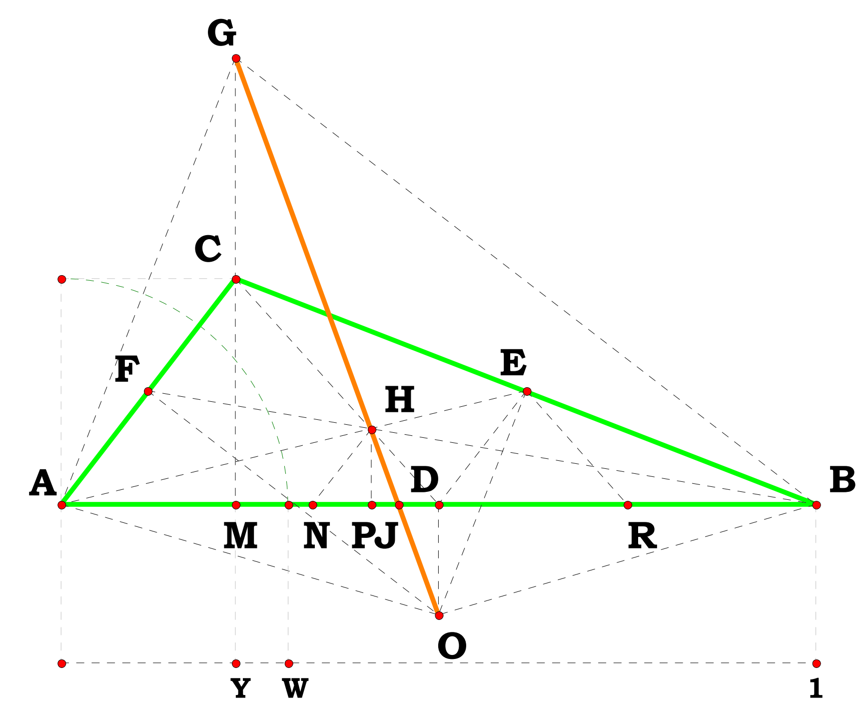
$$HJ - \frac{W\cdot\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{3\cdot X\cdot\left(3\cdot X^2\cdot Y\cdot Z - 3\cdot X^2\cdot Y^2 - W^2\cdot Z^2\right)} = 0$$

$MJ - -\frac{X^2\cdot Y\cdot\left(2\cdot Y^2 - 3\cdot Y\cdot Z + Z^2\right)}{Z\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$ $\quad DO - \frac{X^2\cdot Y\cdot Z - W^2\cdot Z^2 - X^2\cdot Y^2}{2\cdot W\cdot X\cdot Z^2} = 0$

$$GJ - \frac{X\cdot Y\cdot(Y - Z)\cdot\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{W\cdot Z^2\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$$

$$OJ - \frac{\left(W^2\cdot Z^2 + X^2\cdot Y^2 - X^2\cdot Y\cdot Z\right)\cdot\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{2\cdot W\cdot X\cdot Z^2\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$$

$$GO - \frac{\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{2\cdot W\cdot X\cdot Z^2} = 0$$



Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2\,az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.

Given a, n and b for the equation $z^2 =$ $n\,az + b^2 + cd$ find z, c, and d.

121695

Descriptions.

$AD := n \cdot a$ $\quad BE := \sqrt{a^2 + b^2}$ $\quad BC := \dfrac{a^2}{BE}$

$CE := BE - BC$ $\quad CF := \sqrt{BC \cdot CE}$

$FG := \dfrac{AD}{2}$ $\quad CG := \sqrt{FG^2 - CF^2}$

$AG := FG$ $\quad AC := AG + CG$

$BG := CG - BC$ $\quad DG := FG$

$BD := DG - BG$ $\quad AB := AG + BG$

$AE := AB + BE$ $\quad DE := BE - BD$

$DH := \dfrac{b^2}{DE}$ $\quad DI := AE$ $\quad HI := DI - DH$

$z := AE$ $\quad z = 12.621556$

$c := DE$ $\quad c = 0.621556$

$d := HI$ $\quad d = 6.18609$

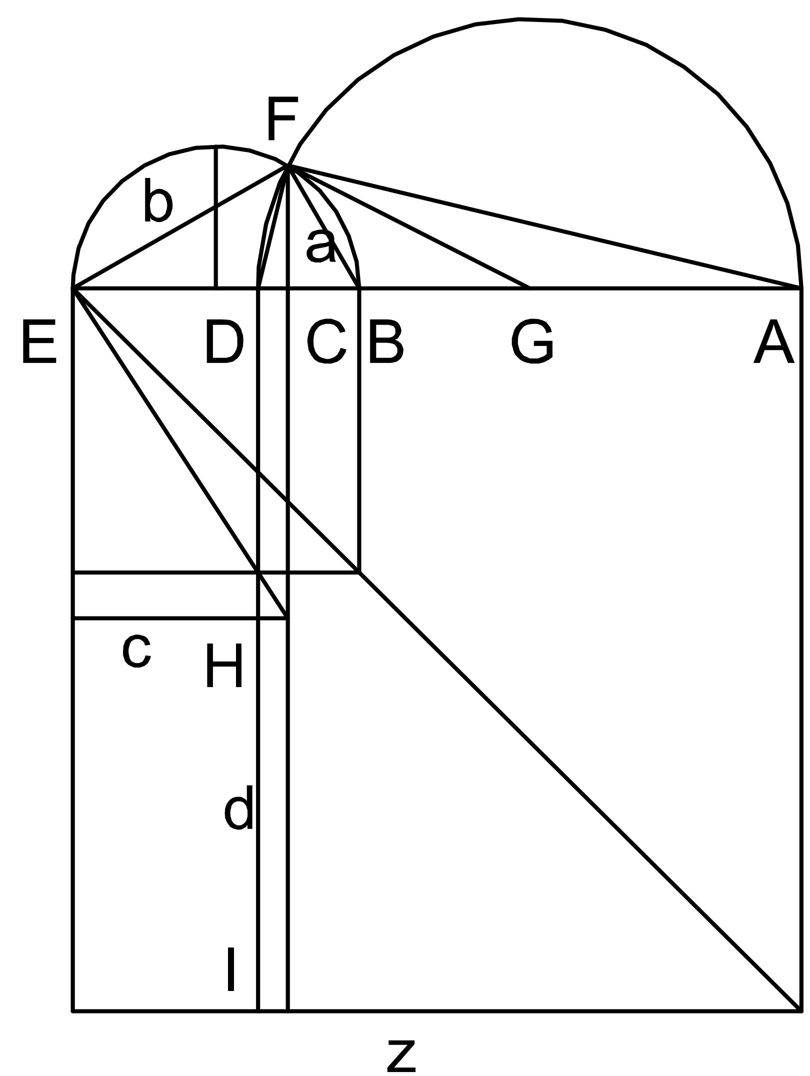
$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$

Given.

$n \equiv 3$
$a \equiv 4$

$b \equiv 2$

Place values here :



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$\mathbf{d}-2\cdot \mathbf{a}\cdot \frac{\mathbf{b}^2}{\sqrt{\mathbf{a}^2+\mathbf{b}^2}}\cdot \frac{\left(2\cdot \mathbf{a}-\sqrt{-2\cdot \mathbf{b}+\mathbf{n}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}}\cdot \sqrt{2\cdot \mathbf{b}+\mathbf{n}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}}\right)}{\left(-2\cdot \mathbf{b}^2+\mathbf{n}\cdot \mathbf{a}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}-\mathbf{a}\cdot \sqrt{-2\cdot \mathbf{b}+\mathbf{n}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}}\cdot \sqrt{2\cdot \mathbf{b}+\mathbf{n}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}}\right)}=0$$

$$\mathbf{c}-\frac{-1}{2}\cdot \frac{\left(-2\cdot \mathbf{b}^2+\mathbf{n}\cdot \mathbf{a}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}-\mathbf{a}\cdot \sqrt{\mathbf{n}^2\cdot \mathbf{a}^2+\mathbf{n}^2\cdot \mathbf{b}^2-4\cdot \mathbf{b}^2}\right)}{\sqrt{\mathbf{a}^2+\mathbf{b}^2}}=0$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$\mathbf{z}-\frac{1}{2}\cdot \frac{\left(\mathbf{n}\cdot \mathbf{a}\cdot \sqrt{\mathbf{a}^2+\mathbf{b}^2}+\mathbf{a}\cdot \sqrt{\mathbf{n}^2\cdot \mathbf{a}^2+\mathbf{n}^2\cdot \mathbf{b}^2-4\cdot \mathbf{b}^2}+2\cdot \mathbf{b}^2\right)}{\sqrt{\mathbf{a}^2+\mathbf{b}^2}}=0$$

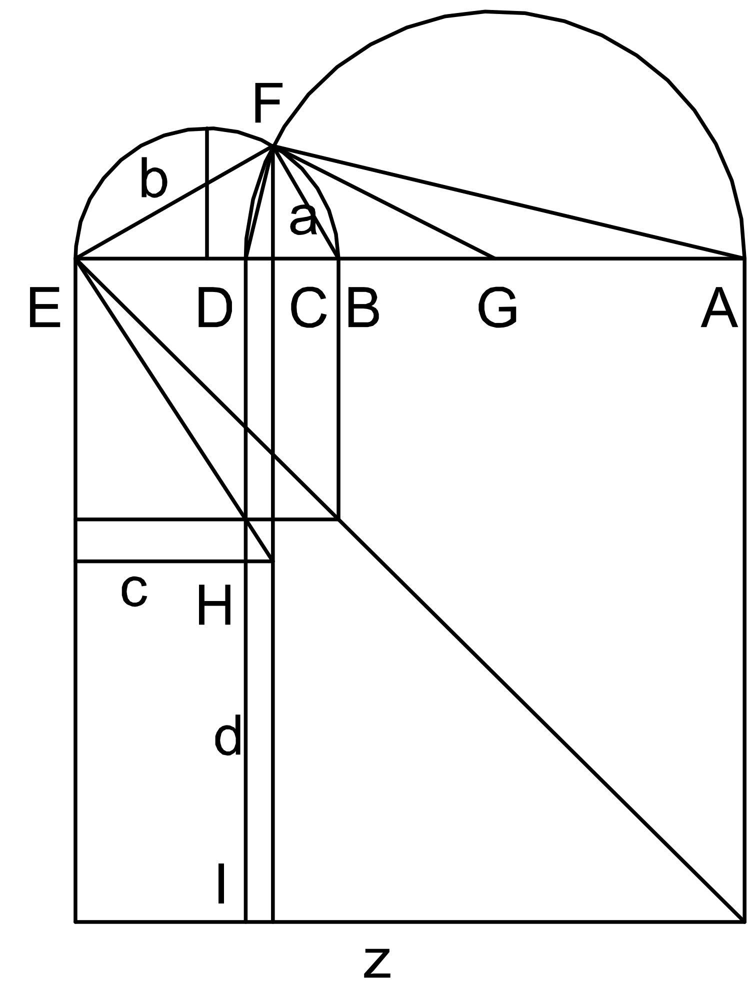
$$\mathbf{z}^2-\left(\mathbf{n}\cdot \mathbf{a}\cdot \mathbf{z}+\mathbf{b}^2+\mathbf{c}\cdot \mathbf{d}\right)=0 \qquad \mathbf{p}:=-\mathbf{a}\cdot \mathbf{b}^2\cdot \frac{\left(2\cdot \mathbf{a}-\sqrt{\mathbf{n}^2\cdot \mathbf{a}^2+\mathbf{n}^2\cdot \mathbf{b}^2-4\cdot \mathbf{b}^2}\right)}{\left(\mathbf{a}^2+\mathbf{b}^2\right)}$$

$$(\mathbf{c}\cdot \mathbf{d})-\mathbf{p}=0$$

$$\mathbf{z}^2-\left[\mathbf{n}\cdot \mathbf{a}\cdot \mathbf{z}+\mathbf{b}^2+\left[-\mathbf{a}\cdot \mathbf{b}^2\cdot \frac{\left(2\cdot \mathbf{a}-\sqrt{\mathbf{n}^2\cdot \mathbf{a}^2+\mathbf{n}^2\cdot \mathbf{b}^2-4\cdot \mathbf{b}^2}\right)}{\left(\mathbf{a}^2+\mathbf{b}^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{pmatrix} \frac{1}{2}\cdot \mathbf{n}\cdot \mathbf{a}+\frac{1}{2}\cdot \frac{\sqrt{\mathbf{n}^2\cdot \mathbf{a}^4+\mathbf{n}^2\cdot \mathbf{a}^2\cdot \mathbf{b}^2-4\cdot \mathbf{a}^2\cdot \mathbf{b}^2+4\cdot \mathbf{b}^4+4\cdot \mathbf{a}\cdot \mathbf{b}^2\cdot \sqrt{\mathbf{n}^2\cdot \mathbf{a}^2+\mathbf{n}^2\cdot \mathbf{b}^2-4\cdot \mathbf{b}^2}}}{\sqrt{\mathbf{a}^2+\mathbf{b}^2}} \\ \frac{1}{2}\cdot \mathbf{n}\cdot \mathbf{a}-\frac{1}{2}\cdot \frac{\sqrt{\mathbf{n}^2\cdot \mathbf{a}^4+\mathbf{n}^2\cdot \mathbf{a}^2\cdot \mathbf{b}^2-4\cdot \mathbf{a}^2\cdot \mathbf{b}^2+4\cdot \mathbf{b}^4+4\cdot \mathbf{a}\cdot \mathbf{b}^2\cdot \sqrt{\mathbf{n}^2\cdot \mathbf{a}^2+\mathbf{n}^2\cdot \mathbf{b}^2-4\cdot \mathbf{b}^2}}}{\sqrt{\mathbf{a}^2+\mathbf{b}^2}} \end{pmatrix}$$



121895

Descriptions.

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$$z^2 := az - b^2$$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

$$b := 2.12 \qquad z := 1.41 \qquad c := \frac{b^2}{z}$$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$$a := z + c$$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.

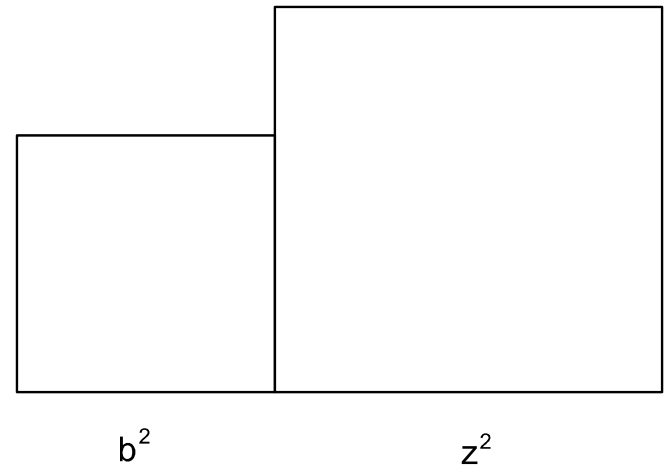
$$\left(c^2 + b^2\right) - a \cdot c = 1.776357 \times 10^{-15}$$

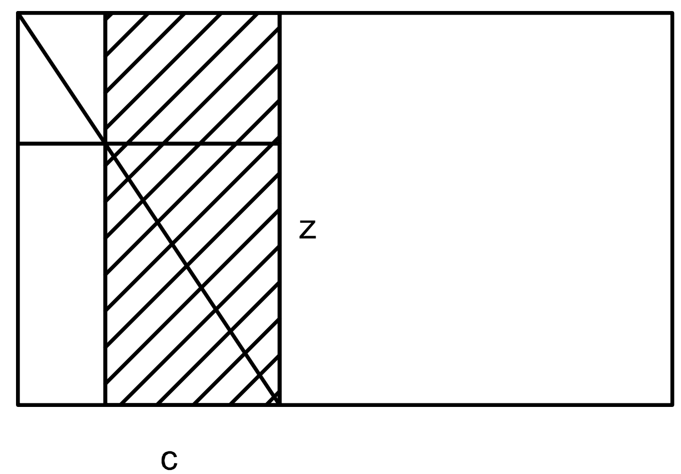
$$\left(c^2 + b^2\right) - [(z + c) \cdot c] = 1.776357 \times 10^{-15}$$

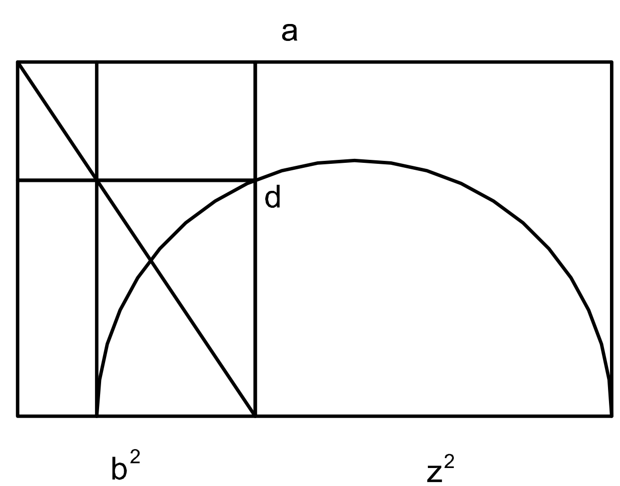
$$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$$

$$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$$

$$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$$

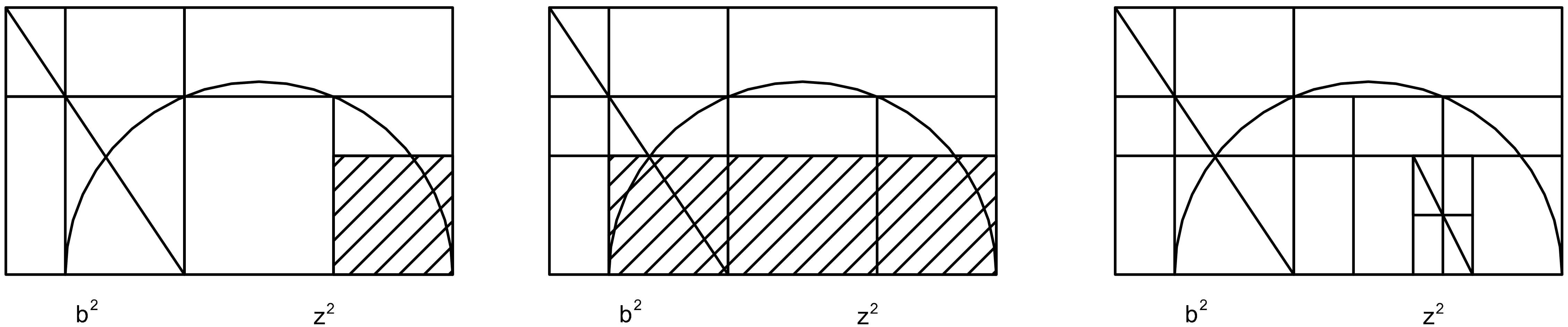




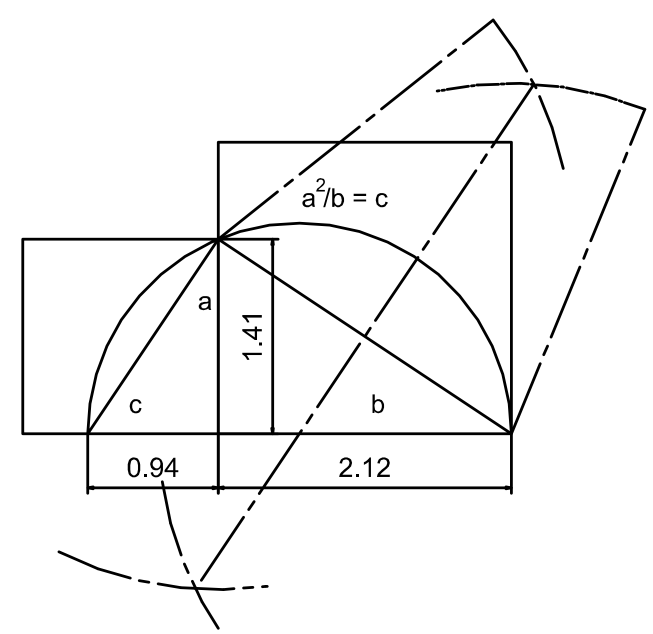


Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.



Unit.

$EF := 1$

Given.

$N := 2$

122095A

Descriptions.

Delian Solution in Every Right Angle

This plate is derived from the fact that for any EJ taken as a square, divided by EF, the answer is AE. And in every case, the small circle OH has a relationship to the circle EJ. This is another plate on geometric progression.

$EJ := EF \cdot N \qquad AE := \dfrac{EJ^2}{EF} \qquad AF := AE + EF$

$AB := \dfrac{AF}{2} \qquad BF := AB \qquad BE := BF - EF \qquad BH := BE$

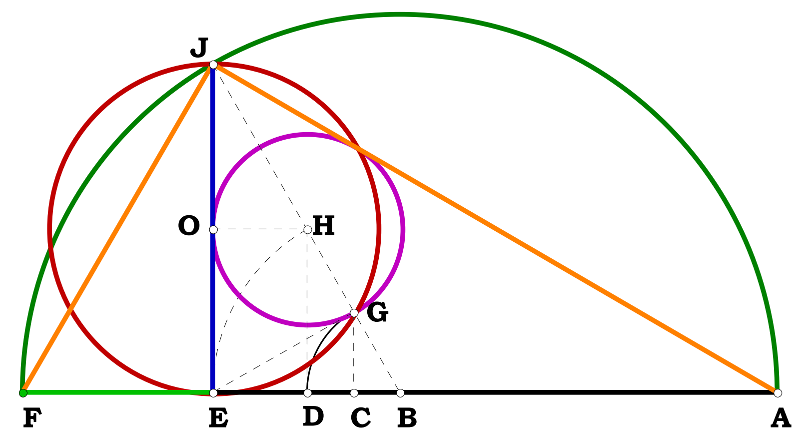
$BJ := \sqrt{EJ^2 + BE^2} \qquad BD := \dfrac{BE \cdot BH}{BJ} \qquad BG := BD$

$BC := \dfrac{BE \cdot BG}{BJ} \qquad GH := BH - BG \qquad DE := BE - BD$

$HO := DE \qquad EG_1 := \sqrt{BE^2 - BG^2} \qquad GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2} \qquad \dfrac{GH}{HO} = 1 \qquad \dfrac{EG_1}{EG_2} = 1$

$\left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \qquad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$



EJ = 1.708 in.
EF = 0.992 in.
AE = 2.943 in.
$\dfrac{EJ^2}{EF}$-AE = 0.000 in.

Definitions.

$EJ - N = 0 \qquad AE - N^2 = 0 \qquad AF - \left(N^2 + 1\right) = 0 \qquad AB - \dfrac{N^2 + 1}{2} = 0 \qquad BF - \dfrac{N^2 + 1}{2} = 0 \qquad BE - \dfrac{(N-1) \cdot (N+1)}{2} = 0$

$BH - \dfrac{(N-1) \cdot (N+1)}{2} = 0 \qquad BJ - \dfrac{\left(N^2 + 1\right)}{2} = 0 \qquad BD - \dfrac{(N-1)^2 \cdot (N+1)^2}{2 \cdot \left(N^2 + 1\right)} = 0 \qquad BG - \dfrac{(N-1)^2 \cdot (N+1)^2}{2 \cdot \left(N^2 + 1\right)} = 0$

$BC - \dfrac{(N-1)^3 \cdot (N+1)^3}{2 \cdot \left(N^2 + 1\right)^2} = 0 \qquad GH - \dfrac{(N-1) \cdot (N+1)}{N^2 + 1} = 0 \qquad DE - \dfrac{(N-1) \cdot (N+1)}{N^2 + 1} = 0 \qquad HO - \dfrac{(N-1) \cdot (N+1)}{N^2 + 1} = 0$

$EG_1 - \dfrac{\left(N^3 - N\right)}{\left(N^2 + 1\right)} = 0 \qquad GJ - \dfrac{2 \cdot N^2}{N^2 + 1} = 0 \qquad EG_2 - \dfrac{N \cdot (N-1) \cdot (N+1)}{\left(N^2 + 1\right)} = 0$

Given.

$X := 7$

$Y := 20$

122095B

Delian Solution in Every Right Angle

This plate is derived from the fact that for any EJ taken as a square, divided by EF, the answer is AE. And in every case, the small circle OH has a relationship to the circle EJ.

Descriptions.

$AF := 2 \quad EF := \frac{X}{Y} \quad EJ := \sqrt{EF \cdot (AF - EF)} \quad AE := \frac{EJ^2}{EF}$

$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF$

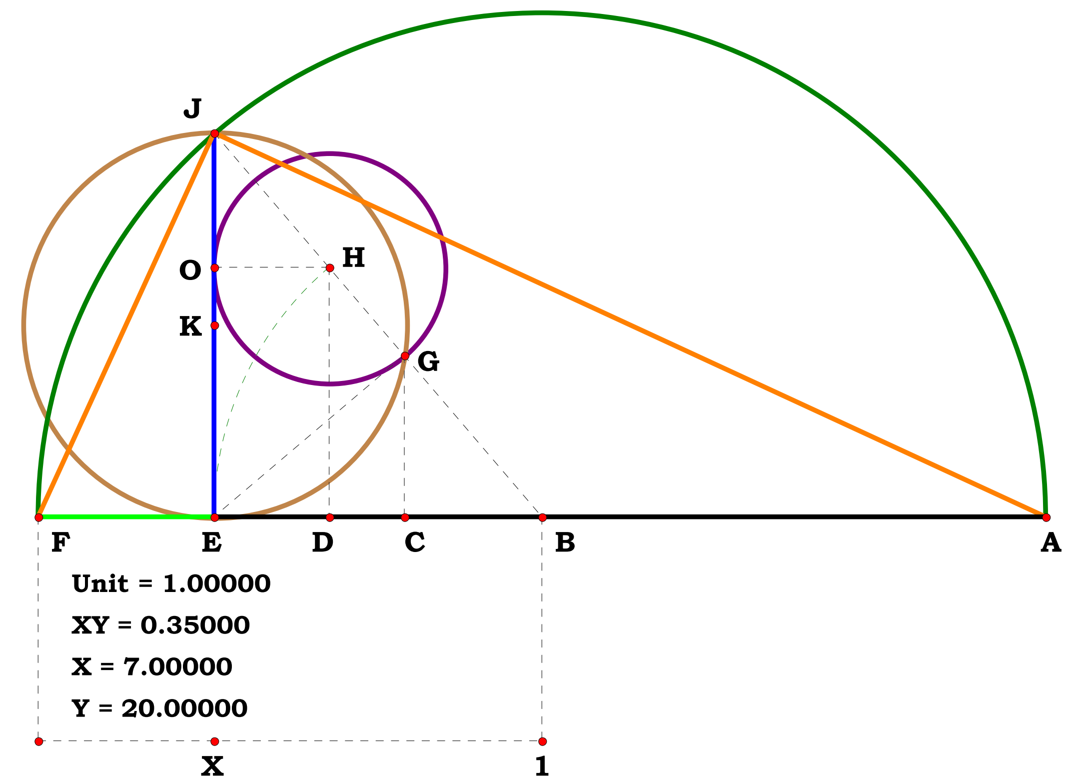
$BH := BE \quad BJ := \sqrt{EJ^2 + BE^2} \quad BD := \frac{BE \cdot BH}{BJ}$

$BG := BD \quad BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD$

$HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2} \quad \frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1$

$\left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$



Definitions.

$AF - 2 = 0 \quad EF - \frac{X}{Y} = 0 \quad EJ - \frac{\sqrt{X \cdot (2 \cdot Y - X)}}{Y} = 0 \quad AE - \frac{2 \cdot Y - X}{Y} = 0 \quad AB - 1 = 0 \quad BF - 1 = 0 \quad BE - \frac{Y - X}{Y} = 0$

$BH - \frac{Y - X}{Y} = 0 \quad BJ - \sqrt{1} = 0 \quad BD - \frac{(X - Y)^2}{Y^2} = 0 \quad BG - \frac{(X - Y)^2}{Y^2} = 0 \quad BC - \frac{(Y - X)^3}{Y^3} = 0 \quad GH - \frac{X \cdot (Y - X)}{Y^2} = 0$

$DE - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad HO - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad EG_1 - \frac{\sqrt{X \cdot (2 \cdot Y - X)} \cdot (Y - X)}{Y^2} = 0 \quad GJ - \frac{X \cdot (2 \cdot Y - X)}{Y^2} = 0 \quad EG_2 - \frac{\sqrt{X \cdot (2 \cdot Y - X)} \cdot (Y - X)}{Y^2} = 0$

122195A1

Unit.

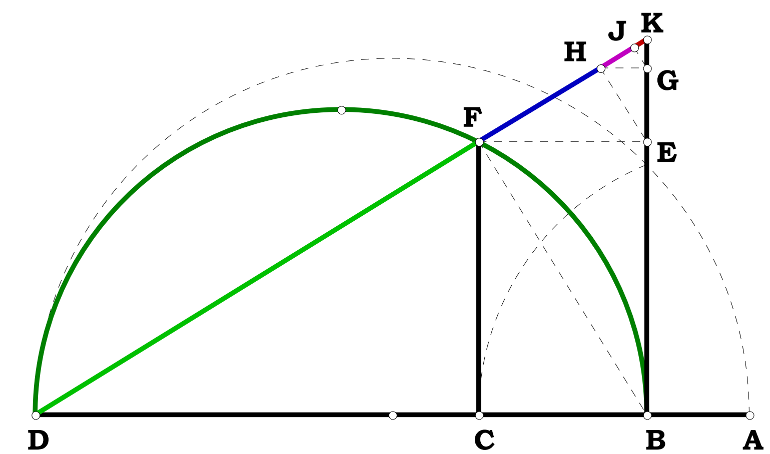
$AB := 1$

Given.

$N := 7$

Pascal's Triangle With Exponential Division

Plate A1



Descriptions.

$AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD} \quad BD := AD - AB \quad CD := AD - AC$

$BC := BD - CD \quad CF := \sqrt{BC \cdot CD} \quad DF := \sqrt{CF^2 + CD^2}$

$$DK := \frac{DF \cdot BD}{CD} \quad FK := \frac{DK \cdot BC}{BD} \quad HK := \frac{FK \cdot FK}{DK} \quad JK := \frac{HK \cdot HK}{FK}$$

Definitions.

$$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \quad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0$$

$AD - N = 0 \quad AC - \sqrt{N} = 0 \quad BD - (N-1) = 0 \quad CD - \left(N - \sqrt{N}\right) = 0$

$$BC - \left(\sqrt{N} - 1\right) = 0 \quad CF - N^{\frac{1}{4}} \cdot \left(\sqrt{N} - 1\right) = 0 \quad DF - \sqrt{\sqrt{N} \cdot \left(\sqrt{N} - 1\right)^2 \cdot \left(\sqrt{N} + 1\right)} = 0$$

$$DK - \frac{\sqrt{N + \sqrt{N}} \cdot (N-1)}{\sqrt{N}} = 0 \quad FK - \frac{\sqrt{N + \sqrt{N}} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N}} = 0$$

$$HK - \frac{\sqrt{N + \sqrt{N}} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N} \cdot \left(\sqrt{N} + 1\right)} = 0 \quad JK - \frac{\sqrt{N + \sqrt{N}} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N} \cdot \left(\sqrt{N} + 1\right)^2} = 0$$

122195A2

Pascal's Triangle With Exponential Division

Plate A2

Given.

$X := 3 \qquad Y := 20$

$AB := \dfrac{X}{Y} \qquad N := \dfrac{Y}{X}$

Descriptions.

$AD := AB \cdot N \qquad AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB \qquad CD := AD - AC$

$BC := BD - CD \qquad CF := \sqrt{BC \cdot CD} \qquad DF := \sqrt{CF^2 + CD^2}$

$DK := \dfrac{DF \cdot BD}{CD} \qquad FK := \dfrac{DK \cdot BC}{BD} \qquad HK := \dfrac{FK \cdot FK}{DK} \qquad JK := \dfrac{HK \cdot HK}{FK}$

Definitions.

$$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0$$

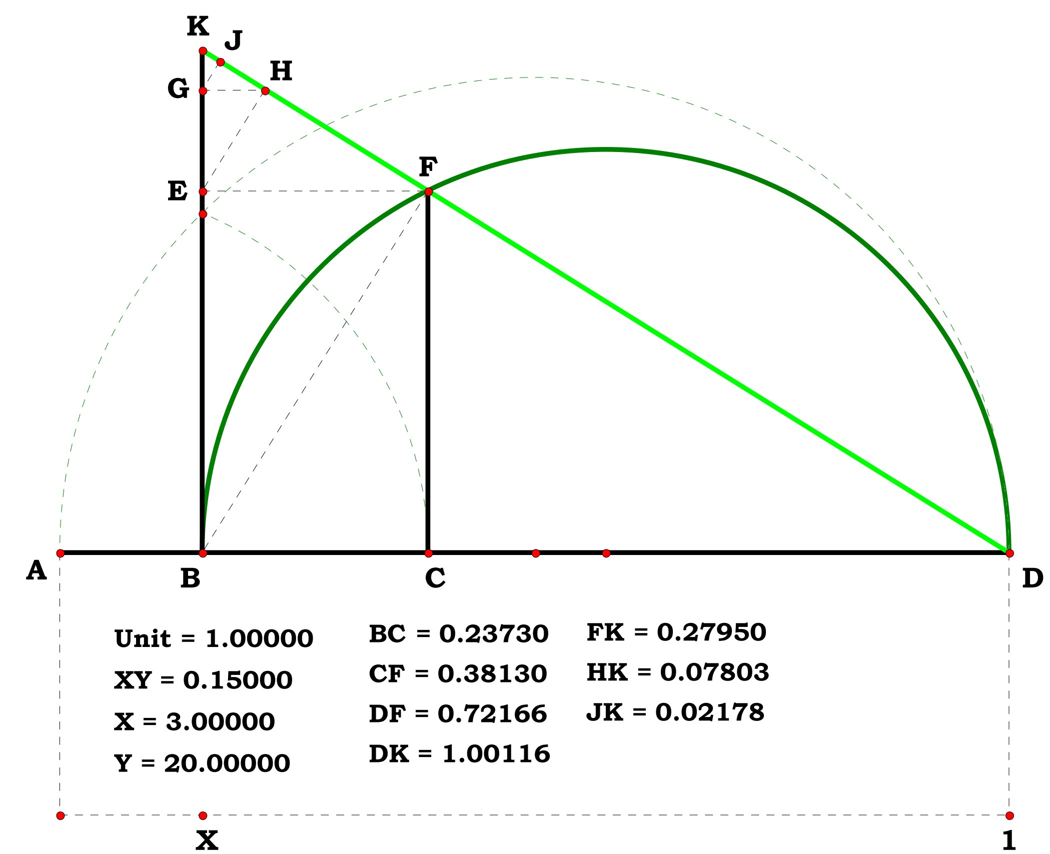
$$AD - 1 = 0 \qquad AC - \frac{\sqrt{X}}{\sqrt{Y}} = 0 \qquad BD - \frac{Y-X}{Y} = 0 \qquad CD - \frac{\sqrt{Y}-\sqrt{X}}{\sqrt{Y}} = 0$$

$$BC - \frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y} = 0 \qquad CF - \frac{\sqrt{\sqrt{X}\cdot Y - 2\cdot X\cdot\sqrt{Y} + X^{\frac{3}{2}}}}{Y^{\frac{3}{4}}} = 0$$

$$DF - \frac{\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y^{\frac{3}{4}}} = 0 \qquad DK - \frac{\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(\sqrt{X}+\sqrt{Y}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}}{Y^{\frac{5}{4}}} = 0$$

$$FK - \frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(\sqrt{\sqrt{X}+\sqrt{Y}}\right)^3}{Y^{\frac{5}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}\right)} = 0$$

$$HK - \frac{X\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y^{\frac{5}{4}}\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} = 0 \qquad JK - \frac{\left(\sqrt{X}\right)^3\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y^{\frac{5}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}\right)^{\frac{3}{2}}} = 0$$



$$\frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y} - BC = 0.00000$$

$$\frac{\sqrt{\left(\sqrt{X}\cdot Y - 2\cdot X\cdot\sqrt{Y}\right)+X^{\frac{3}{2}}}}{Y^{\frac{3}{4}}} - CF = 0.00000$$

$$\frac{\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y^{\frac{3}{4}}} - DF = 0.00000$$

$$\frac{\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(\sqrt{X}+\sqrt{Y}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}}{Y^{\frac{5}{4}}} - DK = 0.00000$$

$$\frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}^{\,3}}{Y^{\frac{5}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}\right)} - FK = 0.00000$$

$$\frac{X\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y^{\frac{5}{4}}\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} - HK = 0.00000$$

$$\frac{\sqrt{X}^3\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y^{\frac{5}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}\right)^{\frac{3}{2}}} - JK = 0.00000$$

122195B1

Descriptions.

Unit.

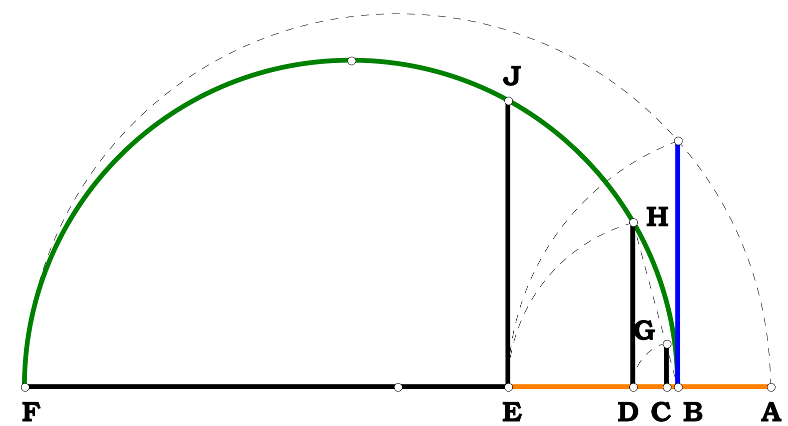
$AB := 1$

Given.

$N := 7$

Pascal's Triangle With Exponential Division

Plate B1



$AF := N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \quad BH := BE$

$BD := \dfrac{BE \cdot BH}{BF} \quad BG := BD \quad BC := \dfrac{BD \cdot BG}{BE}$

Definitions.

$$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$$

$$AF - N = 0 \qquad BF - (N-1) = 0 \qquad AE - \sqrt{N} = 0$$

$$BE - \left(\sqrt{N} - 1\right) = 0 \qquad BH - \left(\sqrt{N} - 1\right) = 0$$

$$BD - \frac{\sqrt{N}-1}{\sqrt{N}+1} = 0 \qquad BG - \frac{\sqrt{N}-1}{\sqrt{N}+1} = 0 \qquad BC - \frac{\sqrt{N}-1}{\left(\sqrt{N}+1\right)^2} = 0$$

122195B2

Given.

$X := 3 \qquad Y := 15$

$AB := \dfrac{X}{Y} \qquad N := \dfrac{Y}{X}$

Descriptions.

$AF := AB \cdot N \qquad BF := AF - AB \qquad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \qquad BH := BE$

$BD := \dfrac{BE \cdot BH}{BF} \qquad BG := BD \qquad BC := \dfrac{BD \cdot BG}{BE}$

$$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{BF}{BD} - \frac{N^2 - 2\cdot N + 1}{N - 2\cdot\sqrt{N} + 1} = 0 \qquad \frac{BF}{BC} - \frac{N^3 - 3\cdot N^2 + 3\cdot N - 1}{N^{\frac{3}{2}} - 3\cdot N + 3\cdot\sqrt{N} - 1} = 0$$

Definitions.

$$AF - 1 = 0 \qquad BF - \frac{Y-X}{Y} = 0 \qquad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$$

$$BE - \frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y} = 0 \qquad BH - \frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y} = 0$$

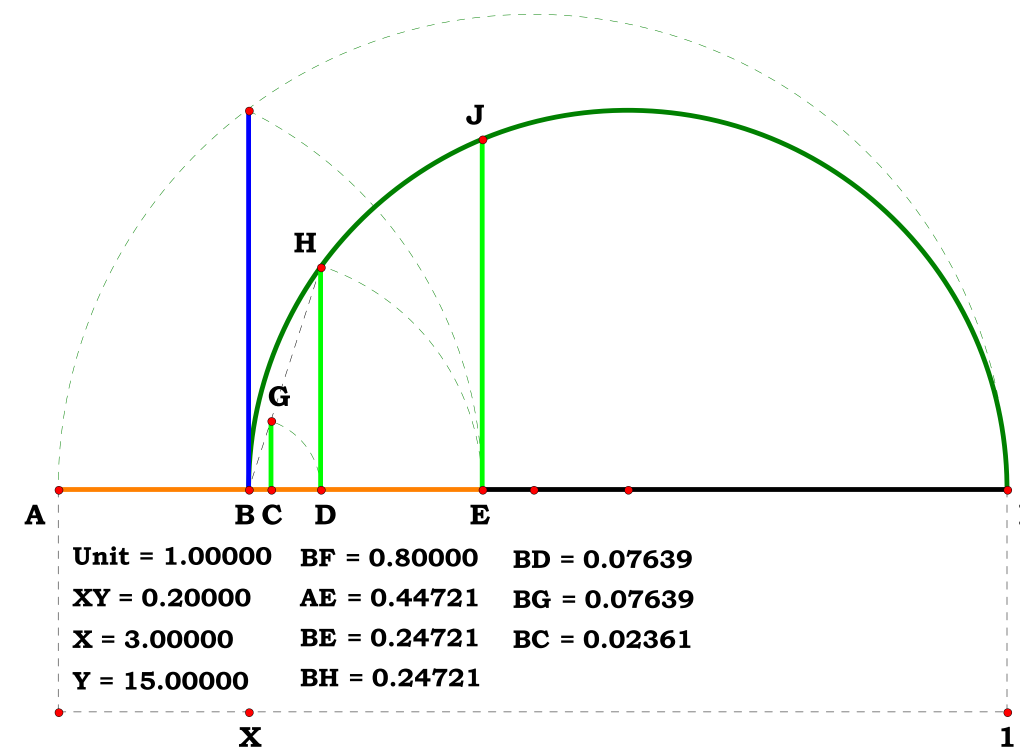
$$BD - \frac{X\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y\cdot\left(\sqrt{X}+\sqrt{Y}\right)} = 0 \qquad BG - \frac{X\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y\cdot\left(\sqrt{X}+\sqrt{Y}\right)} = 0$$

$$BC - \frac{\left(\sqrt{X}\right)^3\cdot\left(\sqrt{Y}-\sqrt{X}\right)}{Y\cdot\left(\sqrt{X}+\sqrt{Y}\right)^2} = 0$$

$$\frac{BF}{BE} - \left(\frac{\sqrt{Y}}{\sqrt{X}} + 1\right) = 0 \qquad \frac{BF}{BD} - \frac{X + Y + 2\cdot\sqrt{X}\cdot\sqrt{Y}}{X} = 0 \qquad \frac{BF}{BC} - \frac{X^3 - 3\cdot X^2\cdot Y + 3\cdot X\cdot Y^2 - Y^3}{X^2\cdot\left[X + 3\cdot Y - X\cdot\left(\frac{Y}{X}\right)^{\frac{3}{2}} - 3\cdot\sqrt{X}\cdot\sqrt{Y}\right]} = 0$$

Pascal's Triangle With Exponential Division

Plate B2



Unit = 1.00000
XY = 0.20000
X = 3.00000
Y = 15.00000

BF = 0.80000
AE = 0.44721
BE = 0.24721
BH = 0.24721

BD = 0.07639
BG = 0.07639
BC = 0.02361

$\frac{Y-X}{Y}$-BF = 0.00000

$\frac{\sqrt{X}}{\sqrt{Y}}$-AE = 0.00000

$\frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})}{Y}$ = 0.24721

$\frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})}{Y}$-BE = 0.00000

$\frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})}{Y}$-BH = 0.00000

$\frac{X\cdot(\sqrt{Y}-\sqrt{X})}{Y\cdot(\sqrt{X}+\sqrt{Y})}$ = 0.07639

$\frac{X\cdot(\sqrt{Y}-\sqrt{X})}{Y\cdot(\sqrt{X}+\sqrt{Y})}$-BD = 0.00000

$\frac{X\cdot(\sqrt{Y}-\sqrt{X})}{Y\cdot(\sqrt{X}+\sqrt{Y})}$-BG = 0.00000

$\frac{\sqrt{X}^3\cdot(\sqrt{Y}-\sqrt{X})}{Y\cdot(\sqrt{X}+\sqrt{Y})^2}$-BC = 0.00000

122195C1

Unit.

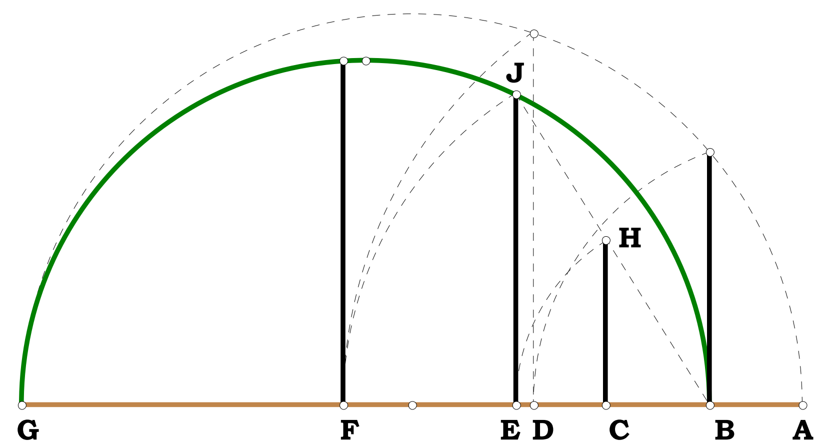
$AB := 1$

Given.

$N := 7$

Pascal's Triangle With Exponential Division

Plate C1



Descriptions.

$AG := N \quad BG := AG - AB \quad AF := \left(AB \cdot AG^{3}\right)^{\frac{1}{4}}$

$BF := AF - AB \quad BJ := BF \quad BE := \dfrac{BJ \cdot BF}{BG}$

$BH := BE \quad BC := \dfrac{BH \cdot BE}{BF}$

Definitions.

$$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{N^{2} - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0 \qquad \frac{BG}{BC} - \frac{N^{3} - 3 \cdot N^{2} + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$$

$$AG - N = 0 \qquad BG - (N-1) = 0 \qquad AF - \left(N^{3}\right)^{\frac{1}{4}} = 0$$

$$BF - \left[\left(N^{3}\right)^{\frac{1}{4}} - 1\right] = 0 \qquad BJ - \left[\left(N^{3}\right)^{\frac{1}{4}} - 1\right] = 0$$

$$BE - \frac{\left[\left(N^{3}\right)^{\frac{1}{4}} - 1\right]^{2}}{N-1} = 0 \qquad BH - \frac{\left[\left(N^{3}\right)^{\frac{1}{4}} - 1\right]^{2}}{N-1} = 0$$

$$BC - \frac{\left[\left(N^{3}\right)^{\frac{1}{4}} - 1\right]^{3}}{(N-1)^{2}} = 0$$

122195C2

Descriptions.

Given.

$X := 3 \qquad Y := 17$

$AB := \dfrac{X}{Y} \qquad N := \dfrac{Y}{X}$

Pascal's Triangle With Exponential Division

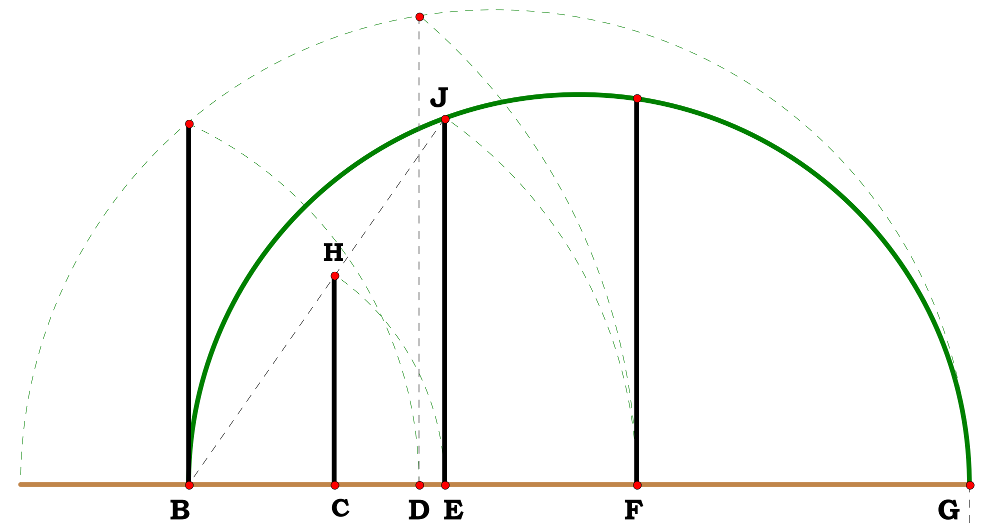
Plate C2

$AG := AB \cdot N \qquad BG := AG - AB \qquad AF := \left(AB \cdot AG^{3}\right)^{\frac{1}{4}} \qquad BF := AF - AB$

$BJ := BF \qquad BE := \dfrac{BJ \cdot BF}{BG} \qquad BH := BE \qquad BC := \dfrac{BH \cdot BE}{BF}$

$$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{N^{2} - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0$$

$$\frac{BG}{BC} - \frac{N^{3} - 3 \cdot N^{2} + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$$



Unit = 1.00000
XY = 0.17647
X = 3.00000
Y = 17.00000

BG = 0.82353
AF = 0.64814
BF = 0.47167
BJ = 0.47167

BE = 0.27014
BH = 0.27014
BC = 0.15472

X

1

Definitions.

$$AG - 1 = 0 \qquad BG - \frac{Y-X}{Y} = 0 \qquad AF - \left(\frac{X}{Y}\right)^{\frac{1}{4}} = 0$$

$$BF - \frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} = 0$$

$$BJ - \frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} = 0$$

$$\frac{Y-X}{Y} - BG = 0.00000$$

$$\left(\frac{X}{Y}\right)^{\frac{1}{4}} - AF = 0.00000$$

$$\frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} = 0.47167$$

$$\frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} - BF = 0.00000$$

$$\frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} - BJ = 0.00000$$

$$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^{2}}{Y \cdot (Y-X) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)} = 0.27014$$

$$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^{2}}{Y \cdot (Y-X) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)} - BE = 0.00000$$

$$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^{2}}{Y \cdot (Y-X) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)} - BH = 0.00000$$

$$\frac{X^{\frac{3}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right)^{2} \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^{3}}{Y \cdot \left(\sqrt{X} + \sqrt{Y}\right)^{2} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)^{2}} - BC = 0.00000$$

$$BE - \frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{2}}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} = 0$$

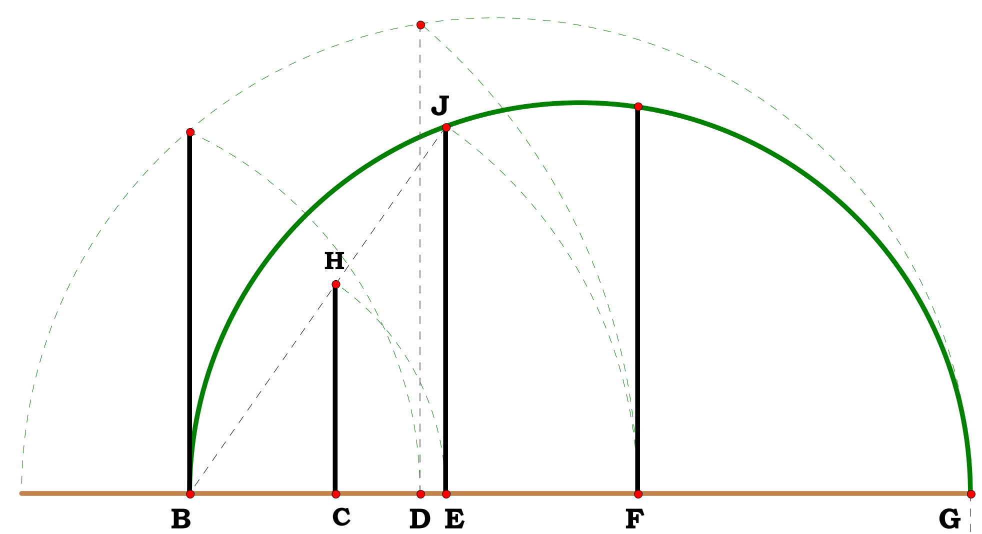
$$BH - \frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{2}}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} = 0$$

$$BC - \frac{X^{\frac{3}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)^{2}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{3}}{\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^{2}\cdot Y\cdot(\sqrt{X}+\sqrt{Y})^{2}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)} = 0$$

$$\frac{BG}{BF} - \frac{(\sqrt{X}+\sqrt{Y})\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)}{X^{\frac{1}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)} = 0$$

$$\frac{BG}{BE} - \frac{\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^{2}\cdot(\sqrt{X}+\sqrt{Y})^{2}}{\sqrt{X}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{2}} = 0$$

$$\frac{BG}{BC} - \frac{(\sqrt{X}+\sqrt{Y})^{3}\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^{3}}{X^{\frac{3}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{3}} = 0$$



Unit = 1.00000
XY = 0.17647
X = 3.00000
Y = 17.00000

BG = 0.82353
AF = 0.64814
BF = 0.47167
BJ = 0.47167

BE = 0.27014
BH = 0.27014
BC = 0.15472

X

1

$$\frac{Y-X}{Y} - BG = 0.00000$$

$$\frac{X^{\frac{1}{4}}}{Y} - AF = 0.00000$$

$$\frac{X^{\frac{1}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y} = 0.47167$$

$$\frac{X^{\frac{1}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y} - BF = 0.00000$$

$$\frac{X^{\frac{1}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y} - BJ = 0.00000$$

$$\frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{2}}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} = 0.27014$$

$$\frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{2}}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} - BE = 0.00000$$

$$\frac{\sqrt{X}\cdot(\sqrt{Y}-\sqrt{X})\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{2}}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} - BH = 0.00000$$

$$\frac{X^{\frac{3}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)^{2}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^{3}}{Y\cdot(\sqrt{X}+\sqrt{Y})^{2}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^{2}} - BC = 0.00000$$

Unit.

$AC := 1$

Given.

$N_1 := 9 \quad CE := N_1$

122995A

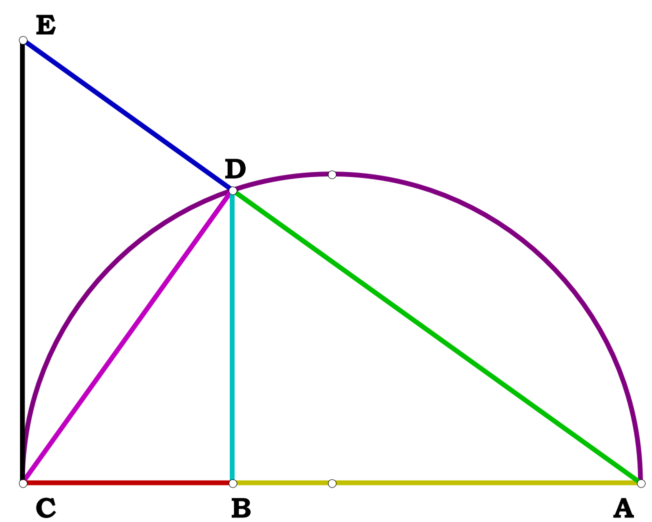
Given AC and CE find BC.

Descriptions.

$$AE := \sqrt{AC^2 + CE^2} \qquad AD := \frac{AC \cdot AC}{AE}$$

$$AB := \frac{AD^2}{AC} \qquad BC := AC - AB \qquad BD := \sqrt{AB \cdot BC}$$

$$CD := \sqrt{BC^2 + BD^2} \qquad DE := AE - AD$$



Definitions.

$$BC - \frac{N_1^{\,2}}{N_1^{\,2} + 1} = 0 \qquad BD - \frac{N_1}{N_1^{\,2} + 1} = 0 \qquad AB - \frac{1}{N_1^{\,2} + 1} = 0$$

$$AD - \frac{1}{\sqrt{N_1^{\,2} + 1}} = 0 \qquad DE - \frac{N_1^{\,2}}{\sqrt{1 + N_1^{\,2}}} = 0$$

Given.

$X := 7 \qquad CE := X$

$Y := 11 \qquad AC := Y$

122995B

Descriptions.

$$AE := \sqrt{AC^2 + CE^2} \qquad AD := \frac{AC \cdot AC}{AE}$$

$$AB := \frac{AD^2}{AC} \qquad BC := AC - AB \qquad BD := \sqrt{AB \cdot BC}$$

$$CD := \sqrt{BC^2 + BD^2} \qquad DE := AE - AD$$

Definitions.

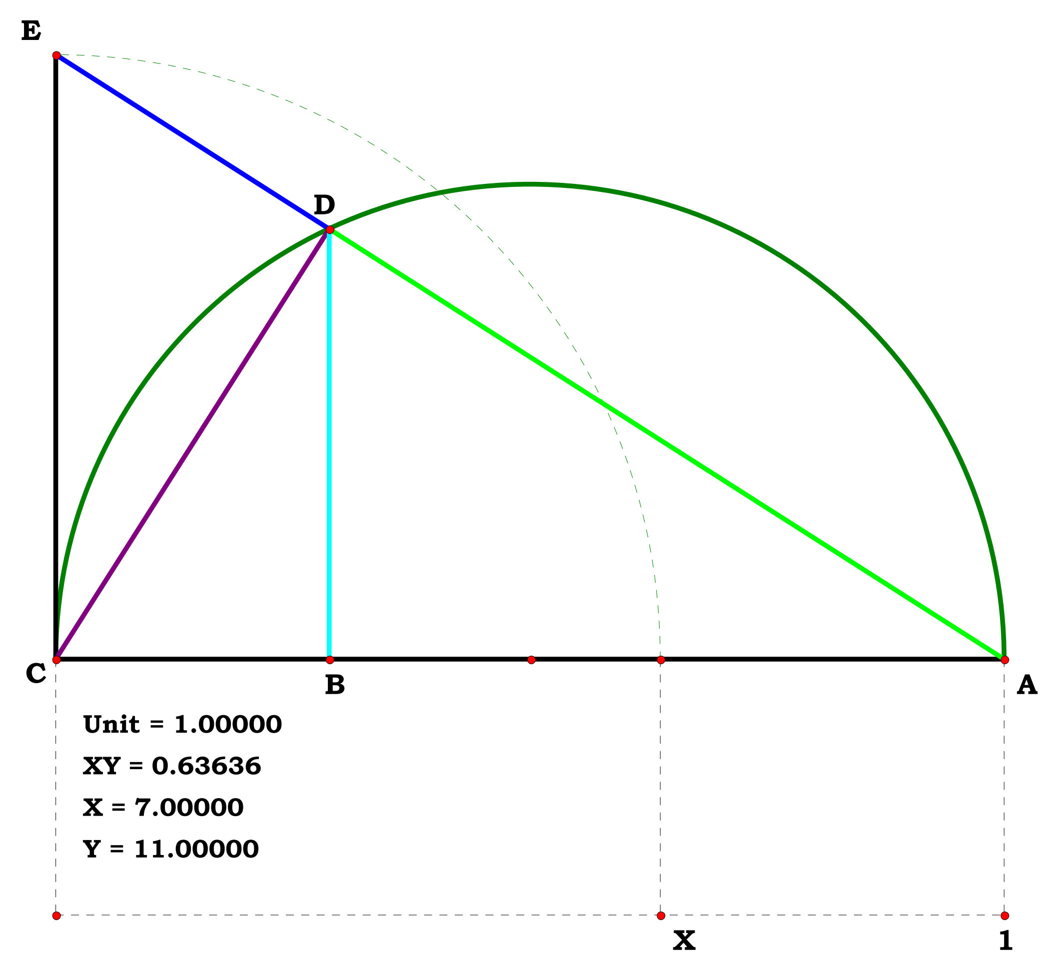
$$AE - \sqrt{Y^2 + X^2} = 0 \qquad AD - \frac{Y^2}{\sqrt{Y^2 + X^2}} = 0$$

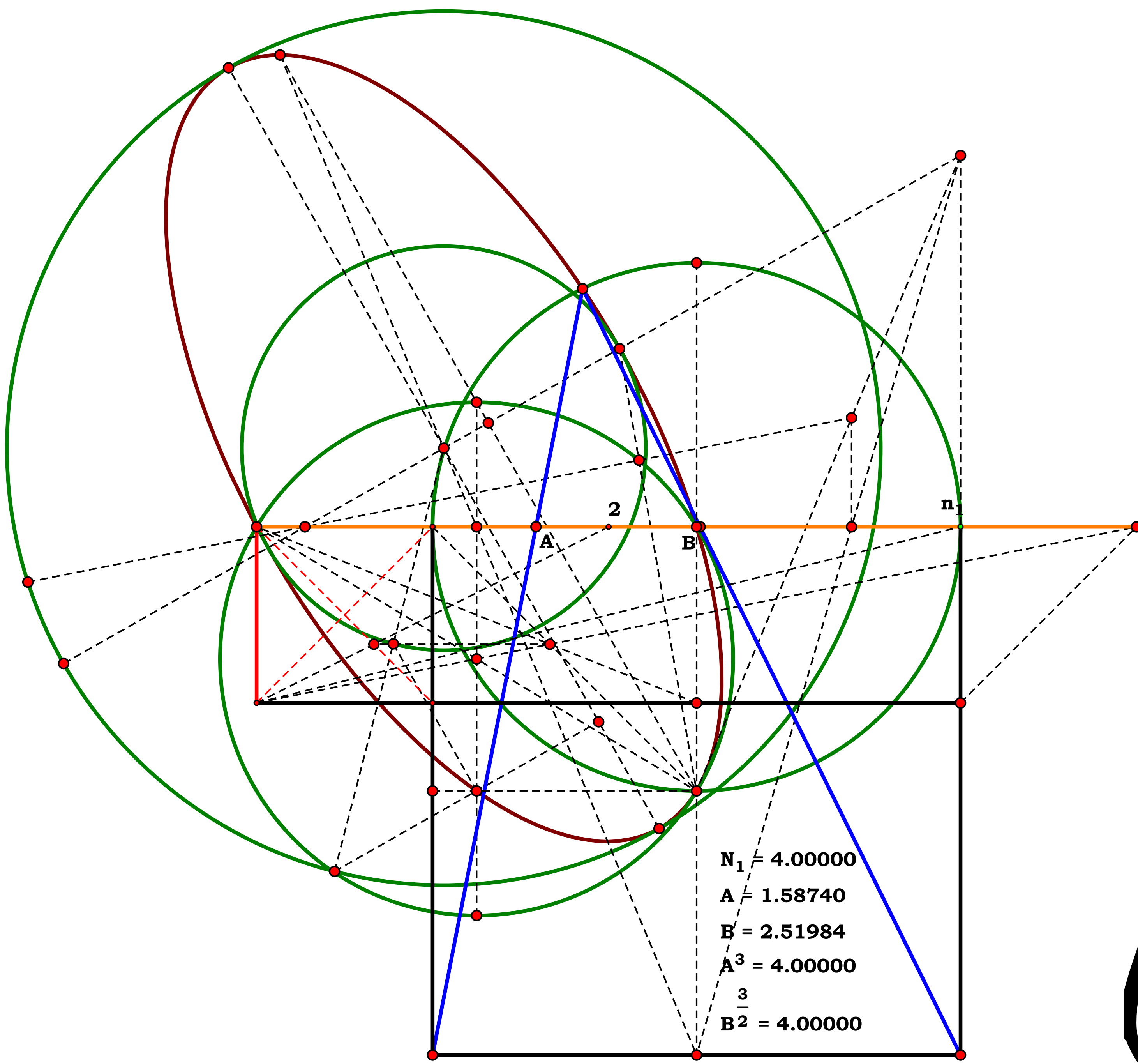
$$AB - \frac{Y^3}{X^2 + Y^2} = 0 \qquad BC - \frac{X^2 \cdot Y}{X^2 + Y^2} = 0$$

$$BD - \frac{X \cdot Y^2}{X^2 + Y^2} = 0 \qquad CD - \frac{X \cdot Y}{\sqrt{X^2 + Y^2}} = 0$$

$$DE - \frac{X^2}{\sqrt{X^2 + Y^2}} = 0$$

Given AC and CE find BC.





The Delian Quest 1996

John Clark

010496

The Archamedian Paper Trisector- Without the Numbers.

Given any circle AB.

Given any circle BC such that BC ≤ 2AB.

Construct AE such that AE = AC.

As AC = AB + BC and AD = AB so too DE = BC.

Construct DH parallel to BD. Construct CE.

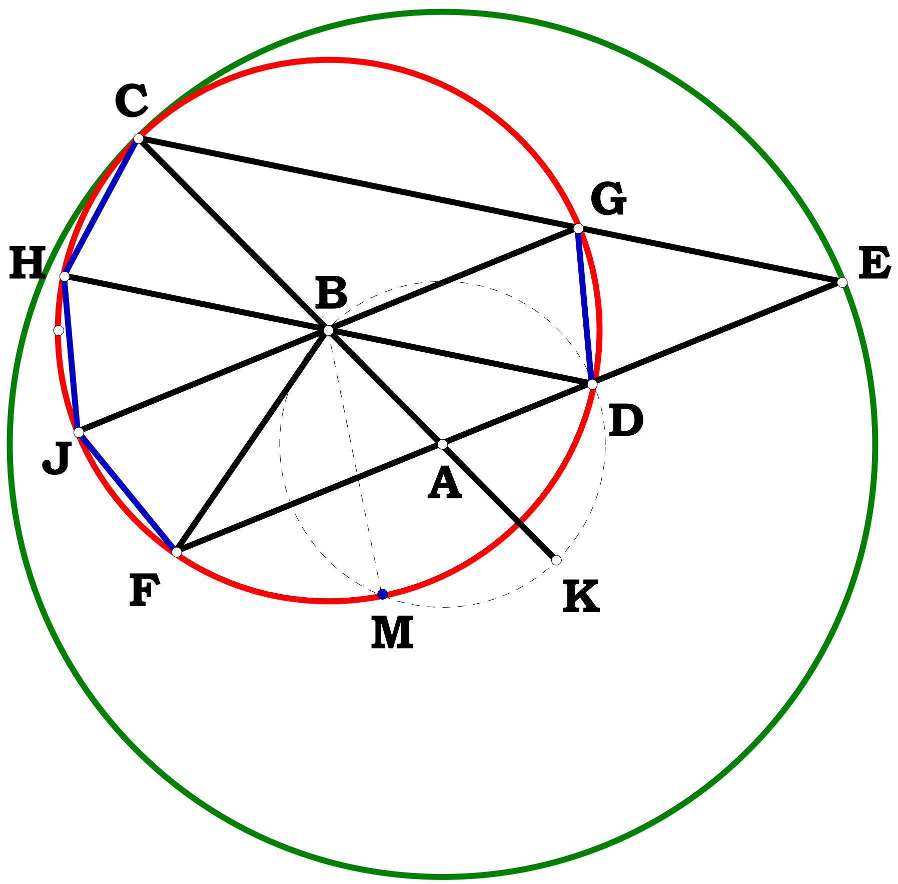
As AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD.

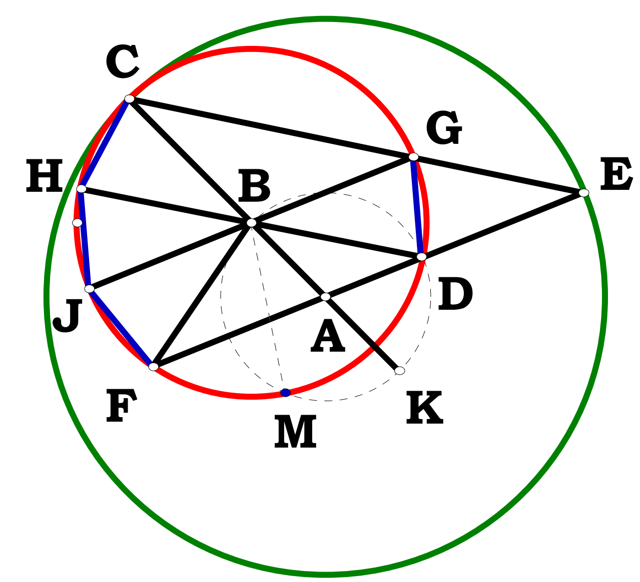
From here one can take two paths.

Construct GJ parallel to EF.

As CE is parallel to DH, DG = CH.

As GJ is parallel to EF, DG = FJ.





As ∠ HBJ is opposite and equal to ∠ GBD, DG = HJ, therefore ∠ DG is $\frac{1}{3}$ CF.

As CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.

By construction DK = KM.

As DH is parallel to CE, CH = DG.

As DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.

Unit.

$AD := 2$

Given.

$N := AD$

010796

A rusty Compass construction for the duplication of the cube.

Descriptions.

$AB := \frac{AD}{2}$ $\quad AG := \sqrt{2 \cdot AB^2}$ $\quad AF := \frac{AG}{9} \cdot 8$

$AC := AF$ $\quad AC = 1.257079$

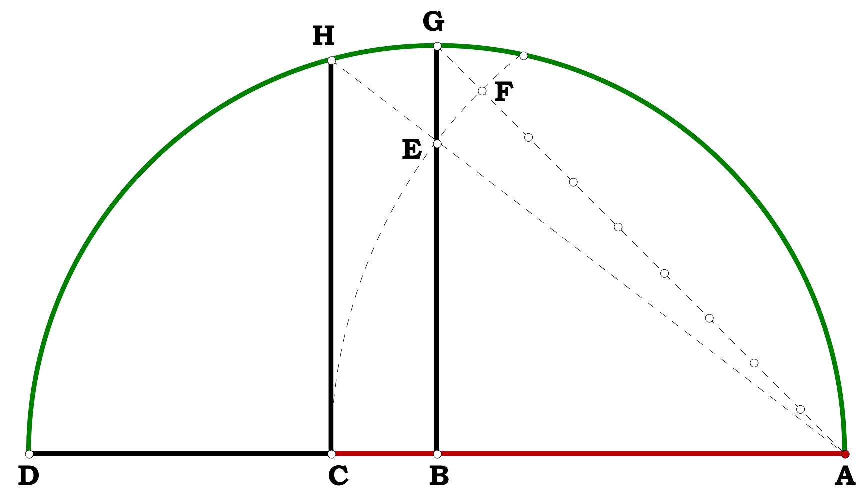
$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.259921$ $\quad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002261$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.

Definitions.

$$\frac{2^{\frac{1}{3}}}{2} \cdot \left(N^3\right)^{\frac{1}{3}} \qquad \frac{9 \cdot 2^{\frac{5}{6}} \cdot \left(N^3\right)^{\frac{1}{3}}}{16 \cdot \sqrt{N^2}}$$

Rusty Cubes



010896A1

Given.

$R_1 := 2$

$R_2 := 5$

$D := 4$

Descriptions.

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF + JK) \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \dfrac{DF \cdot FJ}{DF + JM} \quad GJ := FJ - FG \quad DI := \dfrac{DM}{2}$

$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \dfrac{AD \cdot FG}{DF}$

$GH := \dfrac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

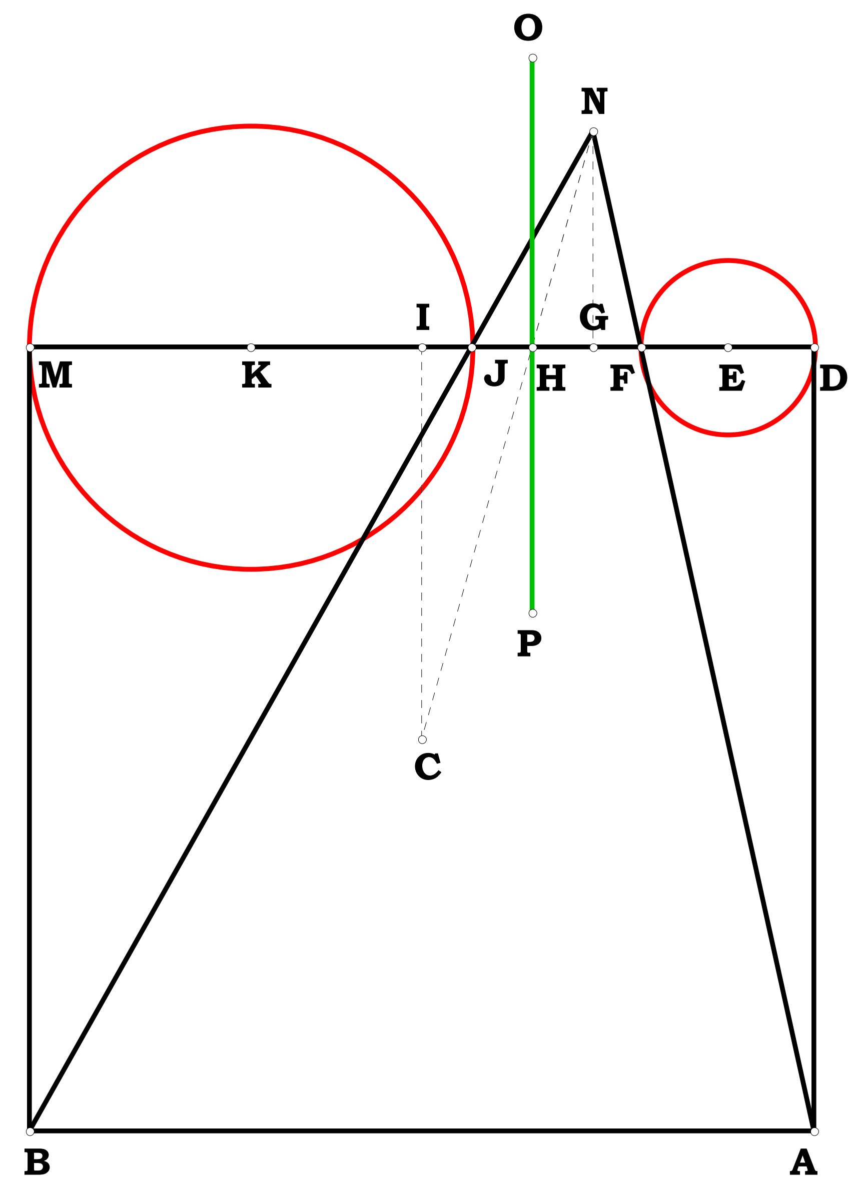
$HM := DM - DH$

Definitions.

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$

Alternate Method Power Line



010896A2

Unit.

$$BD := 1$$

Given.

$$W := 6 \qquad Y := 2$$

$$X := 20 \qquad Z := 17$$

Descriptions.

$$AB := \frac{W}{X} \qquad CD := \frac{Y}{Z} \qquad AC := BD - (AB + CD)$$

$$MN := AC - (AB + CD) \qquad BM := 2 \cdot AB \qquad DN := 2 \cdot CD$$

$$NO := \frac{DN \cdot MN}{BD - MN} \qquad MO := MN - NO \qquad DE := \frac{BD}{2}$$

$$DO := DN + NO \qquad EO := DE - DO \qquad EF := DE$$

$$JO := \frac{BD \cdot NO}{DN} \qquad KO := \frac{EO \cdot JO}{EF + JO}$$

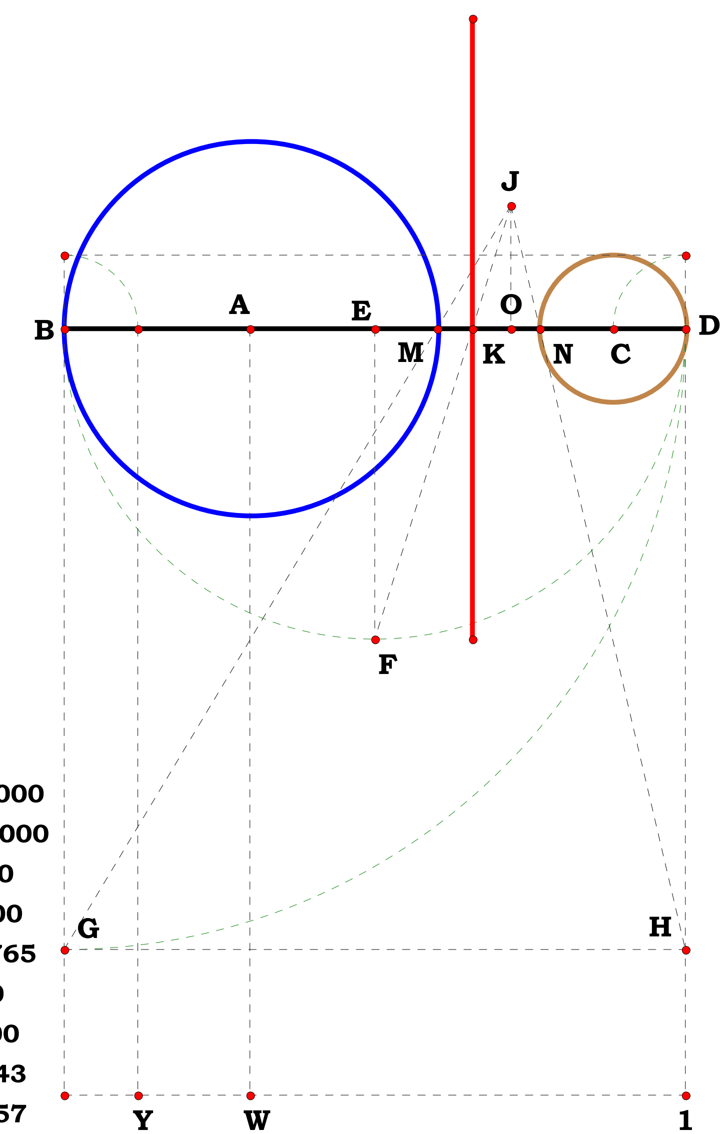
$$DK := DO + KO \qquad BK := BD - DK$$

$$DK = 0.343434 \qquad BK = 0.656566$$

Definitions.

$$DK - \frac{Z \cdot (2 \cdot W - X)}{2 \cdot [W \cdot Z + X \cdot (Y - Z)]} = 0 \qquad BK - \frac{X \cdot (2 \cdot Y - Z)}{2 \cdot [W \cdot Z + X \cdot (Y - Z)]} = 0$$

Alternate Method Power Line



Unit = 1.00000

W/X = 0.30000

W = 6.00000

X = 20.00000

Y/Z = 0.11765

Y = 2.00000

Z = 17.00000

DK = 0.34343

BK = 0.65657

$\frac{Z \cdot (2 \cdot W - X)}{2 \cdot (W \cdot Z + X \cdot (Y - Z))} - DK = 0.00000$

$\frac{X \cdot (2 \cdot Y - Z)}{2 \cdot (W \cdot Z + X \cdot (Y - Z))} - BK = 0.00000$

$$AB - \frac{W}{X} = 0 \qquad CD - \frac{Y}{Z} = 0 \qquad AC - \frac{X\cdot(Z-Y) - W\cdot Z}{X\cdot Z} = 0$$

$$MN - \frac{X\cdot Z - 2\cdot(X\cdot Y + W\cdot Z)}{X\cdot Z} = 0 \qquad BM - \frac{2\cdot W}{X} = 0$$

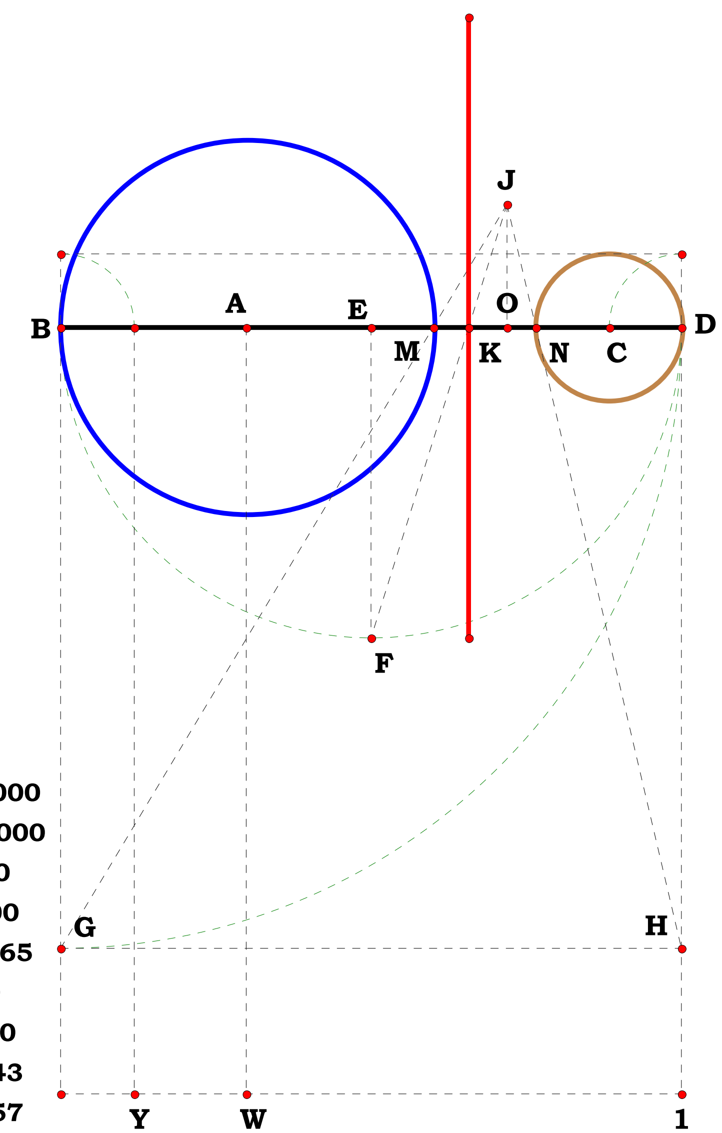
$$DN - \frac{2\cdot Y}{Z} = 0 \qquad NO - \frac{Y\cdot[X\cdot Z - 2\cdot(W\cdot Z + X\cdot Y)]}{Z\cdot(W\cdot Z + X\cdot Y)} = 0$$

$$MO - \frac{W\cdot[X\cdot Z - 2\cdot(W\cdot Z + X\cdot Y)]}{X\cdot(W\cdot Z + X\cdot Y)} = 0 \qquad DE - \frac{1}{2} = 0$$

$$DO - \frac{X\cdot Y}{W\cdot Z + X\cdot Y} = 0 \qquad EO - \frac{W\cdot Z - X\cdot Y}{2\cdot(W\cdot Z + X\cdot Y)} = 0$$

$$JO - \frac{X\cdot Z - 2\cdot(W\cdot Z + X\cdot Y)}{2\cdot(W\cdot Z + X\cdot Y)} = 0 \qquad EF - \frac{1}{2} = 0$$

$$KO - \frac{(W\cdot Z - X\cdot Y)\cdot[2\cdot(W\cdot Z + X\cdot Y) - X\cdot Z]}{2\cdot(W\cdot Z + X\cdot Y)\cdot[W\cdot Z + X\cdot(Y-Z)]} = 0$$



Unit = 1.00000

W/X = 0.30000

W = 6.00000

X = 20.00000

Y/Z = 0.11765

Y = 2.00000

Z = 17.00000

DK = 0.34343

BK = 0.65657

$$\frac{Z\cdot(2\cdot W - X)}{2\cdot(W\cdot Z + X\cdot(Y-Z))} - DK = 0.00000$$

$$\frac{X\cdot(2\cdot Y - Z)}{2\cdot(W\cdot Z + X\cdot(Y-Z))} - BK = 0.00000$$

Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

010896B1

Descriptions.

$BG := AG - AB$

$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

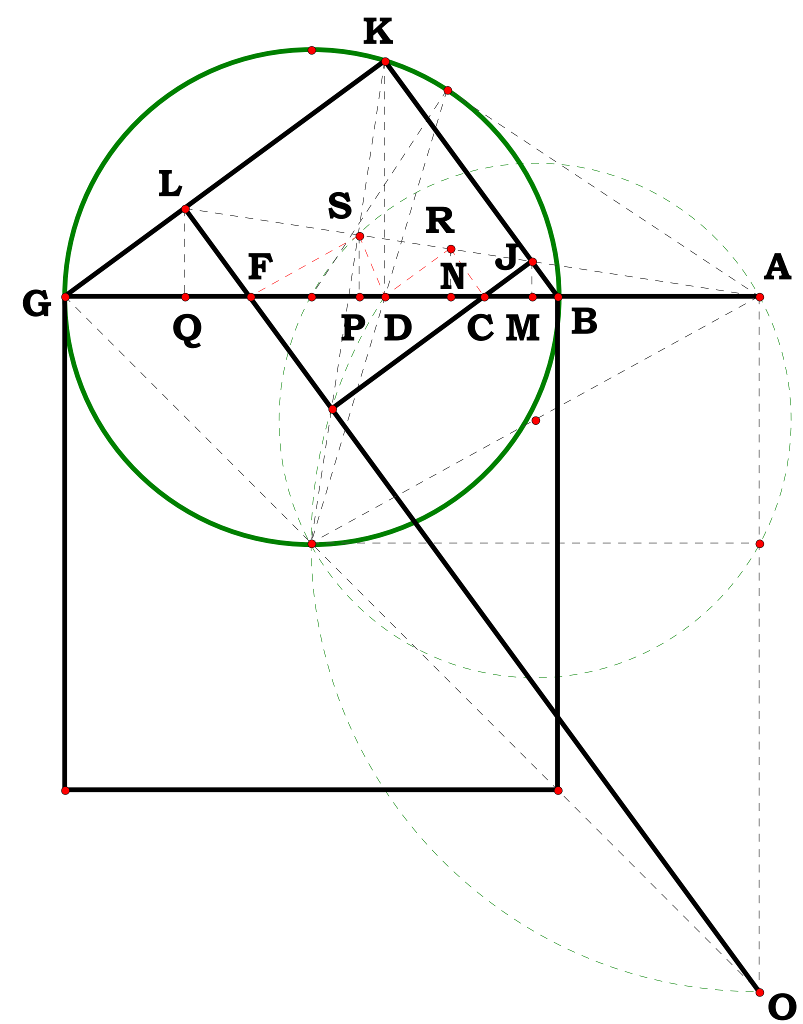
$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$\frac{GL}{BJ} = 5 \quad \frac{AG}{AB} = 5$

Quad Roots

The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.



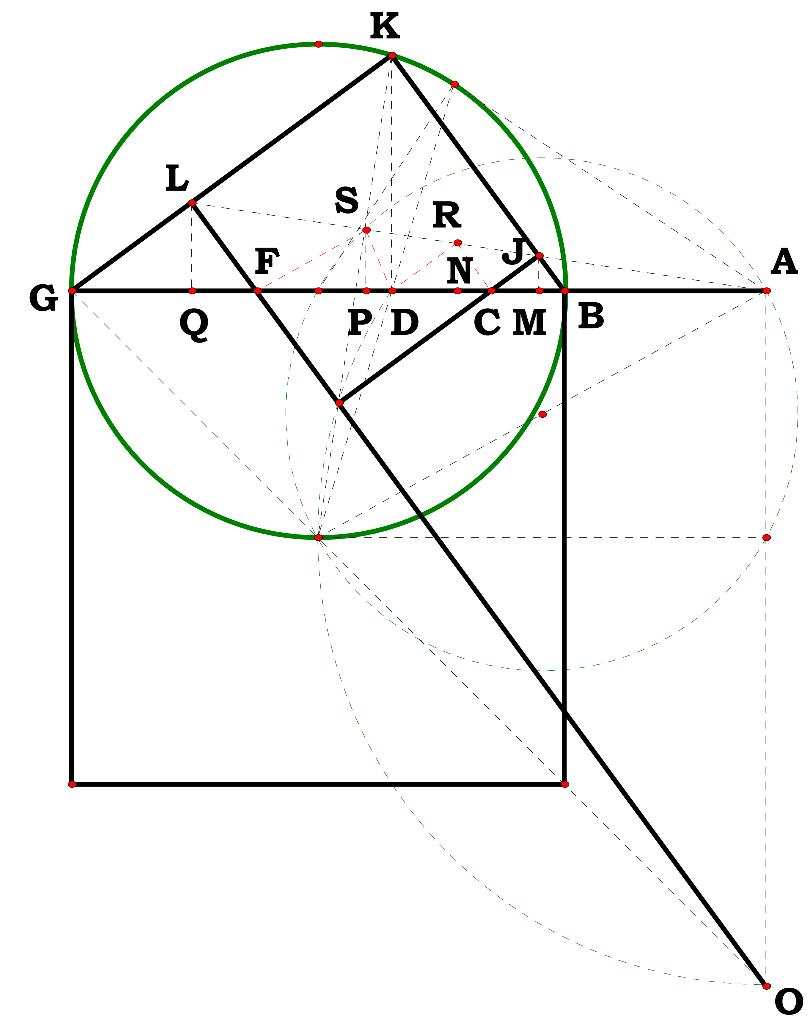
Definitions.

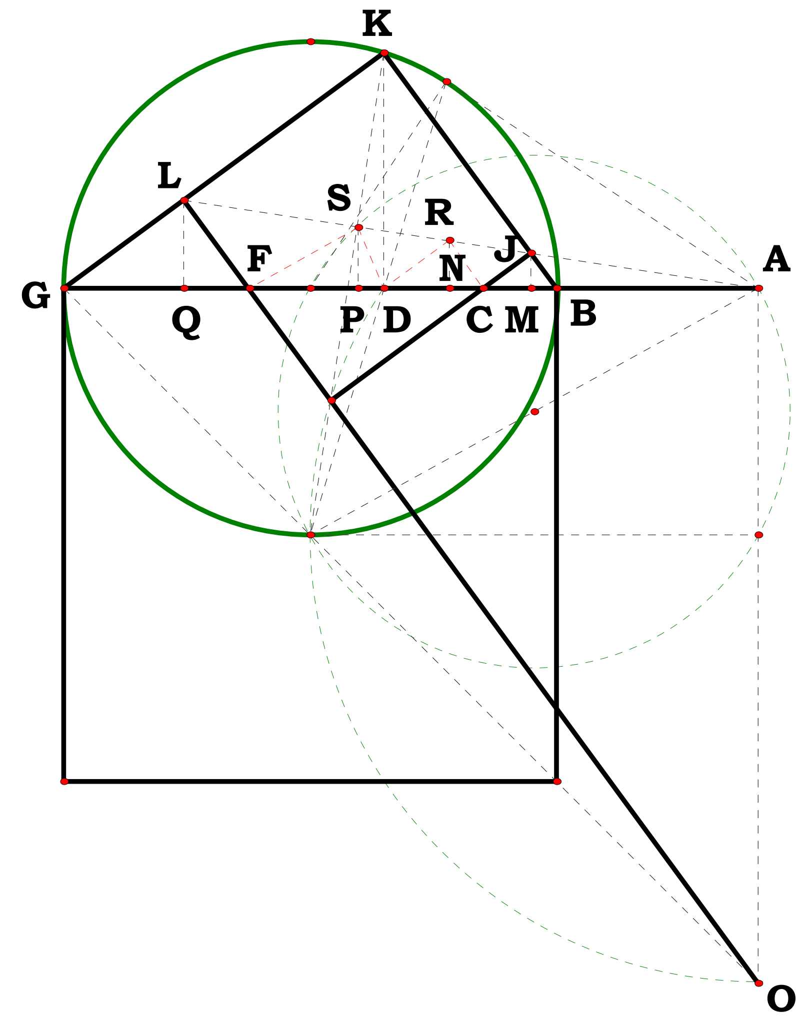
$$\frac{GK}{GL} - \left(\frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} \right) = 0 \qquad \frac{BK}{BJ} - \left(N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} \right) = 0$$

$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG} \quad CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG}$$

$$\frac{BG}{BM} - \left(N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} \right) = 0$$

$$\frac{BG}{CN} - \left(N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} \right) = 0$$





$$\frac{BG}{DP}-\left(N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}+N^{\frac{-2}{4}}\right)=0$$

$$\frac{BG}{FQ}-\left(N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}+N^{\frac{-1}{4}}+N^{\frac{-2}{4}}+N^{\frac{-3}{4}}\right)=0$$

$$\frac{AG}{BM}-\left(\frac{N^{\frac{3}{2}}+N}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0 \qquad \frac{AG}{CN}-\left(\frac{N^{\frac{5}{4}}+N^{\frac{3}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0$$

$$\frac{AG}{DP}-\left(\frac{N+N^{\frac{2}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0 \qquad \frac{AG}{FQ}-\left(\frac{N^{\frac{3}{4}}+N^{\frac{1}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0$$

010896B2

Descriptions.

Unit.

$AB := 1$

Given.

$X := 4$

$Y := 1$

$$AC := \frac{X}{Y} \qquad BC := AC - AB \qquad BD := \frac{BC}{2} \qquad AD := AB + BD$$

$$AE := \sqrt{AD^2 - BD^2} \qquad DE := AD - AE \qquad BE := AE - AB$$

$$CE := BC - BE \qquad EH := \sqrt{BE \cdot CE} \qquad EJ := \frac{DE \cdot EH}{BD + EH}$$

$$AJ := AE + EJ \qquad DJ := DE - EJ \qquad JK := \sqrt{BD^2 + DJ^2}$$

$$JM := \frac{DJ \cdot AJ}{JK} \qquad HK := \sqrt{(BD + EH)^2 + DE^2}$$

$$MH := HK - (JK + JM) \qquad OR := 2 \cdot MH \qquad BH := \sqrt{EH^2 + BE^2}$$

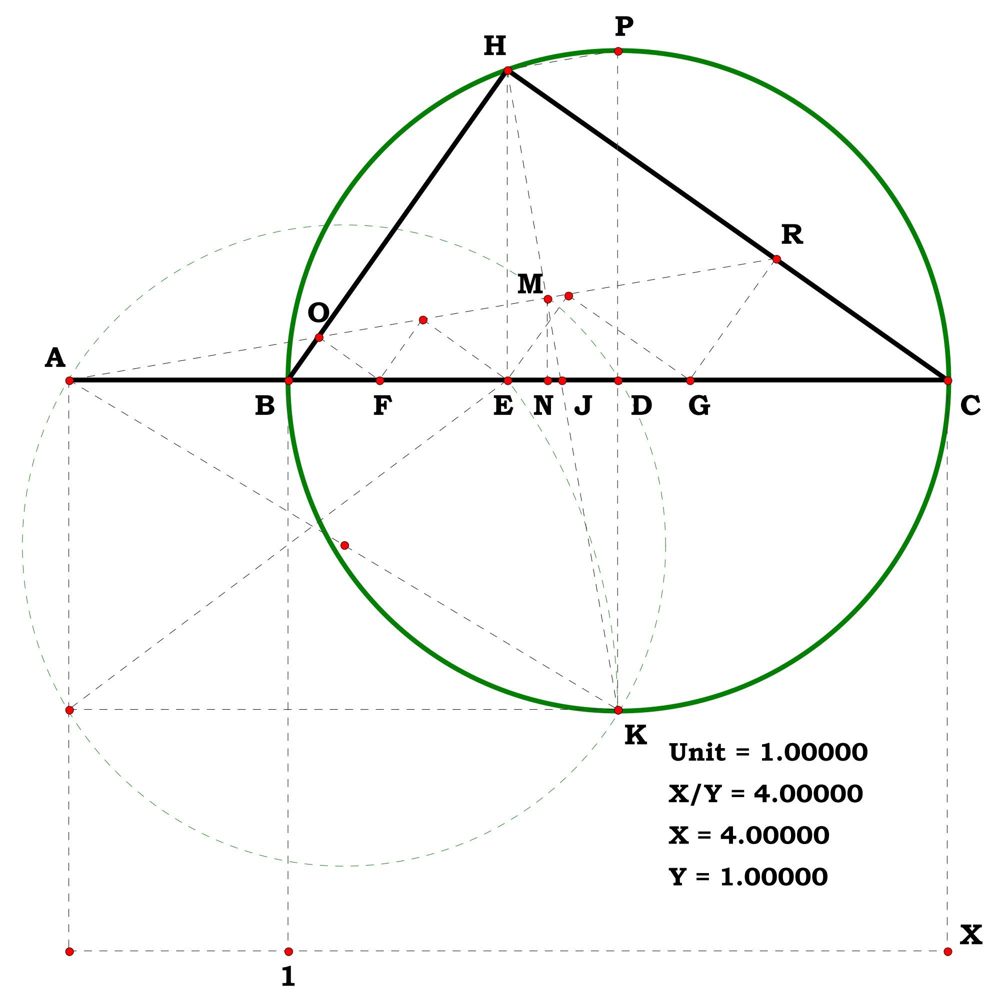
$$HO := \sqrt{2 \cdot MH^2} \qquad BO := BH - HO \qquad CH := \sqrt{CE^2 + EH^2}$$

$$BF := \frac{BC \cdot BO}{BH} \qquad AF := AB + BF \qquad CR := CH - HO$$

$$CG := \frac{BC \cdot CR}{CH} \qquad AG := AC - CG$$

$$AC^{\frac{1}{4}} - AF = 0 \qquad AC^{\frac{2}{4}} - AE = 0 \qquad AC^{\frac{3}{4}} - AG = 0$$

Quad Roots



$\frac{X}{Y}$ = 4.00000

AF = 1.41421

AE = 2.00000

AG = 2.82843

AC = 4.00000

$\frac{X}{Y}^{\frac{1}{4}}$-AF = 0.00000

$\frac{X}{Y}^{\frac{2}{4}}$-AE = 0.00000

$\frac{X}{Y}^{\frac{3}{4}}$-AG = 0.00000

$\frac{X}{Y}^{\frac{4}{4}}$-AC = 0.00000

Definitions.

$$AC - \frac{X}{Y} = 0 \qquad BC - \frac{X-Y}{Y} = 0 \qquad BD - \frac{X-Y}{2\cdot Y} = 0 \qquad AD - \frac{X+Y}{2\cdot Y} = 0 \qquad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$$

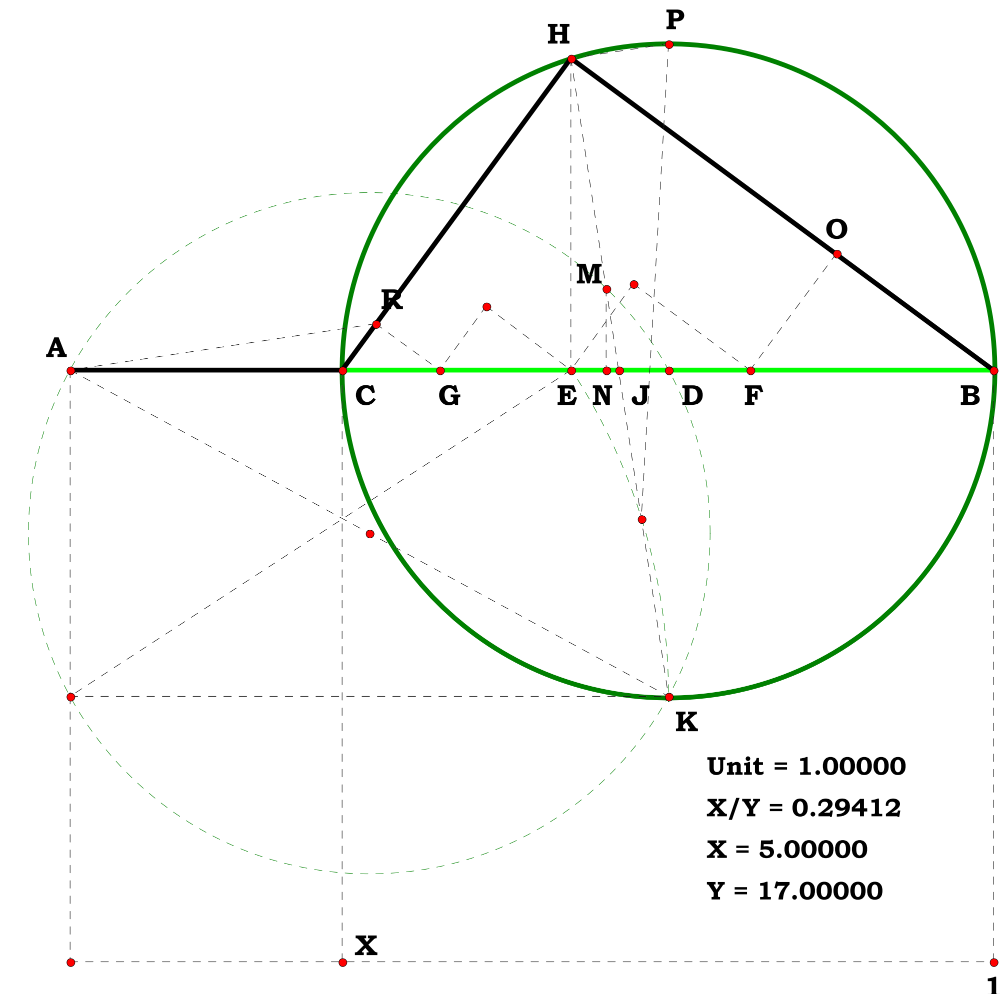
$$DE - \frac{\left(\sqrt{X}-\sqrt{Y}\right)^2}{2\cdot Y} = 0 \qquad BE - \frac{\sqrt{X}-\sqrt{Y}}{\sqrt{Y}} = 0 \qquad CE - \frac{\sqrt{X}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{Y} = 0$$

$$EH - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{Y^{\frac{3}{4}}} = 0 \qquad EJ - \frac{X^{\frac{1}{4}}\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)^2}{Y^{\frac{3}{4}}} = 0 \qquad AJ - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}-X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y^{\frac{3}{4}}} = 0$$

$$DJ - \frac{\left(\sqrt{X}+\sqrt{Y}\right)\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)^2}{2\cdot Y} = 0 \qquad JK - \frac{\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}\right)^{\frac{3}{2}}}{\sqrt{2}\cdot Y} = 0$$

$$JM - \frac{\sqrt{2}\cdot X^{\frac{1}{4}}\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}-X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{2\cdot Y^{\frac{3}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}\right)^{\frac{1}{2}}} = 0$$

$$HK - \frac{\sqrt{2}\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^2}{2\cdot Y} = 0 \qquad MH - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{2}\cdot\sqrt{X}+\sqrt{2}\cdot\sqrt{Y}+\sqrt{2}\cdot X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)}{2\cdot Y^{\frac{3}{4}}\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} = 0$$



$\frac{X}{Y}$ = 0.29412

AF = 0.73643

AE = 0.54233

AG = 0.39938

AC = 0.29412

$\left(\frac{X}{Y}\right)^{\frac{1}{4}}$-AF = 0.00000

$\left(\frac{X}{Y}\right)^{\frac{2}{4}}$-AE = 0.00000

$\left(\frac{X}{Y}\right)^{\frac{3}{4}}$-AG = 0.00000

$\left(\frac{X}{Y}\right)^{\frac{4}{4}}$-AC = 0.00000

$$\mathbf{OR} - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{2}\cdot\sqrt{X}+\sqrt{2}\cdot\sqrt{Y}+\sqrt{2}\cdot X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)}{Y^{\frac{3}{4}}\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} = 0$$

$$\mathbf{BH} - \frac{\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{Y^{\frac{3}{4}}} = 0$$

$$\mathbf{HO} - \frac{\left(X^{\frac{1}{4}}\right)\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y^{\frac{3}{4}}\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} = 0$$

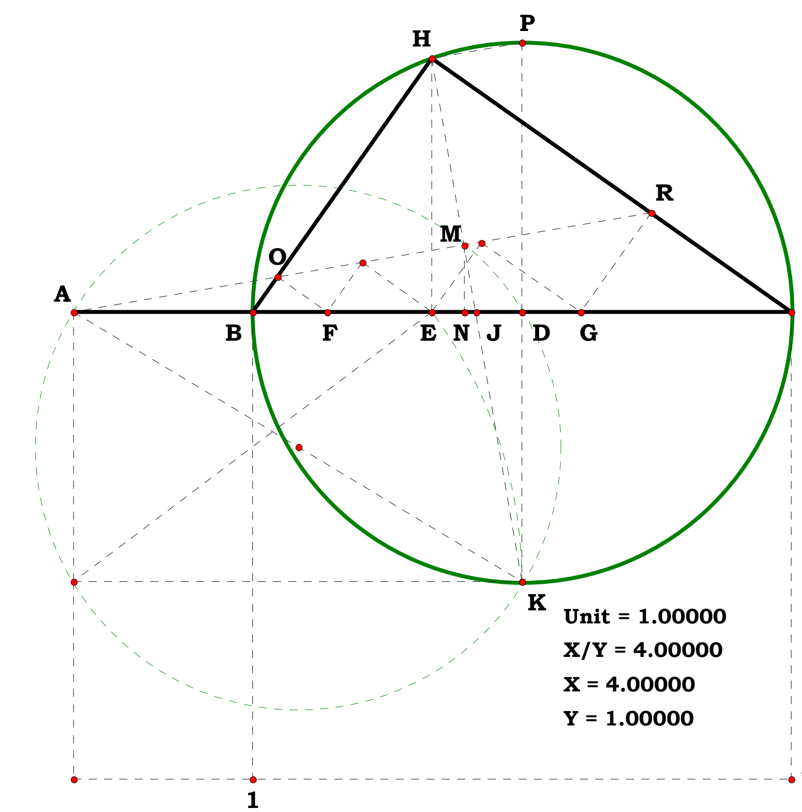
$$\mathbf{BO} - \frac{X^{\frac{1}{4}}-Y^{\frac{1}{4}}}{\sqrt{\sqrt{X}+\sqrt{Y}}} = 0 \qquad \mathbf{CH} - \frac{\sqrt{\sqrt{X}\cdot\left(\sqrt{X}+\sqrt{Y}\right)}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{Y} = 0 \qquad \mathbf{BF} - \frac{X^{\frac{1}{4}}-Y^{\frac{1}{4}}}{Y^{\frac{1}{4}}} = 0 \qquad \mathbf{AF} - \frac{X^{\frac{1}{4}}}{Y^{\frac{1}{4}}} = 0$$

$$\mathbf{CR} - \frac{\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)\cdot\sqrt{X+\sqrt{X}\cdot\sqrt{Y}}-\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}\cdot\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{3}{4}}+X^{\frac{3}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} = 0$$

$$\mathbf{CG} - \frac{\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}\right)\cdot\left[\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)\cdot\sqrt{X+\sqrt{X}\cdot\sqrt{Y}}-\left(\sqrt{X}\cdot\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{3}{4}}+X^{\frac{3}{4}}\cdot Y^{\frac{1}{4}}\right)\right]}{Y\cdot\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\sqrt{\sqrt{X}\cdot\left(\sqrt{X}+\sqrt{Y}\right)}} = 0$$

$$\mathbf{AG} - \frac{Y^{\frac{3}{4}}\cdot\sqrt{X+\sqrt{X}\cdot\sqrt{Y}}+\left(X-X^{\frac{1}{4}}\cdot Y^{\frac{3}{4}}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}}{Y^{\frac{3}{4}}\cdot\sqrt{X+\sqrt{X}\cdot\sqrt{Y}}} = 0$$

This seems to be about the best Mathcad 15 can do for AG.



$\frac{X}{Y} = 4.00000$

AF = 1.41421

AE = 2.00000

AG = 2.82843

AC = 4.00000

$\frac{X}{Y}^{\frac{1}{4}}$-AF = 0.00000

$\frac{X}{Y}^{\frac{2}{4}}$-AE = 0.00000

$\frac{X}{Y}^{\frac{3}{4}}$-AG = 0.00000

$\frac{X}{Y}^{\frac{4}{4}}$-AC = 0.00000

Pyramid of Ratios VI, Moving the Point

011396A

Unit.

$AD := 1$

Given.

$N_1 := 5 \qquad N_a := 2 .. N_1$

$N_2 := 7 \qquad N_b := 2 .. N_2$

Descriptions.

$AB := \dfrac{AD}{N_1} \qquad BD := AD - AB \qquad BG := \sqrt{AB \cdot BD} \qquad BE := \dfrac{BG}{N_2}$

$BC := \dfrac{BD \cdot BE}{BG} \qquad AE := \sqrt{AB^2 + BE^2} \qquad AC := AB + BC \qquad AF := \dfrac{AE \cdot AD}{AC}$

$EF := AF - AE \qquad \dfrac{AF}{EF} - \dfrac{N_1 \cdot N_2}{(N_1 - 1)\cdot(N_2 - 1)} = 0 \qquad \dfrac{AF}{EF} = 1.458333$

Definitions.

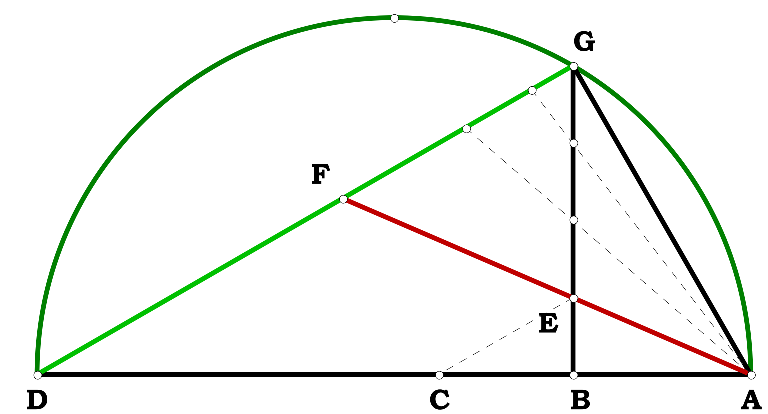
$$SeriesAF_{N_a, N_b} := \frac{N_a \cdot N_b}{(N_a - 1)\cdot(N_b - 1)}$$

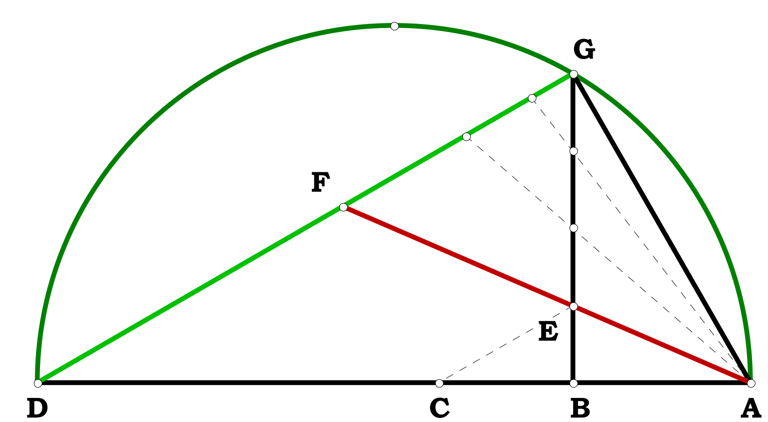
$$SeriesAF = \begin{pmatrix} 4 & 3 & 2.666667 & 2.5 & 2.4 & 2.333333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.666667 & 2 & 1.777778 & 1.666667 & 1.6 & 1.555556 \\ 2.5 & 1.875 & 1.666667 & 1.5625 & 1.5 & 1.458333 \end{pmatrix}$$

$DG := \sqrt{BD^2 + BG^2} \qquad CE := \sqrt{BC^2 + BE^2} \qquad DF := \dfrac{CE \cdot AD}{AC} \qquad GF := DG - DF$

$\dfrac{DG}{GF} - \dfrac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0 \qquad SeriesDG_{N_a, N_b} := \dfrac{(N_b + N_a - 1)}{(N_b - 1)} \qquad \dfrac{DG}{GF} = 1.833333$

$$SeriesDG = \begin{pmatrix} 3 & 2 & 1.666667 & 1.5 & 1.4 & 1.333333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333333 & 2 & 1.8 & 1.666667 \\ 6 & 3.5 & 2.666667 & 2.25 & 2 & 1.833333 \end{pmatrix}$$





011396B

Unit.

$AD := 1$

Given.

$W := 5 \qquad Y := 14$

$X := 20 \qquad Z := 20$

Descriptions.

$$AB := \frac{W}{X} \qquad BD := AD - AB \qquad BG := \sqrt{AB \cdot BD} \qquad BE := \frac{BG \cdot (Z - Y)}{Z}$$

$$BC := \frac{BD \cdot BE}{BG} \qquad AE := \sqrt{AB^2 + BE^2} \qquad AC := AB + BC \qquad AF := \frac{AE \cdot AD}{AC}$$

$$EF := AF - AE \qquad DG := \sqrt{BD^2 + BG^2} \qquad CE := \sqrt{BC^2 + BE^2}$$

$$DF := \frac{CE \cdot AD}{AC} \qquad GF := DG - DF \qquad \frac{AF}{EF} = 1.904762 \qquad \frac{DG}{GF} = 2.714286$$

Definitions.

$$AB - \frac{W}{X} = 0 \qquad BD - \frac{X - W}{X} = 0 \qquad BG - \frac{\sqrt{W \cdot (X - W)}}{X} = 0$$

$$BE - \frac{\sqrt{W \cdot X - W^2} \cdot (Z - Y)}{X \cdot Z} = 0 \qquad BC - \frac{(W - X) \cdot (Y - Z) \cdot \sqrt{W \cdot X - W^2}}{X \cdot Z \cdot \sqrt{-W \cdot (W - X)}} = 0$$

$$AE - \frac{\sqrt{W \cdot \left[Y^2 \cdot (X - W) + X \cdot Z^2 + 2 \cdot Y \cdot Z \cdot (W - X)\right]}}{X \cdot Z} = 0$$

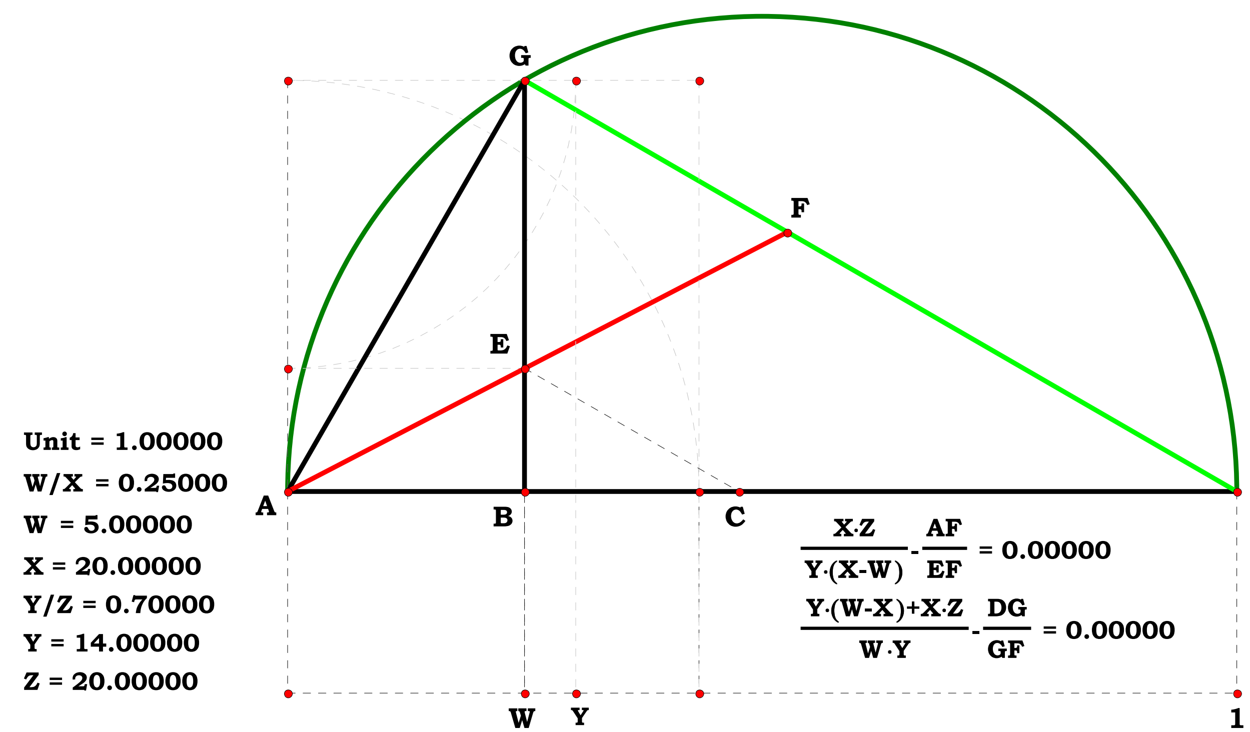
$$AC - \frac{Y \cdot (W - X) + X \cdot Z}{X \cdot Z} = 0 \qquad AF - \frac{\sqrt{W \cdot \left[X \cdot Z^2 - Y \cdot (Y - 2 \cdot Z) \cdot (W - X)\right]}}{W \cdot Y - X \cdot Y + X \cdot Z} = 0$$

$$EF - \frac{\sqrt{W \cdot \left[Y^2 \cdot (X - W) + X \cdot Z^2 + 2 \cdot Y \cdot Z \cdot (W - X)\right]} \cdot Y \cdot (W - X)}{X \cdot Z \cdot [Y \cdot (X - W) - X \cdot Z]} = 0$$

$$DG - \frac{\sqrt{X - W}}{\sqrt{X}} = 0 \qquad CE - \frac{(Z - Y) \cdot \sqrt{X - W}}{\sqrt{X} \cdot Z} = 0 \qquad DF - \frac{\sqrt{X} \cdot \sqrt{X - W} \cdot (Z - Y)}{W \cdot Y - X \cdot Y + X \cdot Z} = 0 \qquad GF - \frac{W \cdot Y \cdot \sqrt{X - W}}{\sqrt{X} \cdot (W \cdot Y - X \cdot Y + X \cdot Z)} = 0$$

$$\frac{AF}{EF} - \frac{X \cdot Z}{Y \cdot (X - W)} = 0 \qquad \frac{AF}{EF} = 1.904762 \qquad \frac{DG}{GF} - \frac{Y \cdot (W - X) + X \cdot Z}{W \cdot Y} = 0 \qquad \frac{DG}{GF} = 2.714286$$

Pyramid of Ratios VI, Moving the Point



011696

Unit.

$$BD := 1$$

$$N_a := 3$$

Descriptions.

$$BC := \frac{BD}{2} \qquad CG := \frac{BC}{N_a} \qquad BG := BC - CG \qquad DG := BC + CG$$

$$GH := \sqrt{BG \cdot DG} \qquad CF := BC \qquad CO := \frac{CG \cdot CF}{GH + CF}$$

$$GJ := \frac{BG \cdot GH}{BD + GH} \qquad GK := \frac{DG \cdot GH}{BD + GH} \qquad JK := GJ + GK$$

$$GT := \frac{GH \cdot JK}{BD} \qquad AG := \frac{CF \cdot GT}{CO} \qquad AD := AG + DG$$

$$AJ := AG - GJ \qquad AK := AG + GK \qquad AB := AD - BD$$

$$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} - AJ = 0 \qquad \left(AB \cdot AD^2\right)^{\frac{1}{3}} - AK = 0$$

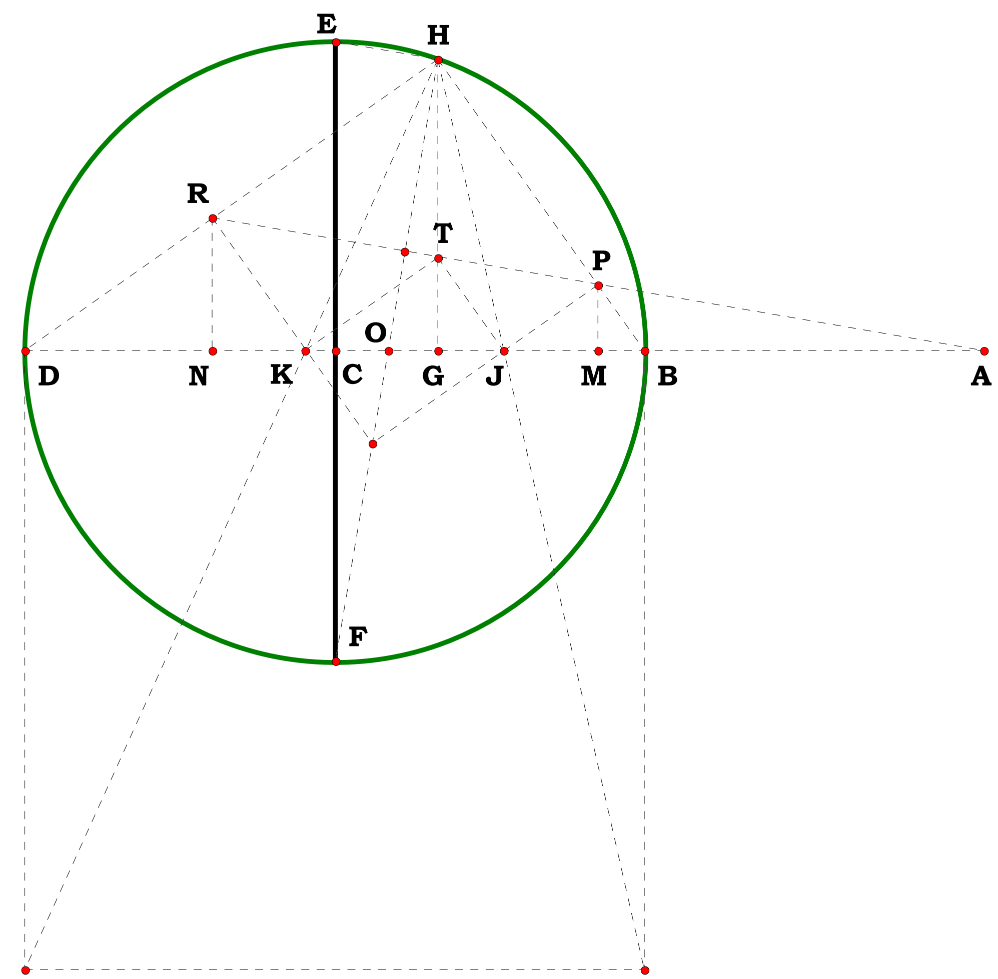
$$BH := \sqrt{GH^2 + BG^2} \qquad DH := \sqrt{GH^2 + DG^2}$$

$$BJ := BG - GJ \qquad DK := DG - GK$$

$$DR := \frac{DH \cdot DK}{BD} \qquad BP := \frac{BH \cdot BJ}{BD} \qquad BM := \frac{BG \cdot BJ}{BD} \qquad KN := \frac{BG \cdot DK}{BD}$$

$$\frac{AD}{AB} = 2.828427 \qquad \frac{DR}{BP} = 2.828427 \qquad N := \frac{AD}{AB}$$

The figure cuts the base in Cube Roots and provides some interesting ratios.



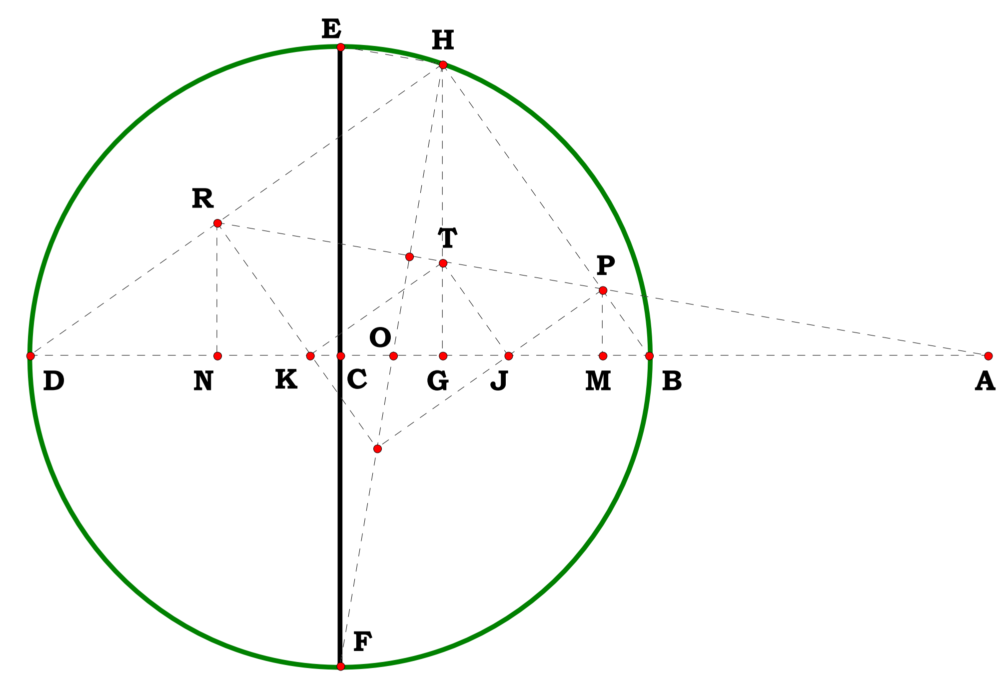
Definitions.

$$\left(\frac{AB}{AD}\right)^{\frac{2}{3}}+\left(\frac{AB}{AD}\right)^{\frac{1}{3}}+\left(\frac{AB}{AD}\right)^{\frac{0}{3}}=2.207107 \qquad \frac{DH}{DR}=2.207107 \qquad \frac{N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}}{N^{\frac{2}{3}}}=2.207107$$

$$\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}=4.414214 \qquad \frac{BH}{BP}=4.414214 \qquad N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}=4.414214$$

$$\left(\frac{AD}{AB}\right)^{\frac{4}{3}}+\left(\frac{AD}{AB}\right)^{\frac{3}{3}}+\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}=13.242641$$

$$N^{\frac{4}{3}}+N^{\frac{3}{3}}+N^{\frac{2}{3}}+N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}=13.242641 \qquad \frac{BD}{BM}=13.242641$$



$$\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{3}{3}}+\left(\frac{AB}{AD}\right)^{\frac{0}{3}}+\left(\frac{AB}{AD}\right)^{\frac{1}{3}}=9.363961$$

$$N^{\frac{3}{3}}+N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}+\frac{1}{N^{\frac{1}{3}}}=9.363961 \qquad \frac{BD}{GJ}=9.363961$$

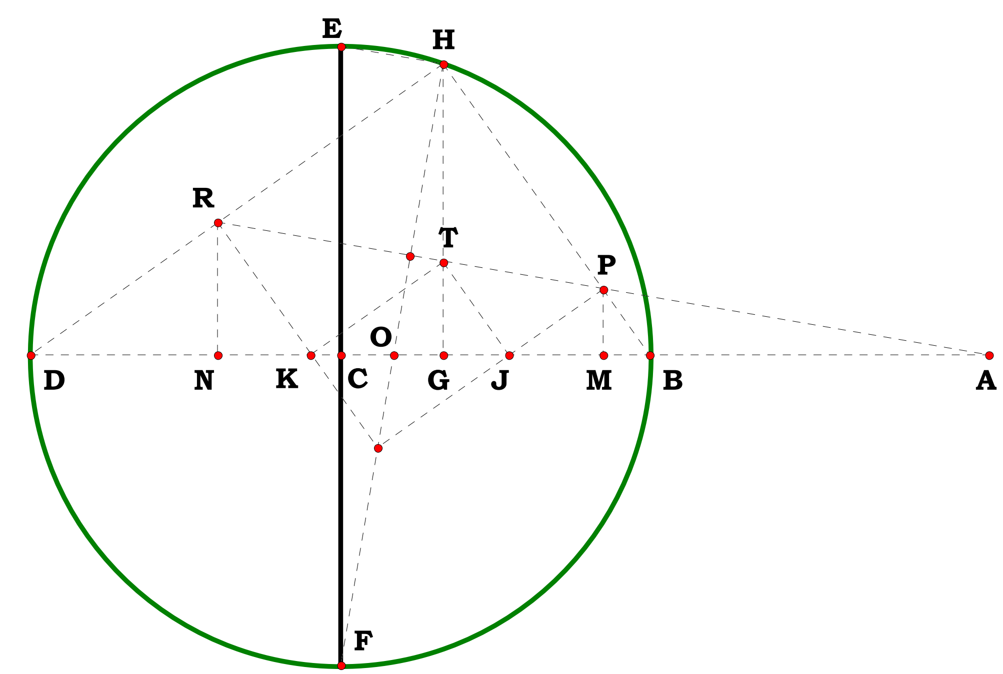
$$\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}+\left(\frac{AB}{AD}\right)^{\frac{1}{3}}+\left(\frac{AB}{AD}\right)^{\frac{2}{3}}=6.62132$$

$$N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}+N^{\frac{0}{3}}+\frac{1}{N^{\frac{1}{3}}}+\frac{1}{N^{\frac{2}{3}}}=6.62132$$

$$\frac{AD^{\frac{5}{3}}+AB^{\frac{2}{3}}\cdot AD}{AD^{\frac{1}{3}}\cdot AB^{\frac{4}{3}}-AB^{\frac{5}{3}}}=20.485281 \qquad \frac{AD}{BM}=20.485281 \qquad \frac{N^{\frac{5}{3}}+N}{N^{\frac{1}{3}}-N^{\frac{0}{3}}}=20.485281$$

$$\frac{AD^{\frac{4}{3}}+AB^{\frac{2}{3}}\cdot AD^{\frac{2}{3}}}{AD^{\frac{1}{3}}\cdot AB-AB^{\frac{4}{3}}}=14.485281 \qquad \frac{AD}{GJ}=14.485281 \qquad \frac{N^{\frac{4}{3}}+N^{\frac{2}{3}}}{N^{\frac{1}{3}}-N^{\frac{0}{3}}}=14.485281$$

$$\frac{AD+AB^{\frac{2}{3}}\cdot AD^{\frac{1}{3}}}{AD^{\frac{1}{3}}\cdot AB^{\frac{2}{3}}-AB}=10.242641 \qquad \frac{AD}{KN}=10.242641 \qquad \frac{N+N^{\frac{1}{3}}}{N^{\frac{1}{3}}-N^{\frac{0}{3}}}=10.242641$$



011796A

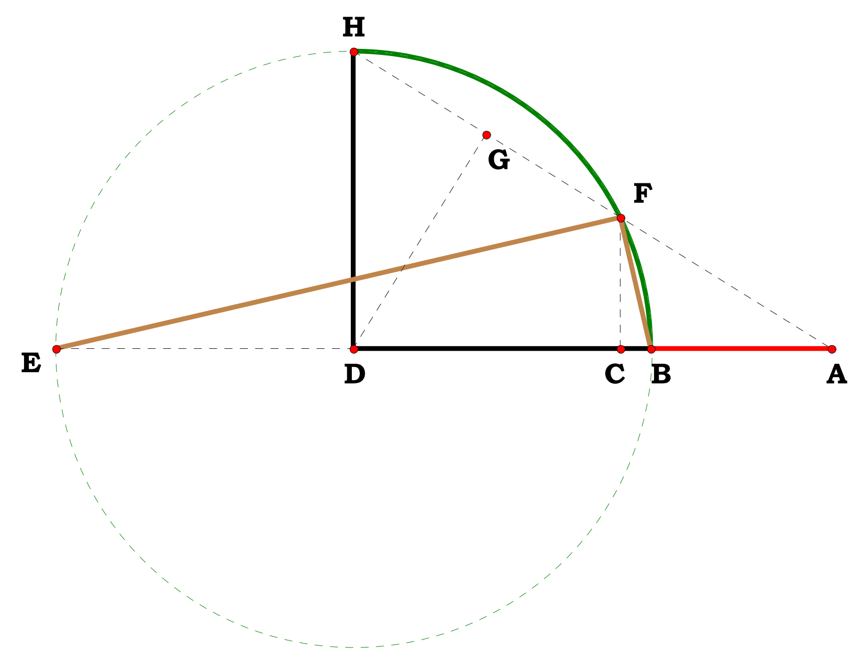
Given.

$AE := 5$

$AB := 3$

Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.

Right Triangle In A Given Ratio



Descriptions.

$BE := AE - AB \quad BD := \dfrac{BE}{2} \quad DF := BD \quad DE := BD$

$AD := AB + BD \quad DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \dfrac{AD \cdot AD}{AH}$

$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$CD := \dfrac{AD^2 - AF^2 + DF^2}{2 \cdot AD} \quad BC := BD - CD \quad CE := CD + DE$

$CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2}$

$\dfrac{AE}{AB} - \dfrac{EF}{BF} = 0$

Definitions.

$BE - (AE - AB) = 0 \quad BD - \dfrac{AE - AB}{2} = 0 \quad DF - \dfrac{AE - AB}{2} = 0 \quad DE - \dfrac{AE - AB}{2} = 0$

$AD - \dfrac{AB + AE}{2} = 0 \quad DH - \dfrac{AE - AB}{2} = 0 \quad AH - \dfrac{\sqrt{AB^2 + AE^2}}{\sqrt{2}} = 0 \quad AG - \dfrac{\sqrt{2} \cdot (AB + AE)^2}{4 \cdot \sqrt{AB^2 + AE^2}} = 0$

$GH - \dfrac{\sqrt{2} \cdot \left(\sqrt{2} \cdot AB - \sqrt{2} \cdot AE\right)^2}{8 \cdot \sqrt{AB^2 + AE^2}} = 0 \quad FG - \dfrac{\sqrt{2} \cdot \left(\sqrt{2} \cdot AB - \sqrt{2} \cdot AE\right)^2}{8 \cdot \sqrt{AB^2 + AE^2}} = 0 \quad AF - \dfrac{\sqrt{2} \cdot AB \cdot AE}{\sqrt{AB^2 + AE^2}} = 0$

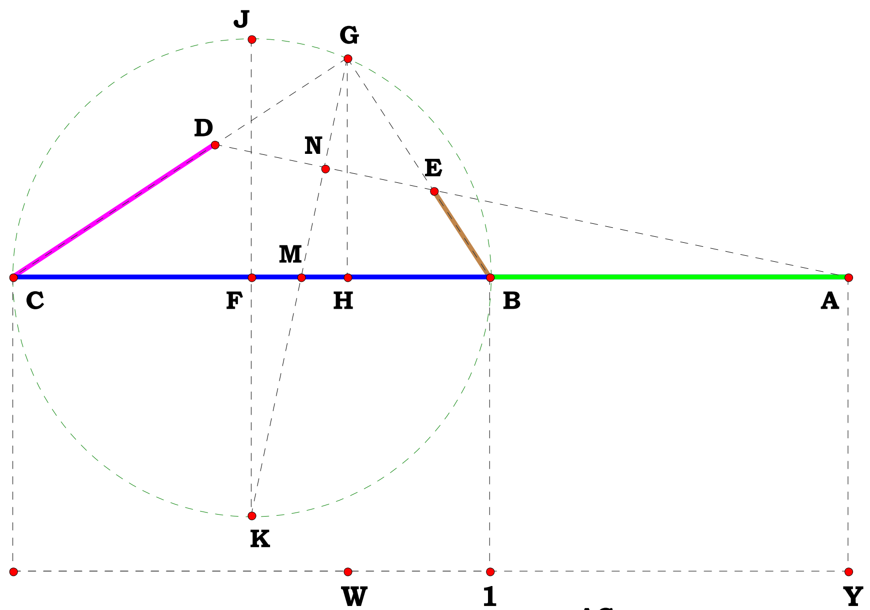
$CD - \dfrac{(AB - AE)^2 \cdot (AB + AE)}{2 \cdot \left(AB^2 + AE^2\right)} = 0 \quad BC - \dfrac{AB^2 \cdot (AE - AB)}{AB^2 + AE^2} = 0 \quad CE - \dfrac{AE^2 \cdot (AE - AB)}{AB^2 + AE^2} = 0$

$CF - \dfrac{AB \cdot AE \cdot (AE - AB)}{\left(AB^2 + AE^2\right)} = 0 \quad BF - \dfrac{AB \cdot (AE - AB)}{\sqrt{AB^2 + AE^2}} = 0 \quad EF - \dfrac{AE \cdot (AE - AB)}{\sqrt{AB^2 + AE^2}} = 0$

011796B1

Divide The Sides Of A Right Triangle In A Given Ratio

Here was another junk write up, not only that, the original plate was defective, not absolutely correct, but, it takes time to learn how to say what one sees and at this revision time, all of it has to be fixed. What is being noted is that in this figure, there is a constant ratio, no matter where GH is on BC, AB is to AC as CD is to BE It is very simple, and originally I over obfuscated the whole thing. In short, it has something to say about the common angle. I suspect, now that I review it, it is very iimportant. There is, and always has been, a physical standard for ratio in this respect.



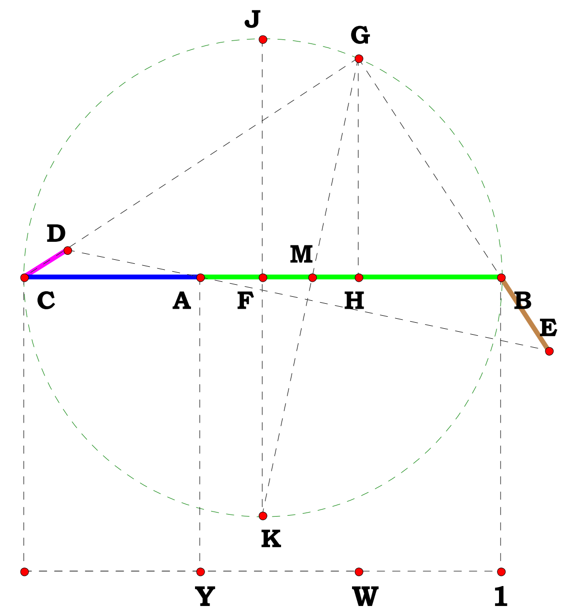
Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AB = 4.73462 cm
AC = 11.04746 cm
CD = 3.19202 cm
BE = 1.36801 cm

$\frac{AC}{AB} = 2.33333$

$\frac{CD}{BE} = 2.33333$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$



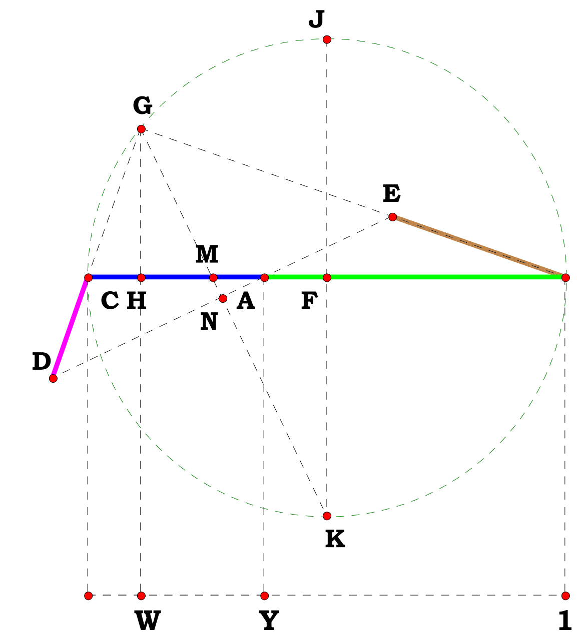
Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 0.67230 cm
BE = 1.15172 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$



Unit = 1.00000
W/X = 0.10988
W = 2.19759
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 1.42395 cm
BE = 2.43938 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$

Unit.

$AB := 1$

Given.

$N_1 := 2.33333 \quad AC := N_1$

$N_2 := .933333 \quad CH := N_2$

011796B1

Descriptions.

$BC := AC - AB \quad BF := \frac{BC}{2} \quad BH := BC - CH \quad GH := \sqrt{CH \cdot BH}$

$FK := BF \quad FH := CH - BF \quad GK := \sqrt{FH^2 + (FK + GH)^2}$

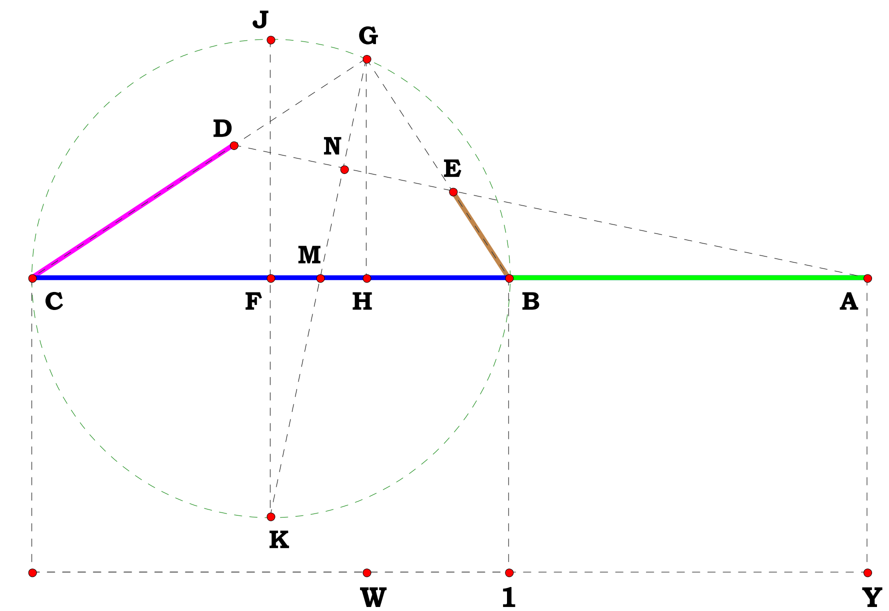
$FM := \frac{FH \cdot FK}{FK + GH} \quad HM := FH - FM \quad AH := AC - CH \quad AM := AH + HM$

$MN := \frac{FH \cdot AM}{GK} \quad KM := \sqrt{FK^2 + FM^2} \quad KN := KM + MN$

$GN := GK - KN \quad CG := \sqrt{CH^2 + GH^2} \quad BG := \sqrt{BH^2 + GH^2}$

$DG := \sqrt{2 \cdot GN^2} \quad EG := DG \quad CD := CG - DG \quad BE := BG - EG$

$\frac{AC}{AB} = 2.33333 \quad \frac{CD}{BE} = 2.33333 \quad \frac{AC}{AB} - \frac{CD}{BE} = 0$



Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AB = 4.73462 cm
AC = 11.04746 cm
CD = 3.19202 cm
BE = 1.36801 cm
CH = 4.41898 cm

$\frac{AC}{AB} = 2.33333$

$\frac{CD}{BE} = 2.33333$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$

$\frac{AB}{AB} = 1.00000$

$\frac{AC}{AB} = 2.33333$

$\frac{CH}{AB} = 0.93333$

$\frac{CD}{AB} = 0.67419$

$\frac{BE}{AB} = 0.28894$

AB = 1.00000
AC = 2.33333
CH = 0.93333
CD = 0.67419
BE = 0.28894

Definitions.

$BC - (N_1 - 1) = 0 \quad BF - \frac{N_1 - 1}{2} = 0 \quad BH - (N_1 - N_2 - 1) = 0 \quad GH - \sqrt{N_2 \cdot (N_1 - N_2 - 1)} = 0 \quad FK - \frac{N_1 - 1}{2} = 0$

$FH - \frac{2 \cdot N_2 - N_1 + 1}{2} = 0 \quad GK - \frac{\sqrt{(N_1 - 1) \cdot \left(N_1 + 2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} - 1\right)}}{\sqrt{2}} = 0 \quad FM - \frac{(N_1 - 1) \cdot (2 \cdot N_2 - N_1 + 1)}{2 \cdot \left[N_1 + 2 \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} - 1\right]} = 0 \quad HM - \frac{\sqrt{N_1 \cdot N_2 - N_2^2 - N_2} \cdot (2 \cdot N_2 - N_1 + 1)}{N_1 + 2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} - 1} = 0$

$AH - (N_1 - N_2) = 0 \quad AM - \frac{N_2 - N_1 + N_1^2 - N_1 \cdot N_2 + \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} \cdot (N_1 + 1)}{2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2 + N_1 - 1}} = 0 \quad MN - \frac{\sqrt{2} \cdot (2 \cdot N_2 - N_1 + 1) \cdot \left[(N_1 + 1) \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} + N_2 - N_1 + N_1^2 - N_1 \cdot N_2\right]}{2 \cdot \sqrt{(N_1 - 1) \cdot \left[N_1 + 2 \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} - 1\right]} \cdot \left[N_1 + 2 \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} - 1\right]} = 0$

$KM - \frac{\sqrt{(N_1 - 1)^3 \cdot \left(N_1 + 2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} - 1\right)}}{\sqrt{2 \cdot \left[(4 \cdot N_1 - 4) \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} + N_1^2 + 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 4 \cdot N_2^2 - 4 \cdot N_2 + 1\right]}} = 0$

$KN - (KM + MN) = 0$

On this equation, Matcad is going to expand it past its viewing ability. At the minimum of 10 percept page size, the results is well over 10 pages and it cannot reduce it. It would take hours to do this manually. A second approach, subing all the expressions that are reduced, and rebuild with that. See last page.

$$KN - \frac{\sqrt{2}\cdot\left[\sqrt{(N_1-1)^3\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]} \cdots + \sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}+N_1^2+4\cdot N_1\cdot N_2-2\cdot N_1-4\cdot N_2^2-4\cdot N_2+1\right]}\cdot(N_1-2\cdot N_2-1)\cdot\left[N_1-N_2-\sqrt{[N_2\cdot(N_1-N_2-1)]}-N_1^2 \cdots + N_1\cdot N_2-N_1\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}\right]\right]}{2\cdot\sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}+N_1^2+4\cdot N_1\cdot N_2-4\cdot N_2-4\cdot N_2^2-2\cdot N_1+1\right]}\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]} = 0$$

$$GN - \left[\frac{\sqrt{(N_1-1)\cdot\left(N_1+2\cdot\sqrt{N_1\cdot N_2-N_2^2-N_2}-1\right)}}{\sqrt{2}} \cdots + -\frac{\sqrt{2}\cdot\left[\sqrt{(N_1-1)^3\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]} \cdots + \sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}+N_1^2+4\cdot N_1\cdot N_2-2\cdot N_1-4\cdot N_2^2-4\cdot N_2+1\right]}\cdot(N_1-2\cdot N_2-1)\cdot\left[N_1-N_2-\sqrt{[N_2\cdot(N_1-N_2-1)]}-N_1^2 \cdots + N_1\cdot N_2-N_1\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}\right]\right]}{2\cdot\sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}+N_1^2+4\cdot N_1\cdot N_2-4\cdot N_2-4\cdot N_2^2-2\cdot N_1+1\right]}\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{[N_2\cdot(N_1-N_2-1)]}-1\right]}\right] = 0$$

$CG - \sqrt{N_2^2 + GH^2} = 0$

Mathcad 15 blows a tire here. After spreading it out, and then asking it to collet, it deletes evreything and comes back, "undefined." I might get back to this some other time, maybe. What you can take away from the distinction between Logic, such as Arithmetic and Algebra, and Analogic, the geometric figure, one of them does the computations instantly and concurrently, and logics require a lot of binary computation not required by analogic. Much of the current problem are the so called invalid principles used in logic today.

$BG - \sqrt{BH^2 + GH^2} = 0$

$DG - \sqrt{2\cdot GN^2} = 0$

$EG - DG = 0$

$CD - (CG - DG) = 0$

$BE - (BG - EG) = 0$

$(\mathbf{KM}+\mathbf{MN})$

$\mathbf{MN} = 0.31209$

$\mathbf{KM} = 0.681031$

$(\mathbf{KM}+\mathbf{MN}) = 0.993121$

$$Z := \sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}$$

$$Y := \sqrt{\left(N_1-1\right)^3\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]}$$

$$X := \sqrt{\left(N_1-1\right)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]}$$

$$W := \sqrt{\left[\left(4\cdot N_1-4\right)\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}+N_1^{\,2}+4\cdot N_1\cdot N_2-4\cdot N_2-4\cdot N_2^{\,2}-2\cdot N_1+1\right]}$$

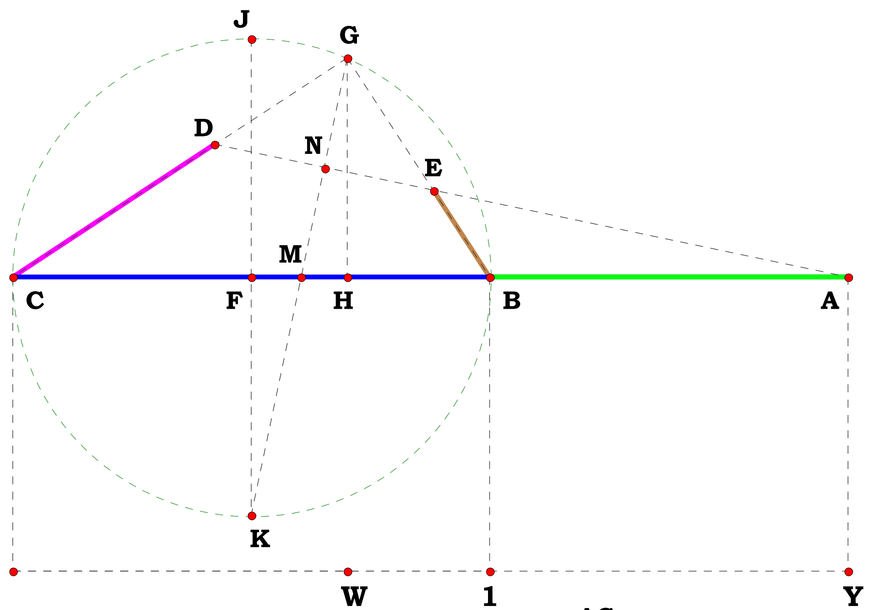
$$\frac{\sqrt{2}\cdot\left[Y\cdot\left(N_1+2\cdot Z-1\right)\cdot X+W\cdot\left(N_1-2\cdot N_2-1\right)\cdot\left(N_1-N_2-Z-N_1^{\,2}+N_1\cdot N_2-N_1\cdot Z\right)\right]}{2\cdot W\cdot X\cdot\left(N_1+2\cdot Z-1\right)} = 0.993121$$

$$\frac{\sqrt{2}\cdot\left[\begin{aligned}&\sqrt{\left(N_1-1\right)^3\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]\cdot\sqrt{\left(N_1-1\right)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]}\ \ldots\\&+\sqrt{\left[\left(4\cdot N_1-4\right)\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}+N_1^{\,2}+4\cdot N_1\cdot N_2-2\cdot N_1-4\cdot N_2^{\,2}-4\cdot N_2+1\right]}\cdot\left(N_1-2\cdot N_2-1\right)\cdot\left[\begin{aligned}&N_1-N_2-\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-N_1^{\,2}\ \ldots\\&+N_1\cdot N_2-N_1\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}\end{aligned}\right]\end{aligned}\right]}{2\cdot\sqrt{\left[\left(4\cdot N_1-4\right)\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}+N_1^{\,2}+4\cdot N_1\cdot N_2-4\cdot N_2-4\cdot N_2^{\,2}-2\cdot N_1+1\right]}\cdot\sqrt{\left(N_1-1\right)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot\left(N_1-N_2-1\right)\right]}-1\right]} = 0.993121$$

011796B2

Divide The Sides Of A Right Triangle In A Given Ratio

Here was another junk write up, not only that, the original plate was defective, not absolutely correct, but, it takes time to learn how to say what one sees and at this revision time, all of it has to be fixed. What is being noted is that in this figure, there is a constant ratio, no matter where GH is on BC, AB is to AC as CD is to BE It is very simple, and originally I over obfuscated the whole thing. In short, it has something to say about the common angle. I suspect, now that I review it, it is very iimportant. There is, and always has been, a physical standard for ratio in this respect.



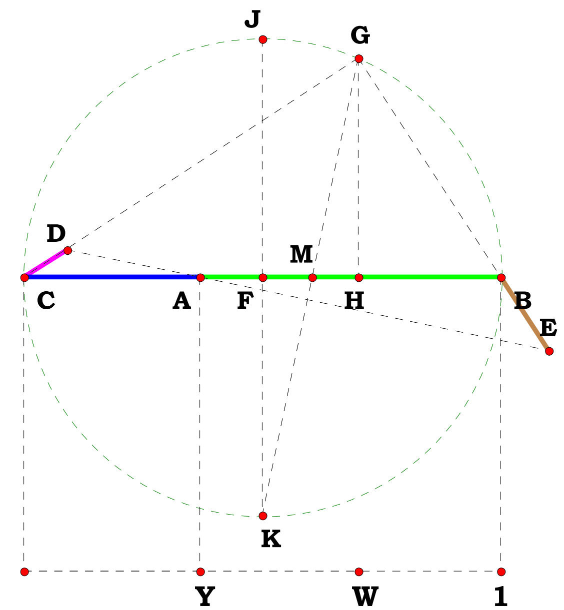
Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AB = 4.73462 cm
AC = 11.04746 cm
CD = 3.19202 cm
BE = 1.36801 cm

$\frac{AC}{AB} = 2.33333$

$\frac{CD}{BE} = 2.33333$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$



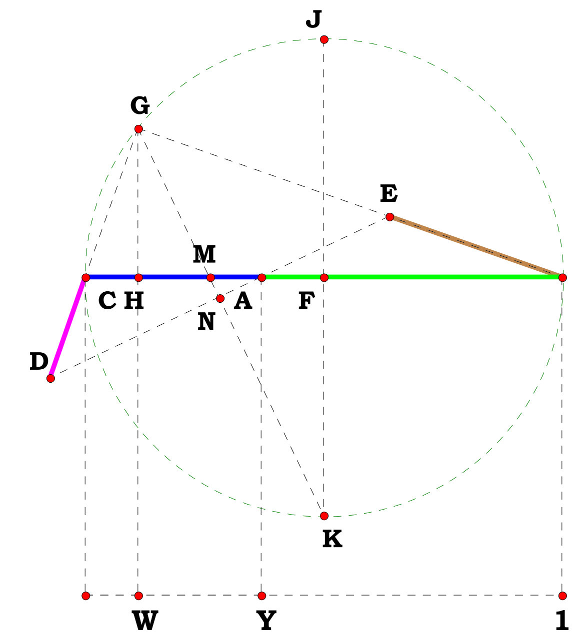
Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 0.67230 cm
BE = 1.15172 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$



Unit = 1.00000
W/X = 0.10988
W = 2.19759
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 1.42395 cm
BE = 2.43938 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$

Given.

$W := 14 \quad Y := 7$

$X := 20 \quad Z := 4$

011796B2

Descriptions.

$$AC := \frac{Y}{Z} \quad AB := \frac{Y - Z}{Z} \quad CH := \frac{W}{X}$$

$$BC := AC - AB \quad BF := \frac{BC}{2} \quad BH := BC - CH \quad GH := \sqrt{CH \cdot BH}$$

$$FK := BF \quad FH := CH - BF \quad GK := \sqrt{FH^2 + (FK + GH)^2}$$

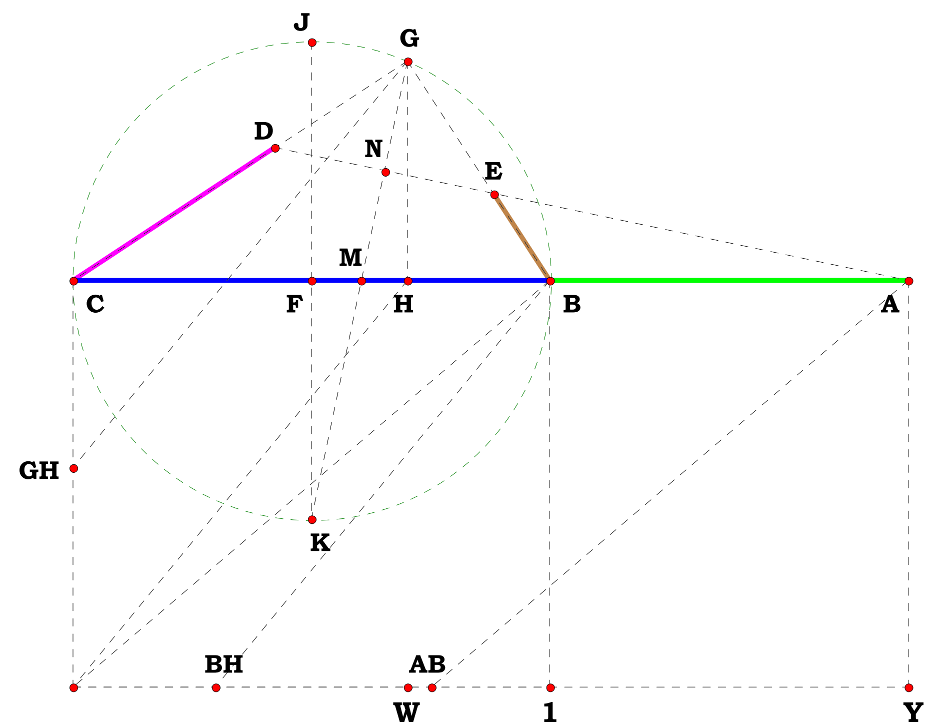
$$FM := \frac{FH \cdot FK}{FK + GH} \quad HM := FH - FM \quad AH := AC - CH \quad AM := AH + HM$$

$$MN := \frac{FH \cdot AM}{GK} \quad KM := \sqrt{FK^2 + FM^2} \quad KN := KM + MN$$

$$GN := GK - KN \quad CG := \sqrt{CH^2 + GH^2} \quad BG := \sqrt{BH^2 + GH^2}$$

$$DG := \sqrt{2 \cdot GN^2} \quad EG := DG \quad CD := CG - DG \quad BE := BG - EG$$

$$\frac{AC}{AB} = 2.333333 \quad \frac{CD}{BE} = 2.333333 \quad \frac{AC}{AB} - \frac{CD}{BE} = 0 \quad \frac{AC}{AB} - \frac{Y}{Y - Z} = 0$$



Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AC = 1.75000
BC = 1.00000
CH = 0.70000
CM = 0.60436
CF = 0.50000
BH = 0.30000
GH = 0.45826

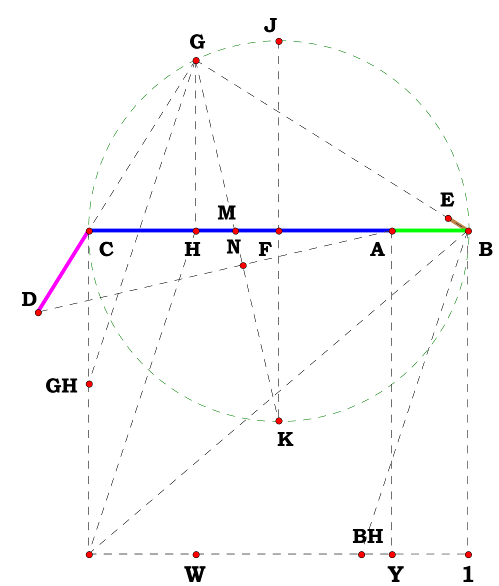
AB = 0.75000

To make this always work, when AB is less than BC: $\frac{AC}{AB} - \frac{Y}{\sqrt{(Y - Z)^2}} = 0$

This is because a difference and a sum are not the same thing.

As one can see, W and X, which form everything about where GH is, and the ratios on it, have disappeared, in short, these particulars have nothing to do with the outcome.

Perhaps if I have the time, I will rework the whole thing using $AB - \frac{\sqrt{(Y - Z)^2}}{Z} = 0$ **and things might work out better because there may, perhaps be five or more operations of math instead of four.**



Unit = 1.00000
W/X = 0.28088
W = 5.61760
X = 20.00000
Y/Z = 0.79776
Y = 3.19104
Z = 4.00000

AC = 0.79776
BC = 1.00000
CH = 0.28088
CM = 0.38460
CF = 0.50000
BH = 0.71912
GH = 0.44943

$\frac{Y}{\sqrt{(Y-Z)^2}} = 3.94462$

$\frac{AC}{AB} - \frac{Y}{\sqrt{(Y-Z)^2}} = 0.00000$

$\frac{CD}{BE} - \frac{Y}{\sqrt{(Y-Z)^2}} = 0.00000$

Definitions.

$$AC - \frac{Y}{Z} = 0 \qquad AB - \frac{Y - Z}{Z} = 0 \qquad CH - \frac{W}{X} = 0 \qquad BC - 1 = 0 \qquad BF - \frac{1}{2} = 0$$

$$BH - \frac{X - W}{X} = 0 \qquad GH - \frac{\sqrt{W \cdot (X - W)}}{X} = 0 \qquad FK - \frac{1}{2} = 0 \qquad FH - \frac{2 \cdot W - X}{2 \cdot X} = 0$$

$$GK - \frac{\sqrt{X + 2 \cdot \sqrt{W \cdot X - W^2}}}{\sqrt{2 \cdot X}} = 0 \qquad FM - \frac{2 \cdot W - X}{2 \cdot \left[X + 2 \cdot \sqrt{-W \cdot (W - X)}\right]} = 0$$

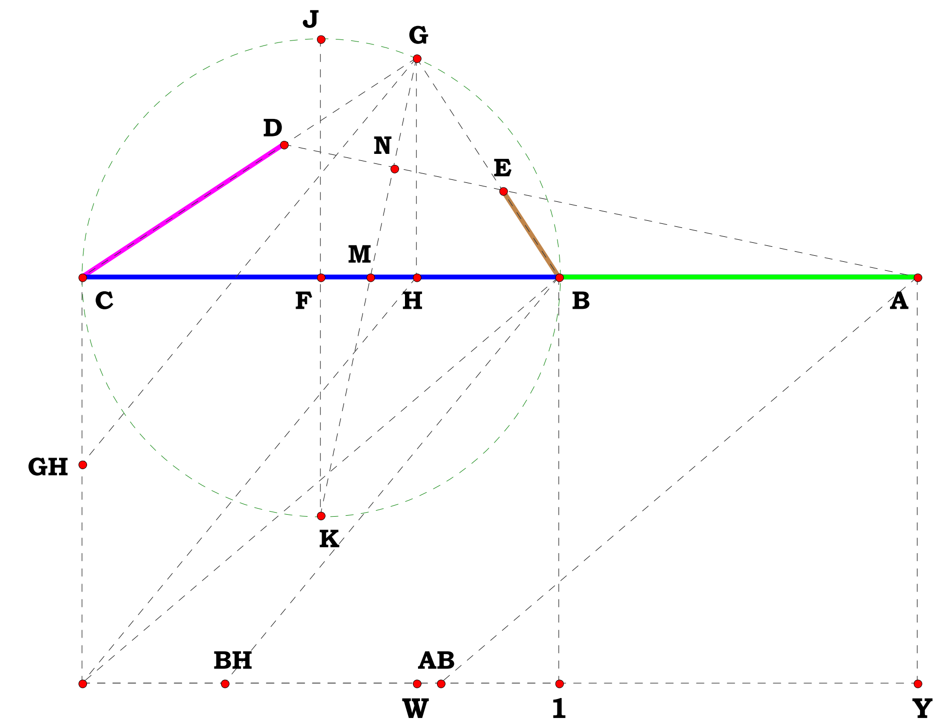
$$HM - \frac{(2 \cdot W - X) \cdot \sqrt{W \cdot X - W^2}}{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)} = 0 \qquad AH - \frac{X \cdot Y - W \cdot Z}{X \cdot Z} = 0 \qquad AM - \frac{\sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z) - W \cdot Z + X \cdot Y}{Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)} = 0$$

$$MN - \frac{\sqrt{2} \cdot (X - 2 \cdot W) \cdot \left[W \cdot Z - X \cdot Y - \sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z)\right]}{2 \cdot \sqrt{X} \cdot Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)^{\frac{3}{2}}} = 0$$

$$KM - \frac{\sqrt{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)}}{\sqrt{2} \cdot \left[\sqrt{X^2 - 4 \cdot W^2 + 4 \cdot X \cdot \left(W + \sqrt{W \cdot X - W^2}\right)}\right]} = 0$$

$$KN - \left[\frac{\sqrt{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)}}{\sqrt{2} \cdot \left[\sqrt{X^2 - 4 \cdot W^2 + 4 \cdot X \cdot \left(W + \sqrt{W \cdot X - W^2}\right)}\right]} + \frac{\sqrt{2} \cdot (X - 2 \cdot W) \cdot \left[W \cdot Z - X \cdot Y - \sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z)\right]}{2 \cdot \sqrt{X} \cdot Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)^{\frac{3}{2}}}\right] = 0$$

$$GN - \left[\frac{\sqrt{X + 2 \cdot \sqrt{W \cdot X - W^2}}}{\sqrt{2 \cdot X}} - \left[\frac{\sqrt{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)}}{\sqrt{2} \cdot \left[\sqrt{X^2 - 4 \cdot W^2 + 4 \cdot X \cdot \left(W + \sqrt{W \cdot X - W^2}\right)}\right]} + \frac{\sqrt{2} \cdot (X - 2 \cdot W) \cdot \left[W \cdot Z - X \cdot Y - \sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z)\right]}{2 \cdot \sqrt{X} \cdot Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)^{\frac{3}{2}}}\right]\right] = 0 \qquad CG - \frac{\sqrt{W}}{\sqrt{X}} = 0 \qquad BG - \frac{\sqrt{X - W}}{\sqrt{X}} = 0$$



Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AC = 1.75000
BC = 1.00000
CH = 0.70000
CM = 0.60436
CF = 0.50000
BH = 0.30000
GH = 0.45826

AB = 0.75000

I am not even going to try, this time, to have Mathcad 15 simplify the following. Mathcad 15 crashes and the programmars nevrer imagined to have this program save backup and restore files. That function has to be manually set and it sometimes works.

$$DG-\sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}}-\left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]}+\frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right]=0$$

$$EG-\sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}}-\left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]}+\frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right]=0$$

$$CD-\left[\frac{\sqrt{W}}{\sqrt{X}}-\sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}}-\left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]}+\frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right]\right]=0$$

$$BE-\left[\frac{\sqrt{X-W}}{\sqrt{X}}-\sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}}-\left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]}+\frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right]\right]=0$$

Mathcad15 cannot simplify the equation, in essence, it cannot produce an algebraic result as easily as Arithmetic and geometry can give their result.

$$\frac{\sqrt{2}\cdot\left[\frac{\sqrt{2}\cdot\sqrt{X^2+2\cdot X\cdot\sqrt{W\cdot X-W^2}}}{2\cdot\sqrt{4\cdot X\cdot\sqrt{-W\cdot(W-X)}-4\cdot W^2+X^2+4\cdot W\cdot X}}-\frac{\sqrt{2}\cdot\sqrt{X+2\cdot\sqrt{-W\cdot(W-X)}}}{2\cdot\sqrt{X}}+\frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[Z\cdot\sqrt{-W\cdot(W-X)}-2\cdot Y\cdot\sqrt{-W\cdot(W-X)}+W\cdot Z-X\cdot Y\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left[X+2\cdot\sqrt{-W\cdot(W-X)}\right]^{\frac{3}{2}}}\right]+\frac{\sqrt{W}}{\sqrt{X}}}{\sqrt{2}\cdot\left[\frac{\sqrt{2}\cdot\sqrt{X^2+2\cdot X\cdot\sqrt{W\cdot X-W^2}}}{2\cdot\sqrt{4\cdot X\cdot\sqrt{-W\cdot(W-X)}-4\cdot W^2+X^2+4\cdot W\cdot X}}-\frac{\sqrt{2}\cdot\sqrt{X+2\cdot\sqrt{-W\cdot(W-X)}}}{2\cdot\sqrt{X}}+\frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[Z\cdot\sqrt{-W\cdot(W-X)}-2\cdot Y\cdot\sqrt{-W\cdot(W-X)}+W\cdot Z-X\cdot Y\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left[X+2\cdot\sqrt{-W\cdot(W-X)}\right]^{\frac{3}{2}}}\right]+\frac{\sqrt{X-W}}{\sqrt{X}}}=2.333333$$

$\frac{AC}{AB}=2.333333$ $\quad\frac{CD}{BE}=2.333333$ $\quad\frac{AC}{AB}-\frac{CD}{BE}=0$ $\quad\frac{AC}{AB}-\frac{Y}{Y-Z}=0$

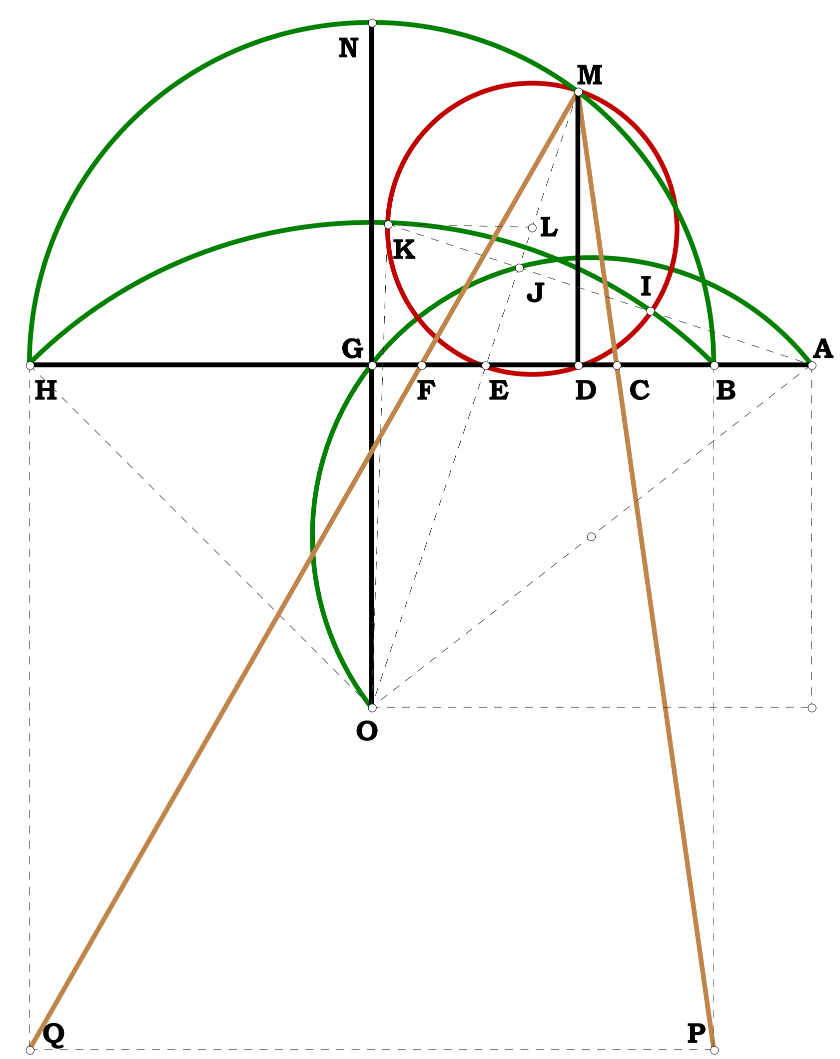
Unit.

$BH := 1$

Given.

$N := 5$

More On Cube Roots



012196A

Descriptions.

$BP := BH \quad HQ := BH \quad BG := \frac{BH}{2} \quad GO := BG \quad GN := BG$

$NO := BH \quad GH := BG \quad BE := \frac{BG}{N} \quad EG := BG - BE$

$EO := \sqrt{EG^2 + GO^2} \quad MO := \frac{GO \cdot NO}{EO} \quad EM := MO - EO$

$EL := \frac{EM}{2} \quad LK := EL \quad LO := EO + EL \quad LJ := \frac{LK^2}{LO}$

$EJ := EL - LJ \quad AE := \frac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH$

$AB := AH - BH \quad DE := \frac{EG \cdot EM}{EO} \quad DM := \frac{GO \cdot EM}{EO}$

$BD := BG - (EG + DE) \quad BC := \frac{BD \cdot BP}{BP + DM} \quad DH := BH - BD$

$DF := \frac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

Definitions.

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$

012196B

Given.

$X := 5$

$Y := 20$

Unit.

$BH := \frac{Y}{Y}$

More On Cube Roots

Descriptions.

$BP := BH \quad HQ := BH \quad BG := \frac{BH}{2} \quad GO := BG \quad GN := BG$

$NO := BH \quad GH := BG \quad BE := BG - \frac{X}{2 \cdot Y} \quad EG := BG - BE$

$EO := \sqrt{EG^2 + GO^2} \quad MO := \frac{GO \cdot NO}{EO} \quad EM := MO - EO$

$EL := \frac{EM}{2} \quad LK := EL \quad LO := EO + EL \quad LJ := \frac{LK^2}{LO}$

$EJ := EL - LJ \quad AE := \frac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH$

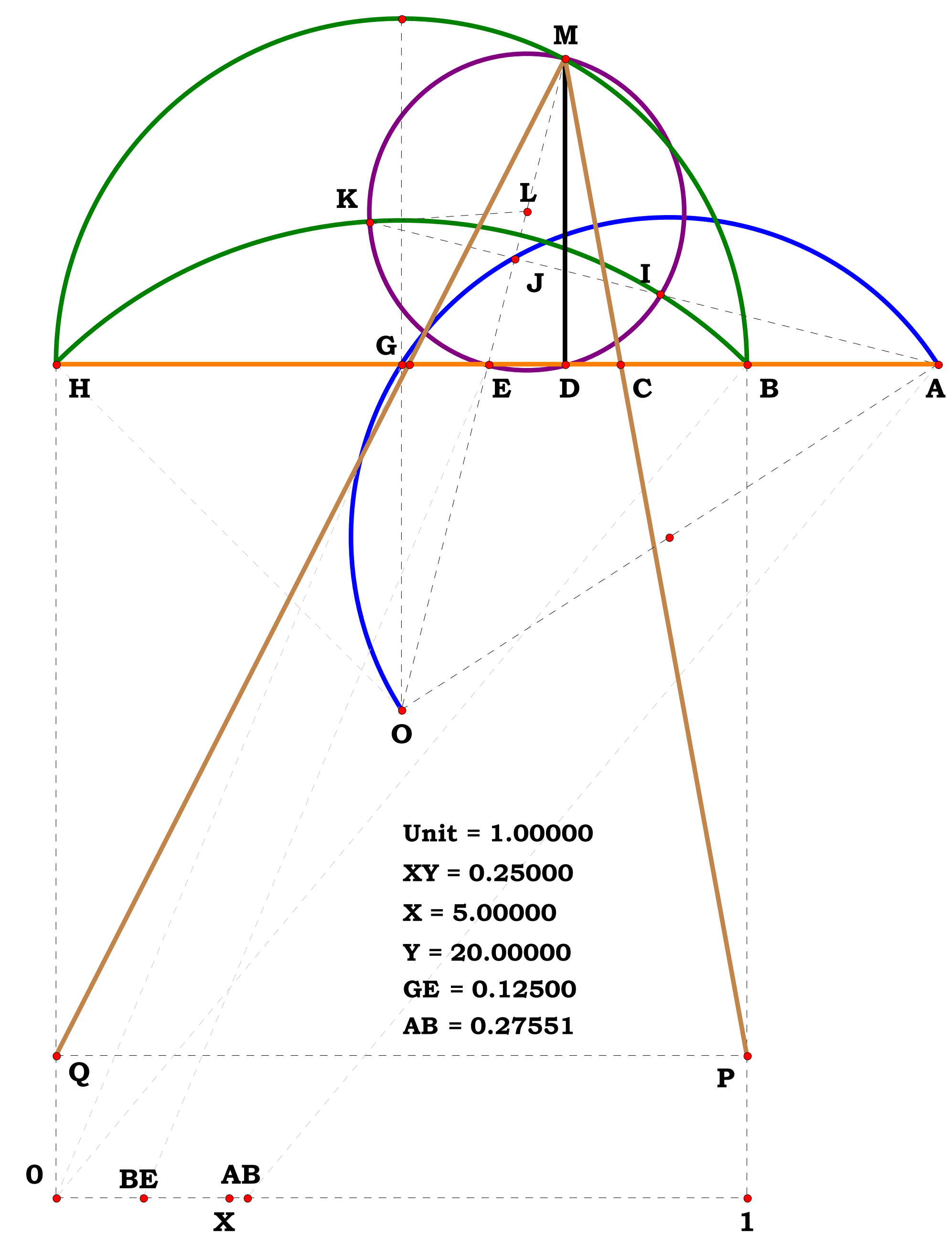
$AB := AH - BH \quad DE := \frac{EG \cdot EM}{EO} \quad DM := \frac{GO \cdot EM}{EO}$

$BD := BG - (EG + DE) \quad BC := \frac{BD \cdot BP}{BP + DM} \quad DH := BH - BD$

$DF := \frac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$$

$GE := BG - BE \quad GE = 0.125 \quad AB = 0.27551$



Definitions.

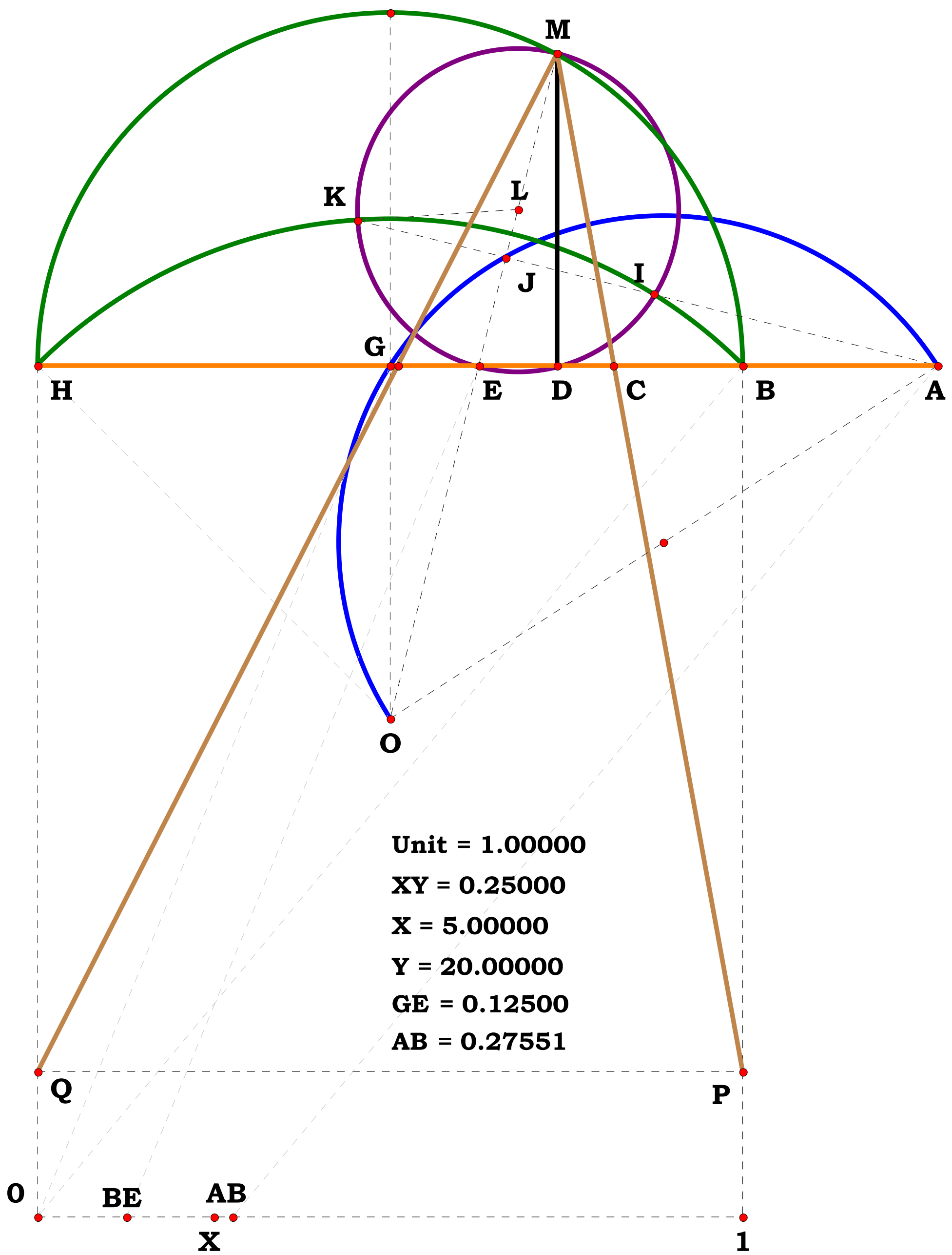
$$BH-1=0 \quad BP-1=0 \quad HQ-1=0 \quad NO-1=0 \quad BG-\frac{1}{2}=0 \quad GO-\frac{1}{2}=0 \quad GN-\frac{1}{2}=0 \quad GH-\frac{1}{2}=0$$

$$BE-\frac{Y-X}{2\cdot Y}=0 \quad EG-\frac{X}{2\cdot Y}=0 \quad EO-\frac{\sqrt{X^2+Y^2}}{2\cdot Y}=0 \quad MO-\frac{Y}{\sqrt{X^2+Y^2}}=0 \quad EM-\frac{(Y-X)\cdot(X+Y)}{2\cdot Y\cdot\sqrt{X^2+Y^2}}=0$$

$$EL-\frac{(Y-X)\cdot(X+Y)}{4\cdot Y\cdot\sqrt{X^2+Y^2}}=0 \quad LK-\frac{(Y-X)\cdot(X+Y)}{4\cdot Y\cdot\sqrt{X^2+Y^2}}=0 \quad LO-\frac{X^2+3\cdot Y^2}{4\cdot Y\cdot\sqrt{X^2+Y^2}}=0$$

$$LJ-\frac{(X-Y)^2\cdot(X+Y)^2}{4\cdot Y\cdot\left(X^2+3\cdot Y^2\right)\cdot\sqrt{X^2+Y^2}}=0 \quad EJ-\frac{(Y-X)\cdot(X+Y)\cdot\left(X^2+Y^2\right)}{2\cdot\sqrt{X^2+Y^2}\cdot Y\cdot\left(X^2+3\cdot Y^2\right)}=0 \quad AE-\frac{(Y-X)\cdot\left(X^2+Y^2\right)\cdot(X+Y)}{2\cdot X\cdot Y\cdot\left(X^2+3\cdot Y^2\right)}=0$$

$$AH-\frac{(X+Y)^3}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}=0 \quad AB-\frac{(Y-X)^3}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}=0 \quad DE-\frac{X\cdot(Y-X)\cdot(X+Y)}{2\cdot Y\cdot\left(X^2+Y^2\right)}=0 \quad DM-\frac{(Y-X)\cdot(X+Y)}{2\cdot\left(X^2+Y^2\right)}=0$$



$$BD-\frac{(X-Y)^2}{2\cdot\left(X^2+Y^2\right)}=0 \quad BC-\frac{(X-Y)^2}{X^2+3\cdot Y^2}=0 \quad DH-\frac{(X+Y)^2}{2\cdot\left(X^2+Y^2\right)}=0 \quad DF-\frac{(Y-X)\cdot(X+Y)^3}{2\cdot\left(X^2+Y^2\right)\cdot\left(X^2+3\cdot Y^2\right)}=0$$

$$AC-\frac{(X+Y)\cdot(X-Y)^2}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}=0 \quad AF-\frac{(Y-X)\cdot(X+Y)^2}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}=0$$

And again, Mathcad 15 cannot reduce the following equations.

$$\left[\left[\frac{(Y-X)^3}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}\right]^2\cdot\frac{(X+Y)^3}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}\right]^{\frac{1}{3}}-\frac{(X+Y)\cdot(X-Y)^2}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}=0 \qquad \left[\frac{(Y-X)^3}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}\cdot\left[\frac{(X+Y)^3}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}\right]^2\right]^{\frac{1}{3}}-\frac{(Y-X)\cdot(X+Y)^2}{2\cdot X\cdot\left(X^2+3\cdot Y^2\right)}=0$$

Unit.

$AF := 1$

Given.

$N_1 := 6$

$N_2 := 3$

012296A

Descriptions.

$AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$

$GL := AL - AG \quad GK := \frac{GL}{N_2} \quad FG := AG - AF$

$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$

$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$

Definitions.

$AL - N_1 = 0 \quad FL - (N_1 - 1) = 0 \quad FJ - \frac{N_1 - 1}{2} = 0 \quad AM - \sqrt{N_1} = 0$

$AJ - \frac{N_1 + 1}{2} = 0 \quad AG - \frac{2 \cdot N_1}{N_1 + 1} = 0 \quad GL - \frac{N_1 \cdot (N_1 - 1)}{N_1 + 1} = 0 \quad GK - \frac{N_1 \cdot (N_1 - 1)}{N_2 \cdot (N_1 + 1)} = 0$

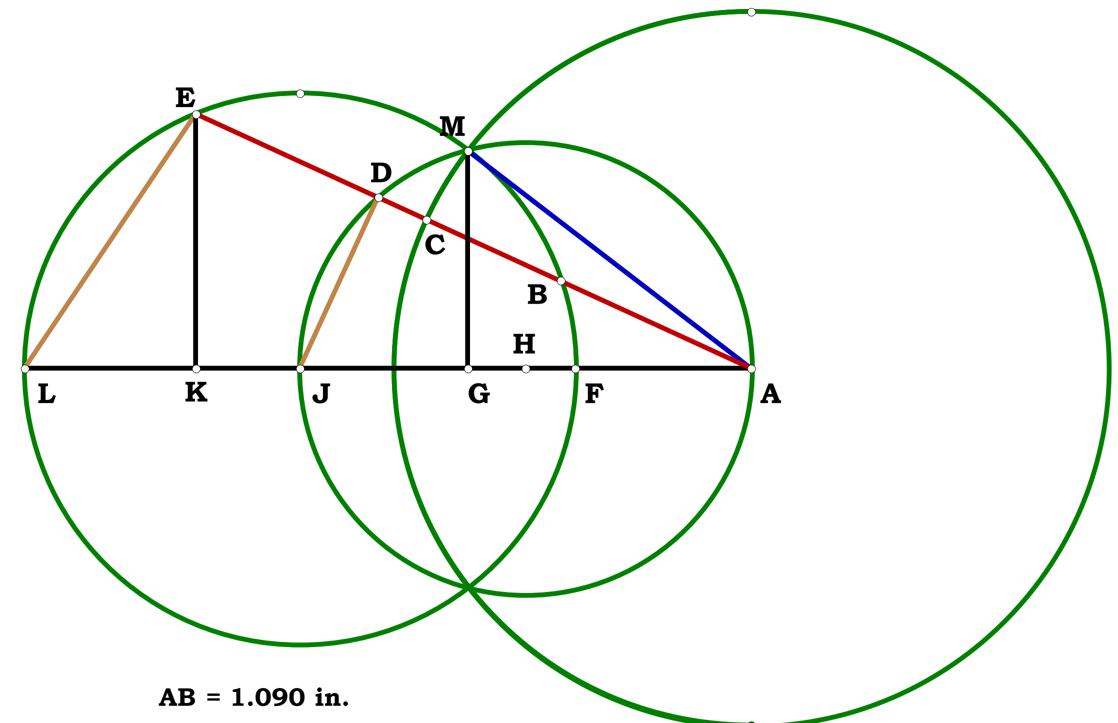
$FG - \frac{N_1 - 1}{N_1 + 1} = 0 \quad FK - \frac{(N_1 - 1) \cdot (N_1 + N_2)}{N_2 \cdot (N_1 + 1)} = 0 \quad KL - \frac{N_1 \cdot (N_2 - 1) \cdot (N_1 - 1)}{N_2 \cdot (N_1 + 1)} = 0$

$EK - \frac{(N_1 - 1) \cdot \sqrt{N_1 \cdot (N_2 - 1) \cdot (N_1 + N_2)}}{N_2 \cdot (N_1 + 1)} = 0 \quad AK - \frac{N_1 \cdot (N_1 + 2 \cdot N_2 - 1)}{N_2 \cdot (N_1 + 1)} = 0 \quad AE - \frac{\sqrt{N_1 \cdot (N_1 + N_2 - 1)}}{\sqrt{N_2}} = 0$

$AD - \frac{N_1 \cdot (N_1 + 2 \cdot N_2 - 1)}{2 \cdot \sqrt{N_2} \cdot \sqrt{N_1 \cdot (N_1 + N_2 - 1)}} = 0 \quad DE - \frac{N_1 \cdot (N_1 - 1)}{2 \cdot \sqrt{N_2} \cdot \sqrt{N_1^2 - N_1 + N_1 \cdot N_2}} = 0 \quad AB - \frac{N_1 \cdot \sqrt{N_2}}{\sqrt{N_1^2 - N_1 + N_1 \cdot N_2}} = 0$

Trivial Method Square Root

For any E between arc M and L, AM is the square root of AB x AE. Perhaps this is one way to construct a logical operator to determine class membrership for E.

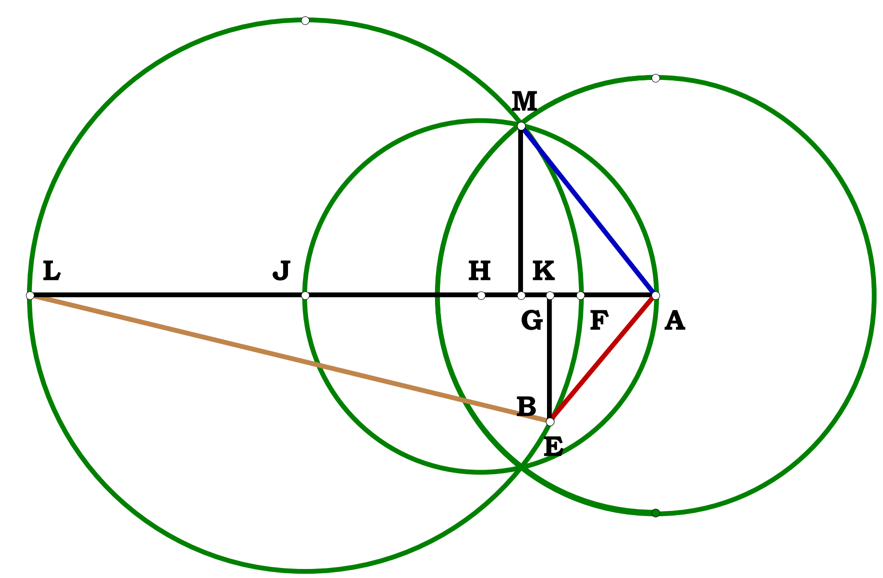


AB = 1.090 in.

AE = 3.182 in.

AM = 1.862 in.

$\sqrt{AB \cdot AE}$-AM = 0.000 in.



AB = 0.857 in.

AE = 0.857 in.

AM = 1.130 in.

$\sqrt{AB \cdot AE}$-AM = -0.273 in.

Unit.

$AB := 1$

Given.

$N := 5$

012496A

Descriptions.

$AE := N \quad BE := AE - AB \quad BD := \dfrac{BE}{2}$

$AD := AB + BD \qquad AJ := \sqrt{AD^2 - BD^2}$

$CJ := \dfrac{BD \cdot AJ}{AD} \qquad AC := \dfrac{AJ^2}{AD}$

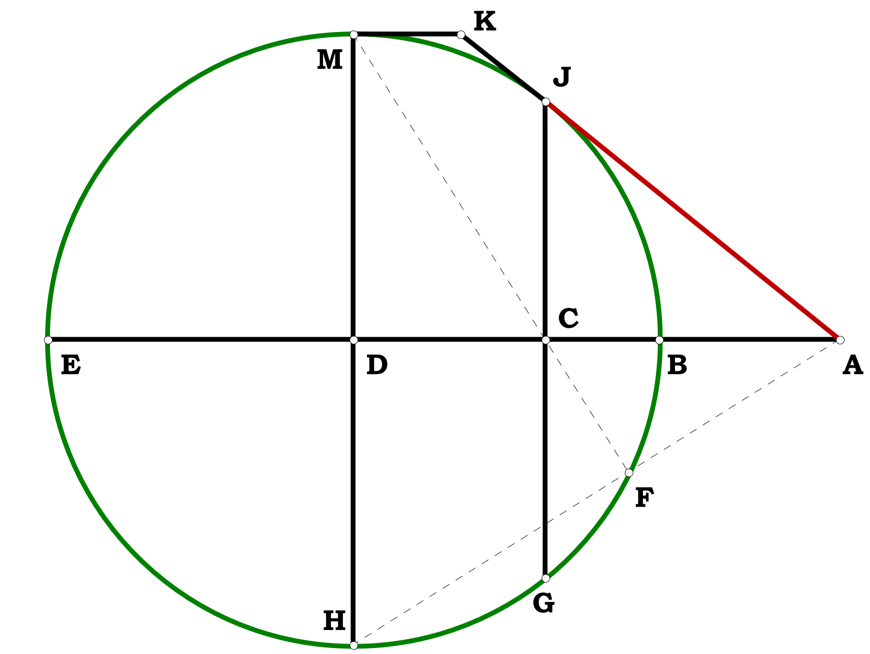
Definitions.

$AE - N = 0 \quad BE - (N - 1) = 0 \quad BD - \dfrac{N-1}{2} = 0$

$AD - \dfrac{N+1}{2} = 0 \qquad AJ - \sqrt{N} = 0$

$CJ - \dfrac{\sqrt{N} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)}{N + 1} = 0 \qquad AC - \dfrac{2 \cdot N}{N + 1} = 0$

Tangent



$N\text{-}1\text{-}BE = 0.000$

$\dfrac{N-1}{2}\text{-}BD = 0.000$

$\sqrt{N}\text{-}AJ = 0.000$

$\dfrac{\sqrt{N} \cdot (\sqrt{N}-1) \cdot (\sqrt{N}+1)}{N+1}\text{-}CJ = 0.000$

$N = 4.381$

$BE = 3.381$

$BD = 1.690$

$AJ = 2.093$

$CJ = 1.315$

Unit. $AB := 1$

Given. $X := 5$

$Y := 20$

012496B

Descriptions.

$AE := \frac{X}{Y}$ $\quad BE := \frac{Y-X}{Y}$ $\quad DE := \frac{BE}{2}$ $\quad AD := DE + AE$

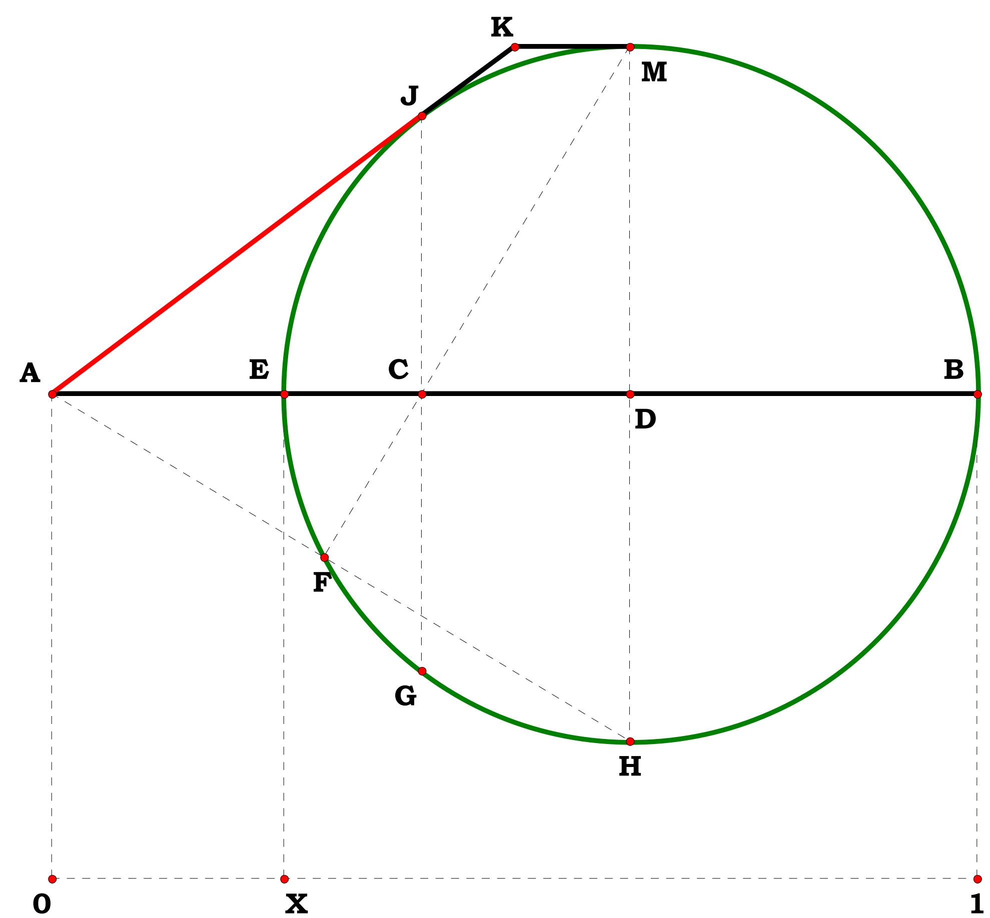
$AJ := \sqrt{AD^2 - DE^2}$ $\quad AC := \frac{AJ^2}{AD}$

Definitions.

$AE - \frac{X}{Y} = 0$ $\quad BE - \frac{Y-X}{Y} = 0$ $\quad DE - \frac{Y-X}{2 \cdot Y} = 0$

$AD - \frac{X+Y}{2 \cdot Y} = 0$ $\quad AJ - \frac{\sqrt{X}}{\sqrt{Y}} = 0$ $\quad AC - \frac{2 \cdot X}{X+Y} = 0$

Tangent



Unit = 1.00000
XY = 0.25000
X = 5.00000
Y = 20.00000

$\frac{\sqrt{X}}{\sqrt{Y}}$-AJ = 0.00000

$\frac{2 \cdot X}{X+Y}$-AC = 0.00000

Unit.

$BE := 1$

Given.

$N := 1.333333$

Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.

012596A

Descriptions.

$BD := \frac{BE}{2} \quad DK := BD \quad DJ := BD \quad JK := BE \quad DE := BD$

$BC := \frac{BD}{N} \quad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$

$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$

$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$

$AB := AE - BE \quad AB = 0.275511 \quad AE = 1.275511$

$BD = 0.5 \quad BC = 0.375$

Definitions.

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

On Cubes

$\frac{BD}{BC} = 1.33333$

$N = 1.33333$

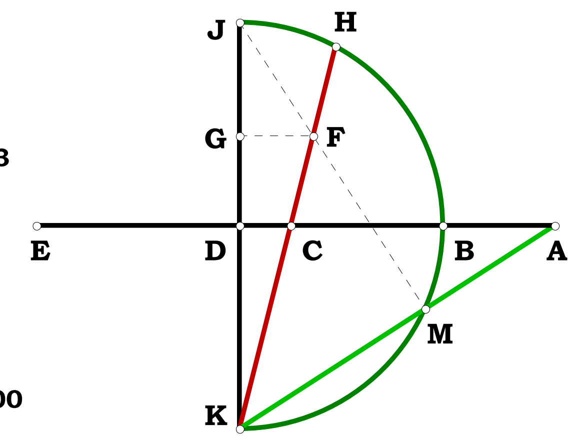
$AE = 1.27551$

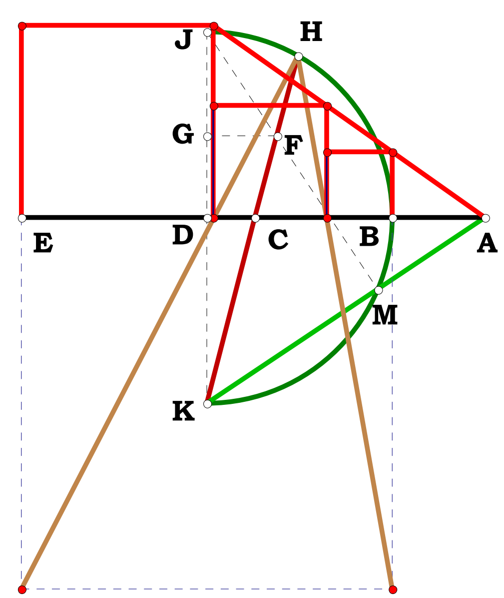
$AB = 0.27551$

$BD = 0.50000$

$\frac{BD}{\frac{BD}{BC}} = 0.37500$

$\frac{(2 \cdot N-1)^3}{2 \cdot (N-1) \cdot ((4 \cdot N^2-2 \cdot N)+1)}-AE = 0.00000$





Quick Roots

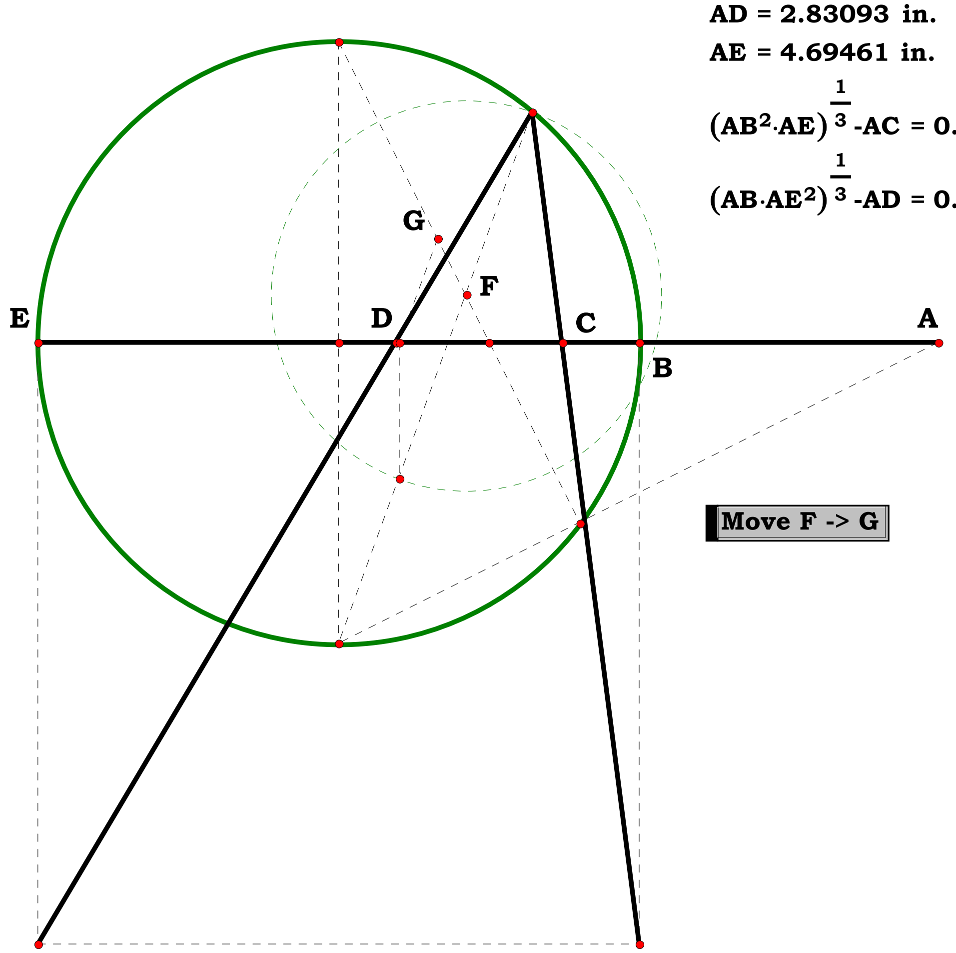
De Button is the replacement for Calculus and it still uses just a straightedge and compass.

Quick roots. Push De Button.

AB = 1.55925 in.
AC = 1.96323 in.
AD = 2.83093 in.
AE = 4.69461 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.28829$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.42023$



Move F -> G

Quick roots. Push De Button.

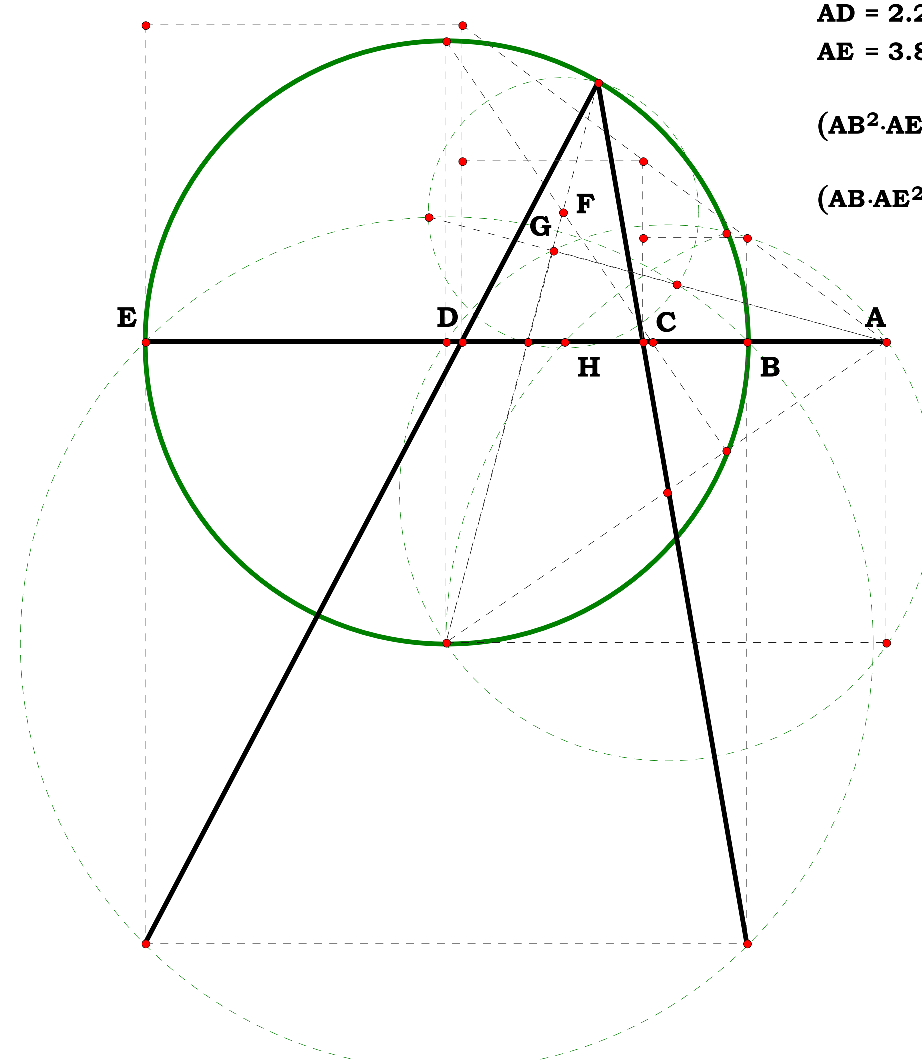
It only takes a few presses to get Sketchpad to fifteen decimal places down.

AB = 0.72571 in.
AC = 1.26691 in.
AD = 2.21171 in.
AE = 3.86108 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

Move F -> G



012596B

Descriptions.

Given.

$X := 5$

$Y := 20$

Unit.

$BE := \frac{Y}{Y}$

Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.

On Cubes

$BD := \frac{BE}{2}$ $\quad DK := BD \quad DJ := BD \quad JK := BE$

$DE := BD \quad CD := \frac{X}{2 \cdot Y} \quad CK := \sqrt{CD^2 + DK^2}$

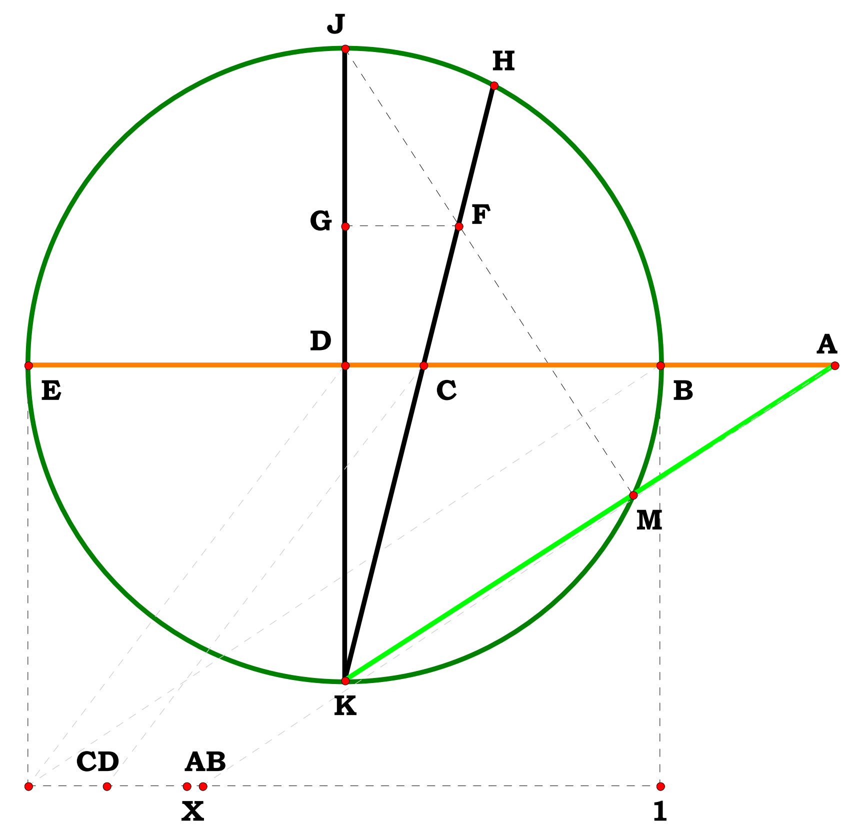
$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$

$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$

$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$

$AB := AE - BE \quad AB = 0.27551 \quad AE = 1.27551$

$BD = 0.5$



Unit = 1.00000
XY = 0.25000
X = 5.00000
Y = 20.00000
CD = 0.12500
AB = 0.27551
AB = 1.27551

$\frac{(Y\text{-}X)^3}{2 \cdot X \cdot (X^2+3 \cdot Y^2)}\text{-}AB = 0.00000$

Definitions.

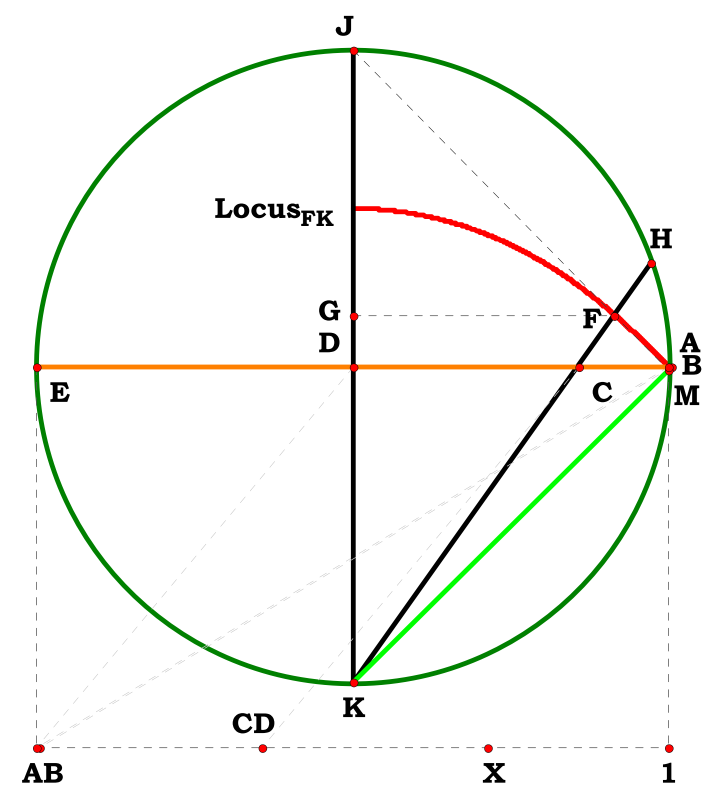
$BD - \frac{1}{2} = 0 \quad DK - \frac{1}{2} = 0 \quad DJ - \frac{1}{2} = 0 \quad DE - \frac{1}{2} = 0$

$JK - 1 = 0 \quad CD - \frac{X}{2 \cdot Y} = 0 \quad CK - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$

$HK - \frac{Y}{\sqrt{X^2 + Y^2}} = 0 \quad CH - \frac{(Y - X) \cdot (X + Y)}{2 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0$

$CF - \frac{(Y - X) \cdot (X + Y)}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0 \quad FK - \frac{X^2 + 3 \cdot Y^2}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0 \quad GK - \frac{DK \cdot FK}{CK} = 0 \quad FG - \frac{X \cdot \left(X^2 + 3 \cdot Y^2\right)}{4 \cdot Y \cdot \left(X^2 + Y^2\right)} = 0$

$GJ - \frac{3 \cdot X^2 + Y^2}{4 \cdot \left(X^2 + Y^2\right)} = 0 \quad AD - \frac{Y \cdot \left(3 \cdot X^2 + Y^2\right)}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0 \quad AE - \frac{(X + Y)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0 \quad AB - \frac{(Y - X)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0$



Unit = 1.00000
XY = 0.71301
X = 14.26010
Y = 20.00000
CD = 0.35650
AB = 0.00472
AB = 1.00472

$\frac{(Y\text{-}X)^3}{2 \cdot X \cdot (X^2+3 \cdot Y^2)}\text{-}AB = 0.00000$

012996A

Given.

$N_1 := 2 \quad N_2 := 3$

$N_3 := 9$

Linear division $\dfrac{N_1 \cdot (N_2 + 2 \cdot N_3)}{2 \cdot (N_2 + N_3)}$

Descriptions.

$AE := N_1 \quad AH := N_2 \quad AC := \dfrac{AE}{2}$

$CF := N_3 \qquad BC := \dfrac{AC \cdot CF}{AH} \quad CE := AC$

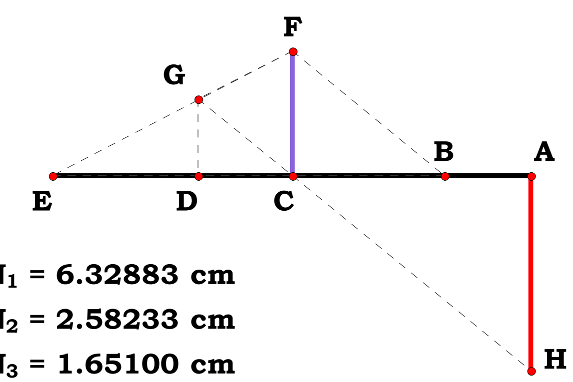
$BE := CE + BC \quad CD := \dfrac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \dfrac{AH \cdot CD}{AC} \quad BC - \dfrac{N_1 \cdot N_3}{2 \cdot N_2} = 0$

Definitions.

$DE - \dfrac{N_1 \cdot N_2}{2 \cdot (N_2 + N_3)} = 0 \quad AD - \dfrac{N_1 \cdot (N_2 + 2 \cdot N_3)}{2 \cdot (N_2 + N_3)} = 0$

$BC - \dfrac{N_1 \cdot N_3}{2 \cdot N_2} = 0 \quad CD - \dfrac{N_1 \cdot N_3}{2 \cdot (N_2 + N_3)} = 0 \quad DG - \dfrac{N_2 \cdot N_3}{N_2 + N_3} = 0$



$N_1 = 6.32883$ cm

$N_2 = 2.58233$ cm

$N_3 = 1.65100$ cm

$\dfrac{N_1 \cdot N_2}{2 \cdot (N_2+N_3)}$-DE = 0.00000 cm

$\dfrac{N_1 \cdot (N_2+2 \cdot N_3)}{2 \cdot (N_2+N_3)}$-AD = 0.00000 cm

$\dfrac{N_1 \cdot N_3}{2 \cdot N_2}$-BC = 0.00000 cm

$\dfrac{N_1 \cdot N_3}{2 \cdot (N_2+N_3)}$-CD = 0.00000 cm

$\dfrac{N_2 \cdot N_3}{N_2+N_3}$-DG = 0.00000 cm

Given.

$N_1 := 2 \quad N_2 := 3$

$N_3 := 9 \quad N_4 := 3$

Linear division $\dfrac{N_2 \cdot (N_1 - N_3)^2}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4}$

012996B

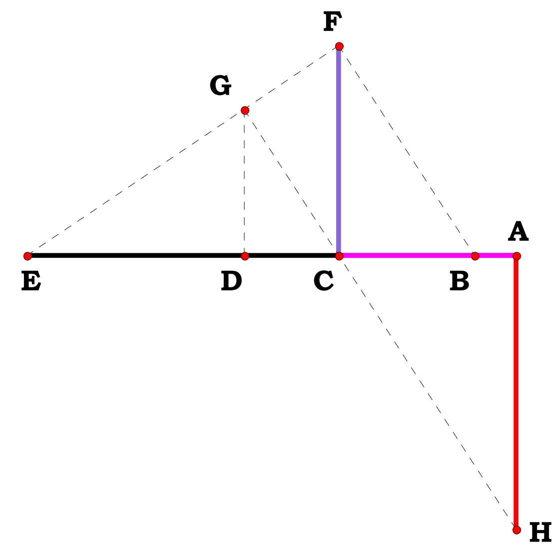
Descriptions.

$AE := N_1 \quad AH := N_2 \quad AC := N_3$

$CF := N_4 \quad BC := \dfrac{AC \cdot CF}{AH} \quad CE := AE - AC$

$BE := CE + BC \quad CD := \dfrac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \dfrac{CF \cdot CE}{BE}$



$\dfrac{N_2 \cdot N_4 \cdot (N_1 - N_3)}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4} - DG = 0.00000 \text{ cm}$

$\dfrac{N_3 \cdot N_4 \cdot (N_1 - N_3)}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4} - CD = 0.00000 \text{ cm}$

$\dfrac{N_3 \cdot ((N_1 \cdot N_2 + N_1 \cdot N_4) - N_2 \cdot N_3)}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4} - AD = 0.00000 \text{ cm}$

$\dfrac{N_2 \cdot (N_1 - N_3)^2}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4} - DE = 0.00000 \text{ cm}$

$\dfrac{N_3 \cdot N_4}{N_2} - BC = 0.00000 \text{ cm}$

Definitions.

$$DE - \frac{N_2 \cdot (N_1 - N_3)^2}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0 \qquad AD - \frac{N_3 \cdot (N_1 \cdot N_2 + N_1 \cdot N_4 - N_2 \cdot N_3)}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0$$

$$BC - \frac{N_3 \cdot N_4}{N_2} = 0$$

$$CD - \frac{N_3 \cdot N_4 \cdot (N_1 - N_3)}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0 \qquad DG - \frac{N_2 \cdot N_4 \cdot (N_1 - N_3)}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0$$

012996C

Given.

$W := 8 \qquad Y := 6$

$X := 20 \qquad Z := 17$

Linear division $\dfrac{W \cdot Z}{2 \cdot (W \cdot Z + X \cdot Y)}$

Descriptions.

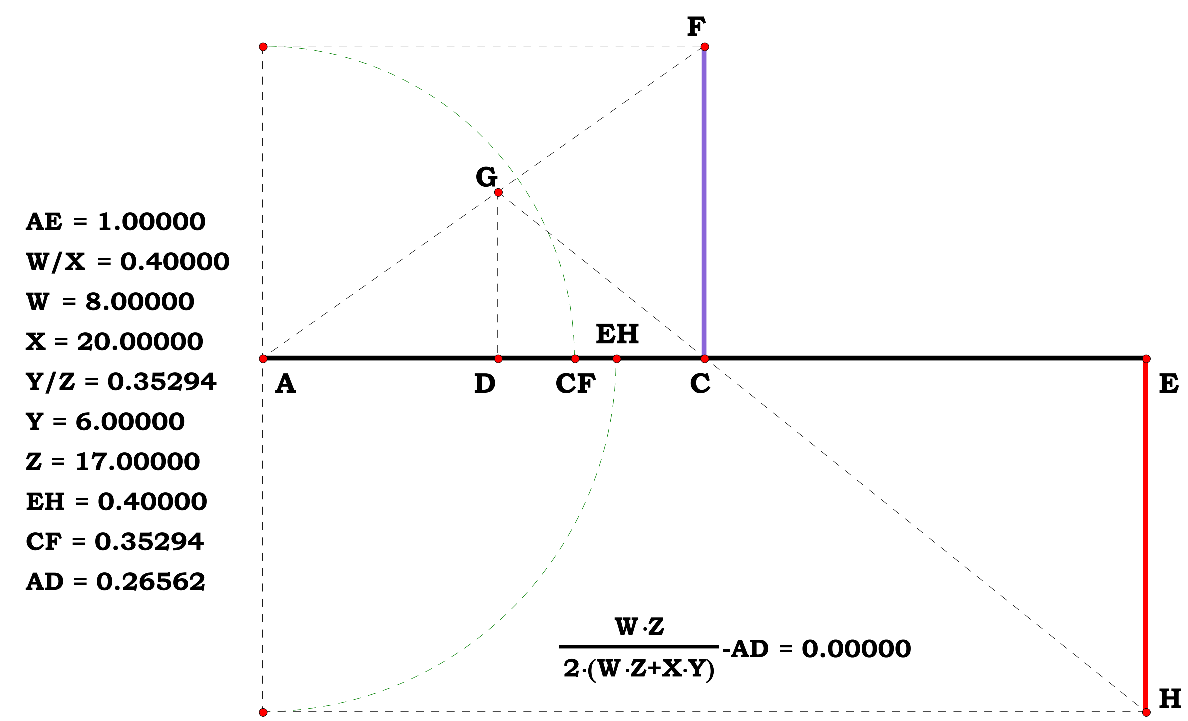
$$AE := \frac{X}{X} \qquad EH := \frac{W}{X} \qquad AC := \frac{AE}{2}$$

$$CF := \frac{Y}{Z} \qquad AD := \frac{AC \cdot EH}{EH + CF} \qquad AD = 0.265625$$

Definitions.

$$AE - \frac{X}{X} = 0 \qquad EH - \frac{W}{X} = 0 \qquad AC - \frac{1}{2} = 0$$

$$CF - \frac{Y}{Z} = 0 \qquad AD - \frac{W \cdot Z}{2 \cdot (W \cdot Z + X \cdot Y)} = 0$$



012996D

Given.

$U := 8 \quad W := 11 \quad Y := 5$

$V := 20 \quad X := 17 \quad Z := 15$

Descriptions.

$$AB := \frac{V}{V} \qquad BC := \frac{U}{V} \qquad AD := \frac{W}{X} \qquad DE := \frac{Y}{Z}$$

$$BD := AB - AD \qquad DH := \frac{BD \cdot DE}{BC}$$

$$AH := AD + DH \qquad AF := \frac{AD \cdot AD}{AH} \qquad AF = 0.444853$$

Definitions.

$$AB - 1 = 0 \qquad BC - \frac{U}{V} \qquad AD - \frac{W}{X} = 0 \qquad DE - \frac{Y}{Z} = 0$$

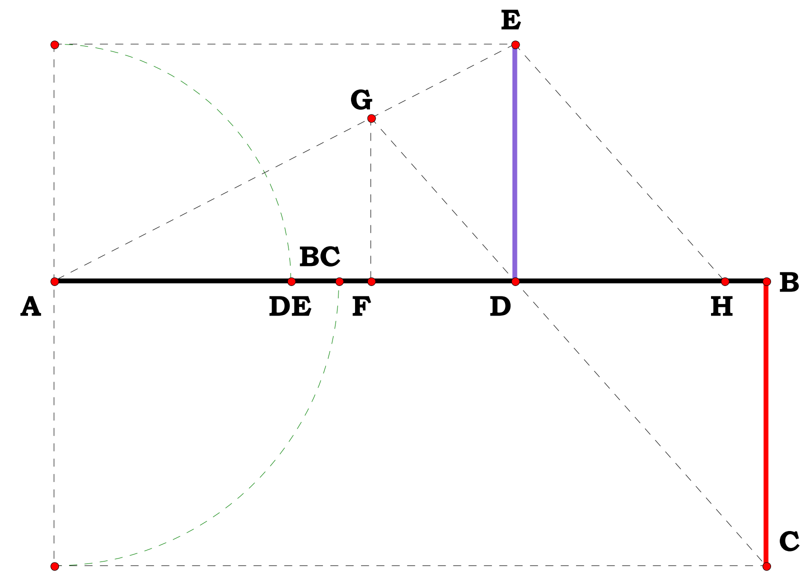
$$BD - \frac{X - W}{X} = 0 \qquad DH - \frac{V \cdot Y \cdot (X - W)}{U \cdot X \cdot Z} = 0$$

$$AH - \frac{U \cdot W \cdot Z + V \cdot Y \cdot (X - W)}{U \cdot X \cdot Z} = 0$$

$$AF - \frac{U \cdot Z \cdot W^2}{X \cdot [U \cdot W \cdot Z + V \cdot Y \cdot (X - W)]} = 0$$

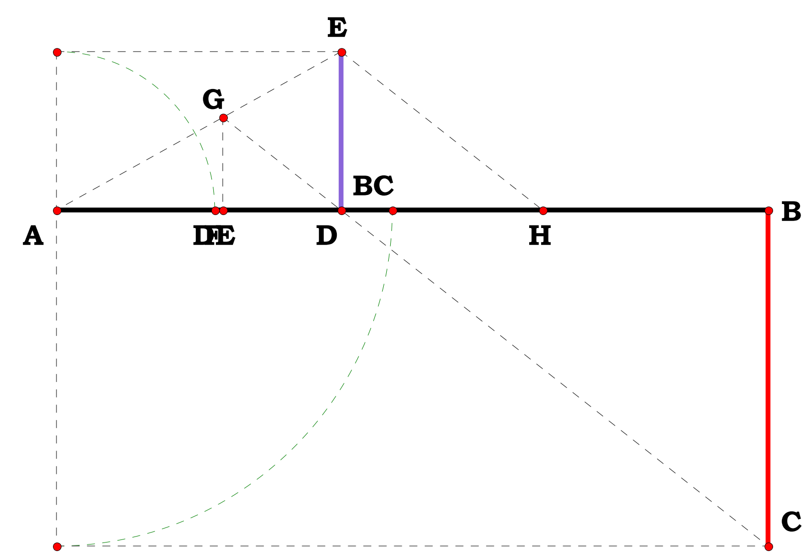
Linear division $\dfrac{U \cdot Z \cdot W^2}{X \cdot [U \cdot W \cdot Z + V \cdot Y \cdot (X - W)]}$

AB = 1.00000
BC = 0.40000
U = 8.00000
V = 20.00000
AD = 0.64706
W = 11.00000
X = 17.00000
DE = 0.33333
Y = 5.00000
Z = 15.00000
AF = 0.44485
FG = 0.22917
AH = 0.94118



$\dfrac{U \cdot Z \cdot W^2}{X \cdot (U \cdot W \cdot Z + V \cdot Y \cdot (X - W))}$ -AF = 0.00000

AB = 1.00000
BC = 0.47059
U = 8.00000
V = 17.00000
AD = 0.40000
W = 8.00000
X = 20.00000
DE = 0.22222
Y = 4.00000
Z = 18.00000
AF = 0.23415
FG = 0.13008
AH = 0.68333



$\dfrac{U \cdot Z \cdot W^2}{X \cdot (U \cdot W \cdot Z + V \cdot Y \cdot (X - W))}$ -AF = 0.00000

013196A

Descriptions.

Unit.

$BC := 1$

Given.

$N_1 := 5$

$N_2 := 3$

On Gemini Roots

$BJ := N_1 \quad CJ := BJ - BC \quad CI := \dfrac{CJ}{2} \quad IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF \quad FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2$

$HS := CR \quad FI := FJ - IJ \quad FG := \dfrac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \dfrac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \dfrac{HS \cdot GO}{OS}$

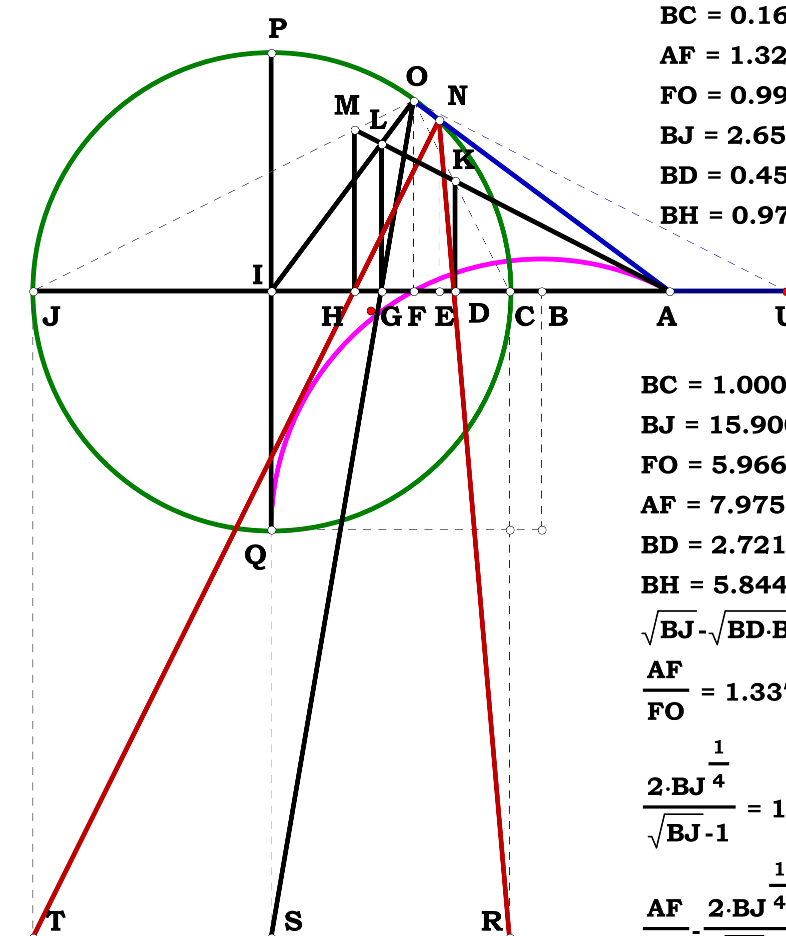
$FU := \dfrac{AG \cdot FO}{GL} \quad AH := \dfrac{FU \cdot AJ}{FU + FJ} \quad DK := \dfrac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \dfrac{AG \cdot DK}{GL} \quad AC := AF - CF$

$CD := AD - AC \quad CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \dfrac{CR \cdot DH}{CD + HJ}$

$CE := \dfrac{CD \cdot (CR + EN)}{CR} \quad AE := AC + CE \quad BD := BC + CD \quad BH := BC + CH$

$\dfrac{AF}{FO} - \dfrac{AE}{EN} = 0 \qquad \sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$

Hitting AO from any RT while maintaining Gemini Roots.



BC = 0.167 in.
AF = 1.329 in.
FO = 0.994 in.
BJ = 2.650 in.
BD = 0.453 in.
BH = 0.974 in.

BC = 1.000
BJ = 15.900
FO = 5.966
AF = 7.975
BD = 2.721
BH = 5.844

$\sqrt{BJ} - \sqrt{BD \cdot BH} = 0.000$

$\dfrac{AF}{FO} = 1.337$

$\dfrac{2 \cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 1.337$

$\dfrac{AF}{FO} - \dfrac{2 \cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 0.000$

Definitions.

$$BJ - N_1 = 0 \quad CJ - (N_1 - 1) = 0 \quad CI - \frac{N_1 - 1}{2} = 0 \quad IJ - \frac{N_1 - 1}{2} = 0 \quad BF - \sqrt{N_1} = 0$$

$$AB - \sqrt{N_1} = 0 \quad AF - 2\cdot\sqrt{N_1} = 0 \quad CF - \left(\sqrt{N_1} - 1\right) = 0 \quad FJ - \left(N_1 - \sqrt{N_1}\right) = 0$$

$$FO - \sqrt{\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)} = 0 \quad CR - \left(N_1\cdot N_2 - N_2\right) = 0$$

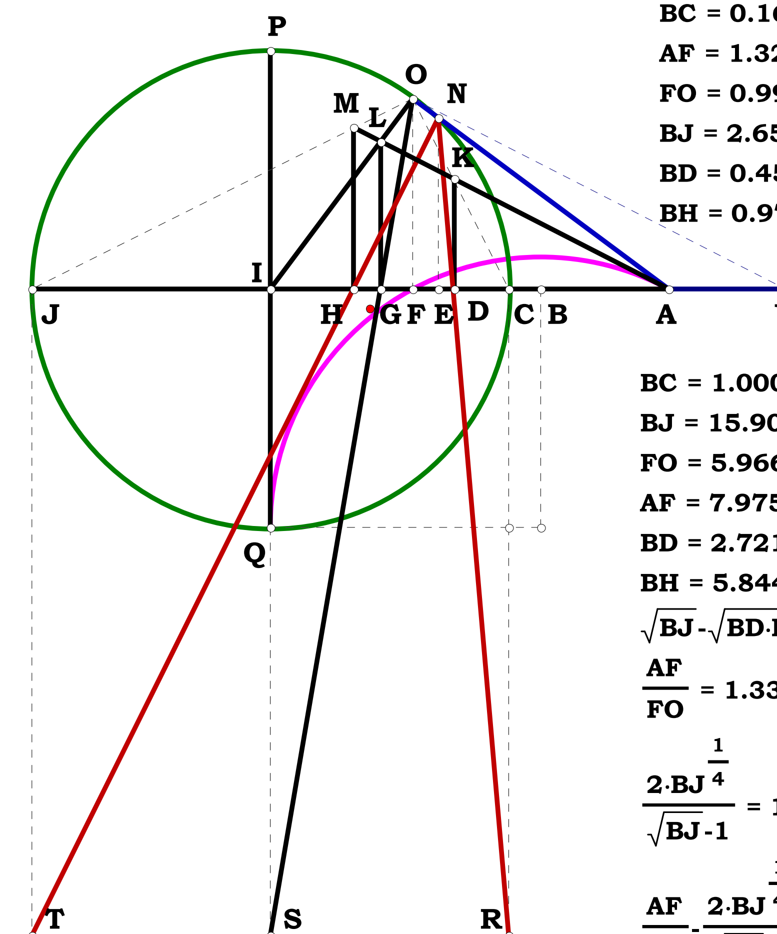
$$HS - \left(N_1\cdot N_2 - N_2\right) = 0 \quad FI - \frac{\left(\sqrt{N_1} - 1\right)^2}{2} = 0 \quad FG - \frac{N_1^{\frac{1}{4}} - 2\cdot N_1^{\frac{3}{4}} + N_1^{\frac{5}{4}}}{2\cdot\left(N_2 + \sqrt{N_1}\cdot N_2 + N_1^{\frac{1}{4}}\right)} = 0$$

$$AG - \frac{N_1^{\frac{1}{4}}\cdot\left(\sqrt{N_1} + 1\right)\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2 + \sqrt{N_1} + 1\right)}{2\cdot\left(N_2 + \sqrt{N_1}\cdot N_2 + N_1^{\frac{1}{4}}\right)} = 0$$

$$OS - \frac{\left(\sqrt{N_1} - 1\right)\cdot\sqrt{\left(\sqrt{N_1} + 1\right)\cdot\left[8\cdot N_1^{\frac{1}{4}}\cdot N_2 + 4\cdot N_2^2 + \sqrt{N_1}\cdot\left(4\cdot N_2^2 + 1\right) + 1\right]}}{2} = 0$$

$$GO - \frac{N_1^{\frac{1}{4}}\cdot\sqrt{\left(\sqrt{N_1} + 1\right)\cdot\left[8\cdot N_1^{\frac{1}{4}}\cdot N_2 + 4\cdot N_2^2 + \sqrt{N_1}\cdot\left(4\cdot N_2^2 + 1\right) + 1\right]}\cdot\left(N_1^{\frac{1}{4}} - 1\right)\cdot\left(N_1^{\frac{1}{4}} + 1\right)}{2\cdot\left(N_2 + \sqrt{N_1}\cdot N_2 + N_1^{\frac{1}{4}}\right)} = 0 \quad AJ - \sqrt{N_1}\cdot\left(\sqrt{N_1} + 1\right) = 0$$

$$GL - \frac{N_1^{\frac{1}{4}}\cdot N_2\cdot\left(N_1 - 1\right)}{N_2 + \sqrt{N_1}\cdot N_2 + N_1^{\frac{1}{4}}} = 0 \quad FU - \frac{N_1^{\frac{1}{4}}\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2 + \sqrt{N_1} + 1\right)}{2\cdot N_2} = 0 \quad AH - \frac{N_1^{\frac{2}{4}}\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2 + \sqrt{N_1} + 1\right)}{2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 1} = 0$$



BC = 0.167 in.
AF = 1.329 in.
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BJ = 2.650 in.
BD = 0.453 in.
BH = 0.974 in.

BC = 1.000
BJ = 15.900
FO = 5.966
AF = 7.975
BD = 2.721
BH = 5.844

$\sqrt{BJ} - \sqrt{BD\cdot BH} = 0.000$

$\frac{AF}{FO} = 1.337$

$\frac{2\cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 1.337$

$\frac{AF}{FO} - \frac{2\cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 0.000$

$$DK-\frac{2\cdot N_1^{\frac{3}{4}}\cdot N_2-2\cdot N_1^{\frac{1}{4}}\cdot N_2}{2\cdot N_2+N_1^{\frac{1}{4}}}=0 \qquad AD-\frac{N_1^{\frac{1}{4}}\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2+\sqrt{N_1}+1\right)}{2\cdot N_2+N_1^{\frac{1}{4}}}=0 \qquad AC-\left(\sqrt{N_1}+1\right)=0$$

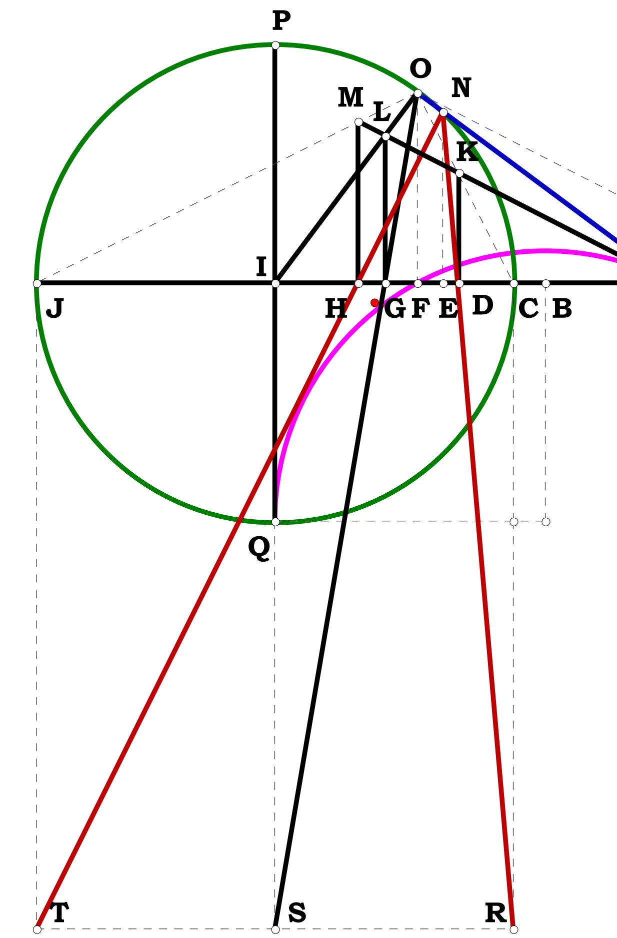
$$CD-\frac{2\cdot N_2\cdot\left(\sqrt{N_1}-1\right)}{2\cdot N_2+N_1^{\frac{1}{4}}}=0 \qquad CH-\frac{N_1-2\cdot N_1^{\frac{1}{4}}\cdot N_2+2\cdot N_1^{\frac{3}{4}}\cdot N_2-1}{2\cdot N_1^{\frac{1}{4}}\cdot N_2+1}=0$$

$$DH-\frac{N_1^{\frac{1}{4}}\cdot\left(\sqrt{N_1}-1\right)\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2+\sqrt{N_1}+1\right)}{2\cdot N_2\cdot\left(\sqrt{N_1}+1\right)+N_1^{\frac{1}{4}}\cdot\left(4\cdot N_2^{2}+1\right)}=0 \qquad HJ-\frac{2\cdot N_1^{\frac{5}{4}}\cdot N_2-2\cdot N_1^{\frac{3}{4}}\cdot N_2}{2\cdot N_1^{\frac{1}{4}}\cdot N_2+1}=0$$

$$EN-\frac{4\cdot N_1^{\frac{3}{2}}\cdot N_2-4\cdot\sqrt{N_1}\cdot N_2-N_1^{\frac{1}{4}}-N_1^{\frac{3}{4}}+N_1^{\frac{5}{4}}+N_1^{\frac{7}{4}}}{2\cdot\left(N_1+2\cdot N_1^{\frac{1}{4}}\cdot N_2+2\cdot N_1^{\frac{3}{4}}\cdot N_2+1\right)}=0 \qquad CE-\frac{N_1-2\cdot N_1^{\frac{1}{4}}\cdot N_2+2\cdot N_1^{\frac{5}{4}}\cdot N_2-1}{N_1+2\cdot N_1^{\frac{1}{4}}\cdot N_2+2\cdot N_1^{\frac{3}{4}}\cdot N_2+1}=0$$

$$AE-\frac{\sqrt{N_1}\cdot\left(\sqrt{N_1}+1\right)\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2+\sqrt{N_1}+1\right)}{N_1+2\cdot N_1^{\frac{1}{4}}\cdot N_2+2\cdot N_1^{\frac{3}{4}}\cdot N_2+1}=0 \qquad BD-\frac{2\cdot\sqrt{N_1}\cdot N_2+N_1^{\frac{1}{4}}}{2\cdot N_2+N_1^{\frac{1}{4}}}=0 \qquad BH-\frac{N_1+2\cdot N_1^{\frac{3}{4}}\cdot N_2}{2\cdot N_1^{\frac{1}{4}}\cdot N_2+1}=0$$

$$\frac{AE}{EN}-\frac{2\cdot N_1^{\frac{1}{4}}}{\left(\sqrt{N_1}-1\right)}=0 \qquad \frac{AF}{FO}-\frac{2\cdot N_1^{\frac{1}{4}}}{\left(\sqrt{N_1}-1\right)}=0$$



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BC = 1.000
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$\sqrt{BJ}-\sqrt{BD\cdot BH} = 0.000$

$\frac{AF}{FO} = 1.337$

$\frac{2\cdot BJ^{\frac{1}{4}}}{\sqrt{BJ}-1} = 1.337$

$\frac{AF}{FO}-\frac{2\cdot BJ^{\frac{1}{4}}}{\sqrt{BJ}-1} = 0.000$

020296A

Descriptions.

Given.

$N_1 := 3.467$

$N_2 := 1.728$

$BE := N_1 \quad BD := \frac{BE}{2} \quad CH := BD \quad BD := \frac{BE}{2}$

$CD := BD - \frac{BD}{N_2} \quad DH := \sqrt{CD^2 + CH^2} \quad DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD} \quad AB := AD - BD \quad BC := BD - CD$

$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$

Definitions.

$BE - N_1 = 0 \quad BD - \frac{N_1}{2} = 0 \quad CH - \frac{N_1}{2} = 0 \quad BD - \frac{N_1}{2} = 0$

$CD - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0 \quad DH - \frac{N_1 \cdot \sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{2 \cdot N_2} = 0$

$DF - \frac{N_1 \cdot \sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{4 \cdot N_2} = 0 \quad AD - \frac{N_1 \cdot (2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{4 \cdot N_2 \cdot (N_2 - 1)} = 0$

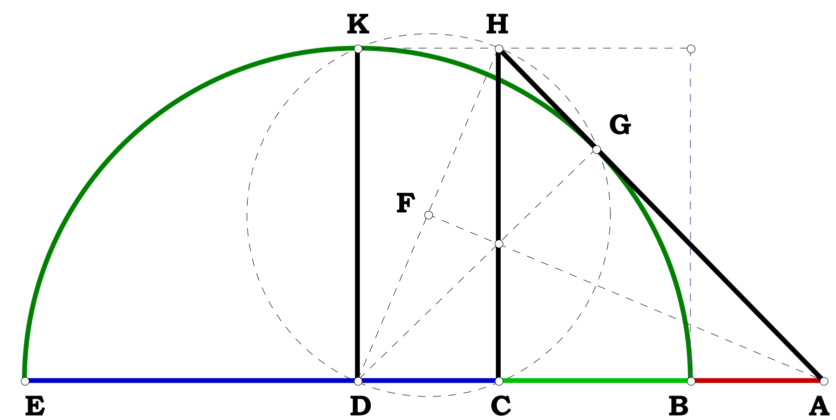
$AB - \frac{N_1}{4 \cdot N_2 \cdot (N_2 - 1)} = 0 \quad BC - \frac{N_1}{2 \cdot N_2} = 0$

$\sqrt{(AB + BE) \cdot AB} - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{4 \cdot N_2 \cdot (N_2 - 1)} = 0 \quad AB + BC - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{4 \cdot N_2 \cdot (N_2 - 1)}$

$\frac{BC^2}{BE - 2 \cdot BC} - AB = 0$

Find A Segment

Find segment AB.



BC = 1.003 in.
BE = 3.467 in.
AB = 0.689 in.
BD = 1.733 in.
BC = 1.003 in.
$\frac{BD}{BC} = 1.728$

$\frac{BC^2}{BE\text{-}2 \cdot BC}\text{-}AB = 0.000$ in.

DH = 1.881 in.
AD = 2.422 in.

Given BE and BC such that $\sqrt{(AB + BE) \cdot AB} = AB + BC$, find AB.

020296B

Descriptions.

$$DE := \frac{Y}{Y} \quad BD := DE \quad BE := 2 \cdot DE$$

$$CH := BD \quad CD := \frac{X}{Y} \quad DH := \sqrt{CD^2 + CH^2}$$

$$DF := \frac{DH}{2} \quad AD := \frac{DH \cdot DF}{CD} \quad AB := AD - BD$$

$$BC := BD - CD \quad AB = 0.603571$$

$$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$$

Given.

$X := 7$

$Y := 20$

Definitions.

$$DE - \frac{Y}{Y} = 0 \quad BD - \frac{Y}{Y} = 0 \quad CH - \frac{Y}{Y} = 0$$

$$BE - 2 = 0 \quad CD - \frac{X}{Y} = 0 \quad DH - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$$

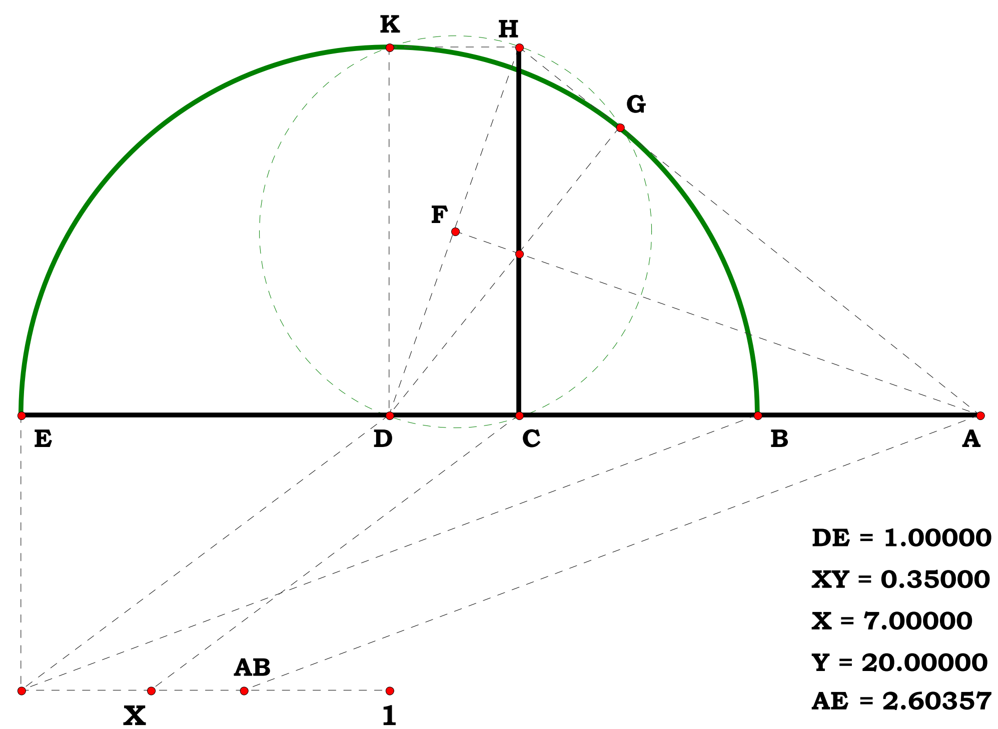
$$DF - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0 \quad AD - \frac{X^2 + Y^2}{2 \cdot X \cdot Y} = 0$$

$$AB - \frac{(Y - X)^2}{2 \cdot X \cdot Y} = 0 \quad BC - \frac{Y - X}{Y} = 0$$

$$\frac{(X + Y) \cdot (Y - X)}{2 \cdot X \cdot Y} - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$$

Find A Segment

Find segment AB.



DE = 1.00000
XY = 0.35000
X = 7.00000
Y = 20.00000
AE = 2.60357
AB = 0.60357

Given BE and BC such that $\sqrt{(AB + BE) \cdot AB} = AB + BC$, find AB.

021496

Descriptions.

Unit.

Or, the 17 decimal place rustic solution.

Given.

$N := 2$

$\Delta := 40 \quad \delta := 0 .. \Delta$

Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

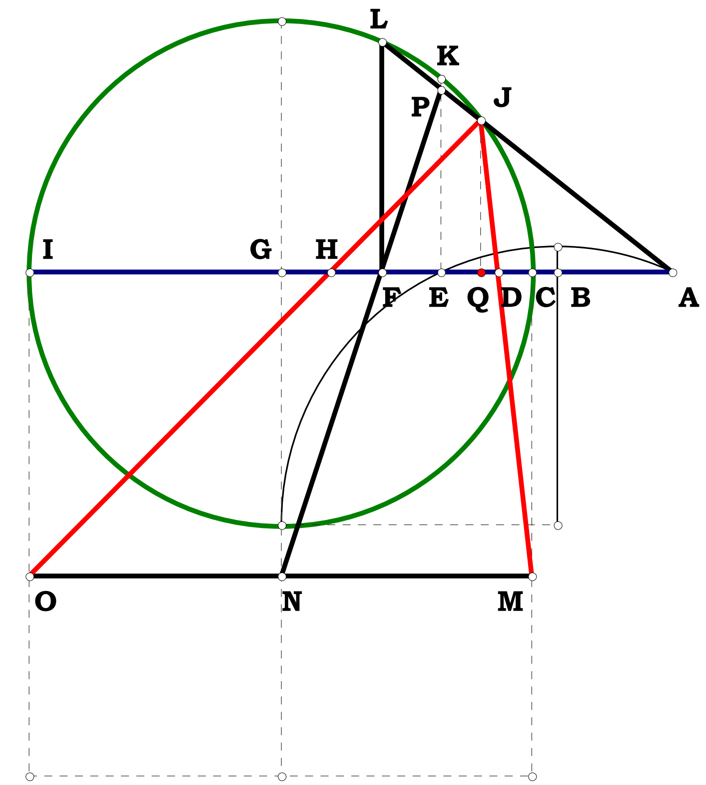
$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$GN := CG \cdot N \quad IO := GN \quad CM := GN$



$$\begin{pmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{pmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC \\ \sqrt{\left[\left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC\right] \cdot \left[GI + \left(\frac{EG \cdot GN}{GN + EK}\right)\right]} \end{bmatrix}$$

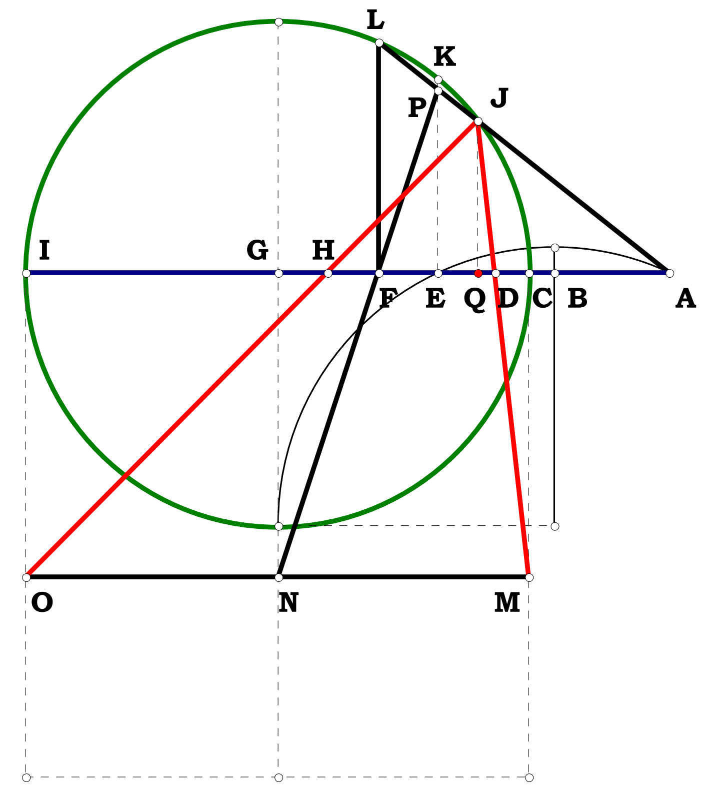
$$\begin{pmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{pmatrix} := \begin{pmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{pmatrix}$$

$$AK := \sqrt{AE^2 + EK^2} \qquad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \qquad AJ := \frac{AK^2}{AL} \qquad AQ := \frac{AF_\Delta \cdot AJ}{AL}$$

$$CQ := AQ - AC \qquad IQ := CI - CQ \qquad JQ := \sqrt{CQ \cdot IQ} \qquad CD := \frac{CQ \cdot CM}{CM + JQ}$$

$$HI := \frac{IQ \cdot IO}{IO + JQ} \qquad DH := CI - (CD + HI) \qquad BD := BC + CD$$

$$BH := BC + CD + DH \qquad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \qquad BE - \sqrt{BD \cdot BH} = 0$$



The next two equations are for the Delian Problem only.
Resolution set to max of the program.

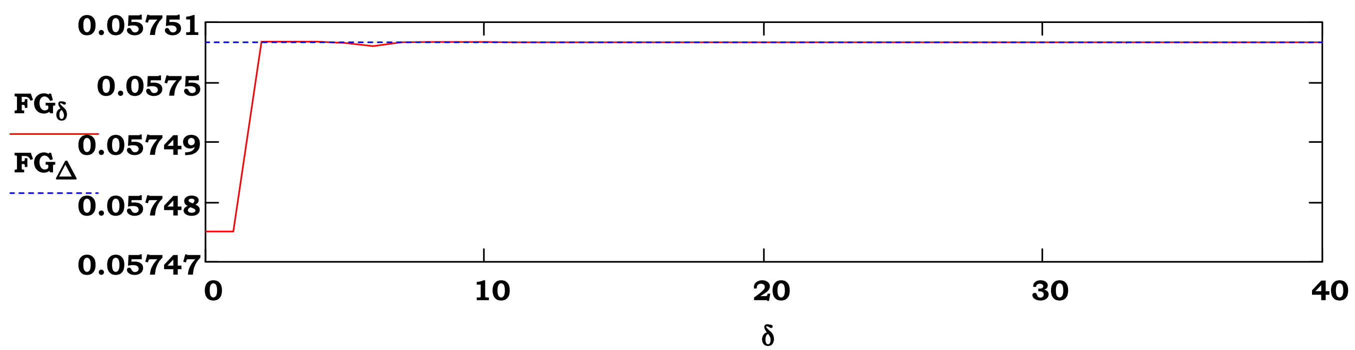
$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0$$

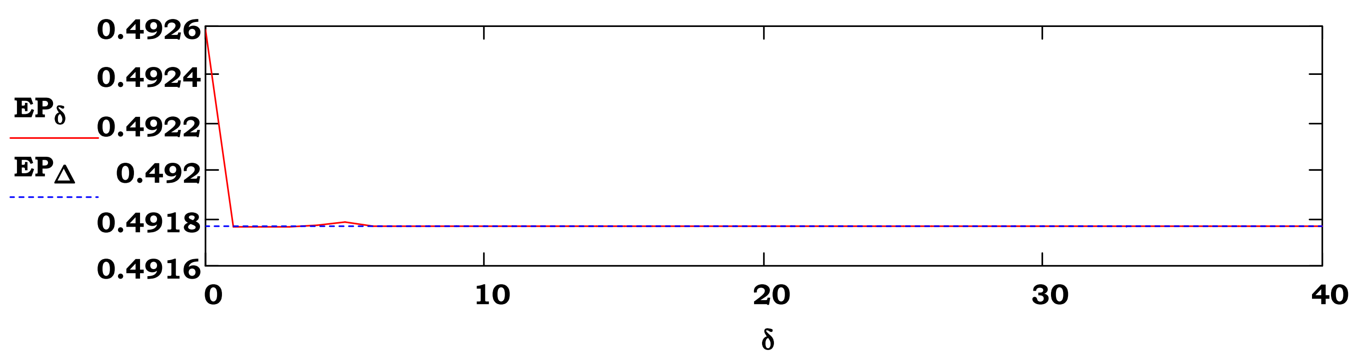
$$BD = 1.2599210498948732 \qquad 2^{\frac{1}{3}} = 1.2599210498948732$$

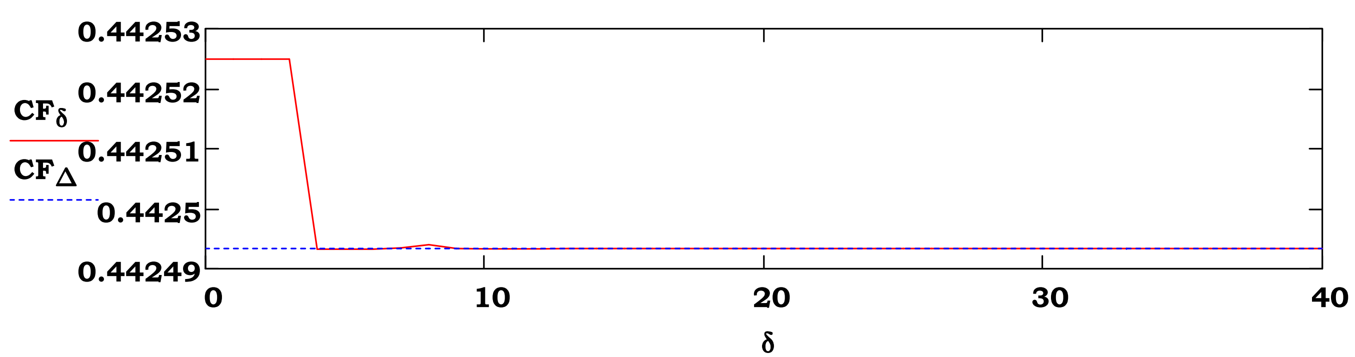
17 decimal places. Good to the limits of the program.

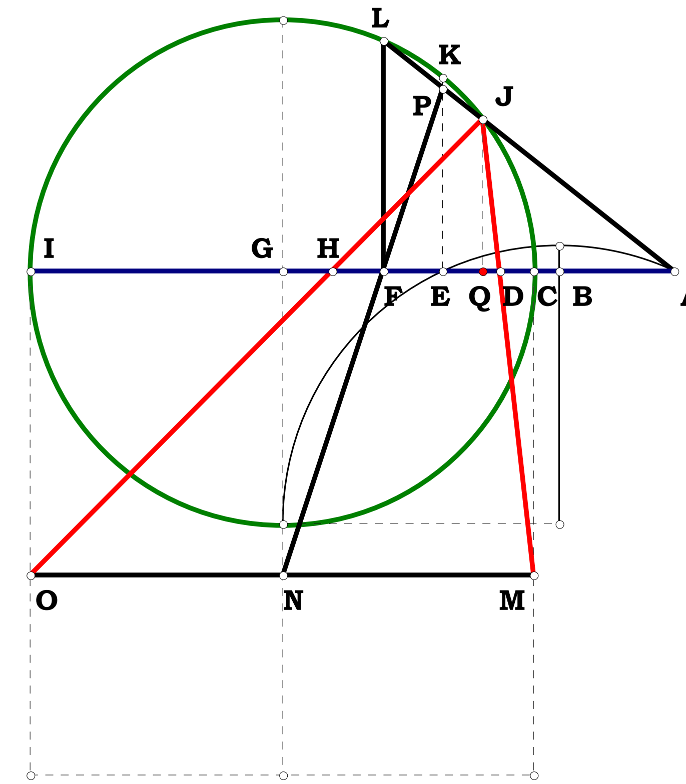
$$BH = 1.5874010519681994 \qquad 4^{\frac{1}{3}} = 1.5874010519681994$$

A graphics peek.









The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist. The solution is only good to material differences, on the atomic level, so to speak.

041496A

Unit.

$CM := 1$

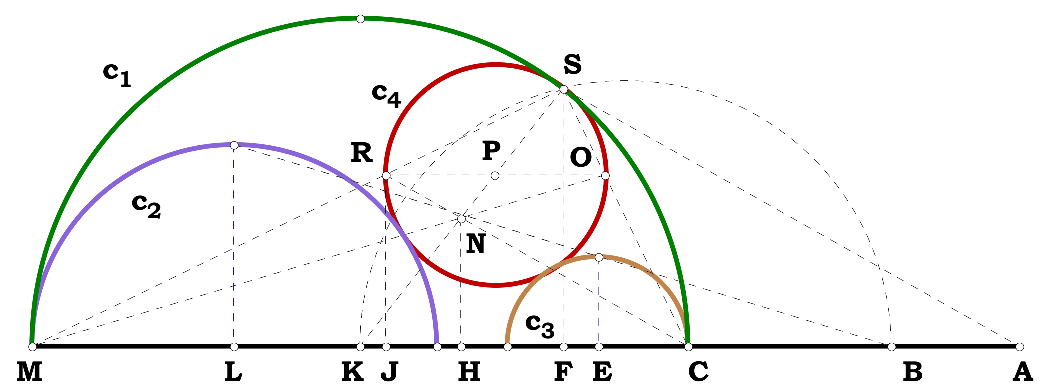
Given.

$N_1 := .13749$

$N_2 := .30814$

Given c_1, c_2, c_3, find c_4. I had this sketched out in 95, but if I put it there I would have had a lot of document links to redo in "The Quest." In my earlier revisions, it seems that I forgot to remove the reciprocals for C2 and 3.

Method for Unequals



CM = 1.00000
CE = 0.13749
LM = 0.30814
PS = 0.16749

$\frac{(2\cdot LM-1)\cdot(2\cdot CE-1)}{2\cdot(1-4\cdot CE\cdot LM)}$-PS = 0.00000

Descriptions.

$CK := \frac{CM}{2}$ $CE := N_1$ $LM := N_2$ $EL := CM-(CE+LM)$

$BL := \frac{EL\cdot LM}{LM-CE}$ $BM := BL+LM$ $BC := BM-CM$ $BK := \frac{CM}{2}+BC$

$R_1 := LM$ $R_2 := CE$ $D := EL$ $KS := CK$ $EH := \frac{\left(R_2^{\,2}+D^2-R_1^{\,2}\right)}{2\cdot D}$

$FK := \frac{KS^2}{BK}$ $CF := CK-FK$ $FM := CM-CF$ $FS := \sqrt{CF\cdot FM}$

$HK := CK-(CE+EH)$ $CH := CK-HK$ $HN := \frac{FS\cdot HK}{FK}$ $AF := \frac{CH\cdot FS}{HN}$

$JR := \frac{FS\cdot CM}{AF+FM}$ $RO := \frac{CM\cdot(FS-JR)}{FS}$ $PS := \frac{RO}{2}$ $PS = 0.167485$

Definitions.

$CK-\frac{1}{2}=0$ $CE-N_1=0$ $LM-N_2=0$ $EL-\left(1-N_2-N_1\right)=0$ $BL-\frac{N_2\cdot\left(N_1+N_2-1\right)}{N_1-N_2}=0$ $BM-\frac{N_2\cdot\left(2\cdot N_1-1\right)}{N_1-N_2}=0$ $BC-\frac{N_1\cdot\left(2\cdot N_2-1\right)}{N_1-N_2}=0$

$BK-\frac{4\cdot N_1\cdot N_2-N_2-N_1}{2\cdot\left(N_1-N_2\right)}=0$ $R_1-N_2=0$ $R_2-N_1=0$ $D-\left(1-N_2-N_1\right)=0$ $KS-\frac{CM}{2}=0$ $EH-\frac{2\cdot N_1+2\cdot N_2-2\cdot N_1^{\,2}-2\cdot N_1\cdot N_2-1}{2\cdot\left(N_1+N_2-1\right)}=0$ $FK-\frac{CM^2\cdot\left(N_1-N_2\right)}{2\cdot\left(4\cdot N_1\cdot N_2-N_2-N_1\right)}=0$

$CF-\frac{N_1\cdot\left(2\cdot N_2-1\right)}{4\cdot N_1\cdot N_2-N_2-N_1}=0$ $FM-\frac{N_2\cdot\left(2\cdot N_1-1\right)}{4\cdot N_1\cdot N_2-N_2-N_1}=0$ $FS-\frac{\sqrt{N_1\cdot N_2\cdot\left(2\cdot N_1-1\right)\cdot\left(2\cdot N_2-1\right)}}{\left(N_1+N_2-4\cdot N_1\cdot N_2\right)}=0$ $HK-\frac{N_1-N_2}{2\cdot\left(N_1+N_2-1\right)}=0$ $CH-\frac{2\cdot N_2-1}{2\cdot\left(N_1+N_2-1\right)}=0$

$HN-\frac{\sqrt{N_1\cdot N_2\cdot\left(2\cdot N_1-1\right)\cdot\left(2\cdot N_2-1\right)}}{\left(1-N_2-N_1\right)}=0$ $AF-\frac{\left(2\cdot N_2-1\right)}{2\cdot\left(4\cdot N_1\cdot N_2-N_2-N_1\right)}=0$ $JR-\frac{2\cdot\sqrt{N_1\cdot N_2\cdot\left(2\cdot N_1-1\right)\cdot\left(2\cdot N_2-1\right)}}{\left(1-4\cdot N_1\cdot N_2\right)}=0$ $RO-\frac{\left(2\cdot N_2-1\right)\cdot\left(2\cdot N_1-1\right)}{1-4\cdot N_1\cdot N_2}=0$

$PS-\frac{\left(2\cdot N_2-1\right)\cdot\left(2\cdot N_1-1\right)}{2\cdot\left(1-4\cdot N_1\cdot N_2\right)}=0$

041496B

Given.

$W := 3 \qquad Y := 6$

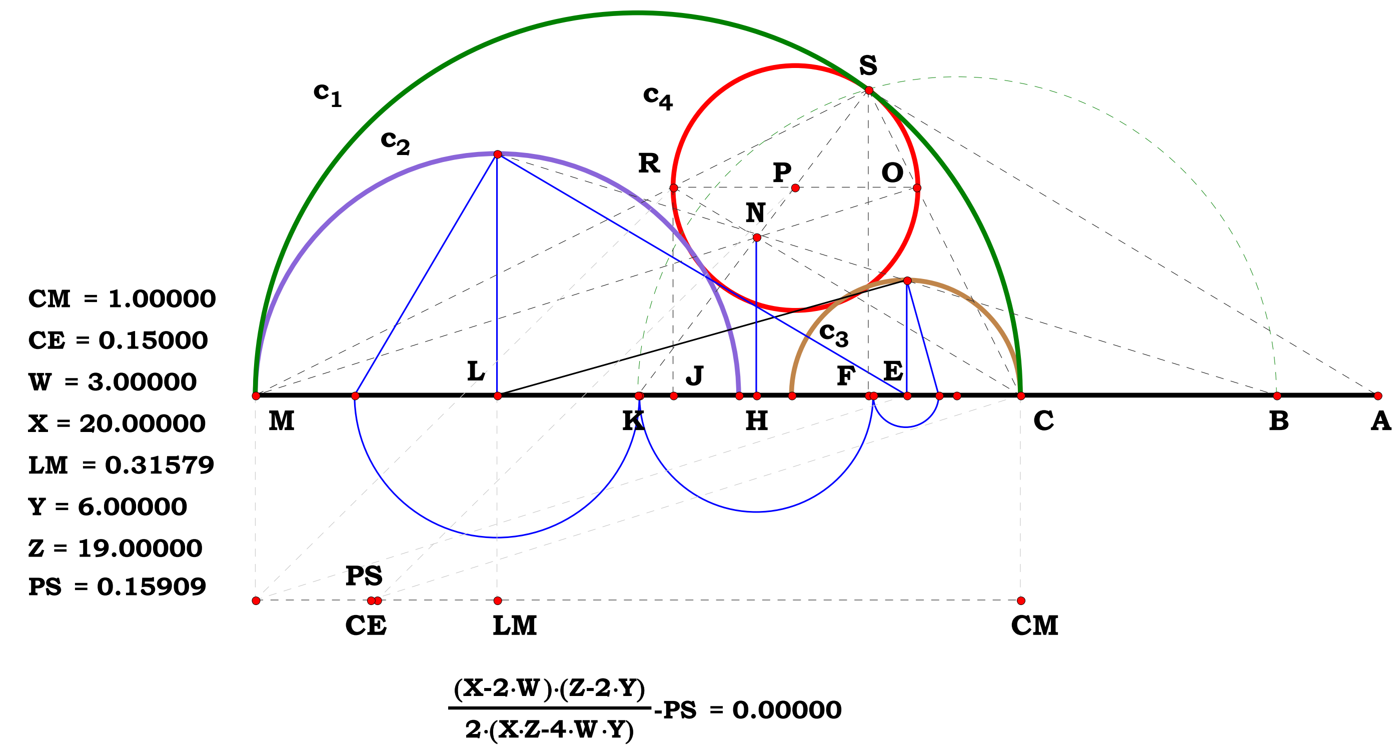
$X := 20 \qquad Z := 19$

Unit.

$CM := \dfrac{X}{X}$

Method for Unequals

Given c_1, c_2, c_3, find c_4. The thin blue lines is the process I developed for finding the powerline between two circles.



Descriptions.

$CK := \dfrac{CM}{2} \qquad CE := \dfrac{W}{X} \qquad LM := \dfrac{Y}{Z} \qquad EL := CM - (CE + LM)$

$BL := \dfrac{EL \cdot LM}{LM - CE} \qquad BM := BL + LM \qquad BC := BM - CM \qquad BK := \dfrac{CM}{2} + BC$

power line. $EH := \dfrac{\left(CE^2 + EL^2 - LM^2\right)}{2 \cdot EL} \qquad KS := CK$

$FK := \dfrac{KS^2}{BK} \qquad CF := CK - FK \qquad FM := CM - CF \qquad FS := \sqrt{CF \cdot FM}$

$HK := CK - (CE + EH) \qquad CH := CK - HK \qquad HN := \dfrac{FS \cdot HK}{FK} \qquad AF := \dfrac{CH \cdot FS}{HN}$

$JR := \dfrac{FS \cdot CM}{AF + FM} \qquad RO := \dfrac{CM \cdot (FS - JR)}{FS} \qquad PS := \dfrac{RO}{2} \qquad PS = 0.159091$

Definitions.

$CM - 1 = 0 \qquad CK - \dfrac{1}{2} = 0 \qquad CE - \dfrac{W}{X} = 0 \qquad LM - \dfrac{Y}{Z} = 0 \qquad EL - \dfrac{(X \cdot Z - X \cdot Y - W \cdot Z)}{X \cdot Z} = 0 \qquad BL - \dfrac{Y \cdot (W \cdot Z + X \cdot Y - X \cdot Z)}{Z \cdot (W \cdot Z - X \cdot Y)} = 0$

$BM - \dfrac{Y \cdot (2 \cdot W - X)}{W \cdot Z - X \cdot Y} = 0 \qquad BC - \dfrac{W \cdot (2 \cdot Y - Z)}{W \cdot Z - X \cdot Y} = 0 \qquad BK - \dfrac{4 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{2 \cdot (W \cdot Z - X \cdot Y)} = 0 \qquad EH - \dfrac{2 \cdot W^2 \cdot Z - 2 \cdot X^2 \cdot Y + X^2 \cdot Z + 2 \cdot W \cdot X \cdot Y - 2 \cdot W \cdot X \cdot Z}{2 \cdot X \cdot (X \cdot Z - X \cdot Y - W \cdot Z)} = 0$

$KS - \dfrac{1}{2} = 0 \qquad FK - \dfrac{W \cdot Z - X \cdot Y}{2 \cdot (4 \cdot W \cdot Y - W \cdot Z - X \cdot Y)} = 0 \qquad CF - \dfrac{W \cdot (2 \cdot Y - Z)}{4 \cdot W \cdot Y - W \cdot Z - X \cdot Y} = 0 \qquad FM - \dfrac{Y \cdot (2 \cdot W - X)}{4 \cdot W \cdot Y - W \cdot Z - X \cdot Y} = 0$

$FS - \dfrac{\sqrt{W \cdot Y \cdot (2 \cdot W - X) \cdot (2 \cdot Y - Z)}}{(W \cdot Z - 4 \cdot W \cdot Y + X \cdot Y)} = 0 \qquad HK - \dfrac{W \cdot Z - X \cdot Y}{2 \cdot (W \cdot Z + X \cdot Y - X \cdot Z)} = 0 \qquad CH - \dfrac{X \cdot (2 \cdot Y - Z)}{2 \cdot (W \cdot Z + X \cdot Y - X \cdot Z)} = 0 \qquad HN - \dfrac{\sqrt{W \cdot Y \cdot (X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}}{(X \cdot Z - X \cdot Y - W \cdot Z)} = 0$

$AF - \dfrac{X \cdot (2 \cdot Y - Z)}{2 \cdot (4 \cdot W \cdot Y - W \cdot Z - X \cdot Y)} = 0 \qquad JR - \dfrac{2 \cdot \sqrt{W \cdot Y \cdot (X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}}{(X \cdot Z - 4 \cdot W \cdot Y)} = 0 \qquad RO := \dfrac{(X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}{X \cdot Z - 4 \cdot W \cdot Y} \qquad PS - \dfrac{(X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}{2 \cdot (X \cdot Z - 4 \cdot W \cdot Y)} = 0$

Unit.

$AB := 1$

Given.

$N_1 := 4$

041596

On Gemini Roots

Descriptions.

$BE := N_1 \qquad BD := \dfrac{BE}{2}$

$AE := AB + BE \qquad AC := \sqrt{AB \cdot AE} \qquad BC := AC - AB \qquad CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \qquad CG := \dfrac{CF^2}{CD} \qquad BG := CG - BC \qquad EG := BG + BE \qquad CH := \dfrac{1}{2} \cdot CF$

$DH := \sqrt{CH^2 + CD^2} \qquad DI := \dfrac{1}{2} \cdot DH \qquad DL := \dfrac{CD \cdot DI}{DH} \qquad BL := BD - DL \qquad EL := BE - BL$

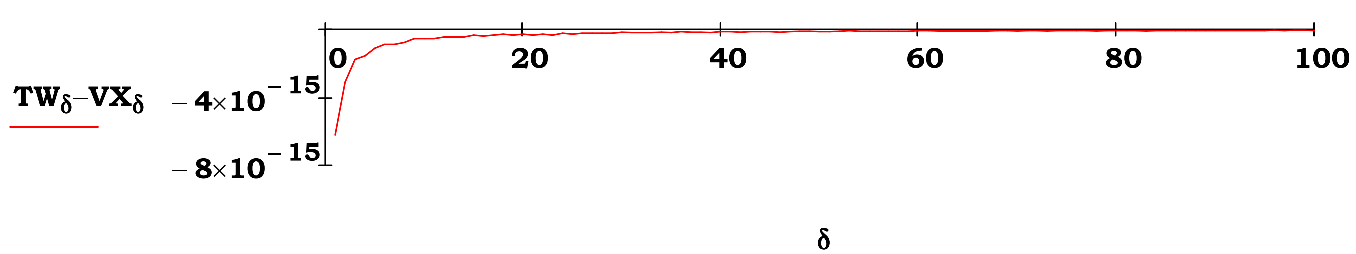
$JL := \sqrt{BL \cdot EL} \qquad GL := BL + BG \qquad GJ := \sqrt{JL^2 + GL^2} \qquad GK := \dfrac{BG \cdot EG}{GJ} \qquad GM := \dfrac{GL \cdot GK}{GJ}$

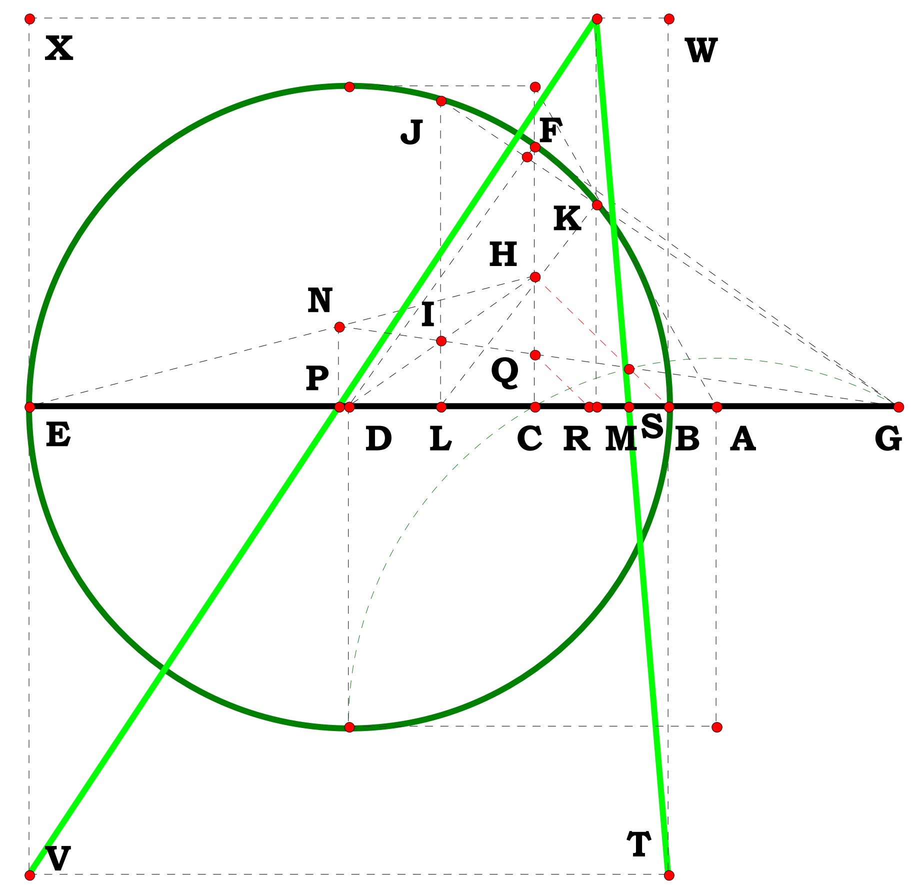
$BM := GM - BG \qquad EM := BE - BM \qquad IL := \sqrt{DI^2 - DL^2} \qquad CO := \dfrac{GL \cdot CH}{IL} \qquad NP := \dfrac{CH \cdot EG}{(CO + CE)}$

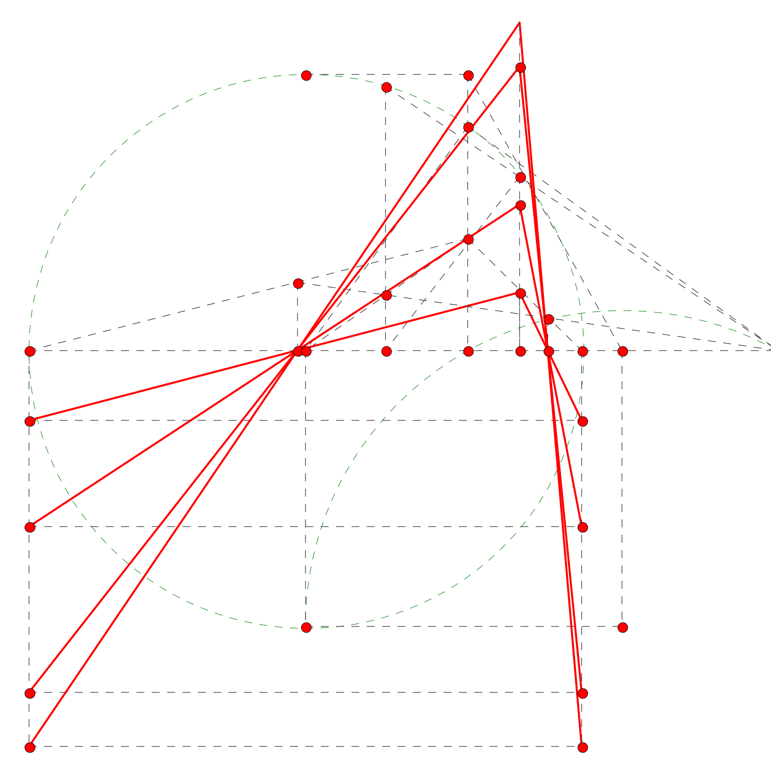
$EP := \dfrac{CE \cdot NP}{CH} \qquad CQ := \dfrac{IL \cdot CG}{GL} \qquad CR := \dfrac{BC \cdot CQ}{CH} \qquad GR := CG - CR \qquad BS := \dfrac{CR \cdot BG}{GR}$

Definitions.

$\delta := 1 .. 100 \qquad E_\delta := \dfrac{BE}{\delta} \qquad BT_\delta := E_\delta \qquad EV_\delta := E_\delta \qquad TW_\delta := \dfrac{BT_\delta \cdot BM}{BS} \qquad VX_\delta := \dfrac{EV_\delta \cdot EM}{EP}$







041696A

Descriptions.

Unit.

$AJ := 1$

Given.

$N_1 := .24440$

$N_2 := .19782$

$AF := \frac{AJ}{2} \quad HJ := N_1 \quad NO := N_2 \quad HM := HJ \quad MO := NO$

$HO := HM + MO \quad FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$EH := \frac{HO^2 + FH^2 - FO^2}{2 \cdot FH} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO$

$AP := \sqrt{AG^2 + GP^2} \quad PL := \frac{AG \cdot (NO + OP)}{AP} \quad AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG}$

$AB = 0.181806$

Definitions.

$AF - \frac{1}{2} = 0 \quad HJ - N_1 = 0 \quad NO - N_2 = 0 \quad HM - N_1 = 0 \quad MO - N_2 = 0$

$HO - (N_1 + N_2) = 0 \quad FO - \frac{1 - 2 \cdot N_2}{2} = 0 \quad AH - (1 - N_1) = 0 \quad FH - \frac{1 - 2 \cdot N_1}{2} = 0$

$EH - \frac{N_1 - N_2 - 2 \cdot N_1^2 - 2 \cdot N_1 \cdot N_2}{2 \cdot N_1 - 1} = 0 \quad EO - \frac{\sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - 2 \cdot N_2 - 2 \cdot N_1)}}{1 - 2 \cdot N_1} = 0$

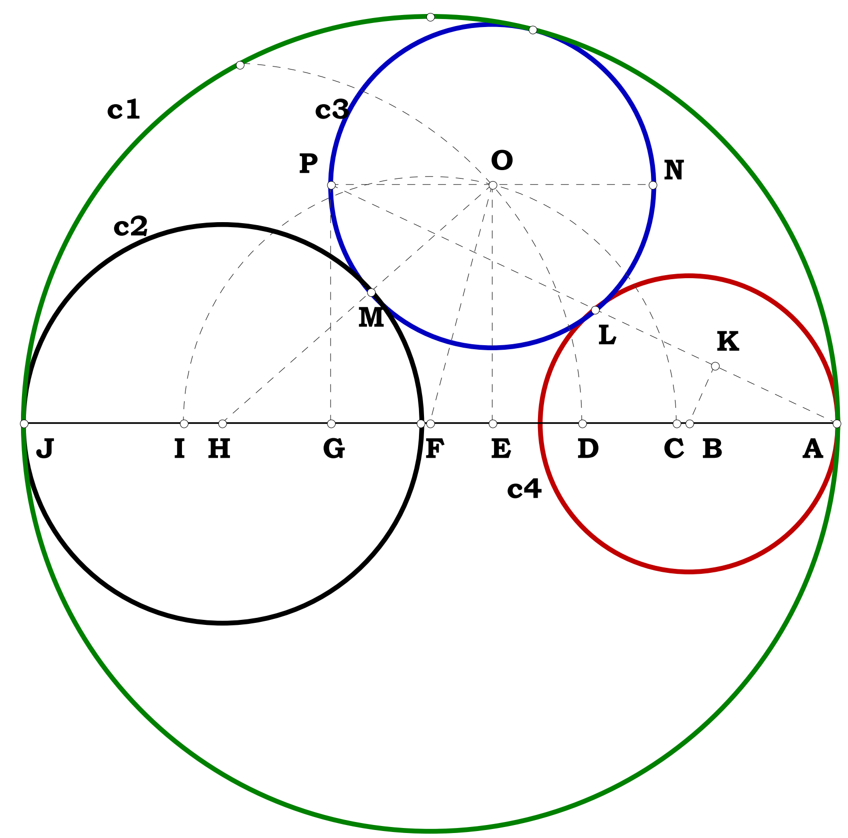
$OP - N_2 = 0 \quad EG - N_2 = 0 \quad AE - \frac{2 \cdot N_1 + N_2 + 2 \cdot N_1 \cdot N_2 - 1}{2 \cdot N_1 - 1} = 0$

$AG - \frac{2 \cdot N_1 + 4 \cdot N_1 \cdot N_2 - 1}{2 \cdot N_1 - 1} = 0 \quad GP - \frac{\sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - 2 \cdot N_2 - 2 \cdot N_1)}}{1 - 2 \cdot N_1} = 0 \quad AP - \frac{\sqrt{\left(1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 8 \cdot N_1 \cdot N_2^2\right)}}{\sqrt{1 - 2 \cdot N_1}} = 0$

$PL - \frac{2 \cdot N_2 \cdot (1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1)}{\sqrt{1 - 2 \cdot N_1} \cdot \sqrt{1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 8 \cdot N_1 \cdot N_2^2}} = 0 \quad AL - \frac{1 - 2 \cdot N_2 - 2 \cdot N_1}{\sqrt{1 - 2 \cdot N_1} \cdot \sqrt{1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 8 \cdot N_1 \cdot N_2^2}} = 0$

$AB := \frac{2 \cdot N_1 + 2 \cdot N_2 - 1}{2 \cdot (2 \cdot N_1 + 4 \cdot N_1 \cdot N_2 - 1)}$

Given Three Radii



$AJ = 4.23333$ in.
$HJ = 1.03462$ in.
$NO = 0.83742$ in.
$AB = 0.76970$ in.

$\frac{AJ}{AJ} = 1.00000 \quad AJ = 1.00000$

$\frac{HJ}{AJ} = 0.24440 \quad HJ = 0.24440$

$\frac{NO}{AJ} = 0.19782 \quad NO = 0.19782$

$\frac{AB}{AJ} = 0.18182 \quad AB = 0.18182$

$\frac{(2 \cdot HJ + 2 \cdot NO) - 1}{2 \cdot ((2 \cdot HJ + 4 \cdot HJ \cdot NO) - 1)} - AB = 0.00000$

Given c1, c2 and c3 find c4 such that
AB is collinear with c1 and c2.

041696B

Descriptions.

Given.

$W := 4 \quad Y := 3$

$X := 20 \quad Z := 17$

Unit.

$AJ := \frac{X}{X}$

$AF := \frac{AJ}{2} \quad HJ := \frac{W}{X} \quad NO := \frac{Y}{Z} \quad HM := HJ \quad MO := NO$

$HO := HM + MO \quad FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$EH := \frac{HO^2 + FH^2 - FO^2}{2 \cdot FH} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO$

$AP := \sqrt{AG^2 + GP^2} \quad PL := \frac{AG \cdot (NO + OP)}{AP} \quad AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG}$

$AB = 0.269231$

Definitions.

$AF - \frac{1}{2} = 0 \quad HJ - \frac{W}{X} = 0 \quad NO - \frac{Y}{Z} = 0 \quad HM - \frac{W}{X} = 0 \quad MO - \frac{Y}{Z} = 0$

$HO - \frac{W \cdot Z + X \cdot Y}{X \cdot Z} = 0 \quad FO - \frac{Z - 2 \cdot Y}{2 \cdot Z} = 0 \quad AH - \frac{X - W}{X} = 0 \quad FH - \frac{X - 2 \cdot W}{2 \cdot X} = 0$

$EH - \frac{2 \cdot W^2 \cdot Z + X^2 \cdot Y + W \cdot X \cdot (2 \cdot Y - Z)}{X \cdot (X - 2 \cdot W) \cdot Z} = 0 \quad EO - \frac{\sqrt{4 \cdot W \cdot Y \cdot (X \cdot Z - 2 \cdot X \cdot Y - 2 \cdot W \cdot Z)}}{Z \cdot (X - 2 \cdot W)} = 0$

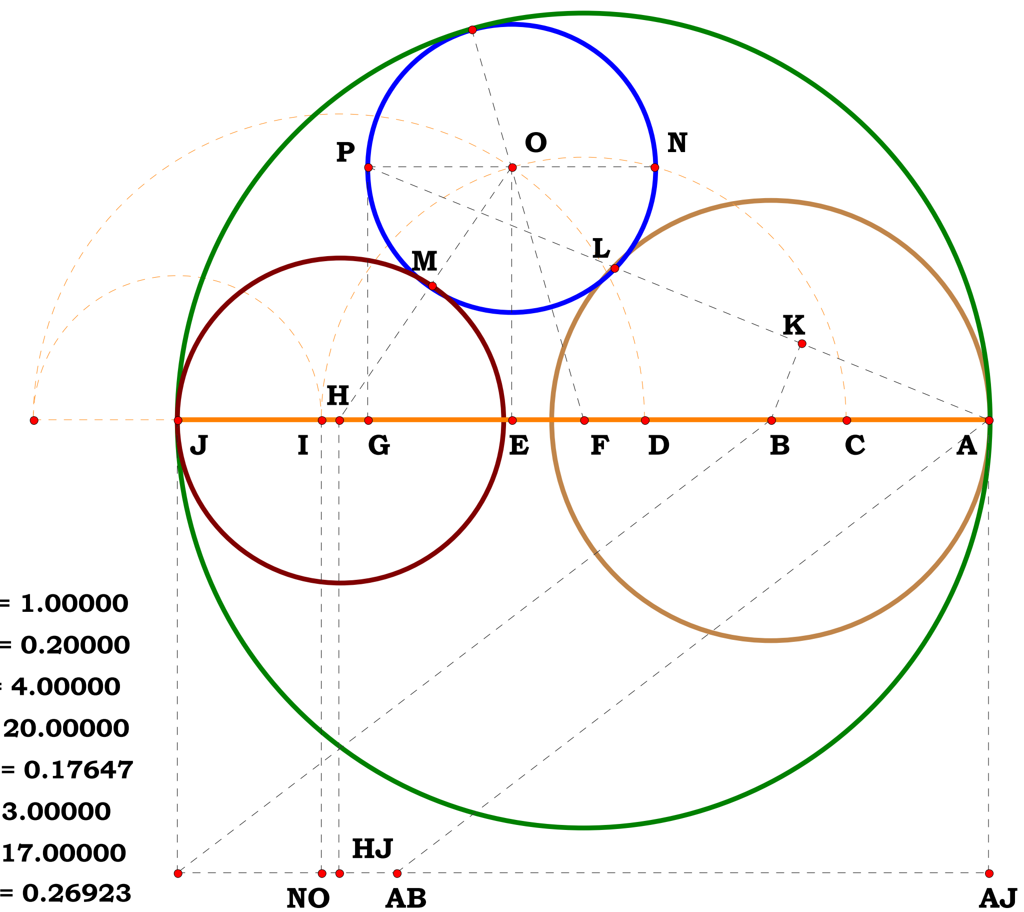
$OP - \frac{Y}{Z} = 0 \quad EG - \frac{Y}{Z} = 0 \quad AE - \frac{2 \cdot W \cdot Y + 2 \cdot W \cdot Z + X \cdot Y - X \cdot Z}{Z \cdot (2 \cdot W - X)} = 0 \quad AG - \frac{4 \cdot W \cdot Y + 2 \cdot W \cdot Z - X \cdot Z}{Z \cdot (2 \cdot W - X)} = 0$

$GP - \frac{\sqrt{4 \cdot W \cdot Y \cdot (X \cdot Z - 2 \cdot X \cdot Y - 2 \cdot W \cdot Z)}}{Z \cdot (X - 2 \cdot W)} = 0 \quad AP - \frac{\sqrt{X \cdot Z^2 - 2 \cdot W \cdot Z^2 - 8 \cdot W \cdot Y^2 - 4 \cdot W \cdot Y \cdot Z}}{Z \cdot \sqrt{X - 2 \cdot W}} = 0$

$PL - \frac{2 \cdot Y \cdot (X \cdot Z - 2 \cdot W \cdot Z - 4 \cdot W \cdot Y)}{Z \cdot \sqrt{X - 2 \cdot W} \cdot \sqrt{Z^2 \cdot (X - 2 \cdot W) - 8 \cdot W \cdot Y^2 - 4 \cdot W \cdot Y \cdot Z}} = 0 \quad AL - \frac{(X \cdot Z - 2 \cdot X \cdot Y - 2 \cdot W \cdot Z)}{\sqrt{X - 2 \cdot W} \cdot \sqrt{X \cdot Z^2 - 2 \cdot W \cdot Z^2 - 8 \cdot W \cdot Y^2 - 4 \cdot W \cdot Y \cdot Z}} = 0 \quad AB - \frac{2 \cdot (W \cdot Z + X \cdot Y) - X \cdot Z}{2 \cdot [2 \cdot W \cdot (2 \cdot Y + Z) - X \cdot Z]} = 0$

Given Three Radii

Given c1, c2 and c3 find c4 such that AB is collinear with c1 and c2.



AJ = 1.00000
HJ = 0.20000
W = 4.00000
X = 20.00000
NO = 0.17647
Y = 3.00000
Z = 17.00000
AB = 0.26923

$\frac{(2 \cdot HJ+2 \cdot NO)-1}{2 \cdot ((2 \cdot HJ+4 \cdot HJ \cdot NO)-1)} - AB = 0.00000$

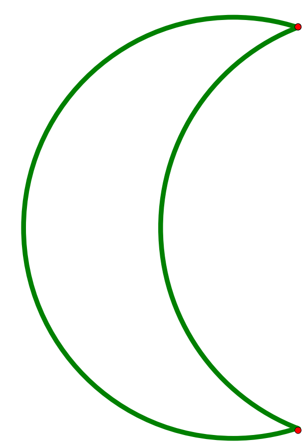
$\frac{2 \cdot (W \cdot Z+X \cdot Y)-X \cdot Z}{2 \cdot (2 \cdot W \cdot (2 \cdot Y+Z)-X \cdot Z)} - AB = 0.00000$

The Man in the Moon; What is his name?

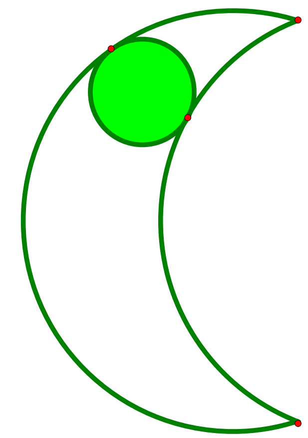
Monday, February 20, 2020

My name is John; I am either insane, or, things have been written about me thousands of years before I was born. These facts, about me, are hardly a mystery and can be dismissed a countless variety of ways. I am not, nor ever have been, of significant importance. I believe that the world was prepared to ignore me so that they would not ponder the most important question there really is, or could ever be: *What is the name of the Man in the Moon*? Let me show you a picture so that you can follow; I have been doing a rather detailed examination of my subject.

Here is the moon, or rather the crescent moon.



From time to time, some observers have spotted the Man in the Moon; fortunately, I have a friend at an observatory which was able to photograph him;



I may have actually seen a commercial of him somewhere. If you were wondering why people say that aliens are little green men, it all started with the Man in the Moon.

This little essay is not about little green men, it is about finding the name of The Man in the Moon. In the following two graphics, if you examine them very, very carefully, you will see the controversy about his name.

041796A

Unit.

$AG := 1$ The radius of the Large, green, Circle, it is taken as the unit of the crescent.

Given.

$CF := .53859$ The radius of the Small, pink circle.

$CG := .65522$ The difference between center of Radius Large and Radius Small.

$BH := 1.15236$ The point on the diameter of the Large Cresant that we want to know the radius of that circle on the perpendicular.

Descriptions.

$AJ := 2 \cdot AG \quad BG := \frac{\left(AG^2 + CG^2 - CF^2\right)}{2 \cdot CG} \quad BC := CG - BG \quad FG := CG - CF$

$AB := AG - BG \quad BJ := AJ - AB$

$AH := BH + AB \quad GH := AH - AG \quad HR := \sqrt{AG^2 - GH^2} \quad BP := HR$

$PR := BH \quad PS := \frac{GH \cdot PR}{HR} \quad BS := BP + PS \quad RS := \sqrt{PR^2 + PS^2} \quad NS := RS$

$CN := CF \quad CS := \sqrt{NS^2 + CN^2} \quad CK := \frac{CN^2}{CS} \quad SK := CS - CK$

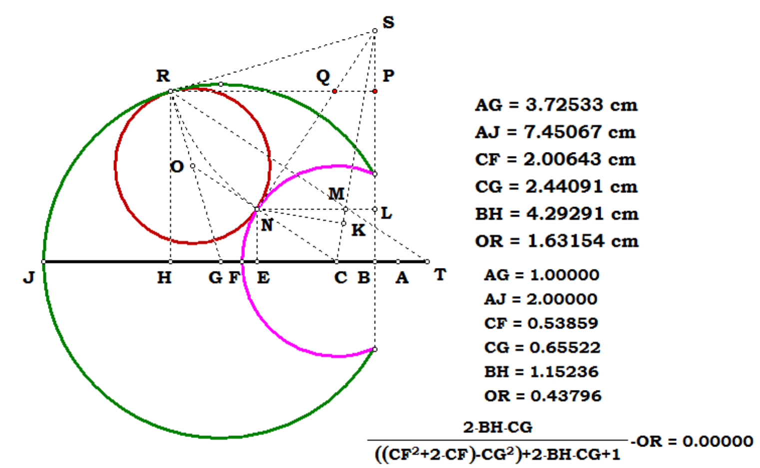
$KN := \sqrt{CN^2 - CK^2} \quad KM := \frac{BC \cdot KN}{BS} \quad SM := SK + KM \quad SL := \frac{BS \cdot SM}{CS}$

$BL := BS - SL \quad EN := BL \quad CE := \sqrt{CN^2 - EN^2} \quad HT := \frac{CE \cdot HR}{EN}$

$GT := HT - GH \quad GO := \frac{AG \cdot CG}{GT} \quad TheManInTheMoon := AG - GO$

$TheManInTheMoon = 0.437958$

The Man in the Moon.



Definitions.

$$TheManInTheMoon - \frac{2 \cdot BH \cdot CG}{CF^2 + 2 \cdot CF - CG^2 + 2 \cdot BH \cdot CG + 1} = 0$$

This name, 2 times BH times CG divided by CF squared added to twice CF removing CG squared added to our denominator with a unit added, which some who are uninformed call an equation, denotes the name of the man in the moon using these very specific set of absolutes, or nouns. It is always correct and always exacting. So, one could say, well, that is the name of the Man in the Moon. If this were the only way to name the Man in the Moon, there would be no controversy. What happens if we use the exact same criteria, same geometry, same arithmetic, same algebra, but make a slight change in our naming convention? Let us ask Seven of Nine or what have you.

041796B

Given.

$U := 7 \quad W := 7 \quad Y := 7$

$V := 20 \quad X := 17 \quad Z := 15$

Unit.

$AB := \frac{X}{X}$

Descriptions.

$AC := 2 \cdot AB \quad CE := \frac{2 \cdot Y}{Z} \quad DE := \frac{2 \cdot W}{X}$

$BC := AB \quad BG := AB \quad CD := CE + DE \quad BD := CD - BC$

$BK := \frac{\left(AB^2 + BD^2 - DE^2\right)}{2 \cdot BD} \quad DK := BD - BK \quad BE := CE - BC$

$AK := AB - BK \quad CK := AC - AK \quad CF := \frac{CK \cdot U}{V} \quad FK := CK - CF$

$BF := BC - CF \quad AF := AC - CF \quad FG := \sqrt{CF \cdot AF} \quad KP := FG$

$GP := FK \quad HP := \frac{BF \cdot GP}{FG} \quad HK := KP + HP \quad GH := \sqrt{GP^2 + HP^2}$

$HN := GH \quad DN := DE \quad DH := \sqrt{DN^2 + HN^2} \quad DS := \frac{DN^2}{DH}$

$HS := DH - DS \quad NS := \sqrt{HN^2 - HS^2} \quad ST := \frac{DK \cdot NS}{HK} \quad HT := HS + ST$

$HU := \frac{HK \cdot HT}{DH} \quad KU := HK - HU \quad JN := KU \quad DJ := \sqrt{DN^2 - JN^2}$

$FV := \frac{DJ \cdot FG}{JN} \quad BV := FV - BF \quad BM := \frac{AB \cdot BD}{BV} \quad TheManInTheMoon := (BG - BM)$

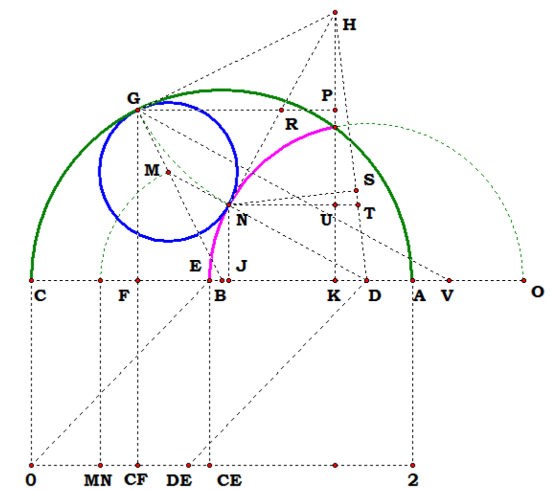
$TheManInTheMoon = 0.36255$

$\frac{2 \cdot FK \cdot BD}{DE^2 + 2 \cdot DE - BD^2 + 2 \cdot FK \cdot BD + 1} = 0.36255 \qquad \frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0.36255$

$TheManInTheMoon - \frac{2 \cdot FK \cdot BD}{DE^2 + 2 \cdot DE - BD^2 + 2 \cdot FK \cdot BD + 1} = 0 \qquad TheManInTheMoon - \frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0$

The Man in the Moon

AB = 1.00000
CF = 0.55686
U = 7.00000
V = 20.00000
DE = 0.82353
W = 7.00000
X = 17.00000
CE = 0.93333
Y = 7.00000
Z = 15.00000
MN = 0.36255



$\frac{2 \cdot FK \cdot BD}{((DE^2+2 \cdot DE) \cdot BD^2)+2 \cdot FK \cdot BD+1} = 0.36255$

$\frac{Y \cdot (U-V)}{U \cdot Y \cdot V \cdot Z} \cdot MN = 0.00000$

Notice that some elements, crucial to the first name, are wholly missing in the second name!!!!!! Where did W and X go?

Along with our first name, we have a second, which is completely different, yet entirely true. Y multiplying U minus V over U times Y subtracting V times Z. How is this possible? The first equation denotes all the givens, which are nowhere to be found in the second!!!!

Here is a little bit about the starting convention for our names. The first method was arithmetic, based on nouns, a one-to-one correspondence. It is like saying, this is such and such, and that is such and such. It is like counting pebbles. The second method is geometric. Same geometry, same arithmetic, same algebra, but instead of just nouns, we use a noun and a verb; for example, instead of saying, it is One, we say, she, or he, is Seven of Nine; in short, nouns and verbs. So, there you have it. The very same

thing to name and by the very same systems of grammar. The very same geometry, the very same arithmetic, the very same algebra, yet we arrive at two different names, both arithmetically identical, meaning of course, they have a one-to-one correspondence between a thing and an arithmetic name. The construct of the naming convention in the first example is very familiar to us; we use it all the time, but the second, well, that is another of my own inventions. Invention does not mean we create anything, only that we have leant to recognize and reproduce something.

Now, people have told stories about me for thousands of years, not one of those who told those stories, save those who were instrumental in creating certain written works, even knew what they were talking about. Well, my name is certainly not important, and in fact, it was written that my name would be a common generic name, John: not interesting at all, I worked from skilled trades to day labor. Yet the name of the Man in the Moon, never worked a day in his life, that, has turned out very interesting; the stories told about that name, well it is well documented by every possible grammar system and like our crescent moon, not even noticed until a shadow appears.

I have created a little work packet for those who want to study the mystic art of names and have placed them in a directory called The Man in the Moon, on the Internet Archive. Anyone wanting to become a true mystic, with real power, will study the art of names, magical incantations which uses only four specific grammar systems; Common Grammar, Arithmetic, Algebra and Geometry. These four grammar systems are the true descendants of Adam and Eve, a Conjugate Binary Pair, often called simply a noun and a verb.

The first set of equations reduce to all of our givens: The second, to a ratio of the givens: Arithmetic and Geometric. Thus, when so called intellectuals tell you, that such and such is THE EQUATION for such and such, well, they are simply illiterate. The elements of every thing are

binary, and in binary, we have both arithmetic and geometric results, absolute and relative. The Relativities of Einstein are proven myths venerated by simpletons; the elements of a thing are physical facts.

Let us do a very brief review of the evolution of binary information processing. Binary is how every possible grammar is effected; we name, and can only name, relatives and correlatives, the two parts of any thing, their shape or boundaries and the relative differences within them. In metaphor, one of the ways it is introduced in the Bible is by a Conjugate Binary Pair called Adam and Eve. The Book, however, is just full of these binary contrasts, very deliberately placed. Then there is Geometry, a simple stop, go, stop, producing a line segment from which all of geometry, unless you are an idiot non-Euclidean Geometer, who cannot spot a contradiction in the words if it bit them in the ass, is produced. Then there is Plato. Plato used the term Dialectic, speaking by 2's, to preserve the science for posterity in dialogs. If he would have put it down plainly, his life would have been forcibly shortened and no dialogs would have remained. There have been relatively few prophets in history whose work has been aimed at bringing into the human mind that all information is a product of binary processing until today we have the computer. Yet every thing, every possible thing, is this binary, so it is not new, in fact, as it defines existence, it could never have been new; it just is.

A mind, by the recursion of this binary, produces exactly four groups of grammar from the intelligible of Language; Common Grammar, Arithmetic, Algebra, of which these three are called Logics, while the remaining one, which can be used to example everything, is an Analogic called Geometry.

Let us use egocentricity as an example. Egocentricity is inversely proportional to intelligence. The reason is very simple. The less you really comprehend of the world, the more one has to play with themselves. It is an evolutionary artifact; unless one is very intelligent, and everyone else

just leaves you to play with yourself. Every animal's behavior is egocentric in terms of simple survival; however, the more one comprehends what it takes to survive, the more of the world one has to be able to comprehend to do it. This is not necessarily linked to large memories. Aristotle had a phenomenal memory, but he was not the sharpest tool in the shed; this also accounts for his vanity.

This fact is what makes me believe that either the Man in the Moon is very stupid, or he is very wise; for thousands of years, all he seems to do is show off. I wager that study will eventually solve the mystery. Maybe he is trying to get someone to toss him a rope so he could climb off the moon. Today, people are still simply smoking the rope.

041796A

Unit.

AG := 1 The radius of the Large, green, Circle, it is taken as the unit of the crescent.

Given.

CF := .53859 The radius of the Small, pink circle.

CG := .65522 The difference between center of Radius Large and Radius Small.

BH := 1.15236 The point on the diameter of the Large Cresant that we want to know the radius of that circle on the perpendicular.

Descriptions.

A Circle In A Crescent

$$AJ := 2 \cdot AG \qquad BG := \frac{\left(AG^2 + CG^2 - CF^2\right)}{2 \cdot CG} \qquad BC := CG - BG \qquad FG := CG - CF$$

$$AB := AG - BG \qquad BJ := AJ - AB$$

$$AH := BH + AB \qquad GH := AH - AG \qquad HR := \sqrt{AG^2 - GH^2} \qquad BP := HR$$

$$PR := BH \qquad PS := \frac{GH \cdot PR}{HR} \qquad BS := BP + PS \qquad RS := \sqrt{PR^2 + PS^2} \qquad NS := RS$$

$$CN := CF \qquad CS := \sqrt{NS^2 + CN^2} \qquad CK := \frac{CN^2}{CS} \qquad SK := CS - CK$$

$$KN := \sqrt{CN^2 - CK^2} \qquad KM := \frac{BC \cdot KN}{BS} \qquad SM := SK + KM \qquad SL := \frac{BS \cdot SM}{CS}$$

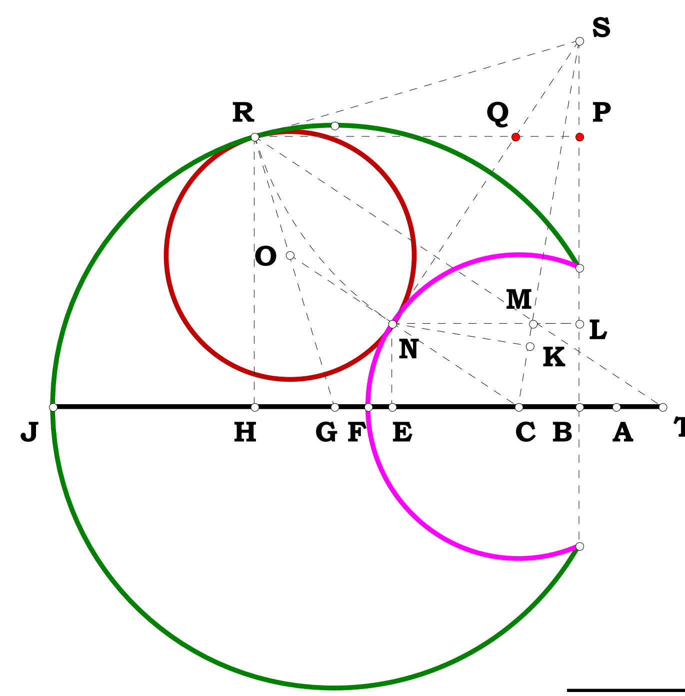
$$BL := BS - SL \qquad EN := BL \qquad CE := \sqrt{CN^2 - EN^2} \qquad HT := \frac{CE \cdot HR}{EN}$$

$$GT := HT - GH \qquad GO := \frac{AG \cdot CG}{GT} \qquad OR := AG - GO$$

$$OR = 0.437958$$

Definitions.

$$OR - \frac{2 \cdot BH \cdot CG}{CF^2 + 2 \cdot CF - CG^2 + 2 \cdot BH \cdot CG + 1} = 0$$



AG = 3.72533 cm
AJ = 7.45067 cm
CF = 2.00643 cm
CG = 2.44091 cm
BH = 4.29291 cm
OR = 1.63154 cm

AG = 1.00000
AJ = 2.00000
CF = 0.53859
CG = 0.65522
BH = 1.15236
OR = 0.43796

$$\frac{2 \cdot BH \cdot CG}{((CF^2+2 \cdot CF)-CG^2)+2 \cdot BH \cdot CG+1} - OR = 0.00000$$

$$AJ - 2 = 0 \qquad BG - \frac{CG^2 - CF^2 + 1}{2 \cdot CG} = 0 \qquad BC - \frac{CF^2 + CG^2 - 1}{2 \cdot CG} = 0 \qquad FG - (CG - CF) = 0$$

$$AB - \frac{CF^2 - CG^2 + 2 \cdot CG - 1}{2 \cdot CG} = 0 \qquad BJ - \frac{CG^2 - CF^2 + 2 \cdot CG + 1}{2 \cdot CG} = 0$$

$$AH - \frac{CF^2 - CG^2 + (2 \cdot BH + 2) \cdot CG - 1}{2 \cdot CG} = 0 \qquad GH - \frac{CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1}{2 \cdot CG} = 0$$

$$HR - \frac{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}}{2 \cdot CG} = 0$$

$$PR - BH = 0 \qquad PS - \frac{BH \cdot \left(CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right)}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0$$

$$BS - \frac{2 \cdot CF^2 \cdot CG^2 - CF^4 - 2 \cdot BH \cdot CF^2 \cdot CG + 2 \cdot CF^2 - CG^4 + 2 \cdot BH \cdot CG^3 + 2 \cdot CG^2 + 2 \cdot BH \cdot CG - 1}{2 \cdot CG \cdot \sqrt{4 \cdot BH \cdot CG^3 - 4 \cdot BH \cdot CF^2 \cdot CG - 4 \cdot BH^2 \cdot CG^2 + 4 \cdot BH \cdot CG - CF^4 + 2 \cdot CF^2 \cdot CG^2 + 2 \cdot CF^2 - CG^4 + 2 \cdot CG^2 - 1}} = 0$$

$$RS - \frac{2 \cdot BH \cdot CG}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0$$

$$NS - \frac{2 \cdot BH \cdot CG}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0 \qquad CN - CF = 0$$

$$CS - \frac{\sqrt{\left(2 \cdot CF^2 - CF^3 + CF \cdot CG^2 - 2 \cdot BH \cdot CF \cdot CG - CF + 2 \cdot BH \cdot CG\right) \cdot \left(CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG\right)}}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0$$

$$CK - \frac{CF^2\cdot\sqrt{(2\cdot CG + CF^2 - CG^2 + 2\cdot BH\cdot CG - 1)\cdot(2\cdot CG - CF^2 + CG^2 - 2\cdot BH\cdot CG + 1)}}{\sqrt{-(CF^3 - 2\cdot CF^2 - CF\cdot CG^2 + 2\cdot BH\cdot CF\cdot CG + CF - 2\cdot BH\cdot CG)\cdot(CF^3 + 2\cdot CF^2 - CF\cdot CG^2 + 2\cdot BH\cdot CF\cdot CG + CF + 2\cdot BH\cdot CG)}} = 0$$

$$SK - \frac{4\cdot BH^2\cdot CG^2}{\left[\sqrt{(2\cdot CG + CF^2 - CG^2 + 2\cdot BH\cdot CG - 1)\cdot\left(2\cdot CG - CF^2 \;\ldots\; + CG^2 - 2\cdot BH\cdot CG + 1\right)}\cdot\sqrt{\left(2\cdot CF^2 - CF^3 + CF\cdot CG^2 \;\ldots\; + -2\cdot BH\cdot CF\cdot CG - CF + 2\cdot BH\cdot CG\right)\cdot\left(CF^3 + 2\cdot CF^2 - CF\cdot CG^2 \;\ldots\; + 2\cdot BH\cdot CF\cdot CG + CF + 2\cdot BH\cdot CG\right)}\right]} = 0$$

$$KN - \frac{2\cdot BH\cdot CF\cdot CG}{\sqrt{(CF^3 + 2\cdot CF^2 - CF\cdot CG^2 + 2\cdot BH\cdot CF\cdot CG + CF + 2\cdot BH\cdot CG)\cdot(2\cdot CF^2 - CF^3 + CF\cdot CG^2 - 2\cdot BH\cdot CF\cdot CG - CF + 2\cdot BH\cdot CG)}} = 0$$

$$KM - \frac{2\cdot BH\cdot CF\cdot CG\cdot(1 - CF^2 - CG^2)\cdot\sqrt{4\cdot BH\cdot CG^3 - 4\cdot BH\cdot CF^2\cdot CG - 4\cdot BH^2\cdot CG^2 + 4\cdot BH\cdot CG - CF^4 + 2\cdot CF^2\cdot CG^2 + 2\cdot CF^2 - CG^4 + 2\cdot CG^2 - 1}}{\sqrt{-\left(CF^3 - 2\cdot CF^2 - CF\cdot CG^2 \;\ldots\; + 2\cdot BH\cdot CF\cdot CG + CF - 2\cdot BH\cdot CG\right)\cdot\left(CF^3 + 2\cdot CF^2 - CF\cdot CG^2 \;\ldots\; + 2\cdot BH\cdot CF\cdot CG + CF + 2\cdot BH\cdot CG\right)}\cdot\left(CF^4 - 2\cdot CF^2\cdot CG^2 + 2\cdot BH\cdot CF^2\cdot CG - 2\cdot CF^2 \;\ldots\; + CG^4 - 2\cdot BH\cdot CG^3 - 2\cdot CG^2 - 2\cdot BH\cdot CG + 1\right)} = 0$$

$$SM - \frac{2\cdot BH\cdot CG\cdot\left(CF^4 + 2\cdot CF^3 + 2\cdot BH\cdot CF^2\cdot CG + 2\cdot CF\cdot CG^2 \;\ldots\; + 4\cdot BH\cdot CF\cdot CG - 2\cdot CF - CG^4 + 2\cdot BH\cdot CG^3 + 2\cdot CG^2 + 2\cdot BH\cdot CG - 1\right)\cdot(CF^3 - 2\cdot CF^2 - CF\cdot CG^2 + 2\cdot BH\cdot CF\cdot CG + CF - 2\cdot BH\cdot CG)}{\sqrt{\left(2\cdot CF^2 - CF^3 + CF\cdot CG^2 \;\ldots\; + -2\cdot BH\cdot CF\cdot CG - CF + 2\cdot BH\cdot CG\right)\cdot\left(CF^3 + 2\cdot CF^2 - CF\cdot CG^2 \;\ldots\; + 2\cdot BH\cdot CF\cdot CG + CF + 2\cdot BH\cdot CG\right)}\cdot\sqrt{\left(2\cdot CG + CF^2 - CG^2 \;\ldots\; + 2\cdot BH\cdot CG - 1\right)\cdot\left(2\cdot CG - CF^2 \;\ldots\; + CG^2 - 2\cdot BH\cdot CG + 1\right)}\cdot\left(CF^4 - 2\cdot CF^2\cdot CG^2 \;\ldots\; + 2\cdot BH\cdot CF^2\cdot CG - 2\cdot CF^2 \;\ldots\; + CG^4 - 2\cdot BH\cdot CG^3 \;\ldots\; + -2\cdot CG^2 - 2\cdot BH\cdot CG + 1\right)} = 0$$

$$SL - \frac{BH\cdot(CF^4 + 2\cdot CF^3 + 2\cdot BH\cdot CF^2\cdot CG + 2\cdot CF\cdot CG^2 + 4\cdot BH\cdot CF\cdot CG - 2\cdot CF - CG^4 + 2\cdot BH\cdot CG^3 + 2\cdot CG^2 + 2\cdot BH\cdot CG - 1)}{(CF^3 + 2\cdot CF^2 - CF\cdot CG^2 + 2\cdot BH\cdot CF\cdot CG + CF + 2\cdot BH\cdot CG)\cdot\sqrt{4\cdot BH\cdot CG^3 - 4\cdot BH\cdot CF^2\cdot CG - 4\cdot BH^2\cdot CG^2 + 4\cdot BH\cdot CG - CF^4 + 2\cdot CF^2\cdot CG^2 + 2\cdot CF^2 - CG^4 + 2\cdot CG^2 - 1}} = 0$$

$$\mathbf{BL} - \left[\frac{CF\cdot(CF+CG+1)\cdot(CF-CG+1)\cdot\left(2\cdot CG+CF^2-CG^2+2\cdot BH\cdot CG-1\right)\cdot\left(2\cdot CG-CF^2+CG^2-2\cdot BH\cdot CG+1\right)}{2\cdot CG\cdot\left(CF^3+2\cdot CF^2-CF\cdot CG^2+2\cdot BH\cdot CF\cdot CG+CF+2\cdot BH\cdot CG\right)\cdot\sqrt{4\cdot BH\cdot CG^3-4\cdot BH\cdot CF^2\cdot CG-4\cdot BH^2\cdot CG^2+4\cdot BH\cdot CG-CF^4+2\cdot CF^2\cdot CG^2+2\cdot CF^2-CG^4+2\cdot CG^2-1}}\right] = 0$$

$$\mathbf{EN} - \left[\frac{CF\cdot(CF+CG+1)\cdot(CF-CG+1)\cdot\left(2\cdot CG+CF^2-CG^2+2\cdot BH\cdot CG-1\right)\cdot\left(2\cdot CG-CF^2+CG^2-2\cdot BH\cdot CG+1\right)}{2\cdot CG\cdot\left(CF^3+2\cdot CF^2-CF\cdot CG^2+2\cdot BH\cdot CF\cdot CG+CF+2\cdot BH\cdot CG\right)\cdot\sqrt{4\cdot BH\cdot CG^3-4\cdot BH\cdot CF^2\cdot CG-4\cdot BH^2\cdot CG^2+4\cdot BH\cdot CG-CF^4+2\cdot CF^2\cdot CG^2+2\cdot CF^2-CG^4+2\cdot CG^2-1}}\right] = 0$$

$$\mathbf{CE} - \frac{CF\cdot\left(CF^4+2\cdot CF^3+2\cdot BH\cdot CF^2\cdot CG+2\cdot CF\cdot CG^2+4\cdot BH\cdot CF\cdot CG-2\cdot CF-CG^4+2\cdot BH\cdot CG^3+2\cdot CG^2+2\cdot BH\cdot CG-1\right)}{2\cdot CG\cdot\left(CF+2\cdot CF^2+CF^3+2\cdot BH\cdot CG-CF\cdot CG^2+2\cdot BH\cdot CF\cdot CG\right)} = 0$$

$$\mathbf{HT} - \frac{\sqrt{\begin{pmatrix}2\cdot CG+CF^2-CG^2 \;\ldots\\ +\;2\cdot BH\cdot CG-1\end{pmatrix}\cdot\begin{pmatrix}2\cdot CG-CF^2+CG^2 \;\ldots\\ +\;-2\cdot BH\cdot CG+1\end{pmatrix}}\cdot\sqrt{\begin{matrix}4\cdot BH\cdot CG^3-4\cdot BH\cdot CF^2\cdot CG-4\cdot BH^2\cdot CG^2 \;\ldots\\ +\;4\cdot BH\cdot CG-CF^4+2\cdot CF^2\cdot CG^2 \;\ldots\\ +\;2\cdot CF^2-CG^4+2\cdot CG^2-1\end{matrix}}\cdot\begin{pmatrix}CF^4+2\cdot CF^3+2\cdot BH\cdot CF^2\cdot CG+2\cdot CF\cdot CG^2 \;\ldots\\ +\;4\cdot BH\cdot CF\cdot CG-2\cdot CF-CG^4 \;\ldots\\ +\;2\cdot BH\cdot CG^3+2\cdot CG^2+2\cdot BH\cdot CG-1\end{pmatrix}}{2\cdot CG\cdot(CF+CG+1)\cdot(CF-CG+1)\cdot\left(2\cdot CG+CF^2-CG^2+2\cdot BH\cdot CG-1\right)\cdot\left(2\cdot CG-CF^2+CG^2-2\cdot BH\cdot CG+1\right)} = 0$$

$$\mathbf{GT} - \frac{CG\cdot\left(CF^2+2\cdot CF-CG^2+2\cdot BH\cdot CG+1\right)}{(CF-CG+1)\cdot(CF+CG+1)} = 0 \qquad \mathbf{GO} - \frac{(CF+CG+1)\cdot(CF-CG+1)}{CF^2+2\cdot CF-CG^2+2\cdot BH\cdot CG+1} = 0 \qquad \mathbf{OR} - \frac{2\cdot BH\cdot CG}{CF^2+2\cdot CF-CG^2+2\cdot BH\cdot CG+1} = 0$$

041796B

Descriptions.

Unit.

$AF := 1$

Given.

$DH := .46274$

$DF := .81564$

$CJ := 1.38714$

$AK := 2 \cdot AF \quad FP := AF \quad FK := AF \quad CD := \dfrac{DH^2 + DF^2 - AF^2}{2 \cdot DF}$

$FH := DF - DH \quad AH := AF - FH \quad AE := \dfrac{AH}{2} \quad EH := AE$

$AD := DF - AF \quad DE := AE - AD \quad DN := DH \quad AC := CD - AD$

$CK := AK - AC \quad CF := CK - FK \quad CF := DF - CD \quad FJ := CJ - CF$

$JP := \sqrt{FP^2 - FJ^2} \quad FS := \dfrac{FP \cdot CF}{FJ} \quad PS := FS + FP \quad QS := \dfrac{FP \cdot PS}{JP}$

$PQ := \dfrac{FJ \cdot QS}{FP} \quad CS := \dfrac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad DQ := \sqrt{CD^2 + CQ^2}$

$DL := \dfrac{DH^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2} \quad LZ := \dfrac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ$

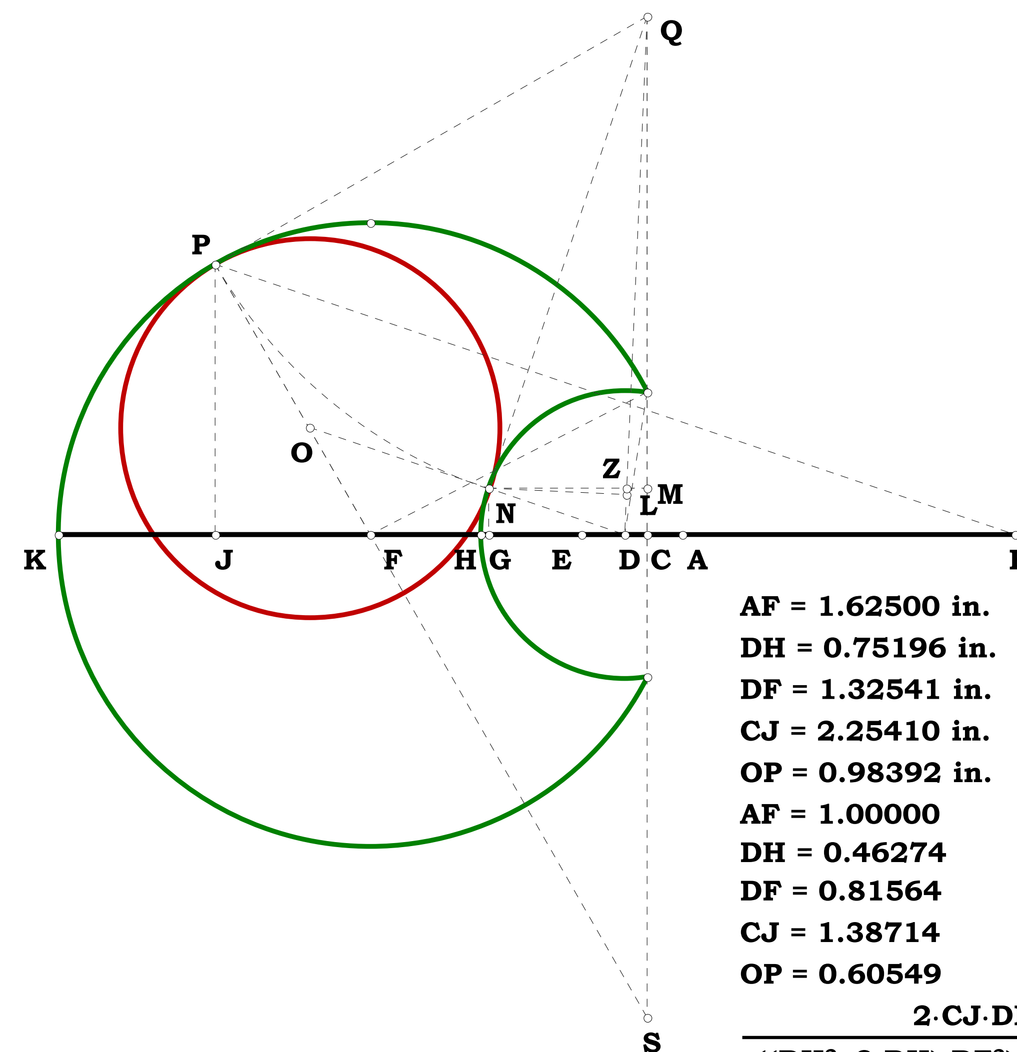
$MQ := \dfrac{CQ \cdot QZ}{DQ} \quad CM := CQ - MQ \quad GN := CM \quad DG := \sqrt{DN^2 - GN^2}$

$BJ := \dfrac{DG \cdot JP}{GN} \quad BF := BJ - FJ \quad FO := \dfrac{FP \cdot DF}{BF} \quad OP := FP - FO$

$OP = 0.605491$

$$\frac{2 \cdot CJ \cdot DF}{DH^2 + 2 \cdot DH - DF^2 + 2 \cdot CJ \cdot DF + 1} = 0.605491$$

A Circle In A Crescent



AF = 1.62500 in.
DH = 0.75196 in.
DF = 1.32541 in.
CJ = 2.25410 in.
OP = 0.98392 in.
AF = 1.00000
DH = 0.46274
DF = 0.81564
CJ = 1.38714
OP = 0.60549

$$\frac{2 \cdot CJ \cdot DF}{((DH^2+2 \cdot DH)-DF^2)+2 \cdot CJ \cdot DF+1} - OP = 0.00000$$

041796C

Given.

$U := 7 \quad W := 7 \quad Y := 7$

$V := 20 \quad X := 17 \quad Z := 15$

Unit.

$AB := \frac{X}{X}$

Descriptions.

$AC := 2 \cdot AB \quad CE := \frac{2 \cdot Y}{Z} \quad DE := \frac{2 \cdot W}{X}$

$BC := AB \quad BG := AB \quad CD := CE + DE \quad BD := CD - BC$

$BK := \frac{\left(AB^2 + BD^2 - DE^2\right)}{2 \cdot BD} \quad DK := BD - BK \quad BE := CE - BC$

$AK := AB - BK \quad CK := AC - AK \quad CF := \frac{CK \cdot U}{V} \quad FK := CK - CF$

$BF := BC - CF \quad AF := AC - CF \quad FG := \sqrt{CF \cdot AF} \quad KP := FG$

$GP := FK \quad HP := \frac{BF \cdot GP}{FG} \quad HK := KP + HP \quad GH := \sqrt{GP^2 + HP^2}$

$HN := GH \quad DN := DE \quad DH := \sqrt{DN^2 + HN^2} \quad DS := \frac{DN^2}{DH}$

$HS := DH - DS \quad NS := \sqrt{HN^2 - HS^2} \quad ST := \frac{DK \cdot NS}{HK} \quad HT := HS + ST$

$HU := \frac{HK \cdot HT}{DH} \quad KU := HK - HU \quad JN := KU \quad DJ := \sqrt{DN^2 - JN^2}$

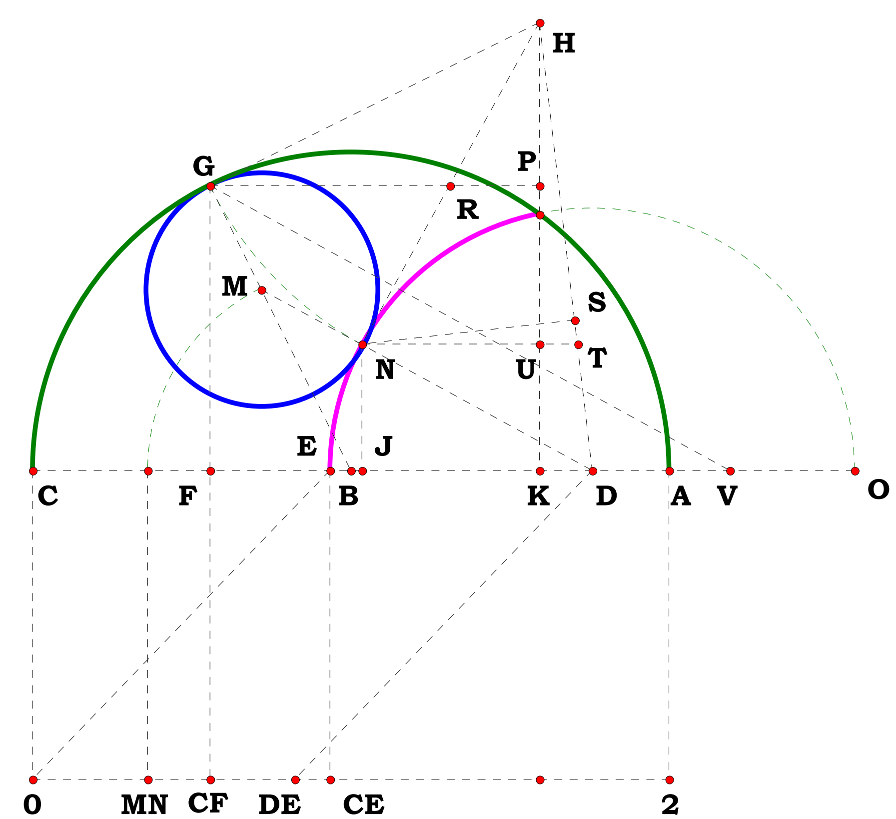
$FV := \frac{DJ \cdot FG}{JN} \quad BV := FV - BF \quad BM := \frac{AB \cdot BD}{BV} \quad GM := (BG - BM)$

$GM = 0.36255$

$\frac{2 \cdot FK \cdot BD}{DE^2 + 2 \cdot DE - BD^2 + 2 \cdot FK \cdot BD + 1} = 0.36255 \qquad \frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0.36255$

This is odd, W and X dissappear out of the equation. In short, this is pure implication in an equation.

A Circle In A Crescent



AB = 1.00000
CF = 0.55686
U = 7.00000
V = 20.00000
DE = 0.82353
W = 7.00000
X = 17.00000
CE = 0.93333
Y = 7.00000
Z = 15.00000
MN = 0.36255

$\frac{2 \cdot FK \cdot BD}{((DE^2+2 \cdot DE)-BD^2)+2 \cdot FK \cdot BD+1} = 0.36255$

$\frac{Y \cdot (U-V)}{U \cdot Y-V \cdot Z} - MN = 0.00000$

$AB - 1 = 0 \quad AC - 2 \cdot AB = 0 \quad CE - \frac{2 \cdot Y}{Z} = 0 \quad DE - \frac{2 \cdot W}{X} = 0$

$BC - 1 = 0 \quad BG - 1 = 0 \quad CD - \frac{2 \cdot (W \cdot Z + X \cdot Y)}{X \cdot Z} = 0$

$$BD - \frac{2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z}{X \cdot Z} = 0$$

$$BK - \frac{2 \cdot X \cdot Y^2 - 2 \cdot W \cdot Z^2 + X \cdot Z^2 + 4 \cdot W \cdot Y \cdot Z - 2 \cdot X \cdot Y \cdot Z}{Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$$

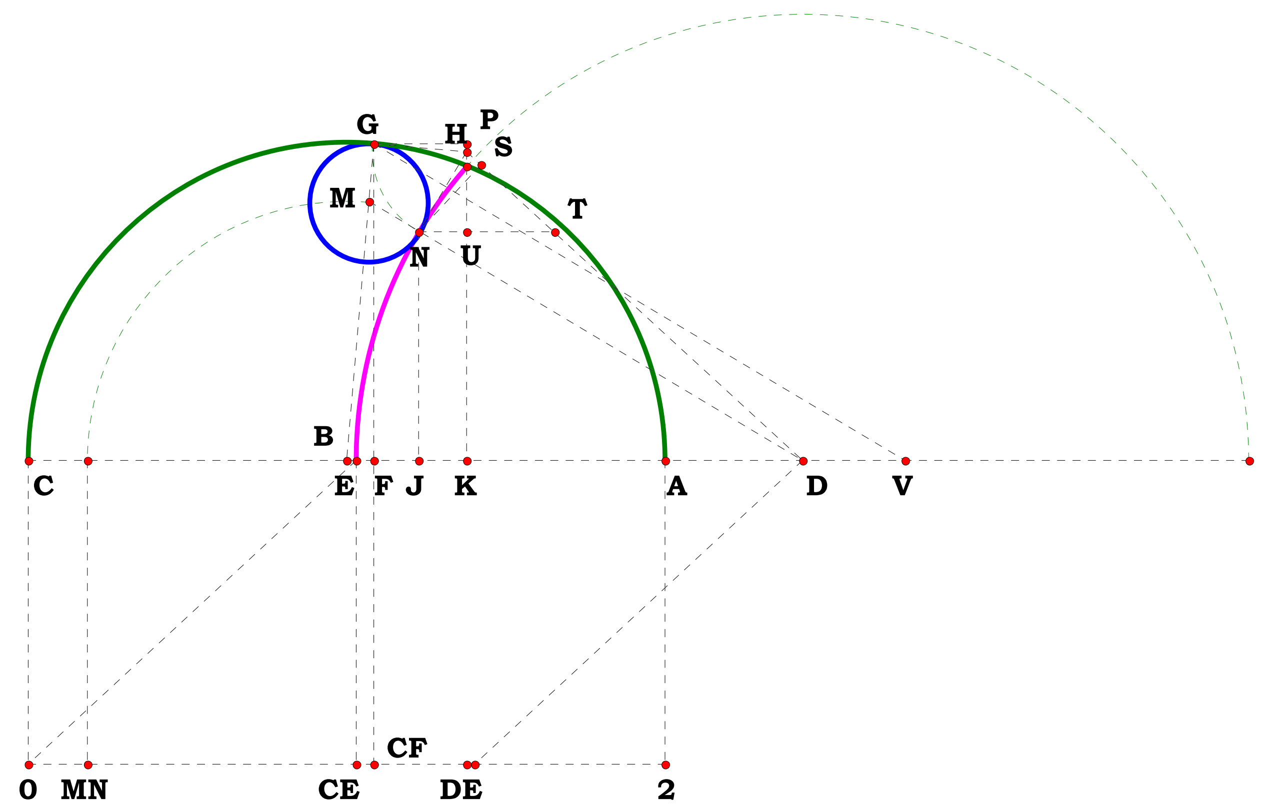
$$DK - \frac{2 \cdot \left(2 \cdot W^2 \cdot Z^2 + 2 \cdot W \cdot X \cdot Y \cdot Z - W \cdot X \cdot Z^2 + X^2 \cdot Y^2 - X^2 \cdot Y \cdot Z\right)}{X \cdot Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$$

$BE - \frac{2 \cdot Y - Z}{Z} = 0 \quad AK - \frac{2 \cdot (Z - Y) \cdot (2 \cdot W \cdot Z + X \cdot Y - X \cdot Z)}{Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$

$CK - \frac{2 \cdot Y \cdot (2 \cdot W \cdot Z + X \cdot Y)}{Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0 \quad CF - \frac{2 \cdot U \cdot Y \cdot (2 \cdot W \cdot Z + X \cdot Y)}{V \cdot Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$

$FK - \frac{2 \cdot Y \cdot (2 \cdot W \cdot Z + X \cdot Y) \cdot (V - U)}{V \cdot Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0 \quad \frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0.36255$

AB = 1.00000
CF = 1.08317
U = 15.73676
V = 20.00000
DE = 1.40309
W = 14.03086
X = 20.00000
CE = 1.02732
Y = 9.75955
Z = 19.00000
MN = 0.18376



$$\frac{2 \cdot FK \cdot BD}{((DE^2 + 2 \cdot DE) - BD^2) + 2 \cdot FK \cdot BD + 1} = 0.18376$$

$$\frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} - MN = 0.00000$$

042296A

Descriptions.

Unit.

$AB := 1$

Given.

$W := 9 \qquad Y := 10$

$X := 20 \qquad Z := 20$

$$MX := 2 \cdot AB \quad BM := AB \quad BE := AB \quad BL := AB \quad BF := \frac{W}{X}$$

$$FX := AB + BF \quad EF := \sqrt{FX \cdot (MX - FX)} \quad EI := 2 \cdot EF$$

$$FG := \frac{Y}{Z} \quad EG := EF + FG \quad BG := \sqrt{BF^2 + FG^2} \quad GL := BL - BG$$

$$DG := GL \quad GH := \frac{FG \cdot GL}{BG} \quad HL := \sqrt{GL^2 - GH^2} \quad EH := EG + GH$$

$$EL := \sqrt{EH^2 + HL^2} \quad JL := \frac{EL}{2} \quad BJ := \sqrt{BL^2 - JL^2} \quad LN := \frac{BL \cdot JL}{BJ}$$

$$GN := \sqrt{LN^2 + GL^2} \quad JN := \sqrt{LN^2 - JL^2} \quad EJ := JL \quad EN := LN$$

$$GO := \frac{GN^2 + EG^2 - EN^2}{2 \cdot EG} \quad NO := \sqrt{GN^2 - GO^2} \quad NR := \frac{NO^2}{GN}$$

$$GS := \frac{DG^2}{GN} \quad RS := GN - (NR + GS) \quad DT := RS \quad DS := \sqrt{DG^2 - GS^2}$$

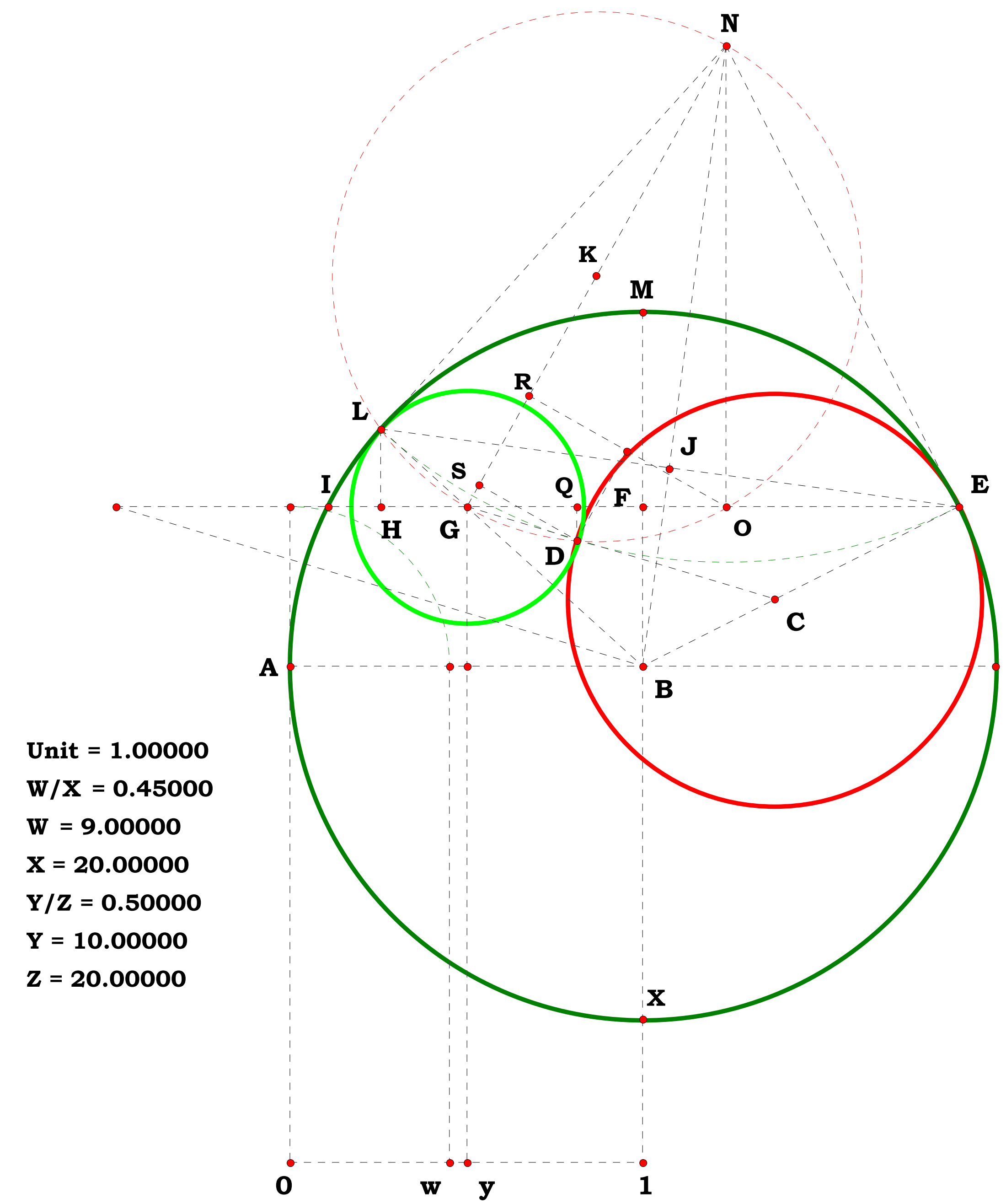
$$RT := DS \quad OR := \sqrt{NO^2 - NR^2} \quad OT := OR - RT \quad DO := \sqrt{DT^2 + OT^2}$$

$$OQ := \frac{DO^2 + GO^2 - DG^2}{2 \cdot GO} \quad GQ := GO - OQ \quad DQ := \sqrt{DO^2 - OQ^2}$$

$$FP := \frac{GQ \cdot BF}{DQ} \quad CE := \frac{BE \cdot EG}{FP + EF} \quad CE = 0.583387$$

Given BF as a ratio to BM and EG as a ratio to EI, what is CE?

Wow. Everything was going well reading this until I came to DT, it did not exist. Seems that in redoing the graphic and editing, I forgot to draw it in. So, I changed MT to MX and put DT back in. One has to find DQ in order to solve for CE and this cannot be done if DT is left out of an earlier update. BP is going to be parallal with CDG and all one has to do is proportion down to find CE.



Definitions.

$$MX - 2 = 0 \qquad BF - \frac{W}{X} = 0 \qquad FX - \frac{W + X}{X} = 0 \qquad EF - \frac{\sqrt{(X - W)\cdot(W + X)}}{X} = 0 \qquad EI - \frac{2\cdot\sqrt{(X - W)\cdot(W + X)}}{X} = 0$$

$$FG - \frac{Y}{Z} = 0 \qquad EG - \frac{Z\cdot\sqrt{X^2 - W^2} + X\cdot Y}{X\cdot Z} = 0 \qquad BG - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}}{X\cdot Z} = 0 \qquad GL - \frac{X\cdot Z - \sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}}{X\cdot Z} = 0$$

$$GH - \frac{Y\cdot\left(X\cdot Z - \sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}\right)}{Z\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}} = 0 \qquad EH - \frac{X^2\cdot Y + \sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}}{X\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}} = 0$$

$$HL - \frac{\sqrt{W^4\cdot Z^3 - 2\cdot X\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} + W^2\cdot X^2\cdot Z^3 + 2\cdot X^3\cdot Y^2\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} + W^2\cdot X^2\cdot Y^2\cdot Z}}{X\cdot\sqrt{Z\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)}} = 0$$

$$EL - \frac{\sqrt{2\cdot\left[X^2\cdot Y^2\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - \left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} + W^2\cdot X\cdot Z^3 + X^3\cdot Y^2\cdot Z + X\cdot Y\cdot Z\cdot\sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}\right]}}{\sqrt{X\cdot Z\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)}} = 0$$

$$JL - \frac{\sqrt{2\cdot\left[X^2\cdot Y^2\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - \left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} + W^2\cdot X\cdot Z^3 + X^3\cdot Y^2\cdot Z + X\cdot Y\cdot Z\cdot\sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}\right]}}{2\cdot\sqrt{X\cdot Z\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)}} = 0$$

$$BJ - \frac{\sqrt{\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} - X^2\cdot Y^2\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} + W^2\cdot X\cdot Z^3 + X^3\cdot Y^2\cdot Z - X\cdot Y\cdot Z\cdot\sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}}}{\sqrt{2\cdot X\cdot Z\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)}} = 0$$

$$LN - \sqrt{\frac{X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z + X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}} = 0$$

$$GN - \sqrt{\left[\frac{\begin{array}{l}\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + 2 \cdot W^2 \cdot X^3 \cdot Z^5 + 2 \cdot X^5 \cdot Y^2 \cdot Z^3 - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z \ \ldots \\ + \ 2 \cdot X^4 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^2 \cdot X^2 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 2 \cdot X^4 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + 2 \cdot W^2 \cdot X^2 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\end{array}}{X^2 \cdot Z^2 \cdot \left[\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right]}\right]} = 0$$

$$JN - \sqrt{\left[\frac{\begin{array}{l}W^6 \cdot Z^5 - 2 \cdot X \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + W^4 \cdot X^2 \cdot Z^5 + 2 \cdot X^3 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + 2 \cdot X^6 \cdot Y^4 \cdot Z + 3 \cdot W^2 \cdot X^4 \cdot Y^2 \cdot Z^3 - W^2 \cdot X^4 \cdot Y^4 \cdot Z \ \ldots \\ + \ -2 \cdot W^2 \cdot X^3 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^4 \cdot X \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} + 2 \cdot X^2 \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\end{array}}{2 \cdot X \cdot \left[\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z + 2 \cdot W^2 \cdot X^3 \cdot Y^2 \cdot Z^3 - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\right]}\right]} = 0$$

$BL = 1$ $\quad DG := GL$

$EN := LN$ $\quad EJ := JL$

$$GO - \frac{\begin{gathered}\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{5}{2}} + 2\cdot W^2\cdot X^3\cdot Z^5 + 2\cdot X^5\cdot Y^2\cdot Z^3 - W^2\cdot Z^2\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} - X^4\cdot Y^4\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}\ \ldots \\ +\ 2\cdot X^2\cdot Z^2\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} - 2\cdot W^4\cdot X\cdot Z^5 + 2\cdot X^5\cdot Y^4\cdot Z + 4\cdot X^4\cdot Y^3\cdot Z^2\cdot\sqrt{X^2-W^2} - 2\cdot W^2\cdot X^2\cdot Z^4\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}\ \ldots \\ +\ -6\cdot X^4\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} + 3\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} - X\cdot Y\cdot Z^3\cdot\left(X^2-W^2\right)^{\frac{3}{2}}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}\ \ldots \\ +\ 4\cdot W^2\cdot X^2\cdot Y\cdot Z^4\cdot\sqrt{X^2-W^2} - X^3\cdot Y\cdot Z^3\cdot\sqrt{X^2-W^2}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} - 3\cdot X^3\cdot Y^3\cdot Z\cdot\sqrt{X^2-W^2}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} + X\cdot Y\cdot Z\cdot\sqrt{X^2-W^2}\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}}\end{gathered}}{2\cdot X\cdot Z\cdot\left(Z\cdot\sqrt{X^2-W^2}+X\cdot Y\right)\cdot\left[\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} - X^2\cdot Y^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} + W^2\cdot X\cdot Z^3 + X^3\cdot Y^2\cdot Z - X\cdot Y\cdot Z\cdot\sqrt{X^2-W^2}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}\right]} = 0$$

$$
NO-\sqrt{\frac{\begin{array}{l}
W^{8}\cdot Z^{5}-W^{2}\cdot X^{6}\cdot Z^{5}+3\cdot W^{4}\cdot X^{4}\cdot Z^{5}-3\cdot W^{6}\cdot X^{2}\cdot Z^{5}-2\cdot X^{5}\cdot Y^{5}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}+2\cdot X^{7}\cdot Y^{5}\cdot\sqrt{X^{2}-W^{2}}-6\cdot W^{2}\cdot X^{6}\cdot Y^{2}\cdot Z^{3}\ \ldots\\
+\ 12\cdot W^{4}\cdot X^{4}\cdot Y^{2}\cdot Z^{3}-6\cdot W^{6}\cdot X^{2}\cdot Y^{2}\cdot Z^{3}-2\cdot W^{2}\cdot X^{5}\cdot Y^{5}\cdot\sqrt{X^{2}-W^{2}}-2\cdot X^{5}\cdot Y^{3}\cdot Z^{2}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}\ \ldots\\
+\ 2\cdot X^{7}\cdot Y^{3}\cdot Z^{2}\cdot\sqrt{X^{2}-W^{2}}-W^{2}\cdot X^{6}\cdot Y^{4}\cdot Z+W^{4}\cdot X^{4}\cdot Y^{4}\cdot Z-2\cdot W^{2}\cdot X^{3}\cdot Y^{3}\cdot Z^{2}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}-4\cdot W^{2}\cdot X^{5}\cdot Y^{3}\cdot Z^{2}\cdot\sqrt{X^{2}-W^{2}}\ \ldots\\
+\ 2\cdot W^{4}\cdot X^{3}\cdot Y^{3}\cdot Z^{2}\cdot\sqrt{X^{2}-W^{2}}+2\cdot W^{4}\cdot X\cdot Y\cdot Z^{4}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}-2\cdot W^{6}\cdot X\cdot Y\cdot Z^{4}\cdot\sqrt{X^{2}-W^{2}}-X^{2}\cdot Y\cdot Z\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}\ \ldots\\
+\ X^{4}\cdot Y\cdot Z\cdot\sqrt{X^{2}-W^{2}}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}-4\cdot W^{2}\cdot X^{3}\cdot Y\cdot Z^{4}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}+2\cdot W^{4}\cdot X^{3}\cdot Y\cdot Z^{4}\cdot\sqrt{X^{2}-W^{2}}+X^{4}\cdot Y\cdot Z^{3}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}\ \ldots\\
+\ 6\cdot X^{4}\cdot Y^{3}\cdot Z\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}-X^{6}\cdot Y\cdot Z^{3}\cdot\sqrt{X^{2}-W^{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}\ \ldots\\
+\ -6\cdot X^{6}\cdot Y^{3}\cdot Z\cdot\sqrt{X^{2}-W^{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}-W^{2}\cdot X^{2}\cdot Y\cdot Z\cdot\sqrt{X^{2}-W^{2}}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}\ \ldots\\
+\ W^{2}\cdot X^{4}\cdot Y\cdot Z^{3}\cdot\sqrt{X^{2}-W^{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}+6\cdot W^{2}\cdot X^{4}\cdot Y^{3}\cdot Z\cdot\sqrt{X^{2}-W^{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}
\end{array}}{X^{2}\cdot\left[\begin{array}{l}
W^{6}\cdot Z^{5}-W^{2}\cdot X^{4}\cdot Z^{5}-2\cdot X^{6}\cdot Y^{2}\cdot Z^{3}-2\cdot X^{3}\cdot Y^{2}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}+2\cdot X^{5}\cdot Y^{4}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}-2\cdot X^{3}\cdot Z^{2}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}\ \ldots\\
+\ -2\cdot X^{6}\cdot Y^{4}\cdot Z+6\cdot W^{2}\cdot X^{4}\cdot Y^{2}\cdot Z^{3}-6\cdot W^{4}\cdot X^{2}\cdot Y^{2}\cdot Z^{3}-2\cdot X^{3}\cdot Y^{3}\cdot Z^{2}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}-2\cdot X^{5}\cdot Y^{3}\cdot Z^{2}\cdot\sqrt{X^{2}-W^{2}}\ \ldots\\
+\ 6\cdot X^{5}\cdot Y^{2}\cdot Z^{2}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}+W^{2}\cdot X^{4}\cdot Y^{4}\cdot Z+2\cdot W^{2}\cdot X\cdot Z^{2}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}+2\cdot W^{2}\cdot X^{3}\cdot Y^{3}\cdot Z^{2}\cdot\sqrt{X^{2}-W^{2}}\ \ldots\\
+\ -6\cdot W^{2}\cdot X^{3}\cdot Y^{2}\cdot Z^{2}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}+2\cdot W^{2}\cdot X\cdot Y\cdot Z^{4}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}-2\cdot W^{4}\cdot X\cdot Y\cdot Z^{4}\cdot\sqrt{X^{2}-W^{2}}\ \ldots\\
+\ -4\cdot X^{2}\cdot Y\cdot Z\cdot\sqrt{X^{2}-W^{2}}\cdot\left(W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}\right)^{\frac{3}{2}}-2\cdot W^{2}\cdot X^{3}\cdot Y\cdot Z^{4}\cdot\sqrt{X^{2}-W^{2}}+2\cdot X^{2}\cdot Y\cdot Z^{3}\cdot\left(X^{2}-W^{2}\right)^{\frac{3}{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}\ \ldots\\
+\ 6\cdot X^{4}\cdot Y^{3}\cdot Z\cdot\sqrt{X^{2}-W^{2}}\cdot\sqrt{W^{2}\cdot Z^{2}+X^{2}\cdot Y^{2}}
\end{array}\right]}}=0
$$

$$
NR - \frac{\sqrt{X^2 \cdot Z^2 \cdot \left[\begin{array}{l}\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\end{array}\right]} \cdot \left[\begin{array}{l} W^8 \cdot Z^5 - W^2 \cdot X^6 \cdot Z^5 + 3 \cdot W^4 \cdot X^4 \cdot Z^5 - 3 \cdot W^6 \cdot X^2 \cdot Z^5 - 2 \cdot X^5 \cdot Y^5 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \ldots \\ + 2 \cdot X^7 \cdot Y^5 \cdot \sqrt{X^2 - W^2} - 6 \cdot W^2 \cdot X^6 \cdot Y^2 \cdot Z^3 \ldots \\ + 12 \cdot W^4 \cdot X^4 \cdot Y^2 \cdot Z^3 - 6 \cdot W^6 \cdot X^2 \cdot Y^2 \cdot Z^3 - 2 \cdot W^2 \cdot X^5 \cdot Y^5 \cdot \sqrt{X^2 - W^2} \ldots \\ + -2 \cdot X^5 \cdot Y^3 \cdot Z^2 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} + 2 \cdot X^7 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - W^2 \cdot X^6 \cdot Y^4 \cdot Z \ldots \\ + W^4 \cdot X^4 \cdot Y^4 \cdot Z - 2 \cdot W^2 \cdot X^3 \cdot Y^3 \cdot Z^2 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} - 4 \cdot W^2 \cdot X^5 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} \ldots \\ + 2 \cdot W^4 \cdot X^3 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} + 2 \cdot W^4 \cdot X \cdot Y \cdot Z^4 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \ldots \\ + -2 \cdot W^6 \cdot X \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} - X^2 \cdot Y \cdot Z \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + X^4 \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - 4 \cdot W^2 \cdot X^3 \cdot Y \cdot Z^4 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \ldots \\ + 2 \cdot W^4 \cdot X^3 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} + X^4 \cdot Y \cdot Z^3 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 6 \cdot X^4 \cdot Y^3 \cdot Z \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + -X^6 \cdot Y \cdot Z^3 \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + -6 \cdot X^6 \cdot Y^3 \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + -W^2 \cdot X^2 \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + W^2 \cdot X^4 \cdot Y \cdot Z^3 \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 6 \cdot W^2 \cdot X^4 \cdot Y^3 \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \end{array}\right]}{X^2 \cdot \sqrt{\begin{array}{l}\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + 2 \cdot W^2 \cdot X^3 \cdot Z^5 + 2 \cdot X^5 \cdot Y^2 \cdot Z^3 - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - W^4 \cdot X \cdot Z^5 \ldots \\ + X^5 \cdot Y^4 \cdot Z + 2 \cdot X^4 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^2 \cdot X^2 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + -2 \cdot X^4 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 2 \cdot W^2 \cdot X^2 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\end{array}} \cdot \left[\begin{array}{l} W^6 \cdot Z^5 - W^2 \cdot X^4 \cdot Z^5 - 2 \cdot X^6 \cdot Y^2 \cdot Z^3 \ldots \\ + -2 \cdot X^3 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + 2 \cdot X^5 \cdot Y^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + -2 \cdot X^3 \cdot Z^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - 2 \cdot X^6 \cdot Y^4 \cdot Z \ldots \end{array}\right.} = 0
$$

$$\left. \begin{array}{l} + 6 \cdot W^2 \cdot X^4 \cdot Y^2 \cdot Z^3 - 6 \cdot W^4 \cdot X^2 \cdot Y^2 \cdot Z^3 \ldots \\ + -2 \cdot X^3 \cdot Y^3 \cdot Z^2 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} - 2 \cdot X^5 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} \ldots \\ + 6 \cdot X^5 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X^4 \cdot Y^4 \cdot Z \ldots \\ + 2 \cdot W^2 \cdot X \cdot Z^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + 2 \cdot W^2 \cdot X^3 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} \ldots \\ + -6 \cdot W^2 \cdot X^3 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 2 \cdot W^2 \cdot X \cdot Y \cdot Z^4 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} - 2 \cdot W^4 \cdot X \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} \ldots \\ + -4 \cdot X^2 \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + -2 \cdot W^2 \cdot X^3 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} \ldots \\ + 2 \cdot X^2 \cdot Y \cdot Z^3 \cdot \left(X^2 - W^2\right)^{\frac{3}{2}} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 6 \cdot X^4 \cdot Y^3 \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \end{array} \right]$$

$$GS - \frac{\left(X \cdot Z - \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right)^2}{X^2 \cdot Z^2 \cdot \sqrt{\dfrac{\begin{array}{l} \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + 2 \cdot W^2 \cdot X^3 \cdot Z^5 + 2 \cdot X^5 \cdot Y^2 \cdot Z^3 - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z \ldots \\ + 2 \cdot X^4 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^2 \cdot X^2 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 2 \cdot X^4 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 2 \cdot W^2 \cdot X^2 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \end{array}}{X^2 \cdot Z^2 \cdot \left[\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right]}}} = 0$$

$$DS - \sqrt{\frac{\begin{array}{l} W^2 \cdot X^3 \cdot Z^5 - 5 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + X^5 \cdot Y^2 \cdot Z^3 + 2 \cdot W^2 \cdot Z^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + 7 \cdot X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + 2 \cdot W^4 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 2 \cdot X^4 \cdot Y^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - X^2 \cdot Z^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + 3 \cdot W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z \ldots \\ + 4 \cdot W^2 \cdot X^3 \cdot Y^2 \cdot Z^3 - 2 \cdot X^4 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^2 \cdot X^2 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - X^4 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 2 \cdot W^2 \cdot X^2 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} \ldots \\ + X^3 \cdot Y \cdot Z^3 \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 4 \cdot X^3 \cdot Y^3 \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \ldots \\ + 5 \cdot X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - 4 \cdot W^2 \cdot X \cdot Y \cdot Z^3 \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \end{array}}{\begin{array}{l} \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + 2 \cdot W^2 \cdot X^3 \cdot Z^5 + 2 \cdot X^5 \cdot Y^2 \cdot Z^3 - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z \ldots \\ + 2 \cdot X^4 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^2 \cdot X^2 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 2 \cdot X^4 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + 2 \cdot W^2 \cdot X^2 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} \end{array}}} = 0$$

This seems to be the limit of my Mathcad

$$OR - \sqrt{NO^2 - NR^2} = 0 \qquad OT - (OR - RT) = 0 \qquad DO - \sqrt{DT^2 + OT^2} = 0 \qquad OQ - \frac{DO^2 + GO^2 - DG^2}{2 \cdot GO} = 0 \qquad GQ - (GO - OQ) = 0$$

$$DQ - \sqrt{DO^2 - OQ^2} = 0 \qquad FP - \frac{GQ \cdot BF}{DQ} = 0$$

$$CE - \frac{BE \cdot EG}{FP + EF} = 0$$

Unit.

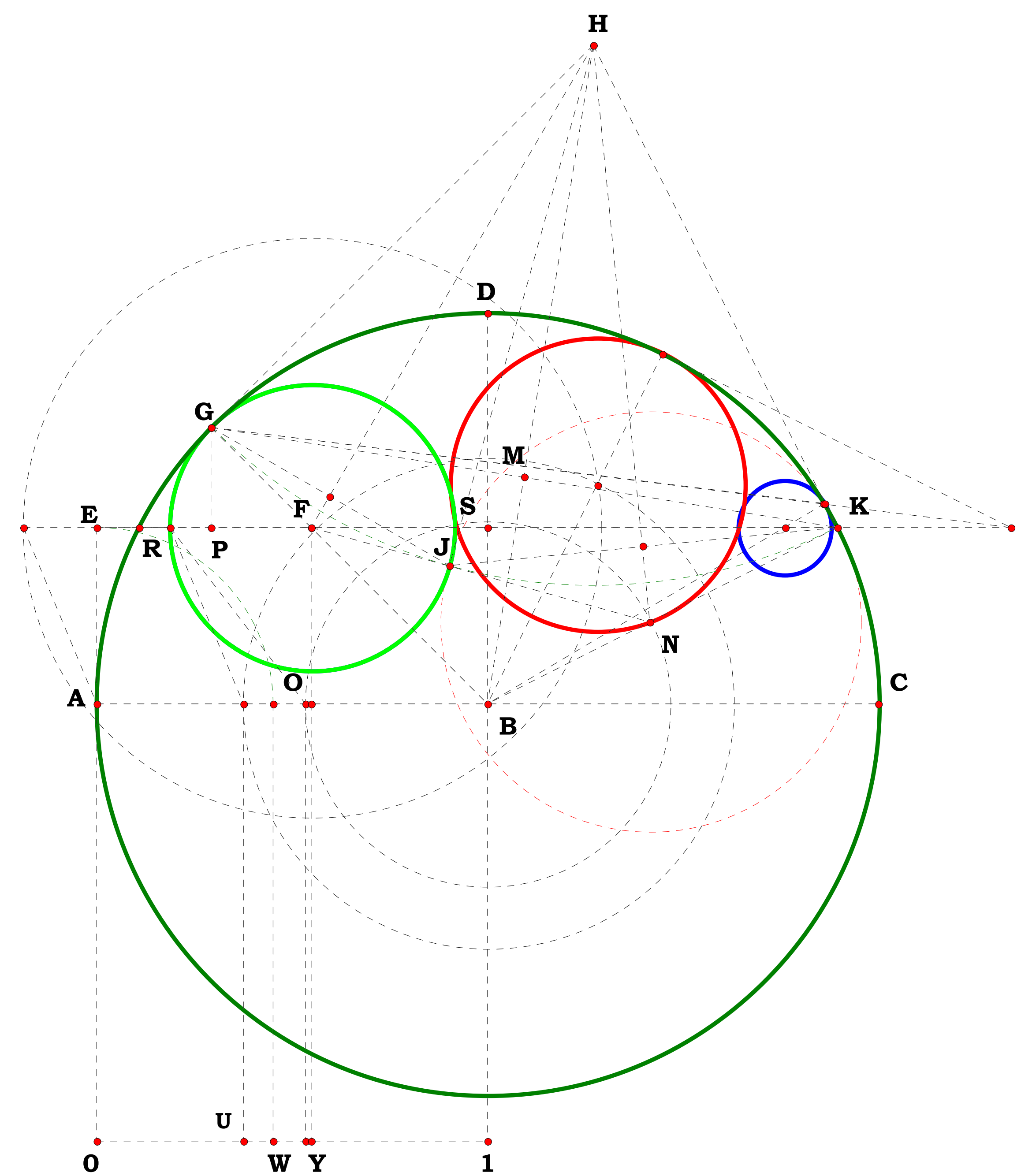
Given.

Place EF and GH and find JK.

I do not think I have drawn this figure correctly since the first time I drew it. Every write up of it after the drawing has been in error. I may get around to writing it up now that I disected and redrew it correctly.

042296B

Definitions.



Unit = 1.00000
W/X = 0.45000
W = 9.00000
X = 20.00000

Y/Z = 0.55000
Y = 11.00000
Z = 20.00000

XY = 0.70000
U = 14.00000
V = 20.00000

042396

Unit.

$AB := 1$

Given.

$N_1 := 3$

Descriptions.

$AH := AB \cdot N_1 \quad BH := AH - AB \quad BG := \frac{BH}{2} \quad BN := BG \quad GO := BG \quad HP := BG$

$GM := BG \quad GH := BG \quad AG := AH - GH \quad AM := \sqrt{GM^2 + AG^2} \quad AL := \frac{AG^2}{AM}$

$LM := AM - AL \quad JL := LM \quad AJ := AM - (JL + LM) \quad AD := \frac{AG \cdot AJ}{AM}$

$BD := AD - AB \quad DH := BH - BD \quad DJ := \sqrt{BD \cdot DH} \quad BC := \frac{BD \cdot BN}{BN + DJ}$

$DF := \frac{DH \cdot DJ}{BN + DJ} \quad CD := BD - BC \quad CF := CD + DF \quad CE := \frac{CF}{2} \quad BE := BC + CE$

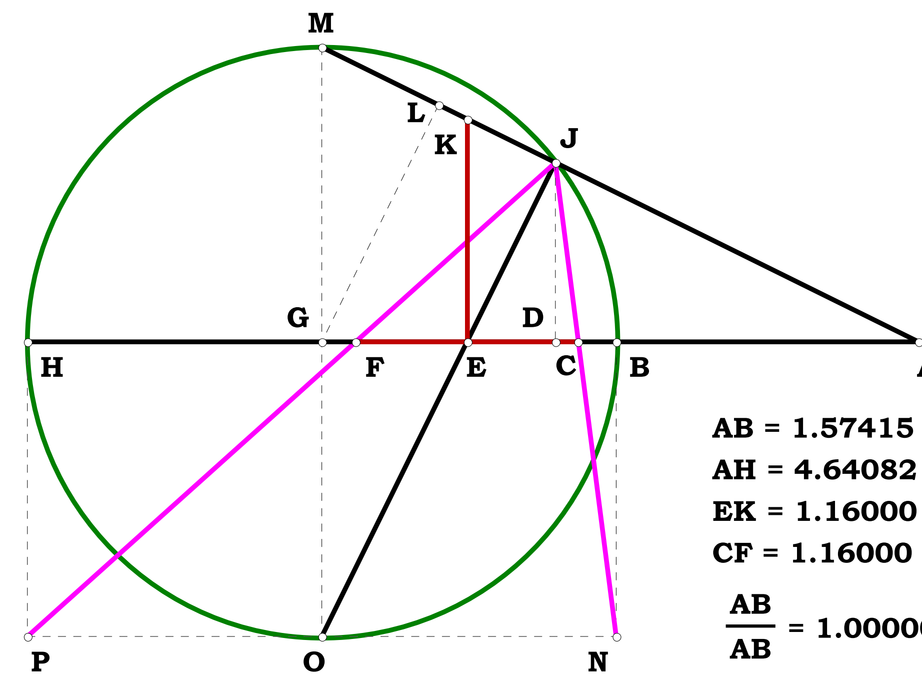
$AE := AB + BE \quad EK := \frac{GM \cdot AE}{AG} \quad EK - CF = 0 \quad EK = 0.75$

Definitions.

$$EK - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{(N_1 + 1)^2} = 0$$

$$CF - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{(N_1 + 1)^2} = 0$$

Is CF always equal to EK?



AB = 1.57415 in.
AH = 4.64082 in.
EK = 1.16000 in.
CF = 1.16000 in.

$\frac{AB}{AB} = 1.00000$

$\frac{AH}{AB} = 2.94814$

$\frac{EK}{AB} = 0.73691$

$\frac{CF}{AB} = 0.73691$

Unit = 1.00000

$N_1 = 2.94814$

$\frac{2 \cdot N_1 \cdot (N_1\text{-Unit})}{(N_1\text{+Unit})^2} = 0.73691$

Sometimes the process of naming becomes very complex and it is impossible given the working constraints to put everything in terms of the givens, actually, they often do not illicit any recognition in the mind as to their truth, so one has to write all the steps down anyway, but sometimes one would like to show all the definitions in a given step by step process.

$$BH-(N_1-1)=0 \qquad BG-\frac{N_1-1}{2}=0 \qquad AG-\left(\frac{N_1+1}{2}\right)=0$$

$$AM-\frac{1}{2}\cdot\sqrt{2\cdot N_1^{\,2}+2}=0 \qquad AL-\frac{1}{2}\cdot\frac{(N_1+1)^2}{\sqrt{2\cdot N_1^{\,2}+2}}=0 \qquad LM-\frac{1}{2}\cdot\frac{(N_1-1)^2}{\sqrt{2\cdot N_1^{\,2}+2}}=0$$

$$AJ-\frac{2}{\sqrt{2\cdot N_1^{\,2}+2}}\cdot N_1=0 \qquad AD-(N_1+1)\cdot\frac{N_1}{(N_1^{\,2}+1)}=0 \qquad BD-\frac{(N_1-1)}{(N_1^{\,2}+1)}=0$$

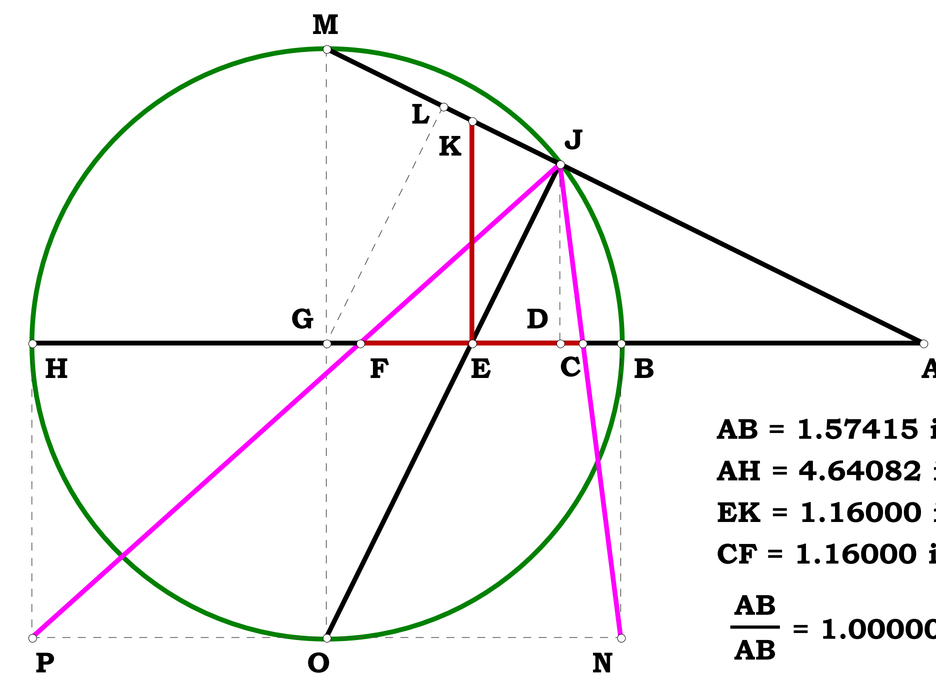
$$DH-N_1^{\,2}\cdot\frac{(N_1-1)}{(N_1^{\,2}+1)}=0 \qquad DJ-\frac{(N_1-1)}{(N_1^{\,2}+1)}\cdot N_1=0 \qquad BC-\frac{(N_1-1)}{(N_1+1)^2}=0$$

$$DF-2\cdot N_1^{\,3}\cdot\frac{(N_1-1)}{\left[(N_1+1)^2\cdot(N_1^{\,2}+1)\right]}=0 \qquad CD-2\cdot(N_1-1)\cdot\frac{N_1}{\left[(N_1^{\,2}+1)\cdot(N_1+1)^2\right]}=0$$

$$CF-2\cdot N_1\cdot\frac{(N_1-1)}{(N_1+1)^2}=0 \qquad CE-N_1\cdot\frac{(N_1-1)}{(N_1+1)^2}=0 \qquad BE-\frac{(N_1-1)}{(N_1+1)}=0$$

$$AE-2\cdot\frac{N_1}{(N_1+1)}=0 \qquad EK-2\cdot(N_1-1)\cdot\frac{N_1}{(N_1+1)^2}$$

$$EK-CF=0 \qquad EK=0.75$$



AB = 1.57415 in.
AH = 4.64082 in.
EK = 1.16000 in.
CF = 1.16000 in.

$\frac{AB}{AB}$ = 1.00000

$\frac{AH}{AB}$ = 2.94814

$\frac{EK}{AB}$ = 0.73691

$\frac{CF}{AB}$ = 0.73691

Unit = 1.00000
N_1 = 2.94814

$\frac{2\cdot N_1\cdot(N_1\text{-Unit})}{(N_1\text{+Unit})^2}$ = 0.73691

Unit.

$AB := 1$

Given.

$N_1 := .36802$

042496

Descriptions.

$AF := AB \qquad AD := \frac{AF}{2} \qquad AC := N_1 \qquad DO := \frac{AB}{2} \qquad OR := AF \qquad CD := AD - AC$

$CO := \sqrt{CD^2 + DO^2} \qquad PO := \frac{DO \cdot OR}{CO} \qquad CP := PO - CO \qquad CK := \frac{DO \cdot CP}{PO}$

$JK := CK \qquad KO := \sqrt{CD^2 + (DO + CK)^2} \qquad JO := \sqrt{KO^2 - JK^2} \qquad KS := \frac{JK^2}{KO}$

$SO := KO - KS \qquad JS := \frac{JK \cdot SO}{JO} \qquad ST := \frac{CD \cdot SO}{DO + CK} \qquad JT := JS + ST$

$TO := \frac{KO \cdot ST}{CD} \qquad TU := \frac{CD \cdot JT}{KO} \qquad DU := TO - (DO + TU)$

$CV := DU \qquad CQ := 2 \cdot CK \qquad QV := CQ - CV \qquad BH := \frac{CK \cdot CV}{QV}$

$CK = 0.232581 \qquad BH = 0.082585$

Definitions.

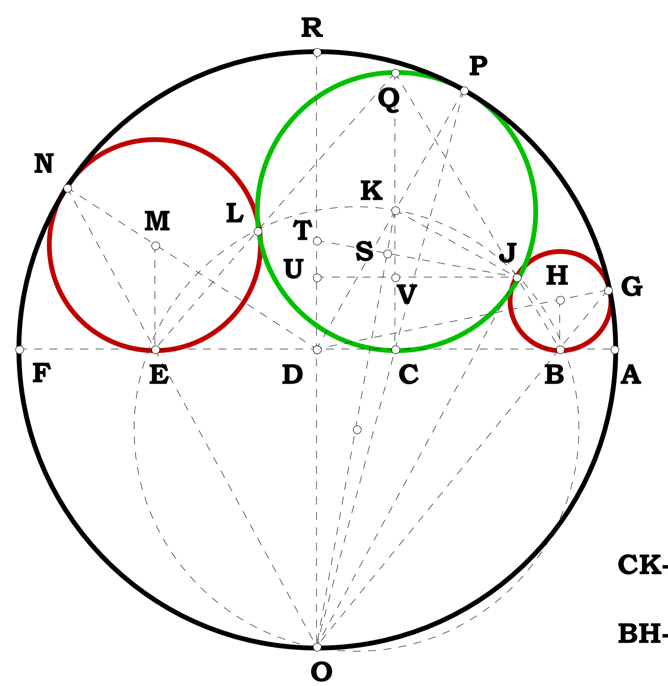
$$CK - \left(N_1 - N_1^{\,2}\right) = 0$$

$$BH - \frac{N_1 \cdot \left(N_1 - 1\right) \cdot \left(2 \cdot N_1 - 2 \cdot N_1^{\,2} - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} - 2\right)}{2 \cdot N_1 + 6 \cdot N_1^{\,2} - 16 \cdot N_1^{\,3} + 8 \cdot N_1^{\,4} - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} + 2} = 0$$

If one takes the time to work MC by hand we get; $BH - \frac{N_1 \cdot \left(1 - N_1\right)}{\left[\sqrt{2} - \left(2N_1 - 1\right)\right]^2} = 0$

Three Circles.

Given AC, find CK and BH.



$AF = 7.87400$ cm

$AC = 2.89777$ cm

$CK = 1.83134$ cm

$BH = 0.65027$ cm

$\frac{AF}{AF} = 1.00000$

$\frac{AC}{AF} = 0.36802$

$\frac{CK}{AF} = 0.23258$

$\frac{BH}{AF} = 0.08258$

$AF = 1.00000$

$N_1 = 0.36802$

$CK = 0.23258$

$BH = 0.08258$

$CK\text{-}N_1\text{-}N_1^2 = 0.00000$

$BH\text{-}\frac{N_1 \cdot (1-N_1)}{(\sqrt{2}\text{-}2 \cdot N_1\text{-}1)^2} = 0.00000$

Some Algebraic Names, or Definitions.

$$AD-\frac{1}{2}=0 \qquad AC-N_1=0 \qquad CD-\frac{\left(1-2\cdot N_1\right)}{2}=0 \qquad CO-\frac{\sqrt{2\cdot N_1{}^2-2\cdot N_1+1}}{\sqrt{2}}=0$$

$$PO-\frac{\sqrt{2}}{2\cdot\sqrt{2\cdot N_1{}^2-2\cdot N_1+1}}=0 \qquad CP-\frac{\sqrt{2}\cdot N_1\cdot\left(1-N_1\right)}{\sqrt{2\cdot N_1{}^2-2\cdot N_1+1}}=0 \qquad CK-\left(N_1-N_1{}^2\right)=0$$

$$KO-\frac{\sqrt{\left(2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1\right)}}{\sqrt{2}}=0 \qquad KS-\frac{\sqrt{2}\cdot N_1{}^2\cdot\left(N_1-1\right)^2}{\sqrt{2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1}}=0$$

$$SO-\frac{\sqrt{2}}{2\cdot\sqrt{2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1}}=0 \qquad JS-\frac{N_1\cdot\left(1-N_1\right)}{\sqrt{2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1}}=0$$

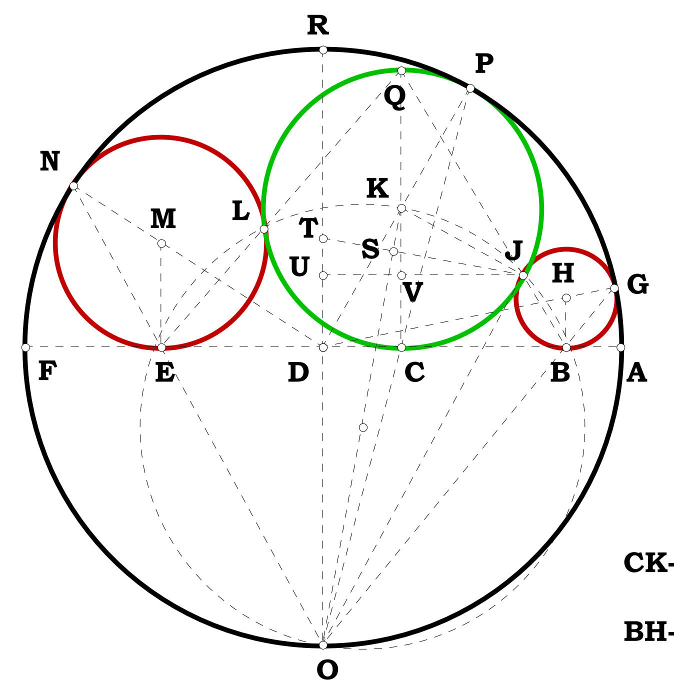
$$ST-\frac{\sqrt{2}\cdot\left(1-2\cdot N_1\right)}{2\cdot\left(2\cdot N_1-2\cdot N_1{}^2+1\right)\cdot\sqrt{2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1}}=0 \qquad JO-\frac{1}{\sqrt{2}}=0$$

$$JT-\frac{N_1+N_1{}^2-4\cdot N_1{}^3+2\cdot N_1{}^4-\sqrt{2}\cdot N_1+\frac{\sqrt{2}}{2}}{\left(2\cdot N_1-2\cdot N_1{}^2+1\right)\cdot\sqrt{2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1}}=0 \qquad TO-\frac{1}{2\cdot N_1-2\cdot N_1{}^2+1}=0$$

$$TU-\left[\frac{1}{2\cdot N_1-2\cdot N_1{}^2+1}-\frac{\left(3\cdot\sqrt{2}-2\right)\cdot N_1{}^2-2\cdot\sqrt{2}\cdot N_1{}^3+\left(2-\sqrt{2}\right)\cdot N_1+1}{2\cdot\left(2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1\right)}\right]=0$$

$$DU-\left[\frac{\left(3\cdot\sqrt{2}-2\right)\cdot N_1{}^2-2\cdot\sqrt{2}\cdot N_1{}^3+\left(2-\sqrt{2}\right)\cdot N_1+1}{2\cdot\left(2\cdot N_1{}^4-4\cdot N_1{}^3+2\cdot N_1{}^2+1\right)}-\frac{1}{2}\right]=0$$

$$CQ-2\cdot\left(N_1-N_1{}^2\right)=0 \qquad QV-\frac{N_1\cdot\left(N_1-1\right)\cdot\left(2\cdot N_1+6\cdot N_1{}^2-16\cdot N_1{}^3+8\cdot N_1{}^4-2\cdot\sqrt{2}\cdot N_1+\sqrt{2}+2\right)}{2\cdot\left(4\cdot N_1{}^3-2\cdot N_1{}^4-2\cdot N_1{}^2-1\right)}=0$$



AF = 7.87400 cm
AC = 2.89777 cm
CK = 1.83134 cm
BH = 0.65027 cm

$\frac{AF}{AF} = 1.00000$

$\frac{AC}{AF} = 0.36802$

$\frac{CK}{AF} = 0.23258$

$\frac{BH}{AF} = 0.08258$

AF = 1.00000

$N_1 = 0.36802$

CK = 0.23258

BH = 0.08258

$CK\text{-}N_1\text{-}N_1{}^2 = 0.00000$

$BH\text{-}\frac{N_1\cdot(1\text{-}N_1)}{(\sqrt{2}\text{-}2\cdot N_1\text{-}1)^2} = 0.00000$

Unit.

$AC := 1$

Given.

$N_1 := 2.817 \qquad AF := N_1$

$N_2 := 3 \qquad FK := N_2$

One Over N + One

Descriptions.
042596

$CF := AF - AC \qquad CE := \frac{CF}{2} \qquad AE := AC + CE$

$EJ := \frac{FK \cdot AE}{AF} \qquad DL := FK \qquad EF := CE \qquad DF := \frac{EF \cdot DL}{EJ}$

$CG := \frac{FK \cdot AC}{AF} \qquad CD := CF - DF \qquad DH := CG$

$HL := DL - DH \qquad BC := \frac{CD \cdot CG}{HL}$

Definitions.

$CF - (N_1 - 1) = 0 \qquad CE - \frac{1}{2} \cdot (N_1 - 1) = 0$

$AE - \frac{1}{2} \cdot (1 + N_1) = 0$

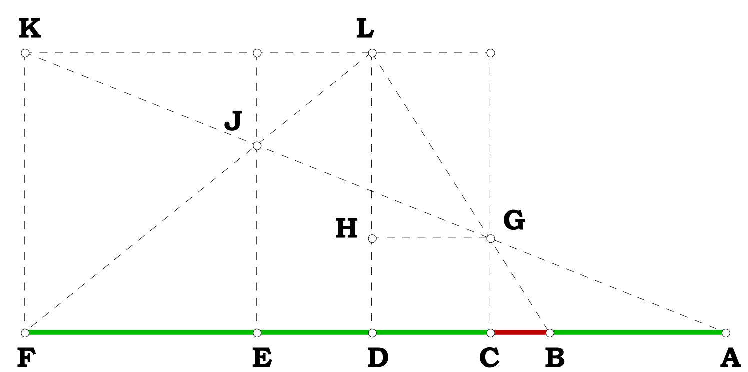
$EJ - \frac{N_2 \cdot (N_1 + 1)}{2 \cdot N_1} = 0 \qquad DF - \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 = 0$

$CG - \frac{N_2}{N_1} = 0 \qquad CD - \frac{(N_1 - 1)}{(1 + N_1)} = 0$

$HL - N_2 \cdot \frac{(N_1 - 1)}{N_1} = 0 \qquad \frac{1}{N_1 + 1} - BC = 0$

Construct 1/(N+1)

In this construction, N_2 has to be something, it can be anything and will not change the results BC.



$N_1 = 2.96971$
$N_2 = 1.18938$

AC = 1.22908 in.
AF = 3.65000 in.
CF = 2.42092 in.
CE = 1.21046 in.
AE = 2.43954 in.
FK = 1.46184 in.
EJ = 0.97705 in.
DL = 1.46184 in.
EF = 1.21046 in.
DF = 1.81107 in.
CG = 0.49225 in.
CD = 0.60985 in.
DH = 0.49225 in.
HL = 0.96959 in.
BC = 0.30961 in.

$\frac{AC}{AC} = 1.00000$

$\frac{AF}{AC} - N_1 = 0.00000$

$\frac{FK}{AC} - N_2 = 0.00000$

$\frac{CF}{AC} - N_1 - 1 = 0.00000$

$\frac{CE}{AC} - \frac{N_1 - 1}{2} = 0.00000$

$\frac{AE}{AC} - \frac{N_1 + 1}{2} = 0.00000$

$\frac{FK}{AC} - N_2 = 0.00000$

$\frac{EJ}{AC} - \frac{N_2 \cdot (N_1 + 1)}{2 \cdot N_1} = 0.00000$

$\frac{DL}{AC} - N_2 = 0.00000$

$\frac{EF}{AC} - \frac{N_1 - 1}{2} = 0.00000$

$\frac{DF}{AC} - \frac{N_1 \cdot (N_1 - 1)}{N_1 + 1} = 0.00000$

$\frac{CG}{AC} - \frac{N_2}{N_1} = 0.00000$

$\frac{CD}{AC} - \frac{N_1 - 1}{N_1 + 1} = 0.00000$

$\frac{DH}{AC} - \frac{N_2}{N_1} = 0.00000$

$\frac{HL}{AC} - \frac{N_2 \cdot (N_1 - 1)}{N_1} = 0.00000$

$\frac{BC}{AC} - \frac{1}{N_1 + 1} = 0.00000$

042696

Unit. $AB := 1$

Given. $W := 10$ $Y := 7$

$X := 18$ $Z := 20$

Three Bases.

082621 Each time I looked at the original write-up done in 96, I have promised myself to redo it as it was not done right. So, I finally got it done.

Descriptions.

$AC := \frac{AB}{2}$ $BC := AC$ $CJ := AC$ $CD := BC \cdot \frac{W}{X}$ $AD := AC + CD$ $CE := CD \cdot \frac{Y}{Z}$ $AE := AC + CE$ $BD := AB - AD$ $BE := AB - AE$ $DJ := \sqrt{AD \cdot BD}$ $EM := \sqrt{AE \cdot BE}$ $DG := \frac{DJ^2}{CD}$

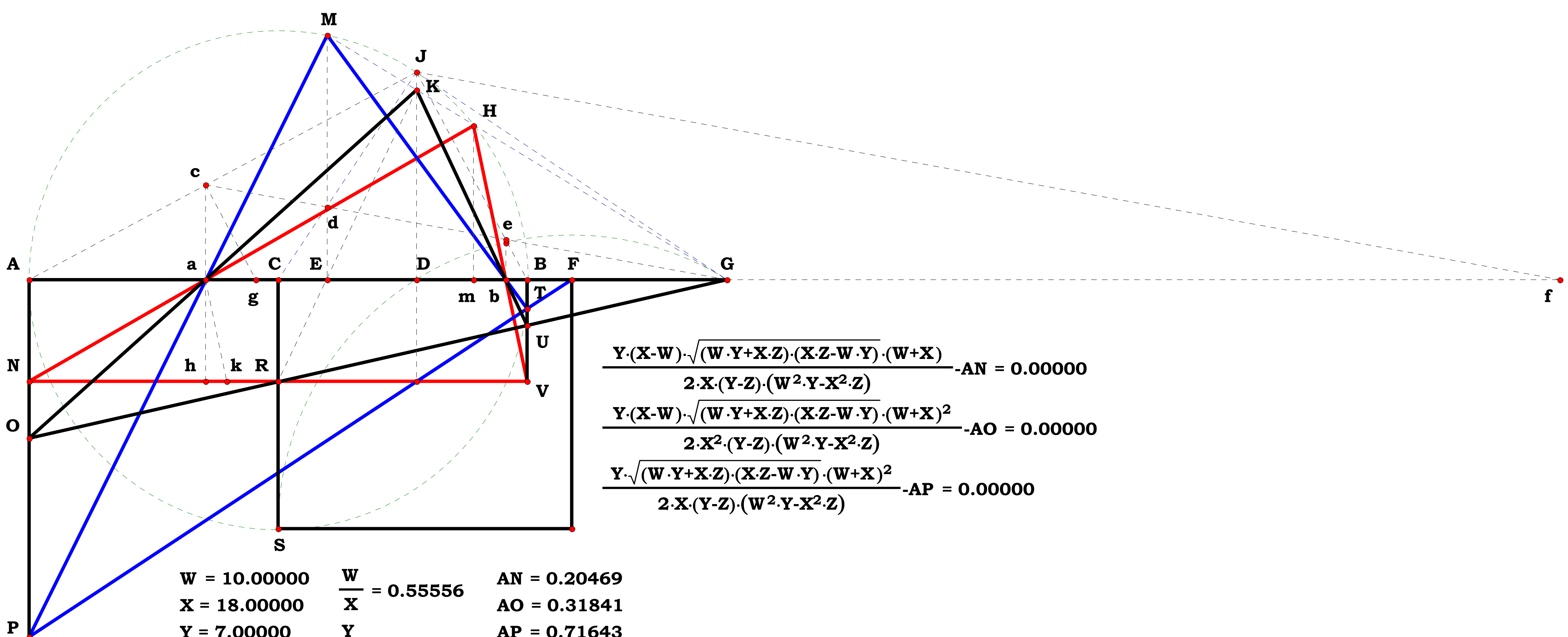
$AG := AD + DG$ $DF := \frac{DG}{2}$ $AF := AD + DF$ $BF := AF - AB$ $Ed := DJ \cdot \frac{CE}{CD}$ $EG := AG - AE$ $Df := EG \cdot \frac{DJ}{Ed}$ $Af := Df + AD$ $Aa := \frac{AD \cdot AG}{Af}$ $Ag := \frac{AB \cdot Aa}{AD}$ $Gg := AG - Ag$

$ag := Ag - Aa$ $BG := AG - AB$ $Bb := \frac{ag \cdot BG}{Gg}$ $DK := EM \cdot \frac{DG}{EG}$ $DE := CD - CE$ $CR := DK \cdot \frac{CE}{DE}$ $ah := CR$ $Nh := Aa$ $hk := Nk := Nh + hk$ $ab := AB - (Aa + Bb)$ $Hm := ah \cdot \frac{ab}{Nk}$

$am := \frac{Nh \cdot ab}{Nk}$ $Am := Aa + am$ $Bm := AB - Am$ $AN := CR$ $Da := AD - Aa$ $AO := \frac{DK \cdot Aa}{Da}$ $Ea := AE - Aa$ $AP := EM \cdot \frac{Aa}{Ea}$ $Eb := ab - Ea$ $BT := EM \cdot \frac{Bb}{Eb}$ $Db := ab - Da$

$BU := DK \cdot \frac{Bb}{Db}$

$BV := CR$



$$\frac{Y \cdot (X-W) \cdot \sqrt{(W \cdot Y+X \cdot Z) \cdot (X \cdot Z-W \cdot Y)} \cdot (W+X)}{2 \cdot X \cdot (Y-Z) \cdot (W^2 \cdot Y-X^2 \cdot Z)} - AN = 0.00000$$

$$\frac{Y \cdot (X-W) \cdot \sqrt{(W \cdot Y+X \cdot Z) \cdot (X \cdot Z-W \cdot Y)} \cdot (W+X)^2}{2 \cdot X^2 \cdot (Y-Z) \cdot (W^2 \cdot Y-X^2 \cdot Z)} - AO = 0.00000$$

$$\frac{Y \cdot \sqrt{(W \cdot Y+X \cdot Z) \cdot (X \cdot Z-W \cdot Y)} \cdot (W+X)^2}{2 \cdot X \cdot (Y-Z) \cdot (W^2 \cdot Y-X^2 \cdot Z)} - AP = 0.00000$$

$W = 10.00000$ $\frac{W}{X} = 0.55556$ $AN = 0.20469$

$X = 18.00000$ $AO = 0.31841$

$Y = 7.00000$ $\frac{Y}{Z} = 0.35000$ $AP = 0.71643$

$Z = 20.00000$

Definitions.

$$AC-\frac{1}{2}=0 \quad BC-\frac{1}{2}=0 \quad CJ-\frac{1}{2}=0 \quad CD-\frac{W}{2\cdot X}=0 \quad AD-\frac{W+X}{2\cdot X}=0 \quad CE-\frac{W\cdot Y}{2\cdot X\cdot Z}=0 \quad AE-\frac{W\cdot Y+X\cdot Z}{2\cdot X\cdot Z}=0 \quad BD-\frac{X-W}{2\cdot X}=0 \quad BE-\frac{X\cdot Z-W\cdot Y}{2\cdot X\cdot Z}=0 \quad DJ-\frac{\sqrt{(X-W)\cdot(W+X)}}{2\cdot X}=0$$

$$EM-\frac{\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}}{2\cdot X\cdot Z}=0 \quad DG-\frac{(X-W)\cdot(W+X)}{2\cdot W\cdot X}=0 \quad AG-\frac{W+X}{2\cdot W}=0 \quad DF-\frac{(X-W)\cdot(W+X)}{4\cdot W\cdot X}=0 \quad AF-\frac{(W+X)^2}{4\cdot W\cdot X}=0 \quad BF-\frac{(W-X)^2}{4\cdot W\cdot X}=0 \quad Ed-\frac{Y\cdot\sqrt{(X-W)\cdot(W+X)}}{2\cdot X\cdot Z}=0$$

$$EG-\frac{X^2\cdot Z-W^2\cdot Y}{2\cdot W\cdot X\cdot Z}=0 \quad Df-\frac{X^2\cdot Z-W^2\cdot Y}{2\cdot W\cdot X\cdot Y}=0 \quad Af-\frac{W\cdot Y+X\cdot Z}{2\cdot W\cdot Y}=0 \quad Aa-\frac{Y\cdot(W+X)^2}{2\cdot X\cdot(W\cdot Y+X\cdot Z)}=0 \quad Ag-\frac{Y\cdot(W+X)}{W\cdot Y+X\cdot Z}=0 \quad Gg-\frac{(W+X)\cdot(X\cdot Z-W\cdot Y)}{2\cdot W\cdot(W\cdot Y+X\cdot Z)}=0 \quad ag-\frac{Y\cdot(X-W)\cdot(W+X)}{2\cdot X\cdot(W\cdot Y+X\cdot Z)}=0$$

$$BG-\frac{X-W}{2\cdot W}=0 \quad Bb-\frac{Y\cdot(W-X)^2}{2\cdot X\cdot(X\cdot Z-W\cdot Y)}=0 \quad DK-\frac{(W-X)\cdot\sqrt{-(W\cdot Y+X\cdot Z)\cdot(W\cdot Y-X\cdot Z)}\cdot(W+X)}{2\cdot X\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad DE-\frac{W\cdot(Z-Y)}{2\cdot X\cdot Z}=0$$

$$CR-\frac{Y\cdot(X-W)\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)}{2\cdot X\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad ah-\frac{Y\cdot(X-W)\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)}{2\cdot X\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad Nh-\frac{Y\cdot(W+X)^2}{2\cdot X\cdot(W\cdot Y+X\cdot Z)}=0 \quad hk-\frac{Y\cdot(W-X)^2}{2\cdot X\cdot(X\cdot Z-W\cdot Y)}=0$$

$$Nk-\frac{Y\cdot\left(2\cdot W^2\cdot Y-W^2\cdot Z-X^2\cdot Z\right)}{(W\cdot Y-X\cdot Z)\cdot(W\cdot Y+X\cdot Z)}=0 \quad ab-\frac{(Z-Y)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}{(W\cdot Y-X\cdot Z)\cdot(W\cdot Y+X\cdot Z)}=0 \quad Hm-\frac{(W-X)\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)}{2\cdot X\cdot\left(2\cdot W^2\cdot Y-W^2\cdot Z-X^2\cdot Z\right)}=0$$

$$am-\frac{(Z-Y)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)\cdot(W+X)^2}{2\cdot X\cdot(W\cdot Y+X\cdot Z)\cdot\left(2\cdot W^2\cdot Y-W^2\cdot Z-X^2\cdot Z\right)}=0 \quad Am-\frac{(W+X)^2\cdot(W\cdot Y-X\cdot Z)}{2\cdot X\cdot\left(2\cdot W^2\cdot Y-W^2\cdot Z-X^2\cdot Z\right)}=0 \quad Bm-\frac{(W-X)^2\cdot(W\cdot Y+X\cdot Z)}{2\cdot X\cdot\left(W^2\cdot Z-2\cdot W^2\cdot Y+X^2\cdot Z\right)}=0$$

$$AN-\frac{Y\cdot(X-W)\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)}{2\cdot X\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad Da-\frac{(Z-Y)\cdot(W+X)}{2\cdot(W\cdot Y+X\cdot Z)}=0 \quad AO-\frac{Y\cdot(X-W)\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)^2}{2\cdot X^2\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad Ea-\frac{(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}{2\cdot X\cdot Z\cdot(W\cdot Y+X\cdot Z)}=0$$

$$AP-\frac{Y\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)^2}{2\cdot X\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad Eb-\frac{(Z-Y)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}{2\cdot X\cdot Z\cdot(W\cdot Y-X\cdot Z)}=0 \quad BT-\frac{Y\cdot(W-X)^2\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}}{2\cdot X\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad Db-\frac{(Z-Y)\cdot(W-X)}{2\cdot(W\cdot Y-X\cdot Z)}=0$$

$$BU-\frac{Y\cdot(W-X)^2\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)}{2\cdot X^2\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0 \quad BV-\frac{Y\cdot(X-W)\cdot\sqrt{(W\cdot Y+X\cdot Z)\cdot(X\cdot Z-W\cdot Y)}\cdot(W+X)}{2\cdot X\cdot(Y-Z)\cdot\left(W^2\cdot Y-X^2\cdot Z\right)}=0$$

042796

Unit.

$CF := 1$

Given.

$N_1 := 4$

$N_2 := 9$

Descriptions.

$CE := \frac{CF}{2}$ $\quad CD := \frac{CE}{N_1}$ $\quad FK := N_2$ $\quad DM := FK$ $\quad EL := FK$

$DF := CF - CD$ $\quad EF := \frac{CF}{2}$ $\quad EJ := \frac{DM \cdot EF}{DF}$ $\quad JL := EL - EJ$

$KL := EF$ $\quad AF := \frac{KL \cdot FK}{JL}$ $\quad AC := AF - CF$ $\quad CG := \frac{FK \cdot AC}{AF}$

$DH := CG$ $\quad HM := DM - DH$ $\quad BC := \frac{CD \cdot DH}{HM}$ $\quad BF := BC + CF$

$BD := BC + CD$ $\quad AB := AF - BF$

$AB^2 - BC \cdot BF = 0$ $\quad \sqrt{BC \cdot BF} - BD = 0$ $\quad BD - (CD + BC) = 0$

$AB - BD = 0$ $\quad \sqrt{BC \cdot BF} - AB = 0$ $\quad AB = 0.145833$

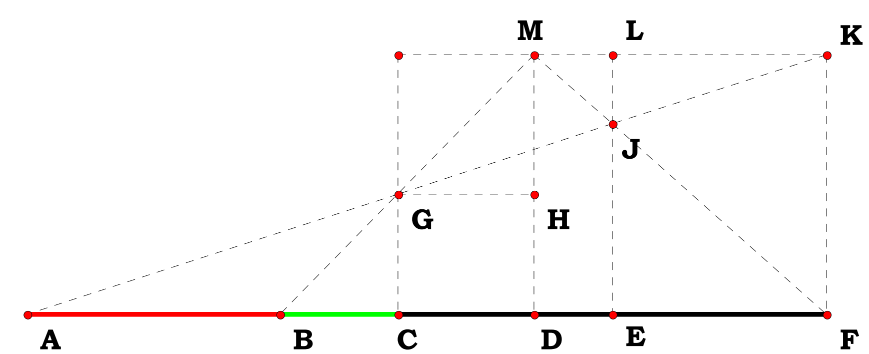
One may notice that can be any value at all, it just has to be some value. Every use of symbols require prue intelligble concepts which are perceptible in the grammar, but not all people can see them intelligibly.

A Root Figure

The difference between the perceptible and the intelligible.

CD + BC is the square root of $\sqrt{BC \cdot BF}$. What is BC?

When I originally did this story, I said to myself, well I can name that tune in 2 variables, which it should be, however, it is one of those things one says that they will do later as it has little to do with the main goal of the Delian Quest. Sometimes later means later in the day, sometimes later means decades away, as in this case. What are revisions for? One should also be aware, while learning, that one is not going to have the experience required to do a story justice until they mature, thus waiting is certainly an option.



$CF = 5.67267$ cm
$CE = 2.83633$ cm
$CD = 1.79928$ cm
$AB = 3.36014$ cm
$BC = 1.56086$ cm
$BF = 7.23353$ cm
$N_1 = 1.57637$
$AB = 0.59234$
$BC = 0.27515$
$BF = 1.27515$

$\frac{CF}{CF} = 1.00000$

$\frac{CE}{CD} = 1.57637$

$\frac{AB}{CF} = 0.59234$

$\frac{BC}{CF} = 0.27515$

$\frac{BF}{CF} = 1.27515$

$\frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.59234$ $\quad AB - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$\frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.27515$ $\quad BC - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$\frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 1.27515$ $\quad BF - \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$AB - \sqrt{BC \cdot BF} = 0.00000$

$AB - \sqrt{BC \cdot BF} = 0.00000$ cm

This equation is in the universal and any relatative difference can use it.

This equation is particular. It is from these particular examples, perceptible examples, that one acquires what Plato called, the similie in multis, or the Universal, or again, the intelligible. One aims for three things then, the correct range for a varaible built in to the equation, the answer expressed only in the givens, and the equation be free from any particular perceptible as grammar is always applicable universally. Not all my examples fill that bill, but then this is my storybook, my rather odd novel.

Definitions.

$$CF - 1 = 0 \qquad CE - \frac{1}{2} = 0 \qquad CD - \frac{1}{2 \cdot N_1} = 0 \qquad FK - N_2 = 0 \qquad DM - N_2 = 0 \qquad EL - N_2 = 0 \qquad DF - \frac{2 \cdot N_1 - 1}{2 \cdot N_1} = 0 \qquad EF - \frac{1}{2} = 0 \qquad EJ - \frac{N_1 \cdot N_2}{2 \cdot N_1 - 1} = 0$$

$$JL - \frac{N_2 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0 \qquad KL - \frac{1}{2} = 0 \qquad AF - \frac{2 \cdot N_1 - 1}{2 \cdot (N_1 - 1)} = 0 \qquad AC - \frac{1}{2 \cdot (N_1 - 1)} = 0 \qquad CG - \frac{N_2}{2 \cdot N_1 - 1} = 0 \qquad DH := \frac{N_2}{2 \cdot N_1 - 1} \qquad HM - \frac{2 \cdot N_2 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0$$

$$BC - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad BF - \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad BD - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad AB - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0$$

$$\left[\frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)}\right]^2 - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} \cdot \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad \sqrt{\frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} \cdot \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)}} - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} - \left[\frac{1}{2 \cdot N_1} + \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)}\right] = 0$$

$$AB - BD = 0 \qquad \sqrt{BC \cdot BF} - AB = 0 \qquad AB = 0.145833$$

Unit.

Given.

$N_1 := 2$

$N_2 := 6$

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

042896

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Process Summary will use a 5th root series for an example.

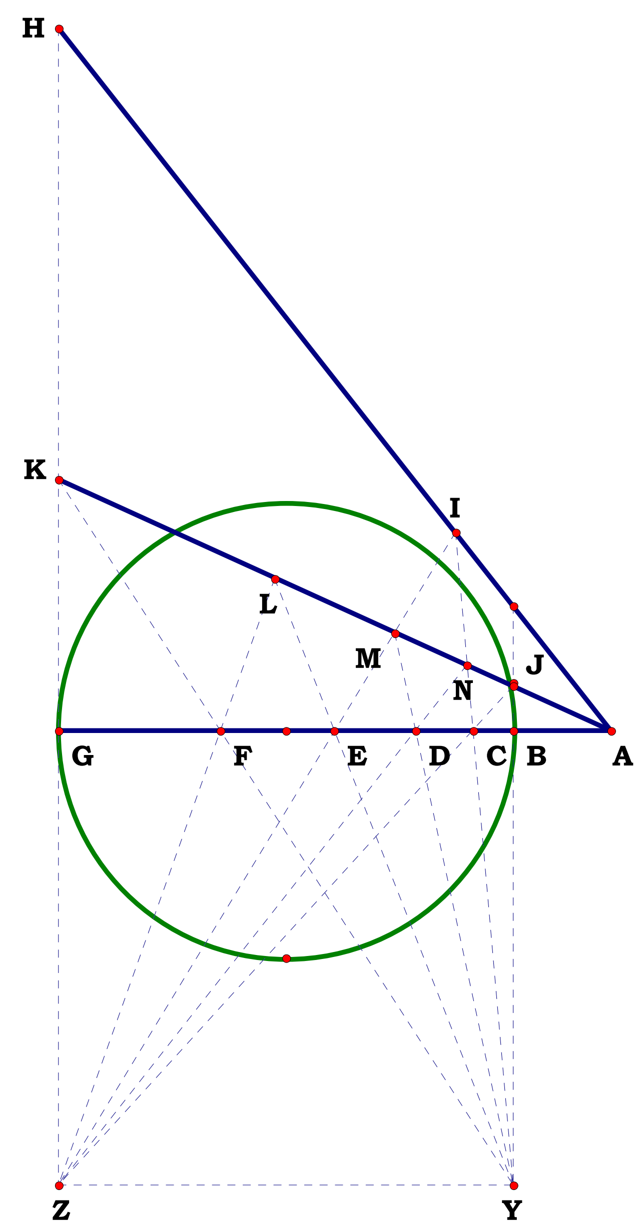
Descriptions.

$AB := N_1$ $\quad AG := N_2$ $\quad AE := \left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}$

$BG := AG - AB$ $\quad GZ := BG$ $\quad YZ := BG$

$BY := BG$ $\quad BE := AE - AB$ $\quad EG := BG - BE$ $\quad$ Definitions.

$$GH := \frac{BY \cdot EG}{BE}$$



AB = 1.29117 cm
AG = 7.30394 cm
AC = 1.82599 cm
AD = 2.58233 cm
AE = 3.65197 cm
AF = 5.16467 cm

$(AB^4 \cdot AG^1)^{\frac{1}{5}} - AC = 0.00000$

$(AB^3 \cdot AG^2)^{\frac{1}{5}} - AD = 0.00000$

$(AB^2 \cdot AG^3)^{\frac{1}{5}} - AE = 0.00000$

$(AB^1 \cdot AG^4)^{\frac{1}{5}} - AF = 0.00000$

$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$

$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$

$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$

$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$

$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$

$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$

$\frac{AG}{AB}^{\frac{1\,2}{5}} = 2.00000$

$\frac{AG}{AB}^{\frac{1\,3}{5}} = 2.82843$

$\frac{AG}{AB}^{\frac{1\,4}{5}} = 4.00000$

$\frac{AG}{AB}^{\frac{1\,5}{5}} = 5.65685$

$\frac{AG}{AB} = 5.65685$

$\frac{AF}{AB} = 4.00000$

$\frac{AE}{AB} = 2.82843$

$\frac{AD}{AB} = 2.00000$

$\frac{AC}{AB} = 1.41421$

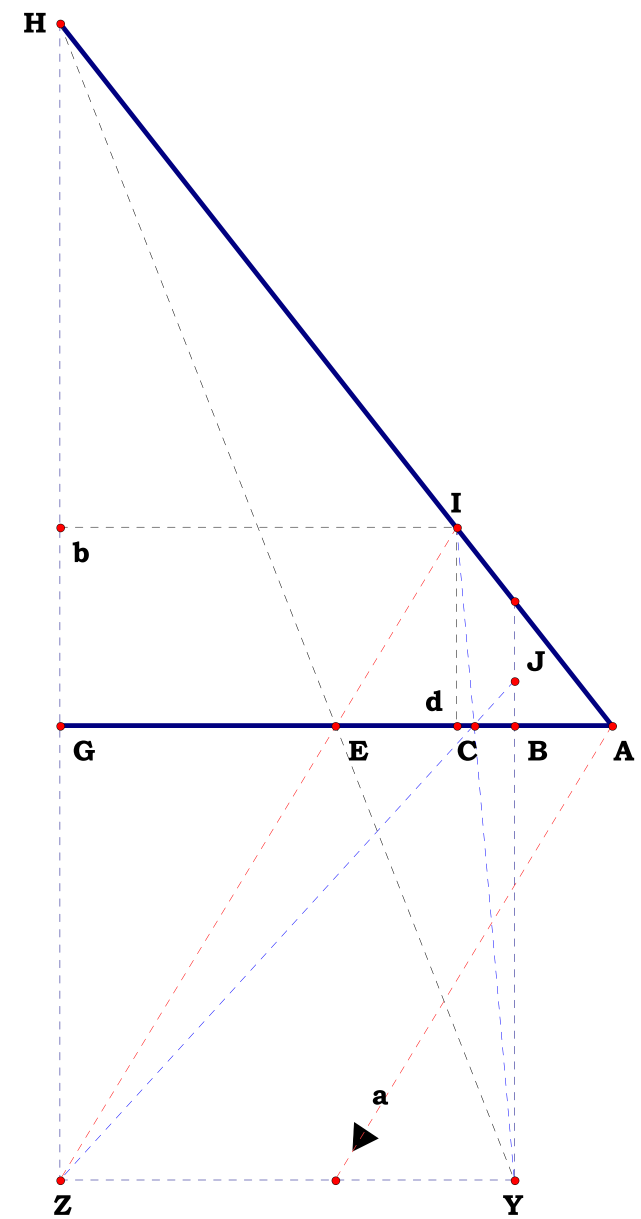
$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$



AB = 1.29117 cm
AG = 7.30394 cm
AC = 1.82599 cm
AD = 2.58233 cm
AE = 3.65197 cm
AF = 5.16467 cm

$(AB^4 \cdot AG^1)^{\frac{1}{5}} - AC = 0.00000$

$(AB^3 \cdot AG^2)^{\frac{1}{5}} - AD = 0.00000$

$(AB^2 \cdot AG^3)^{\frac{1}{5}} - AE = 0.00000$

$(AB^1 \cdot AG^4)^{\frac{1}{5}} - AF = 0.00000$

$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$

$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$

$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$

$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$

$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$

$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$

$\frac{AG}{AB}^{\frac{1\ 2}{5}} = 2.00000$

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$\frac{AG}{AB}^{\frac{1\ 4}{5}} = 4.00000$

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$\frac{AG}{AB} = 5.65685$

$\frac{AF}{AB} = 4.00000$

$\frac{AE}{AB} = 2.82843$

$\frac{AD}{AB} = 2.00000$

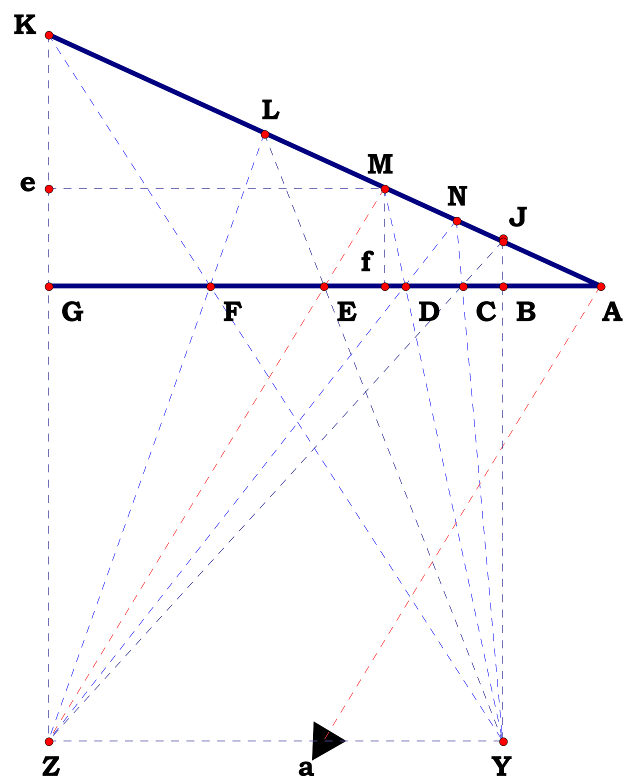
$\frac{AC}{AB} = 1.41421$

$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



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$\frac{AG}{AB} = 5.65685$

$\frac{AF}{AB} = 4.00000$

$\frac{AE}{AB} = 2.82843$

$\frac{AD}{AB} = 2.00000$

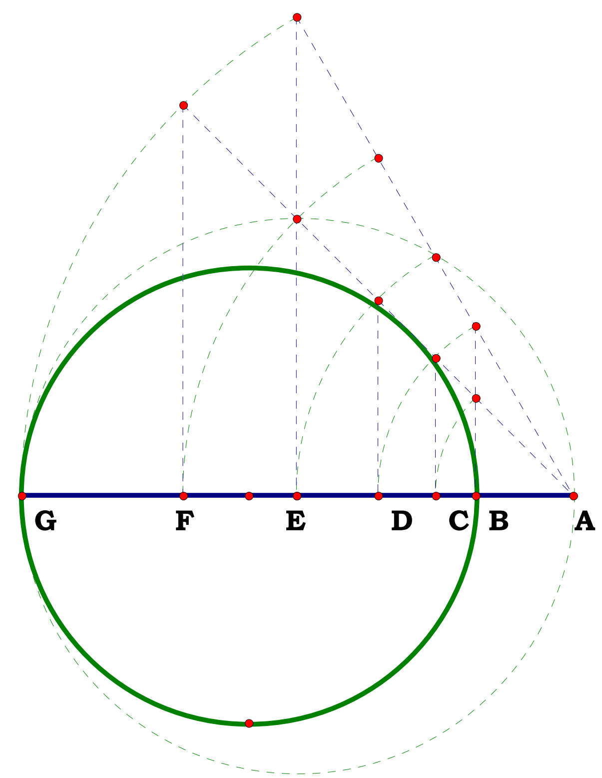
$\frac{AC}{AB} = 1.41421$

If any of a prime root series can be given exactly, every root of the series can be determined exactly.

$$\frac{\left(AB^{5}\cdot AG^{0}\right)^{\frac{1}{5}}}{AB}=1 \qquad \frac{\left(AB^{4}\cdot AG^{1}\right)^{\frac{1}{5}}}{AC}=1$$

$$\frac{\left(AB^{3}\cdot AG^{2}\right)^{\frac{1}{5}}}{AD}=1 \qquad \frac{\left(AB^{2}\cdot AG^{3}\right)^{\frac{1}{5}}}{AE}=1$$

$$\frac{\left(AB^{1}\cdot AG^{4}\right)^{\frac{1}{5}}}{AF}=1 \qquad \frac{\left(AB^{0}\cdot AG^{5}\right)^{\frac{1}{5}}}{AG}=1$$



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$$\left(AB^{3}\cdot AG^{2}\right)^{\frac{1}{5}}-AD = 0.00000$$

$$\left(AB^{2}\cdot AG^{3}\right)^{\frac{1}{5}}-AE = 0.00000$$

$$\left(AB^{1}\cdot AG^{4}\right)^{\frac{1}{5}}-AF = 0.00000$$

$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$

$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$$

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$$\frac{AG}{AB} = 5.65685$$

$$\frac{AF}{AB} = 4.00000$$

$$\frac{AE}{AB} = 2.82843$$

$$\frac{AD}{AB} = 2.00000$$

$$\frac{AC}{AB} = 1.41421$$

Unit.

$AB := 1$

Given.

$N := 5$

042996

Descriptions.

$AG := AB \cdot N \qquad BG := AG - AB$

$BF := \dfrac{BG}{2} \qquad FK := B{:}FO := BF \qquad AF := BF + AB$

$DF := \dfrac{FK \cdot FO}{AF} \qquad AK := \sqrt{AF^2 + FK^2} \qquad KO := BG$

$HO := \dfrac{AF \cdot KO}{AK} \qquad DO := \dfrac{AK \cdot FO}{AF} \qquad DH := HO - DO$

$DJ := \sqrt{DH \cdot DO}$

Definitions.

$AG - N = 0 \qquad BG - (N-1) = 0 \qquad BF := \dfrac{N-1}{2} \qquad AF - \left(\dfrac{1}{2} \cdot N + \dfrac{1}{2}\right) = 0$

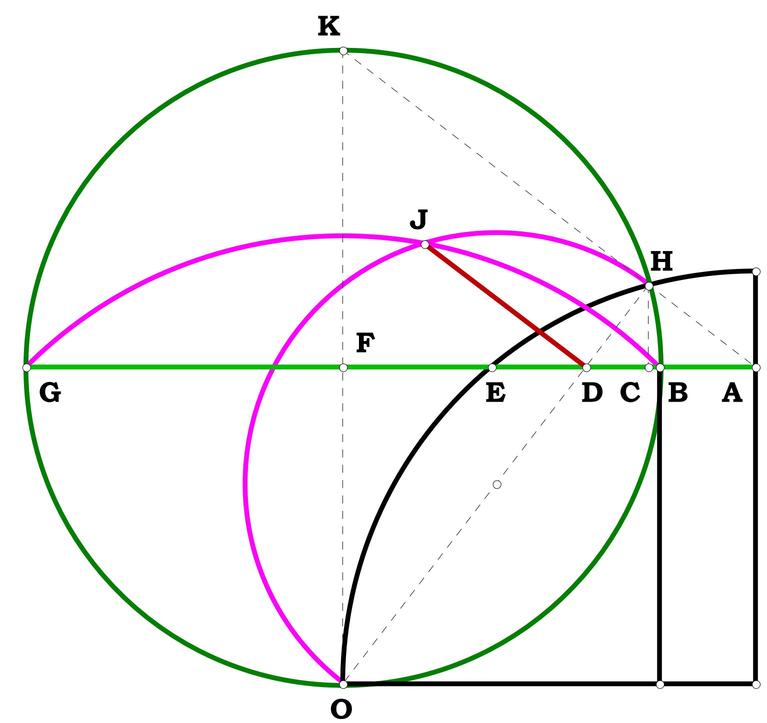
$DF - \dfrac{(N-1)^2}{2 \cdot (N+1)} = 0 \qquad AK - \dfrac{\sqrt{2 \cdot \left(N^2+1\right)}}{2} = 0$

$HO - \dfrac{\sqrt{2} \cdot \left(N^2-1\right)}{2 \cdot \sqrt{N^2+1}} = 0 \qquad DO - \dfrac{\sqrt{2} \cdot (N-1) \cdot \sqrt{N^2+1}}{2 \cdot (N+1)} = 0$

$DH - \dfrac{\sqrt{2} \cdot N \cdot (N-1)}{(N+1) \cdot \sqrt{N^2+1}} = 0 \qquad DJ - \sqrt{N} \cdot \dfrac{(N-1)}{(N+1)} = 0$

DJ is the Geometric name, what is its Algebraic name?

What is DJ?



Unit.

$AB := 1$

Given.

$N_1 := 128 \quad Root := 8$

$\delta := 1 .. Root$

043096

Descriptions.

Geometric Exponential Series of the form

$$\frac{\sum_{\delta} N^{\frac{Root-\delta}{Root}}}{N^{\frac{Root-1}{Root}}} \;,\; \sum_{\delta} N^{\frac{Root-\delta}{Root}} \quad \text{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 010896 and 011696 for the sides of the right triangle.

$BG := N_1 \quad AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

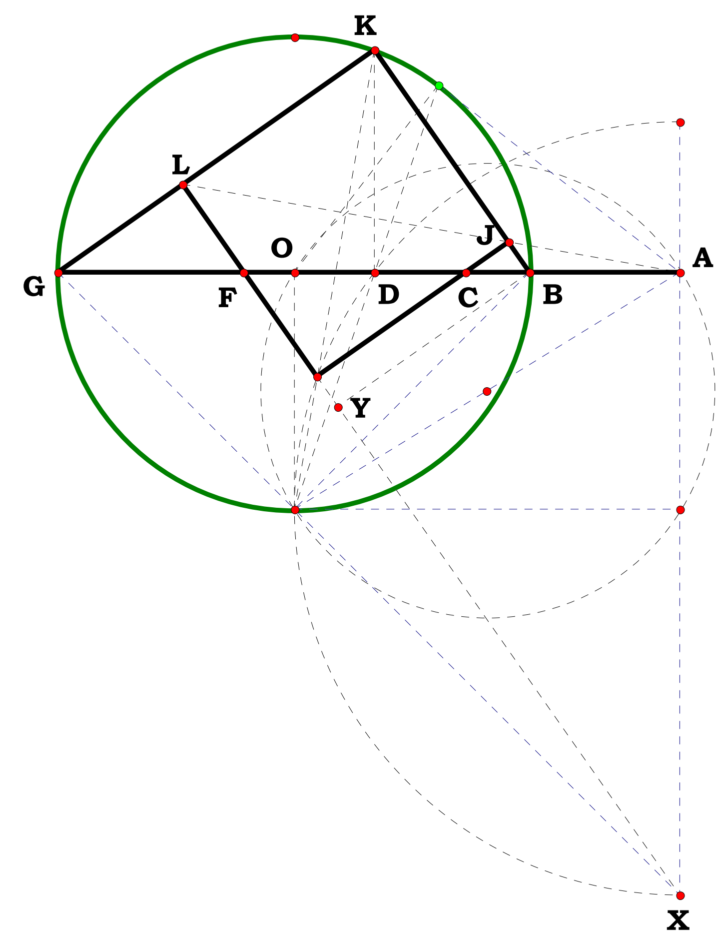
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



$\frac{\delta+2}{Root}$ =	$\frac{\delta}{Root}$ =	$\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}}$ =	$\left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}$ =
0.375	0.125	6.186872	1.835793
0.5	0.25	11.357817	3.370136
0.625	0.375	20.850601	6.186872
0.75	0.5	38.277387	11.357817
0.875	0.625	70.26936	20.850601
1	0.75	129	38.277387
1.125	0.875	236.817299	70.26936
1.25	1	434.747545	129

$$\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} = 1.835793$$

$$\frac{1}{Root} = 0.125$$

$$\frac{0}{Root} = 0$$

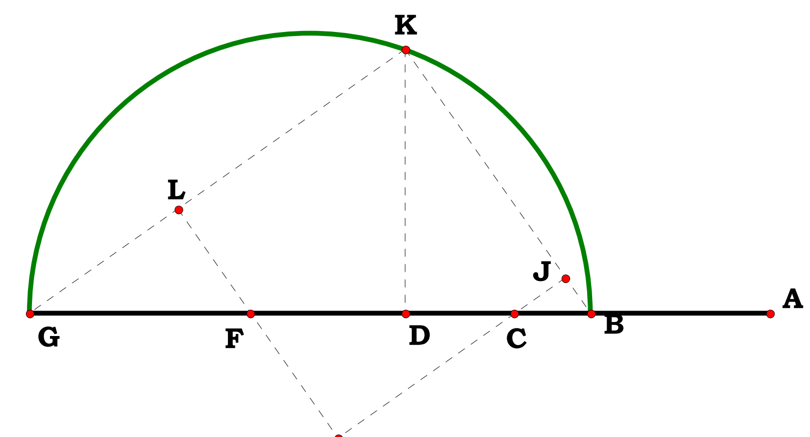
Definitions.

$GL = 51.575206 \qquad BJ = 0.399808$

$$\frac{GL}{BJ} = 129 \qquad \frac{AG}{AB} = 129$$

$$\frac{\sum_{\delta}\left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root-1}{Root}}} = 2.179442 \qquad \frac{GK}{GL} = 2.179442 \qquad \sum_{\delta}\left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}} = 153.147965$$

$$\frac{BK}{BJ} = 153.147965$$



$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

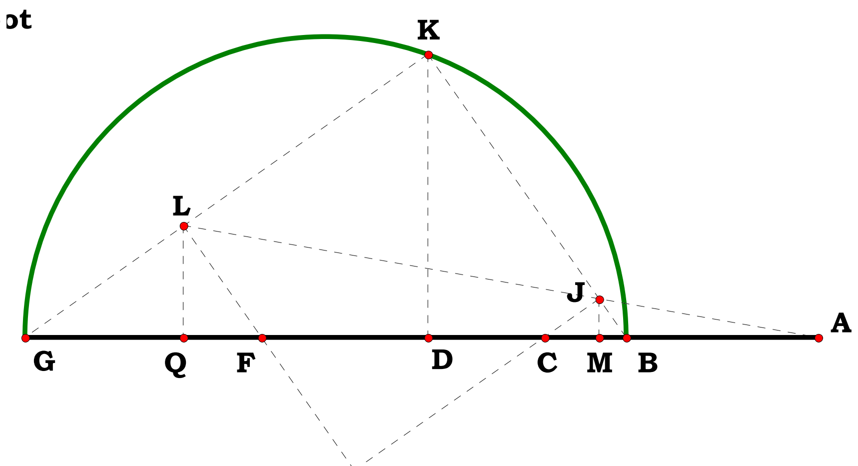
$$\frac{AG}{FQ} = 9.598866$$

$$\frac{AG}{BM} = 674.506167$$

On the left is the first and last of the series, on the right is the entire series.

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}} =$$

9.598866
17.621531
32.349484
59.386957
109.02216
200.142121
367.419508
674.506167



122096

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 2 \quad AL := N_1$

$N_2 := .2$

$BL := AL - AB \qquad BS := BL \qquad LT := BL$

$BH := \dfrac{BL}{2} \qquad HL := BH \qquad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \qquad FL := AL - AF \qquad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \qquad FN := \dfrac{BQ \cdot FP}{BS} \qquad EF := \dfrac{BF \cdot FN}{BQ}$

$EL := EF + FL \qquad FG := \dfrac{EF \cdot FL}{EL} \qquad GO := \dfrac{FN \cdot FG}{EF}$

$GL := FL - FG \qquad LR := BQ \qquad JL := \dfrac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL \qquad AJ = 1.681793$

Definitions.

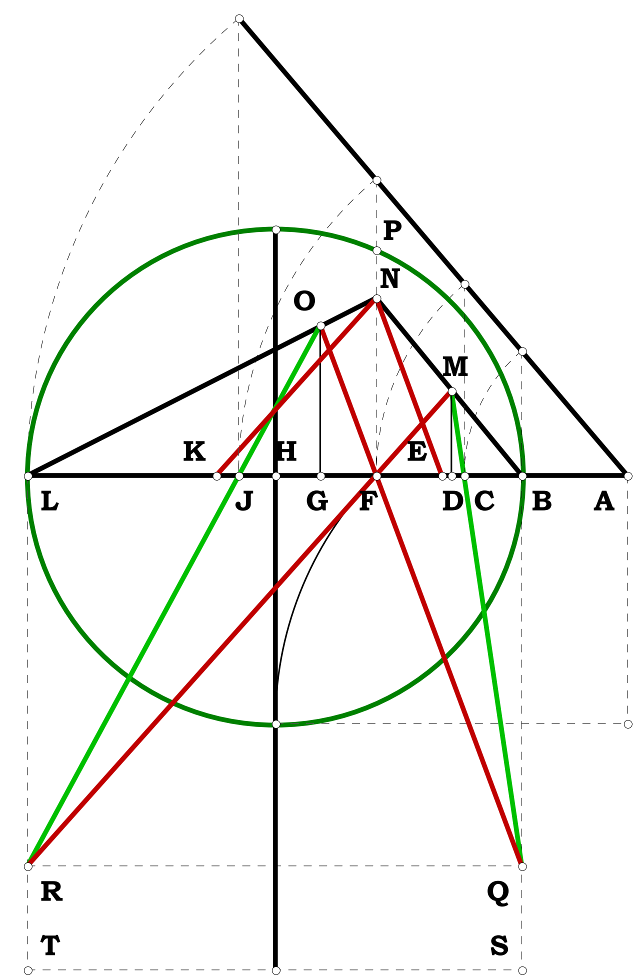
$$\left(AL^3\right)^{\frac{1}{4}} - AJ = 0$$

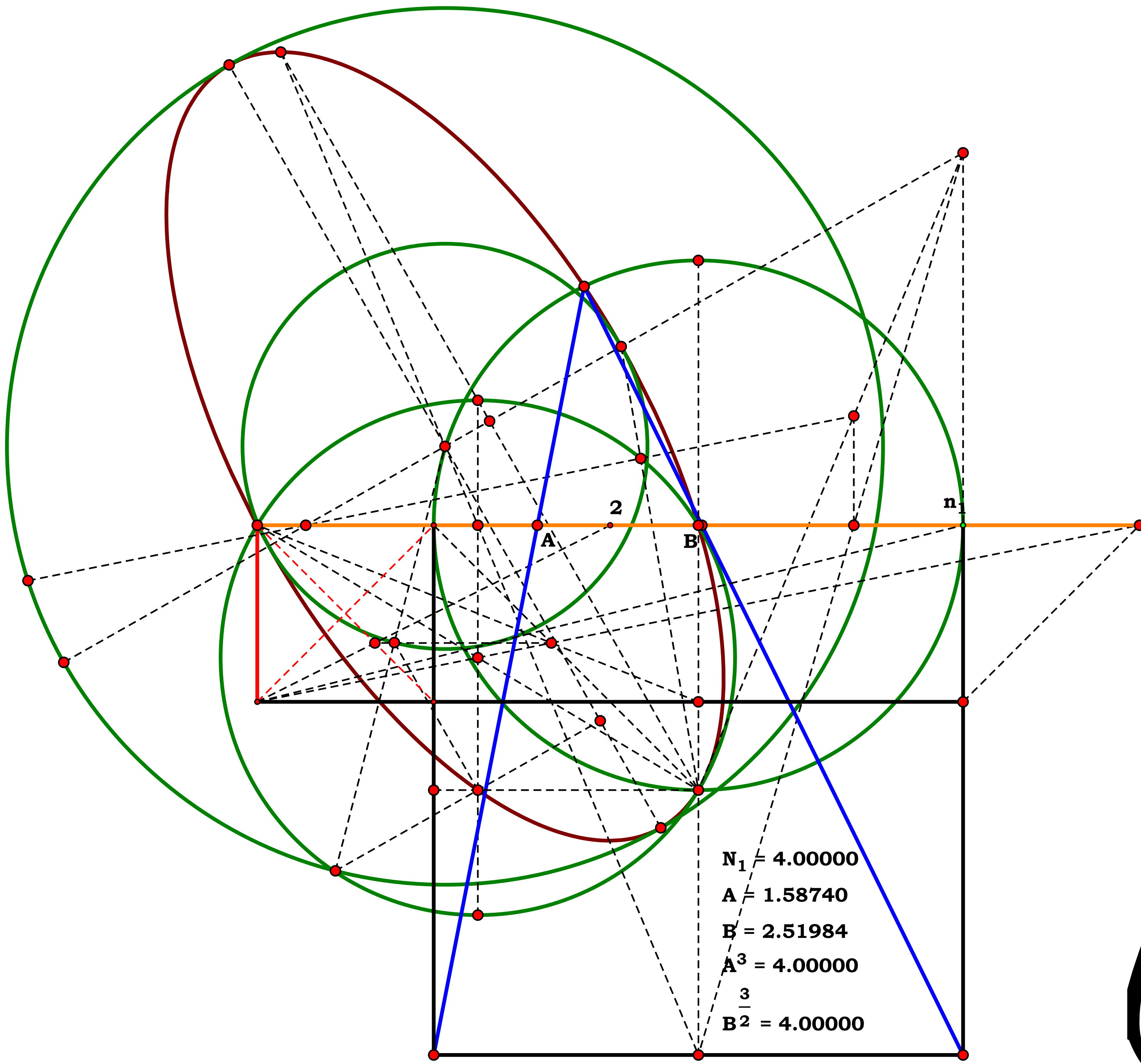
$$AJ - \left(N_1^{\frac{1}{4}}\right)^3 = 0$$

$$\left(N_1^3\right)^{\frac{1}{4}} - \left(N_1^{\frac{1}{4}}\right)^3 = 0$$

Alternate Method Quad Roots

If FN:FP as BQ:BS then quad roots series can be divided off in the figure.





The Delian Quest 1997

John Clark

040397

Descriptions.

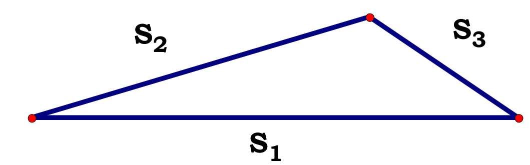
Unit.

Given.

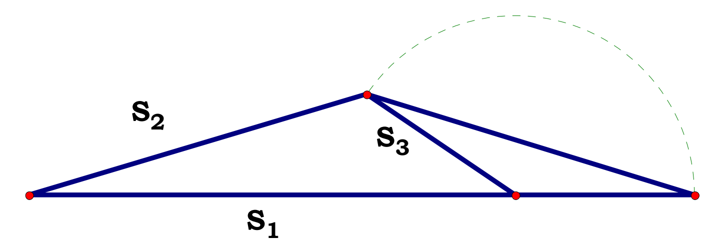
$n := 1 .. 3$

$$S_1 := \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad S_2 := \begin{pmatrix} b \\ c \\ a \end{pmatrix} \quad S_3 := \begin{pmatrix} c \\ a \\ b \end{pmatrix}$$

Not changing the height of a given triangle, or the length of the subtended side, what happens to it's area if we halve the angle of one side?



$$\text{Is\_This\_a\_Triangle} := \left({S_1}_1 + {S_2}_1 > {S_3}_1\right)\cdot\left({S_1}_1 + {S_3}_1 > {S_2}_1\right)\cdot\left({S_2}_1 + {S_3}_1 > {S_1}_1\right)$$



As was learned in school, the area of a triagle is given by $\frac{1}{2}\cdot B\cdot H.$

From 04_02_97.MCD I show that, for a given side, the height is given by;

$$H_n := \frac{\sqrt{{S_1}_n + {S_2}_n + {S_3}_n}\cdot\sqrt{-{S_1}_n + {S_2}_n + {S_3}_n}\cdot\sqrt{{S_1}_n - {S_2}_n + {S_3}_n}\cdot\sqrt{{S_1}_n + {S_2}_n - {S_3}_n}}{2\cdot {S_1}_n}$$

And since $B := S_1$ Area is defined as

$$A_n := \frac{\sqrt{{S_1}_n + {S_2}_n + {S_3}_n}\cdot\sqrt{-{S_1}_n + {S_2}_n + {S_3}_n}\cdot\sqrt{{S_1}_n - {S_2}_n + {S_3}_n}\cdot\sqrt{{S_1}_n + {S_2}_n - {S_3}_n}}{4}$$

$\frac{1\cdot B_n\cdot H_n}{2} - A_n =$

0
0
0

$A_n =$

2.904738
2.904738
2.904738

$H_n =$

2.904738
1.936492
1.452369

What is the definition of acute, solely in terms of the sides of a triangle? Basically from this it can be argued that Euclid's definition of acute or obtuse was out of order.

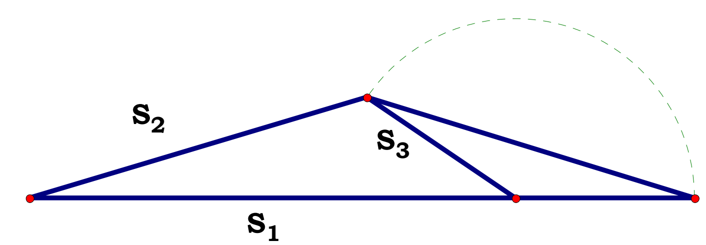
$$\mathbf{Acute_n} := \mathbf{if}\left[\sqrt{\left(\mathbf{S_{1_n}}\right)^2 + \left(\mathbf{S_{2_n}}\right)^2} > \mathbf{S_{3_n}}, \mathbf{1}, \mathbf{0}\right] \qquad \mathbf{Acute2_n} := \mathbf{if}\left[\sqrt{\left(\mathbf{S_{1_n}}\right)^2 + \left(\mathbf{S_{3_n}}\right)^2} > \mathbf{S_{2_n}}, \mathbf{1}, \mathbf{0}\right]$$

$\mathbf{Acute_n} =$

0
1
1

$\mathbf{Acute2_n} =$

1
0
1



$\mathbf{H_n} =$

2.904738
1.936492
1.452369

$\mathbf{S_{1_n}} =$

2
3
4

$\mathbf{S_{2_n}} =$

3
4
2

$\mathbf{S_{3_n}} =$

4
2
3

if we halve $\angle\, \mathbf{S_1 S_2}$

$$\frac{\left(\mathbf{S_{1_n}} + \mathbf{S_{2_n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n} =$$

4.357106
3.872983
1.452369

if we halve $\angle \mathbf{S_1 S_3}$

$$\frac{\left(\mathbf{S_{1_n}} + \mathbf{S_{3_n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n} =$$

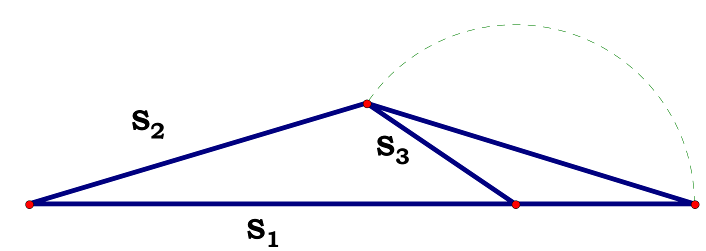
5.809475
1.936492
2.178553

Is_This_a_Triangle = 1

Since the greater angle is subtended by the greater side, halving the lesser angle increases the area of the triangle by the greater amount.

$$\left(\mathbf{S_{2_n}} > \mathbf{S_{3_n}}\right) - \left[\frac{\left(\mathbf{B_n} + \mathbf{S_{2_n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n} > \frac{\left(\mathbf{B_n} + \mathbf{S_{3_n}}\right)\cdot \mathbf{H_n}}{\mathbf{2}} - \mathbf{A_n}\right] =$$

0
0
0



Given two sides of a triangle, the height and if the angle contained by the two sides is acute or not, find the remaining side. What would happen if you were given just the equation and had no idea what the equation represented? You could not possibly solve it so quickly.

$$H_n = \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2 \cdot S_{1_n}}$$

Given S_1, S_2 and $\sqrt{S_1^{\,2} + S_2^{\,2}} > S_3$, find S_3.

$Acute_n =$

0
1
1

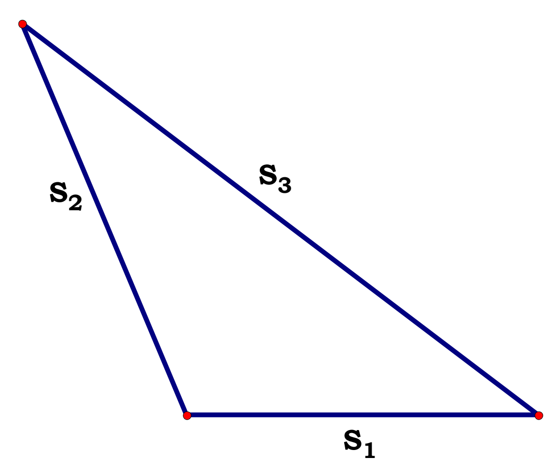
$Is\_This\_a\_Triangle = 1$

$$S_{4_n} := \sqrt{\left(S_{2_n}\right)^2 - \left(H_n\right)^2}$$

$$S_{X_n} := \text{if}\left(Acute_n, S_{1_n} - S_{4_n}, S_{1_n} + S_{4_n}\right)$$

$a \equiv 2 \quad b \equiv 3 \quad c \equiv 4$ ← Plug your values in here.

$$S_{3_n} := \sqrt{\left(H_n\right)^2 + \left(S_{X_n}\right)^2}$$



$S_{1_n} =$

2
3
4

$S_{2_n} =$

3
4
2

$S_{3_n} =$

4
2
3

Unit.

Given.

$N_1 := 5 \quad AB := N_1$
$N_2 := 4 \quad AC := N_2$
$N_3 := 3 \quad CD := N_3$

040497

Descriptions.

Triangles

Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.

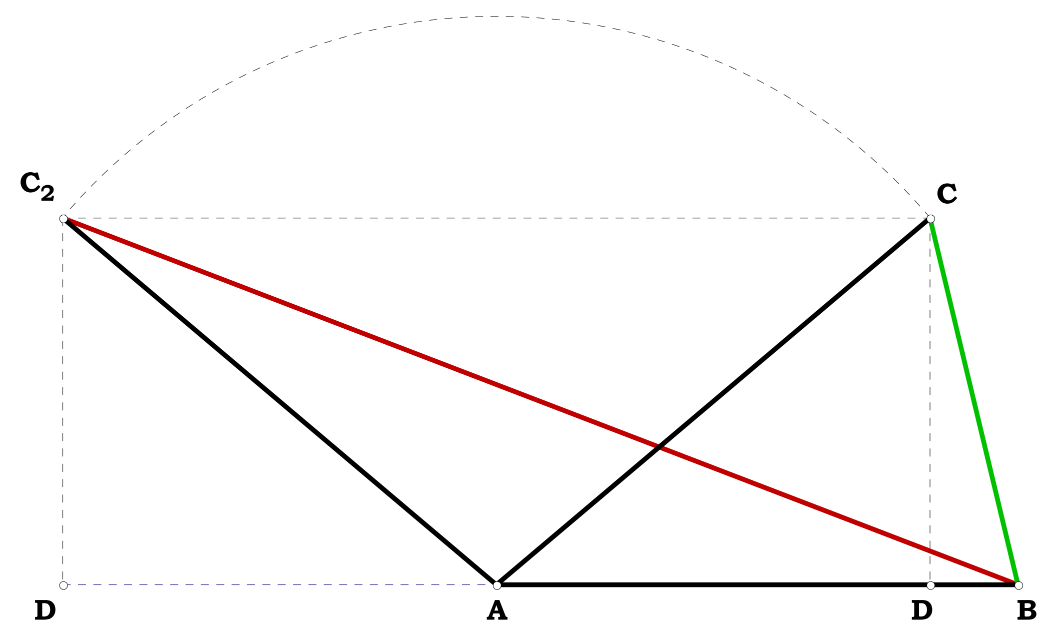
$$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$$

$$BC_1 := \sqrt{CD^2 + BD_1{}^2} \qquad BC_2 := \sqrt{CD^2 + BD_2{}^2}$$

$$BC_1 = 8.213252 \qquad BC_2 = 3.813461$$

$$BC_1 - \sqrt{N_1{}^2 + N_2{}^2 + 2 \cdot N_1 \cdot \sqrt{N_2{}^2 - N_3{}^2}} = 0$$

$$BC_2 - \sqrt{\left(N_1{}^2 + N_2{}^2 - 2 \cdot N_1 \cdot \sqrt{N_2{}^2 - N_3{}^2}\right)} = 0$$



AB = 2.71667 in. $\quad N_1$ = 2.71667 in.
AC = 2.95665 in. $\quad N_2$ = 2.95665 in.
CD = 1.90833 in. $\quad N_3$ = 1.90833 in.
BC = 1.96260 in. $\quad N_1^2+N_2^2$ = 16.12208 in²
BC_2 = 5.32845 in. $\quad 2\cdot N_1\cdot\sqrt{N_2^2-N_3^2}$ = 12.27028 in²

$BC-\sqrt{(N_1^2+N_2^2)-\left(2\cdot N_1\cdot\sqrt{N_2^2-N_3^2}\right)}$ = 0.00000 in.
$BC_2-\sqrt{(N_1^2+N_2^2)+\left(2\cdot N_1\cdot\sqrt{N_2^2-N_3^2}\right)}$ = 0.00000 in.

Definitions.

$$S_1 := AB \qquad S_2 := AC \qquad S_3 := BC_1$$

$$\frac{\sqrt{S_1+S_2+S_3}\cdot\sqrt{-S_1+S_2+S_3}\cdot\sqrt{S_1-S_2+S_3}\cdot\sqrt{S_1+S_2-S_3}}{2\cdot S_1} - CD = 0$$

$$S_1 := AB \qquad S_2 := AC \qquad S_3 := BC_2$$

$$\frac{\sqrt{S_1+S_2+S_3}\cdot\sqrt{-S_1+S_2+S_3}\cdot\sqrt{S_1-S_2+S_3}\cdot\sqrt{S_1+S_2-S_3}}{2\cdot S_1} - CD = 0$$

Unit.

Given.

$N_1 := 3.14854$ $\quad AD := N_1$

$N_2 := 6.50875$ $\quad AC := N_2$

042897

Descriptions.

$CD := \sqrt{AD^2 + AC^2} \qquad DH := CD$

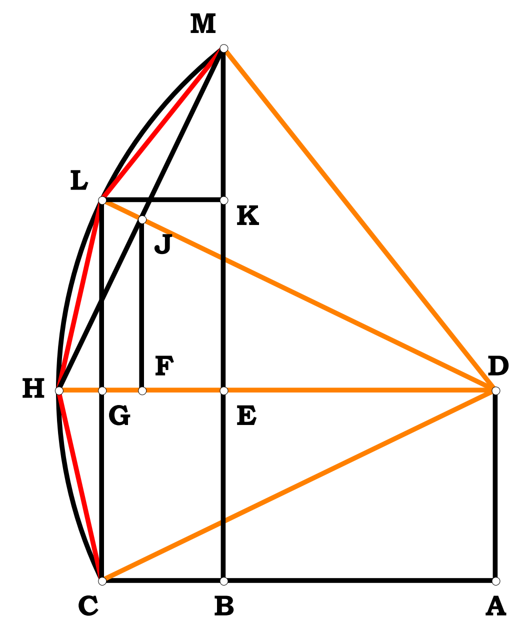
$CG := AD \qquad DG := AC \qquad GH := DH - DG \qquad CH := \sqrt{GH^2 + CG^2}$

$HJ := CG \qquad DJ := DG$

$FH := \dfrac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH} \qquad EF := FH \qquad DE := DH - (EF + FH)$

$AB := DE \qquad EG := DG - DE \qquad LM := CH \qquad LK := EG$

$KM := \sqrt{LM^2 - LK^2} \qquad BE := AD \qquad BK := 2 \cdot BE \qquad BM := BK + KM$



AD = 2.51883 cm $\quad N_1 = 2.51883$ cm

AC = 5.20700 cm $\quad N_2 = 5.20700$ cm

CH = 2.58413 cm $\quad \sqrt{(2 \cdot N_1^2 + 2 \cdot N_2^2) - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} = 2.58413$ cm

$CH - \sqrt{(2 \cdot N_1^2 + 2 \cdot N_2^2) - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} = 0.00000$ cm

$m\angle DCA = 25.81490°$

$m\angle CDM = 77.44471°$

$\dfrac{m\angle CDM}{m\angle DCA} = 3.00000$

Some definitions:

$\sqrt{N_1^2 + N_2^2} - CD = 0 \qquad \sqrt{N_1^2 + N_2^2} - N_2 - GH = 0 \qquad \sqrt{2 \cdot N_1^2 + 2 \cdot N_2^2 - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} - CH = 0$

$\dfrac{N_1^2}{\sqrt{N_1^2 + N_2^2}} - FH = 0 \qquad \dfrac{\left(N_2^2 - N_1^2\right)}{\sqrt{N_1^2 + N_2^2}} - DE = 0$

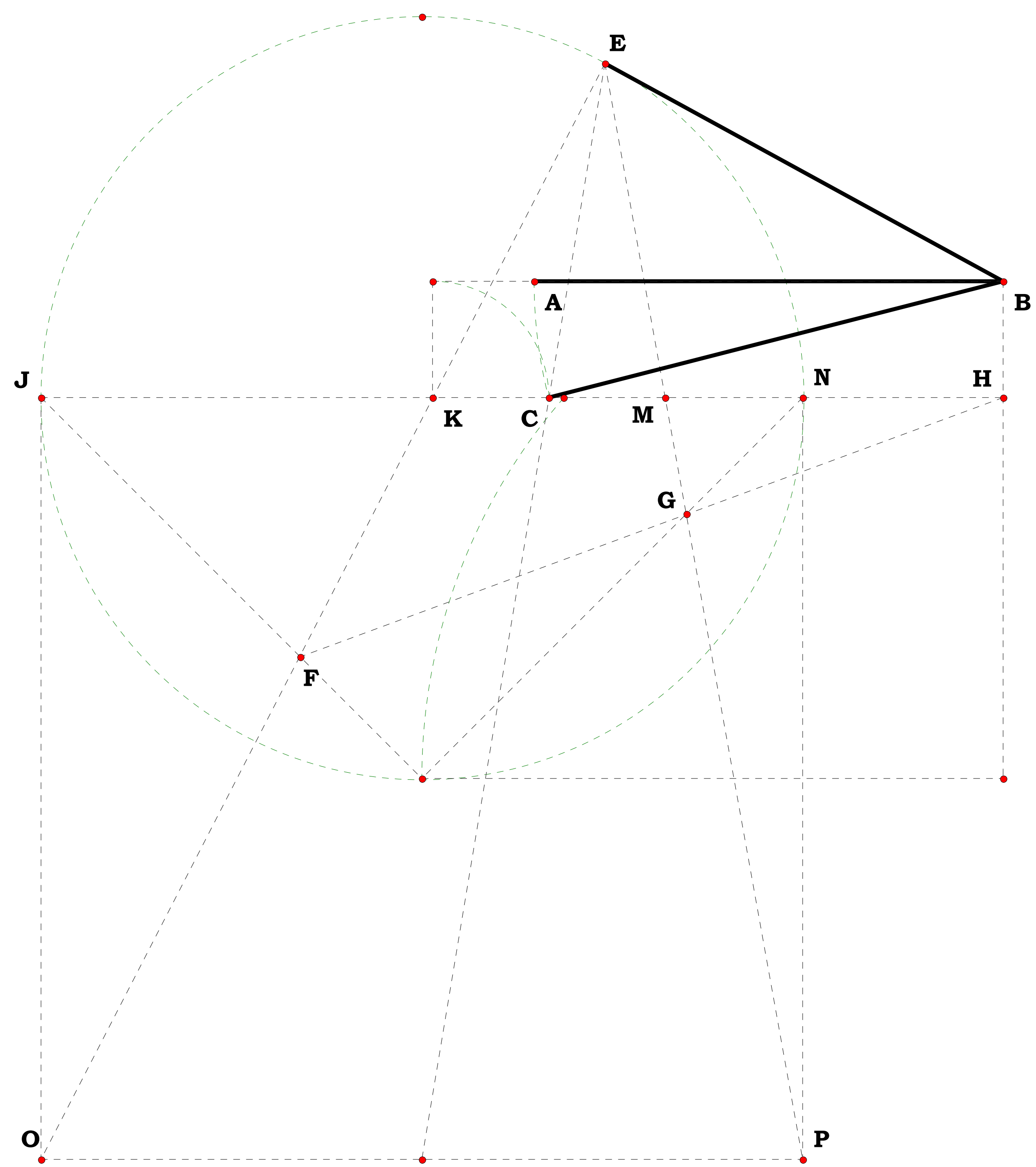
$$2 \cdot N_1 + \frac{\sqrt{\left(2 \cdot N_2^3 - 2 \cdot N_1^2 \cdot N_2\right) \cdot \sqrt{N_1^2 + N_2^2} + N_1^4 + 5 \cdot N_1^2 \cdot N_2^2 - 2 \cdot N_2 \cdot \left(\sqrt{N_1^2 + N_2^2}\right)^3}}{\sqrt{N_1^2 + N_2^2}} - BM = 0$$

042997

Trisection and the Cube Roots

Exploring trisection produced in the cube root figure. If one places E where they will, they would find the origin of the root series H by projecting through FG. Taking half of KM for BH we find that angle ABC is 1/3 of angle EBC. So, we can not only produce a cube root series from OP but also a trisection as well from the orign which is the same origin for the square root of the figure.

I should write up some plates concerning the point G or F and the different relationships they form with the finished plate.



$m\angle ABC = 14.33586°$

$m\angle EBA = 28.67173°$

$\frac{m\angle EBA}{m\angle ABC} = 2.00000$

$m\angle EBC = 43.00759°$

$\frac{m\angle EBC}{m\angle ABC} = 3.00000$

Unit.

$AB := 1$

Given.

$N := 5$

If trisection can be placed at RUE, then PV is proportional to RW.

Descriptions.

$AH := N \qquad BH := AH - AB \qquad BJ := BH$

$$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$$

$$CF := AF - AC \qquad CE := \frac{CF}{2} \qquad AE := AC + CE$$

$AU := CE \qquad NV := AU \qquad MW := AU$

(For the next two equations see 042897.)

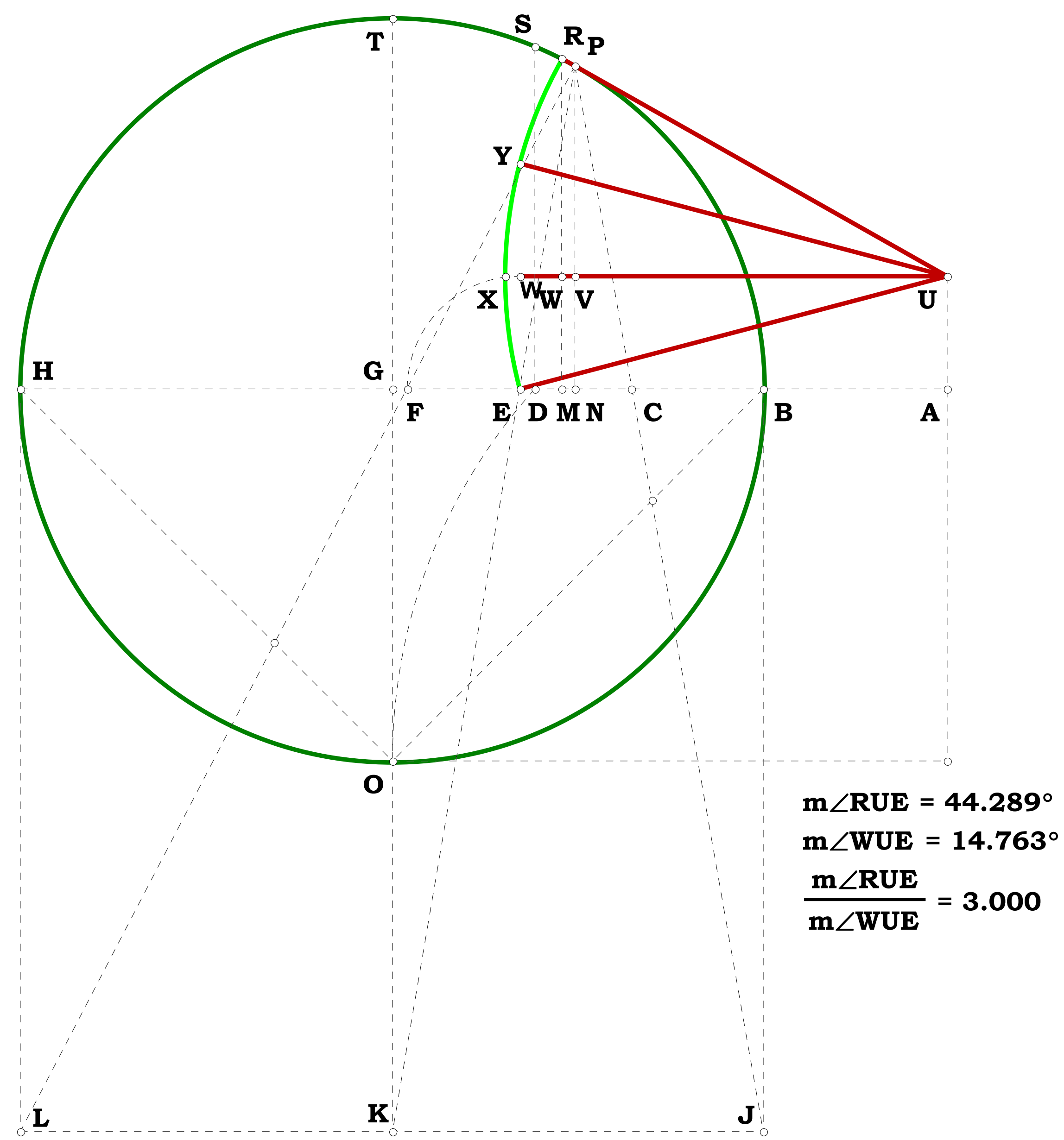
$$AM := \frac{\left(\frac{AE}{AU} - 1\right)\cdot\left(\frac{AE}{AU} + 1\right)\cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{\left(AU^2\right)^{\frac{3}{2}} + 5 \cdot AE^2 \cdot \sqrt{AU^2} - 4 \cdot AE \cdot AU \cdot \sqrt{AE^2 + AU^2}}{\left(AE^2 + AU^2\right) \cdot \sqrt{AU^2}}}$$

$$RW := MR - MW \qquad BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH}$$

$$NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV \qquad AN := AC + CN \qquad UV := AN$$

$$UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$$

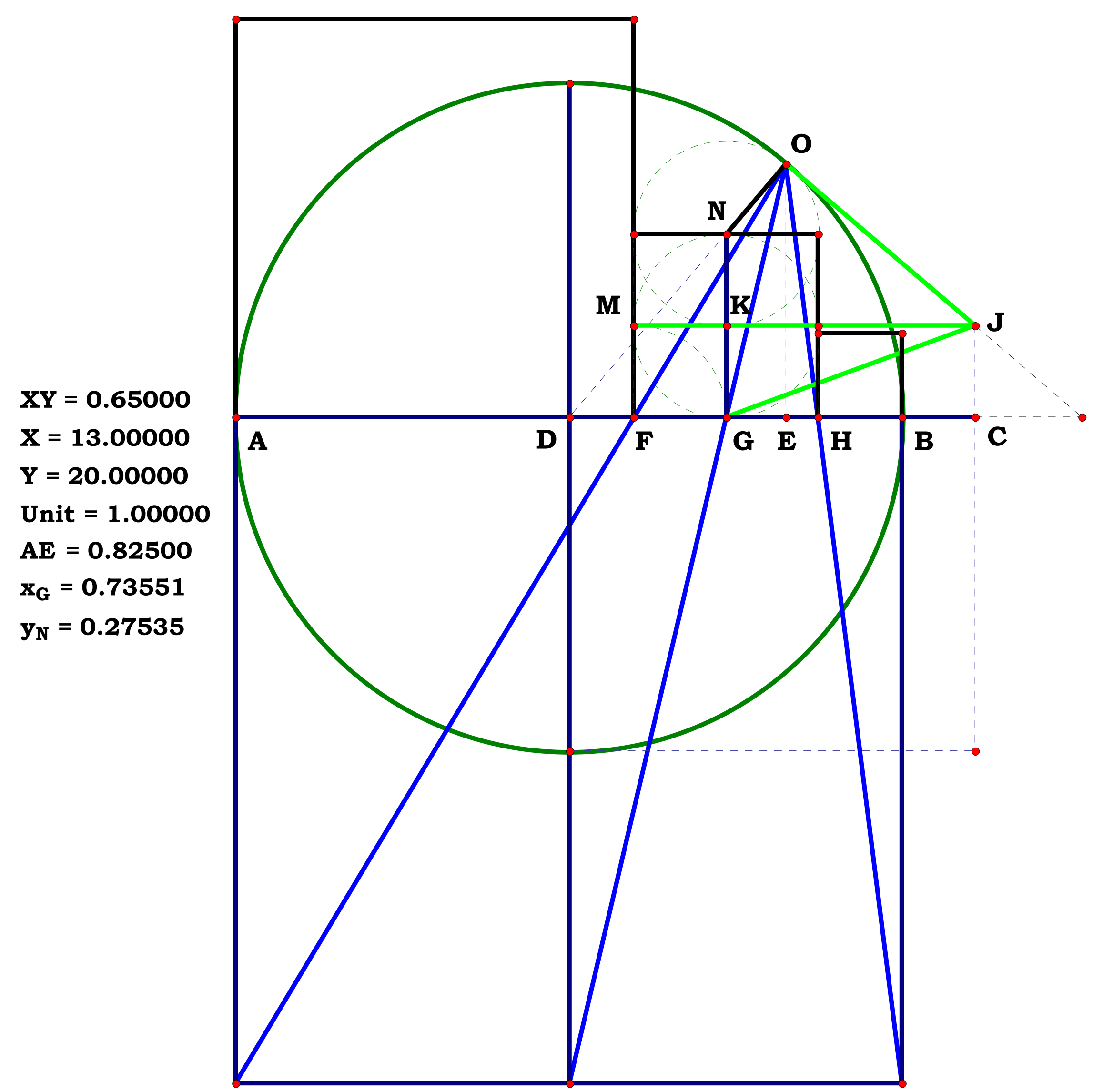


081921 Study

Unit. $AB := 1$

Given. $X := 12$

$Y := 20$



Descriptions.

$AD := \frac{AB}{2}$ $\quad DE := \frac{X}{2 \cdot Y}$ $\quad AE := DE + AD$ $\quad AE = 0.8$

$BE := AB - AE$ $\quad EO := \sqrt{AE \cdot BE}$ $\quad DG := \frac{DE \cdot AB}{AB + EO}$

$GN := \frac{EO \cdot DG}{DE}$ $\quad FH := GN$ $\quad GF := \frac{GN}{2}$ $\quad GH := \frac{GN}{2}$

$DO := \frac{1}{2}$ $\quad DP := \frac{DO^2}{DE}$ $\quad BP := DP - AD$ $\quad PE := BE + BP$

$CP := \frac{GN \cdot PE}{2 \cdot EO}$ $\quad BC := BP - CP$ $\quad BG := AD - DG$ $\quad CH := BG + BC - GH$

$CF := BG + BC + GH$ $\quad AC := AB + BC$ $\quad \frac{AC}{BC} = 8$ $\quad \frac{CH}{BC} = 2$ $\quad \frac{CF}{BC} = 4$

$CH - \left(BC^2 \cdot AC\right)^{\frac{1}{3}} = 0$ $\quad CF - \left(BC \cdot AC^2\right)^{\frac{1}{3}} = 0$

Definitions.

$AD - \frac{1}{2} = 0$ $\quad DE - \frac{X}{2 \cdot Y} = 0$ $\quad AE - \frac{X + Y}{2 \cdot Y} = 0$ $\quad BE - \frac{Y - X}{2 \cdot Y} = 0$ $\quad EO - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot Y} = 0$ $\quad DG - \frac{X}{2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}} = 0$ $\quad GN - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot Y + \sqrt{(Y - X) \cdot (X + Y)}} = 0$

$FH - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}} = 0$ $\quad GF - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot \left[2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}\right]} = 0$ $\quad GH - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot \left[2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}\right]} = 0$ $\quad DO - \frac{1}{2} = 0$ $\quad DP - \frac{Y}{2 \cdot X} = 0$ $\quad BP - \frac{Y - X}{2 \cdot X} = 0$

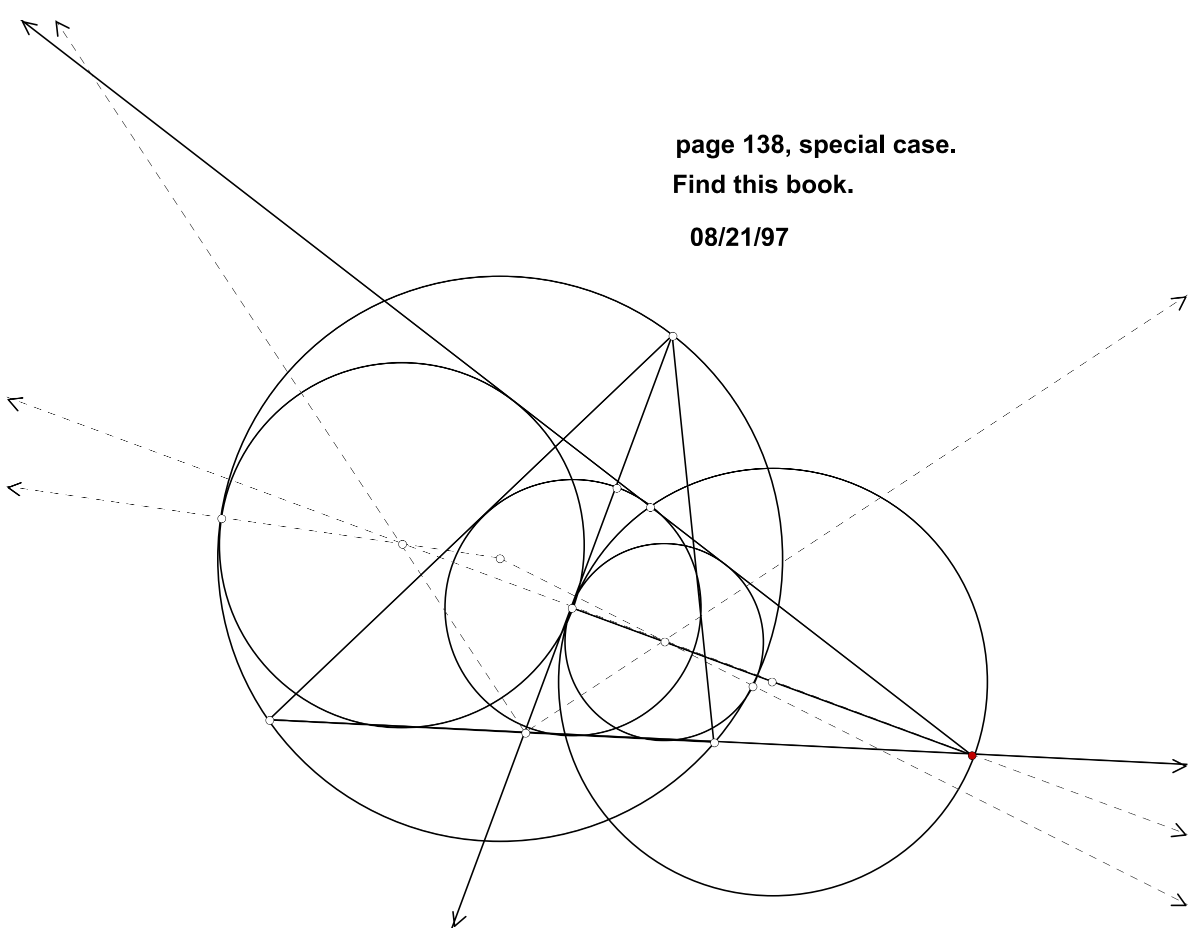
$PE - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $\quad CP - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot \left[2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}\right]} = 0$ $\quad BC - \frac{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $\quad BG - \frac{2 \cdot Y - 2 \cdot X + \sqrt{Y^2 - X^2}}{2 \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $\quad CH - \frac{(Y - X) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$

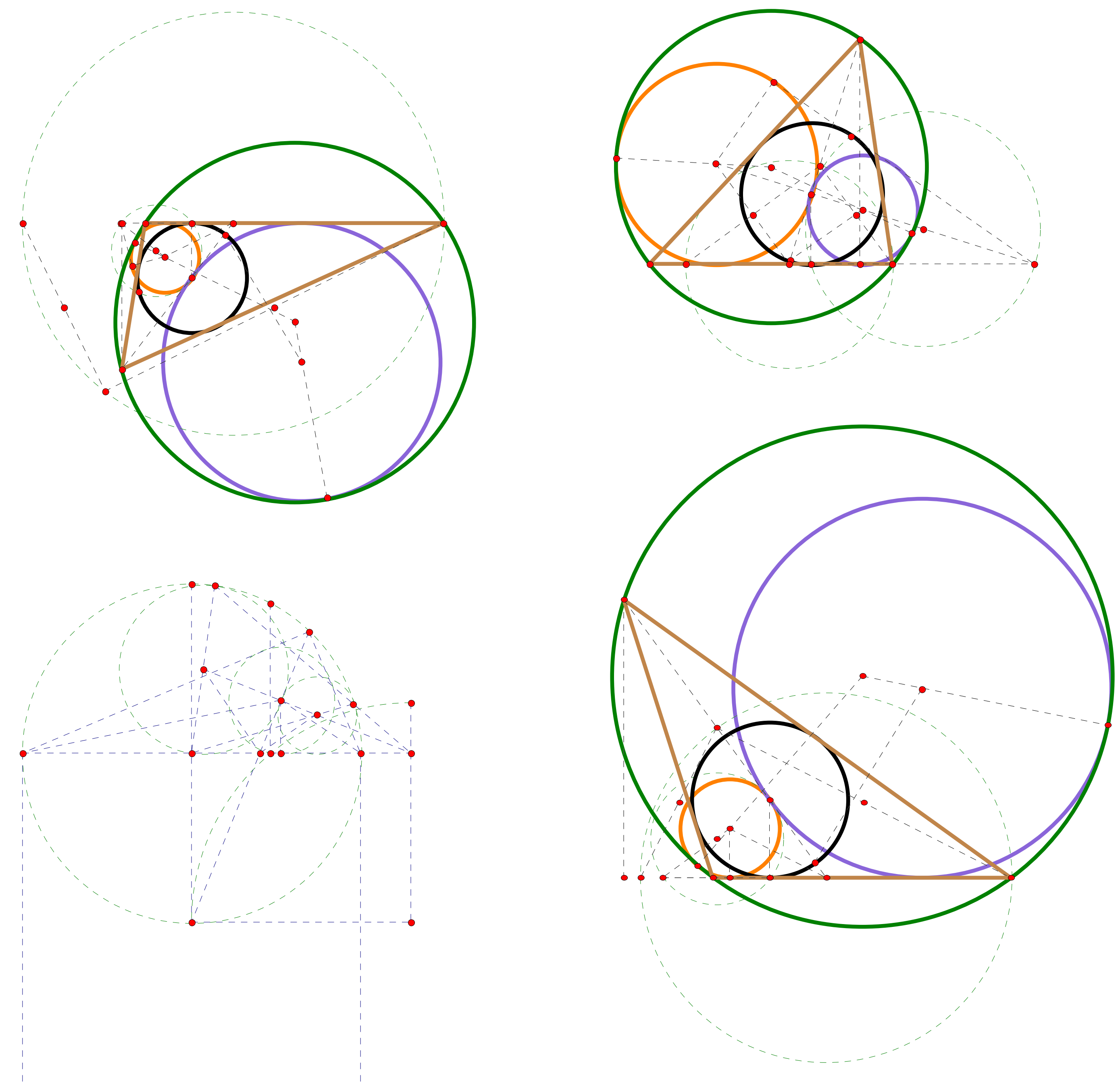
$CF - \frac{(X + Y) \cdot \left(Y - X + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $\quad AC - \frac{(X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $\quad \frac{(X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)} = 8$ $\quad \frac{X + Y + \sqrt{Y^2 - X^2}}{Y - X + \sqrt{Y^2 - X^2}} = 2$ $\quad \frac{X + Y}{Y - X} = 4$

Eight Circles and a Forgotten Book.

082197

In my files I have the following plate, from what book I do not recall, however, the figure as it was written up there was listed as a "special case," of what I do not recall either. But I want to write it up because it is not a special case of anything, it is actually a plate showing that one can treat every triangle as an eight circle problem, simply add the remaining two sides by recursion of the first. I suspect now that if someone thought this was a special case of something, then they did not comprehend the actual relationships, they are easily found by compass.
The project would start with the equations from 062793 and 040694. So, this is a project I am interested in doing and have been for a long time. I might even find the book it came from. Might be interesting to find the equations for all eight circles. Maybe some day.

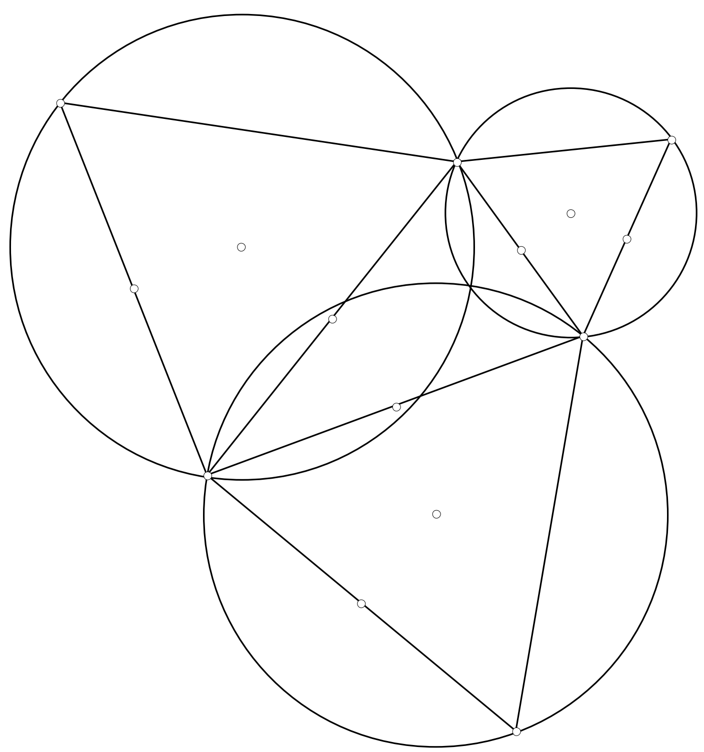




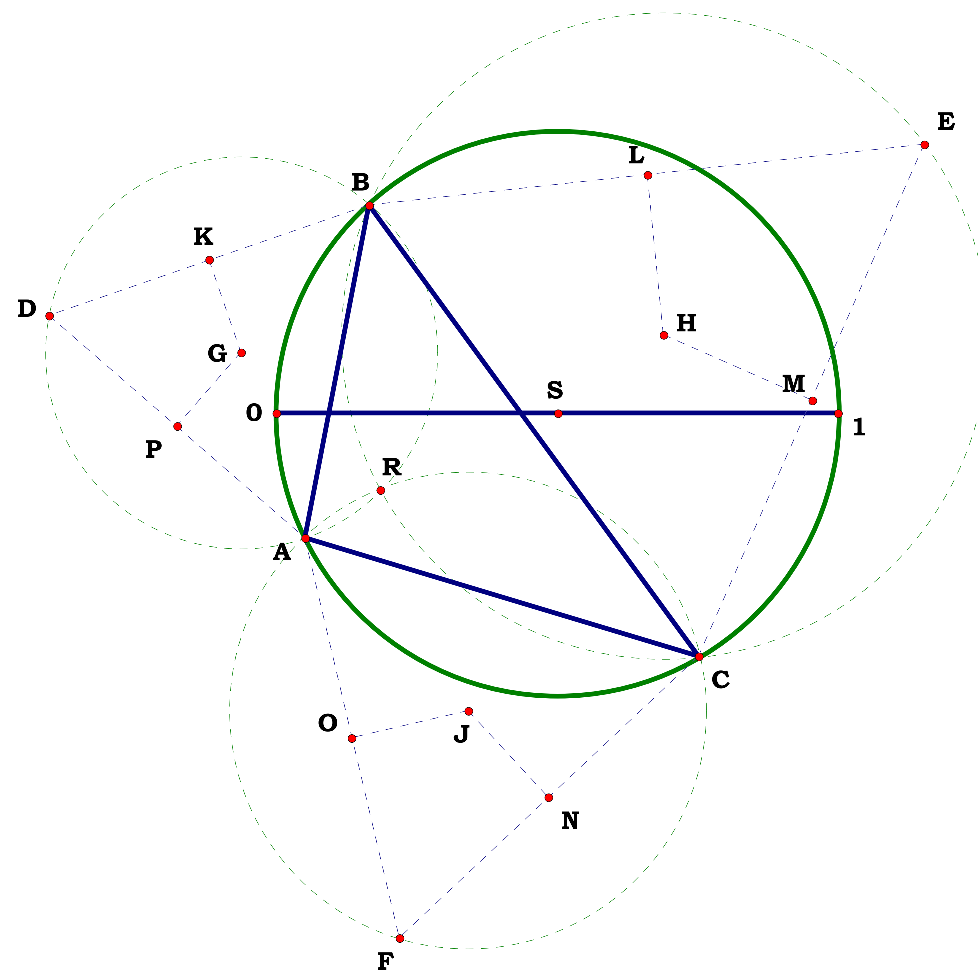
Unit.
Given.

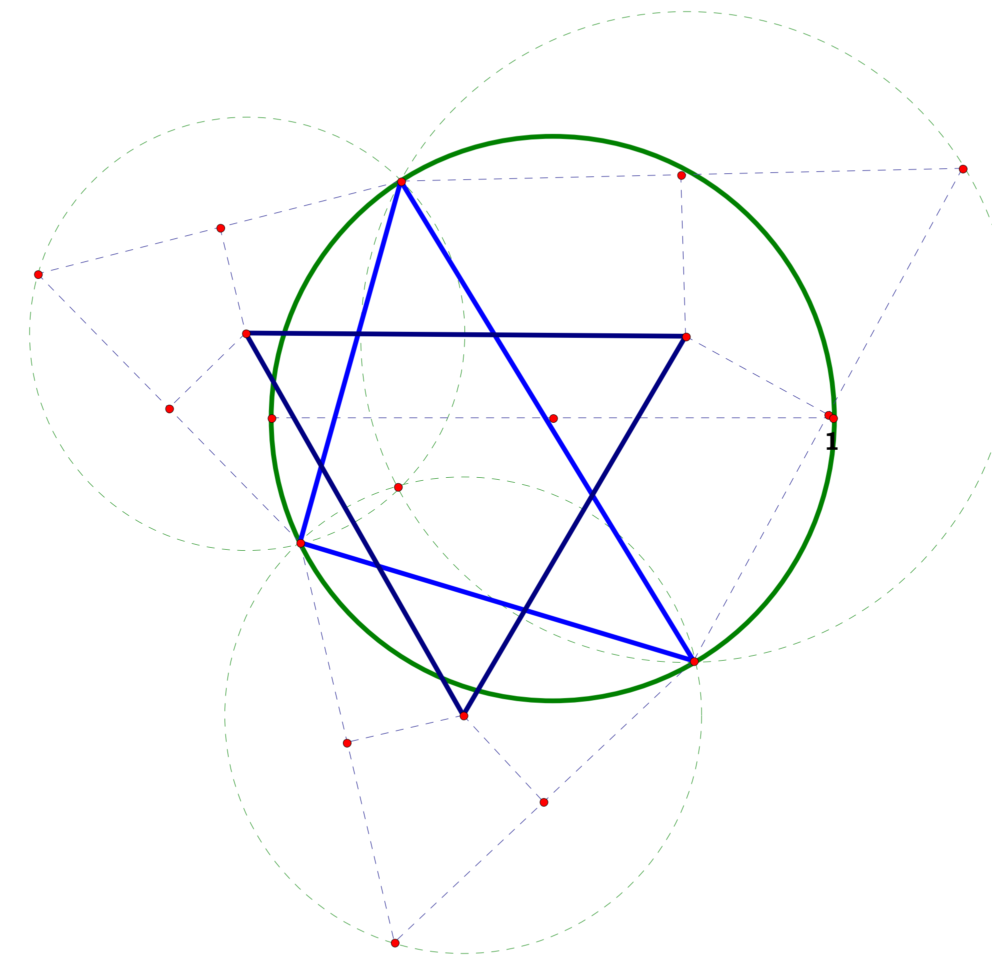
Steiner Point

082297
Descriptions.
Definitions.



Steiner point solution (a) page 361 Computing in
Euclidean Geometry Du and Hwang 1985





091197A

Unit.

Given.

$N_1 := 2.66666 \quad AB := N_1$

$N_2 := 1.31473 \quad EF := N_2$

$N_3 := 1.26711$

The Ellipse

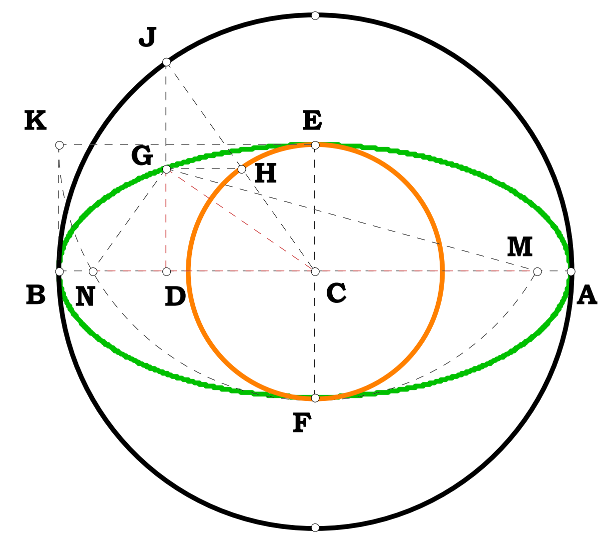
Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.

Descriptions.

$$AD := \frac{N_1}{N_3} \quad BD := AB - AD \quad DJ := \sqrt{AD \cdot BD} \quad AC := \frac{AB}{2}$$

$$DC := \frac{(AC - AD)^2}{\sqrt{(AC-AD)^2}} \quad CH := \frac{EF}{2} \quad CJ := AC$$

$$DG := \frac{DJ \cdot CH}{CJ} \quad CG := \sqrt{DG^2 + DC^2} \quad MN := 2 \cdot \sqrt{\left(\frac{AB}{2}\right)^2 - CH^2}$$



AB = 2.66667 in.

EF = 1.31473 in.

AD = 2.10453 in.

$\frac{AB}{AD}$ = 1.26711

CH = 0.65736 in.

DG = 0.53625 in.

MN = 2.32004 in.

$MN - 2 \cdot \sqrt{\frac{AB}{2}^2 - CH^2}$ = 0.00000 in.

$MN - \sqrt{(AB-EF) \cdot (AB+EF)}$ = 0.00000 in.

Definitions.

$$BD - \frac{N_1 \cdot (N_3 - 1)}{N_3} = 0 \quad DJ - \frac{N_1 \cdot \sqrt{(N_3 - 1)}}{N_3} = 0 \quad AC - \frac{N_1}{2} = 0$$

$$DC - \frac{N_1 \cdot \sqrt{(N_3 - 2)^2}}{2 \cdot N_3} = 0 \quad CH - \frac{N_2}{2} = 0 \quad CJ - \frac{N_1}{2} = 0 \quad DG - \frac{N_2 \cdot \sqrt{N_3 - 1}}{N_3} = 0$$

$$CG - \frac{\sqrt{\left(N_3^{\,2} - 4 \cdot N_3 + 4\right) \cdot N_1^{\,2} + 4 \cdot N_2^{\,2} \cdot (N_3 - 1)}}{2 \cdot N_3} = 0 \quad MN - \sqrt{(N_1 - N_2) \cdot (N_1 + N_2)} = 0$$

Unit.

Given.

$S_1 := 8.14917 \quad AB := S_1$

$S_2 := 7.23745 \quad AC := S_2$

$S_3 := 2.58277 \quad BC := S_3$

091197B

Descriptions.

$$DE := AC + BC \quad AH := \frac{DE}{2} \quad AG := \frac{AB}{2} \quad FG := \sqrt{AH^2 - AG^2}$$

Definitions.

$$FG - \frac{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2} = 0$$

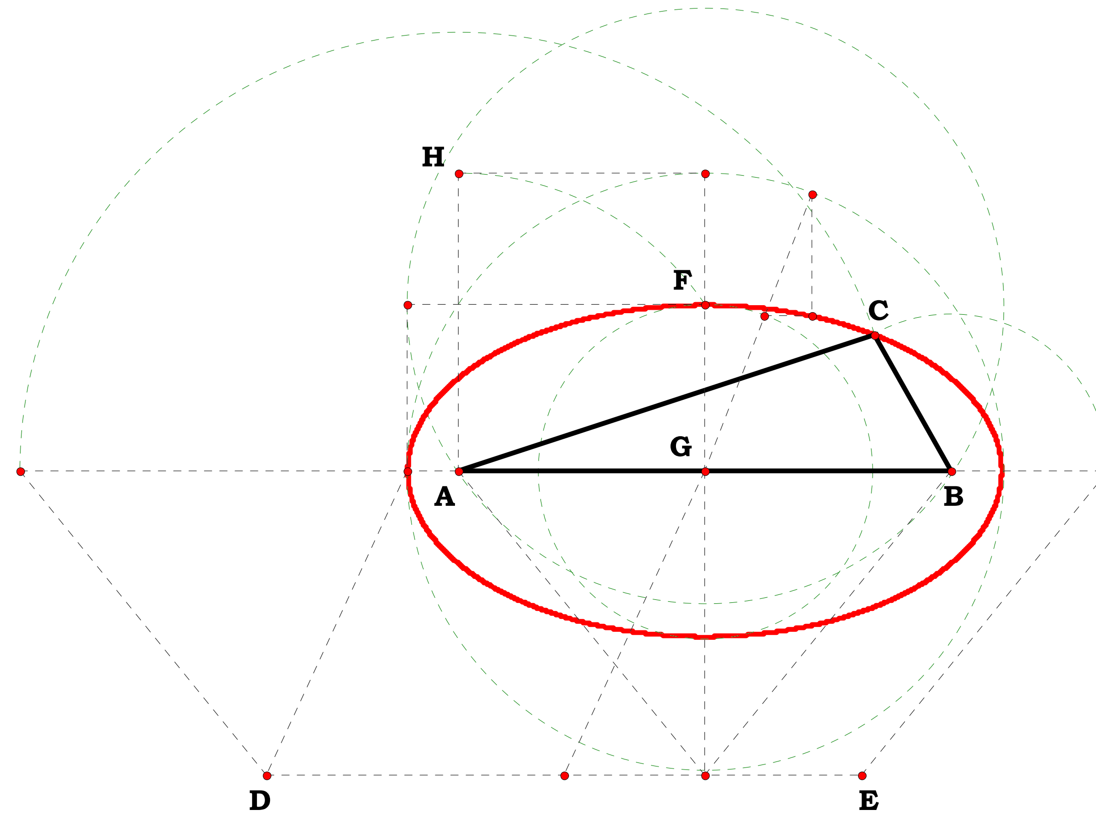
The ratio of the ellipse is thus;

$$\frac{AH}{FG} - \frac{(S_2 + S_3)}{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}} = 0$$

From any point on DE, one can find everything
and not once think about x and y.

The Ellipse

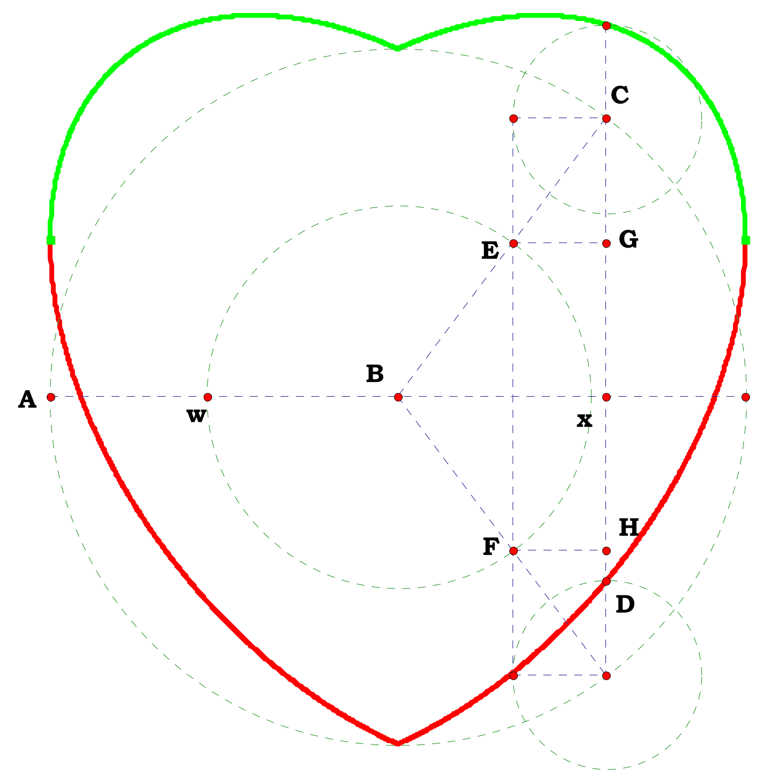
Given triangle ABC, and AB as base, describe the Ellipse



Unit.
Given.

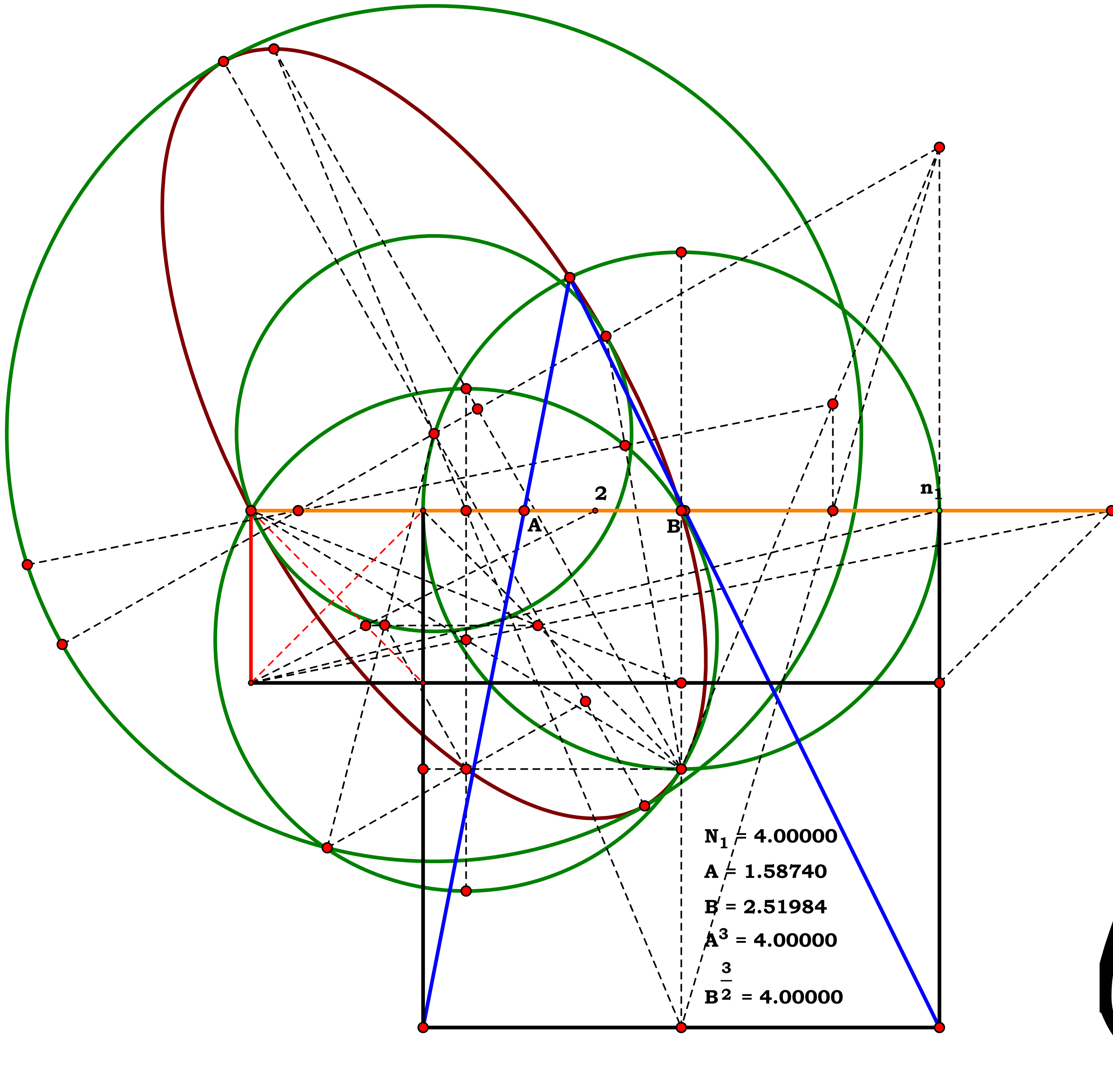
091197C
Descriptions.

The Perfect Heart



The Delian Quest 1998

John Clark



020298

Descriptions.

Unit.

Given.

$N := .656$

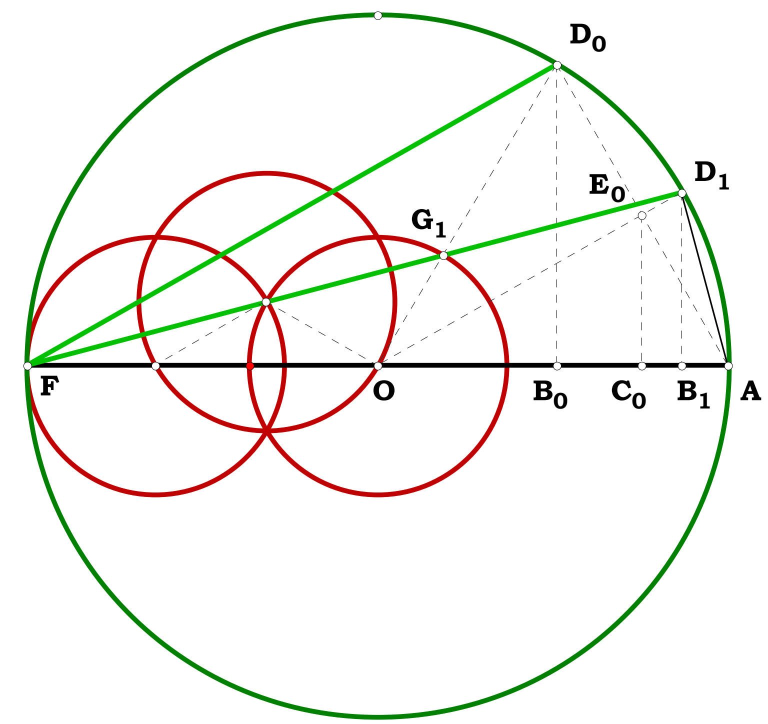
$\Delta := 4$

$\delta := 0 .. \Delta - 1$

$AF := 2.3754 \quad AO := \frac{AF}{2} \quad AB_0 := N \quad BD_0 := \sqrt{AB_0 \cdot (AF - AB_0)}$

$CE_0 := \frac{BD_0}{2} \quad AC_0 := \frac{AB_0}{2} \quad OC_0 := AO - AC_0 \quad OE_0 := \sqrt{(CE_0)^2 + (OC_0)^2}$

$$\begin{pmatrix} OB_{\delta+1} \\ AB_{\delta+1} \\ BD_{\delta+1} \\ CE_{\delta+1} \\ AC_{\delta+1} \\ OC_{\delta+1} \\ OE_{\delta+1} \end{pmatrix} := \begin{bmatrix} OC_\delta \cdot \frac{AO}{OE_\delta} \\ AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{OE_\delta} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{(2 \cdot OE_\delta)} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta + OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot \sqrt{AO \cdot \frac{(AF \cdot OE_\delta + 4 \cdot OC_\delta \cdot AO - OC_\delta \cdot AF)}{OE_\delta}} \end{bmatrix}$$



$AD_\delta := \sqrt{(AB_\delta)^2 + (BD_\delta)^2}$

Definitions.

$$AD = \begin{pmatrix} 1.248304 \\ 0.648825 \\ 0.327541 \\ 0.164163 \end{pmatrix}$$

Length of cord by progressive bisections.

I have no idea why I did this figure, it was so long ago. I don't know what I would do with a cord series unless one wanted to see how close one could get to PI starting from some particular angle or the length of some cord of a circle of some lenght at some commensurable angle. I wonder if I will ever get that bored?

021098

Descriptions.

$AB := \frac{AE}{2}$ $BJ := \frac{EG}{2}$ $BD := BJ$

$AD := AB + BD$ $CE := BD \cdot \frac{AE}{AD}$

$AC := AE - CE$ $FG := EG - CE$

Unit.

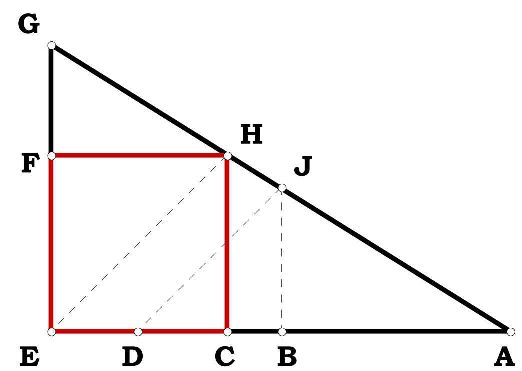
Given.

$N_1 := 2.98958$ $AE := N_1$

$N_2 := 1.86690$ $EG := N_2$

A Square In A Triangle

What is the Algebraic Name for the square as given in a right triangle? What is the Algebraic name for the ratio AE/AC?



AE = 2.39167 in.
EG = 1.49352 in.
CE = 0.91939 in.
AC = 1.47228 in.

N_1 = 2.39167 in.
N_2 = 1.49352 in.

$CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0.00000$ in. $\frac{AE}{AC} - \frac{N_1 + N_2}{N_1} = 0.00000$

Definitions.

$$AB - \frac{N_1}{2} = 0 \qquad BJ - \frac{N_2}{2} = 0 \qquad AD - \left(\frac{N_1}{2} + \frac{N_2}{2}\right) = 0$$

$$CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0 \qquad AC - \frac{N_1^{\ 2}}{N_1 + N_2} = 0$$

$$FG - \frac{N_2^{\ 2}}{N_1 + N_2} = 0 \qquad \frac{AE}{AC} - \left(\frac{N_1 + N_2}{N_1}\right) = 0 \qquad \frac{EG}{FG} - \frac{N_1 + N_2}{N_2} = 0$$

Unit.

Given.

$N_1 := 1.67500 \quad AH := N_1$

$N_2 := 1.55441 \quad HN := N_1 \cdot N_2$

Alternate Method Root Series

Given a length and a unit, raise that length to any whole power.

022598A

Descriptions.

Given for the third power.

$HJ := HN - AH \quad FH := \dfrac{AH \cdot HJ}{AH + HJ} \quad FG := FH$

$AF := AH - FH \quad DF := \dfrac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \dfrac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

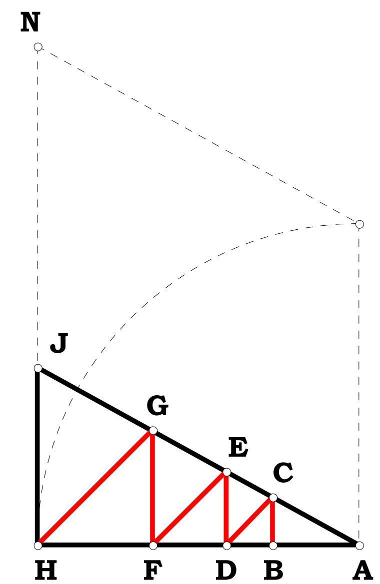
Definitions.

$\dfrac{AH}{AF} - N_2^{\,1} = 0 \quad \dfrac{AH}{AD} - N_2^{\,2} = 0 \quad \dfrac{AH}{AB} - N_2^{\,3} = 0$

$HN - N_1 \cdot N_2 = 0 \quad HJ - \left(N_1 \cdot N_2 - N_1\right) = 0 \quad FH - \dfrac{N_1 \cdot \left(N_2 - 1\right)}{N_2} = 0$

$FG - \dfrac{N_1 \cdot \left(N_2 - 1\right)}{N_2} = 0 \quad AF - \dfrac{N_1}{N_2} = 0 \quad DF - \dfrac{N_1 \cdot \left(N_2 - 1\right)}{N_2^{\,2}} = 0$

$AD - \dfrac{N_1}{N_2^{\,2}} = 0 \quad DE - \dfrac{N_1 \cdot \left(N_2 - 1\right)}{N_2^{\,2}} = 0 \quad BD - \dfrac{N_1 \cdot \left(N_2 - 1\right)}{N_2^{\,3}} = 0 \quad AB - \dfrac{N_1}{N_2^{\,3}} = 0$



$AH = 1.67500$ in. $\quad N_1 = 1.67500$ in.

$HN = 2.60364$ in.

$\dfrac{HN}{AH} = 1.55441 \quad N_2 = 1.55441$

$HJ = 0.92864$ in.

$AB = 0.44598$ in.

$AF = 1.07758$ in.

$AD = 0.69324$ in.

$\dfrac{AH}{AF} - N_2 = 0.00000 \quad \dfrac{AH}{AD} - N_2^2 = 0.00000 \quad \dfrac{AH}{AB} - N_2^3 = 0.00000$

022598B

Unit.

$AH := 1$

Given.

$N_1 := 5$ $\quad HM := N_1$

$N_2 := 6$ $\quad AN := N_2$

Sum Divided by One Powered

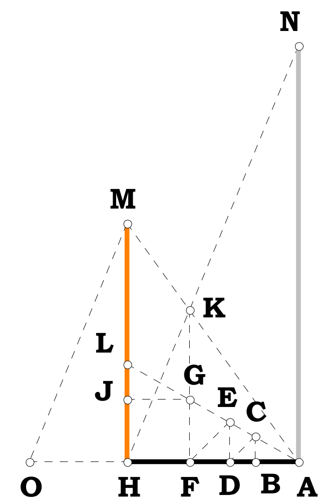
Descriptions.

$$HO := \frac{AH \cdot HM}{AN} \qquad AO := AH + HO \qquad AF := \frac{AH^2}{AO}$$

$$FH := AH - AF \qquad FG := FH \qquad DF := \frac{AF \cdot FG}{AF + FG}$$

$$AD := AF - DF \qquad DE := DF \qquad BD := \frac{AD \cdot DE}{AD + DE}$$

$$AB := AD - BD$$



AH = 0.89167 in.
HM = 1.24469 in.
AN = 2.16687 in.
AB = 0.22848 in.

$\frac{HM}{AH} = 1.39591$ $\quad N_1 = 1.39591$

$\frac{AN}{AH} = 2.43013$ $\quad N_2 = 2.43013$

$\frac{AB}{AH} = 0.25624$ $\quad AB = 0.25624$

$$AB - \frac{N_2^3}{(N_1+N_2)^3} = 0.00000$$

Definitions.

$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0 \qquad HO - \frac{N_1}{N_2} = 0 \qquad AO - \frac{N_1 + N_2}{N_2} = 0$$

$$AF - \frac{N_2}{N_1 + N_2} = 0 \qquad FH - \frac{N_1}{N_1 + N_2} = 0 \qquad FG - \frac{N_1}{N_1 + N_2} = 0$$

$$DF - \frac{N_1 \cdot N_2}{(N_1 + N_2)^2} = 0 \qquad AD - \frac{N_2^2}{(N_1 + N_2)^2} = 0 \qquad DE - \frac{N_1 \cdot N_2}{(N_1 + N_2)^2} = 0$$

$$BD - \frac{N_1 \cdot N_2^2}{(N_1 + N_2)^3} = 0 \qquad AB - \frac{N_2^3}{(N_1 + N_2)^3} = 0 \qquad AB^{\frac{1}{3}} - \frac{N_2}{(N_1 + N_2)} = 0$$

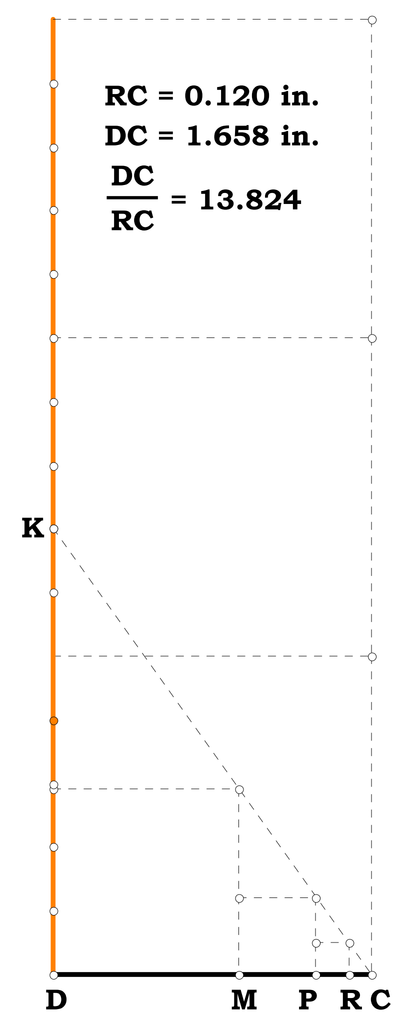
022598C **Doing the Math**

Descriptions.

These plates sat, I never actually wrote them up, in their directory, but included in the Delian Quest. Because they are so elementary, I assumed that doing math with a geometric figure was known. I was a bit conflicted about this, however, working 12 hours a day for years on end tends to dull the senses. Then, I got to thinking about them again in 2007. I even did a couple of searches on the internet to see if anyone had actually developed doing the math with a simple geometric figure and could not find anything. Then I found scraps in old books found on the Internet Archive where certain operations were fragmented and really undeveloped. Then I started to realize and understand that BAM was not developed as a grammar If it had been, there would be no talk of non-Euclidean Geometry, there would only be embarrassment of its memory.

One can see that it is directly derived from plate A on this date. BAM (Basic Analog Mathematics) has its roots in exponential series.

Definitions.



$4^3 = 64.000$

$\left(3+\frac{4}{5}\right)^3 = 54.872$

$\left(3+\frac{3}{5}\right)^3 = 46.656$

$\left(3+\frac{2}{5}\right)^3 = 39.304$

$\left(3+\frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2+\frac{4}{5}\right)^3 = 21.952$

$\left(2+\frac{3}{5}\right)^3 = 17.576$

$\left(2+\frac{2}{5}\right)^3 = 13.824$

$\left(2+\frac{1}{5}\right)^3 = 10.648$

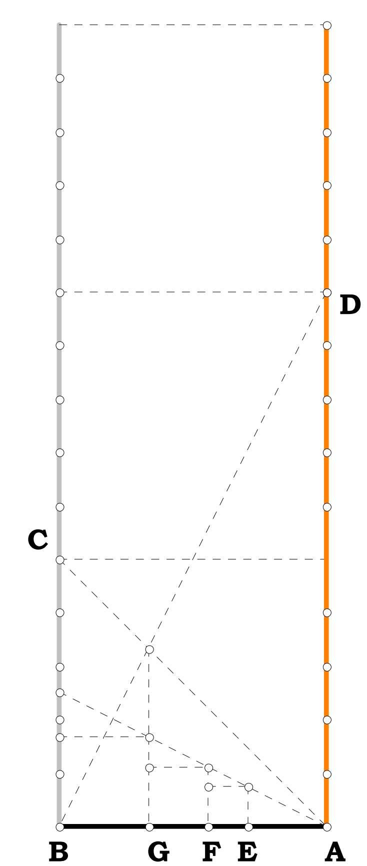
$\left(1+\frac{4}{5}\right)^3 = 5.832$

$\left(1+\frac{3}{5}\right)^3 = 4.096$

$\left(1+\frac{2}{5}\right)^3 = 2.744$

$\left(1+\frac{1}{5}\right)^3 = 1.728$

Given line AB, divide it by the equation $\frac{BC+AD^{W}}{AD} = 5.06250$ where W is a whole number.



$\frac{AB}{AG} = 1.50000$

$\frac{AB}{AF} = 2.25000$

$\frac{AB}{AE} = 3.37500$

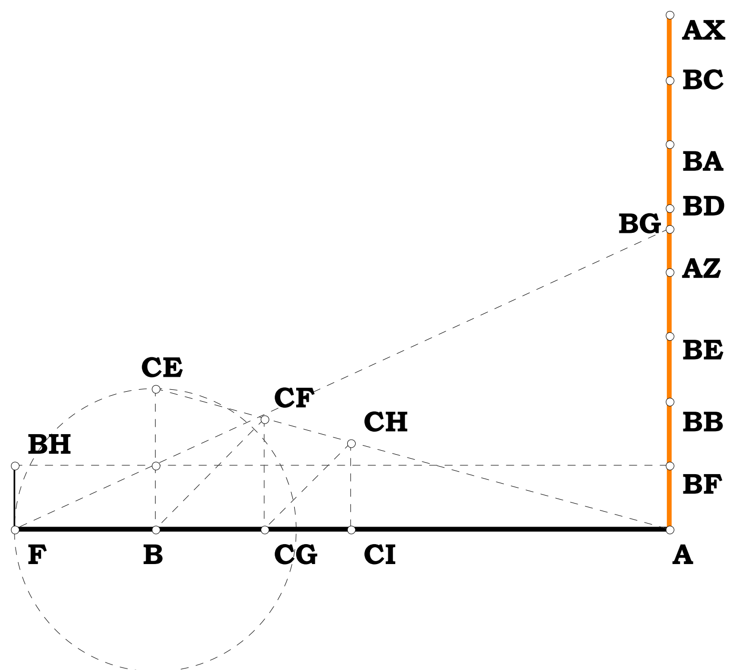
$\frac{BC+AD^{1}}{AD} = 1.50000$

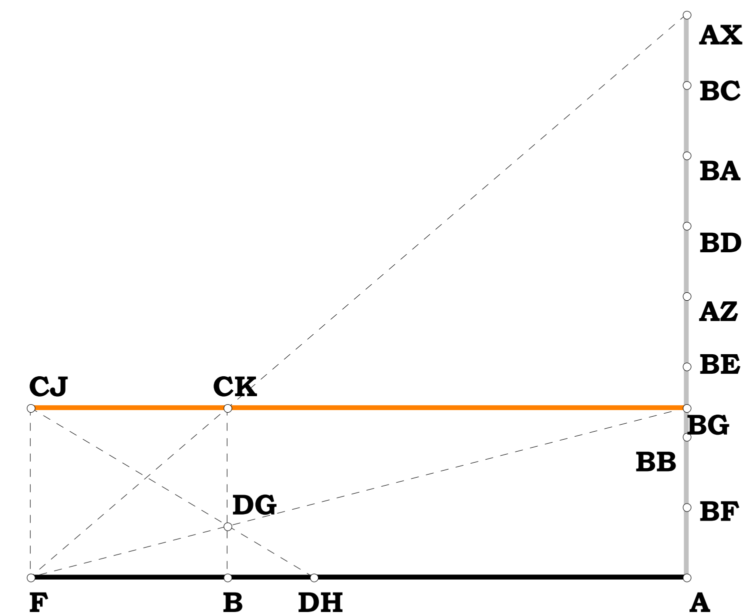
$\frac{BC+AD^{2}}{AD} = 2.25000$

$\frac{BC+AD^{3}}{AD} = 3.37500$

022698

Now ain't this just typically human. Some of the more defined plates that led to Basic Analog Mathematics promptly get wrote up in 0816 2015. I am on the ball! It appears I never even bothered to do a pdf file of these. They are, however, not fundamentally distinct from some previous write ups except, these are series format. So, I think I will forgo the write ups again!





$\frac{AF}{FB} = 4.672$

$AB = 2.679$ in.

$FB = 0.730$ in.

$AF = 3.408$ in.

$\frac{AF}{AB} = 1.272$

$ACG = 2.105$ in.

$ACI = 1.655$ in.

$\frac{AF}{ACI} = 2.060$

$\frac{AF}{ACG} = 1.619$

$\frac{AF^3}{AB} = 2.060$

$\frac{AF^2}{AB} = 1.619$

$AF = 3.417$ in.

$AB = 2.388$ in.

$\frac{AF}{AB} = 1.431$

$\frac{AF}{DHF} = 2.321$

$\frac{DHF}{AF} = 0.431$

$DHF = 1.472$ in.

$BF = 1.029$ in.

$\frac{AAX}{BGA} = 3.321$

$\frac{AAX}{BGA} - 1 = 2.321$

$BGA = 0.884$ in.

$AAX = 2.936$ in.

$\left(\frac{1}{\frac{AAX}{BGA} - 1}\right) \cdot BF = 0.443$ in.

042398

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 4.27 \quad AF := N_1$

$$BF := AF - AB \quad AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2}$$

$$BD := AD - AB \quad DE := BE - BD \quad EQ := BE$$

$$DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \quad PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ}$$

$$DM := QM - DQ \quad AE := AB + BE \quad AC := \frac{AE}{2}$$

$$Db := \frac{DM}{2} \quad CM := AC \quad ab := \frac{CM \cdot Db}{DM}$$

$$CD := AD - AC \quad Ca := \frac{CD}{2} \quad Aa := AC + Ca$$

$$CH := \frac{ab \cdot AC}{Aa} \quad AM := AD \quad Ac := \frac{AM \cdot CH}{CM}$$

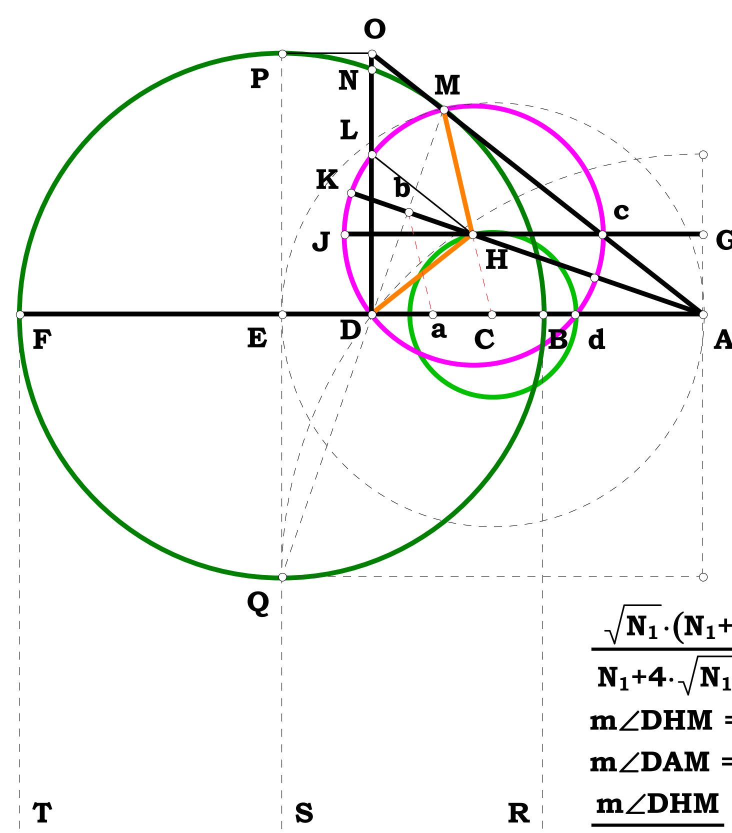
$$HM := CM - CH \quad HM - Ac = 0$$

Definitions.

$$Ac - \frac{\sqrt{N_1} \cdot \left(N_1 + 1\right)}{N_1 + 4 \cdot \sqrt{N_1} + 1} = 0 \quad HM - \frac{\sqrt{N_1} \cdot \left(N_1 + 1\right)}{N_1 + 4 \cdot \sqrt{N_1} + 1} = 0$$

A Square Root Figure And Triseciton

In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?



$AB = 2.11667$ cm

$AF = 9.03817$ cm

$Ac = 1.70294$ cm

$\frac{AB}{AB} = 1.00000$

$\frac{AF}{AB} = 4.27000 \quad N_1 = 4.27000$

$\frac{Ac}{AB} = 0.80454 \quad Ac = 0.80454$

$\frac{\sqrt{N_1} \cdot (N_1+1)}{N_1+4 \cdot \sqrt{N_1}+1} - Ac = 0.00000$

$m\angle DHM = 115.05651°$

$m\angle DAM = 38.35217°$

$\frac{m\angle DHM}{m\angle DAM} = 3.00000$

The traditional paper trisector fits right into this figure.
This figure seems to be just full of surprises.

$$AF - N_1 = 0 \qquad BF - \left(N_1 - 1\right) = 0 \qquad AD - \sqrt{N_1} = 0$$

$$BE - \frac{N_1 - 1}{2} = 0 \qquad BD - \left(\sqrt{N_1} - 1\right) = 0 \qquad DE - \frac{\left(\sqrt{N_1} - 1\right)^2}{2} = 0$$

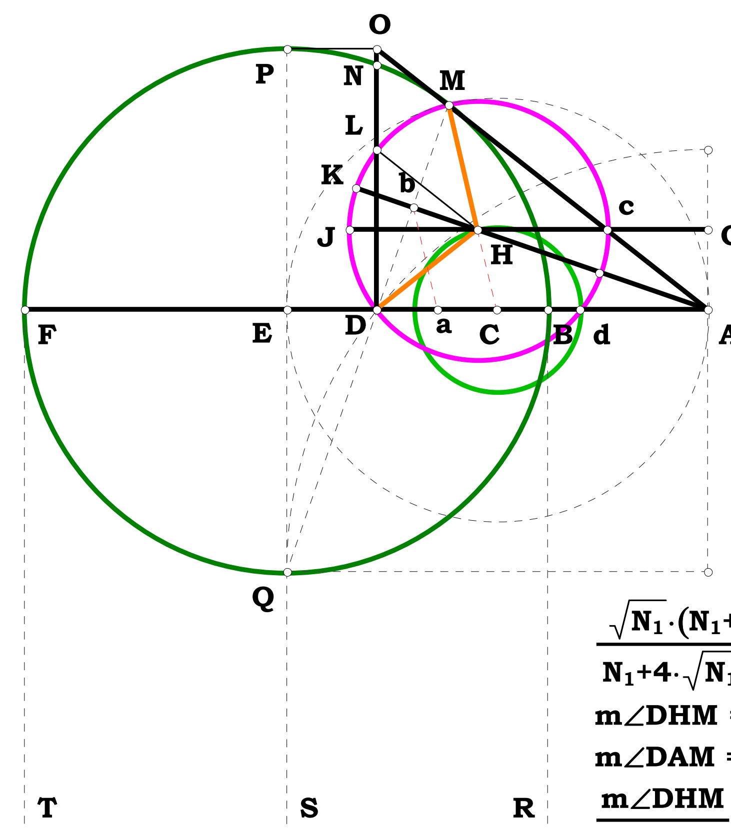
$$DQ - \frac{\sqrt{\left(N_1 + 1\right)\cdot\left(\sqrt{N_1} - 1\right)^2}}{\sqrt{2}} = 0 \qquad QM - \frac{\sqrt{2}\cdot\left(\sqrt{N_1} - 1\right)^2\cdot\left(\sqrt{N_1} + 1\right)^2}{2\cdot\sqrt{\left(N_1 + 1\right)\cdot\left(\sqrt{N_1} - 1\right)^2}} = 0$$

$$DM - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)^2}{\sqrt{\left(N_1 + 1\right)\cdot\left(\sqrt{N_1} - 1\right)^2}} = 0 \qquad AE - \frac{1 + N_1}{2} = 0 \qquad AC - \frac{1 + N_1}{4} = 0$$

$$Db - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)^2}{2\cdot\sqrt{\left(N_1 + 1\right)\cdot\left(N_1 - 2\cdot\sqrt{N_1} + 1\right)}} = 0 \qquad CD - \frac{4\cdot\sqrt{N_1} - N_1 - 1}{4} = 0$$

$$Ca - \frac{4\cdot\sqrt{N_1} - N_1 - 1}{8} = 0 \qquad Aa - \frac{N_1 + 4\cdot\sqrt{N_1} + 1}{8} = 0$$

$$CH - \frac{\left(1 + N_1\right)^2}{4\cdot\left(N_1 + 4\cdot\sqrt{N_1} + 1\right)} = 0$$



$AB = 2.11667$ cm

$AF = 9.03817$ cm

$Ac = 1.70294$ cm

$\frac{AB}{AB} = 1.00000$

$\frac{AF}{AB} = 4.27000$ $\qquad N_1 = 4.27000$

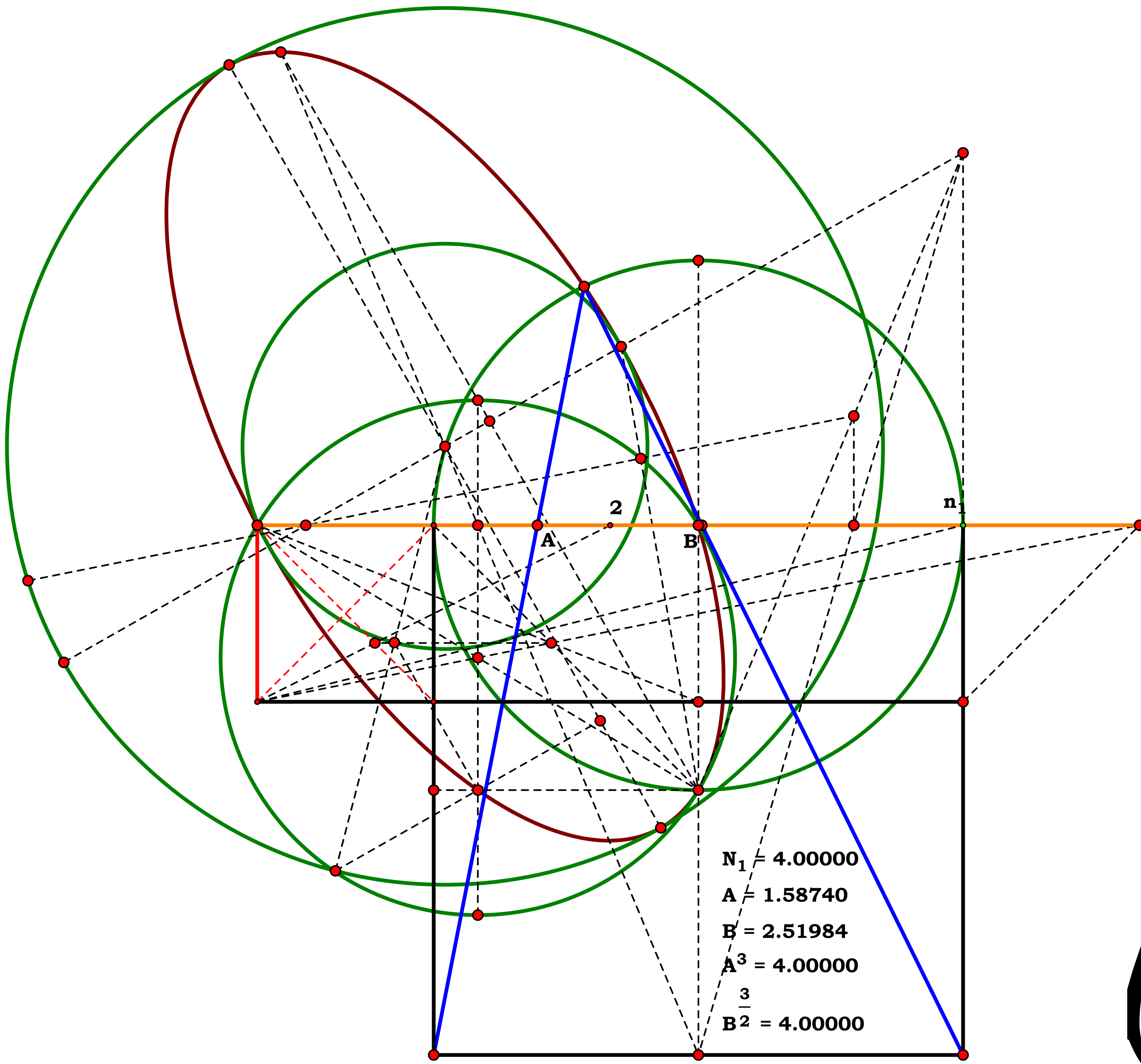
$\frac{Ac}{AB} = 0.80454$ $\qquad Ac = 0.80454$

$\frac{\sqrt{N_1}\cdot(N_1+1)}{N_1+4\cdot\sqrt{N_1}+1} - Ac = 0.00000$

$m\angle DHM = 115.05651°$

$m\angle DAM = 38.35217°$

$\frac{m\angle DHM}{m\angle DAM} = 3.00000$



The Delian Quest 1999

John Clark

072499

Unit.

$AB := 1$

Given.

$N_1 := 6.429 \qquad AD := N_1$

$N_2 := 2$

On Gemini Roots

CE is to EF as CY is to CW

Descriptions.

$BD := AD - AB \qquad BC := \dfrac{BD}{2} \qquad CW := BC \qquad CT := BC$

$FV := BC \qquad AI := \sqrt{AB \cdot AD} \qquad BI := AI - AB \qquad DI := BD - BI$

$IR := \sqrt{BI \cdot DI} \qquad AE := AI \qquad AC := AB + BC \qquad ED := AD + AE$

$CE := AC + AE \qquad EF := \dfrac{CE}{N_2} \qquad BE := AE + AB \qquad EI := AE + AI$

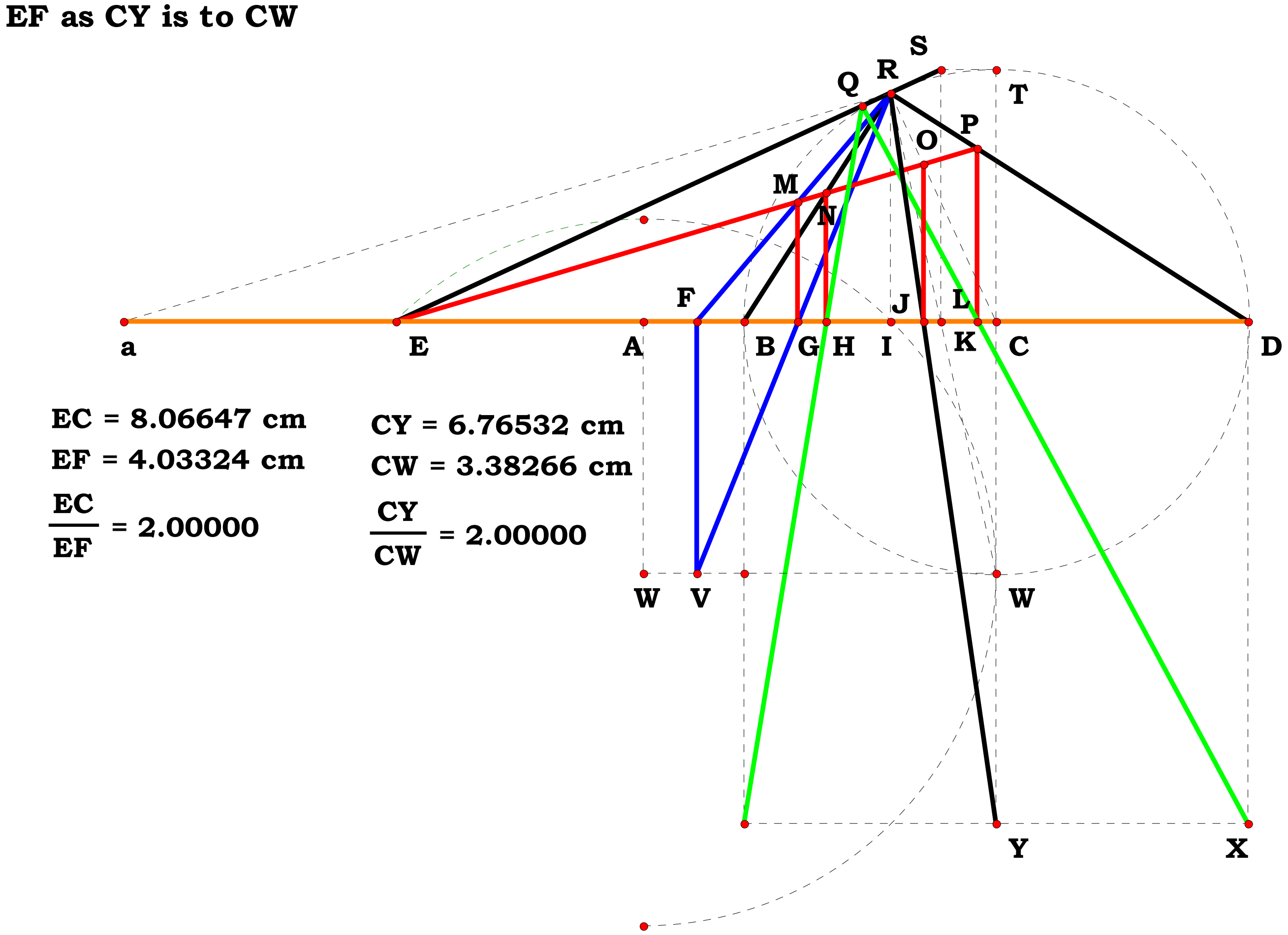
$FI := EI - EF \qquad FG := \dfrac{FI \cdot FV}{FV + IR} \qquad EG := EF + FG \qquad GI := FI - FG$

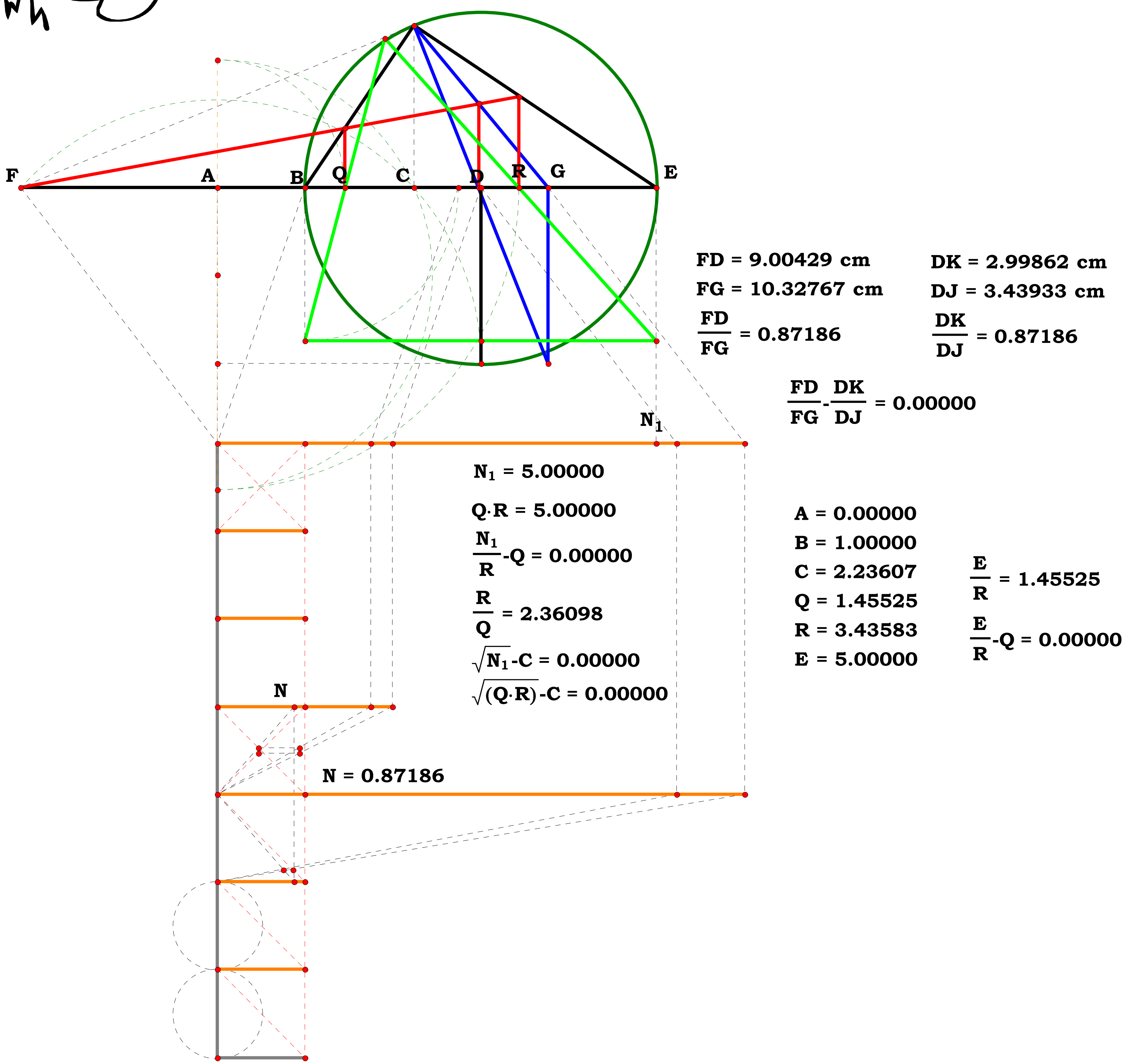
$GM := \dfrac{FV \cdot GI}{FI} \qquad Ia := \dfrac{EG \cdot IR}{GM} \qquad EL := \dfrac{Ia \cdot ED}{Ia + DI} \qquad BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI \qquad BH := \dfrac{BI \cdot BE}{Ba} \qquad EH := BE + BH \qquad CI := AC - AI$

$JO := \dfrac{IR \cdot CE}{CI + Ia} \qquad CJ := \dfrac{CI \cdot JO}{IR} \qquad JI := CI - CJ$

$CY := \dfrac{IR \cdot CJ}{JI} \qquad \dfrac{CY}{CW} - \dfrac{CE}{EF} = 0$




F
A
B
Q
C
D
R
G
E
FD = 9.00429 cm
FG = 10.32767 cm
FD/FG = 0.87186
DK = 2.99862 cm
DJ = 3.43933 cm
DK/DJ = 0.87186
FD/FG - DK/DJ = 0.00000
N1
N1 = 5.00000
Q·R = 5.00000
N1/R - Q = 0.00000
R/Q = 2.36098
√N1 - C = 0.00000
√(Q·R) - C = 0.00000
A = 0.00000
B = 1.00000
C = 2.23607
Q = 1.45525
R = 3.43583
E = 5.00000
E/R = 1.45525
E/R - Q = 0.00000
N
N = 0.87186

081199

A Delian Solution

In this revision, I have dimmed down the names of those points which are not used in a section which greately facilitates reading of each chapter.

As in the past, you can compare the Arithmetic results produced by both the Sketchpad and Mathcad. One will then have Geometric Names, which is the figure, Algebraic Names used in MathCad, and Arithmetic Names, produced by both.

$AF^{\frac{1}{3}} - AC = 0.00000$

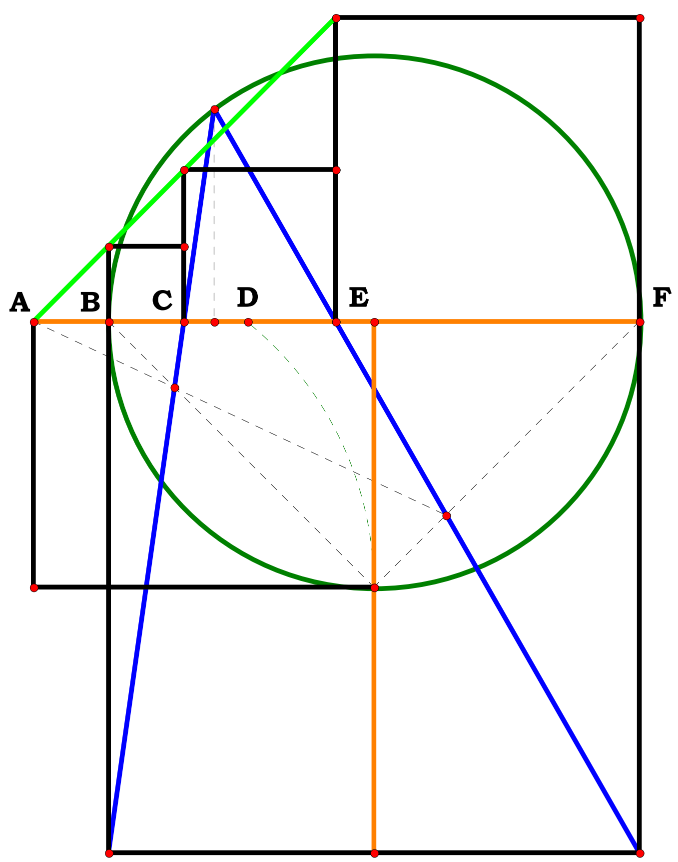
$AF^{\frac{1}{2}} - AD = 0.00000$

$AF^{\frac{2}{3}} - AE = 0.00000$

$af^{\frac{1}{3}} - ac = 0.00000$

$af^{\frac{1}{2}} - ad = 0.00000$

$af^{\frac{2}{3}} - ae = 0.00000$



AB = 1.00310 cm
AC = 2.00619 cm
AD = 2.83718 cm
AE = 4.01238 cm
AF = 8.02476 cm

$\frac{AB}{AB} = 1.00000$ AB = 1.00000

$\frac{AC}{AB} = 2.00000$ AC = 2.00000

$\frac{AD}{AB} = 2.82843$ AD = 2.82843

$\frac{AE}{AB} = 4.00000$ AE = 4.00000

$\frac{AF}{AB} = 8.00000$ AF = 8.00000

$\frac{AB}{AC} = 0.50000$ ac = 0.50000

$\frac{AB}{AD} = 0.35355$ ad = 0.35355

$\frac{AB}{AE} = 0.25000$ ae = 0.25000

$\frac{AB}{AF} = 0.12500$ af = 0.12500

Unit.

AB := 2.59047

Given.

AG := 11.81347

A Delian Solution

Definitions.

What are the minor and major axis for the ellipse that will give point Z for the cube root?

081199

Descriptions.

$BG := AG - AB \qquad BF := \dfrac{BG}{2} \qquad FG := BF \qquad AF := AB + BF$

$FX := BF \qquad Mf := \dfrac{\sqrt{AF^2 + FX^2}}{2} \qquad Lf := \dfrac{FX}{2} \qquad ML := Mf - Lf$

$FL := \dfrac{AF}{2} \qquad Xd := FL \qquad df := Lf \qquad IX := FX \qquad Md := Mf + df$

Definitions.

$BG = 9.22300 \qquad BF = 4.61150 \qquad FG = 4.61150 \qquad AF = 7.20197$

$FX = 4.61150 \qquad Mf = 4.27593 \qquad Lf = 2.30575 \qquad ML = 1.97018$

$FL = 3.60099 \qquad Xd = 3.60099 \qquad df = 2.30575 \qquad IX = 4.61150$

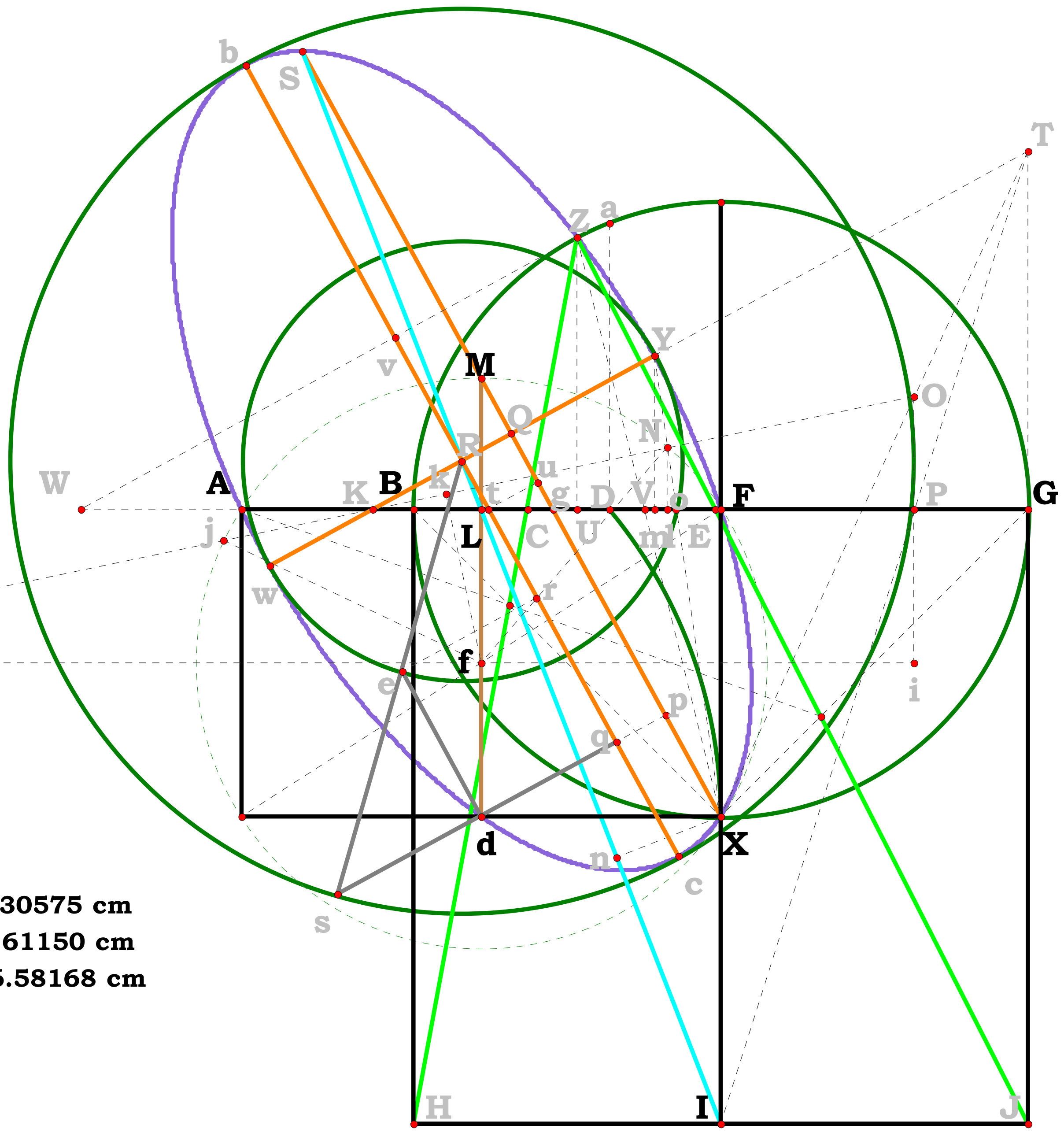
$Md = 6.58168$

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036

$AG^{\frac{1}{3}}\text{-}AC = 0.00000$

$AG^{\frac{1}{2}}\text{-}AD = 0.00000$

$AG^{\frac{2}{3}}\text{-}AE = 0.00000$



BG = 9.22300 cm
BF = 4.61150 cm
FG = 4.61150 cm
AF = 7.20197 cm
FX = 4.61150 cm
Mf = 4.27593 cm
Lf = 2.30575 cm
ML = 1.97018 cm
FL = 3.60098 cm
Xd = 3.60098 cm

df = 2.30575 cm
XI = 4.61150 cm
Md = 6.58168 cm

Descriptions.

$$MX := \sqrt{Xd^2 + Md^2} \qquad SX := \frac{MX \cdot IX}{IX - ML} \qquad Lg := \frac{FL \cdot ML}{ML + FX}$$

$$QX := \frac{SX}{2} \qquad Fg := FL - Lg \qquad Xg := \frac{MX \cdot Fg}{Xd} \qquad Qg := QX - Xg$$

$$Kg := \frac{Xg \cdot Qg}{Fg} \qquad GK := FG + Fg + Kg \qquad GJ := BG \qquad GT := \frac{Fg \cdot GK}{FX}$$

$$JT := GJ + GT \qquad IJ := BF \qquad FP := \frac{IJ \cdot GJ}{JT} \qquad OP := \frac{IX \cdot GT}{JT}$$

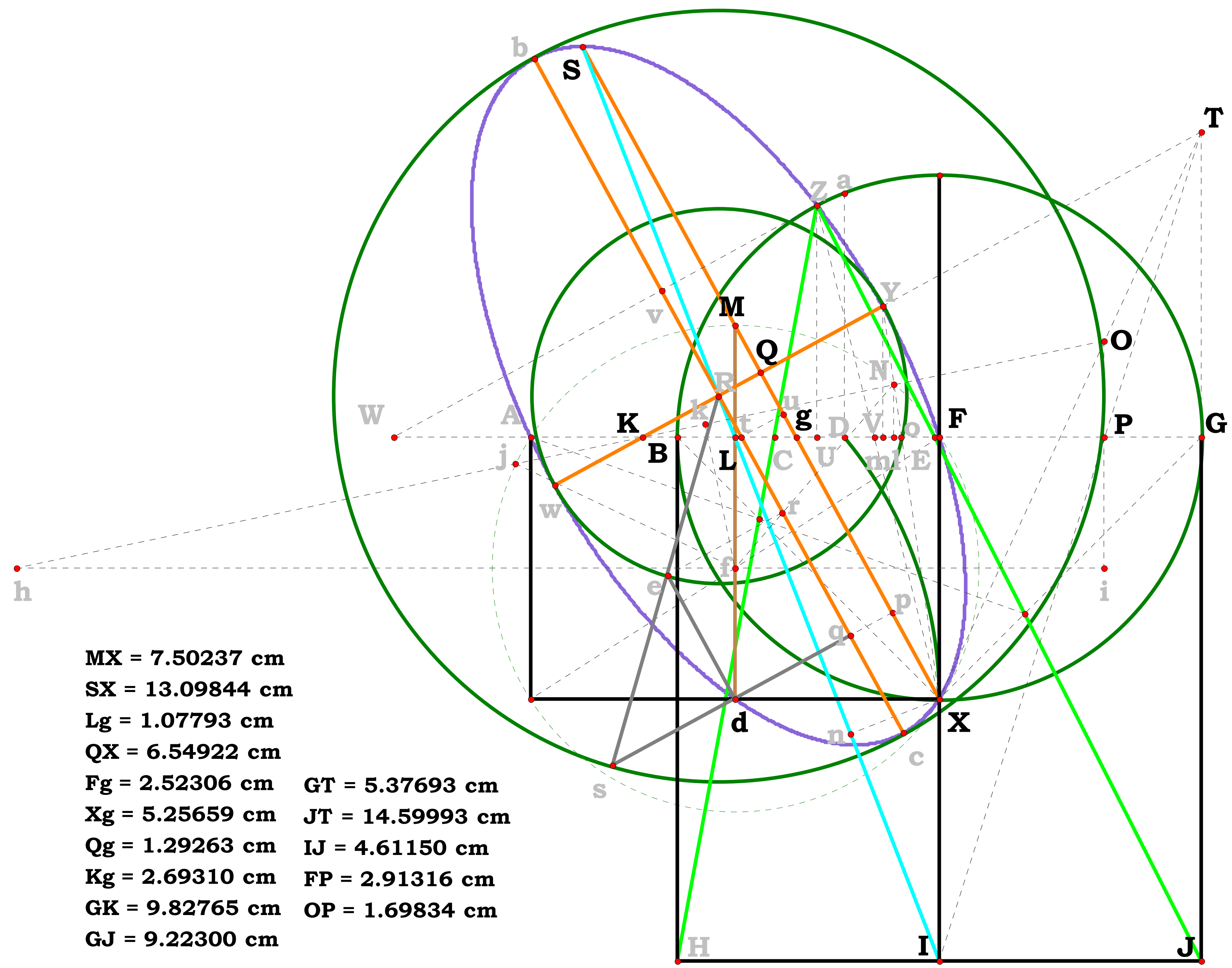
Definitions.

$$MX = 7.50237 \qquad SX = 13.09845 \qquad Lg = 1.07793 \qquad QX = 6.54922$$

$$Fg = 2.52306 \qquad Xg = 5.25659 \qquad Qg = 1.29263$$

$$Kg = 2.69310 \qquad GK = 9.82766 \qquad GJ = 9.22300 \qquad GT = 5.37693$$

$$JT = 14.59993 \qquad IJ = 4.61150 \qquad FP = 2.91315 \qquad OP = 1.69835$$



MX = 7.50237 cm
SX = 13.09844 cm
Lg = 1.07793 cm
QX = 6.54922 cm
Fg = 2.52306 cm
Xg = 5.25659 cm
Qg = 1.29263 cm
Kg = 2.69310 cm
GK = 9.82765 cm
GJ = 9.22300 cm
GT = 5.37693 cm
JT = 14.59993 cm
IJ = 4.61150 cm
FP = 2.91316 cm
OP = 1.69834 cm

Descriptions.

$\mathbf{KP} := \mathbf{Fg} + \mathbf{Kg} + \mathbf{FP} \qquad \mathbf{Pi} := \mathbf{Lf} \qquad \mathbf{Oi} := \mathbf{OP} + \mathbf{Pi} \qquad \mathbf{hi} := \dfrac{\mathbf{KP} \cdot \mathbf{Oi}}{\mathbf{OP}}$

$\mathbf{fi} := \mathbf{FP} + \mathbf{FL} \qquad \mathbf{fh} := \mathbf{hi} - \mathbf{fi} \qquad \mathbf{KO} := \sqrt{\mathbf{KP}^2 + \mathbf{OP}^2} \qquad \mathbf{hk} := \dfrac{\mathbf{KP} \cdot \mathbf{fh}}{\mathbf{KO}}$

$\mathbf{Nf} := \mathbf{Mf} \qquad \mathbf{fk} := \dfrac{\mathbf{OP} \cdot \mathbf{fh}}{\mathbf{KO}} \qquad \mathbf{Nk} := \sqrt{\mathbf{Nf}^2 - \mathbf{fk}^2} \qquad \mathbf{Nh} := \mathbf{hk} + \mathbf{Nk}$

$\mathbf{Oh} := \dfrac{\mathbf{KO} \cdot \mathbf{Oi}}{\mathbf{OP}} \qquad \mathbf{NO} := \mathbf{Oh} - \mathbf{Nh} \qquad \mathbf{KN} := \mathbf{KO} - \mathbf{NO}$

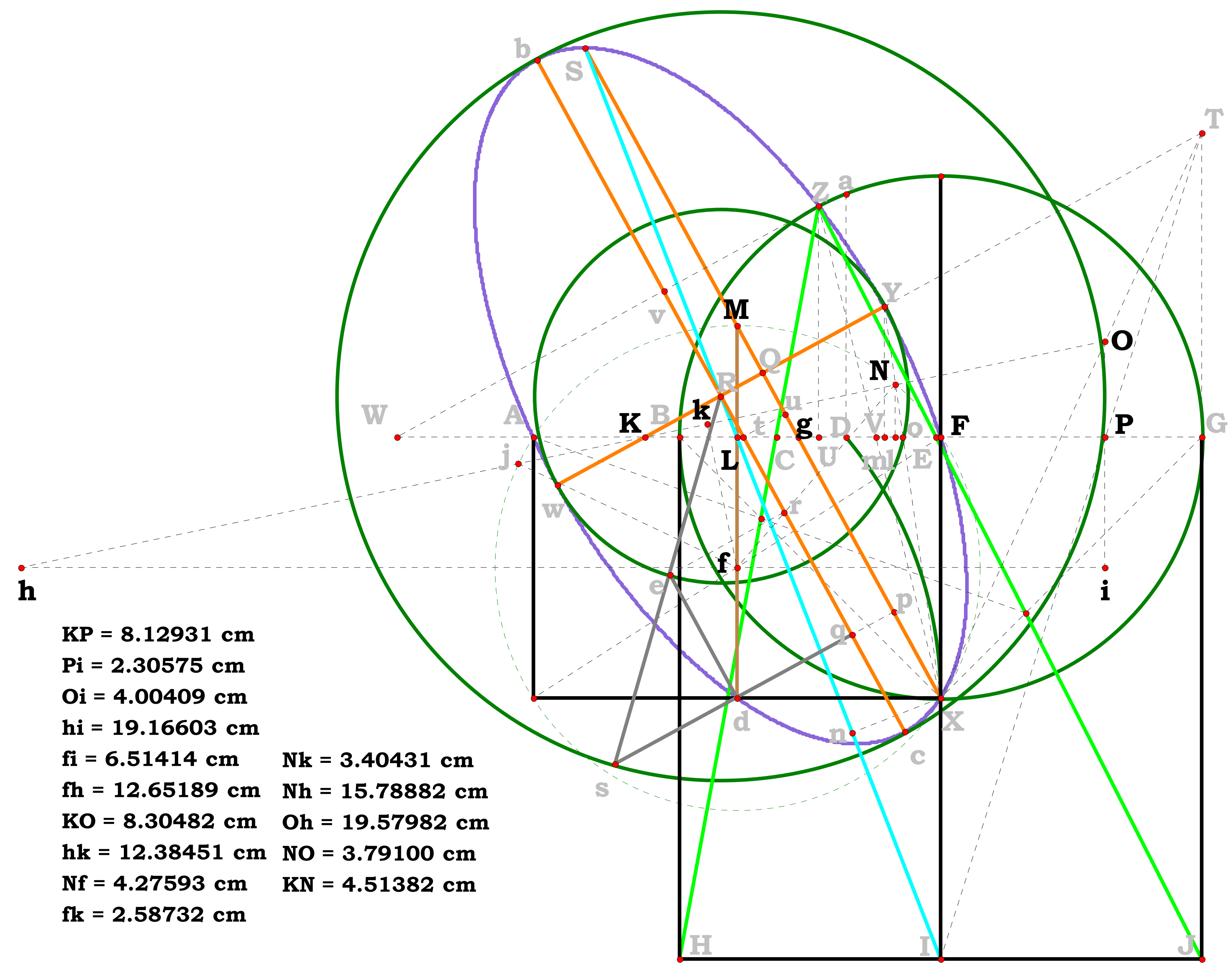
Definitions.

$\mathbf{KP} = 8.12931 \qquad \mathbf{Pi} = 2.30575 \qquad \mathbf{Oi} = 4.00410 \qquad \mathbf{hi} = 19.16603$

$\mathbf{fi} = 6.51414 \qquad \mathbf{fh} = 12.65189 \qquad \mathbf{KO} = 8.30482 \qquad \mathbf{hk} = 12.38451$

$\mathbf{Nf} = 4.27593 \qquad \mathbf{fk} = 2.58733 \qquad \mathbf{Nk} = 3.40431 \qquad \mathbf{Nh} = 15.78882$

$\mathbf{Oh} = 19.57983 \qquad \mathbf{NO} = 3.79100 \qquad \mathbf{KN} = 4.51382$



Descriptions.

$$Nl := \frac{OP \cdot KN}{KO} \qquad Kl := \frac{KP \cdot KN}{KO} \qquad FK := KP - FP \qquad Fl := FK - Kl$$

$$NX := \sqrt{(FX + Nl)^2 + Fl^2} \qquad XY := \frac{NX \cdot IX}{IX - Nl} \qquad Fm := \frac{Fl \cdot XY}{NX} \qquad Km := FK - Fm$$

$$Fo := \frac{Fl \cdot FX}{FX + Nl} \qquad Xo := \frac{NX \cdot Fo}{Fl} \qquad mo := Fm - Fo \qquad Ym := \frac{FX \cdot mo}{Fo}$$

$$FI := 2 \cdot BF \qquad IL := \sqrt{FL^2 + FI^2} \qquad IS := \frac{IL \cdot IX}{IX - ML} \qquad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

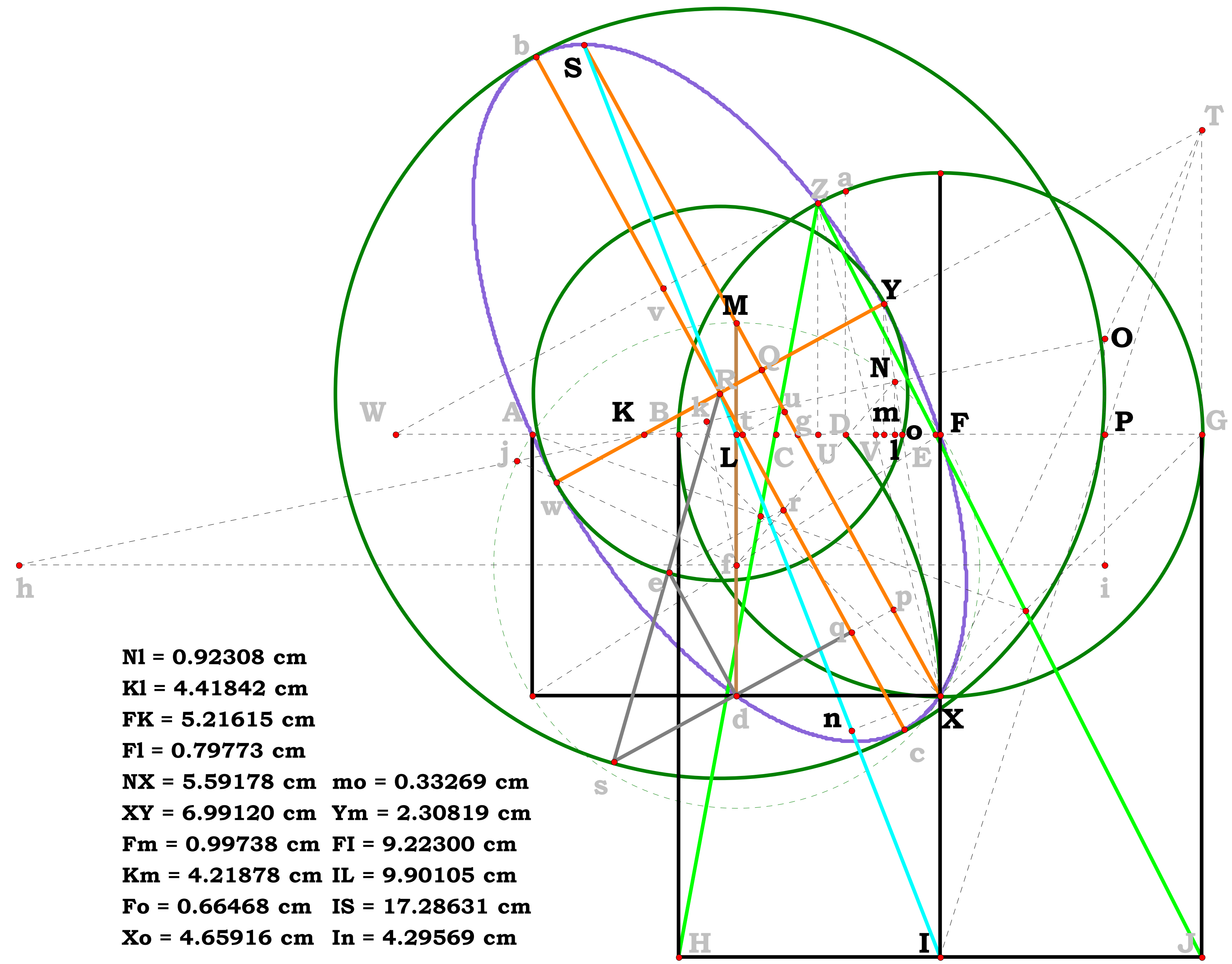
Definitions.

$Nl = 0.92308$ $\quad Kl = 4.41843$ $\quad FK = 5.21616$ $\quad Fl = 0.79773$

$NX = 5.59178$ $\quad XY = 6.99120$ $\quad Fm = 0.99738$ $\quad Km = 4.21878$

$Fo = 0.66468$ $\quad Xo = 4.65916$ $\quad mo = 0.33269$ $\quad Ym = 2.30819$

$FI = 9.22300$ $\quad IL = 9.90105$ $\quad IS = 17.28632$ $\quad In = 4.29569$



Nl = 0.92308 cm
Kl = 4.41842 cm
FK = 5.21615 cm
Fl = 0.79773 cm
NX = 5.59178 cm mo = 0.33269 cm
XY = 6.99120 cm Ym = 2.30819 cm
Fm = 0.99738 cm FI = 9.22300 cm
Km = 4.21878 cm IL = 9.90105 cm
Fo = 0.66468 cm IS = 17.28631 cm
Xo = 4.65916 cm In = 4.29569 cm

Descriptions.

$$Xn := \sqrt{IX^2 - In^2} \qquad QR := \frac{Xn \cdot QX}{IS - In} \qquad KT := \sqrt{GK^2 + GT^2} \qquad KY := \frac{KT \cdot Ym}{GT}$$

$$KQ := \frac{GK \cdot Qg}{GT} \qquad RY := KY - KQ + QR \qquad Xd := FL \qquad dp := \frac{GK \cdot Xd}{KT}$$

$$pq := QR \qquad dq := dp - pq \qquad Xp := \frac{GT \cdot Xd}{KT} \qquad Qp := QX - Xp \qquad Rq := Qp$$

$$er := dq \qquad Re := RY \qquad Rr := \sqrt{Re^2 - er^2} \qquad Rs := \frac{Re \cdot Rq}{Rr}$$

RY is the radius of the Minor Axis.

Rs is the radius of the Major Axis.

Definitions.

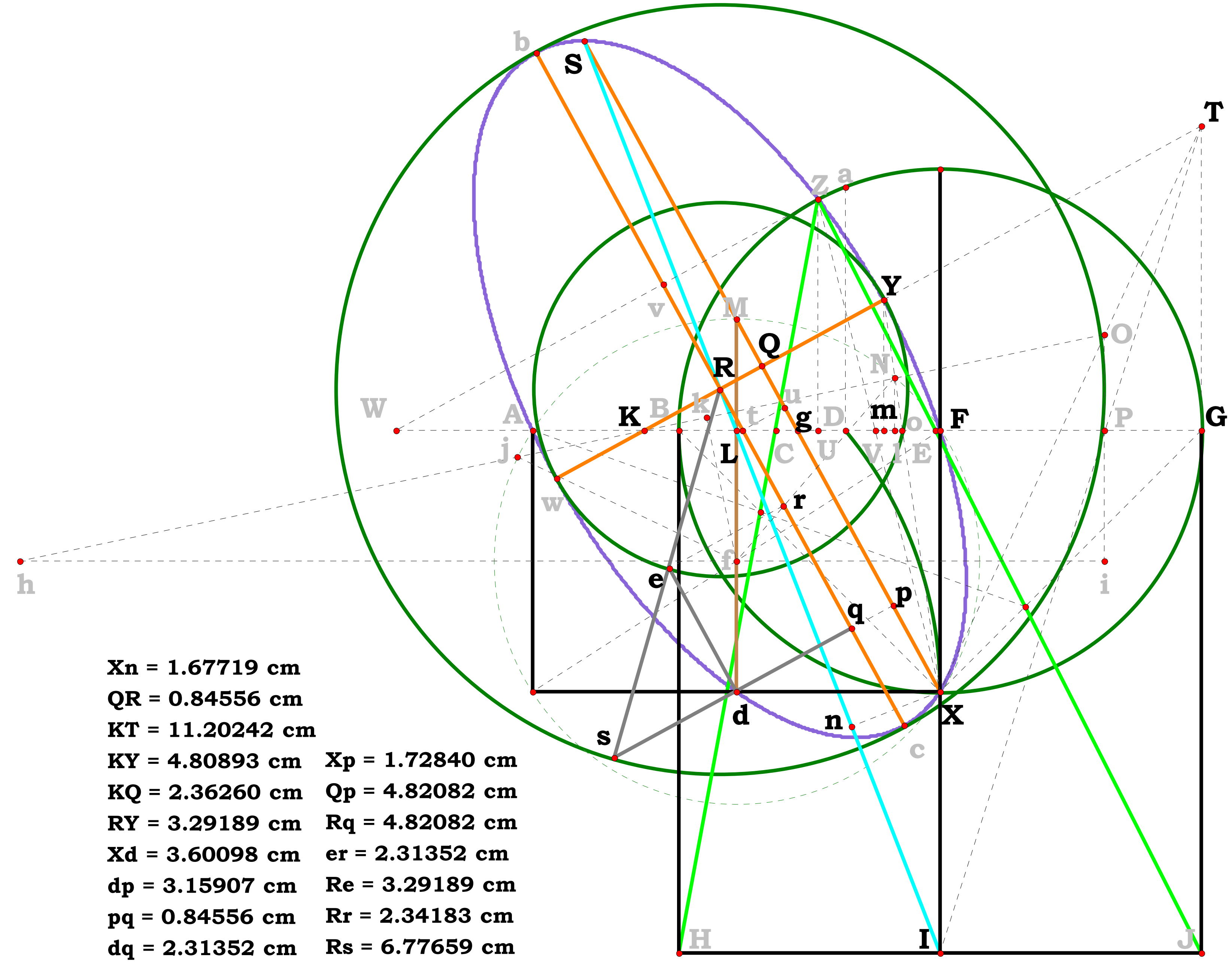
$Xn = 1.67719$ $\quad QR = 0.84556$ $\quad KT = 11.20242$ $\quad KY = 4.80894$

$KQ = 2.36260$ $\quad RY = 3.29189$ $\quad Xd = 3.60099$ $\quad dp = 3.15907$

$pq = 0.84556$ $\quad dq = 2.31352$ $\quad Xp = 1.72840$ $\quad Qp = 4.82082$

$Rq = 4.82082$ $\quad er = 2.31352$ $\quad Re = 3.29189$ $\quad Rr = 2.34183$

$Rs = 6.77659$



Xn = 1.67719 cm
QR = 0.84556 cm
KT = 11.20242 cm
KY = 4.80893 cm
KQ = 2.36260 cm
RY = 3.29189 cm
Xd = 3.60098 cm
dp = 3.15907 cm
pq = 0.84556 cm
dq = 2.31352 cm

Xp = 1.72840 cm
Qp = 4.82082 cm
Rq = 4.82082 cm
er = 2.31352 cm
Re = 3.29189 cm
Rr = 2.34183 cm
Rs = 6.77659 cm

The first two equations should have been learnt by prior explorations. And they will be repeated demonstrated in following demonstrations. Therefore, here we can, like other equations, simply recall them for the figure.

Descriptions.

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}}$ $\quad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}}$ $\quad BC := AC - AB$ $\quad BE := AE - AB$

$CE := BE - BC$ $\quad EG := BG - BE$ $\quad CU := \dfrac{BC \cdot CE}{BC + EG}$ $\quad BU := BC + CU$

$GU := BG - BU$ $\quad UZ := \sqrt{BU \cdot GU}$ $\quad UW := \dfrac{GK \cdot UZ}{GT}$ $\quad Gg := GK - Kg$

$tu := QR$ $\quad gt := \dfrac{KT \cdot tu}{GK}$ $\quad Gt := Gg + gt$ $\quad GW := GU + UW$

Is the segment Zv equal to the perpendicular for the ellipse?

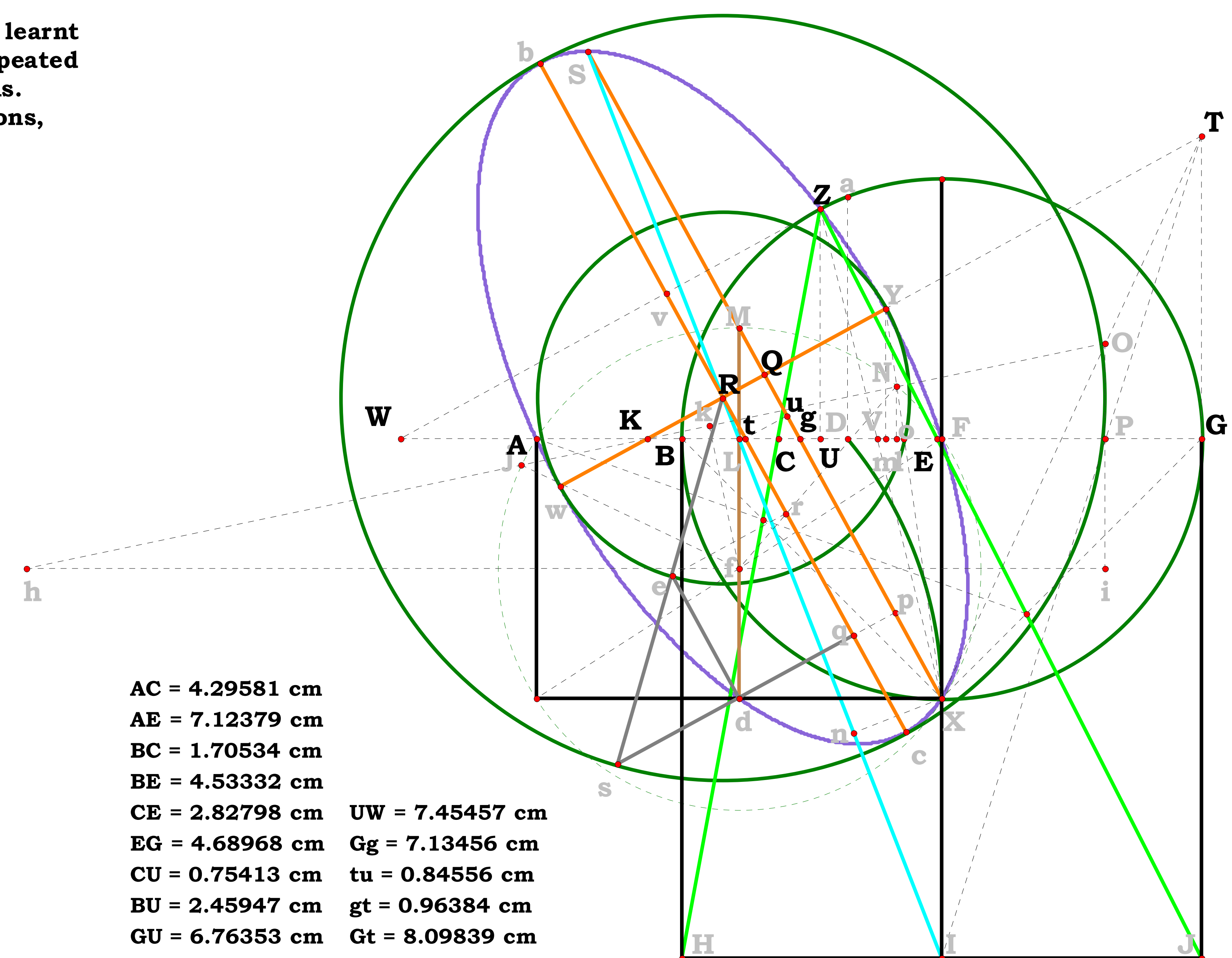
Definitions.

$AC = 4.29581$ $\quad AE = 7.12379$ $\quad BC = 1.70534$ $\quad BE = 4.53332$

$CE = 2.82798$ $\quad EG = 4.68968$ $\quad CU = 0.75413$ $\quad BU = 2.45947$

$GU = 6.76353$ $\quad UZ = 4.07856$ $\quad UW = 7.45457$ $\quad Gg = 7.13456$

$tu = 0.84556$ $\quad gt = 0.96384$ $\quad Gt = 8.09839$ $\quad GW = 14.21811$



AC = 4.29581 cm
AE = 7.12379 cm
BC = 1.70534 cm
BE = 4.53332 cm
CE = 2.82798 cm
EG = 4.68968 cm
CU = 0.75413 cm
BU = 2.45947 cm
GU = 6.76353 cm
UZ = 4.07856 cm

UW = 7.45457 cm
Gg = 7.13456 cm
tu = 0.84556 cm
gt = 0.96384 cm
Gt = 8.09839 cm
GW = 14.21811 cm

Descriptions.

$Wt := GW - Gt \qquad tv := \dfrac{GT \cdot Wt}{KT} \qquad Kt := GK - Gt \qquad Rt := \dfrac{GT \cdot Kt}{KT}$

$Rv := tv - Rt \qquad bc := 2 \cdot Rs \qquad Rc := Rs \qquad cv := Rc + Rv$

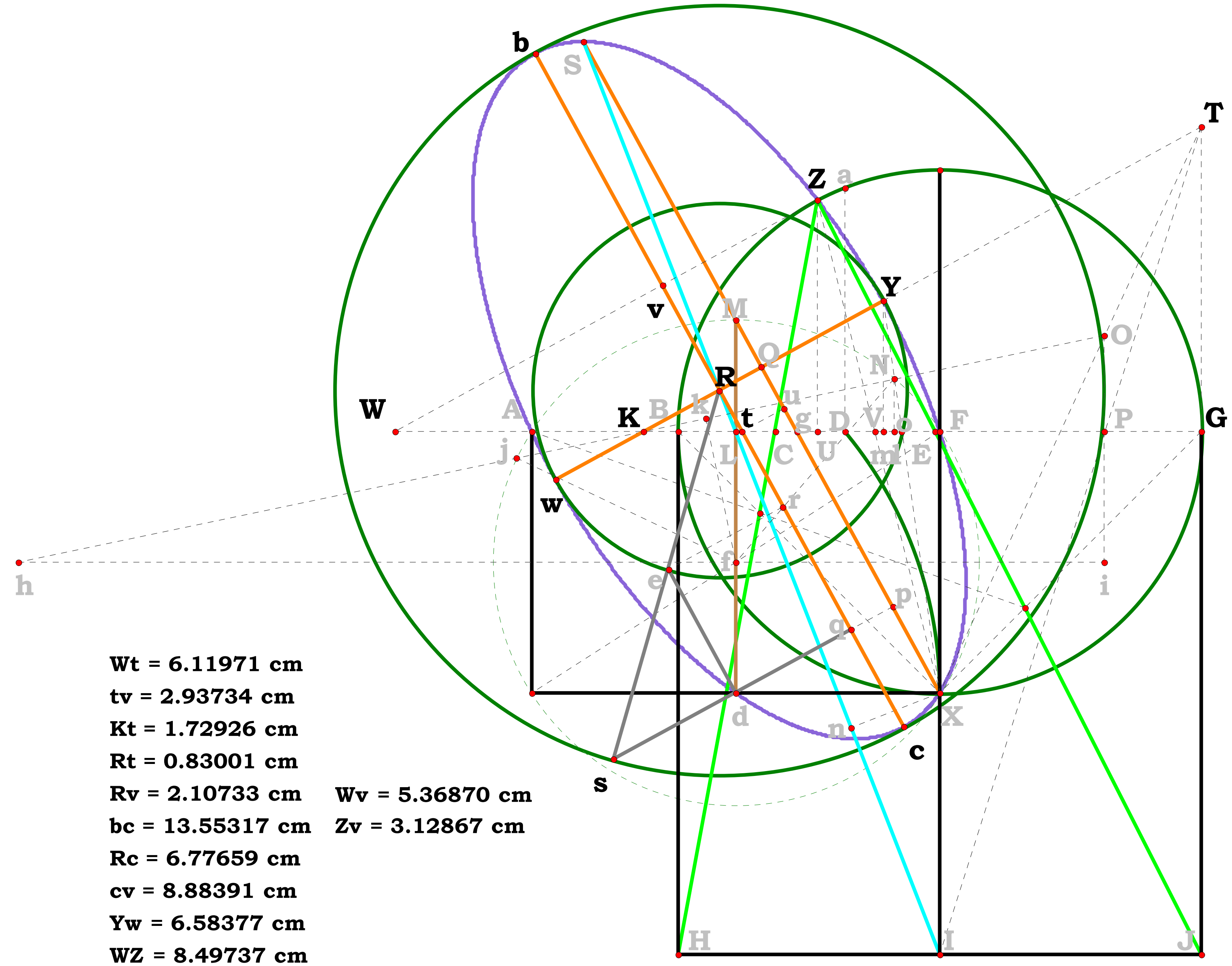
$Yw := 2 \cdot RY \qquad WZ := \dfrac{KT \cdot UZ}{GT} \qquad Wv := \dfrac{GK \cdot tv}{GT} \qquad Zv := |WZ - Wv|$

Definitions.

$Wt = 6.11971 \qquad tv = 2.93734 \qquad Kt = 1.72926 \qquad Rt = 0.83001$

$Rv = 2.10733 \qquad bc = 13.55318 \qquad Rc = 6.77659 \qquad cv = 8.88391$

$Yw = 6.58377 \qquad WZ = 8.49737 \qquad Wv = 5.36870 \qquad Zv = 3.12867$



Wt = 6.11971 cm
tv = 2.93734 cm
Kt = 1.72926 cm
Rt = 0.83001 cm
Rv = 2.10733 cm
bc = 13.55317 cm
Rc = 6.77659 cm
cv = 8.88391 cm
Yw = 6.58377 cm
WZ = 8.49737 cm
Wv = 5.36870 cm
Zv = 3.12867 cm

Given.

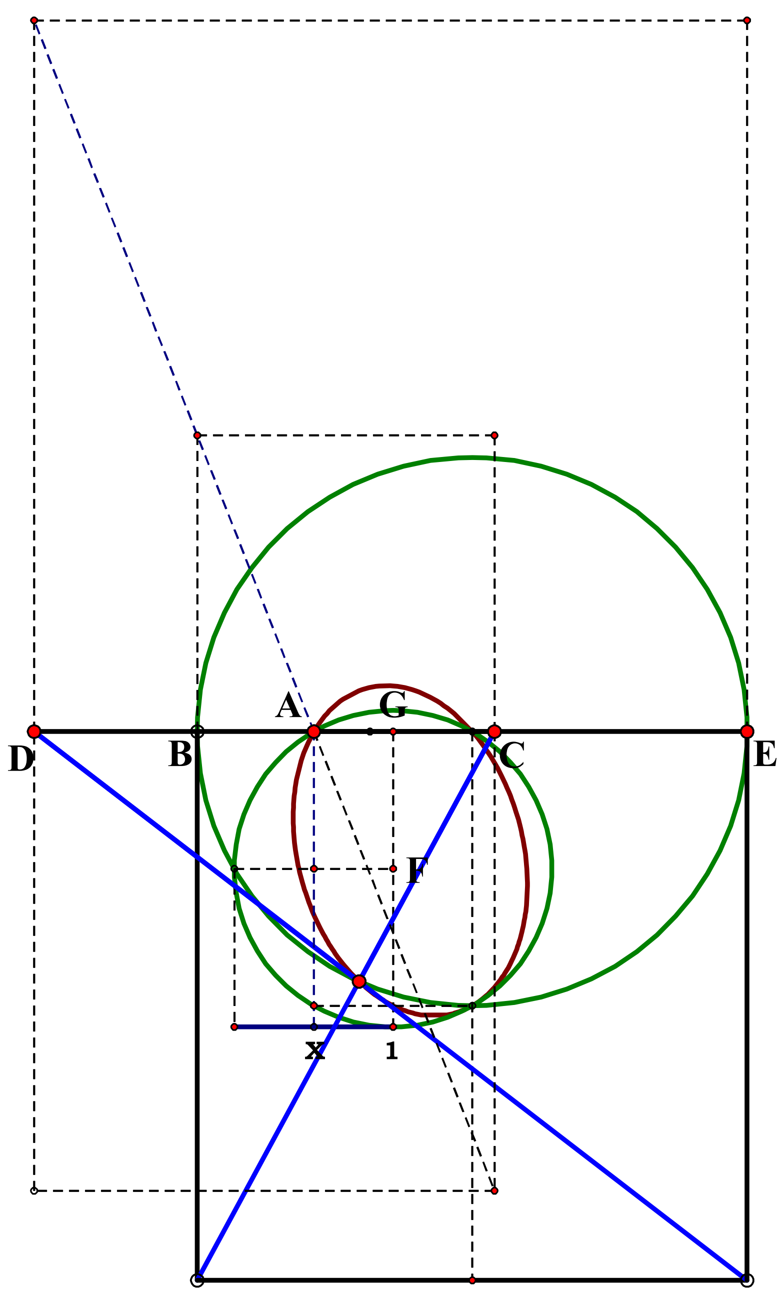
$N_1 := Yw \qquad N_2 := bc \qquad N_3 := cv \qquad N_4 := bc$

Definitions.

$$\sqrt{N_3 \cdot (N_4 - N_3) \cdot \frac{N_1}{N_2}} - Zv = 0.00000$$

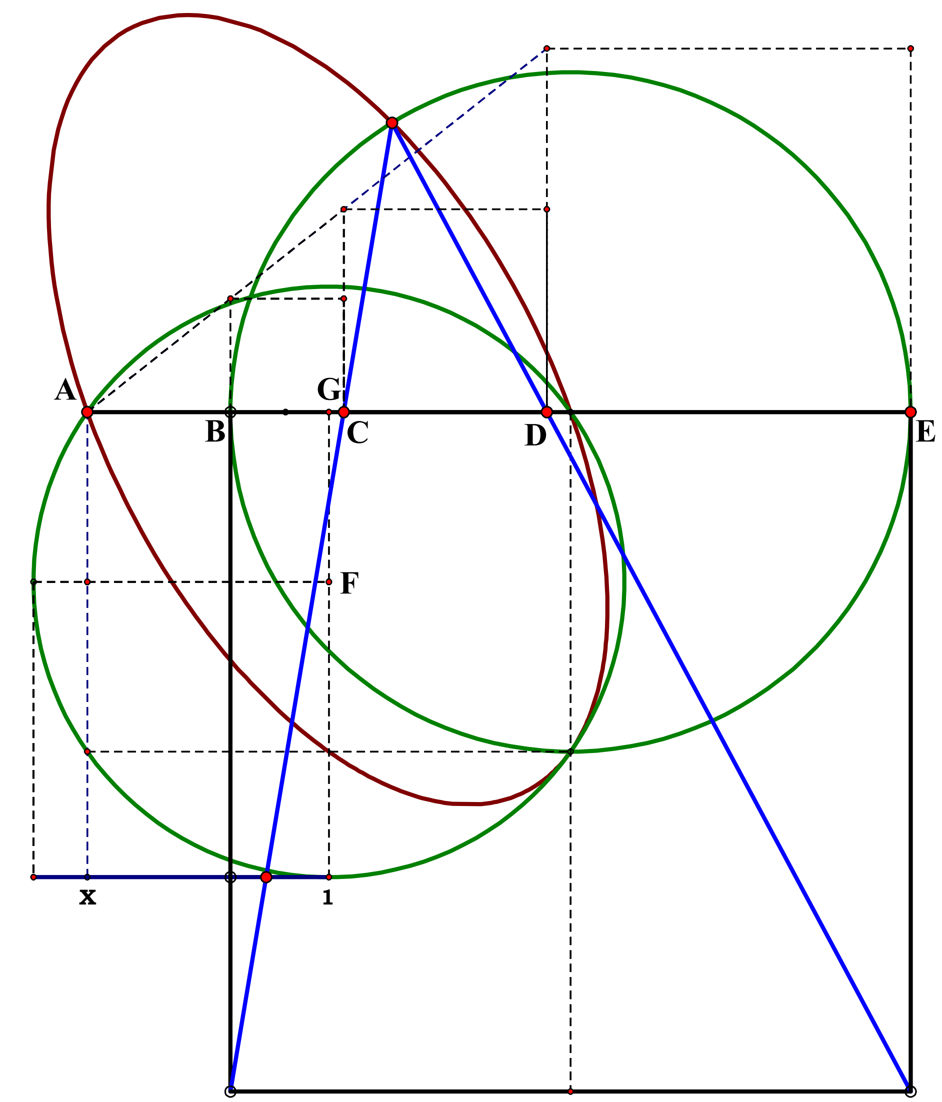
$\sqrt{N_3 \cdot (N_4 - N_3) \cdot \frac{N_1}{N_2}}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

As one can see, the figure covers the complete range of cubes and the full range of intersection of the ellipse with the major circle.



Unit = 1.00000
XY = 0.50076
X = 10.01523
Y = 20.00000

AB = 1.54284 cm
AC = 2.39035 cm
AD = 3.70342 cm
AE = 5.73777 cm
$(AB^2 \bullet AE)^{\frac{1}{3}}$ AC = 0.00000
$(AB \bullet AE^2)^{\frac{1}{3}}$ AD = 0.00000
AB = 1.00000
AC = 1.54932
AD = 2.40039
AE = 3.71897
$AE^{\frac{1}{3}}$ AC = 0.00000
$AE^{\frac{2}{3}}$ AD = 0.00000



Unit = 1.00000
XY = 0.18220
X = 3.64391
Y = 20.00000

AB = 2.24895 cm
AC = 4.02907 cm
AD = 7.21822 cm
AE = 12.93169 cm
$(AB^2 \bullet AE)^{\frac{1}{3}}$ AC = 0.00000
$(AB \bullet AE^2)^{\frac{1}{3}}$ AD = 0.00000
AB = 1.00000
AC = 1.79154
AD = 3.20960
AE = 5.75011
$AE^{\frac{1}{3}}$ AC = 0.00000
$AE^{\frac{2}{3}}$ AD = 0.00000

081899

Unit.

Given.

$N_1 := 5.05354 \qquad AC := N_1$

$N_2 := 14.49917 \qquad AF := N_2$

$N_3 := 5.71500 \qquad FN := N_3$

Promptly writing this up in 0816 2015

Exponential series by changing the unit, in other words, the same way as done inside a circle only this circle is getting smaller.

Descriptions.

$$CF := AF - AC \qquad CG := \frac{FN \cdot AC}{AF} \qquad EF := \frac{CF}{2} \qquad EH := \frac{FN \cdot (AC + EF)}{AF}$$

$$KN := \frac{EF \cdot FN}{EH} \qquad JK := CF - KN \qquad BC := \frac{JK \cdot CG}{FN - CG} \qquad BF := BC + CF$$

$$AD := AC + JK \qquad BD := BC + JK$$

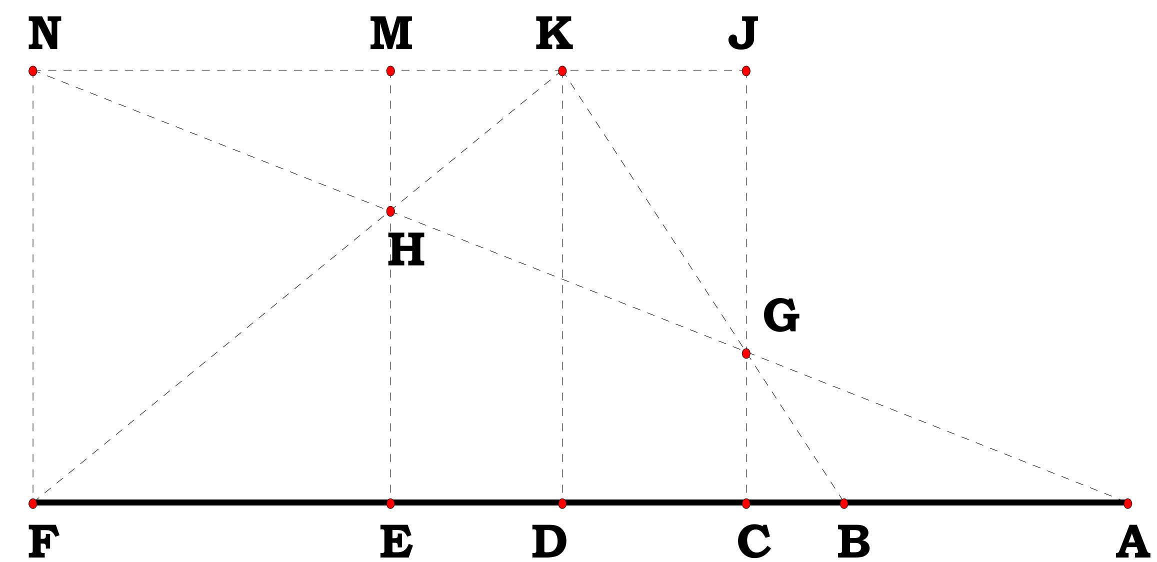
Definitions.

$$CF - (N_2 - N_1) = 0 \qquad \left(\frac{AF}{AC}\right)^2 - \frac{BF}{BC} = 0 \qquad \sqrt{\frac{BF}{BC}} - \frac{AF}{AC} = 0$$

$$CG - \frac{N_3 \cdot N_1}{N_2} = 0 \qquad EF - \frac{N_2 - N_1}{2} = 0 \qquad EH - \frac{N_3 \cdot (N_1 + N_2)}{2 \cdot N_2} = 0$$

$$KN - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \qquad JK - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \qquad BC - \frac{N_1^{\,2}}{N_1 + N_2} = 0$$

$$BF := \frac{N_2^{\,2}}{N_1 + N_2} \qquad AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0 \qquad BD - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$$



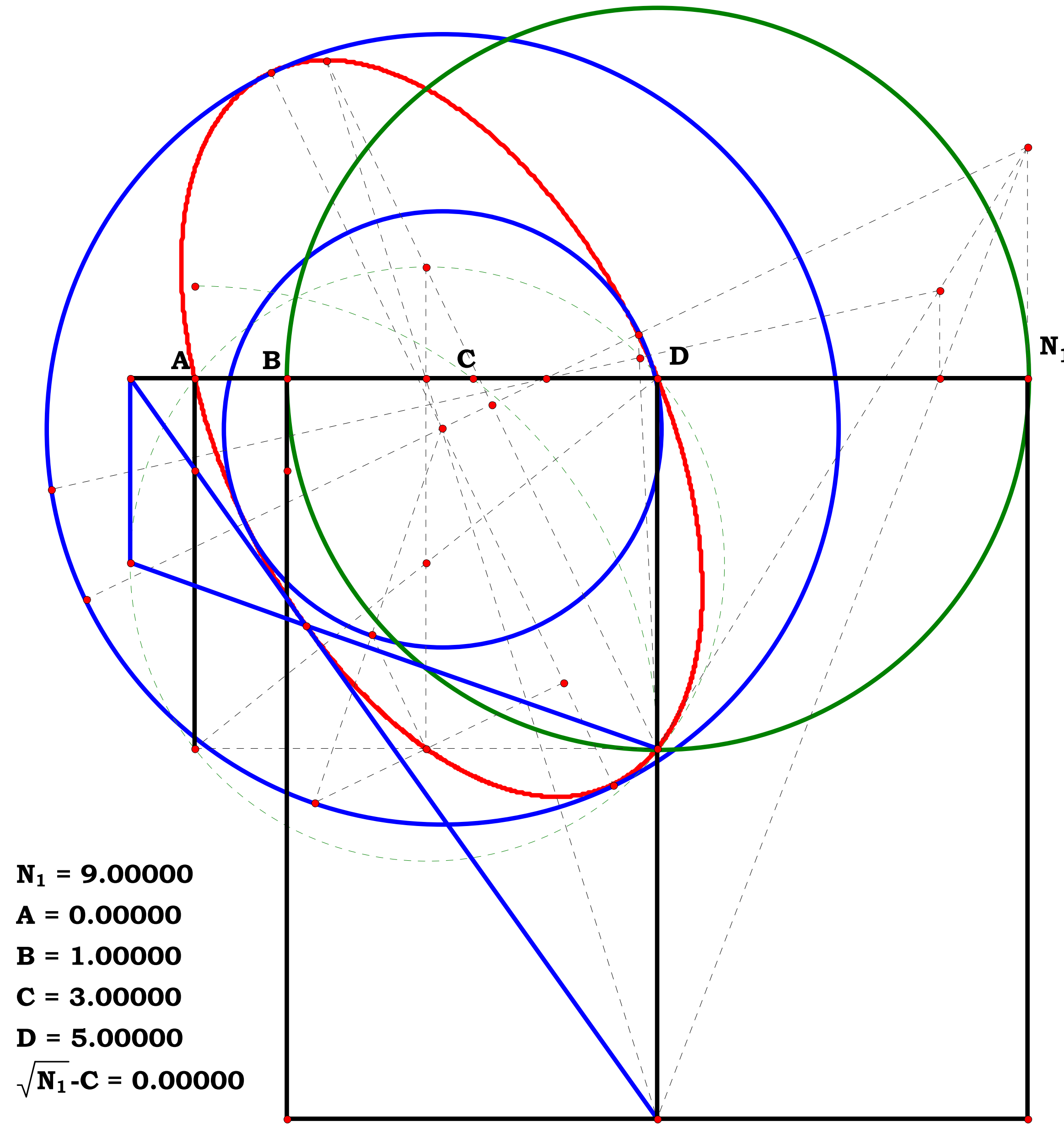
What is the name of the Ellipse which gives us the cube roots of any number?

081999

What about the Names?

Geometry, Algebra and Arithmetic are each and all binary grammars. All we do when writing up a figure is pair the name for any particular thing or the parts of a thing in terms of each of the convention of names provided by these three systems of grammar to each other. Each of these names are a binary expression, but they each use the convention of names given by a particular grammar. Traditionally, people have approached the topic in terms of precision, which is not right. The distinction is between the perceptible and the intelligible provided by each system of grammar which is expressible by our ability in that grammar system. Grammar cannot, in any wise, change the facts, nor can our ability with a grammar. Stupid people speak of proof as if proof determines the reality, the facts, which is wholly bizzar. All we are doing when pairing names is exercising our ability to do so. Proofing is only our ability to follow the intelligible using perceptible systems of grammar. So, by writing up complex figures one not only pushes their limits, but also learn how to fall back and go on naming in all three conventions. We go from universal expressions to expressions particular to where we are naming.

It is quite natural to become wholly frustrated when on reaches their own particular limits and the limits of even the computer that they may be using.



Unit.

$AB := 1$

Given.

$N_1 := 9$

Descriptions.

$$BN_1 := N_1 - AB \qquad BD := \frac{BN_1}{2}$$

$$DO := BD \qquad AD := BD + AB$$

$$AC := \sqrt{AD^2 - DO^2}$$

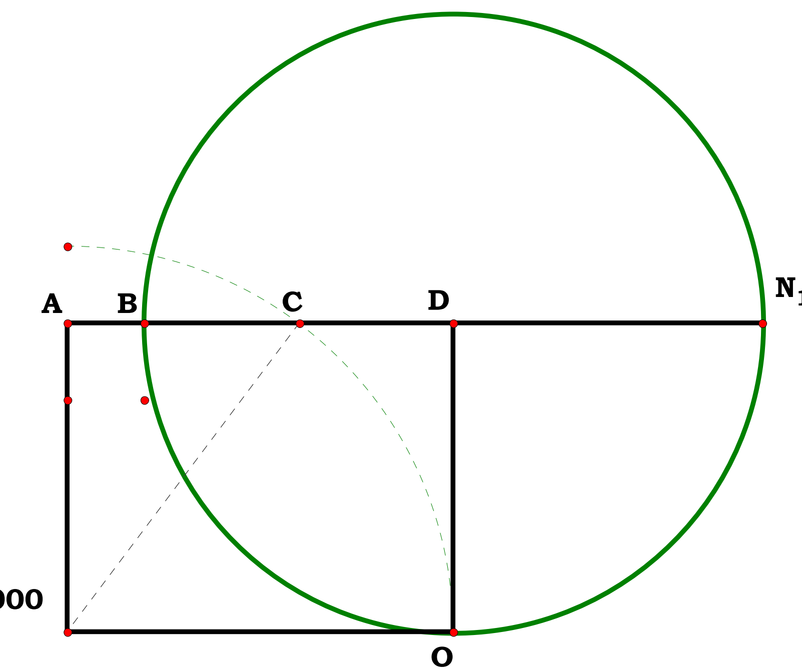
Definitions.

$$BN_1 - (N_1 - 1) = 0 \qquad BD - \frac{N_1 - 1}{2} = 0$$

$$DO - \frac{N_1 - 1}{2} = 0 \qquad AD - \frac{N_1 + 1}{2} = 0$$

$$AC - \sqrt{\left(\frac{N_1 + 1}{2}\right)^2 - \left(\frac{N_1 - 1}{2}\right)^2} = 0$$

$$AC - \sqrt{N_1} = 0$$



$N_1 = 9.00000$
$A = 0.00000$
$B = 1.00000$
$C = 3.00000$
$D = 5.00000$
$\sqrt{N_1}\text{-}C = 0.00000$

AC is the square root of N_1

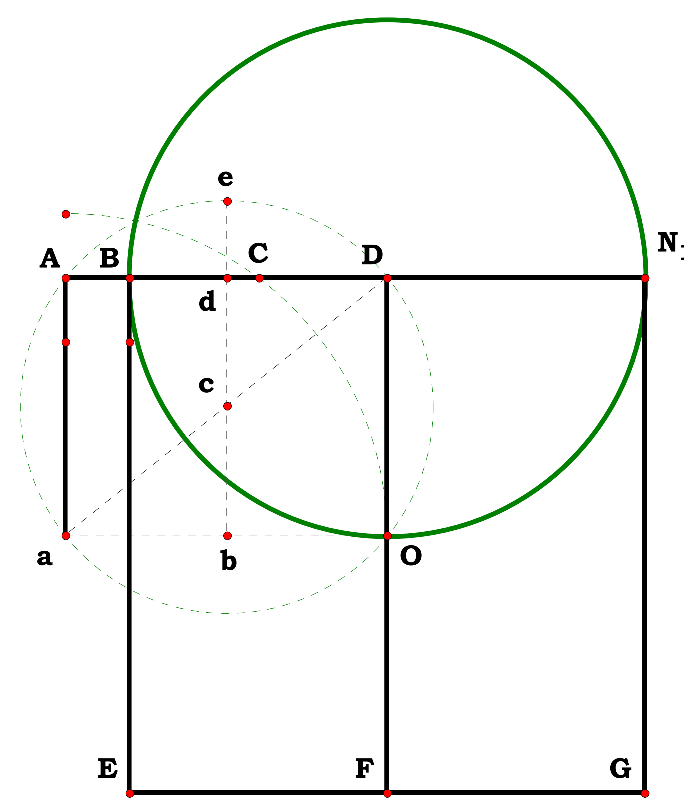
Descriptions.

$$Dd := \frac{AD}{2} \qquad Da := \sqrt{AD^2 + DO^2} \qquad ce := \frac{Da}{2} \qquad de := ce - \frac{BD}{2}$$

Definitions.

$$Dd - \frac{N_1 + 1}{4} = 0 \qquad Da - \frac{\sqrt{N_1^{\,2} + 1}}{\sqrt{2}} = 0 \qquad ce - \frac{\sqrt{2 \cdot (N_1^{\,2} + 1)}}{4} = 0$$

$$de - \frac{\sqrt{2 \cdot (N_1^{\,2} + 1)} - N_1 + 1}{4} = 0$$



$N_1 = 9.00000$

$Dd = 2.50000 \quad \frac{N_1+1}{4}\text{-}Dd = 0.00000$

$Da = 6.40312 \quad \frac{\sqrt{N_1^2+1}}{\sqrt{2}}\text{-}Da = 0.00000$

$ce = 3.20156 \quad \frac{\sqrt{2\cdot(N_1^2+1)}}{4}\text{-}ce = 0.00000$

$de = 1.20156 \quad \frac{(\sqrt{2\cdot(N_1^2+1)}\text{-}N_1)+1}{4}\text{-}de = 0.00000$

Descriptions.

$$DF := BN_1 \qquad Fd := \sqrt{Dd^2 + DF^2}$$

$$Df := DO + de \qquad df := \sqrt{Dd^2 + Df^2} \qquad Ff := DF - Df$$

$$Og := \frac{df \cdot DO}{Ff}$$

Definitions.

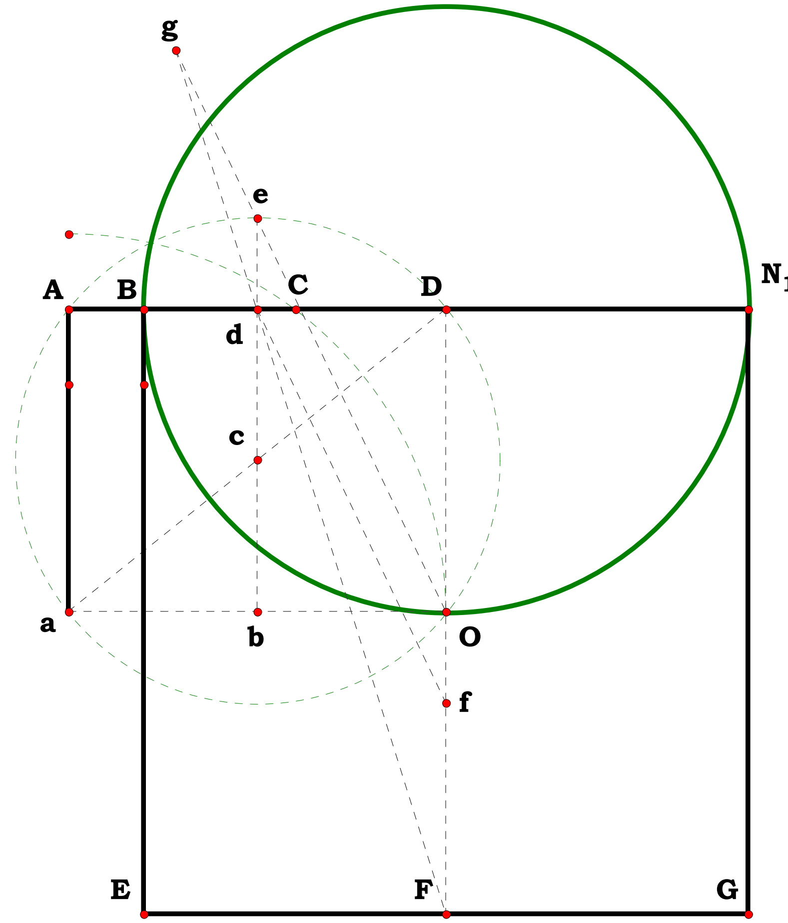
$$DF - (N_1 - 1) = 0 \qquad Fd - \frac{\sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}}{4} = 0$$

$$Df - \frac{N_1 + \sqrt{2 \cdot (N_1^2 + 1)} - 1}{4} = 0$$

$$df - \frac{\sqrt{2 \cdot N_1^2 + \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1) + 2}}{\sqrt{8}} = 0$$

$$Ff - \frac{3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3}{4} = 0$$

$$Og - \frac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)} + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{2 \left[3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3\right]} = 0$$



$N_1 = 9.00000$

$DF = 8.00000 \qquad N_1\text{-}1 = 8.00000$

$Fd = 8.38153 \qquad \dfrac{\sqrt{(17 \cdot N_1^2 - 30 \cdot N_1) + 17}}{4} - Fd = 0.00000$

$Df = 5.20156 \qquad \dfrac{\sqrt{2 \cdot N_1^2 + \sqrt{2 \cdot (N_1^2+1)} \cdot (N_1-1) + 2}}{\sqrt{8}} - df = 0.00000$

$df = 5.77116 \qquad \dfrac{\left(N_1 + \sqrt{2 \cdot (N_1^2+1)}\right) - 1}{4} - Df = 0.00000$

$Ff = 2.79844 \qquad \dfrac{3 \cdot N_1 - \sqrt{2 \cdot (N_1^2+1)} - 3}{4} - Ff = 0.00000$

$Og = 8.24911 \qquad \dfrac{\sqrt{2} \cdot (N_1-1) \cdot \sqrt{\left(2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2+1)}\right) + N_1 \cdot \sqrt{2 \cdot (N_1^2+1)} + 2}}{2 \cdot \left(3 \cdot N_1 - \sqrt{2 \cdot (N_1^2+1)} - 3\right)} - Og = 0.00000$

Descriptions.

$$OI := \frac{Og}{2} \qquad FO := DO \qquad Fh := \frac{DF \cdot FO}{Fd}$$

$$Oh := \sqrt{FO^2 - Fh^2} \qquad gh := \sqrt{Og^2 - Oh^2}$$

$$IJ := \frac{Oh \cdot OI}{gh}$$

Definitions.

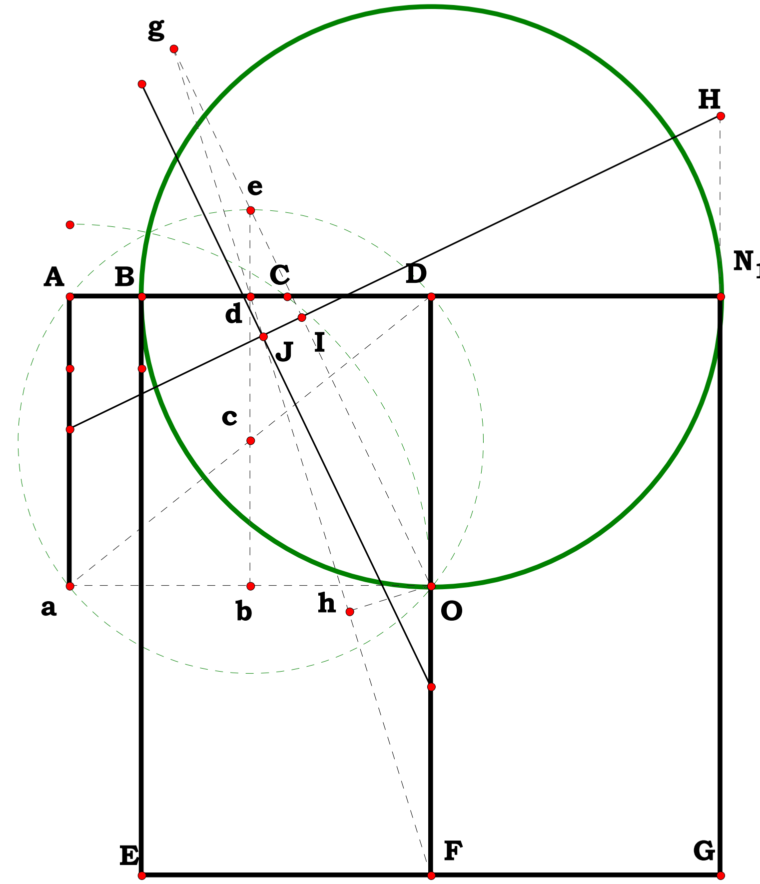
$$FO - \frac{N_1 - 1}{2} = 0 \qquad Fh - \frac{2 \cdot (N_1 - 1)^2}{\sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$

$$Oh - \frac{\sqrt{\left(N_1^2 - 1\right)^2}}{2 \cdot \sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$

$$OI - \frac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2\left(N_1^2 + 1\right)} + N_1 \cdot \sqrt{2\left(N_1^2 + 1\right)} + 2}}{4 \cdot \left[3 \cdot N_1 - \sqrt{2 \cdot \left(N_1^2 + 1\right)} - 3\right]} = 0$$

$$gh - \frac{\sqrt{(N_1 - 1)^2 \cdot \left[\left[8 \cdot (N_1 - 1) \cdot \left(5 \cdot N_1^2 - 6 \cdot N_1 + 5\right)\right] \cdot \sqrt{2\left(N_1^2 + 1\right)} + 57 \cdot \left(N_1^4 + 1\right) + 150 \cdot N_1^2 - 124 \cdot N_1^3 - 124 \cdot N_1\right]}}{\sqrt{\left[4 \cdot \left(17 \cdot N_1^2 - 30 \cdot N_1 + 17\right)\right] \cdot \left[\left[11 \cdot \left(N_1^2 + 1\right) - 18 \cdot N_1\right] - 6 \cdot \sqrt{2 \cdot \left(N_1^2 + 1\right)} \cdot (N_1 - 1)\right]}} = 0$$

$$IJ - \frac{\sqrt{2} \cdot \sqrt{-\left(68 \cdot N_1^2 - 120 \cdot N_1 + 68\right) \cdot \left[18 \cdot N_1 - 11 \cdot N_1^2 + 6 \cdot \sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{N_1^2 + 1} - 11\right]} \cdot \sqrt{\left(N_1^2 - 1\right)^2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2} \cdot \sqrt{N_1^2 + 1} + \sqrt{2} \cdot N_1 \cdot \sqrt{N_1^2 + 1} + 2}}{8 \cdot \sqrt{(N_1 - 1)^2 \cdot \left[150 \cdot N_1^2 - 124 \cdot N_1 - 124 \cdot N_1^3 + 57 \cdot N_1^4 + \sqrt{2} \cdot (8 \cdot N_1 - 8) \cdot \sqrt{N_1^2 + 1} \cdot \left(5 \cdot N_1^2 - 6 \cdot N_1 + 5\right) + 57\right]} \cdot \left(3 \cdot N_1 - \sqrt{2} \cdot \sqrt{N_1^2 + 1} - 3\right) \cdot \sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$



$$N_1 = 9.00000$$

$$\frac{OI}{AB} = 4.12456 \quad \frac{Fh}{AB} = 3.81792 \quad \frac{gh}{AB} = 8.16237 \quad \frac{FO}{AB} = 4.00000 \quad \frac{Oh}{AB} = 1.19310 \quad \frac{IJ}{AB} = 0.60289$$

Definitions (Arithmetic).

$OI = 4.124556$ $\quad FO = 4$

$Fh = 3.81792$ $\quad Oh = 1.1931$

$gh = 8.162374$ $\quad IJ = 0.602889$

At this point of complexity, I have to set the definitions aside and just compare the arithmetic. I really do not have all the time there is. This is the major reason I set this write-up aside for a later date, one which is after mine. As we find in the name of Shadows (Babylon 5,) a definition could become over 10,000 symbols long.

$$Oj := \frac{Df \cdot OI}{df} \qquad Ij := \frac{Dd \cdot Oj}{Df} \qquad Ik := DO + Ij$$

$$Hk := \frac{Dd \cdot Ik}{Df} \qquad Dj := DO - Oj \qquad HN_1 := Hk - Dj$$

$$HI := \frac{df \cdot Ik}{Df} \qquad HJ := HI + IJ \qquad lm := \frac{DO \cdot HN_1}{DF + HN_1}$$

$$nN_1 := \frac{Df \cdot HN_1}{Dd}$$

Definitions (Arithmetic).

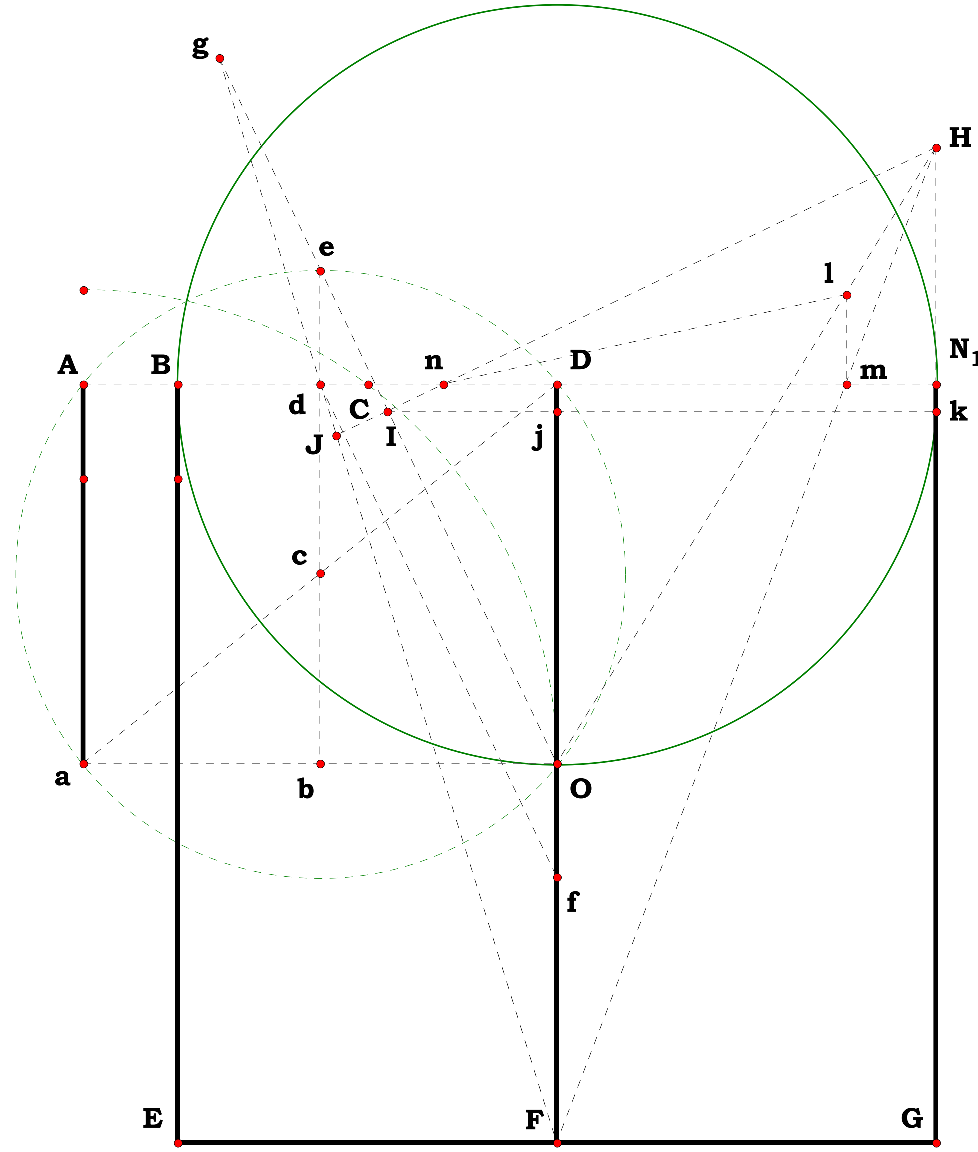
$Oj = 3.717475$ $\quad Ij = 1.786711$

$Ik = 5.786711$ $\quad Hk = 2.781237$

$Dj = 0.282525$ $\quad HN_1 = 2.498713$

$HI = 6.420382$ $\quad HJ = 7.023271$

$lm = 0.952007$ $\quad nN_1 = 5.198884$



$N_1 = 9.00000$

$AB = 1.25400$ cm

$\frac{Oj}{AB} = 3.71748$ $\quad \frac{Ij}{AB} = 1.78671$

$\frac{Ik}{AB} = 5.78671$ $\quad \frac{Hk}{AB} = 2.78124$

$\frac{Dj}{AB} = 0.28252$ $\quad \frac{HN_1}{AB} = 2.49871$

$\frac{HI}{AB} = 6.42038$ $\quad \frac{HJ}{AB} = 7.02327$

$\frac{ml}{AB} = 0.95201$ $\quad \frac{nN_1}{AB} = 5.19888$

082999A

The first section of this is simply demonstrating that cube roots are producable as a ratio between BE and BO. In fact, using whole numbers, and a lot of time, one can actually construct an Arithmetic dictionary of such ratios.

Then sometime I am want to I show that AW passes through R, Y, S, V, S and T.

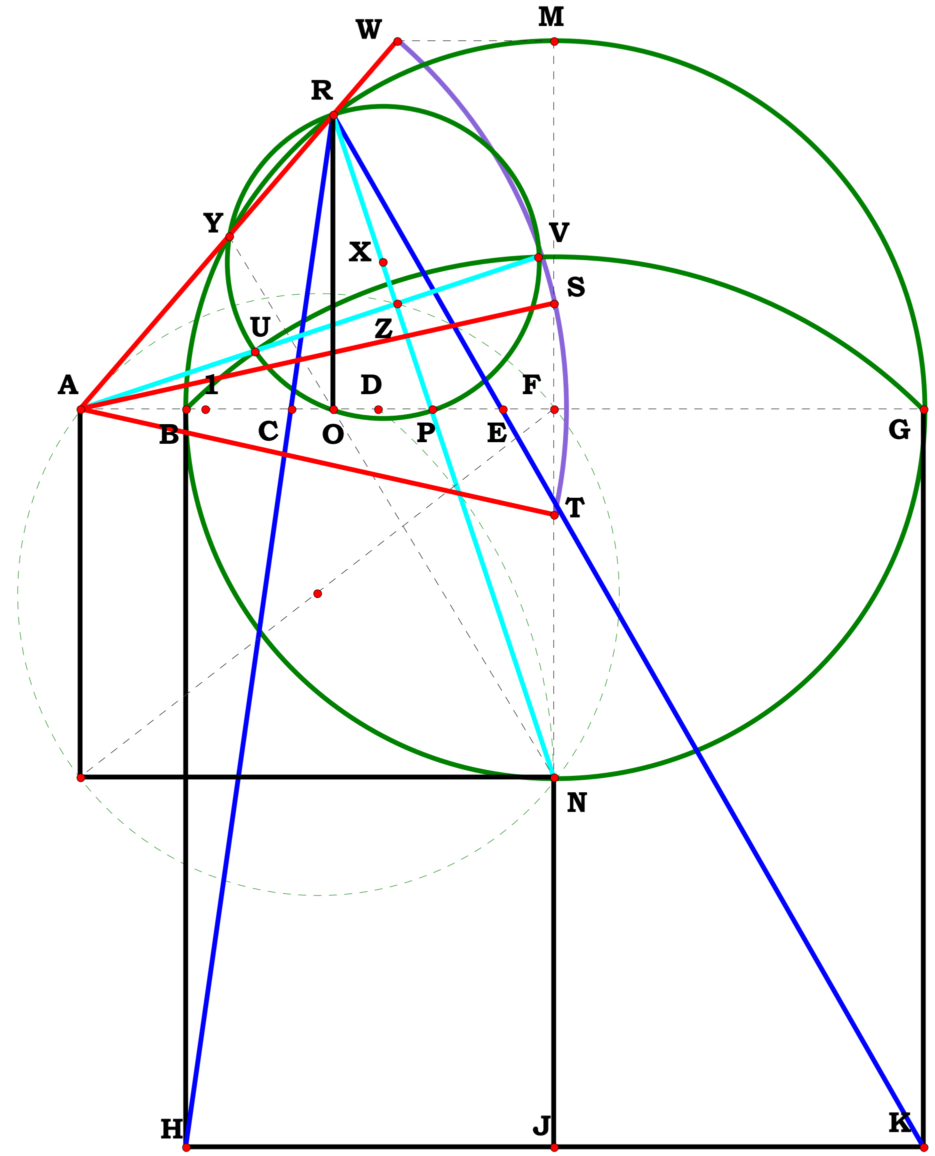
UV originates at A.

and;

The cord ST is equal to CE.

The circle RX pass through R, Y, O, P and V.

See E Go!



Unit.

$BG := 1$

Given.

$N_1 := 2$

$N_2 := 5$

Descriptions.

$BF := \frac{BG}{2}$ $\quad BO := \frac{N_1 \cdot BF}{N_2}$ $\quad GO := BG - BO$ $\quad OR := \sqrt{BO \cdot GO}$

$FN := BF$ $\quad FO := BF - BO$ $\quad FP := \frac{FO \cdot FN}{FN + OR}$ $\quad GK := BG$ $\quad BH := BG$

$BC := \frac{BO \cdot BH}{BH + OR}$ $\quad GE := \frac{GO \cdot GK}{GK + OR}$ $\quad CE := BG - (BC + GE)$

$BE := BC + CE$ $\quad AB := \frac{BC^2}{CE - BC}$ $\quad AG := AB + BG$ $\quad \frac{AG}{AB} = 8$

Definitions.

$BF - \frac{1}{2} = 0$ $\quad BO - \frac{N_1}{2 \cdot N_2} = 0$ $\quad GO - \frac{2 \cdot N_2 - N_1}{2 \cdot N_2} = 0$

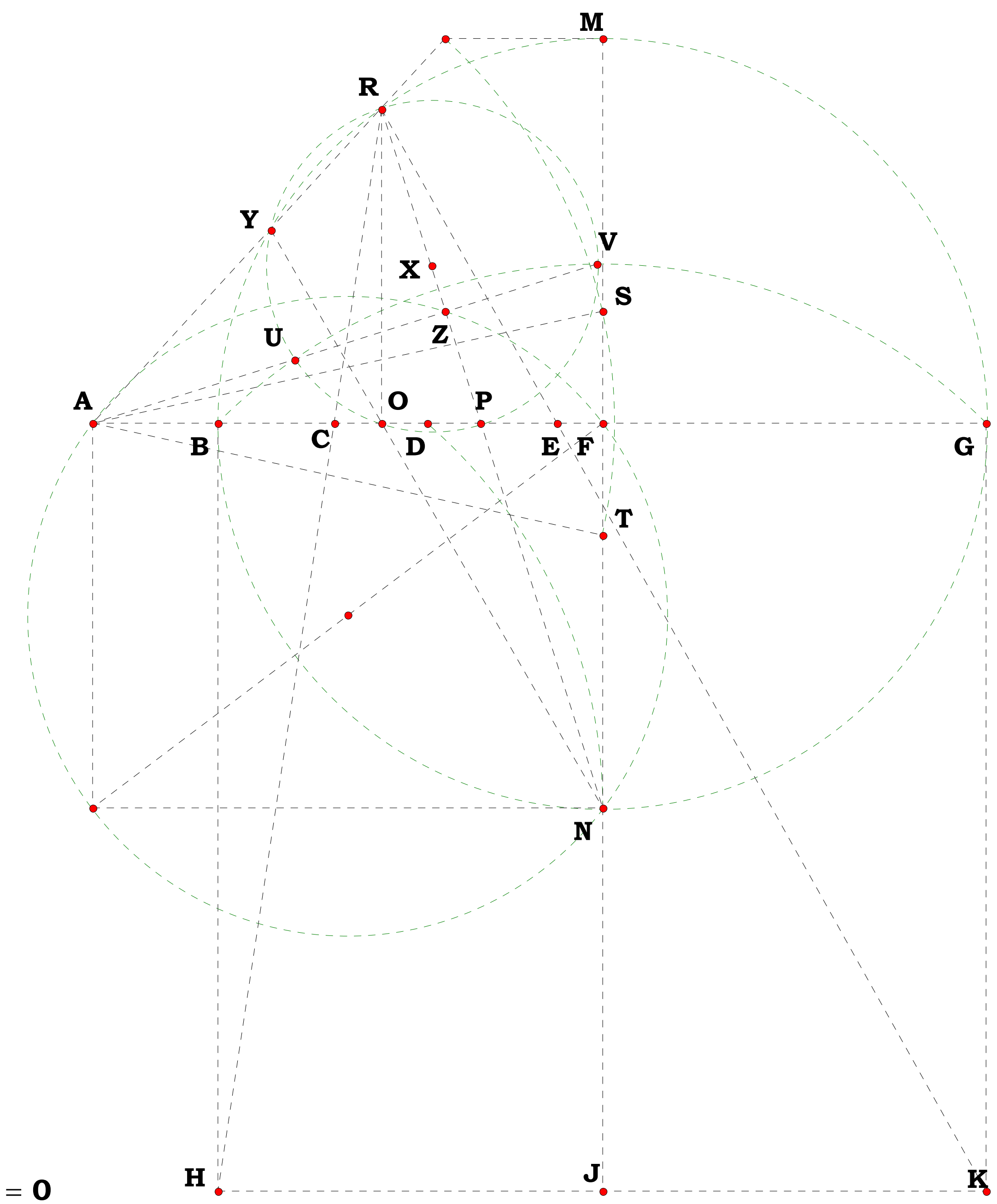
$OR - \frac{\sqrt{N_1 \cdot (2 \cdot N_2 - N_1)}}{2 \cdot N_2} = 0$ $\quad FN - \frac{1}{2} = 0$ $\quad FO - \frac{N_2 - N_1}{2 \cdot N_2} = 0$

$FP - \frac{N_2 - N_1}{2 \cdot \left(N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}\right)} = 0$ $\quad GK - 1 = 0$ $\quad BH - 1 = 0$

$BC - \frac{N_1}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $\quad GE - \frac{(N_1 - 2 \cdot N_2) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - 2 \cdot N_2\right)}{N_1^2 - 2 \cdot N_1 \cdot N_2 + 4 \cdot N_2^2} = 0$

$CE - \frac{\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $\quad BE - \frac{N_1 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $\quad AB - \frac{N_1^2}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}\right) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - N_1\right)} = 0$

$$AG - \frac{\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} \cdot (N_1 - 2 \cdot N_2)}{N_1^2 + N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$$



$$AC := AB + BC$$

$$AE := AB + BE$$

$$\left(\frac{AG}{AB}\right)^{\left(\frac{1}{3}\right)} - \frac{AC}{AB} = 0 \qquad \left(\frac{AG}{AB}\right)^{\left(\frac{2}{3}\right)} - \frac{AE}{AB} = 0$$

I find it very strange that so called mathematicians claim that one cannot abstract cube roots in geomety when every grammar is a binary expression and, since cube roots is simply a two dimensional ratio. I grant that the process to most is complicated, however, complicated and impossible are not the same concept.

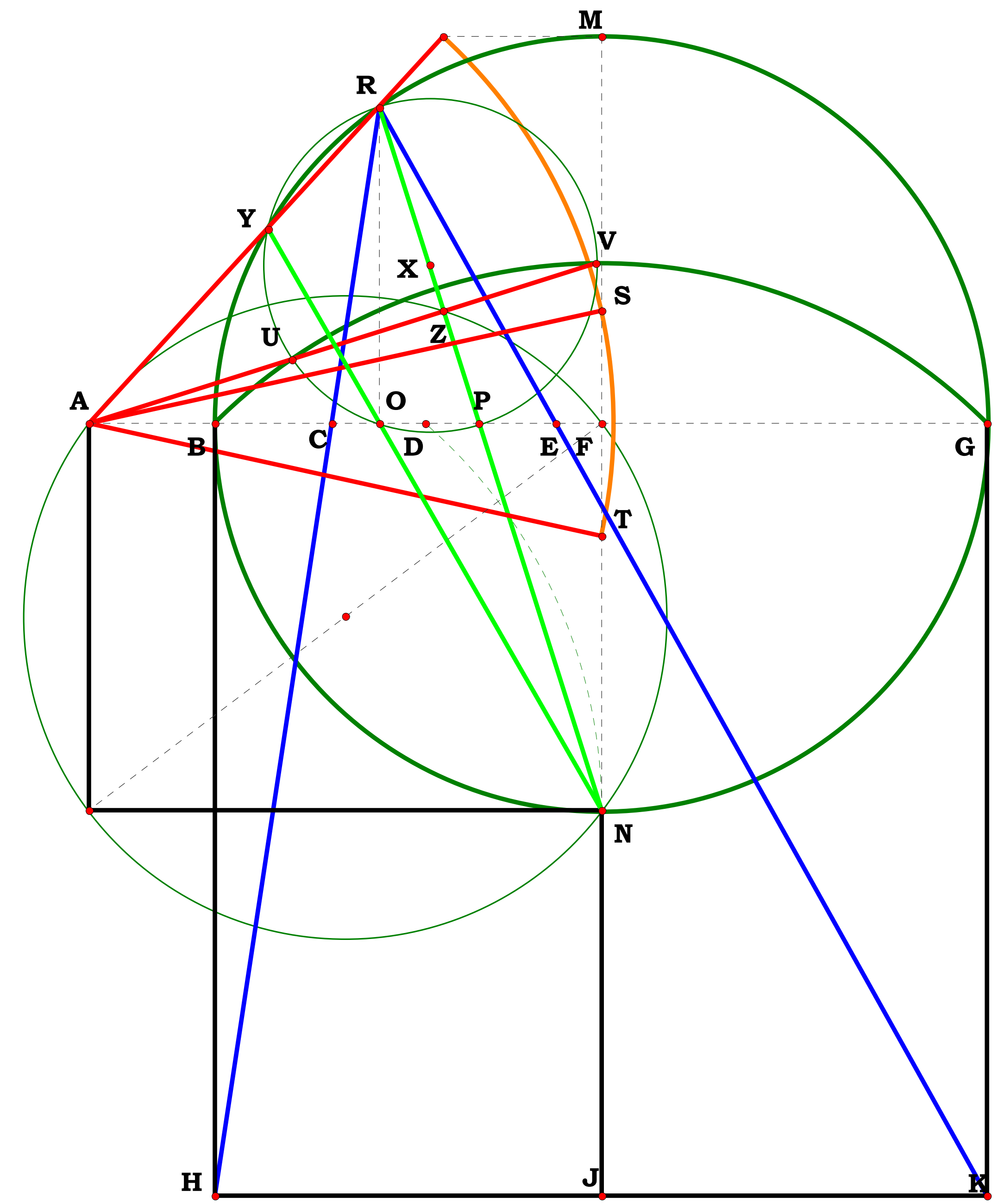
I have, by no means, finished the write-up of this figure as I want to find the equations to the remaining structures pointed out in the opening graphic, however, I am still in the early stages of this revision and may come back to it at some later date.

$$N_1 = 2 \qquad N_2 = 5$$

$$AB - \frac{N_1^{\,2}}{\left(2\cdot N_2 + \sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}}\right)\cdot\left(\sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}} - N_1\right)} = 0$$

$$AC - \frac{N_1\cdot\sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}}}{\left(2\cdot N_2\cdot\sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}} - N_1\cdot\sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}} - N_1^{\,2}\right)} = 0$$

$$AE - \frac{N_1\cdot\left(N_1 - 2\cdot N_2\right)}{N_1^{\,2} + N_1\cdot\sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}} - 2\cdot N_2\cdot\sqrt{2\cdot N_1\cdot N_2 - N_1^{\,2}}} = 0$$



083099A

Unit. $By := 1$

Given. $y := 20$

$x := 7$

Comming Through the Front Door.

One can prove the Solution to Cube Roots with this figure in a manner suggested as far back as the early Greeks, start with it the figure as proven, and work backward.

Descriptions.

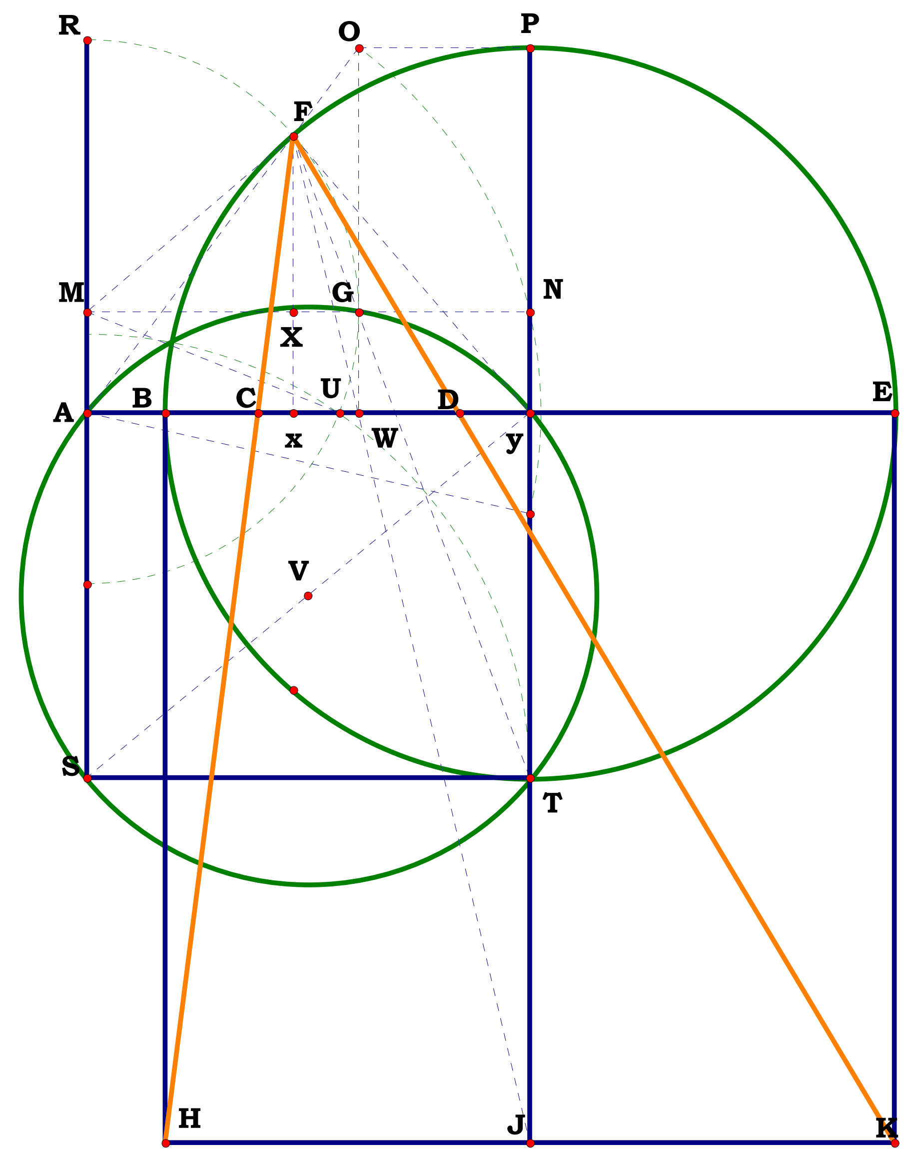
$Bx := By \cdot \frac{x}{y}$ $\quad BE := 2 \cdot By$ $\quad Fx := \sqrt{Bx \cdot (BE - Bx)}$ $\quad JT := By$ $\quad GW := \frac{Fx \cdot By}{(Fx + BE)}$

$Xx := GW$ $\quad xy := By - Bx$ $\quad FX := Fx - Xx$ $\quad MX := \frac{FX \cdot Fx}{xy}$ $\quad AB := MX - Bx$

$BC := \frac{Bx \cdot BE}{Fx + BE}$ $\quad Ex := BE - Bx$ $\quad DE := \frac{Ex \cdot BE}{Fx + BE}$ $\quad AE := AB + BE$ $\quad AC := BC + AB$

$AD := AE - DE$ $\quad \left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\quad \left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$$\frac{AE}{AB} - \frac{(x - 2 \cdot y) \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{x \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$$



$x = 7.00000$
$y = 20.00000$

$AB = 1.04618$ cm
$AC = 2.27150$ cm
$AD = 4.93198$ cm
$AE = 10.70851$ cm

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

Descriptions.

$By - 1 = 0$ $\quad Bx - \frac{x}{y} = 0$ $\quad BE - 2 = 0$ $\quad Fx - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{y} = 0$ $\quad JT - 1 = 0$

$GW - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{2 \cdot y + \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $\quad Xx - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{2 \cdot y + \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $\quad xy - \frac{(y - x)}{y} = 0$

$FX - \frac{y \cdot \sqrt{2 \cdot x \cdot y - x^2} - x^2 + 2 \cdot x \cdot y}{y \cdot \left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $\quad MX - \frac{\sqrt{x \cdot (2 \cdot y - x)} \cdot \left(x^2 - y \cdot \sqrt{2 \cdot x \cdot y - x^2} - 2 \cdot x \cdot y\right)}{y \cdot (x - y) \cdot \left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $\quad AB - \left[\frac{x \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (y - x)}\right] = 0$ $\quad BC - \frac{2 \cdot x}{2 \cdot y + \sqrt{-x \cdot (x - 2 \cdot y)}} = 0$

$Ex - \frac{(2 \cdot y - x)}{y} = 0$ $\quad DE - \frac{2 \cdot (2 \cdot y - x)}{2 \cdot y + \sqrt{-x \cdot (x - 2 \cdot y)}} = 0$ $\quad AE - \frac{(2 \cdot y - x) \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{(x - y) \cdot \left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $\quad AC := \frac{x \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)}$ $\quad AD - \left[\frac{(x - 2 \cdot y) \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)}\right] = 0$

083099B

Unit. $By := 1$

Given. $y := 20$

$x := 10$

Comming Through the Front Door.

Descriptions.

$Bx := By\cdot\frac{x}{y}$ $\quad BE := 2\cdot By$ $\quad Fx := \sqrt{Bx\cdot(BE-Bx)}$ $\quad JT := By$ $\quad GW := \frac{Fx\cdot By}{(BE-Fx)}$

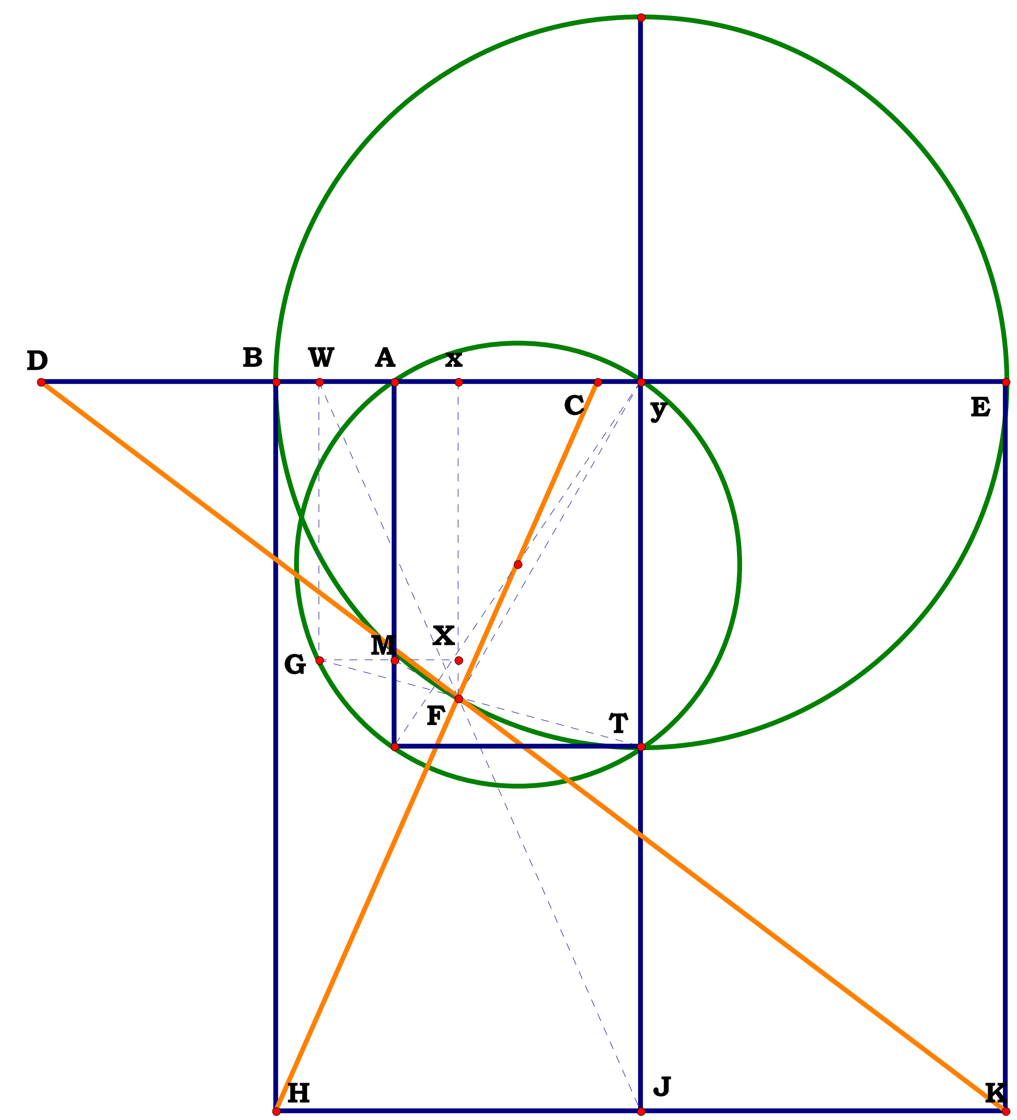
$Xx := GW$ $\quad xy := By - Bx$ $\quad FX := Fx - Xx$ $\quad MX := \frac{FX\cdot Fx}{xy}$ $\quad AB := Bx - MX$

$BC := \frac{Bx\cdot BE}{BE-Fx}$ $\quad Ex := BE - Bx$ $\quad DE := \frac{Ex\cdot BE}{BE-Fx}$ $\quad AE := BE - AB$ $\quad AC := BC - AB$

$AD := DE - AE$ $\quad \left(AB^2\cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\quad \left(AB\cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$$\frac{AE}{AB} - \frac{(x-2\cdot y)\cdot\left(x-2\cdot y-\sqrt{2\cdot x\cdot y-x^2}\right)}{x\cdot\left(x+\sqrt{2\cdot x\cdot y-x^2}\right)} = 0$$

One can see that the final equations are identical.



XY = 0.50000
X = 10.00000
Y = 20.00000

AB = 1.55941 cm
AC = 2.70097 cm
AD = 4.67823 cm
AE = 8.10292 cm
$(AB^2\cdot AE)^{\frac{1}{3}}$-AC = 0.00000
$(AB\cdot AE^2)^{\frac{1}{3}}$-AD = 0.00000
By = 4.83117 cm

Descriptions.

$By - 1 = 0$ $\quad Bx - \frac{x}{y} = 0$ $\quad BE - 2 = 0$ $\quad Fx - \frac{\sqrt{x\cdot(2\cdot y-x)}}{y} = 0$ $\quad JT - 1 = 0$

$GW - \frac{\sqrt{x\cdot(2\cdot y-x)}}{2\cdot y-\sqrt{x\cdot(2\cdot y-x)}} = 0$ $\quad Xx - \frac{\sqrt{x\cdot(2\cdot y-x)}}{2\cdot y-\sqrt{x\cdot(2\cdot y-x)}} = 0$ $\quad xy - \frac{y-x}{y} = 0$

$FX - \frac{y\cdot\sqrt{2\cdot x\cdot y-x^2}+x^2-2\cdot x\cdot y}{y\cdot\left(2\cdot y-\sqrt{2\cdot x\cdot y-x^2}\right)} = 0$ $\quad MX - \frac{\sqrt{x\cdot(2\cdot y-x)}\cdot\left(y\cdot\sqrt{2\cdot x\cdot y-x^2}+x^2-2\cdot x\cdot y\right)}{y\cdot(x-y)\cdot\left(\sqrt{2\cdot x\cdot y-x^2}-2\cdot y\right)} = 0$ $\quad AB - \frac{x\cdot\left(x-\sqrt{2\cdot x\cdot y-x^2}\right)}{\left(2\cdot y-\sqrt{2\cdot x\cdot y-x^2}\right)\cdot(x-y)} = 0$

$BC - \frac{2\cdot x}{2\cdot y-\sqrt{-x\cdot(x-2\cdot y)}} = 0$ $\quad Ex - \frac{(2\cdot y-x)}{y} = 0$ $\quad DE - \frac{2\cdot(2\cdot y-x)}{2\cdot y-\sqrt{x\cdot(2\cdot y-x)}} = 0$ $\quad AE - \left[2-\frac{x\cdot\left(x-\sqrt{2\cdot x\cdot y-x^2}\right)}{\left(2\cdot y-\sqrt{2\cdot x\cdot y-x^2}\right)\cdot(x-y)}\right] = 0$ $\quad AC - \left[\frac{x\cdot\left(x-2\cdot y+\sqrt{2\cdot x\cdot y-x^2}\right)}{\left(2\cdot y-\sqrt{2\cdot x\cdot y-x^2}\right)\cdot(x-y)}\right] = 0$

$AD := DE - AE$ $\quad \left(AB^2\cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\quad \left(AB\cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

100299

Enough here to keep one busy for a while.

The Circle and Segment, which if one can see it is fundamental to Jacob's Ladder which I use for BAM and BAG. They represent induction and deduction, arithmetic and geometric processing. This is why so many of my plates use it.

Again we see that trisection is directly related to square roots. However, there is just a lot of beauty seeing all of the interactions in the figure.

$AB = 5.42961$ cm

$AC = 9.87931$ cm

$AD = 17.97565$ cm

$\sqrt{AB \cdot AD} - AC = 0.00000$ cm

$m\angle DJB = 90.00000°$

$m\angle KJL = 30.00000°$

$\frac{m\angle DJB}{m\angle KJL} = 3.00000$

$m\angle JBH = 68.31951°$

$m\angle JBE = 22.77317°$

$m\angle EBG = 22.77317°$

$m\angle GBH = 22.77317°$

$\frac{m\angle JBH}{m\angle JBE} = 3.00000$

$m\angle JDH = 21.68049°$

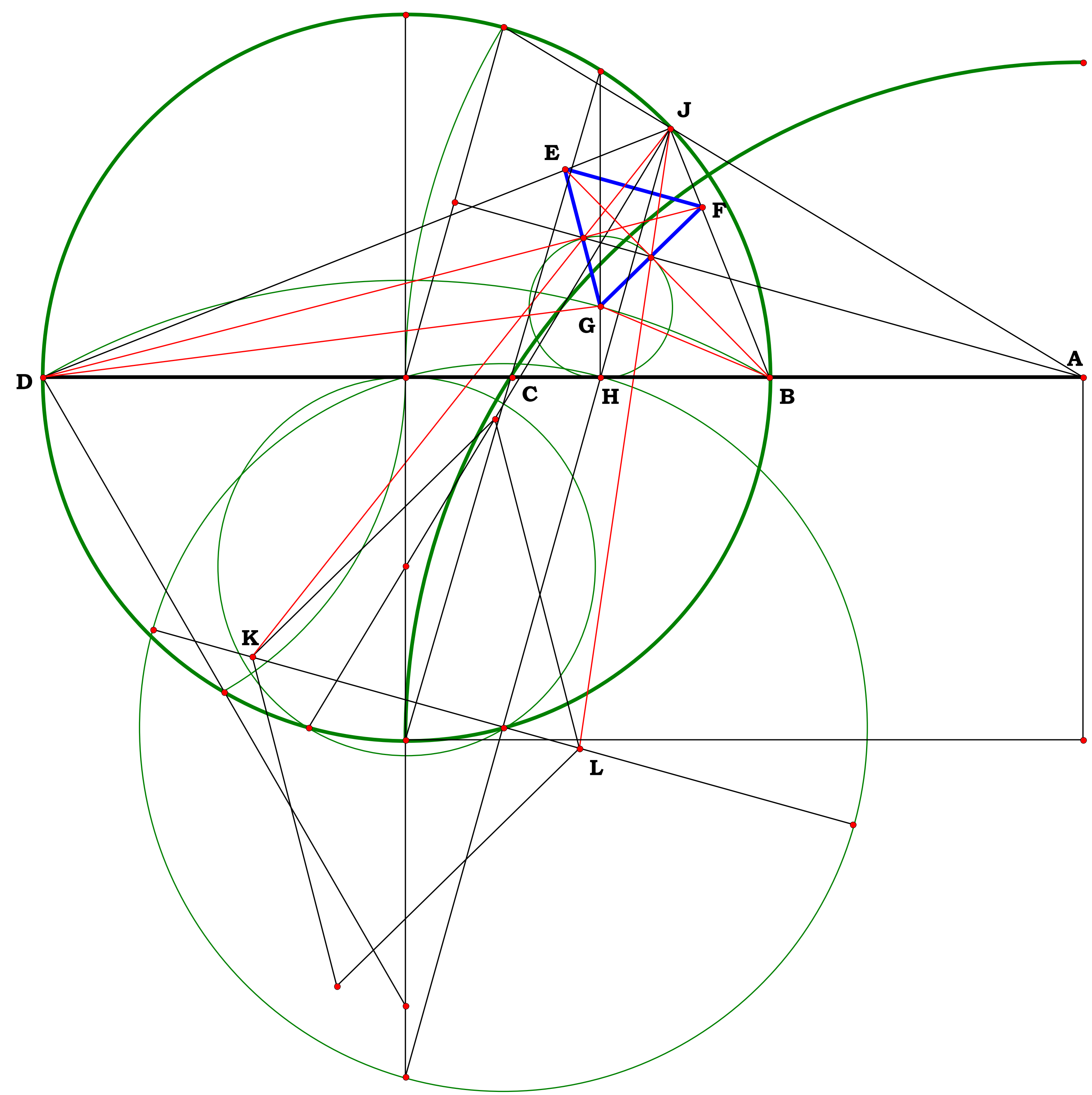
$m\angle JDF = 7.22683°$

$m\angle FDG = 7.22683°$

$m\angle GDH = 7.22683°$

$\frac{m\angle JDH}{m\angle JDF} = 3.00000$

Animate Point



100499

Parcing project

m∠KGB = 66.41899°
m∠KGI = 22.13966°

$$\frac{m\angle KGB}{m\angle KGI} = 3.00000$$

m∠LMN = 60.00000°
m∠MNL = 60.00000°
m∠NLM = 60.00000°

m∠PC1 = 66.41899°
m∠PCQ = 22.13966°

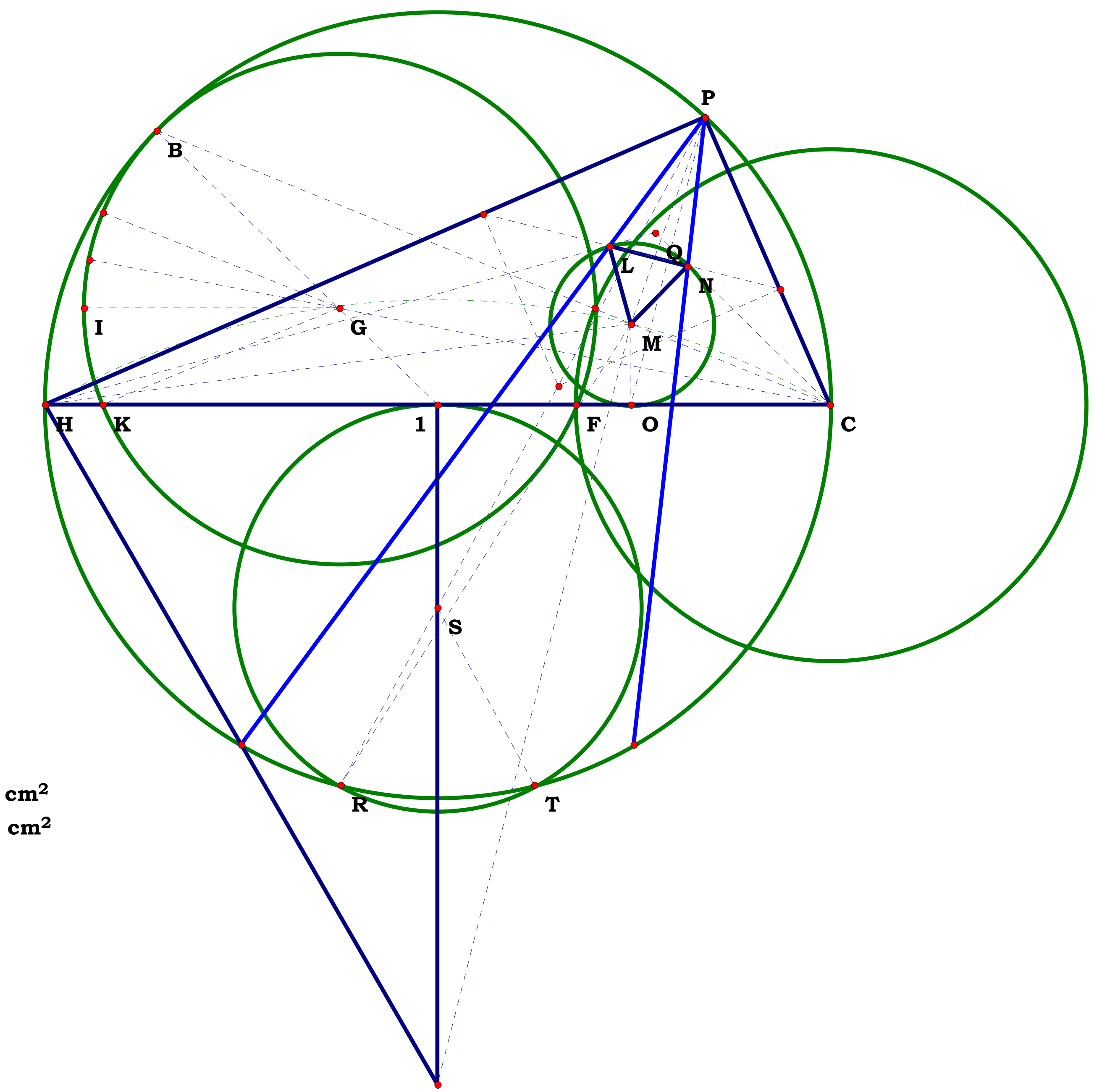
$$\frac{m\angle PC1}{m\angle PCQ} = 3.00000$$

m∠PH1 = 23.58101°
m∠PHL = 7.86034°

$$\frac{m\angle PH1}{m\angle PHL} = 3.00000$$

Area ⨀CF = 50.70109 cm²
Area ⨀GB = 50.70109 cm²

m∠RST = 57.11732°

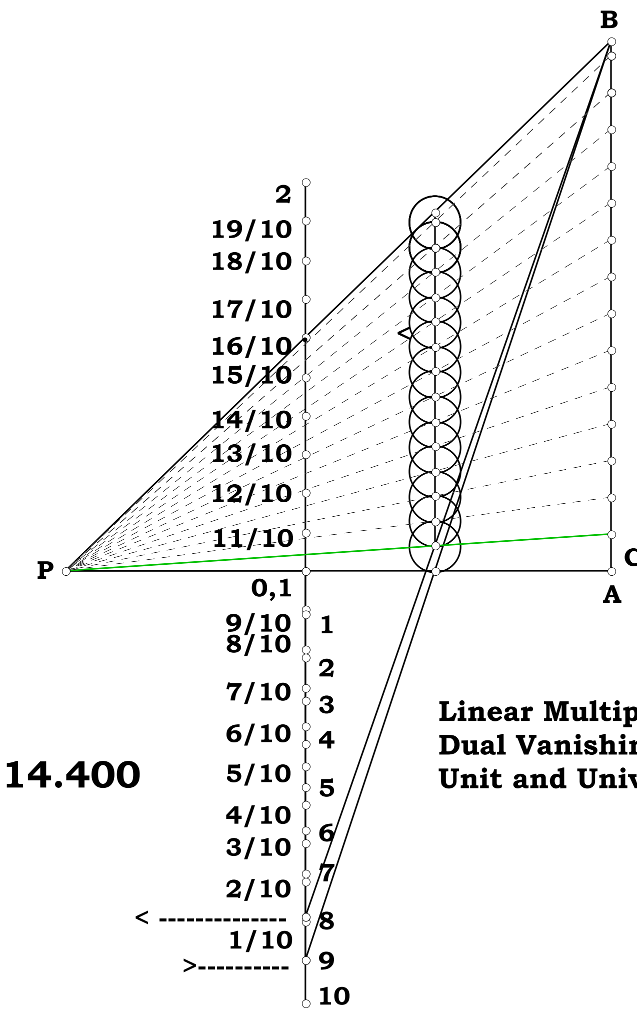


100799

1
2
3
4
5
6
7
8
9
10

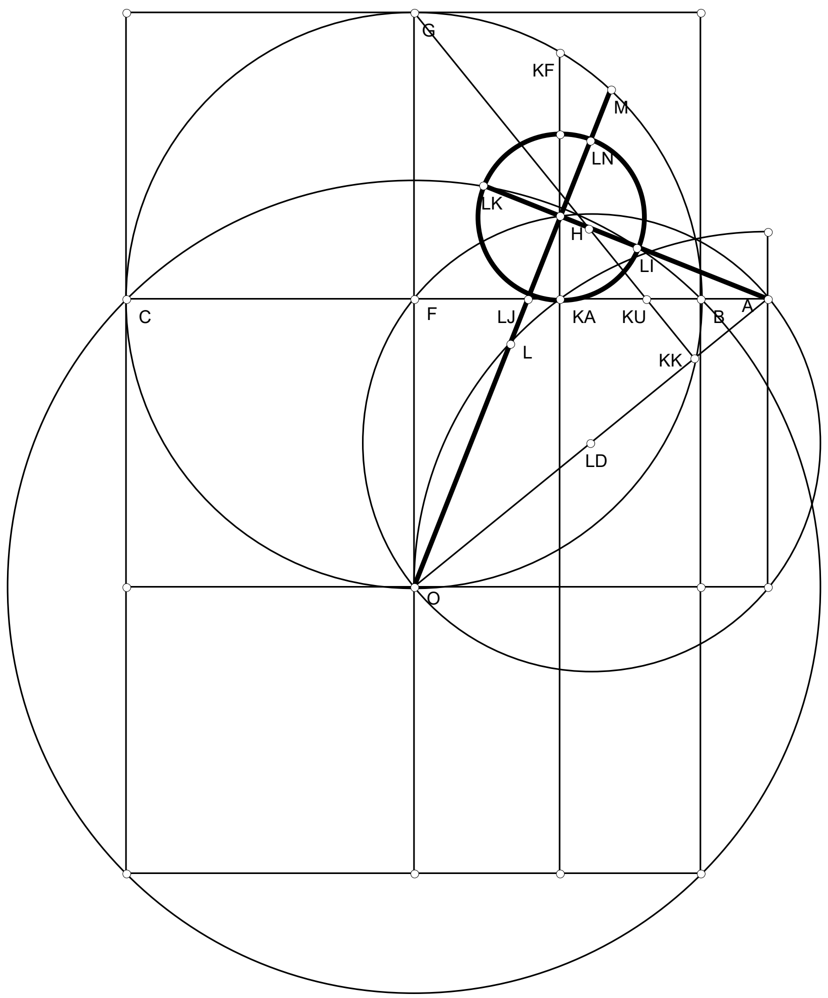
X 1/10
X 2/10
X 3/10
X 4/10
X 5/10
X 6/10
X 7/10
X 8/10
X 9/10
X 1
X 11/10
X 12/10
X 13/10
X 14/10
X 15/10
X 16/10
X 17/10
X 18/10
X 19/10
X 2

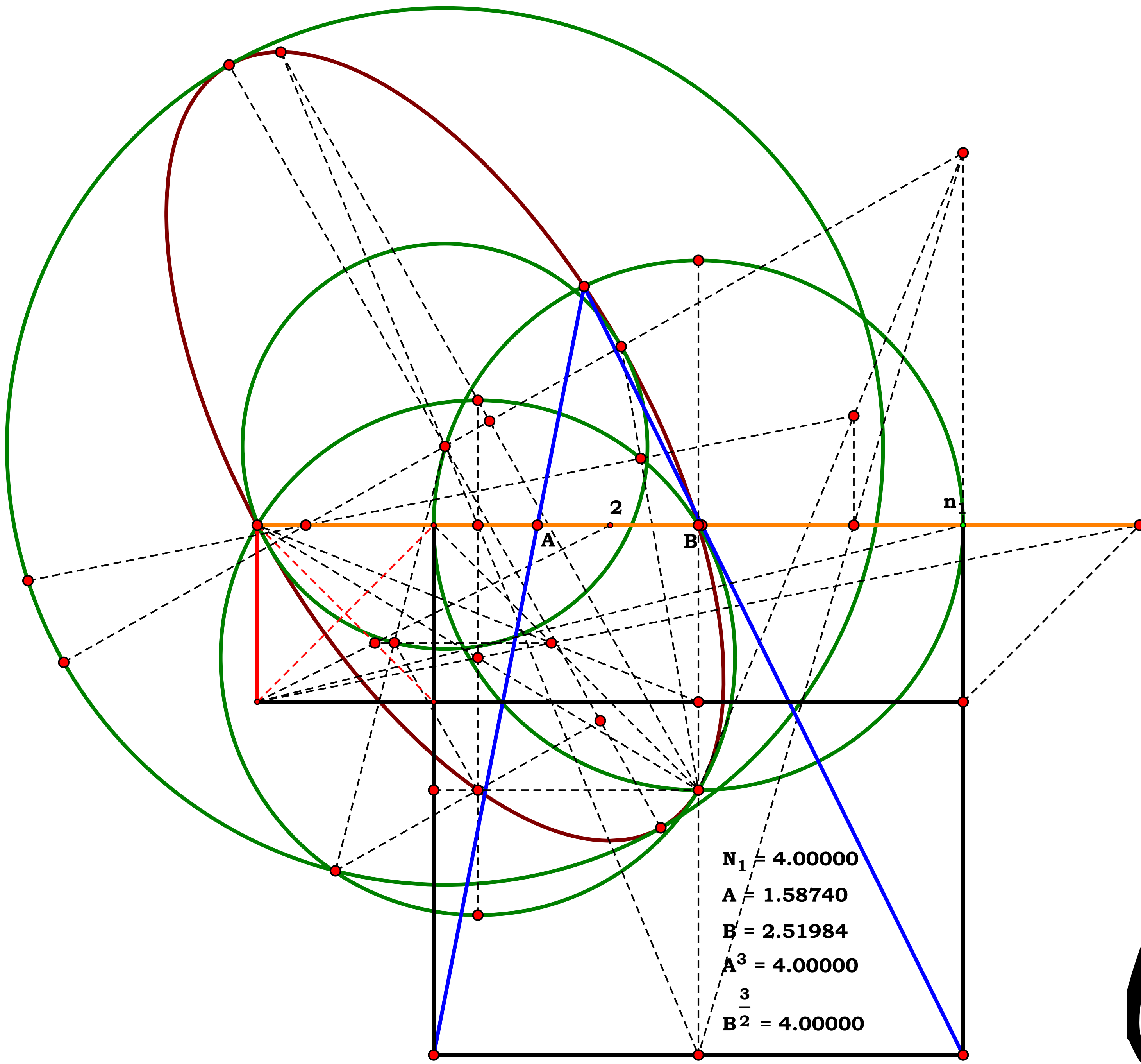
$$\frac{AB}{AC} = 14.400$$



Linear Multiplication and Division,
Dual Vanishing Point Method
Unit and Universe of Discourse.

101799





The Delian Quest 2000

John Clark

070200

Unit.

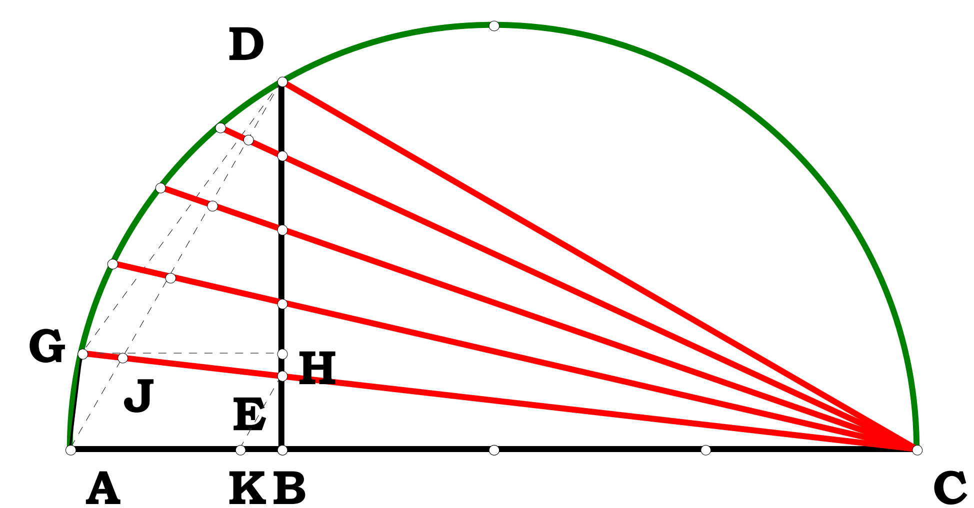
$AC := 1$

Given.

$N_1 := 4$

$N_2 := 5 \qquad \delta := 1 .. N_2$

In process. POR something or other.



Descriptions.

$AB := \dfrac{AC}{N_1} \qquad BC := AC - AB \qquad BD := \sqrt{AB \cdot BC} \qquad BE_\delta := \dfrac{BD \cdot \delta}{N_2} \qquad CE_\delta := \sqrt{(BE_\delta)^2 + BC^2}$

$CG_\delta := \dfrac{BC \cdot AC}{CE_\delta} \qquad AD := \sqrt{AB^2 + BD^2} \qquad AG_\delta := \dfrac{BE_\delta \cdot AC}{CE_\delta} \qquad EG_\delta := CG_\delta - CE_\delta \qquad EH_\delta := \dfrac{BE_\delta \cdot EG_\delta}{CE_\delta}$

$GH_\delta := \dfrac{BC \cdot EH_\delta}{BE_\delta} \qquad BH_\delta := BE_\delta + EH_\delta \qquad DH_\delta := BD - BH_\delta \qquad DG_\delta := \sqrt{(GH_\delta)^2 + (DH_\delta)^2}$

$BK_\delta := \dfrac{AB \cdot BE_\delta}{BD} \qquad CK_\delta := BC + BK_\delta \qquad CJ_\delta := \dfrac{CE_\delta \cdot AC}{CK_\delta} \qquad EJ_\delta := CJ_\delta - CE_\delta$

Definitions.

$AG_\delta =$

0.114708
0.225018
0.327327
0.419314
0.5

$\dfrac{\delta}{\sqrt{\delta^2 + N_1 \cdot N_2^2 - N_2^2}} =$

0.114708
0.225018
0.327327
0.419314
0.5

$DG_\delta =$

0.39736
0.292306
0.188982
0.090784
0

$(N_2 - \delta) \cdot \sqrt{\dfrac{(N_1 - 1)}{\left[N_1 \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)\right]}} =$

0.39736
0.292306
0.188982
0.090784
0

$EG_\delta =$

0.238416
0.204614
0.151186
0.081706
0

$\dfrac{\left(N_2^2 - \delta^2\right) \cdot \sqrt{N_1 - 1}}{N_2 \cdot N_1 \cdot \sqrt{\delta^2 + N_2^2 \cdot N_1 - N_2^2}} =$

0.238416
0.204614
0.151186
0.081706
0

$CG_\delta =$

0.993399
0.974355
0.944911
0.907841
0.866025

$N_2 \cdot \dfrac{(N_1 - 1)}{\sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}} =$

0.993399
0.974355
0.944911
0.907841
0.866025

$BE_\delta =$

0.086603
0.173205
0.259808
0.34641
0.433013

$\sqrt{\frac{(N_1-1)}{(N_1 \cdot N_1)}} \cdot \frac{\delta}{N_2} =$

0.086603
0.173205
0.259808
0.34641
0.433013

$EJ_\delta =$

0.188746
0.135837
0.088192
0.043481
0

$\frac{(N_2-\delta) \cdot \sqrt{(N_1-1) \cdot \left(\delta^2 + N_2{}^2 \cdot N_1 - N_2{}^2\right)}}{N_2 \cdot N_1 \cdot (N_2 \cdot N_1 - N_2 + \delta)} =$

0.188746
0.135837
0.088192
0.043481
0

$AD = 0.5$ $\sqrt{\frac{1}{N_1}} = 0.5$

$CJ_\delta =$

0.943729
0.905577
0.881917
0.869616
0.866025

$\frac{\sqrt{(N_1-1) \cdot \left(\delta^2 + N_2{}^2 \cdot N_1 - N_2{}^2\right)}}{N_2 \cdot N_1 - N_2 + \delta}$

0.943729
0.905577
0.881917
0.869616
0.866025

$\frac{AD}{AG_\delta} =$

4.358899
2.222049
1.527525
1.192424
1

$\frac{\sqrt{\frac{\left(\delta^2 + N_2{}^2 \cdot N_1 - N_2{}^2\right)}{N_1}}}{\delta} =$

4.358899
2.222049
1.527525
1.192424
1

$\frac{CG_\delta}{EG_\delta} =$

4.166667
4.761905
6.25
11.111111
$7.800463 \cdot 10^{15}$

$\mathrm{if}\left(N_2{}^2 - \delta^2, \frac{N_2{}^2 \cdot N_1}{N_2{}^2 - \delta^2}, 0\right) =$

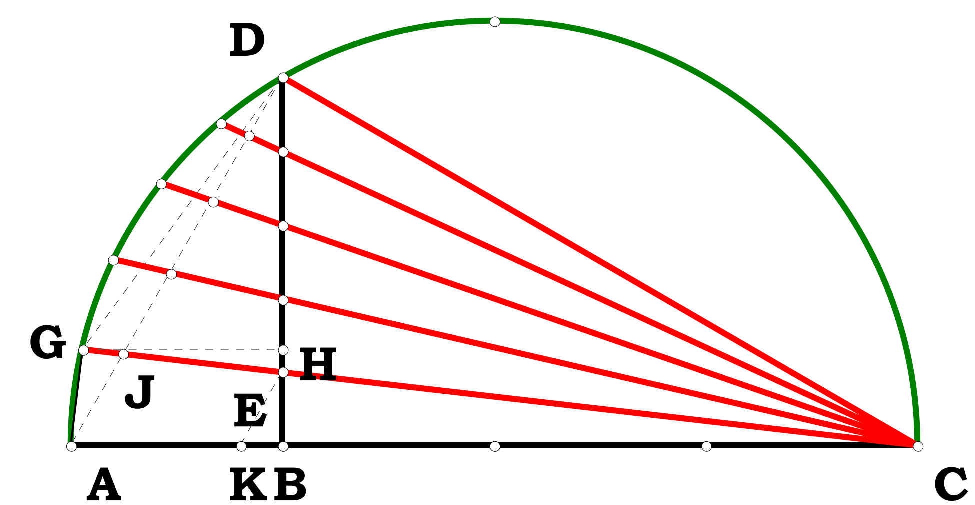
4.166667
4.761905
6.25
11.111111
0

$\mathrm{if}\left(EJ_\delta, \frac{CJ_\delta}{EJ_\delta}, 0\right) =$

5
6.666667
10
20
0

$\mathrm{if}\left(EJ_\delta, \frac{N_2 \cdot N_1}{N_2 - \delta}, 0\right) =$

5
6.666667
10
20
0



070900

Descriptions.

Unit.

Given.

$N_1 := 1.79201 \quad AB := N_1$

$N_2 := 10.41743 \quad AG := N_2$

Alternate Method Quad Roots

$AD := \sqrt{AB \cdot AG}$

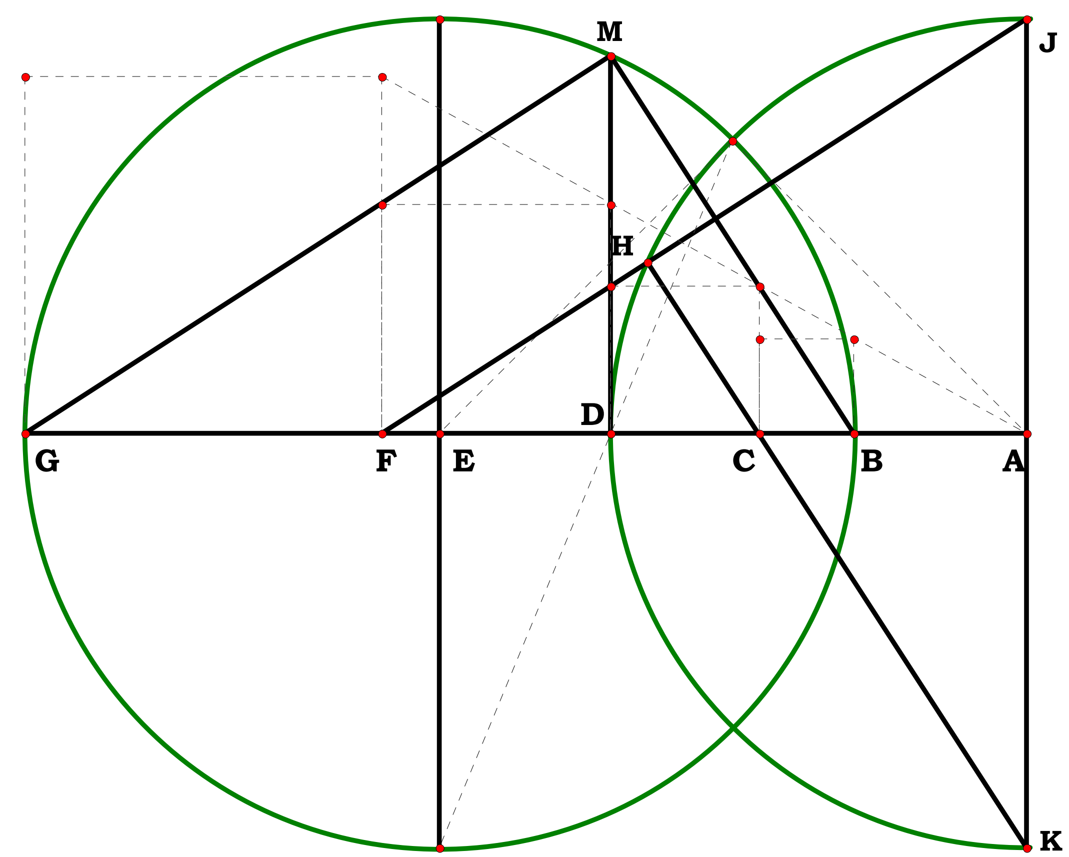
$BD := AD - AB \qquad BG := AG - AB$

$DG := BG - BD \qquad DM := \sqrt{BD \cdot DG}$

$AJ := AD \qquad AK := AD \qquad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \qquad AF := \dfrac{GM \cdot AJ}{BM}$

$AC := \dfrac{BM \cdot AK}{GM}$



AB = 2.27795 cm
AC = 3.53711 cm
AD = 5.49228 cm
AF = 8.52821 cm
AG = 13.24228 cm
AB = 1.00000
AC = 1.55276
AD = 2.41107
AF = 3.74382
AG = 5.81325

$AG^{\frac{3}{4}} - AF = 0.00000$

$AG^{\frac{2}{4}} - AD = 0.00000$

$AG^{\frac{1}{4}} - AC = 0.00000$

$AG^{\frac{0}{4}} - AB = 0.00000$

Definitions.

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \qquad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

Unit.
Given.

$N_1 := 3.73926 \qquad AB := N_1$

$N_2 := 11.78259 \qquad AF := N_2$

Quad Roots via Tangent Circles.

Descriptions.
000720a

$AD := \sqrt{AB \cdot AF} \qquad BF := AF - AB \qquad Bd := \dfrac{BF}{2} \qquad BD := AD - AB$

$Dd := Bd - BD \qquad DH := \sqrt{Bd^2 + Dd^2} \qquad HL := \dfrac{Bd \cdot BF}{DH}$

$DL := HL - DH \qquad Dk := \dfrac{Dd \cdot DL}{DH} \qquad Bk := Bd - (Dd + Dk) \qquad Lk := \dfrac{Bd \cdot Dk}{Dd}$

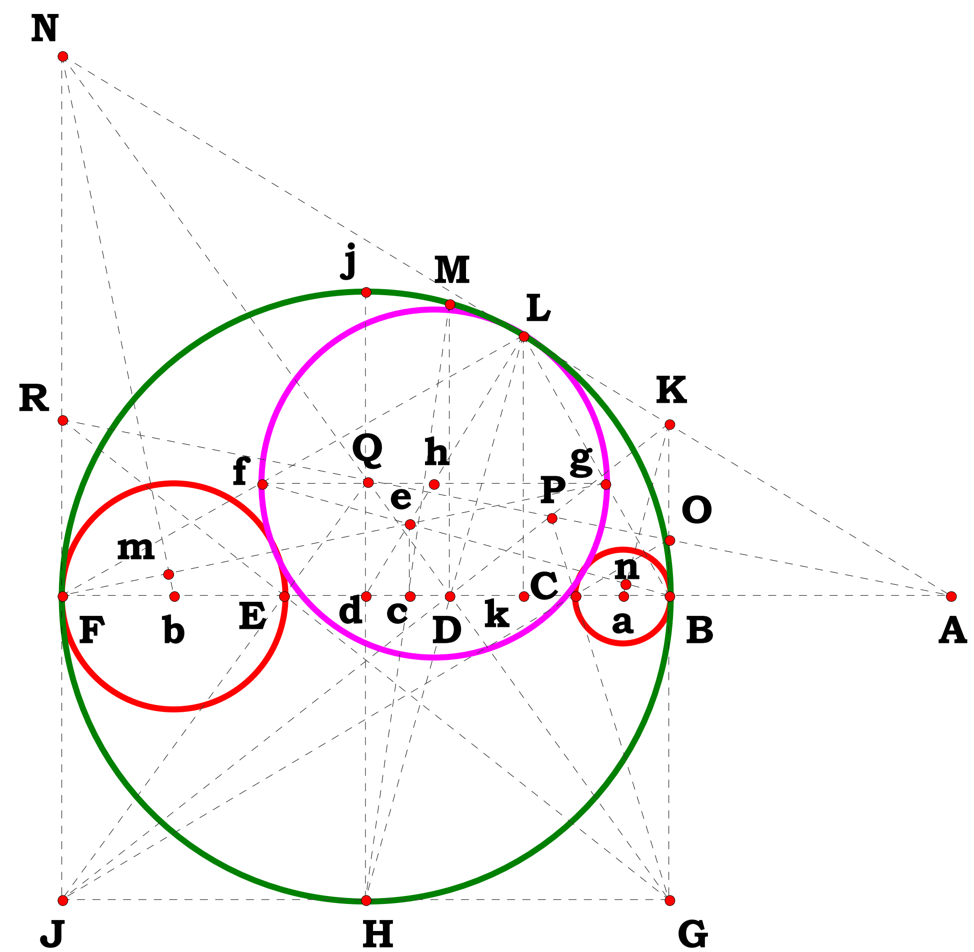
$JN := \dfrac{Bd \cdot BF}{BD} \qquad GK := \dfrac{Bd \cdot BF}{(Bd + Dd)} \qquad FN := JN - Bd \qquad BK := GK - Bd$

$Ak := Bk + AB \qquad AF - \dfrac{BF \cdot FN}{FN - BK} = 0 \qquad \dfrac{AF}{FN} - \dfrac{Ak}{Lk} = 0$

$DF := Bd + Dd \qquad DM := \sqrt{BD \cdot DF} \qquad cd := \dfrac{Dd \cdot Bd}{Bd + DM} \qquad dk := Bd - Bk$

$ce := \dfrac{Lk \cdot cd}{dk} \qquad Fb := \dfrac{ce \cdot FN}{Bd + cd} \qquad Ba := \dfrac{ce \cdot BK}{Bd - cd} \qquad AE := AF - 2 \cdot Fb$

$AC := AB + 2 \cdot Ba$



i.e., A, K, L and N are colinear.

Definitions.

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB \cdot AF^3\right)^{\frac{1}{4}} - AE = 0 \qquad$ etc., etc.

Unit.

Quad Roots by equal angles.

Given.

$N_1 := 2.07320$ $AB := N_1$

$N_2 := 10.53987$ $AF := N_2$

OOO720b

Descriptions.

$AD := \sqrt{AB \cdot AF}$ $BF := AF - AB$

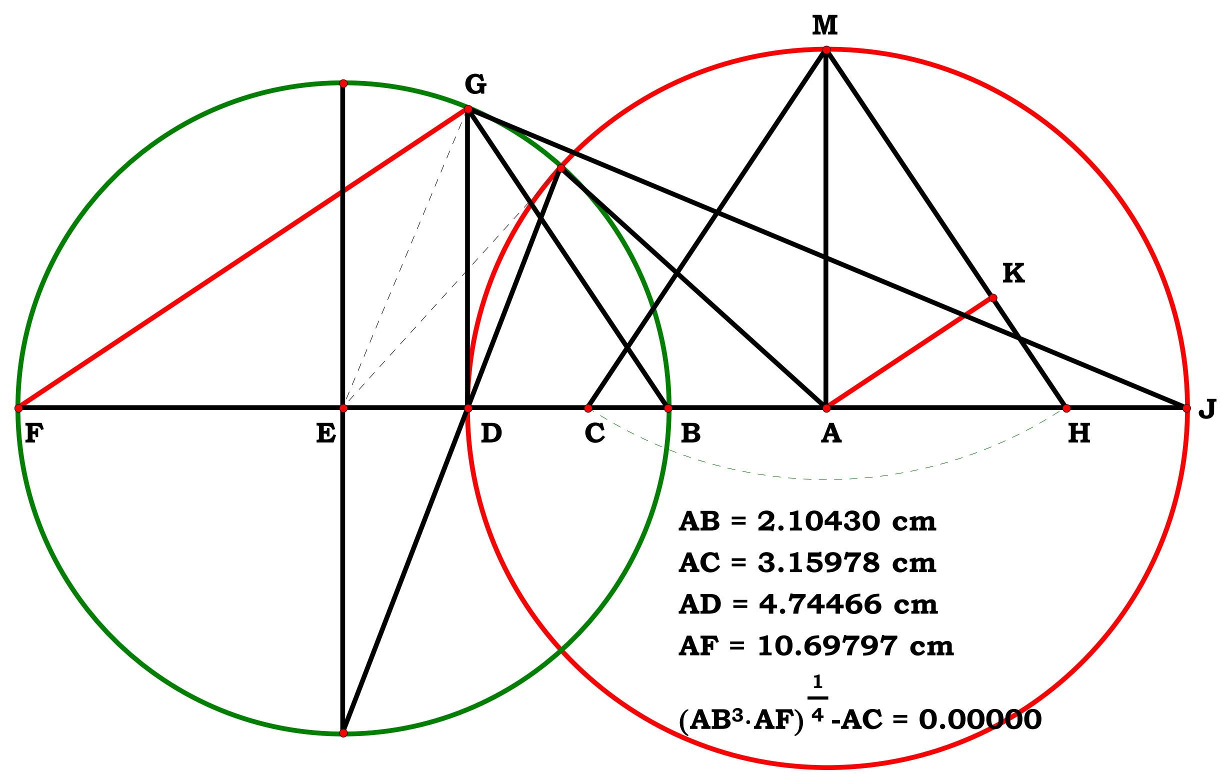
$AM := AD$ $DF := AF - AD$

$BD := AD - AB$ $DG := \sqrt{DF \cdot BD}$

$BG := \sqrt{DG^2 + BD^2}$ $AC := \frac{BD \cdot AD}{DG}$

Definitions.

$$AC - \left(AB^3 \cdot AF\right)^{\frac{1}{4}} = 0$$



Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

000801a

Descriptions.

$BG := AG - AB \quad BE := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \quad DE := AE - AD \quad NY := DE$

$BD := AD - AB \quad DG := BG - BD \quad EQ := BE$

$DN := \sqrt{BD \cdot DG} \quad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \quad OQ := BG \quad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \quad NP := NQ - PQ \quad MN := \sqrt{\frac{NP^2}{2}}$

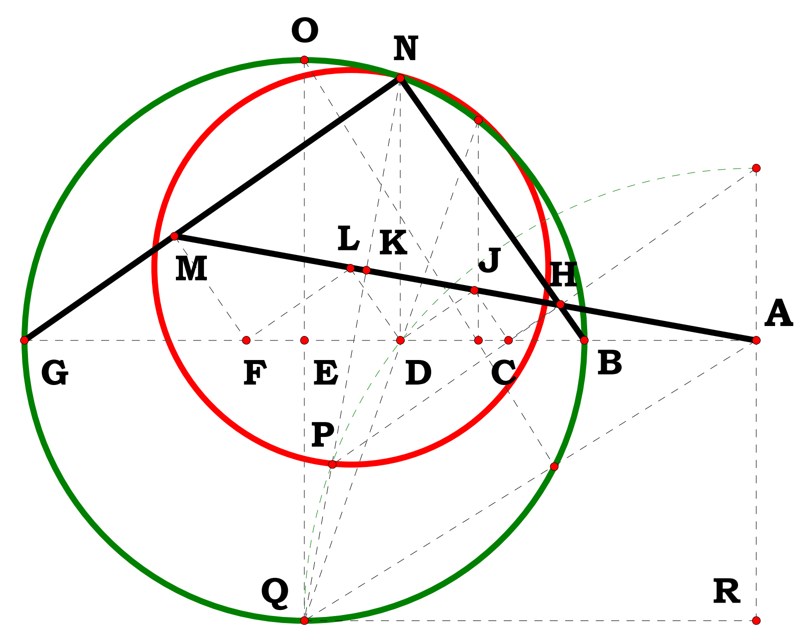
$BN := \sqrt{BD^2 + DN^2} \quad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \quad FG := \frac{BG \cdot GM}{GN} \quad AF := AG - FG$

Definitions.

$$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$$

Alternate Method Quad Roots



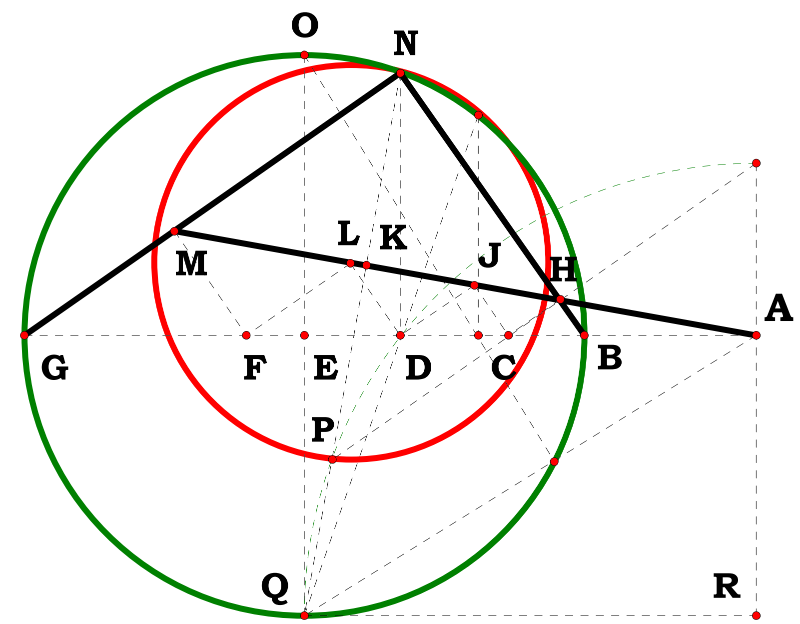
Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

Alternate Method Quad Roots



080100B

Descriptions.

$BG := AG - AB \quad BE := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad DG := BG - BD \quad EQ := BE$

$DN := \sqrt{BD \cdot DG} \quad BN := \sqrt{BD^2 + DN^2}$

$GN := \sqrt{DG^2 + DN^2} \quad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \quad AN := \sqrt{AD^2 + DN^2}$

$AE := AB + BE \quad AQ := \sqrt{AE^2 + EQ^2}$

$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \quad KM := KN \quad MN := \sqrt{KN^2 + KM^2}$

$GM := GN - MN \quad GF := \frac{BG \cdot GM}{GN} \quad AF := AG - GF$

Definitions.

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

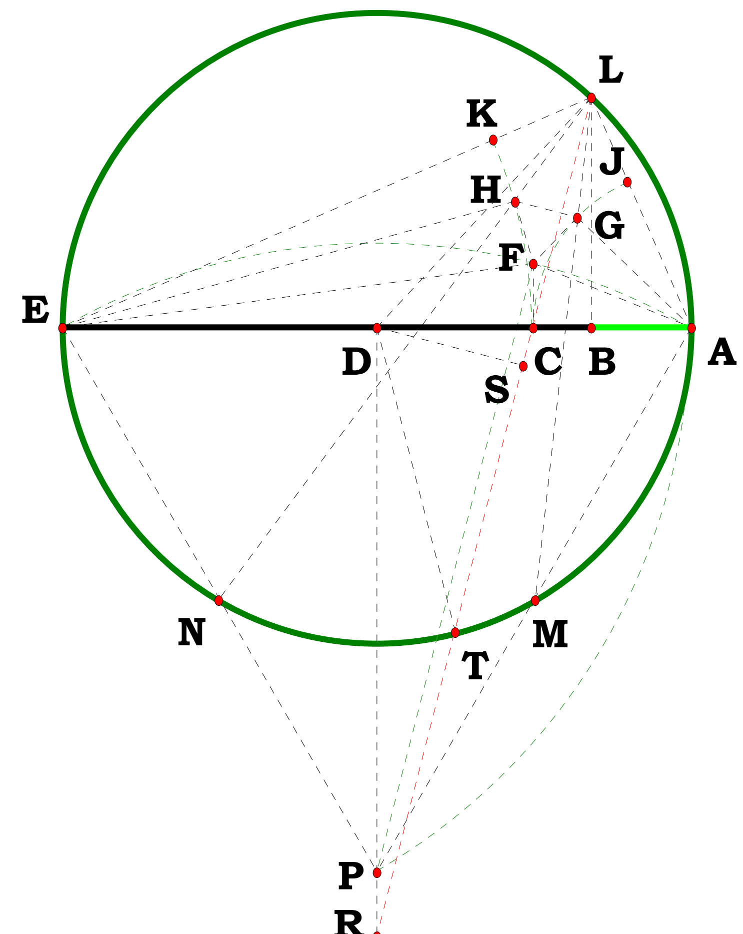
In Trisection What Is AB?

Given.

$AC := .884$

$AE := 3.521$

In the trisection figure given and given AC as the Unit what is AB?



080200

Descriptions.

$AD := \frac{AE}{2}$ $\quad EP := AE$ $\quad DE := AD$ $\quad DP := \sqrt{EP^2 - DE^2}$

$FP := EP$ $\quad CE := AE - AC$ $\quad CD := CE - DE$ $\quad CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF$ $\quad DR := DP + PR$ $\quad CR := \sqrt{CD^2 + DR^2}$ $\quad CS := \frac{CD^2}{CR}$

$DS := \sqrt{CD^2 - CS^2}$ $\quad DL := AD$ $\quad LS := \sqrt{DL^2 - DS^2}$ $\quad RS := CR - CS$

$LR := RS + LS$ $\quad BD := \frac{CD \cdot LR}{CR}$ $\quad AB := AD - BD$ $\quad ST := LS$ $\quad RT := RS - ST$

In trisection the length RT to the similarity point is equal to the radius of the circle.

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

Definitions.

$$AD - \frac{AE}{2} = 0 \qquad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \qquad CF - \left(\frac{\sqrt{4 \cdot AC \cdot AE - 4 \cdot AC^2 + 3 \cdot AE^2}}{2} - \frac{\sqrt{3} \cdot AE}{2}\right) = 0 \qquad CE - (AE - AC) = 0 \qquad CD - \left(\frac{1}{2} \cdot AE - AC\right) = 0$$

$$DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad CR - AE = 0 \qquad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \qquad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0 \qquad RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0$$

$$BD - \left(\frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3\right) = 0 \qquad AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \qquad AB \cdot AE^2 - AC^2(3 \cdot AE - 2 \cdot AC) = 0$$

Unit.

$AB := 1$

Given.

$N_1 := 3$

$N_2 := 2$

080300A

Descriptions.

$AD := AB \quad AP := \frac{AD}{2} \quad BP := AB + AP \quad BO := \frac{BP}{N_1} \quad AE := AB$

$DO := N_2 - BO \quad GO := \sqrt{BO \cdot DO} \quad BG := \sqrt{GO^2 + BO^2} \quad BS := \frac{BG}{2} \quad ER := BS \quad TO := ER$

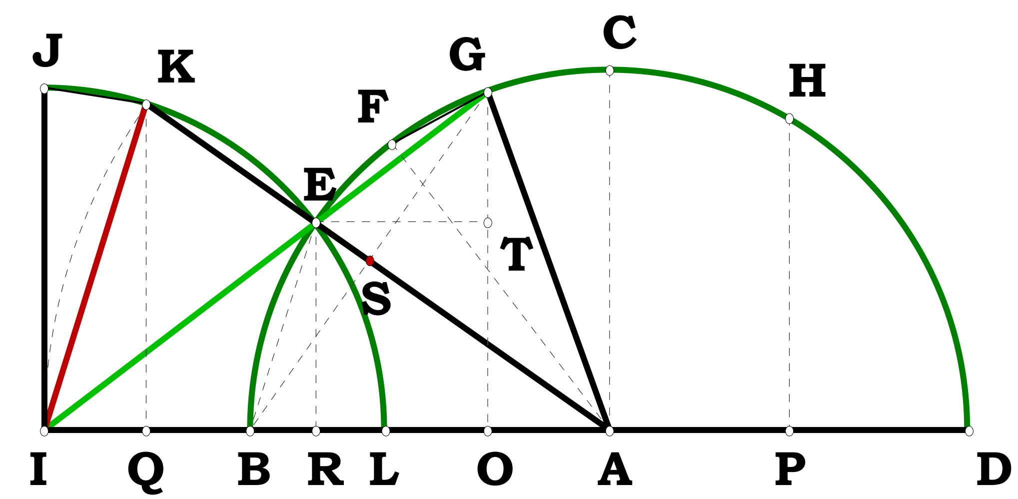
$GT := GO - TO \quad AS := \sqrt{AB^2 - BS^2} \quad ES := AE - AS \quad BR := ES \quad OR := BO - BR$

$ET := OR \quad IO := \frac{ET \cdot GO}{GT} \quad BI := IO - BO \quad AI := BI + AB \quad BE := \sqrt{ER^2 + BR^2}$

$GE := BE \quad GI := \frac{GE \cdot GO}{GT} \quad EI := GI - GE \quad AK := AI \quad IK := EI \quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

$EK := AK - AE \quad \frac{EK}{IQ} = 2$

On Trisection



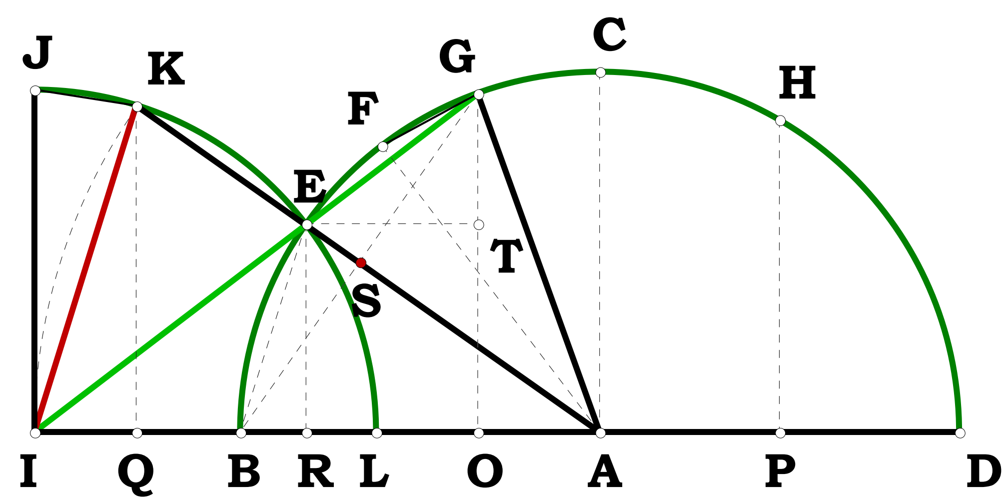
If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.

Definitions.

$$\frac{3 \cdot N_2}{4 \cdot N_1} - BO = 0 \qquad \frac{N_2}{4} \cdot \frac{(4 \cdot N_1 - 3)}{N_1} - DO = 0 \qquad \frac{N_2}{(4 \cdot N_1)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N_1 - 3} - GO = 0 \qquad \frac{N_2}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BG = 0 \qquad \frac{N_2}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BS = 0$$

$$\frac{\sqrt{3} \cdot N_2 \cdot \left(\sqrt{4 \cdot N_1 - 3} - \sqrt{N_1}\right)}{4 \cdot N_1} - GT = 0 \qquad \frac{N_2}{4} \cdot \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} - AS = 0 \qquad \frac{N_2}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}}\right] - ES = 0 \qquad \frac{N_2}{4} \cdot \frac{\left[3 - 2 \cdot N_1 + \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} \cdot N_1\right]}{N_1} - OR = 0$$

$$\frac{-N_2}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N_1-3}\cdot\sqrt{N_1}-2\cdot\sqrt{4\cdot N_1-3}\cdot N_1^{\left(\frac{3}{2}\right)}+4\cdot N_1^{2}-3\cdot N_1\right]}{\left[N_1^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)\right]}-IO=0$$



$$\frac{-N_2}{2}\cdot\frac{\left(\sqrt{4\cdot N_1-3}-2\cdot\sqrt{N_1}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-BI=0$$

$$\frac{-N_2}{\left(-2\cdot\sqrt{4\cdot N_1-3}+2\cdot\sqrt{N_1}\right)}\cdot\sqrt{N_1}-AI=0$$

$$\frac{N_2}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N_1}}\cdot\sqrt{4\cdot N_1-3}}-BE=0$$

$$\frac{N_2}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N_1}+\sqrt{4\cdot N_1-3}\right)}{\sqrt{N_1}}}\cdot\frac{\sqrt{4\cdot N_1-3}}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-GI=0$$

$$\frac{-N_2}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N_1-3}}{\sqrt{N_1}}}\cdot\frac{\sqrt{N_1}}{\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)}-EI=0$$

$$\frac{N_2}{4}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-IQ=0$$

$$\frac{N_2}{2}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-EK=0$$

080300B

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 3$

$N_2 := 2$

$AD := AB \qquad AP := \frac{AD}{2} \qquad BP := AB + AP \qquad BO := \frac{BP}{N_1} \qquad AE := AB$

$DO := N_2 - BO \qquad GO := \sqrt{BO \cdot DO} \qquad BG := \sqrt{GO^2 + BO^2} \qquad BS := \frac{BG}{2} \qquad ER := BS \qquad TO := ER$

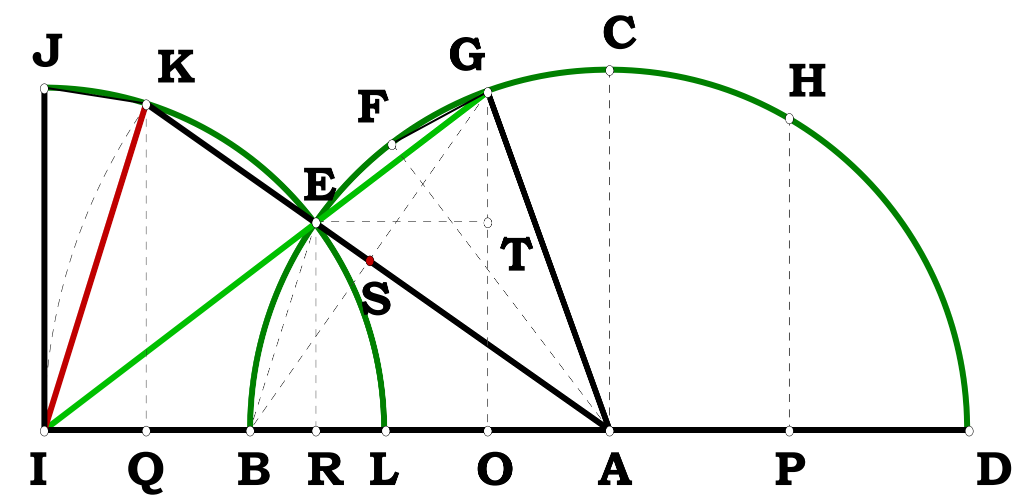
$GT := GO - TO \qquad AS := \sqrt{AB^2 - BS^2} \qquad ES := AE - AS \qquad BR := ES \qquad OR := BO - BR$

$ET := OR \qquad IO := \frac{ET \cdot GO}{GT} \qquad BI := IO - BO \qquad AI := BI + AB \qquad BE := \sqrt{ER^2 + BR^2}$

$GE := BE \qquad GI := \frac{GE \cdot GO}{GT} \qquad EI := GI - GE \qquad AK := AI \qquad IK := EI \qquad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

$EK := AK - AE \qquad \frac{EK}{IQ} = 2$

On Trisection

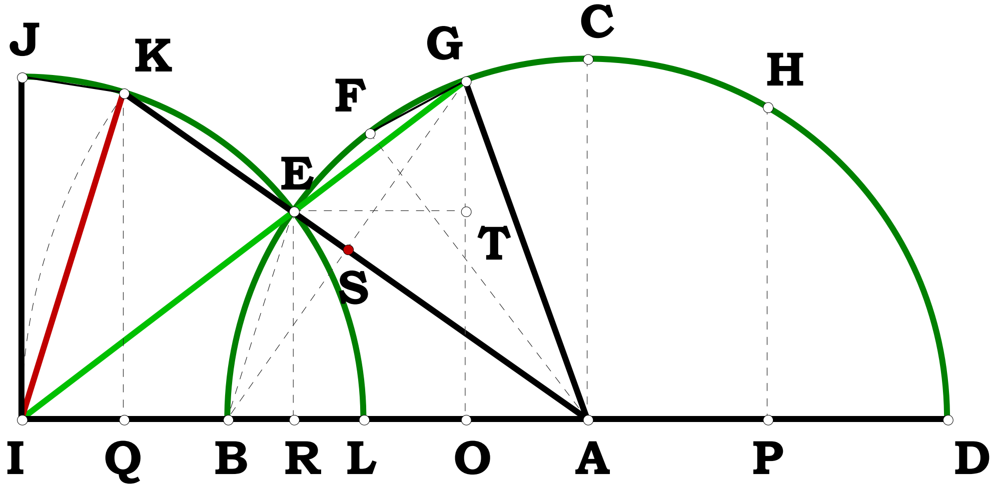


If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.

Definitions.

$$\frac{3 \cdot N_2}{4 \cdot N_1} - BO = 0 \qquad \frac{N_2}{4} \cdot \frac{(4 \cdot N_1 - 3)}{N_1} - DO = 0 \qquad \frac{N_2}{(4 \cdot N_1)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N_1 - 3} - GO = 0 \qquad \frac{N_2}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BG = 0 \qquad \frac{N_2}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BS = 0$$

$$\frac{\sqrt{3} \cdot N_2 \cdot \left(\sqrt{4 \cdot N_1 - 3} - \sqrt{N_1}\right)}{4 \cdot N_1} - GT = 0 \qquad \frac{N_2}{4} \cdot \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} - AS = 0 \qquad \frac{N_2}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}}\right] - ES = 0 \qquad \frac{N_2}{4} \cdot \frac{\left[3 - 2 \cdot N_1 + \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} \cdot N_1\right]}{N_1} - OR = 0$$



$$\frac{-N_2}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N_1-3}\cdot\sqrt{N_1}-2\cdot\sqrt{4\cdot N_1-3}\cdot N_1^{\left(\frac{3}{2}\right)}+4\cdot N_1^{2}-3\cdot N_1\right]}{\left[N_1^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)\right]}-IO=0$$

$$\frac{-N_2}{2}\cdot\frac{\left(\sqrt{4\cdot N_1-3}-2\cdot\sqrt{N_1}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-BI=0$$

$$\frac{-N_2}{\left(-2\cdot\sqrt{4\cdot N_1-3}+2\cdot\sqrt{N_1}\right)}\cdot\sqrt{N_1}-AI=0$$

$$\frac{N_2}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N_1}}\cdot\sqrt{4\cdot N_1-3}}-BE=0$$

$$\frac{N_2}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N_1}+\sqrt{4\cdot N_1-3}\right)}{\sqrt{N_1}}}\cdot\frac{\sqrt{4\cdot N_1-3}}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-GI=0$$

$$\frac{-N_2}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N_1-3}}{\sqrt{N_1}}}\cdot\frac{\sqrt{N_1}}{\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)}-EI=0$$

$$\frac{N_2}{4}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-IQ=0$$

$$\frac{N_2}{2}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-EK=0$$

080400A

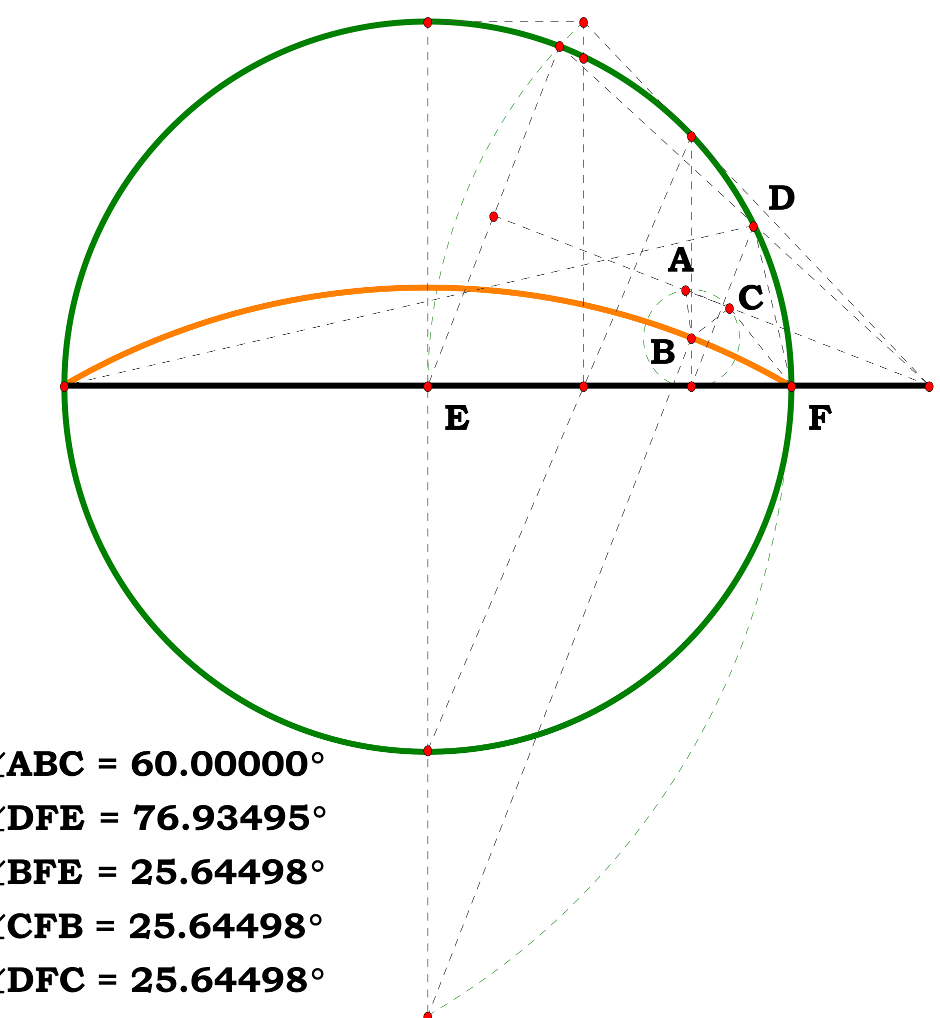
With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?

I am going to figure this out and then I am going to order the equations a little different at the start to see what happens to all the definitions.

This is the A plate, or the first of six.

What will be demonstrated are the differences in the choice of what one uses for a unit to write the figure up.

Trisection and Square Roots



$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

Unit.
Given.

$N_1 := 1.90557 \qquad AB := N_1$

$N_2 := 12.01265 \qquad AF := N_2$

Descriptions.

$AD := \sqrt{AB \cdot AF} \qquad BD := AD - AB$

$BF := AF - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

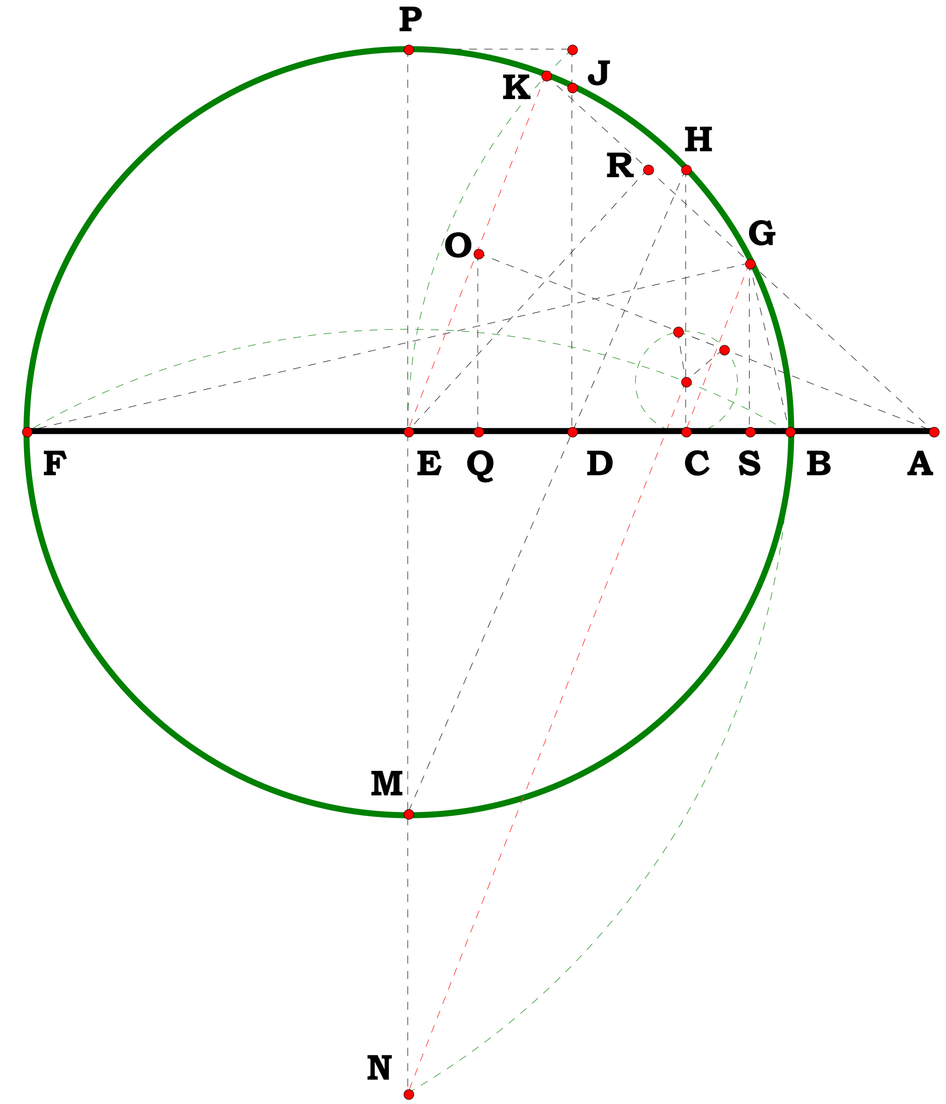
$BE := \dfrac{BF}{2} \qquad EO := \dfrac{BE}{2} \qquad AE := BE + AB \qquad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \dfrac{DE \cdot DH}{DM} \qquad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$



Definitions:

$AB - N_1 = 0 \qquad AF - N_2 = 0 \qquad AD - \sqrt{N_1 \cdot N_2} = 0 \qquad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0$

$BF - \left(N_2 - N_1\right) = 0 \qquad DF - \left(N_2 - \sqrt{N_1 \cdot N_2}\right) = 0 \qquad DJ - \sqrt{\sqrt{N_1 \cdot N_2} \cdot \left(N_1 + N_2\right) - 2 \cdot N_1 \cdot N_2} = 0$

$BE - \dfrac{N_2 - N_1}{2} \qquad EO - \dfrac{N_2 - N_1}{4} = 0 \qquad AE - \dfrac{N_1 + N_2}{2} = 0 \qquad EQ - \dfrac{\left(N_1 - N_2\right)^2}{8 \cdot \left(N_1 + N_2\right)} = 0$

$$DE - \frac{N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}}{2} = 0 \qquad DM - \frac{\sqrt{\left[N_1^{\,2} + N_2^{\,2} + 2\cdot N_1\cdot N_2 - 2\cdot\sqrt{N_1\cdot N_2}\cdot\left(N_1 + N_2\right)\right]}}{\sqrt{2}} = 0$$

$$HM - \frac{\sqrt{2}\cdot\left(N_1 - N_2\right)^2}{2\cdot\sqrt{N_1^{\,2} + N_2^{\,2} - 2\cdot\sqrt{N_1\cdot N_2}\cdot\left(N_1 + N_2\right) + 2\cdot N_1\cdot N_2}} = 0$$

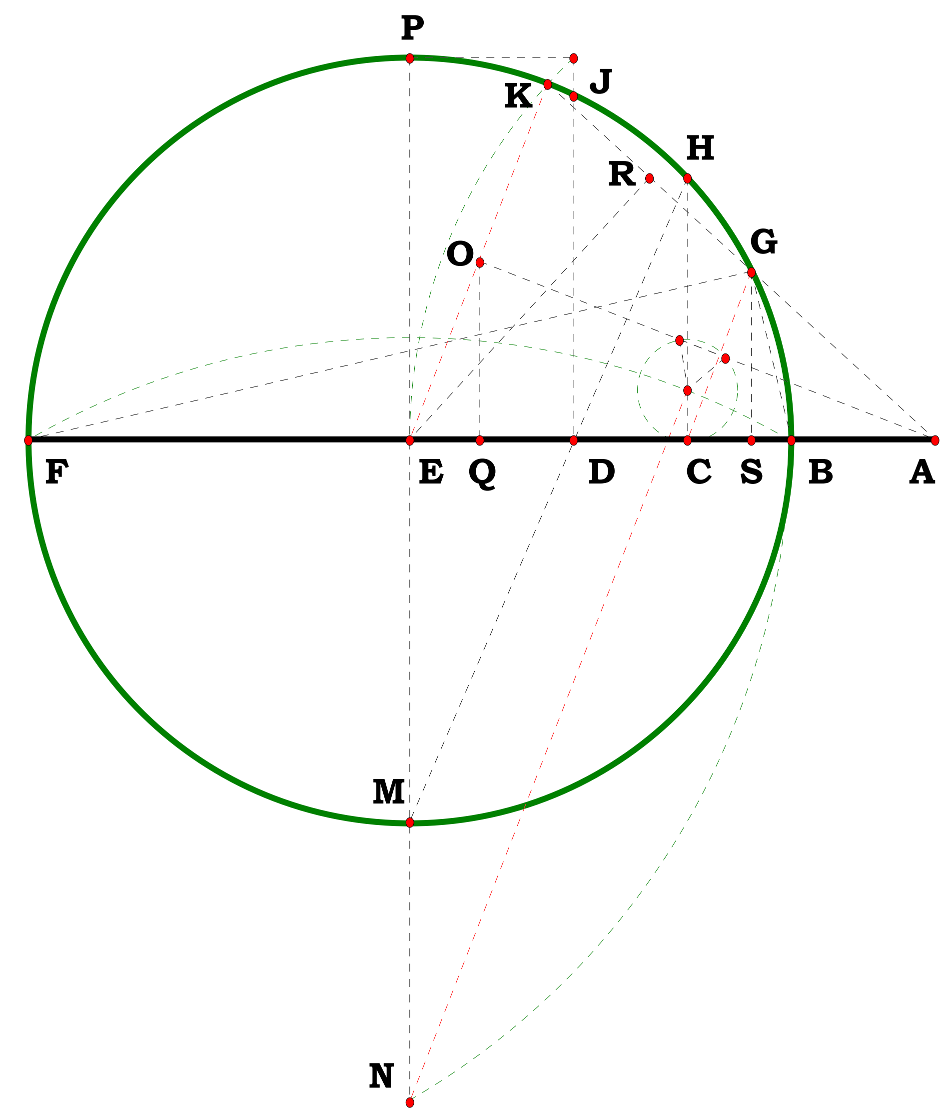
$$DH - \frac{\left(N_1 + N_2\right)\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{\sqrt{\left(N_1 + N_2\right)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}\right)}} = 0$$

$$CD - \frac{\sqrt{N_1\cdot N_2}\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right) - 4\cdot N_1^{\,2}\cdot N_2 - 4\cdot N_1\cdot N_2^{\,2}}{\left(N_1 + N_2\right)\cdot\left(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}\right)} = 0$$

$$CE - \frac{\left(N_1 - N_2\right)^2}{2\cdot\left(N_1 + N_2\right)} = 0 \qquad BC - \frac{N_1\cdot\left(N_2 - N_1\right)}{N_1 + N_2} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{\left(N_1 - N_2\right)^2}}{2} = 0$$

$$KG - \frac{\left(N_1 - N_2\right)^2}{2\cdot\left(N_1 + N_2\right)} = 0 \qquad AG - \frac{2\cdot N_1\cdot N_2}{N_1 + N_2} = 0 \qquad CS - \frac{N_1\cdot N_2\cdot\left(N_1 - N_2\right)^2}{\left(N_1 + N_2\right)^3} = 0$$

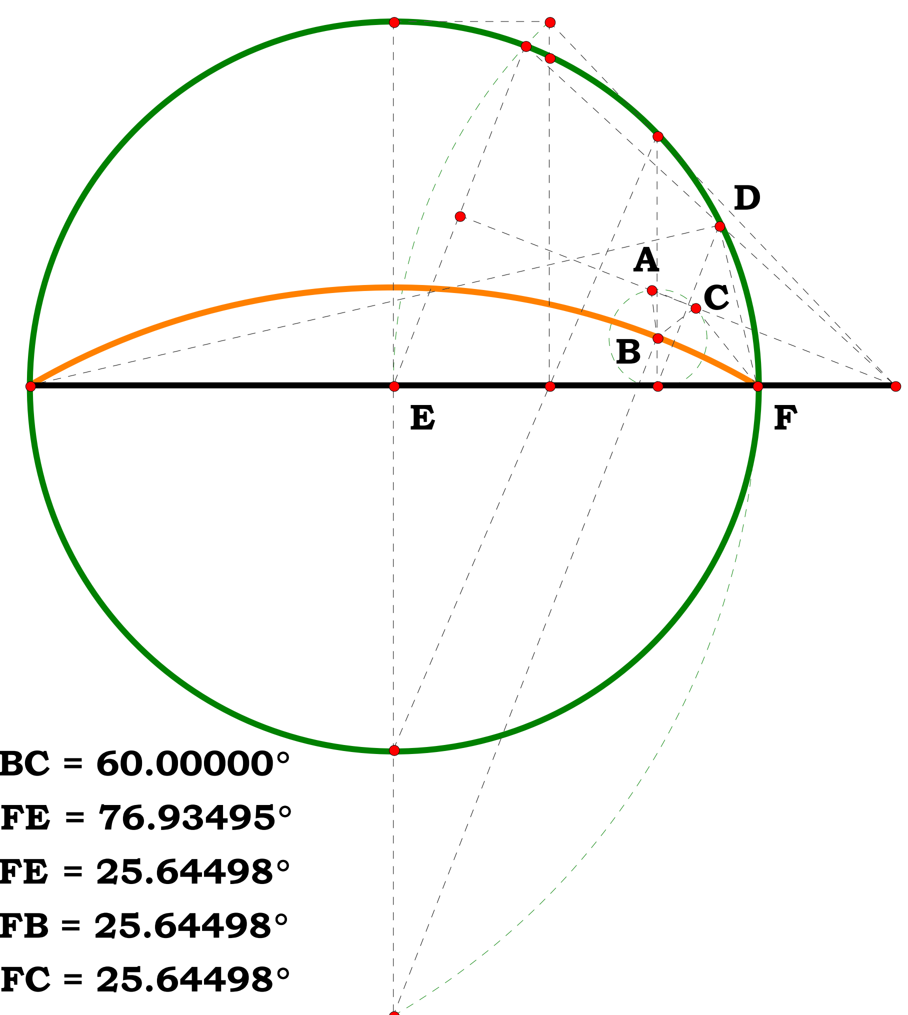
$$AS - \frac{N_1\cdot N_2\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right)}{\left(N_1 + N_2\right)^3} = 0 \qquad BS - \frac{N_1^{\,2}\cdot\left(N_2 - N_1\right)\cdot\left(N_1 + 3\cdot N_2\right)}{\left(N_1 + N_2\right)^3} = 0$$



080400B

This is the B plate, or the second of six.

Trisection and Square Roots



$m\angle ABC = 60.00000^\circ$

$m\angle DFE = 76.93495^\circ$

$m\angle BFE = 25.64498^\circ$

$m\angle CFB = 25.64498^\circ$

$m\angle DFC = 25.64498^\circ$

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

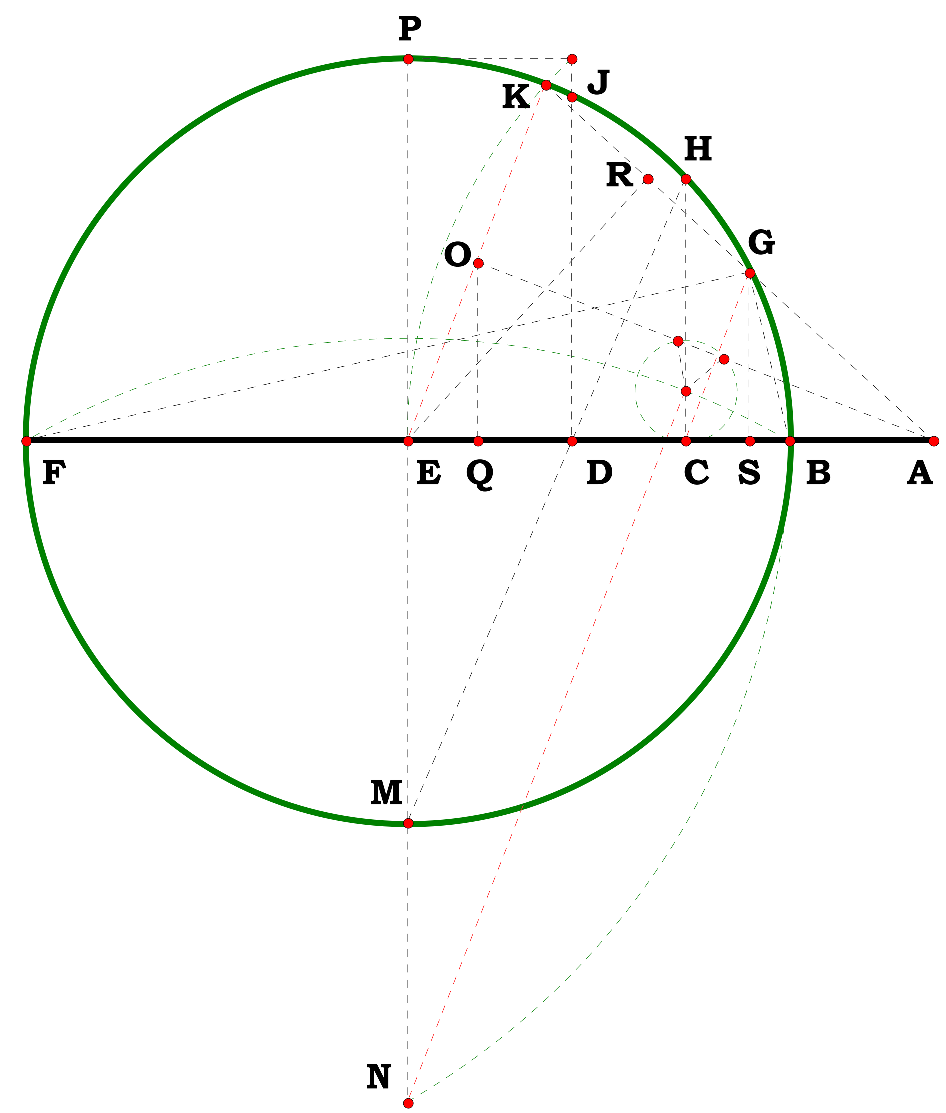
$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000^\circ$

Unit.

Given.

$N_1 := 1.90557$ $\quad AB := N_1$

$N_2 := 10.10708$ $\quad BF := N_2$



Descriptions.

$AF := AB + BF \qquad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

$BE := \dfrac{BF}{2} \qquad EO := \dfrac{BE}{2} \qquad AE := BE + AB \qquad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \dfrac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$

Definitions:

$AB - N_1 = 0 \qquad BF - N_2 = 0 \qquad AF - \left(N_1 + N_2\right) = 0$

$AD - \sqrt{N_1 \cdot \left(N_1 + N_2\right)} = 0 \qquad BD - \left[\sqrt{N_1 \cdot \left(N_1 + N_2\right)} - N_1\right] \qquad DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot \left(N_1 + N_2\right)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^{\,2} + N_2 \cdot N_1} \cdot \left(2 \cdot N_1 + N_2\right) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^{\,2}\right]} = 0 \qquad BE - \dfrac{N_2}{2} = 0 \qquad EO - \dfrac{N_2}{4} = 0$

$AE - \dfrac{2 \cdot N_1 + N_2}{2} = 0 \qquad EQ - \dfrac{N_2^{\,2}}{8 \cdot \left(2 \cdot N_1 + N_2\right)} = 0 \qquad DE - \dfrac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}}{2} = 0$

$$DM - \frac{\sqrt{2\cdot(2\cdot N_1+N_2)^2-4\cdot\sqrt{N_1{}^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)}}{2}=0$$

$$HM - \frac{N_2{}^2}{\sqrt{2\cdot(2\cdot N_1+N_2)^2-4\cdot\sqrt{N_1{}^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)}}=0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{N_1{}^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)-2\cdot N_1{}^2-2\cdot N_1\cdot N_2\right]}{\sqrt{(2\cdot N_1+N_2)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot(N_1+N_2)}\right]}}=0$$

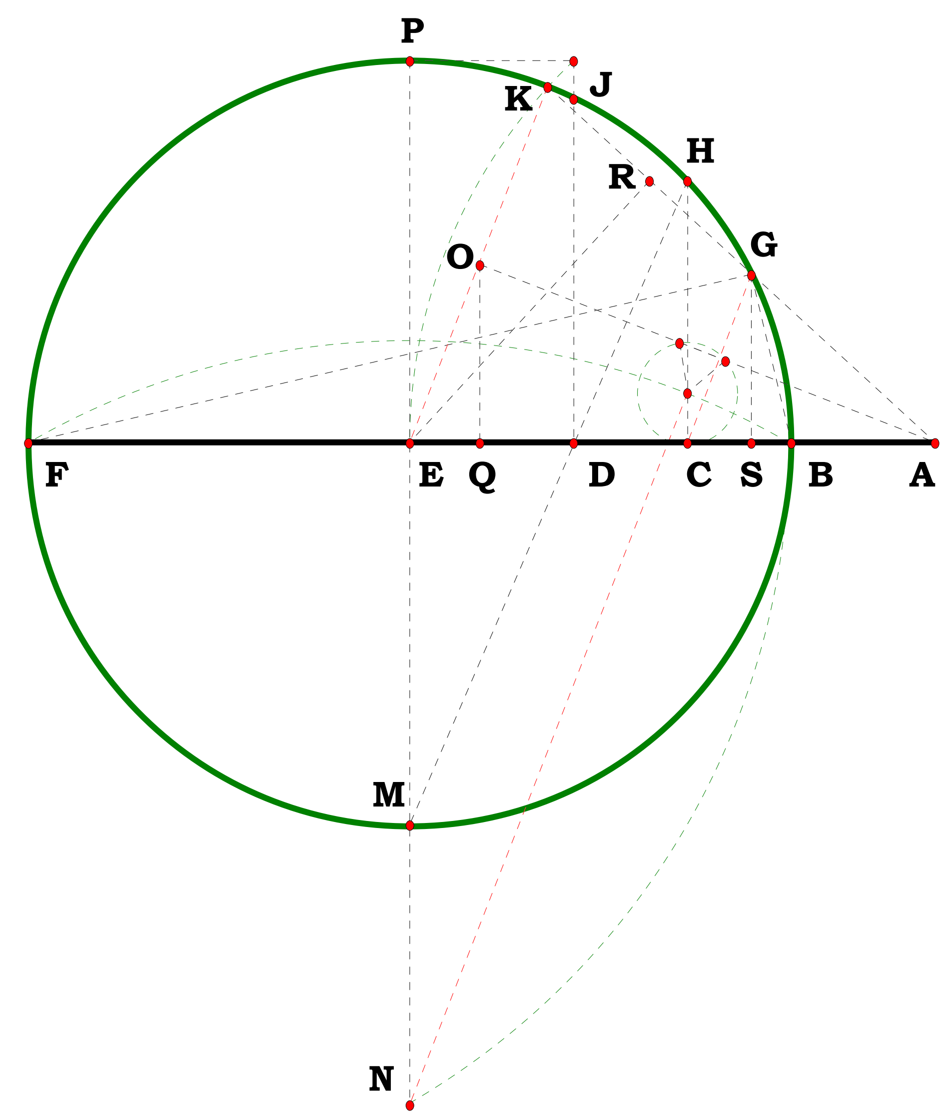
$$CD - \frac{\sqrt{N_1{}^2+N_2\cdot N_1}\cdot\left(8\cdot N_1{}^2+8\cdot N_1\cdot N_2+N_2{}^2\right)-4\cdot N_1\cdot(2\cdot N_1+N_2)\cdot(N_1+N_2)}{(2\cdot N_1+N_2)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{N_1{}^2+N_2\cdot N_1}\right)}=0$$

$$CE - \frac{N_2{}^2}{2\cdot(2\cdot N_1+N_2)}=0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{N_2{}^2}}{2}=0$$

$$KG - \frac{N_2{}^2}{2\cdot(2\cdot N_1+N_2)}=0 \qquad AG - \frac{2\cdot N_1\cdot(N_1+N_2)}{2\cdot N_1+N_2}=0 \qquad CS - \frac{N_1\cdot N_2{}^2\cdot(N_1+N_2)}{(2\cdot N_1+N_2)^3}=0$$

$$AS - \frac{N_1\cdot(N_1+N_2)\cdot\left(8\cdot N_1{}^2+8\cdot N_1\cdot N_2+N_2{}^2\right)}{(2\cdot N_1+N_2)^3}=0$$

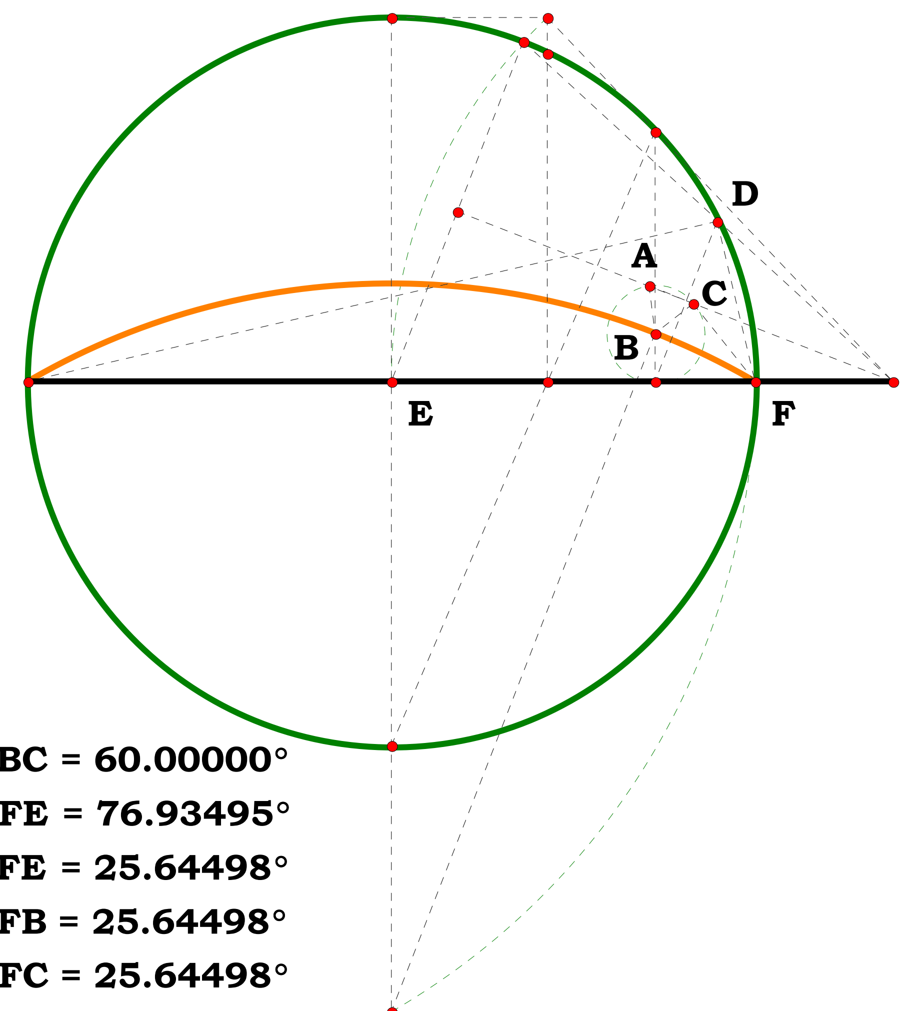
$$BS - \frac{N_1{}^2\cdot N_2\cdot(4\cdot N_1+3\cdot N_2)}{(2\cdot N_1+N_2)^3}=0$$



080400C

Trisection and Square Roots

This is the C plate, or the third of six.



m∠ABC = 60.00000°

m∠DFE = 76.93495°

m∠BFE = 25.64498°

m∠CFB = 25.64498°

m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°

Unit.

$AB := 1$

Given.

$N_1 := 12.01265 \qquad AF := N_1$

Descriptions.

$AD := \sqrt{AB \cdot AF} \qquad BD := AD - AB$

$BF := AF - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

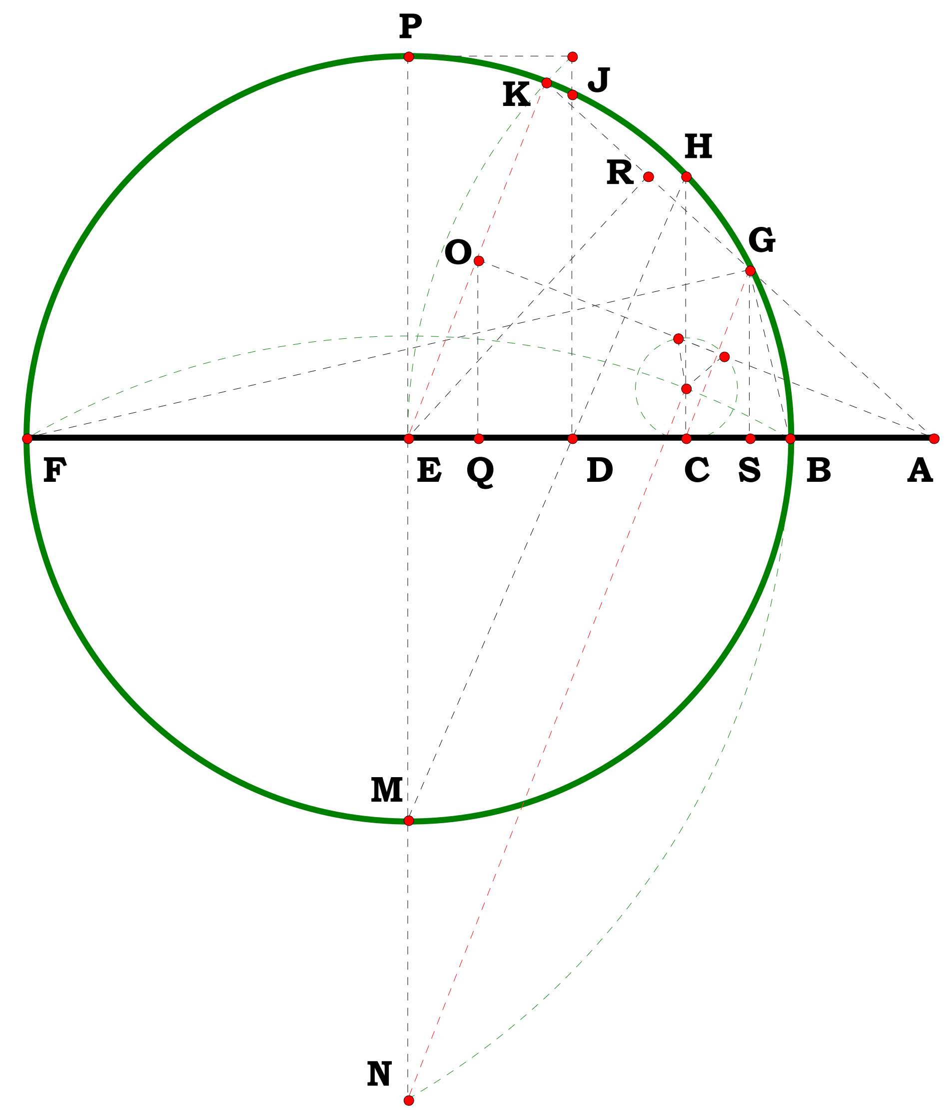
$BE := \dfrac{BF}{2} \qquad EO := \dfrac{BE}{2} \qquad AE := BE + AB \qquad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \dfrac{DE \cdot DH}{DM} \qquad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$



Definitions:

$1 - 1 = 0 \qquad AF - N_1 = 0 \qquad AD - \sqrt{N_1} = 0 \qquad BD - \left(\sqrt{N_1} - 1\right) = 0$

$BF - \left(N_1 - 1\right) = 0 \qquad DF - \left(N_1 - \sqrt{N_1}\right) = 0 \qquad DJ - \sqrt{\sqrt{N_1} \cdot \left(\sqrt{N_1} - 1\right)^2} = 0$

$BE - \dfrac{N_1 - 1}{2} \qquad EO - \dfrac{N_1 - 1}{4} = 0 \qquad AE - \dfrac{1 + N_1}{2} = 0 \qquad EQ - \dfrac{\left(1 - N_1\right)^2}{8 \cdot \left(1 + N_1\right)} = 0$

$$DE - \frac{1 + N_1 - 2\cdot\sqrt{N_1}}{2} = 0 \qquad DM - \frac{\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}}{\sqrt{2}} = 0$$

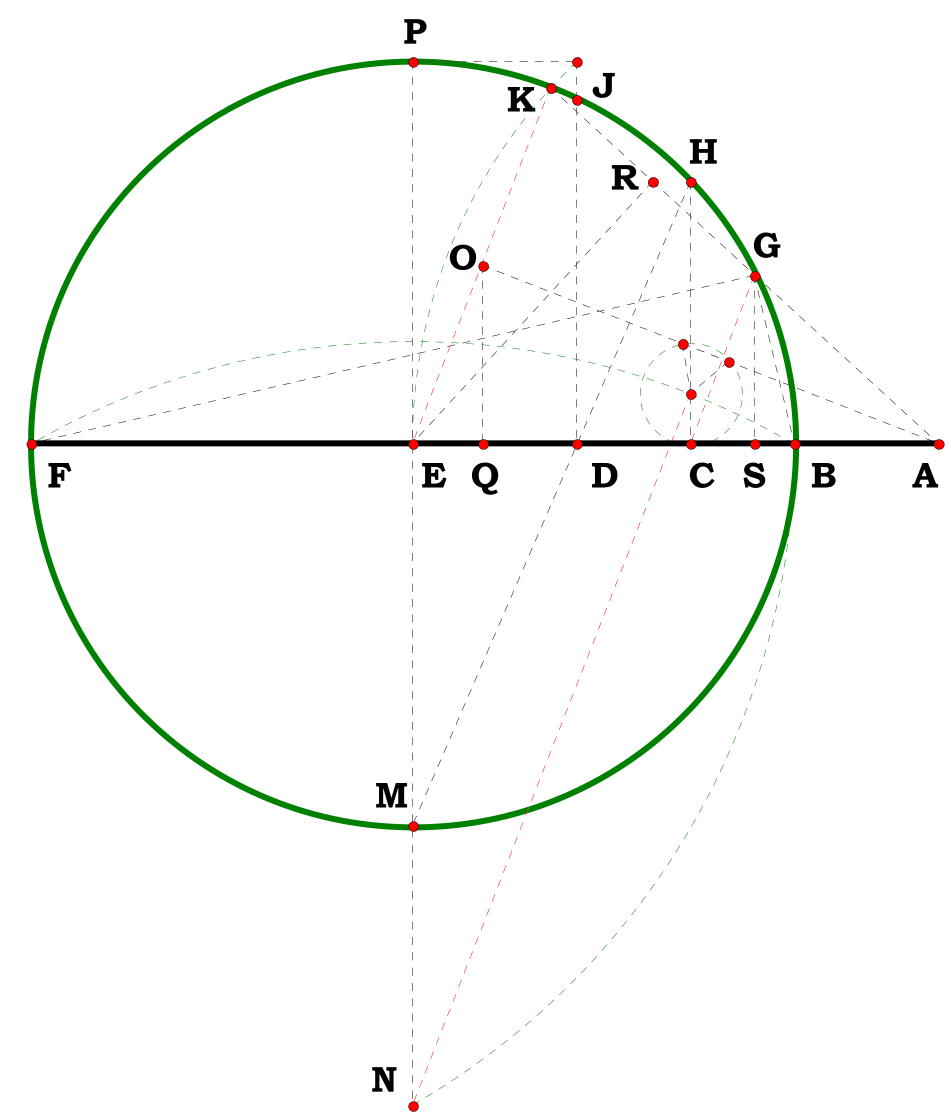
$$HM - \frac{\sqrt{2}\cdot(1-N_1)^2}{2\cdot\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}} = 0 \qquad DH - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1}-1\right)^2}{\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}} = 0$$

$$CD - \frac{\sqrt{N_1}\cdot\left(\sqrt{N_1}-1\right)^4}{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2} = 0 \qquad CE - \frac{(1-N_1)^2}{2\cdot(1+N_1)} = 0$$

$$BC - \frac{1\cdot(N_1-1)}{1+N_1} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{(1-N_1)^2}}{2} = 0 \qquad KG - \frac{(1-N_1)^2}{2\cdot(1+N_1)} = 0$$

$$AG - \frac{2\cdot 1\cdot N_1}{1+N_1} = 0 \qquad CS - \frac{1\cdot N_1\cdot(1-N_1)^2}{(1+N_1)^3} = 0$$

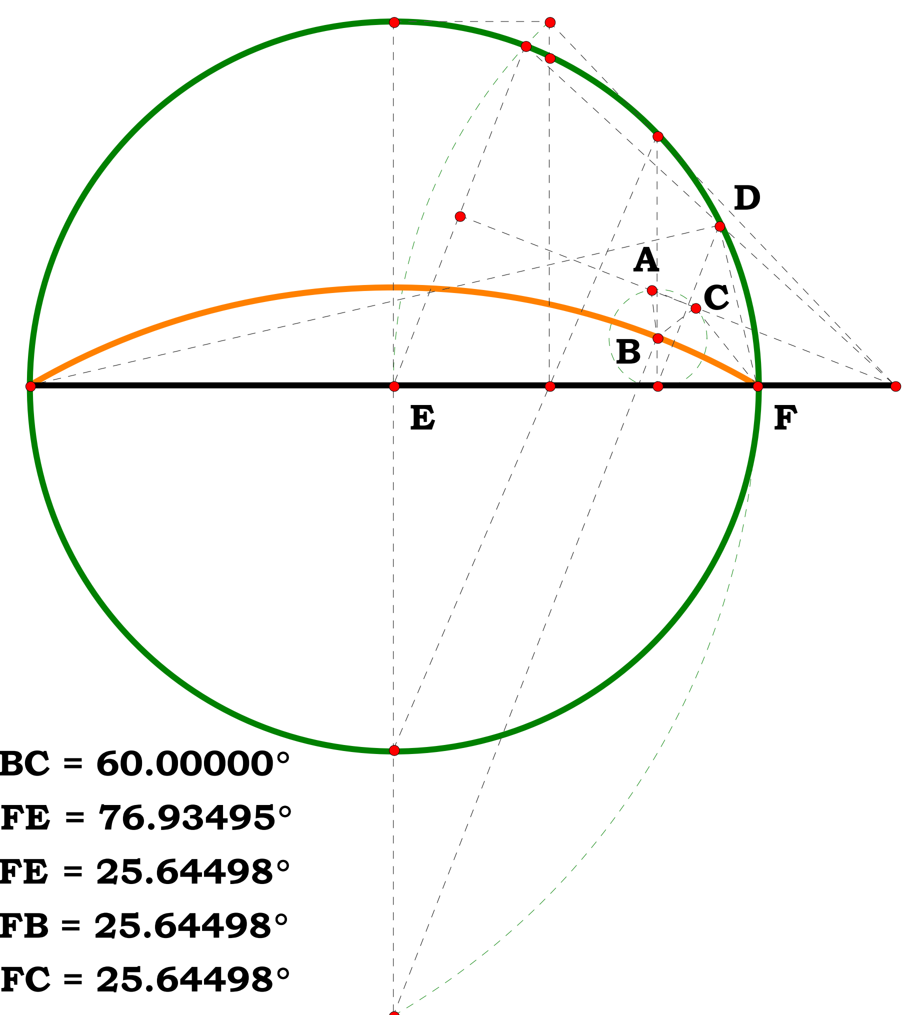
$$AS - \frac{N_1\cdot\left(N_1^{\,2}+6\cdot N_1+1\right)}{(1+N_1)^3} = 0 \qquad BS - \frac{(N_1-1)\cdot(3\cdot N_1+1)}{(1+N_1)^3} = 0$$



080400D

This is the D plate, or the forth of six.

Trisection and Square Roots



m∠ABC = 60.00000°

m∠DFE = 76.93495°

m∠BFE = 25.64498°

m∠CFB = 25.64498°

m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

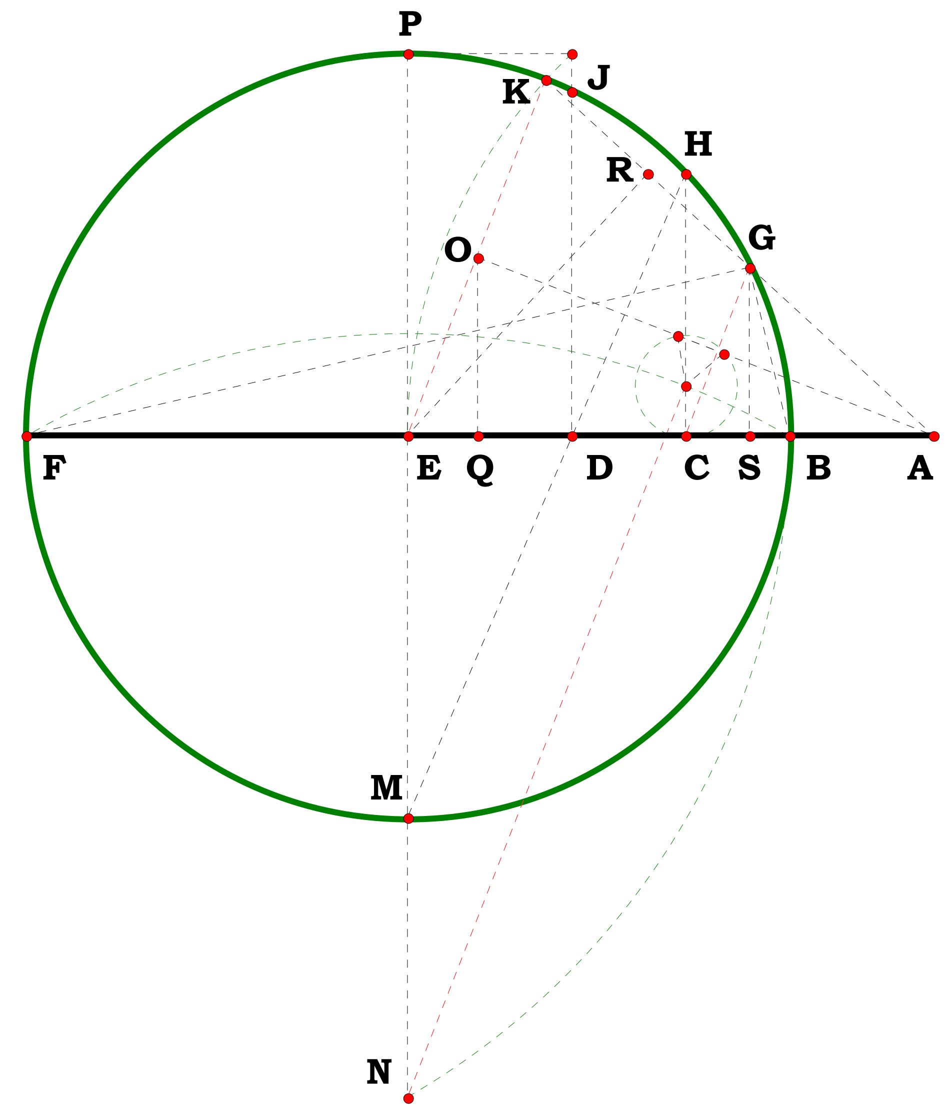
m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°

Unit.

Given.

$N_1 := 1.90557 \qquad AB := N_1$

$N_2 := 10.10708 \qquad BF := N_2$



Descriptions.

$AF := AB + BF \qquad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

$BE := \dfrac{BF}{2} \qquad EO := \dfrac{BE}{2} \qquad AE := BE + AB \qquad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \dfrac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$

Definitions:

$AB - N_1 = 0 \qquad BF - N_2 = 0 \qquad AF - \left(N_1 + N_2\right) = 0$

$AD - \sqrt{N_1 \cdot \left(N_1 + N_2\right)} = 0 \qquad BD - \left[\sqrt{N_1 \cdot \left(N_1 + N_2\right)} - N_1\right] \qquad DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot \left(N_1 + N_2\right)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot \left(2 \cdot N_1 + N_2\right) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0 \qquad BE - \dfrac{N_2}{2} = 0 \qquad EO - \dfrac{N_2}{4} = 0$

$AE - \dfrac{2 \cdot N_1 + N_2}{2} = 0 \qquad EQ - \dfrac{N_2^2}{8 \cdot \left(2 \cdot N_1 + N_2\right)} = 0 \qquad DE - \dfrac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$

$$DM-\frac{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{N_1{}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}{2}=0$$

$$HM-\frac{N_2{}^2}{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{N_1{}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}=0$$

$$DH-\frac{\sqrt{2}\cdot\left[\sqrt{N_1{}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)-2\cdot N_1{}^2-2\cdot N_1\cdot N_2\right]}{\sqrt{\left(2\cdot N_1+N_2\right)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot\left(N_1+N_2\right)}\right]}}=0$$

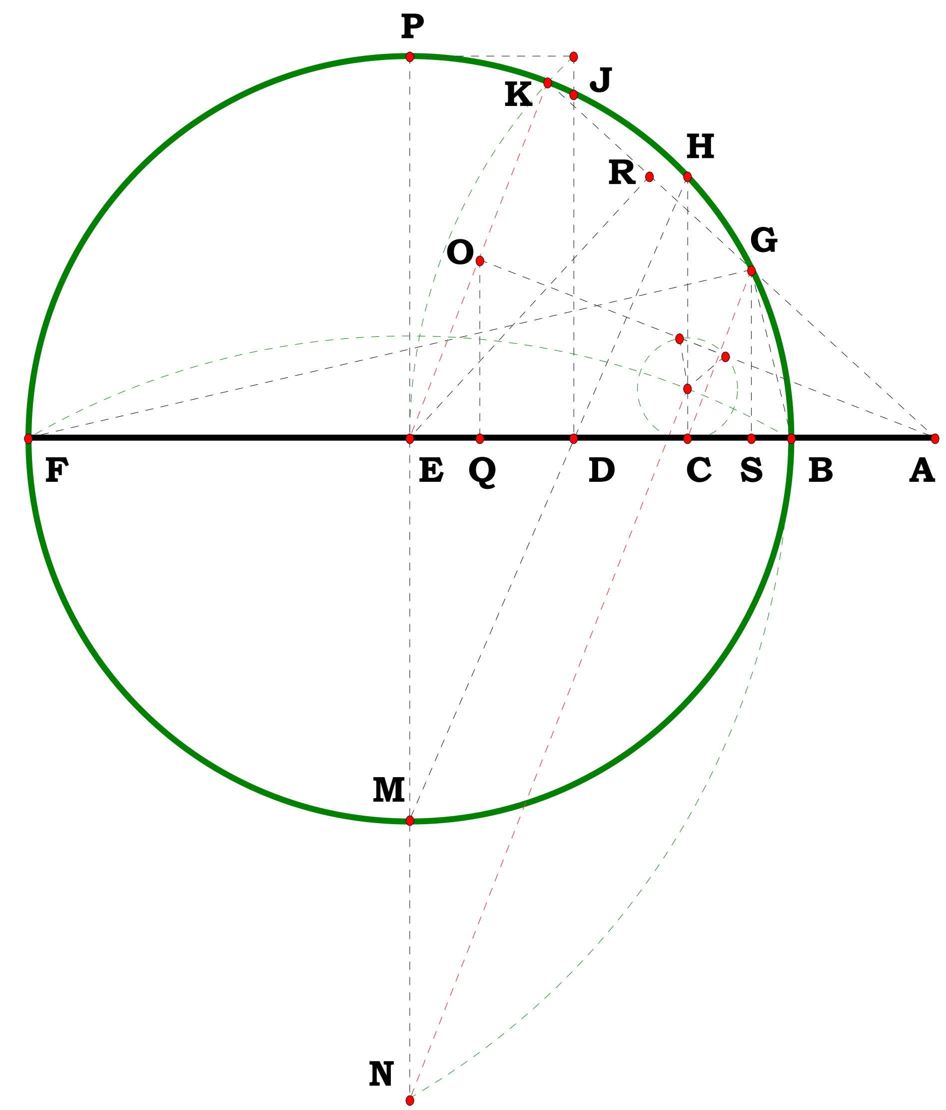
$$CD-\frac{\sqrt{N_1{}^2+N_2\cdot N_1}\cdot\left(8\cdot N_1{}^2+8\cdot N_1\cdot N_2+N_2{}^2\right)-4\cdot N_1\cdot\left(2\cdot N_1+N_2\right)\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{N_1{}^2+N_2\cdot N_1}\right)}=0$$

$$CE-\frac{N_2{}^2}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad BC-\frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN-\frac{\sqrt{3}\cdot\sqrt{N_2{}^2}}{2}=0$$

$$KG-\frac{N_2{}^2}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad AG-\frac{2\cdot N_1\cdot\left(N_1+N_2\right)}{2\cdot N_1+N_2}=0 \qquad CS-\frac{N_1\cdot N_2{}^2\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$AS-\frac{N_1\cdot\left(N_1+N_2\right)\cdot\left(8\cdot N_1{}^2+8\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

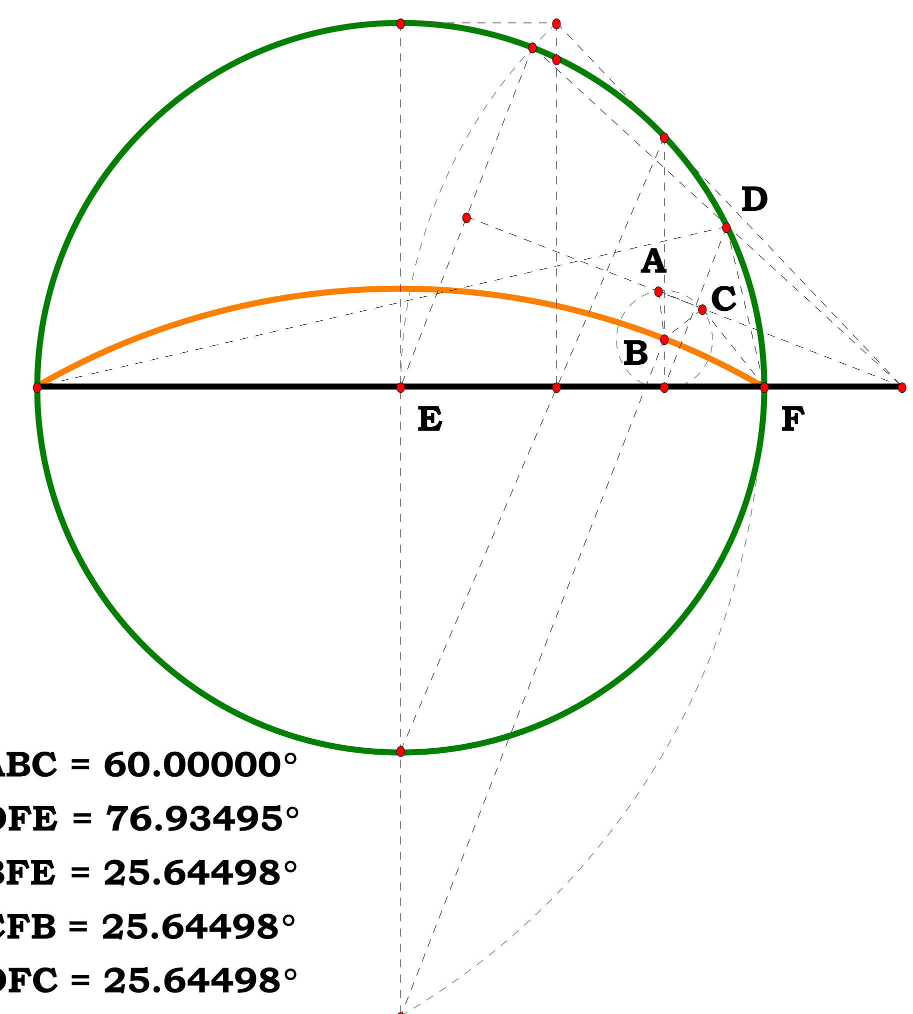
$$BS-\frac{N_1{}^2\cdot N_2\cdot\left(4\cdot N_1+3\cdot N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$



080400E

Trisection and Square Roots

This is the E plate, or the fifth of six.



m∠ABC = 60.00000°

m∠DFE = 76.93495°

m∠BFE = 25.64498°

m∠CFB = 25.64498°

m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°

P

Unit.

$BF := 1$

Given.

$N := 4$

$AF := N + BF \qquad AD := \sqrt{N \cdot AF} \qquad BD := AD - N$

$BF := AF - N \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

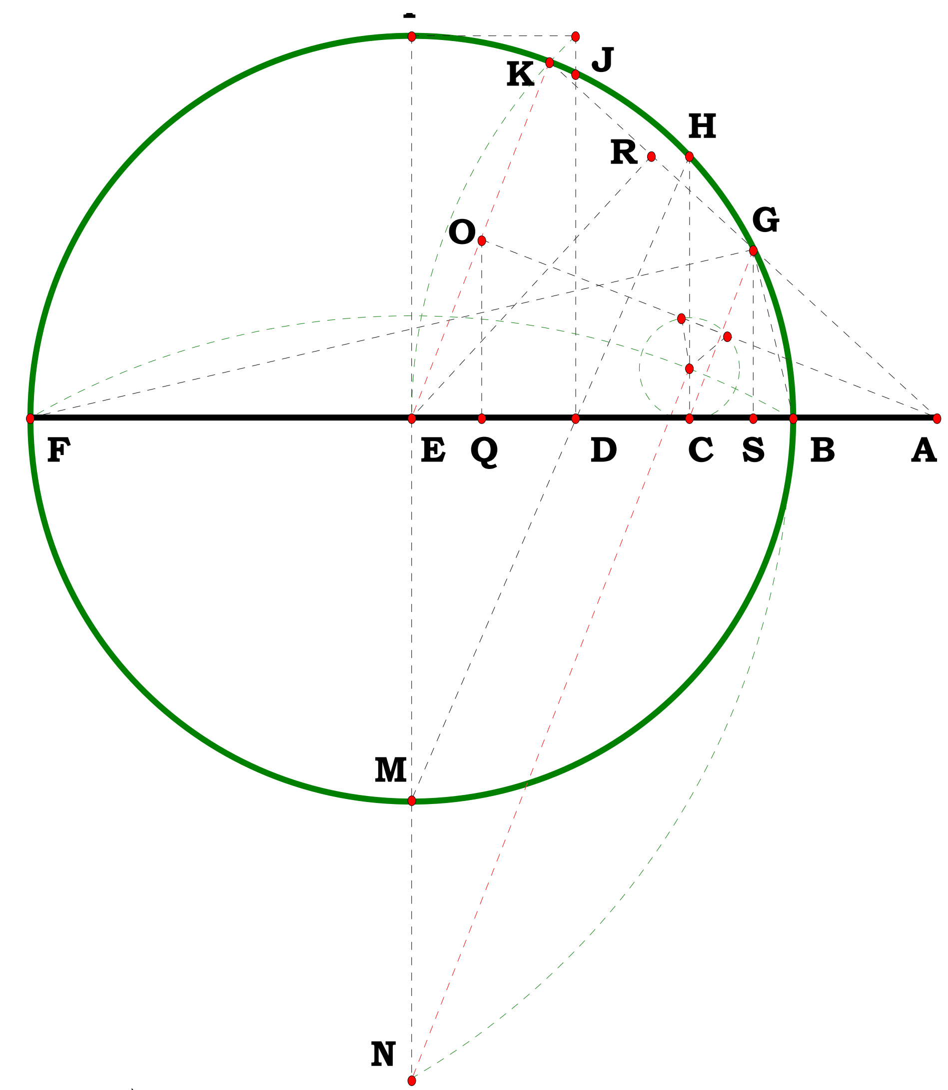
$BE := \dfrac{BF}{2} \qquad EO := \dfrac{BE}{2} \qquad AE := BE + N \qquad EQ := \dfrac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \dfrac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \dfrac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \dfrac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \dfrac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - N$



Definitions:

$N - N = 0 \qquad AF - (N + 1) = 0 \qquad AD - \sqrt{N^2 + N} = 0 \qquad BD - \left(\sqrt{N^2 + N} - N\right) = 0$

$BF - (N + 1 - N) = 0 \qquad DF - \left[N + 1 - \sqrt{N \cdot (N + 1)}\right] = 0 \qquad DJ - \sqrt{(2 \cdot N + 1) \cdot \sqrt{N^2 + N} - \left(2 \cdot N^2 + 2 \cdot N\right)} = 0$

$BE - 2^{-1} \qquad EO - 2^{-2} = 0 \qquad AE - \dfrac{2 \cdot N + 1}{2} = 0 \qquad EQ - \dfrac{1}{8 \cdot (2 \cdot N + 1)} = 0$

$$DE - \frac{2\cdot N - 2\cdot\sqrt{N^2 + N} + 1}{2} = 0 \qquad DM - \frac{\sqrt{2\cdot(2\cdot N + 1)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)} + 1\right]}}{2} = 0$$

$$HM - \left[(4\cdot N + 2)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)} + 1\right]\right]^{\frac{-1}{2}} = 0$$

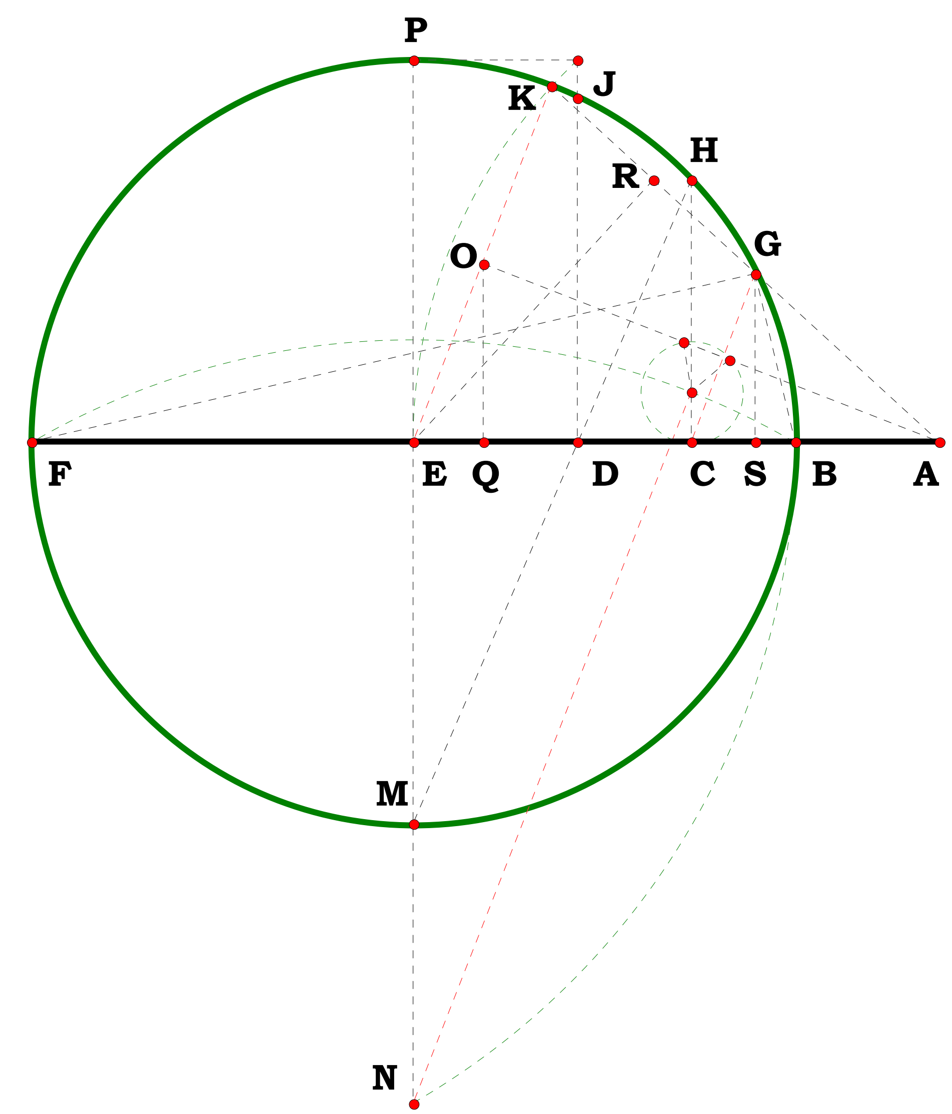
$$DH - \frac{(2\cdot N + 1)\cdot\left(\sqrt{2}\cdot\sqrt{N^2 + N}\right) - 2\cdot\sqrt{2}\cdot N\cdot(N+1)}{\sqrt{\left[(2\cdot N + 1)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)} + 1\right]\right]}} = 0$$

$$CD - \frac{\left(8\cdot N^2 + 8\cdot N + 1\right)\cdot\sqrt{N^2 + N} - \left(8\cdot N^3 + 12\cdot N^2 + 4\cdot N\right)}{(2\cdot N + 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 + N} + 1\right)} = 0$$

$$CE - \frac{1}{2\cdot(2\cdot N + 1)} = 0 \qquad BC - \frac{N}{2\cdot N + 1} = 0 \qquad EN - \frac{\sqrt{3}}{2} = 0$$

$$KG - \frac{1}{2\cdot(2\cdot N + 1)} = 0 \qquad AG - \frac{2\cdot N\cdot(N+1)}{2\cdot N + 1} = 0 \qquad CS - \frac{N\cdot(N+1)}{(2\cdot N + 1)^3} = 0$$

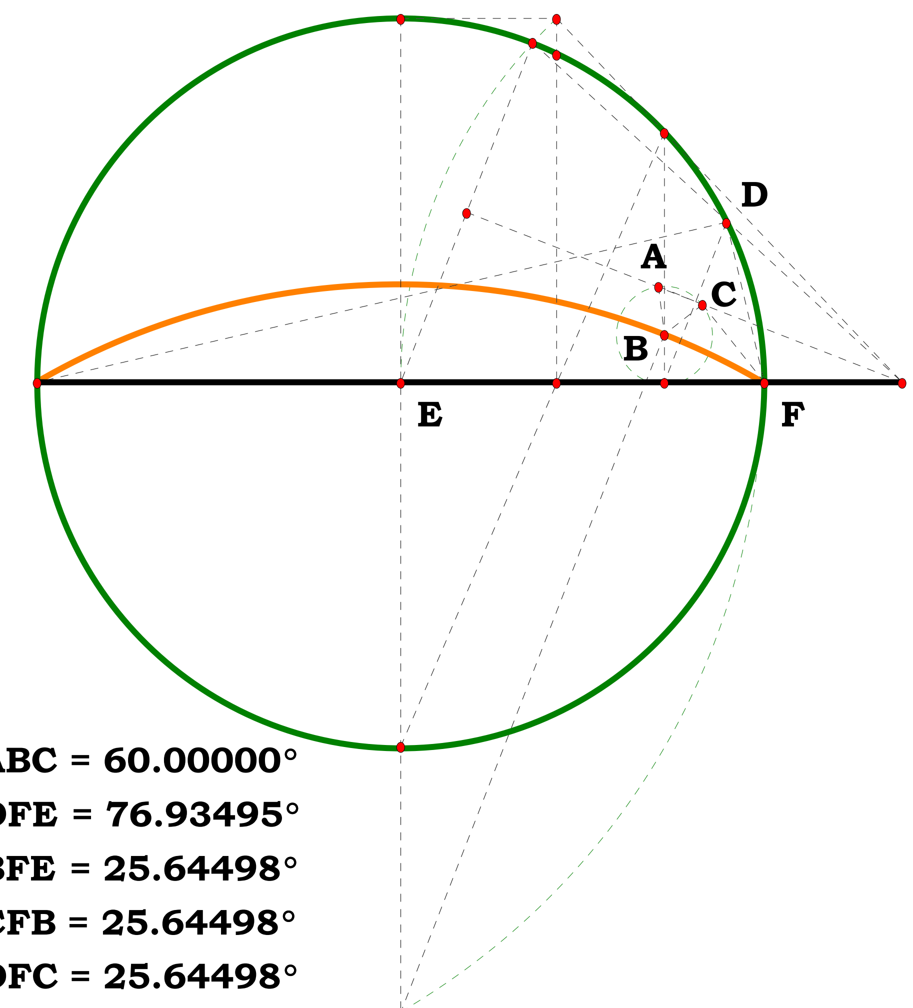
$$AS - \frac{N\cdot(N+1)\cdot\left(8\cdot N^2 + 8\cdot N + 1\right)}{(2\cdot N + 1)^3} = 0 \qquad BS - \frac{N^2\cdot(4\cdot N + 3)}{(2\cdot N + 1)^3} = 0$$



080400F

Trisection and Square Roots

This is the F plate, or the sixth of six.



m∠ABC = 60.00000°

m∠DFE = 76.93495°

m∠BFE = 25.64498°

m∠CFB = 25.64498°

m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°

Unit.

$BF := 1$

Given.

$N := 3 \quad AB := N$

$AF := AB + BF \quad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB \quad DF := AF - AD \quad DJ := \sqrt{BD \cdot DF}$

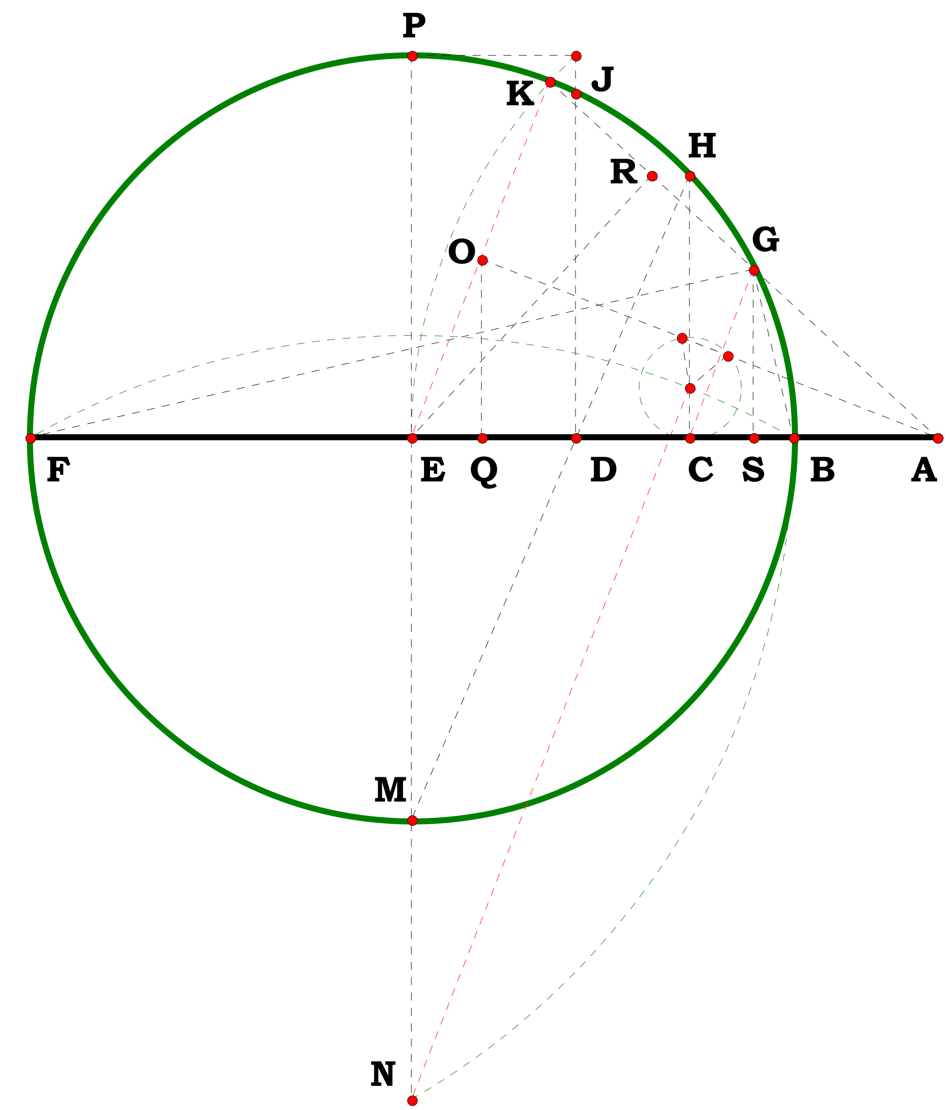
$BE := \frac{BF}{2} \quad EO := \frac{BE}{2} \quad AE := BE + AB \quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \quad DM := \sqrt{DE^2 + BE^2} \quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \quad CD := \frac{DE \cdot DH}{DM} \quad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \quad KG := \frac{2 \cdot EQ \cdot BE}{EO} \quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \quad AS := AE - (DE + CD + CS) \quad BS := AS - AB$



Definitions:

$AB - N = 0 \quad BF - 1 = 0 \quad AF - (N + 1) = 0$

$AD - \sqrt{N \cdot (N+1)} = 0 \quad BD - \sqrt{N \cdot (N+1)} - N \quad DF - \left[N + 1 - \sqrt{N \cdot (N+1)}\right] = 0$

$DJ - \sqrt{\sqrt{N^2 + 1 \cdot N} \cdot (2 \cdot N + 1) - 2 \cdot N - 2 \cdot N^2} = 0 \quad BE - \frac{1}{2} = 0 \quad EO - \frac{1}{4} = 0$

$AE - \frac{2 \cdot N + 1}{2} = 0 \quad EQ - \frac{1^2}{8 \cdot (2 \cdot N + 1)} = 0 \quad DE - \frac{2 \cdot N + 1 - 2 \cdot \sqrt{N^2 + 1 \cdot N}}{2} = 0$

$$DM - \frac{\sqrt{(4\cdot N+2)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}}{2} = 0$$

$$HM - \frac{\sqrt{2}}{2\cdot\sqrt{(2\cdot N+1)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}} = 0$$

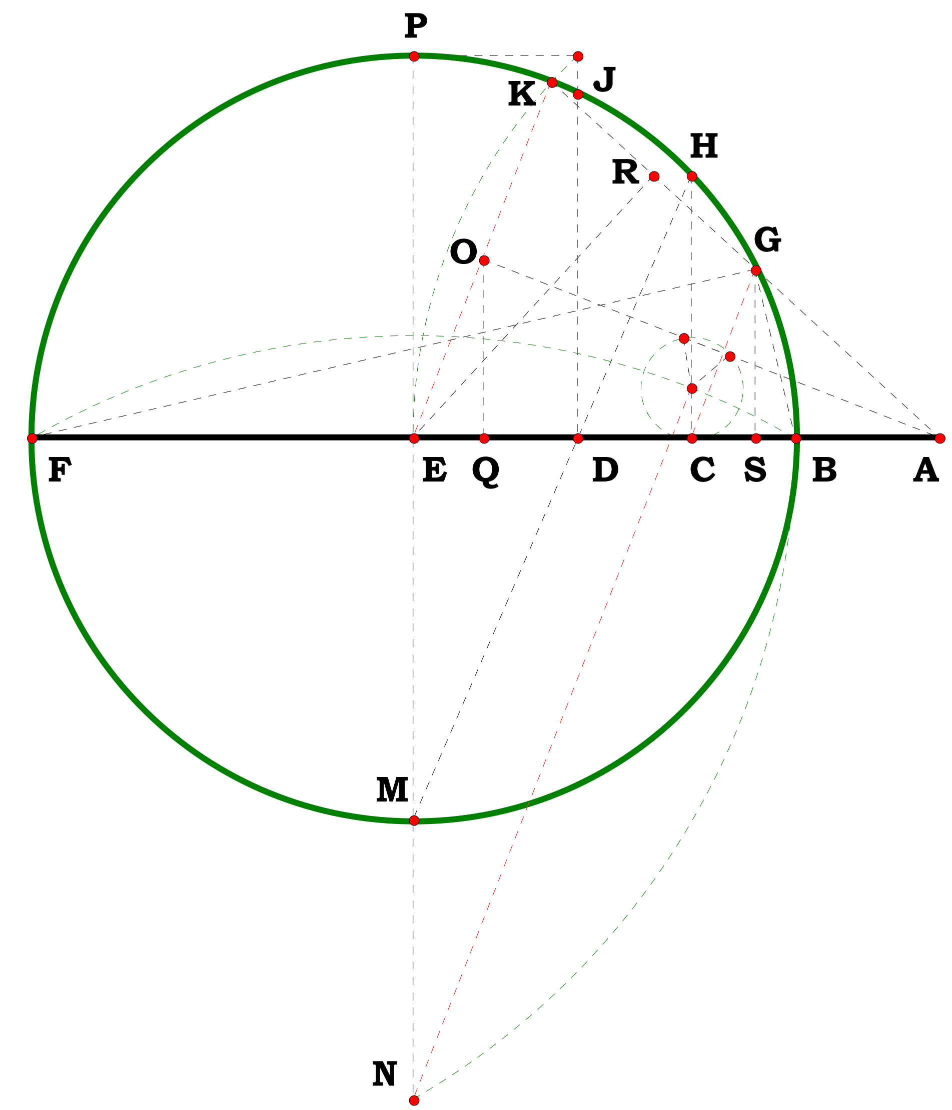
$$DH - \frac{(2\cdot N+1)\cdot\sqrt{2\cdot N^2+2\cdot N}-\left[2\cdot\sqrt{2}\cdot N\cdot(N+1)\right]}{\sqrt{(2\cdot N+1)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}} = 0$$

$$CD - \frac{\left(8\cdot N^2+8\cdot N+1\right)\cdot\sqrt{N^2+N}-4\cdot N\cdot(N+1)\cdot(2\cdot N+1)}{(2\cdot N+1)\cdot\left(2\cdot N+1-2\cdot\sqrt{N^2+N}\right)} = 0$$

$$CE - \frac{1}{4\cdot N+2} = 0 \qquad BC - \frac{N}{2\cdot N+1} = 0 \qquad EN - \frac{\sqrt{3}}{2} = 0$$

$$KG - \frac{1}{4\cdot N+2} = 0 \qquad AG - \frac{2\cdot N\cdot(N+1)}{2\cdot N+1} = 0 \qquad CS - \frac{N\cdot(N+1)}{(2\cdot N+1)^3} = 0$$

$$AS - \frac{N\cdot(N+1)\cdot\left(8\cdot N^2+8\cdot N+1\right)}{(2\cdot N+1)^3} = 0 \qquad BS - \frac{N^2\cdot(4\cdot N+3)}{(2\cdot N+1)^3} = 0$$



080700

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 9 \quad AC := N_1$

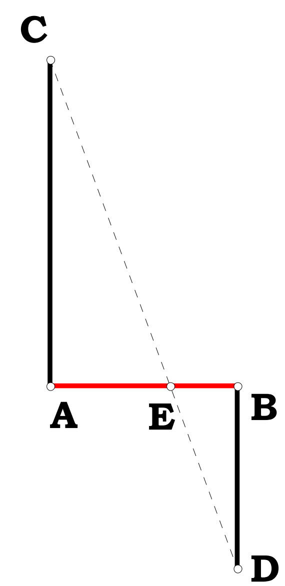
$N_2 := 2 \quad BD := N_2$

Proportion Series II

Divide AB into the same ratio as AB:CD.

$$BE := \frac{AC \cdot AB}{AC + BD} \qquad CE := \frac{BD \cdot AB}{AC + BD}$$

$$BE + CE - AB = 0 \qquad \frac{AC}{BD} - \frac{BE}{CE} = 0$$



Definitions.

$$BE - \frac{N_1}{N_1 + N_2} = 0 \qquad CE - \frac{N_2}{N_1 + N_2} = 0$$

Unit.

$AB := 1$

Given.

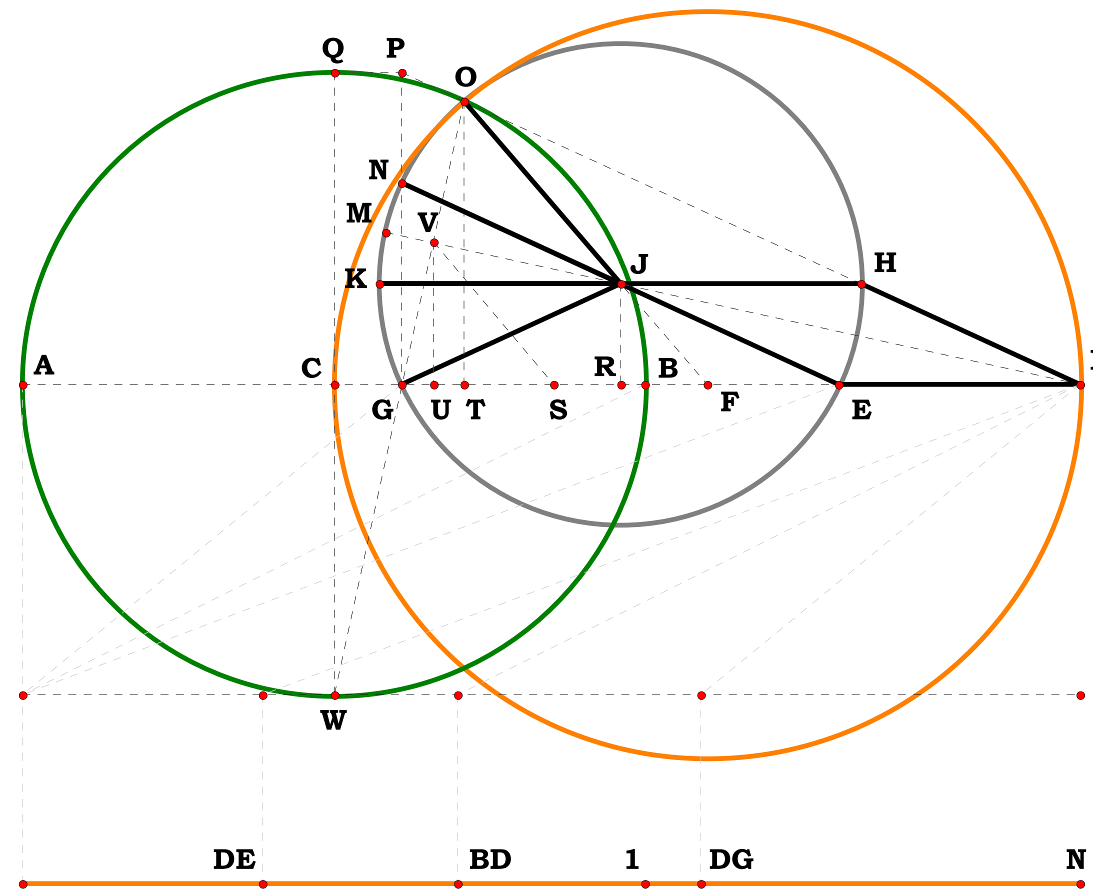
$N := 3 \qquad AD := N$

000822A

Descriptions.

This square root figure affords another approach to proofing the Archimedean Paper Trisecter.

Square Root and the Archimedean Paper Trisecter.



$m\angle OJG = 74.01600°$

$m\angle NJK = 24.67200°$

$\frac{m\angle OJG}{m\angle NJK} = 3.00000$

$\sqrt{N \cdot BD} - DG = 0.00000$

DE = 0.38634 BD = 0.69783 DG = 1.08848 N = 1.69783

$$\frac{(2\cdot N-1)\cdot\left(\left((2\cdot N-1)\cdot\sqrt{N^2-N}+2\cdot N\right)-2\cdot N^2\right)}{\left(\left((4\cdot N-2)\cdot\sqrt{N^2-N}+4\cdot N\right)-4\cdot N^2\right)+1} = 0.38634 \qquad \frac{(2\cdot N-1)\cdot\left(\left((2\cdot N-1)\cdot\sqrt{N^2-N}+2\cdot N\right)-2\cdot N^2\right)}{\left(\left((4\cdot N-2)\cdot\sqrt{N^2-N}+4\cdot N\right)-4\cdot N^2\right)+1} - DE = 0.00000$$

$$BD := AD - AB \qquad DG := \sqrt{BD \cdot AD} \qquad BC := \frac{AB}{2}$$

$$CD := BD + BC \qquad DF := \frac{CD}{2} \qquad BG := DG - BD$$

$$CG := BC - BG \qquad GW := \sqrt{CG^2 + BC^2} \qquad OW := \frac{BC \cdot AB}{GW}$$

$$GO := OW - GW \qquad GU := \frac{CG \cdot GO}{2 \cdot GW} \qquad DU := DG - GU$$

$$OT := \frac{BC \cdot GO}{GW} \qquad US := \sqrt{\left(\frac{DF}{2}\right)^2 - \left(\frac{OT}{2}\right)^2} \qquad DS := DU - US$$

$$JR := \frac{OT \cdot DF}{2 \cdot DS} \qquad JF := \frac{DF \cdot JR}{OT} \qquad EJ := DF - JF$$

$$FR := \frac{2 \cdot US \cdot JR}{OT} \qquad DR := DF + FR \qquad ER := \sqrt{EJ^2 - JR^2}$$

$$DE := DR - ER \qquad DE - EJ = 0$$

As one can see, the APT actually multiplies an angle.

Definitions.

$$BD - (N-1) = 0 \qquad DG - \sqrt{N^2 - N} = 0 \qquad BC - \frac{1}{2} = 0 \qquad CD - \frac{2\cdot N-1}{2} = 0 \qquad DF - \frac{2\cdot N-1}{4} = 0 \qquad BG - \left(\sqrt{N^2-N} - N + 1\right) = 0 \qquad CG - \frac{2\cdot N - 2\cdot\sqrt{N^2-N} - 1}{2} = 0$$

$$GW - \frac{\sqrt{(2\cdot N-1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2-N} - 1\right)}}{\sqrt{2}} = 0 \qquad OW - \frac{\sqrt{2}}{2\cdot\sqrt{(2\cdot N-1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2-N} - 1\right)}} = 0 \qquad GO - \frac{(2\cdot N-1)\cdot\left(\sqrt{2}\cdot\sqrt{N^2-N}\right) - 2\cdot\sqrt{2}\cdot N\cdot(N-1)}{\sqrt{(2-4\cdot N)\cdot\sqrt{N^2-N} + (2\cdot N-1)^2}} = 0$$

$$GU - \frac{\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot\sqrt{N^2 - N} + \left(12\cdot N^2 - 8\cdot N^3 - 4\cdot N\right)}{2\cdot\left[(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N + 1\right]} = 0 \qquad DU - \frac{\sqrt{N^2 - N}}{2\cdot(2\cdot N - 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0 \qquad OT - \frac{(2\cdot N - 1)\cdot\sqrt{N\cdot(N-1)} + 2\cdot N - 2\cdot N^2}{(2\cdot N - 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0$$

$$US - \frac{4\cdot N^2 - 4\cdot N - 1}{8\cdot(2\cdot N - 1)} = 0 \qquad DS - \frac{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1}{8\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0 \qquad JR - \frac{(1 - 2\cdot N)\cdot\sqrt{N\cdot(N-1)} + 2\cdot N^2 - 2\cdot N}{(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N - 1} = 0 \qquad JF - \frac{(2\cdot N - 1)^2\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)}{4\cdot\left[(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1\right]} = 0$$

$$EJ - \frac{(2\cdot N - 1)\cdot\left[(2\cdot N - 1)\cdot\sqrt{N^2 - N} + 2\cdot N - 2\cdot N^2\right]}{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1} = 0 \qquad FR - \frac{\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)\cdot\left(4\cdot N^2 - 4\cdot N - 1\right)}{4\cdot\left[(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1\right]} = 0 \qquad DR - \frac{\sqrt{N^2 - N}}{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1} = 0$$

$$ER - \frac{\sqrt{4\cdot N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N^2 - N} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N^2 - N} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}} = 0$$

Here is where Mathcad 15 Uncle call Uncle! It cannot reduce the following equation and I am at a loss as to how to effect reductions for it in the time I am willing to spend on it.

$$DE - \left[\frac{\sqrt{N\cdot(N-1)}}{(4\cdot N - 2)\cdot\sqrt{N\cdot(N-1)} + 4\cdot N - 4\cdot N^2 + 1} - \frac{2\cdot\sqrt{N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N\cdot(N-1)} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N\cdot(N-1)} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}}\right] = 0$$

$$DE - EJ = 0$$

$$\left[\frac{\sqrt{N\cdot(N-1)}}{(4\cdot N - 2)\cdot\sqrt{N\cdot(N-1)} + 4\cdot N - 4\cdot N^2 + 1} - \frac{2\cdot\sqrt{N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N\cdot(N-1)} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N\cdot(N-1)} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}} - \frac{(2\cdot N - 1)\cdot\left[(1 - 2\cdot N)\cdot\sqrt{N^2 - N} + 2\cdot N^2 - 2\cdot N\right]}{(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N - 1}\right] = 0$$

If you ask Mathcad to reduce the above equation, it will simply spred it out over several pages and quit.

Unit.

$AB := 1$

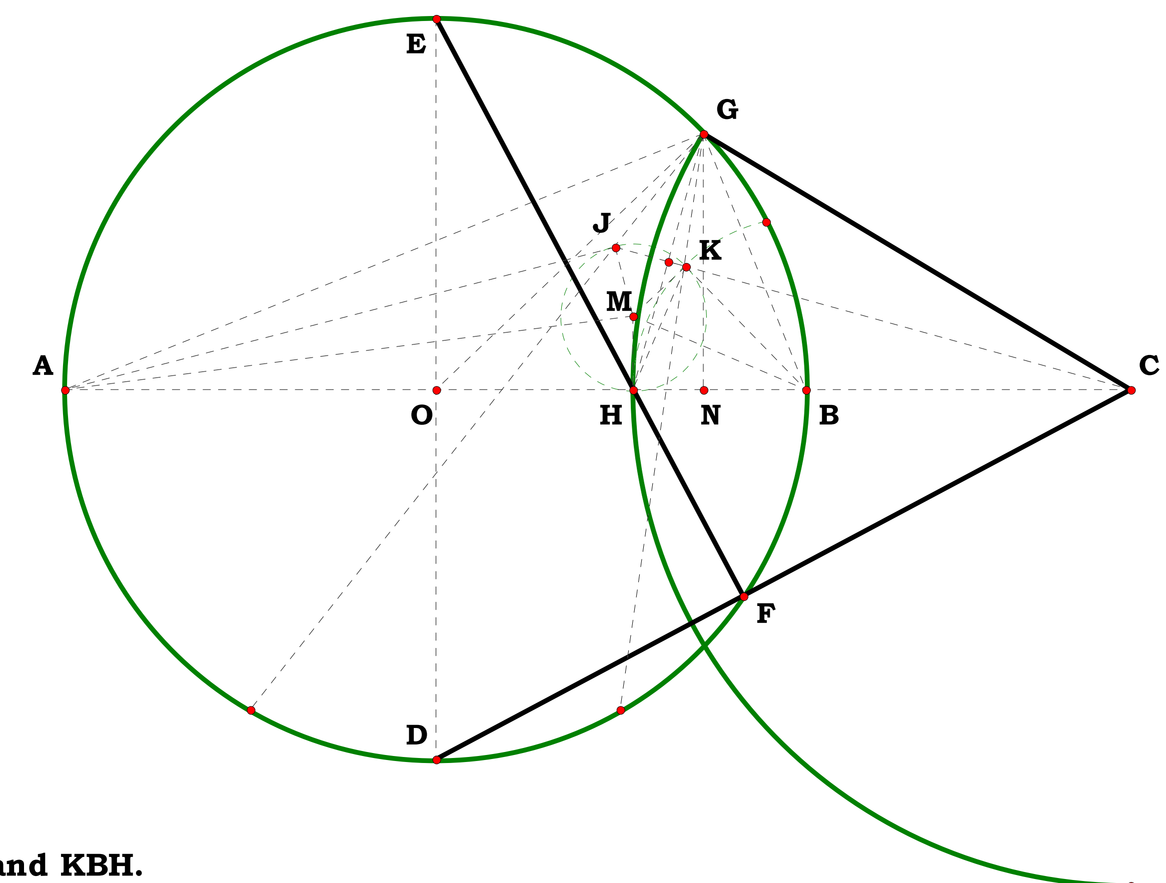
Given.

$N := 1.43693 \quad AC := N$

Show the trisection in a circle for any square root that also divides the circle into six equal cords.

000822B

In the 2015 revision of the DQ, I got as fars as a blank Mathcad template like this one. So, yea, I have put doing this off for a special long time.



$m\angle KCH = 15.47574°$
$m\angle MBH = 22.73787°$

$m\angle GBH = 68.21360°$
$m\angle MBH = 22.73787°$
$\dfrac{m\angle GBH}{m\angle MBH} = 3.00000$
$m\angle EGF = 30.00000°$
$m\angle KJM = 60.00000°$
$N = 1.43693$

Descriptions.

$BO := \dfrac{AB}{2} \quad BC := AC - AB \quad DO := BO$

$CO := BO + BC \quad CD := \sqrt{CO^2 + DO^2}$

$DF := \dfrac{DO \cdot AB}{CD} \quad CF := CD - DF \quad CH := \dfrac{CD \cdot CF}{CO}$

$BH := CH - BC \quad GO := BO \quad CN := \dfrac{CO^2 - GO^2 + CH^2}{2 \cdot CO}$

$GN := \sqrt{CH^2 - CN^2} \quad BN := CN - BC$

To be completed by simply dividing the cords, GCH and KBH. This should help show that angle trisection, the entire developed figure is a proportion to the square root figure.

Definitions.

Unit.

$AB := 1$

Given.

$N := .3 \quad BC := N$

Trisection In A Square Root Figure

Given the square root figure drawn for trisection, what is AR given AB and AD? A slightly different appraach than the one on 04.

082300

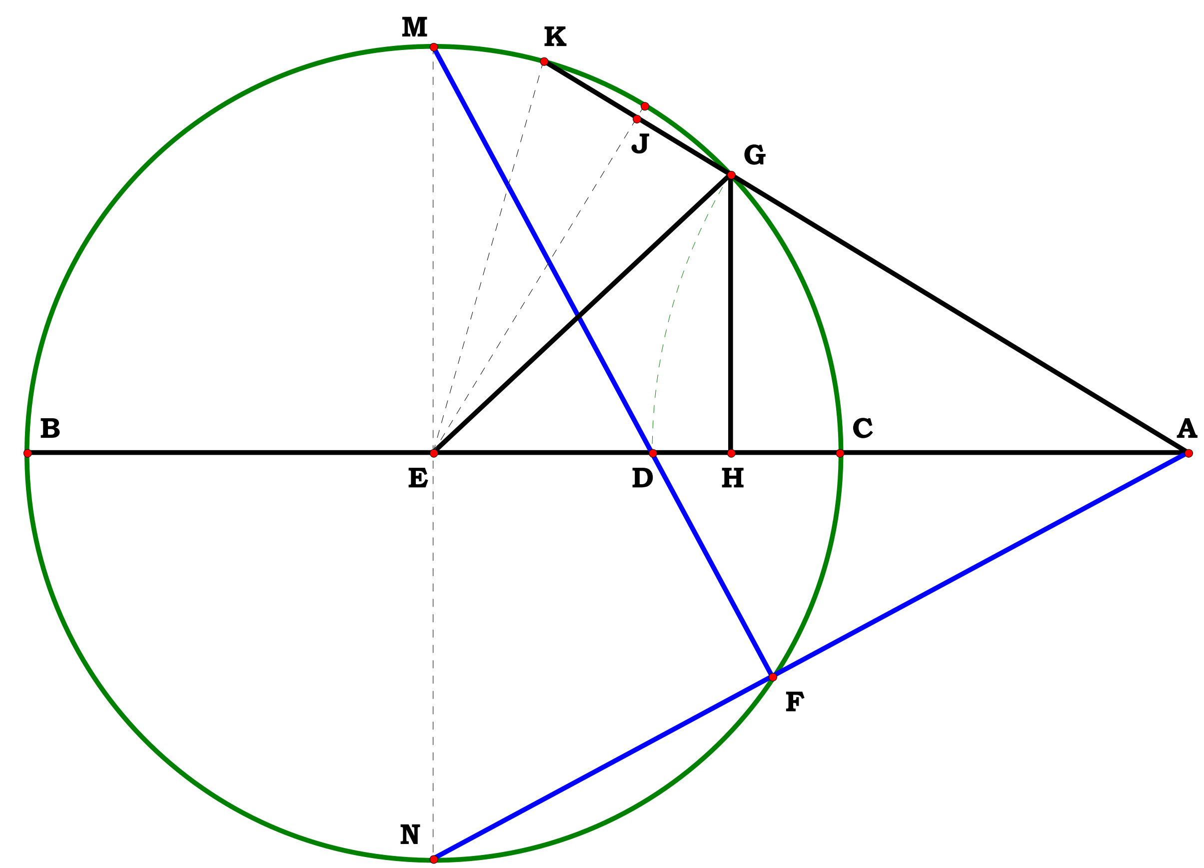
Descriptions.

$AC := AB - N \qquad BE := \frac{BC}{2} \qquad AE := AC + BE$

$AN := \sqrt{BE^2 + AE^2} \qquad FN := \frac{BE \cdot BC}{AN} \qquad DE := \frac{BE^2}{AE}$

$BD := BE + DE \qquad AD := AB - BD \qquad AH := \frac{AE^2 - BE^2 + AD^2}{2 \cdot AE}$

$BH := AB - AH \qquad CH := BC - BH$



Definitions.

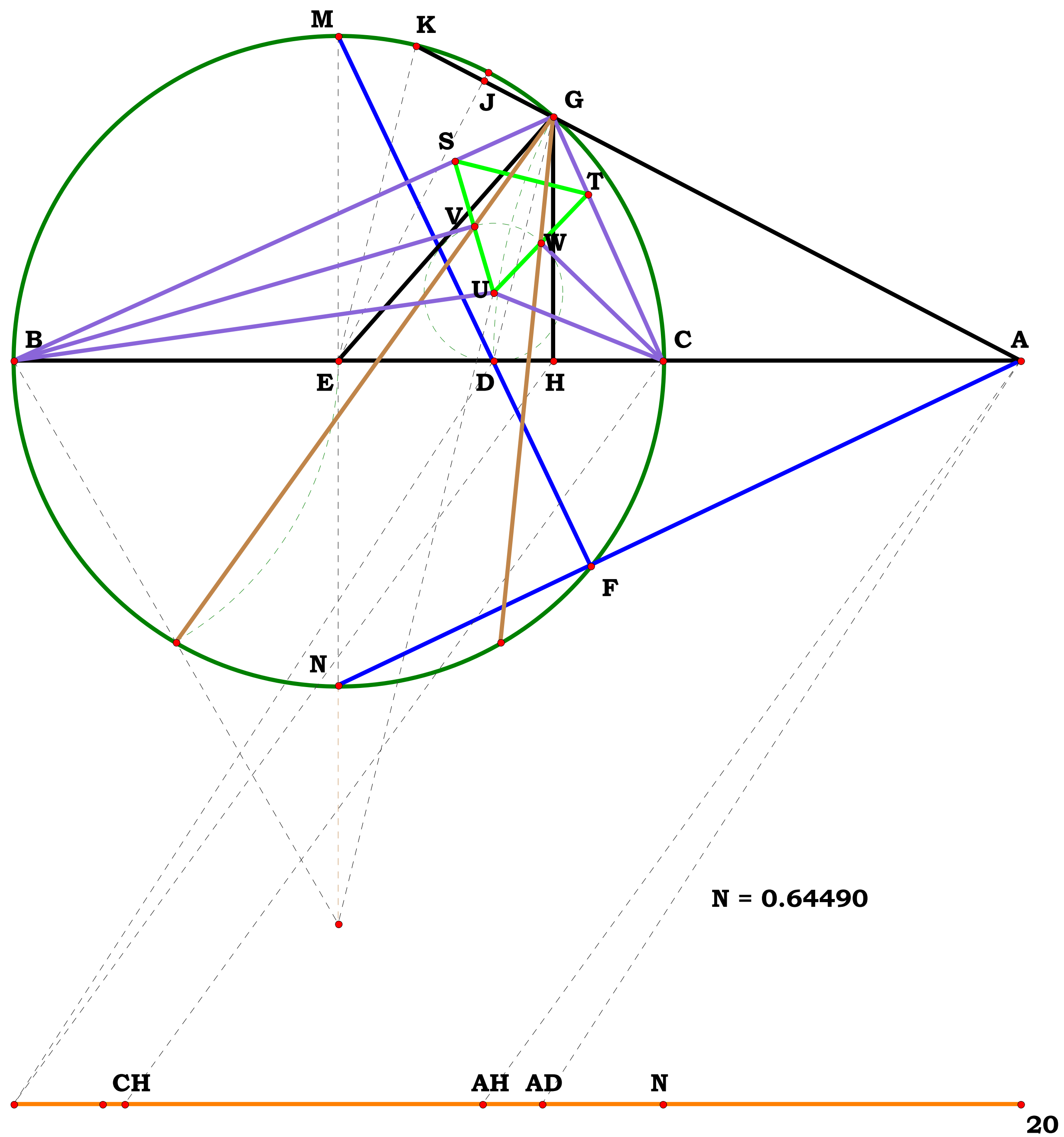
$AC - (1 - N) = 0 \qquad BE - \frac{N}{2} = 0 \qquad AE - \left[\frac{(2 - N)}{2}\right] = 0$

$AN - \frac{\sqrt{N^2 - 2 \cdot N + 2}}{\sqrt{2}} = 0 \qquad FN - \frac{\sqrt{2} \cdot N^2}{2 \cdot \sqrt{N^2 - 2 \cdot N + 2}} = 0$

$DE - \frac{N^2}{2 \cdot (2 - N)} = 0$

$BD - \left[\frac{N}{(2 - N)}\right] = 0 \qquad AD - \left[\frac{2 \cdot (1 - N)}{(2 - N)}\right] = 0 \qquad AH - \frac{(1 - N) \cdot \left(N^2 - 8 \cdot N + 8\right)}{(2 - N)^3} = 0$

$BH - \left[\frac{N \cdot (4 - 3 \cdot N)}{(2 - N)^3}\right] = 0 \qquad CH - \frac{N \cdot (4 - N) \cdot (1 - N)^2}{(2 - N)^3} = 0$



$BD = 0.47590$

$\frac{N}{2-N} = 0.47590$

$\frac{N}{2-N} - BD = 0.00000$

$AD = 0.52410$

$\frac{2\cdot(N-1)}{N-2} = 0.52410$

$\frac{2\cdot(N-1)}{N-2} - AD = 0.00000$

$AH = 0.46475$

$\frac{(N-1)\cdot((N^2-8\cdot N)+8)}{(N-2)^3} = 0.46475$

$\frac{(N-1)\cdot((N^2-8\cdot N)+8)}{(N-2)^3} - AH = 0.00000$

$BH = 0.53525$

$\frac{N\cdot(3\cdot N-4)}{(N-2)^3} = 0.53525$

$\frac{N\cdot(3\cdot N-4)}{(N-2)^3} - BH = 0.00000$

$CH = 0.10965$

$\frac{N\cdot(N-4)\cdot(N-1)^2}{(N-2)^3} = 0.10965$

$\frac{N\cdot(N-4)\cdot(N-1)^2}{(N-2)^3} - CH = 0.00000$

$m\angle GEM = 41.29679°$

$m\angle KEM = 13.76560°$

$\frac{m\angle GEM}{m\angle KEM} = 3.00000$

$m\angle BGC = 90.00000°$

$m\angle VGW = 30.00000°$

$\frac{m\angle BGC}{m\angle VGW} = 3.00000$

$m\angle GCD = 65.64840°$

$m\angle UCD = 21.88280°$

$\frac{m\angle GCD}{m\angle UCD} = 3.00000$

$m\angle GBD = 24.35160°$

$m\angle GBV = 8.11720°$

$\frac{m\angle GBD}{m\angle GBV} = 3.00000$

$m\angle UST = 60.00000°$

$BC = 0.64490$

$BC - N = 0.00000$

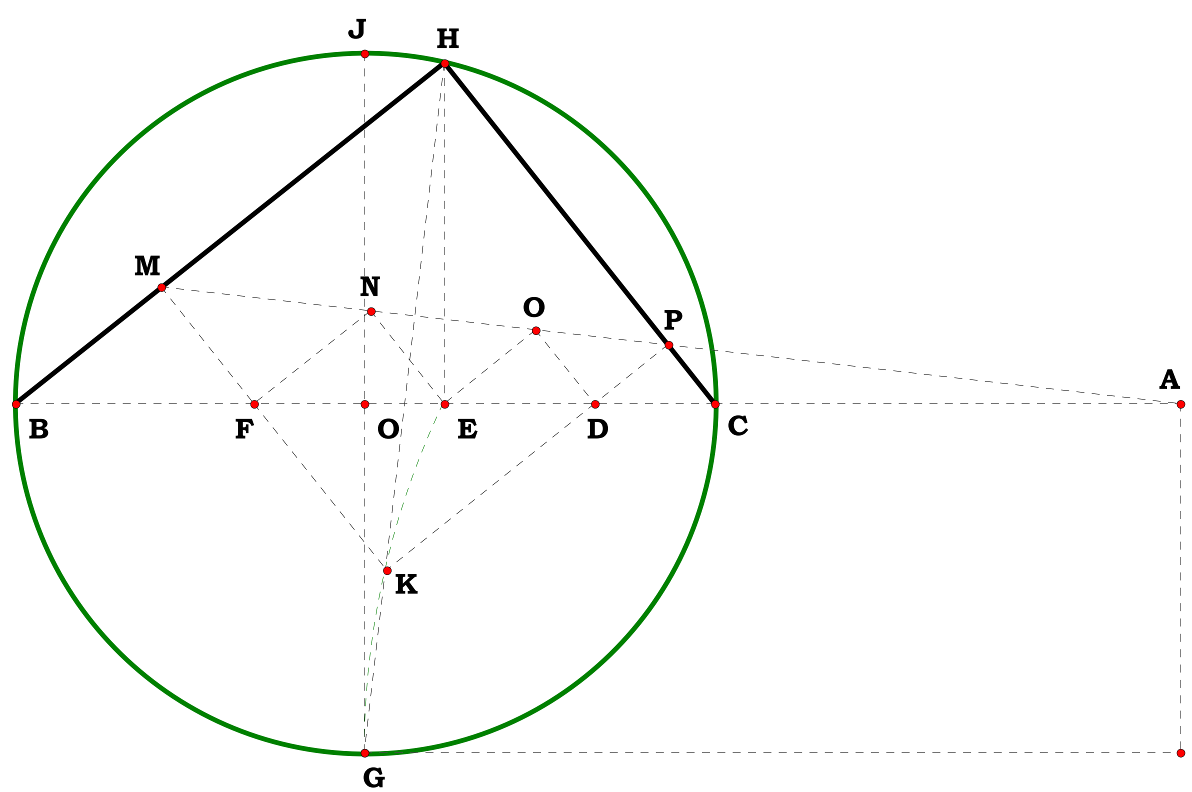
090100Z

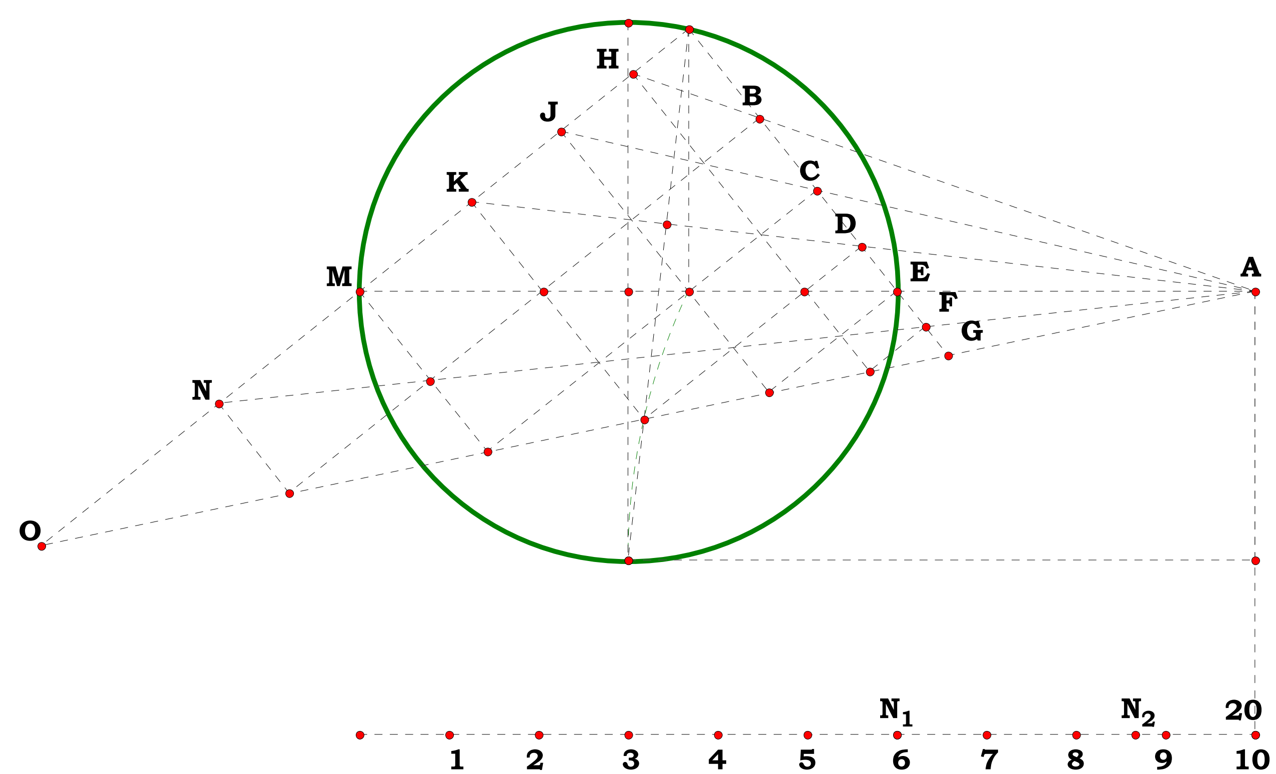
Unit.
Given.
Descriptions.
Definitions.

Quad Roots Etc.

I have had this plate in and out of revisions since 00 but it is so simple and straight forward, I just like looking at it.

And the figure can be expanded and this I find interesting.





$\left(\frac{AB}{AC}\right)^{\frac{1}{4}} = 1.25743$ $\left(\frac{AB}{AC}\right)^{\frac{1}{4}} - \frac{AD}{AC} = 0.00000$ $\frac{AC}{AC} = 1.00000$

$\left(\frac{AB}{AC}\right)^{\frac{2}{4}} = 1.58114$ $\left(\frac{AB}{AC}\right)^{\frac{2}{4}} - \frac{AE}{AC} = 0.00000$ $\frac{AD}{AC} = 1.25743$

$\left(\frac{AB}{AC}\right)^{\frac{3}{4}} = 1.98818$ $\left(\frac{AB}{AC}\right)^{\frac{3}{4}} - \frac{AF}{AC} = 0.00000$ $\frac{AE}{AC} = 1.58114$

$\frac{AF}{AC} = 1.98818$

$\frac{AB}{AC} = 2.50000$

AC = 6.16580 cm
AD = 7.75308 cm
AE = 9.74899 cm
AO = 10.79015 cm
AF = 12.25870 cm
AB = 15.41450 cm

090300A

Unit.

$BG := 1$

Given.

$N_1 := 8$

$N_2 := 10$

Ratios In Trisection

How does BF vary with BC? How does DF vary with BC?

Descriptions.

$BE := \frac{BG}{2}$ $\quad EM := BE$ $\quad BO := \sqrt{2 \cdot BE^2}$ $\quad EN := BE$ $\quad EK := \frac{BE \cdot BE}{BO}$

$KN := EN - EK$ $\quad BK := \frac{BO}{2}$ $\quad BN := \sqrt{BK^2 + KN^2}$ $\quad BD := \frac{BN^2}{BG}$ $\quad BC := BD - BD \cdot \frac{N_1}{N_2}$

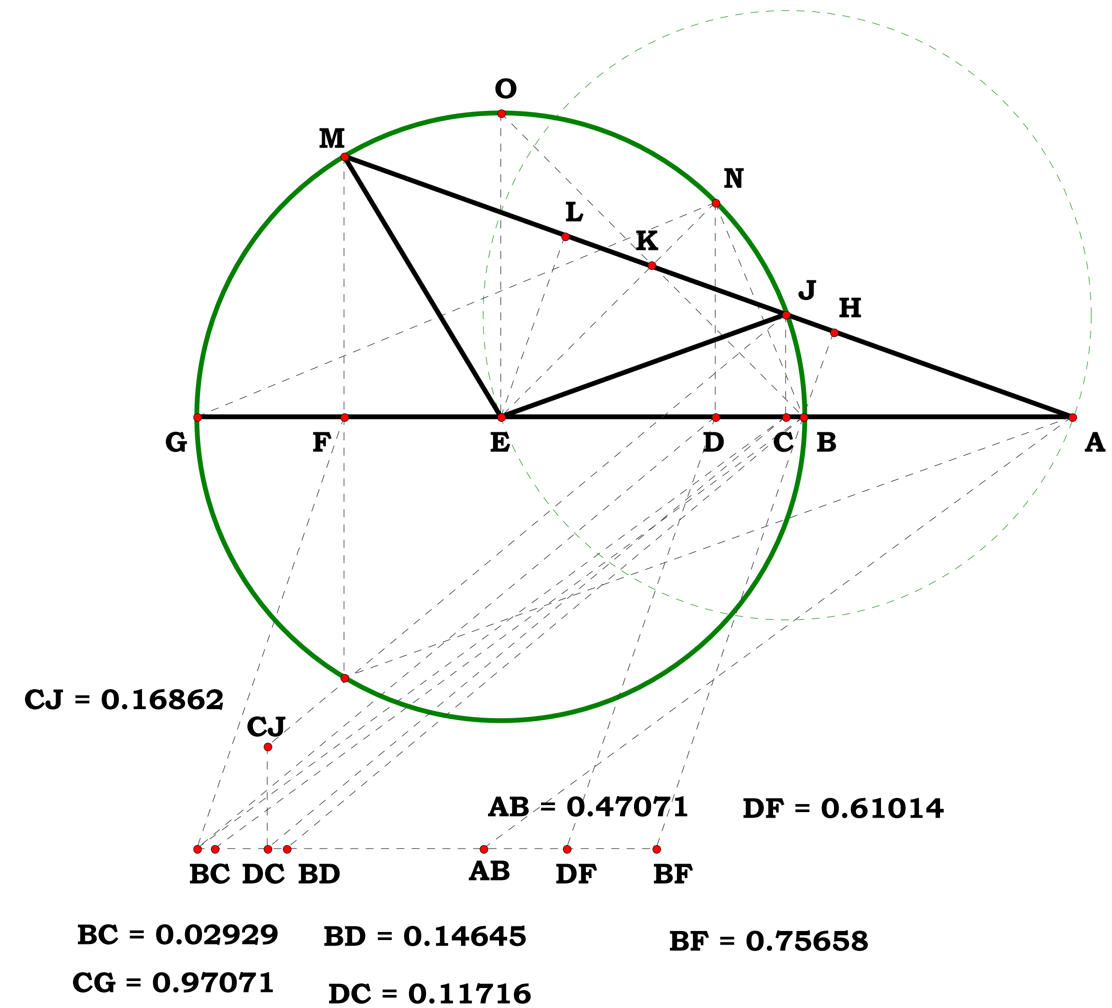
$CG := BG - BC$ $\quad CJ := \sqrt{BC \cdot CG}$ $\quad AJ := BE$ $\quad AC := \sqrt{AJ^2 - CJ^2}$ $\quad AB := AC - BC$

$AE := AB + BE$ $\quad JH := \frac{CJ^2}{AJ}$ $\quad AH := AJ - JH$ $\quad AL := \frac{AH \cdot AE}{AC}$ $\quad JL := AL - AJ$

$LM := JL$ $\quad AM := AL + LM$ $\quad AF := \frac{AH \cdot AM}{AC}$ $\quad BF := AF - AB$ $\quad DF := BF - BD$

$CJ = 0.168616$ $\quad BD = 0.146447$ $\quad DF = 0.610136$ $\quad BF = 0.756583$

$BC = 0.029289$ $\quad CG = 0.970711$ $\quad AB = 0.441421$ $\quad DF = 0.610136$



Definitions.

$BC - \frac{(\sqrt{2}-2)\cdot(N_1 - N_2)}{4 \cdot N_2} = 0$ $\quad BE - \frac{1}{2} = 0$ $\quad EM - \frac{1}{2} = 0$ $\quad BO - \frac{\sqrt{2}}{2} = 0$ $\quad EN - \frac{1}{2} = 0$

$EK - \frac{\sqrt{2}}{4} = 0$ $\quad KN - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) = 0$ $\quad BK - \frac{\sqrt{2}}{4} = 0$ $\quad BN - \frac{\sqrt{2-\sqrt{2}}}{2} = 0$

$BD - \frac{2-\sqrt{2}}{4} = 0$ $\quad BC - \frac{(\sqrt{2}-2)\cdot(N_1 - N_2)}{4 \cdot N_2} = 0$ $\quad CG - \frac{(N_2 - N_1)\cdot(\sqrt{2}+2) + 4 \cdot N_1}{4 \cdot N_2} = 0$ $\quad CJ - \frac{\sqrt{(N_1 - N_2)\cdot\left[N_1 \cdot (2 \cdot \sqrt{2} - 3) - N_2\right]}}{2\sqrt{2} \cdot N_2} = 0$ $\quad AJ - \frac{1}{2} = 0$ $\quad AC - \frac{N_2 + N_1 \cdot (\sqrt{2}-1)}{2\sqrt{2} \cdot N_2} = 0$

$AB - \frac{N_2 \cdot (\sqrt{2}-1) - N_1 \cdot (\sqrt{2}-2)}{2 \cdot N_2} = 0$ $\quad AE - \frac{\sqrt{2} \cdot N_2 - N_1 \cdot (\sqrt{2}-2)}{2 \cdot N_2} = 0$ $\quad JH - \frac{(N_1 - N_2)\cdot\left[N_1 \cdot (2 \cdot \sqrt{2} - 3) - N_2\right]}{4 \cdot N_2^2} = 0$ $\quad AH - \frac{\left[N_2 + N_1 \cdot (\sqrt{2}-1)\right]^2}{4 \cdot N_2^2} = 0$

$AL - \frac{(2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2)\cdot\sqrt{2} + 3 \cdot N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2}{2 \cdot N_2^2} = 0$ $\quad JL - \frac{N_1 \cdot \left[N_2 \cdot (2 \cdot \sqrt{2} - 2) - N_1 \cdot (2 \cdot \sqrt{2} - 3)\right]}{2 \cdot N_2^2} = 0$ $\quad LM - \frac{N_1 \cdot \left[N_2 \cdot (2 \cdot \sqrt{2} - 2) - N_1 \cdot (2 \cdot \sqrt{2} - 3)\right]}{2 \cdot N_2^2} = 0$

$$BF - \frac{\left(\frac{7\cdot\sqrt{2}}{4}+\frac{5}{2}\right)\cdot\left(3\cdot N_1+N_2-2\cdot\sqrt{2}\cdot N_1\right)\cdot\left(4\cdot N_1-3\cdot N_2-3\cdot\sqrt{2}\cdot N_1+2\cdot\sqrt{2}\cdot N_2\right)^2}{N_2^{\,3}} = 0$$

$$AM - \left[\frac{\left(N_2+\sqrt{2}\cdot N_1\right)\cdot\left[N_2+N_1\cdot\left(3\cdot\sqrt{2}-4\right)\right]}{2\cdot N_2^{\,2}}\right] = 0 \qquad AF - \frac{\sqrt{2}\cdot\left(N_2+\sqrt{2}\cdot N_1\right)\cdot\left(N_2-N_1+\sqrt{2}\cdot N_1\right)\cdot\left(N_2-4\cdot N_1+3\cdot\sqrt{2}\cdot N_1\right)}{4\cdot N_2^{\,3}} = 0$$

$$DF - \left[\frac{3\cdot N_1\cdot\left[\left(\frac{20}{3}-\frac{14\cdot\sqrt{2}}{3}\right)\cdot N_1^{\,2}+\left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2+\left(2-\sqrt{2}\right)\cdot N_2^{\,2}\right]}{4\cdot N_2^{\,3}}\right] = 0$$

$$\frac{BF}{BC} = 25.831354 \qquad \frac{DF}{BC} = 20.831354$$

$$\frac{BF}{BC} - \frac{4\cdot\left(\frac{7\cdot\sqrt{2}}{4}+\frac{5}{2}\right)\cdot\left(3\cdot N_1+N_2-2\cdot\sqrt{2}\cdot N_1\right)\cdot\left(4\cdot N_1-3\cdot N_2-3\cdot\sqrt{2}\cdot N_1+2\cdot\sqrt{2}\cdot N_2\right)^2}{N_2^{\,2}\cdot\left(\sqrt{2}-2\right)\cdot\left(N_1-N_2\right)} = 0$$

$$\frac{DF}{BC} - \frac{\left(6\cdot N_1^{\,2}\cdot N_2-3\cdot N_1\cdot N_2^{\,2}+4\cdot\sqrt{2}\cdot N_1^{\,3}-6\cdot N_1^{\,3}-6\cdot\sqrt{2}\cdot N_1^{\,2}\cdot N_2\right)}{N_2^{\,2}\cdot\left(N_1-N_2\right)} = 0$$

090300B

Descriptions.

Unit.

$BC := 1$

Given.

$N_1 := 10$

$N_2 := 8$

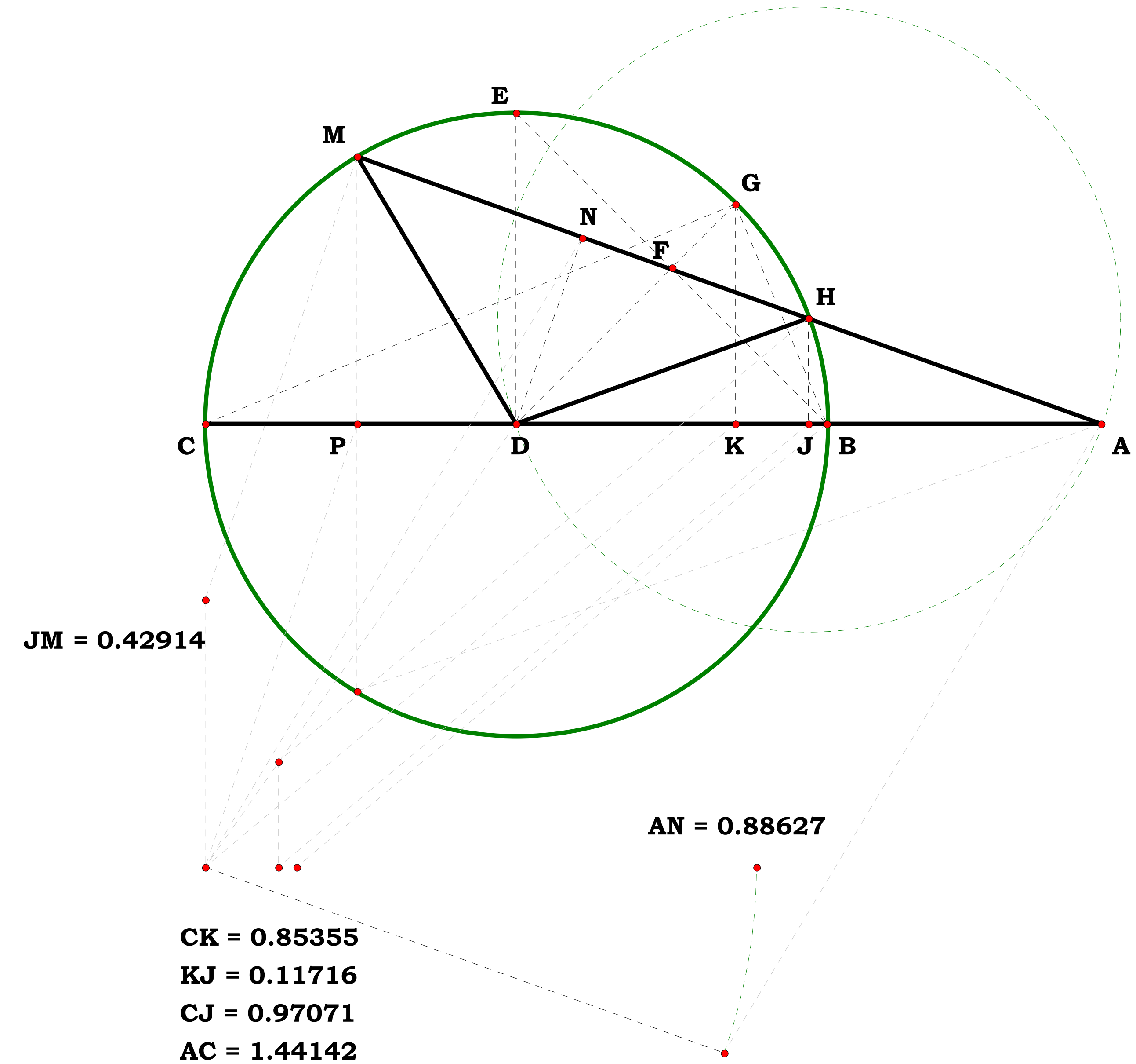
$$BD := \frac{BC}{2} \quad DE := BD \quad BE := \sqrt{2 \cdot BD^2} \quad DK := \frac{BD^2}{BE} \quad BK := BD - DK \quad KJ := BK \cdot \frac{N_2}{N_1}$$

$$CK := BD + DK \quad CJ := CK + KJ \quad BJ := BC - CJ \quad HJ := \sqrt{CJ \cdot BJ} \quad AJ := \sqrt{BD^2 - HJ^2}$$

$$AB := AJ - BJ \quad AB = 0.441421 \quad AN := \frac{AJ \cdot (BD + AB)}{BD} \quad HN := AN - BD \quad MN := HN$$

$$AM := AN + MN \quad MP := \frac{HJ \cdot AM}{BD} \quad MP = 0.429145 \quad \frac{HJ}{MP} = 0.392912$$

$$BR := \frac{HJ \cdot AB}{AJ} \quad AP := \frac{AB \cdot MP}{BR} \quad BP := AP - AB \quad CP := BC - BP \quad KP := BP - BK$$



Definitions.

$$BD - \frac{1}{2} = 0 \quad DE - \frac{1}{2} = 0 \quad BE - \frac{1}{\sqrt{2}} = 0 \quad DK - \frac{\sqrt{2}}{4} = 0 \quad BK - \frac{2-\sqrt{2}}{4} = 0$$

$$KJ - \frac{N_2 \cdot \left(2-\sqrt{2}\right)}{4 \cdot N_1} = 0 \quad CK - \frac{2+\sqrt{2}}{4} = 0 \quad CJ - \frac{N_1 \cdot \left(\sqrt{2}+2\right) - N_2 \cdot \left(\sqrt{2}-2\right)}{4 \cdot N_1} = 0$$

$$BJ - \frac{\left(N_2 - N_1\right) \cdot \left(\sqrt{2}-2\right)}{4 \cdot N_1} = 0 \quad HJ - \frac{\sqrt{\left(N_1 - N_2\right) \cdot \left(N_1 + 3 \cdot N_2 - 2 \cdot \sqrt{2} \cdot N_2\right)}}{2 \cdot \sqrt{2} \cdot N_1} = 0$$

$$AJ - \frac{N_1 + N_2 \cdot \left(\sqrt{2}-1\right)}{2 \cdot \sqrt{2} \cdot N_1} = 0 \quad AB - \left[\frac{N_1 \cdot \left(\sqrt{2}-1\right) - N_2 \cdot \left(\sqrt{2}-2\right)}{2 \cdot N_1}\right] = 0$$

$$AN - \frac{2 \cdot N_2 \cdot \left(N_1 - N_2\right) \cdot \sqrt{2} + N_1^{\,2} - 2 \cdot N_1 \cdot N_2 + 3 \cdot N_2^{\,2}}{2 \cdot N_1^{\,2}} = 0 \quad HN - \frac{N_2 \cdot \left[N_1 \cdot \left(2 \cdot \sqrt{2} - 2\right) - N_2 \cdot \left(2 \cdot \sqrt{2} - 3\right)\right]}{2 \cdot N_1^{\,2}} = 0 \quad AM - \left[\frac{\left(N_1 + \sqrt{2} \cdot N_2\right) \cdot \left[N_1 + N_2 \cdot \left(3 \cdot \sqrt{2} - 4\right)\right]}{2 \cdot N_1^{\,2}}\right] = 0$$

$MP = 0.429145$

$HJ = 0.168616$

$$MP - \frac{\left(N_1+\sqrt{2}\cdot N_2\right)\cdot\sqrt{2\cdot\left(N_1-N_2\right)\cdot\left(N_1+3\cdot N_2-2\cdot\sqrt{2}\cdot N_2\right)}\cdot\left(N_1-4\cdot N_2+3\cdot\sqrt{2}\cdot N_2\right)}{4\cdot N_1^{3}} = 0$$

$$BR - \frac{\sqrt{\left(N_1-N_2\right)\cdot\left(N_1+3\cdot N_2-2\cdot\sqrt{2}\cdot N_2\right)}\cdot\left[N_1\cdot\left(\sqrt{2}-1\right)-N_2\cdot\left(\sqrt{2}-2\right)\right]}{2\cdot N_1\cdot\left[N_1+N_2\cdot\left(\sqrt{2}-1\right)\right]} = 0$$

$$AP - \frac{\left(N_1+\sqrt{2}\cdot N_2\right)\cdot\sqrt{\left(2\cdot N_1-2\cdot N_2\right)\cdot\left(N_1+3\cdot N_2-2\cdot\sqrt{2}\cdot N_2\right)}\cdot\left[N_1+N_2\cdot\left(\sqrt{2}-1\right)\right]\cdot\left[N_1+N_2\cdot\left(3\cdot\sqrt{2}-4\right)\right]}{4\cdot N_1^{3}\cdot\sqrt{\left(N_1-N_2\right)\cdot\left(N_1+3\cdot N_2-2\cdot\sqrt{2}\cdot N_2\right)}} = 0$$

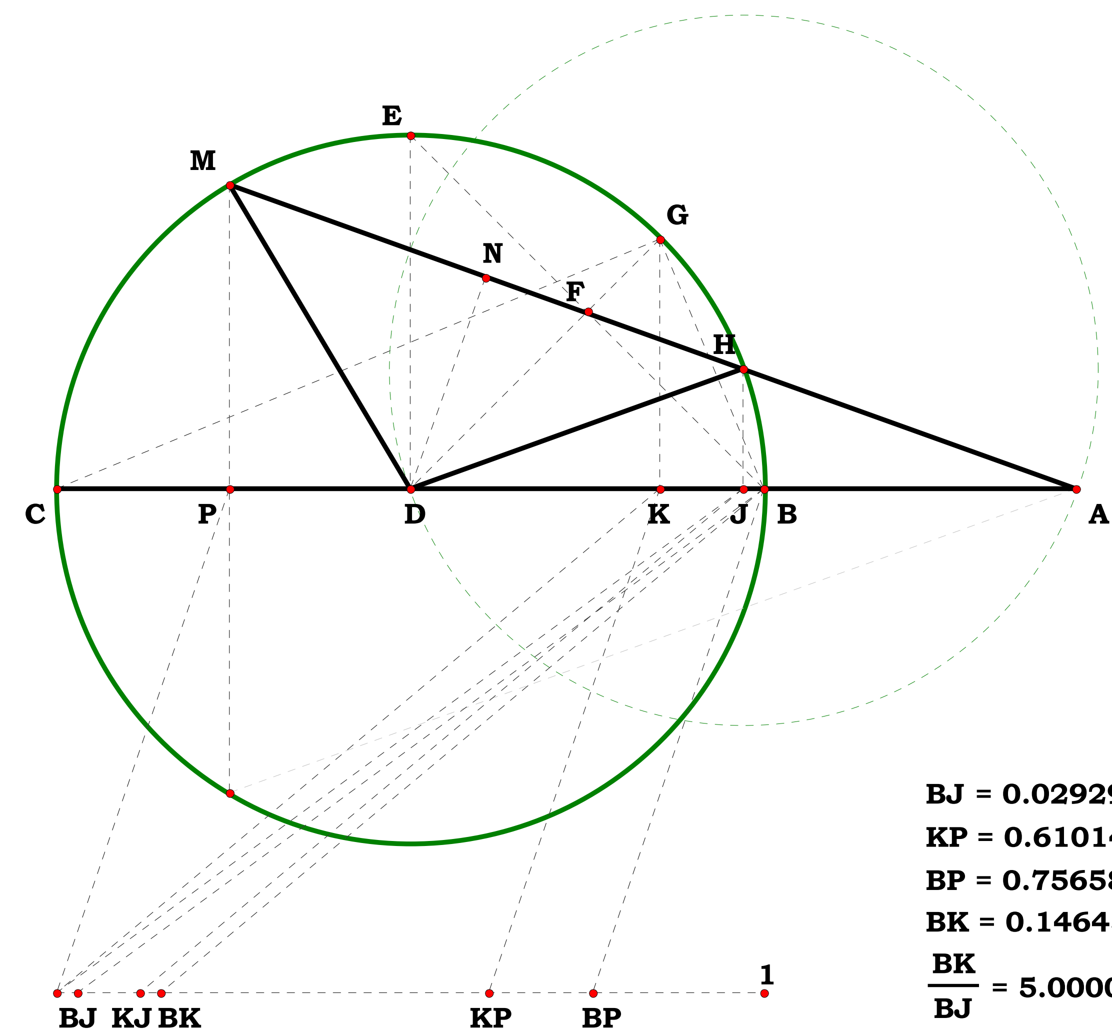
$$BP - \frac{\left[\left[\left(\frac{\sqrt{2}}{8}+\frac{1}{4}\right)\cdot\left[N_1-N_2\cdot\left(2\cdot\sqrt{2}-3\right)\right]\right]\cdot\left[N_2\cdot\left(2\cdot\sqrt{2}-2\right)-N_1\cdot\left(\sqrt{2}-2\right)\right]^{2}\right]}{N_1^{3}} = 0$$

$$CP - \frac{\left(\frac{1}{4}-\frac{\sqrt{2}}{8}\right)\cdot\left(N_1-N_2\right)\cdot\left[N_1\cdot\left(\sqrt{2}+2\right)+N_2\cdot\left(2\cdot\sqrt{2}-2\right)\right]^{2}}{N_1^{3}} = 0$$

$$KP - \frac{3\cdot N_2\cdot\left[\left(2-\sqrt{2}\right)\cdot N_1^{2}+\left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2+\left(\frac{20}{3}-\frac{14\cdot\sqrt{2}}{3}\right)\cdot N_2^{2}\right]}{4\cdot N_1^{3}} = 0$$

$$\frac{BP}{BJ} - \frac{\left(12\cdot\sqrt{2}+17\right)\cdot\left[N_1-N_2\cdot\left(2\cdot\sqrt{2}-3\right)\right]\cdot\left[N_2\cdot\left(3\cdot\sqrt{2}-4\right)-N_1\cdot\left(2\cdot\sqrt{2}-3\right)\right]^{2}}{N_1^{2}\cdot\left(N_1-N_2\right)} = 0$$

$$\frac{KP}{BJ} - \frac{\left(\frac{3\cdot\sqrt{2}}{2}+3\right)\cdot N_2\cdot\left[\left(2-\sqrt{2}\right)\cdot N_1^{2}+\left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2+\left(\frac{20}{3}-\frac{14\cdot\sqrt{2}}{3}\right)\cdot N_2^{2}\right]}{N_1^{2}\cdot\left(N_1-N_2\right)} = 0$$



$BJ = 0.02929$

$KP = 0.61014$

$BP = 0.75658$

$BK = 0.14645$

$\frac{BK}{BJ} = 5.00000$

$KJ = 0.11716$

$\frac{KJ}{BK} = 0.80000$

$\frac{BK}{BK} = 1.00000$

$\frac{KJ}{BK} = 0.80000$

$\frac{BJ}{BK} = 0.20000$

$\frac{2}{10} = 0.20000$

$$\frac{BP}{BJ} = 25.831354$$

$$\frac{KP}{BJ} = 20.831354$$

$$BJ - \frac{(N_2 - N_1)\cdot(\sqrt{2} - 2)}{4\cdot N_1} = 0 \qquad BK - \frac{2 - \sqrt{2}}{4} = 0$$

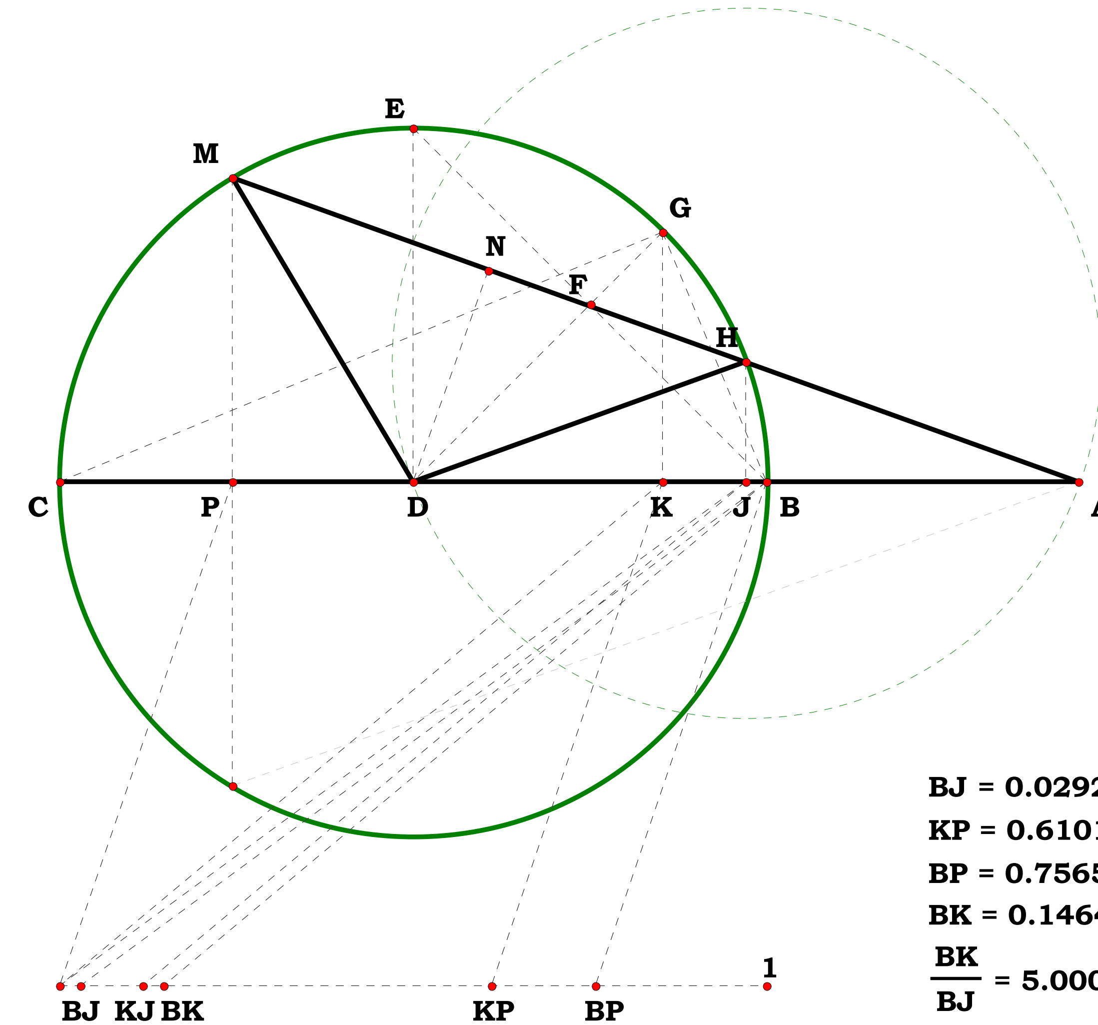
$$BP - \frac{\left[\left(\frac{\sqrt{2}}{8} + \frac{1}{4}\right)\cdot\left[10 - 8\cdot(2\cdot\sqrt{2} - 3)\right]\cdot\left[8\cdot(2\cdot\sqrt{2} - 2) - 10\cdot(\sqrt{2} - 2)\right]^2\right]}{10^3} = 0$$

Which means that you can write a simple program using whole numbers for N1 and N2 to find BJ as a unit to name BP and then you will know the name of BJ. A ratio is a unit conversion.

$$\frac{BJ}{BK} = 0.2 \qquad \frac{BJ}{BK} - \frac{N_1 - N_2}{N_1} = 0 \qquad N_1 = 10 \qquad N_2 = 8 \qquad \frac{1}{1000000000000000} = 1\times 10^{-15}$$

Which is the ratio one uses all the time when dividing a simple segment. I trust one can write a program to find any given ratio with it? N1 simply sets the precision of the Arithmetic name.

$$KP - \frac{3\cdot 8\cdot\left[(2 - \sqrt{2})\cdot 10^2 + (6\cdot\sqrt{2} - 8)\cdot 10\cdot 8 + \left(\frac{20}{3} - \frac{14\cdot\sqrt{2}}{3}\right)\cdot 8^2\right]}{4\cdot 10^3} = 0$$



$BJ = 0.02929$

$KP = 0.61014$

$BP = 0.75658$

$BK = 0.14645$

$\frac{BK}{BJ} = 5.00000$

$KJ = 0.11716$

$\frac{KJ}{BK} = 0.80000$

$\frac{BK}{BK} = 1.00000$

$\frac{KJ}{BK} = 0.80000$

$\frac{BJ}{BK} = 0.20000$

$\frac{2}{10} = 0.20000$

Unit.

$BC := 1$

Given.

$N := 2$

Goshdarn Good Pencil

091600

Descriptions.

$BJ := N \quad BE := \sqrt{BC \cdot BJ} \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IO := CI \quad NO := CJ \quad CR := CJ \quad CE := BE - BC$

$EI := CI - CE \quad EJ := CJ - CE \quad EL := \sqrt{CE \cdot EJ}$

$EG := \frac{EI \cdot EL}{EL + IO} \quad GI := EI - EG \quad GO := \sqrt{GI^2 + IO^2}$

$OP := GO \quad IP := IO + OP \quad EF := \frac{EI \cdot EL}{EL + IP} \quad FI := EI - EF$

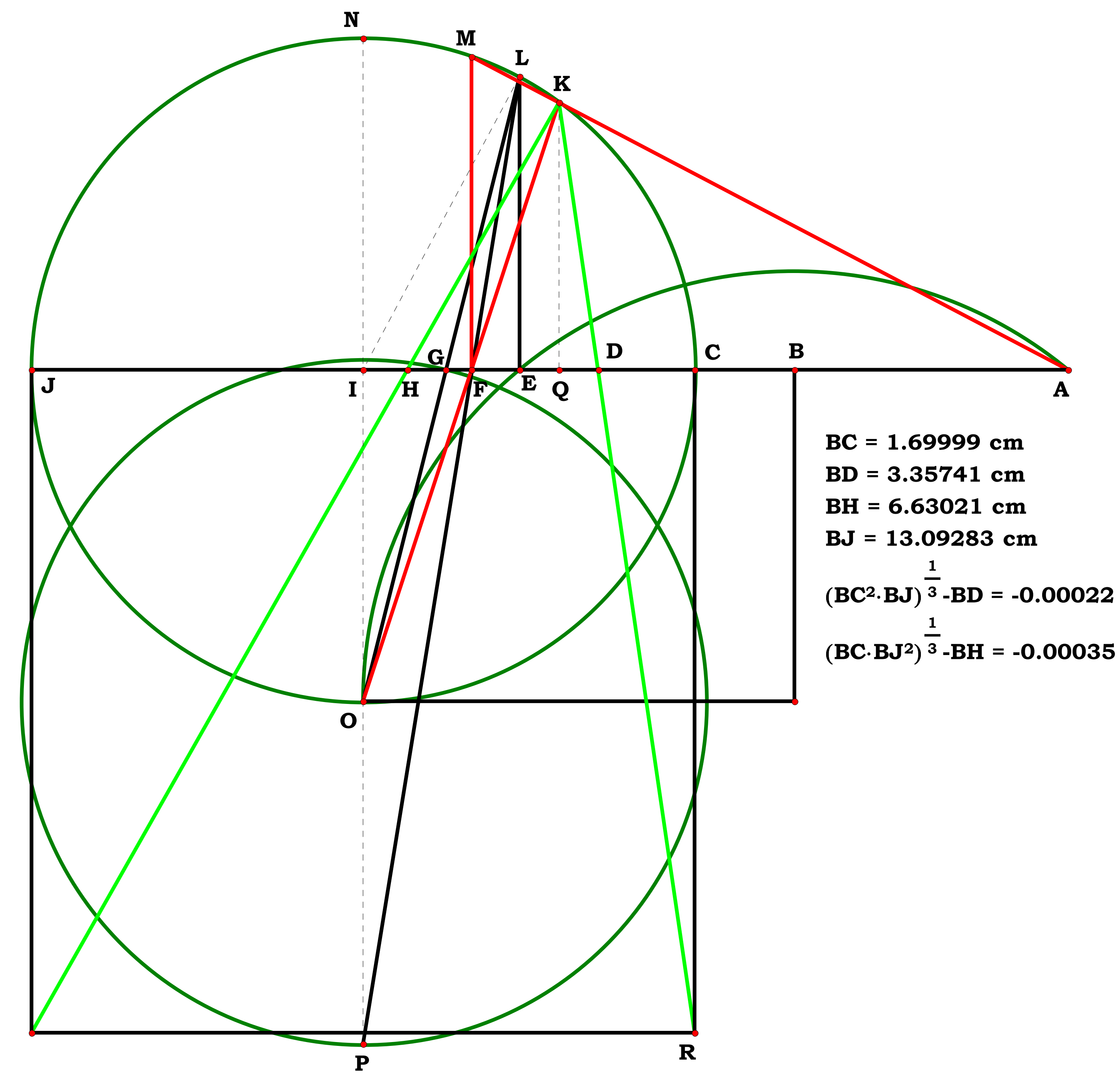
$FO := \sqrt{FI^2 + IO^2} \quad OK := \frac{IO \cdot NO}{FO} \quad FK := OK - FO$

$FQ := \frac{FI \cdot FK}{FO} \quad QI := FQ + FI \quad CQ := CI - QI \quad QJ := CJ - CQ$

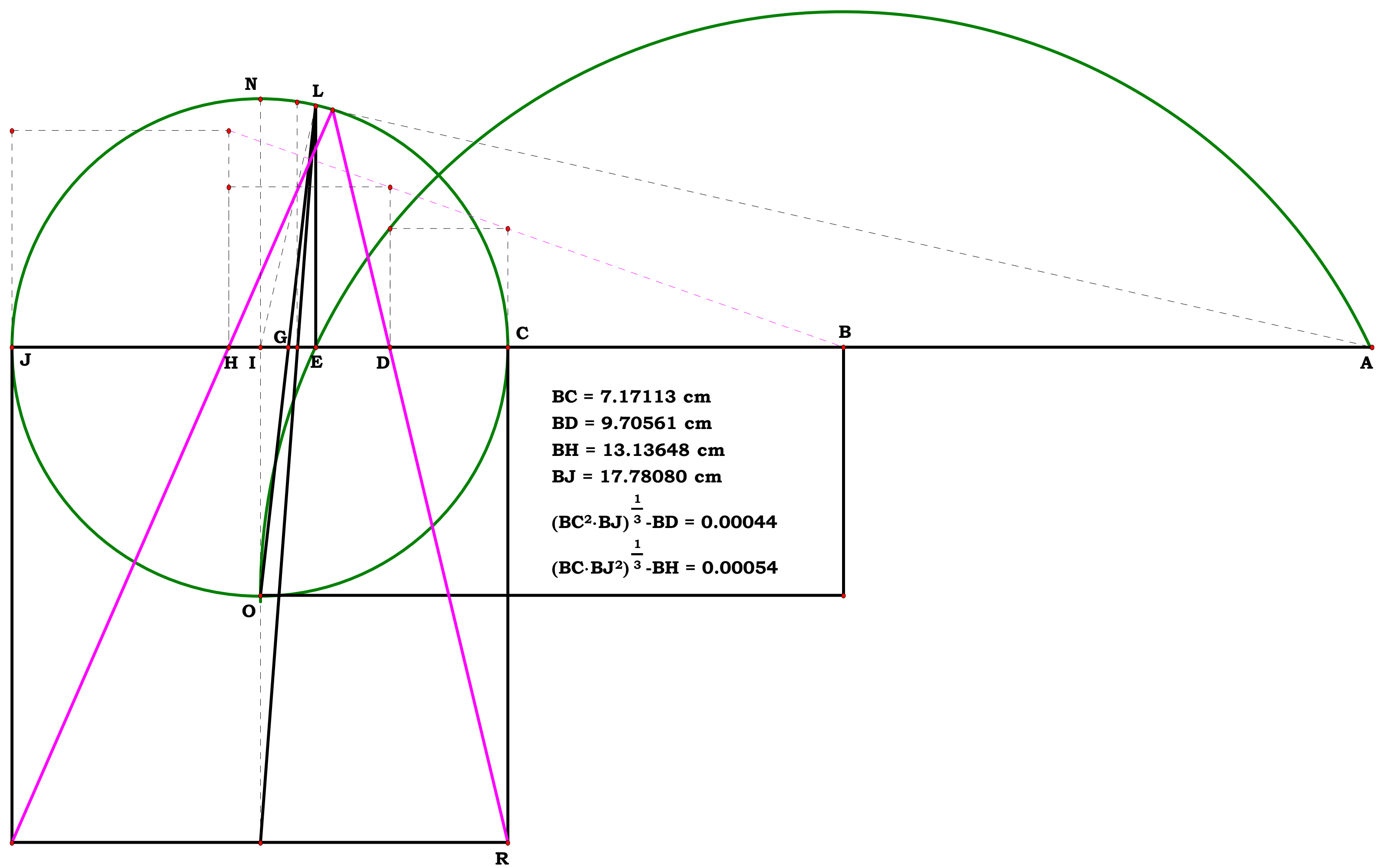
$QK := \sqrt{CQ \cdot QJ} \quad CD := \frac{CQ \cdot CR}{CR + QK} \quad BD := CD + BC$

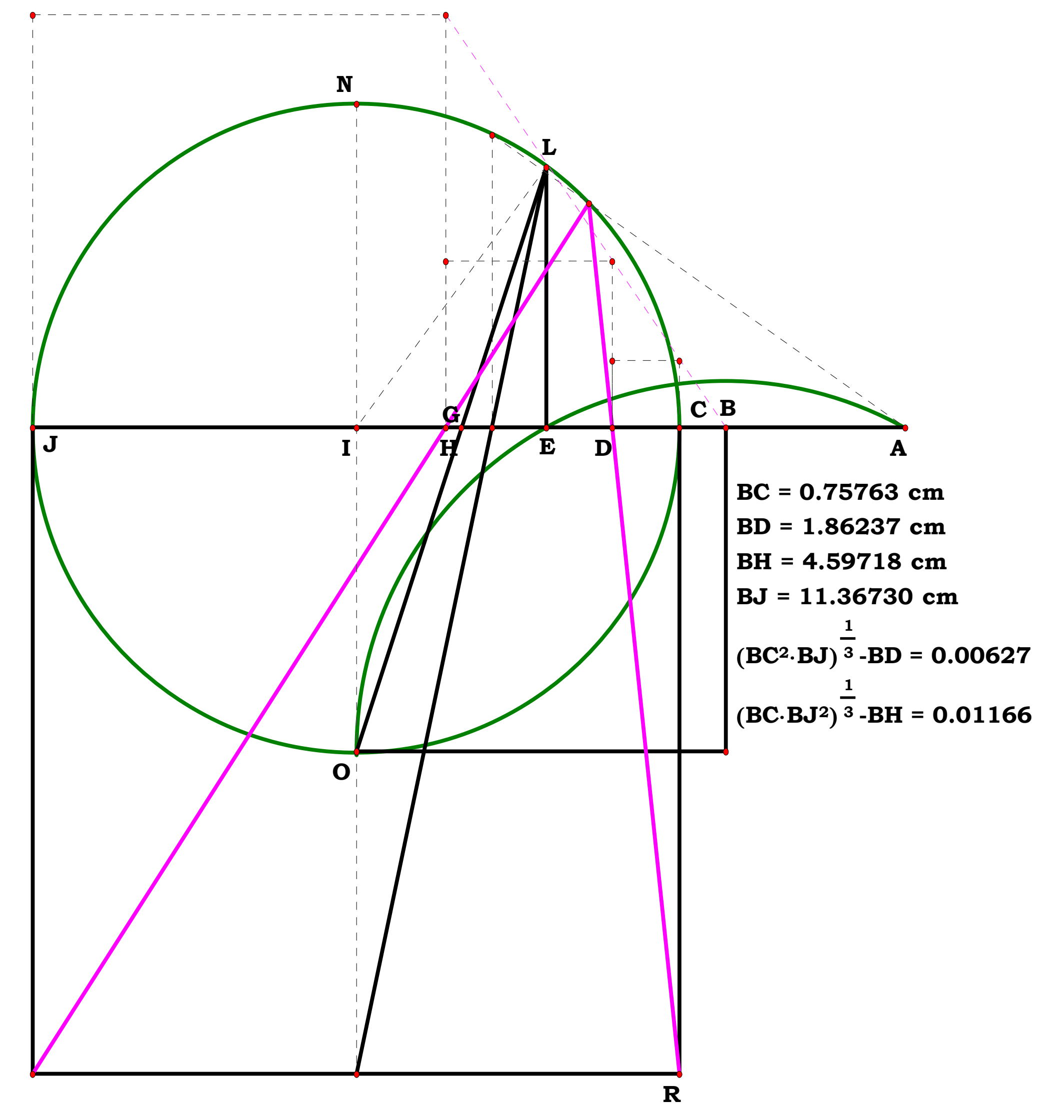
$$\left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 4.486958 \times 10^{-6}$$

Definitions.



Compare to the following which is a great deal less accurate. When you work the figure, you cannot tell it is off:





091800A

Descriptions.

Unit.

Given.

$N_1 := 2 \quad BC := N_1 \quad CJ := BC \quad GK := CJ$

$N_2 := 3 \quad GH := N_2 \quad GM := GH$

$N_3 := 8 \quad CG := N_3 \quad JK := CG$

$BH := BC + CG + GH \qquad BE := \dfrac{BH}{2} \qquad KM := GM - GK$

$JK := CG \qquad AG := \dfrac{JK \cdot GM}{KM} \qquad AH := AG + GH$

$AB := AH - BH \qquad AE := AB + BE$

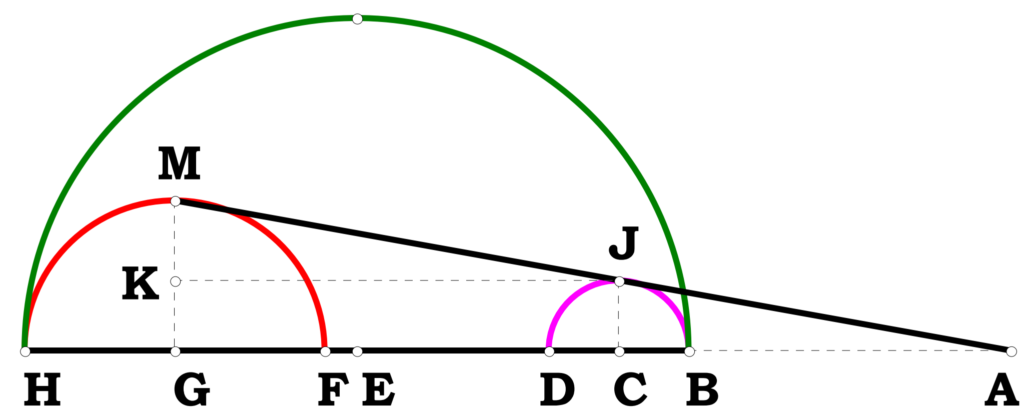
Definitions.

$$BH - \left(N_1 + N_3 + N_2\right) = 0 \qquad BE - \frac{N_1 + N_3 + N_2}{2} = 0 \qquad KM - \left(N_2 - N_1\right) = 0$$

$$AG - \frac{N_3 \cdot N_2}{N_2 - N_1} = 0 \qquad AH - \frac{N_2 \cdot \left(N_1 - N_2 - N_3\right)}{N_1 - N_2} = 0$$

$$AB - \frac{N_1 \cdot \left(N_1 - N_2 + N_3\right)}{\left(N_2 - N_1\right)} = 0 \qquad AE - \frac{\left(N_1 - N_2\right)^2 + N_3 \cdot \left(N_1 + N_2\right)}{2 \cdot \left(N_2 - N_1\right)} = 0$$

Midpoints and Similarity Points



What is AE given the radius of the two circles and the difference between their centers? (External Unit).

091800B

Descriptions.

Unit.

$BH := 1$

Given.

$N_1 := 3 \quad N_3 := 4$

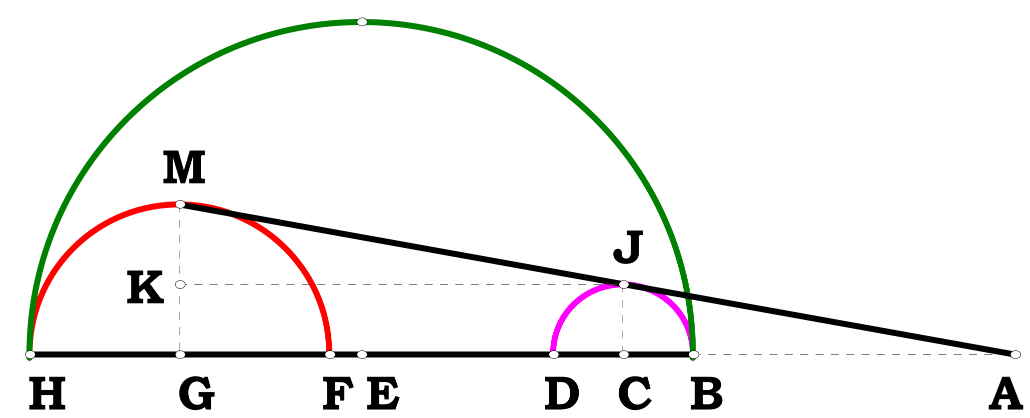
$N_2 := 7 \quad N_4 := 9$

$$BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2} \quad GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$$

$$CJ := BC \quad GM := GH \quad GK := CJ \quad KM := GM - GK \quad JK := CG$$

$$AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$$

Midpoints and Similarity Points



What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).

Definitions.

$$BE - \frac{1}{2} = 0 \quad BC - \frac{N_1}{N_2} = 0 \quad GH - \frac{N_3}{N_4} = 0 \quad CG - \frac{\left(N_2 \cdot N_4 - N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0$$

$$CJ - \frac{N_1}{N_2} = 0 \quad GM - \frac{N_3}{N_4} = 0 \quad GK - \frac{N_1}{N_2} = 0 \quad KM - \frac{\left(N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0 \quad JK - \frac{\left(N_2 \cdot N_4 - N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0$$

$$AG - \frac{N_3 \cdot \left(N_1 \cdot N_4 + N_2 \cdot N_3 - N_2 \cdot N_4\right)}{N_4 \cdot \left(N_1 \cdot N_4 - N_2 \cdot N_3\right)} = 0 \quad AH - \frac{N_3 \cdot \left(2 \cdot N_1 - N_2\right)}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0 \quad AB - \frac{N_1 \cdot \left(2 \cdot N_3 - N_4\right)}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0$$

$$AE - \frac{4 \cdot N_1 \cdot N_3 - N_1 \cdot N_4 - N_2 \cdot N_3}{2 \cdot \left(N_1 \cdot N_4 - N_2 \cdot N_3\right)} = 0$$

000920A

Descriptions.

$$BD := \frac{BE}{2} \qquad CD := BD - BC \qquad CH := BD$$

$$DH := \sqrt{BD^2 + CD^2} \qquad DG := \frac{DH^2}{2BD} \qquad AD := \frac{BD \cdot DG}{CD}$$

$$AE := AD + BD \qquad AB := AE - BE \qquad AC := BC + AB$$

$$AC - \sqrt{AB \cdot AE} = 0$$

Definitions.

$$BD - \frac{N_1}{2} = 0 \qquad CD - \left(\frac{N_1 - 2 \cdot N_2}{2}\right) = 0 \qquad CH - \frac{N_1}{2} = 0$$

$$DH - \frac{\sqrt{N_1{}^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2{}^2}}{\sqrt{2}} = 0$$

$$DG - \frac{N_1{}^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2{}^2}{2 \cdot N_1} = 0$$

$$AD - \frac{N_1{}^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2{}^2}{2 \cdot (N_1 - 2 \cdot N_2)} = 0 \qquad AE - \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0$$

$$AB - \frac{N_2{}^2}{N_1 - 2 \cdot N_2} = 0 \qquad AC - \frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0$$

Unit.

Given.

$$N_1 := 11.69458 \qquad BE := N_1$$

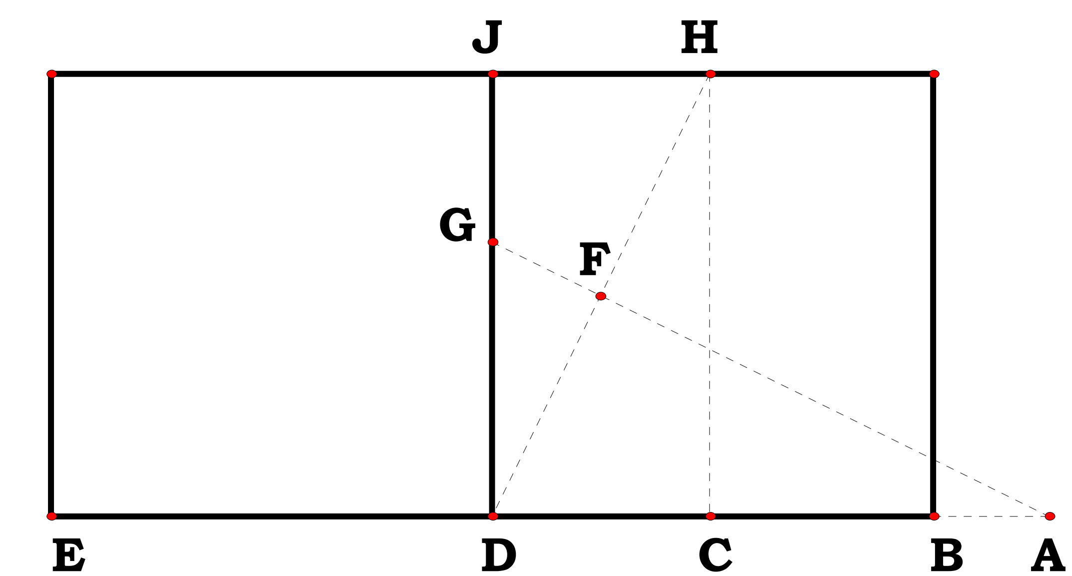
$$N_2 := 2.96916 \qquad BC := N_2$$

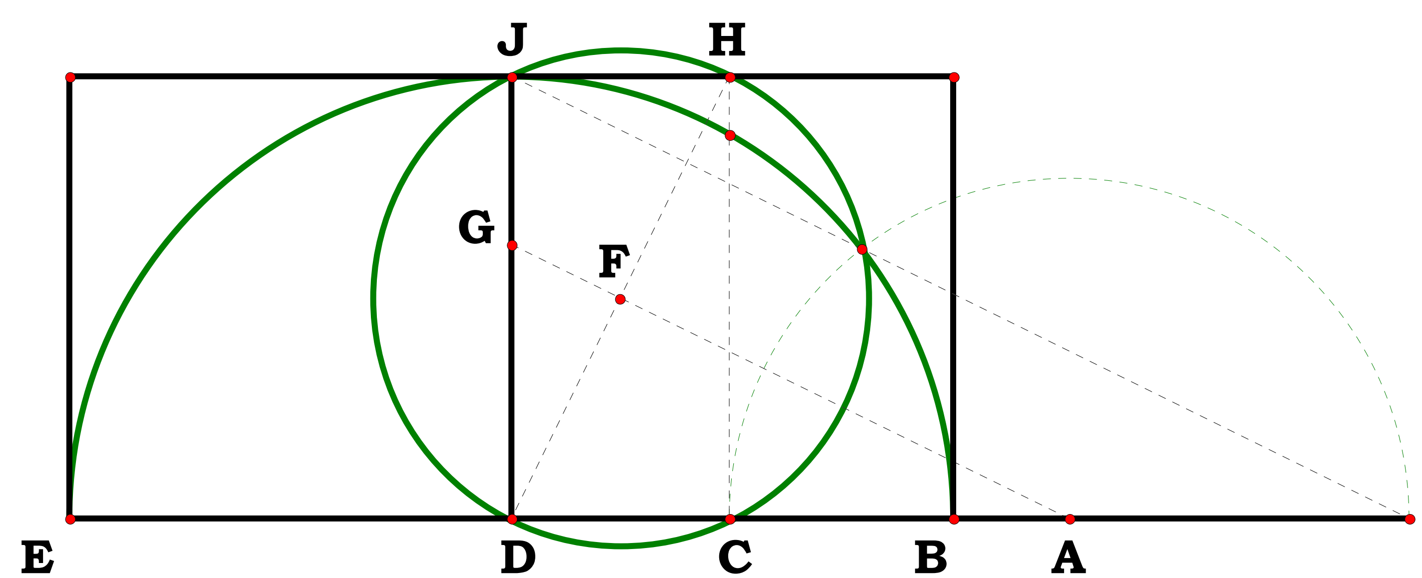
Squaring

Is AC the square root of AB x AE?

Given BC, find AB such that AB x AE is the square root.

This plate solves for the figure using a traditional square root figure.
The next two plates, which I put off doing until now, will approach the figure differently and the last might be a surprise but it was what I was pondering when I set up the original figure for solving cube roots. The last figure will actuall glue both of these figures together. Now, if one had glued them both together, one would be hard pressed to prove that cube roots are impossible in geometry as the compound figure makes the claim rather dubious.





Unit.
$BE := 1$

Given.

$N_1 := 5$

$N_2 := 20$

092000B

Descriptions.

$DE := \frac{BE}{2}$ $\quad BD := DE$ $\quad CD := BD \cdot \frac{N_1}{N_2}$

$CE := CD + DE$ $\quad DF := \sqrt{CD^2 + BD^2}$

$DG := \frac{DF}{2}$ $\quad AD := \frac{DF \cdot DG}{CD}$ $\quad AE := AD + DE$

$AB := AD - BD$ $\quad AC := AD - CD$

Definitions.

$$DE - \frac{1}{2} = 0 \qquad BD - \frac{1}{2} = 0 \qquad CD - \frac{N_1}{2 \cdot N_2}$$

$$CE - \frac{N_1 + N_2}{2 \cdot N_2} = 0 \qquad DF - \frac{\sqrt{N_1^{\,2} + N_2^{\,2}}}{2 \cdot N_2} = 0$$

$$DG - \frac{\sqrt{N_1^{\,2} + N_2^{\,2}}}{4 \cdot N_2} = 0 \qquad AD - \frac{N_1^{\,2} + N_2^{\,2}}{4 \cdot N_1 \cdot N_2} = 0 \qquad AE - \frac{(N_1 + N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$$

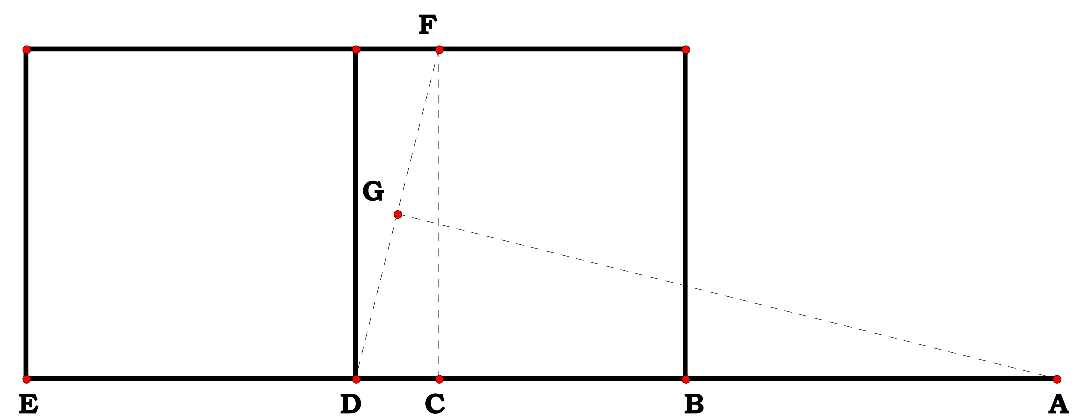
$$AB - \frac{(N_1 - N_2)^2}{4 \cdot N_1 \cdot N_2} = 0 \qquad AC - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$$

$$AB \cdot AE - \frac{(N_1 - N_2)^2}{4 \cdot N_1 \cdot N_2} \cdot \frac{(N_1 + N_2)^2}{4 \cdot N_1 \cdot N_2} = 0 \qquad AC - \sqrt{\frac{(N_1 + N_2)^2 \cdot (N_1 - N_2)^2}{16 \cdot N_1^{\,2} \cdot N_2^{\,2}}} = 0 \qquad AC - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$$

Squaring

Is AC the square root of AB x AE?

Given BC, find AB such that AB x AE is the square root.



$BE = 1.00000$

$DE = 0.50000$

$CE = 0.62500$

$AE = 1.56250$

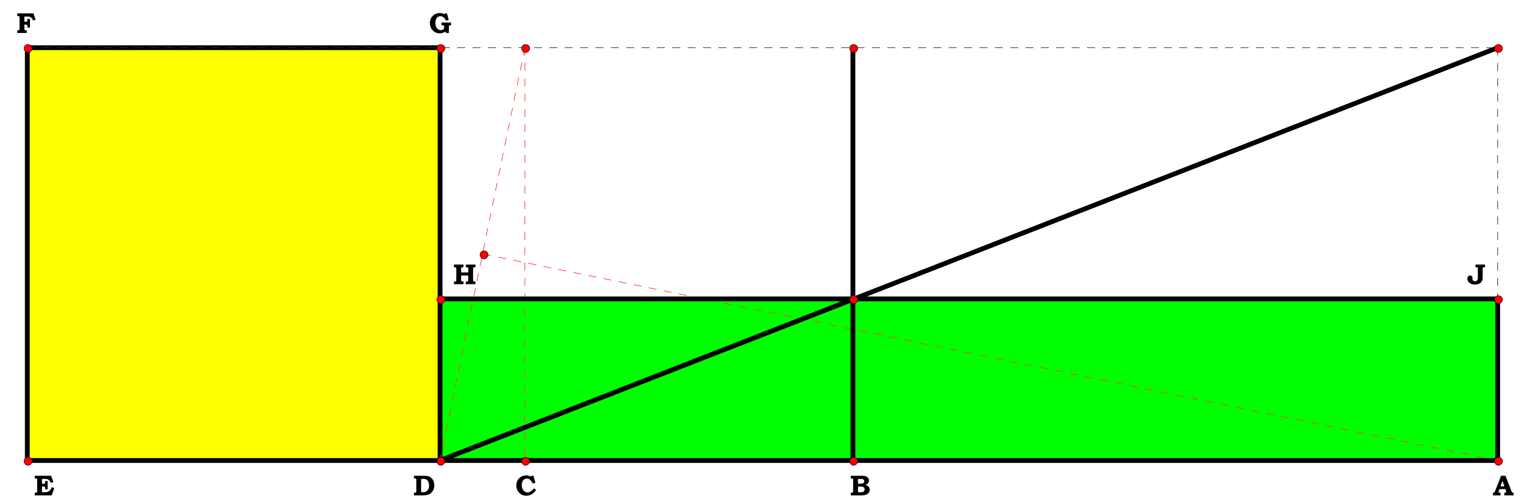
$\frac{DC}{DB} = 0.25000$

Unit.
Given.
Descriptions.
Definitions.

092000B

I am not going to write this up as it is obvious. One will notice that the part of the figure with the green in it is a figure in the Elements involving complements. The original two ways to express the same figure produces a cube root figure, but one which is not exactly as we would hope for.
The figure does, however, disprove the claim that cube roots cannot be done at all when it is actually a primive.

Cube Root Primitive



Area FGDE = 29.87898 cm²
Area HJAD = 29.87898 cm²

DE = 5.46617 cm
DH = 2.13559 cm
AD = 13.99098 cm

AB = 8.52482 cm
AC = 12.87900 cm
AE = 19.45715 cm

$(AB \cdot AE)^{\frac{1}{2}}$-AC = 0.00000
AB·AC·AE = 2136.22303 cm³

DE·DH·AD = 163.32347 cm³

$(AB \cdot AC \cdot AE)^{\frac{1}{3}}$-AC = 0.00000

$(DE \cdot DH \cdot AD)^{\frac{1}{3}}$-DE = 0.00000

111300
Descriptions.

Unit.
Given.

$N_1 := 3$

$N_2 := 5$

$N_3 := 1$

Definitions.

$AB := N_1$

$AD := N_2$

$DE := N_3$

For Two Right Triangles.

Given AB, DE, AD find BE, AC, CD, CE, BC.

BAD and BED are right.

$BD := \sqrt{AB^2 + AD^2}$ $\quad BF := \dfrac{AB^2}{BD}$ $\quad DG := \dfrac{DE^2}{BD}$ $\quad BE := \sqrt{BD^2 - DE^2}$

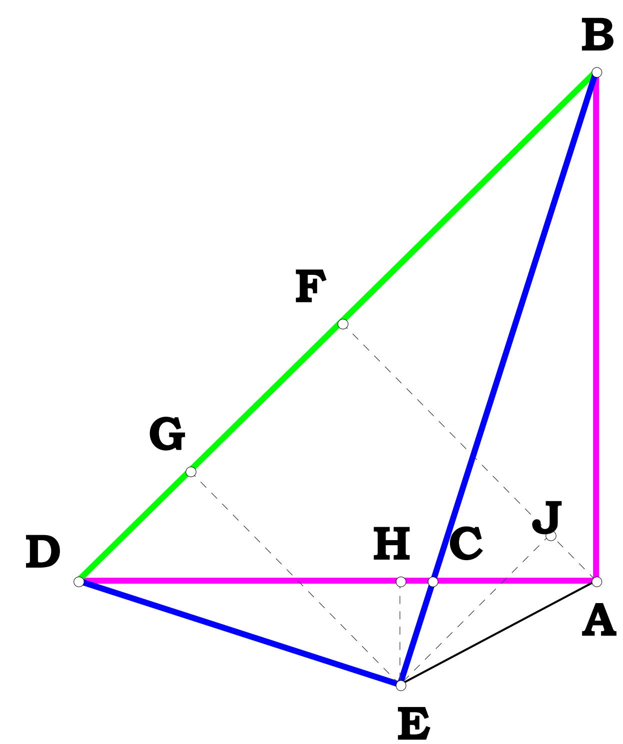
$AF := \sqrt{AB^2 - BF^2}$ $\quad EG := \sqrt{DE^2 - DG^2}$ $\quad FG := BD - (BF + DG)$

$EJ := FG$ $\quad FJ := EG$ $\quad AJ := AF - FJ$ $\quad AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD$ $\quad S_2 := DE$ $\quad S_3 := AE$ $\quad AH := \dfrac{S_3{}^2 + S_1{}^2 - S_2{}^2}{2 \cdot S_1}$

$EH := \sqrt{AE^2 - AH^2}$ $\quad CH := \dfrac{EH \cdot AH}{AB + EH}$ $\quad AC := AH - CH$

$CE := \dfrac{AC \cdot DE}{AB}$ $\quad CD := AD - AC$ $\quad BC := BE - CE$



Definitions:

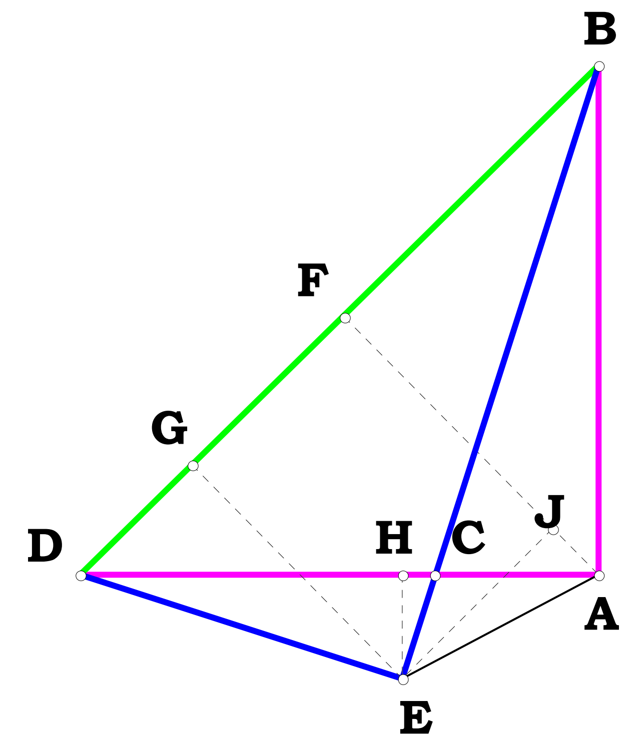
$$BE - \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}} = 0$$

$$\frac{N_1 \cdot \left(N_1^{\,2} \cdot N_2 - N_2 \cdot N_3^{\,2} + N_2^{\,3} - N_1 \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}\right)}{N_1 \cdot N_2^{\,2} + N_1^{\,3} + N_3 \cdot \sqrt{N_2^{\,4} + N_1^{\,2} \cdot N_2^{\,2} + N_1^{\,2} \cdot N_3^{\,2} - N_2^{\,2} \cdot N_3^{\,2} - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}}} - AC = 0$$

$$N_2 + \frac{N_1 \cdot N_2 \cdot N_3^{\,2} - N_2 \cdot \left(N_1^{\,3} + N_1 \cdot N_2^{\,2}\right) + N_1^{\,2} \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}}{N_1 \cdot N_2^{\,2} + N_1^{\,3} + N_3 \cdot \sqrt{N_2^{\,4} + N_1^{\,2} \cdot N_2^{\,2} + N_1^{\,2} \cdot N_3^{\,2} - N_2^{\,2} \cdot N_3^{\,2} - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}}} - CD = 0$$

$$\frac{N_3 \cdot \left(N_1^{\,2} \cdot N_2 - N_2 \cdot N_3^{\,2} + N_2^{\,3} - N_1 \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}\right)}{N_1 \cdot N_2^{\,2} + N_1^{\,3} + N_3 \cdot \sqrt{N_2^{\,4} + N_1^{\,2} \cdot N_2^{\,2} + N_1^{\,2} \cdot N_3^{\,2} - N_2^{\,2} \cdot N_3^{\,2} - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}}} - CE = 0$$

$$BC - \left(\sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}} + \frac{N_2 \cdot N_3^{\,3} - N_2^{\,3} \cdot N_3 - N_1^{\,2} \cdot N_2 \cdot N_3 + N_1 \cdot N_3^{\,2} \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}}{N_1 \cdot N_2^{\,2} + N_1^{\,3} + N_3 \cdot \sqrt{N_2^{\,4} + N_1^{\,2} \cdot N_2^{\,2} + N_1^{\,2} \cdot N_3^{\,2} - N_2^{\,2} \cdot N_3^{\,2} - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - N_3^{\,2}}}}\right) = 0$$



112700

Descriptions.

Unit.

$A1 := 1$

Given.

$N := 8$

Let us suppose that we have any unit, or thing and we want to parse that thing into units, any number of units at all, we must first define the unit. In the following case, our starting unit, or thing is A to 1. Now when we parse it, we are defining smaller units. We do not want to call them fractions, or have to deal with fractions, so we rename our thing in terms of a given number of our new chosen unit, AB. This figure shows how to proceed to parse A1 into a 2N exponential series. We do this, as said, by creating a fraction, and giving the unit of that fraction the name of our new working unit. In short, we are converting base systems. We go from base 1, always to some other base. The base is named for the number of units it contains, or subsets of our given set.
In this case, 8 is a subset of 1, and every member of 1, is defined in terms of 1.

$AB := \frac{A1}{N}$ AB is a member of the set A1.

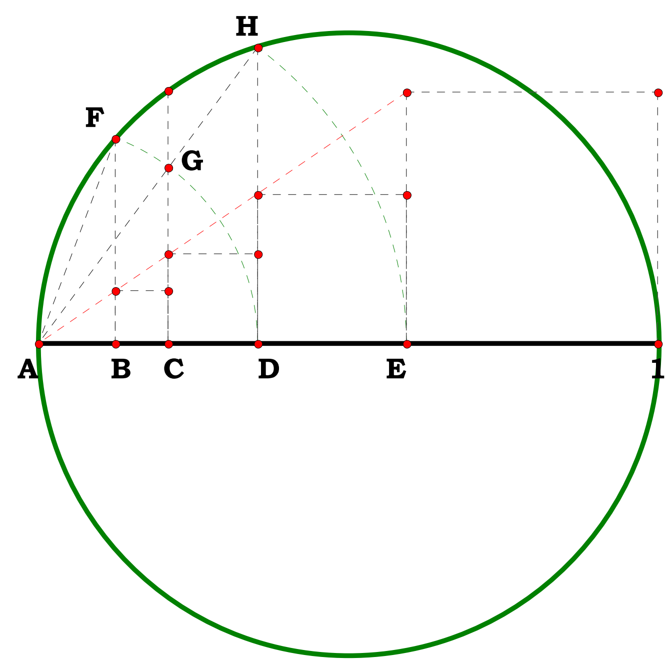
$BF := \sqrt{AB \cdot (A1 - AB)}$ $\quad AF := \sqrt{AB^2 + BF^2}$ $\quad AD := AF$

$DH := \sqrt{AD \cdot (A1 - AD)}$ $\quad AH := \sqrt{AD^2 + DH^2}$ $\quad AE := AH$

$AG := AD$ $\quad AC := \frac{AD \cdot AD}{AH}$

Any 2N root series.

This is just 112293 with lipstick and a dress. I have always felt a bit of annoyance with those who write Algebra books claiming that exponential series is a pure conceptual abstraction which has no geometric figure to demonstrate it. Apparently those writers do not even know simple geometry. I do not mind someone being ignorant, but when such words are in school books it is disinformation and misleading of students. I had never lernt geometry when I read that in a school book, however I was still amazed at the author putting words into a text which could not have possibly been true. Euclid gave his readers individual components to work with, the readers inability to combine those components together to figure our their interaction is not the fault of Euclid, it is the stupidity and lazyness of the reader.



$\frac{Unit}{XY} = 8.00000$

$x_B = 0.12500$

$x_C = 0.21022$

$x_D = 0.35355$

$x_E = 0.59460$

$\frac{Unit}{x_B} = 8.00000$

$\frac{Unit}{x_C} = 4.75683$

$\frac{Unit}{x_D} = 2.82843$

$\frac{Unit}{x_E} = 1.68179$

$B = 8.00000$ $\quad B^{\frac{4}{4}} = 8.00000$

$C = 4.75683$ $\quad B^{\frac{3}{4}} = 4.75683$

$D = 2.82843$ $\quad B^{\frac{2}{4}} = 2.82843$

$E = 1.68179$ $\quad B^{\frac{1}{4}} = 1.68179$

Definitions.

$\frac{A1}{AB} = 8$ $\quad \frac{A1}{AC} = 4.756828$ $\quad \frac{A1}{AD} = 2.828427$ $\quad \frac{A1}{AE} = 1.681793$ $\quad C := \frac{A1}{AC}$ $\quad D := \frac{A1}{AD}$ $\quad E := \frac{A1}{AE}$ $\quad N^{\frac{3}{4}} - C = 0$ $\quad N^{\frac{2}{4}} - D = 0$ $\quad N^{\frac{1}{4}} - E = 0$ Etc.

Now we can think of A1 as being a class or a noun, and B, C, D, E, members of that class, or its defining characteristics.

112800A

Unit.

$AE := 1 \quad JK := AE$

Given.

$N := 3 \quad EJ := N$

Means On Means

Modify 02/28/98 for Mean proportionals between E and J.

As I have always found this little exercise quite useless, I have decided on a B writeup aimed at helping to explain how to use things like this in template making for geometric progression. My templates tend all to be arithmetic in expression, however, using them to construct one which effects geometric progression is quite easy. So, try to reflect on what the plate is demonstrating.

Descriptions.

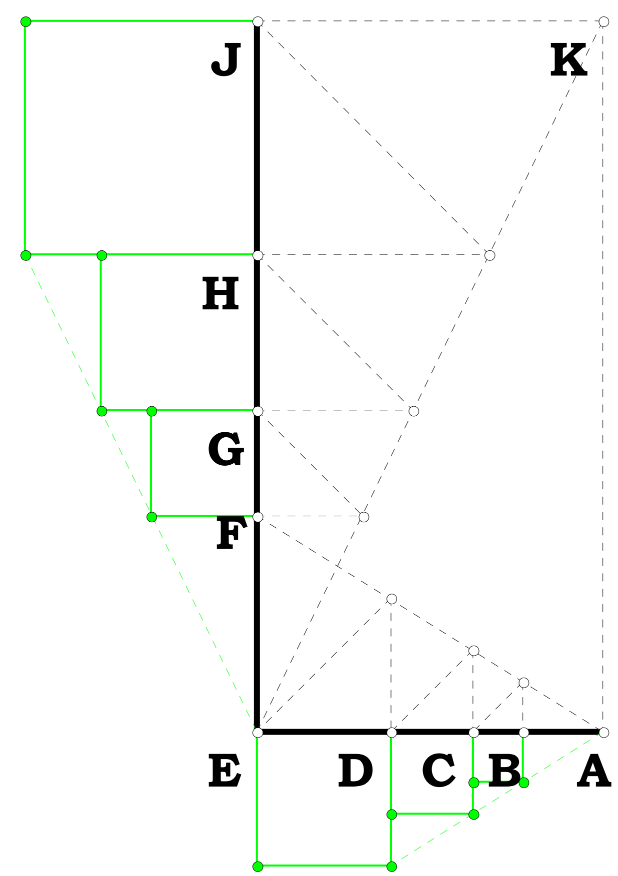
$$HJ := \frac{JK \cdot EJ}{JK + EJ} \quad EH := EJ - HJ \quad GH := \frac{EH \cdot HJ}{EH + HJ}$$

$$EG := EH - GH \quad FG := \frac{EG \cdot GH}{EG + GH} \quad EF := EG - FG$$

$$DE := \frac{EF \cdot AE}{EF + AE} \quad AD := AE - DE \quad CD := \frac{AD \cdot DE}{AD + DE}$$

$$AC := AD - CD \quad BC := \frac{AC \cdot CD}{AC + CD} \quad AB := AC - BC$$

$$M := 0 .. 3 \quad P := 0 .. 3 \qquad AEAB_{M,P} := \left[\frac{N^{M+1}}{(N+1)^M} + 1\right]^P$$



Definitions.

$$AEAB = \begin{pmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.5625 & 34.328125 \\ 1 & 2.6875 & 7.222656 & 19.410889 \\ 1 & 2.265625 & 5.133057 & 11.629581 \end{pmatrix}$$

$$AEAB_{3,3} - \frac{AE}{AB} = 0 \quad AEAB_{3,2} - \frac{AE}{AC} = 0 \quad AEAB_{3,1} - \frac{AE}{AD} = 0 \quad AEAB_{3,0} - \frac{AE}{AE} = 0$$

112900A

Unit.

$AC := 1$

Given.

$N_1 := 3 \qquad AH := N_1$

$N_2 := 12 \qquad CJ := N_2$

Descriptions for Division.

$$AB := \frac{AH}{(CJ + AH)} \cdot AC \qquad BC := AC - AB$$

$$BD := BC \qquad CG := \frac{BD \cdot AC}{AB} \qquad CG = 4$$

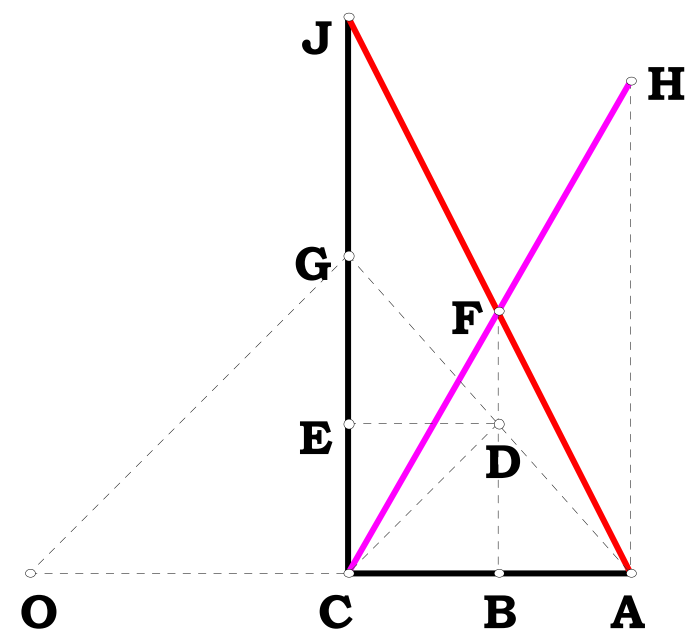
Definitions.

$$AB - \frac{N_1}{\left(N_2 + N_1\right)} = 0 \qquad BC - \frac{N_2}{N_1 + N_2} = 0$$

$$BD - \frac{N_2}{N_1 + N_2} = 0 \qquad CG - \frac{N_2}{N_1} = 0$$

Multiplication and Division-Line By A Line

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



Unit.

$AC := 1$

Given.

$N_1 := 5 \quad AH := N_1$

$N_2 := 7 \quad CG := N_2$

112900B

Descriptions for Multiplication.

$$CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$$

$$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$$

$$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$$

Definitions.

$$CO - N_2 = 0 \qquad BD - \frac{N_2}{N_2 + 1} = 0$$

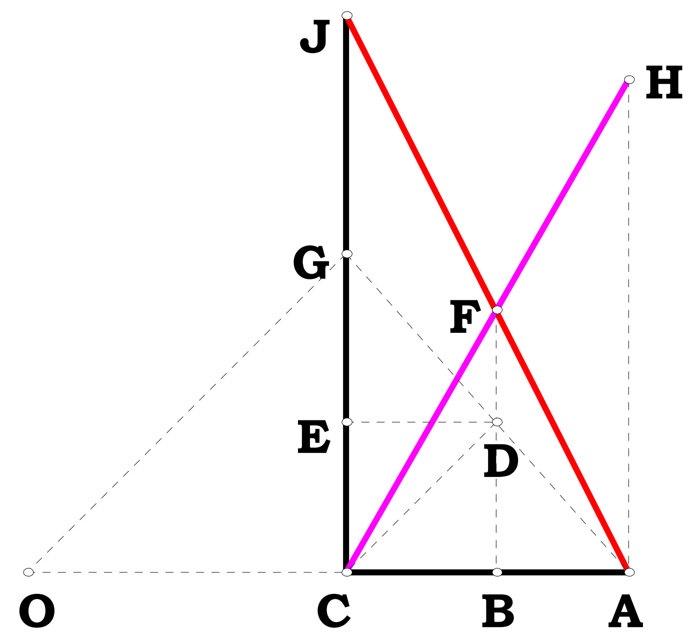
$$BC - \frac{N_2}{N_2 + 1} = 0 \qquad AB - \frac{1}{N_2 + 1} = 0$$

$$BF - \frac{N_1 \cdot N_2}{N_2 + 1} = 0 \qquad CJ - N_1 \cdot N_2 = 0$$

$$CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$$

Multiplication and Division-Line By A Line

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



120500

Descriptions.

Unit.

$N_3 := 1 \qquad CE := N_3$

Given.

$N_1 := 5 \qquad BC := N_1$

$N_2 := 25 \qquad AC := N_2$

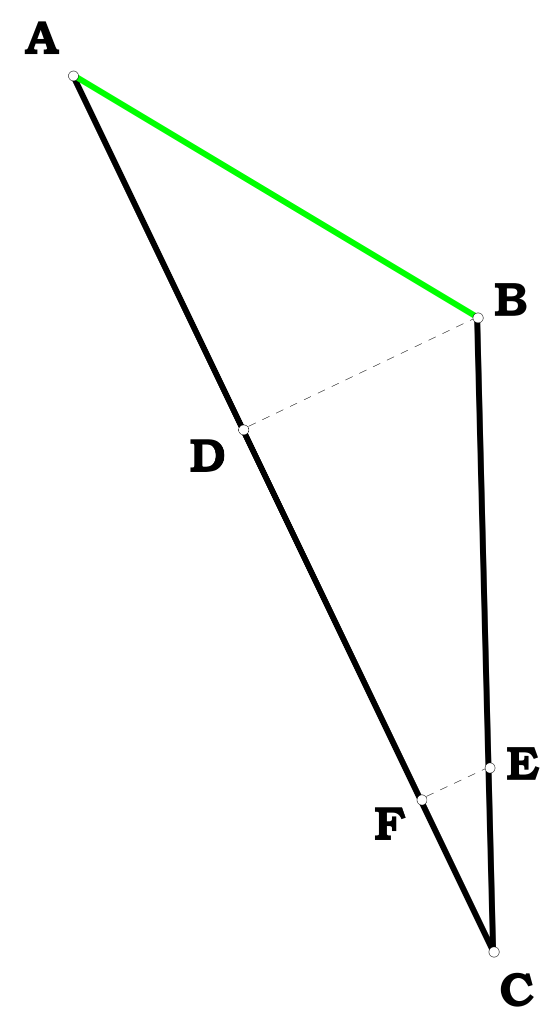
$N_4 := .5 \qquad EF := N_4$

From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.

$$BD := \frac{EF \cdot BC}{CE} \qquad CF := \sqrt{CE^2 - EF^2}$$

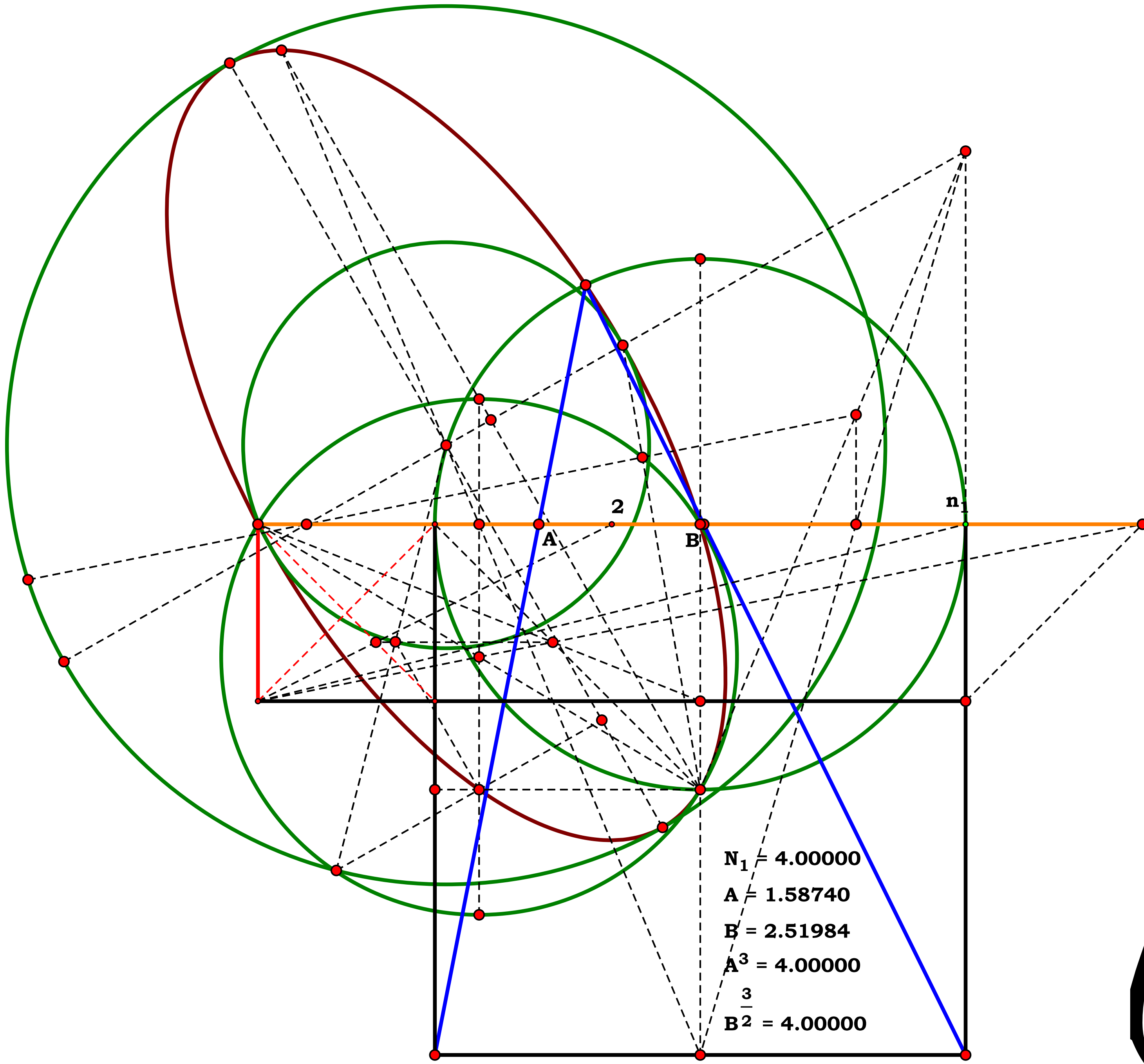
$$CD := \frac{CF \cdot BC}{CE} \qquad AD := AC - CD$$

$$AB := \sqrt{BD^2 + AD^2} \qquad AB = 20.82051$$



Definitions.

$$AB - \frac{\sqrt{N_1^{\,2} \cdot N_3 + N_2^{\,2} \cdot N_3 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_3^{\,2} - N_4^{\,2}}}}{\sqrt{N_3}} = 0$$



The Delian Quest 2001

John Clark

010101

Unit.

Given.

$N_1 := 5$ $AC := N_1$

$N_2 := 3$ $BC := N_2$

Square Root, common segment common endpoint.

Alternate method for common segment common endpoint square root. $\sqrt{AC \cdot BC} = CD$

Descriptions.

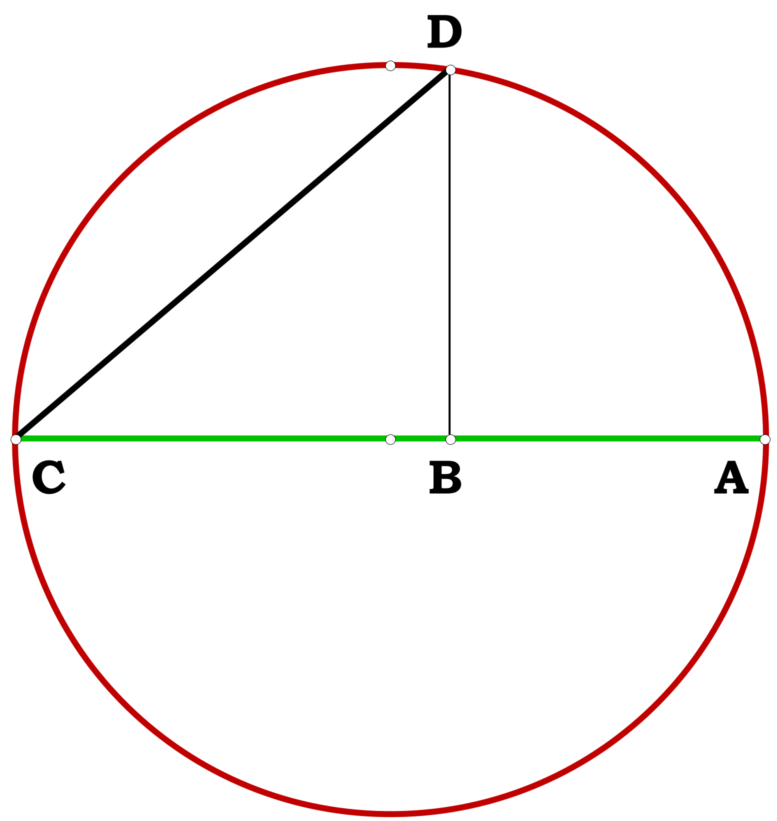
$AB := AC - BC$

$BD := \sqrt{AB \cdot BC}$

$CD := \sqrt{BD^2 + BC^2}$

Definitions.

$\sqrt{(N_1 \cdot N_2)} - CD = 0$



042101
Descriptions.

Unit.
Given.

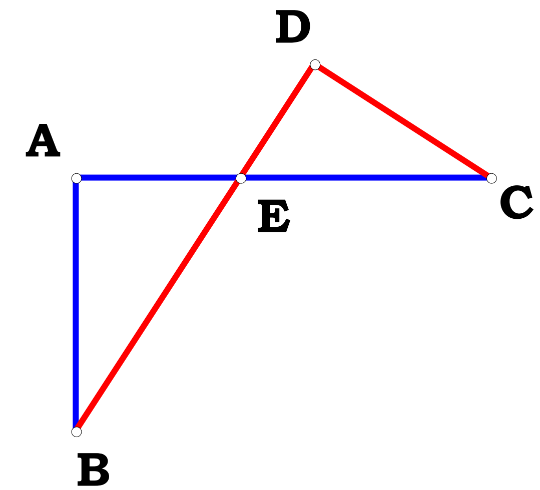
$N_1 := 2 \quad AB := N_1$

$N_2 := 3 \quad CD := N_2$

$N_3 := 4 \quad AC := N_3$

Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?

Three Given Five Taken



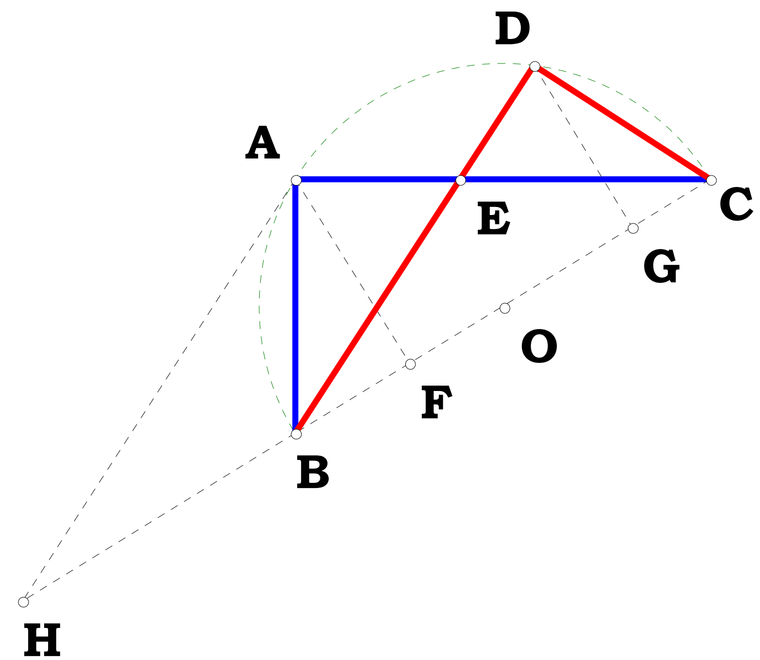
$$BC := \sqrt{AB^2 + AC^2} \qquad CG := \frac{CD^2}{BC} \qquad BF := \frac{AB^2}{BC}$$

$$BG := BC - CG \qquad CF := BC - BF \qquad AF := \sqrt{AB^2 - BF^2}$$

$$DG := \sqrt{CD^2 - CG^2} \qquad FH := \frac{BG \cdot AF}{DG} \qquad CH := CF + FH$$

$$BD := \sqrt{BG^2 + DG^2} \qquad AH := \frac{BD \cdot FH}{BG} \qquad BE := \frac{AH \cdot BC}{CH}$$

$$DE := BD - BE \qquad CE := \frac{AC \cdot BC}{CH} \qquad AE := AC - CE$$



Definitions.

$$\sqrt{N_1^2 + N_3^2} - BC = 0 \qquad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$$

$$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0 \qquad \frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0 \qquad \frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0 \qquad \frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$$

$$\frac{N_2 \cdot \sqrt{N_1^2 - N_2^2 + N_3^2}}{\sqrt{N_1^2 + N_3^2}} - DG = 0 \qquad \frac{N_2 \cdot \sqrt{N_1^2 - N_2^2 + N_3^2}}{\sqrt{N_1^2 + N_3^2}} \qquad \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0 \qquad N_3 \cdot \frac{N_1}{N_2} - AH = 0$$

$$\frac{N_3 \cdot \left[N_1 \cdot \left(N_1^2 - N_2^2 + N_3^2\right) \cdot \sqrt{N_1^2 + N_3^2} + N_2 \cdot N_3 \cdot \sqrt{\left[\left(N_1^2 + N_3^2\right) \cdot \left(N_1^2 - N_2^2 + N_3^2\right)\right]}\right]}{N_2 \cdot \sqrt{N_1^2 + N_3^2} \cdot \sqrt{\left(N_1^2 + N_3^2\right) \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}} - CH = 0$$

The Five Sought:

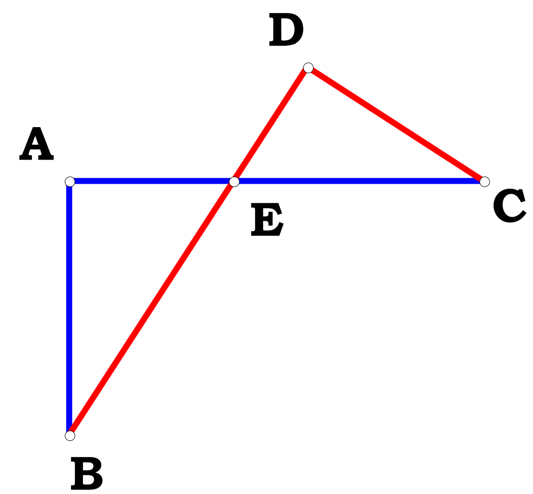
$$\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} - BD = 0$$

$$N_2 \cdot \frac{\left(\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - DE = 0$$

$$N_1 \cdot \frac{\left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - BE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - AE = 0$$

$$\frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1} - CE = 0$$



Unit.

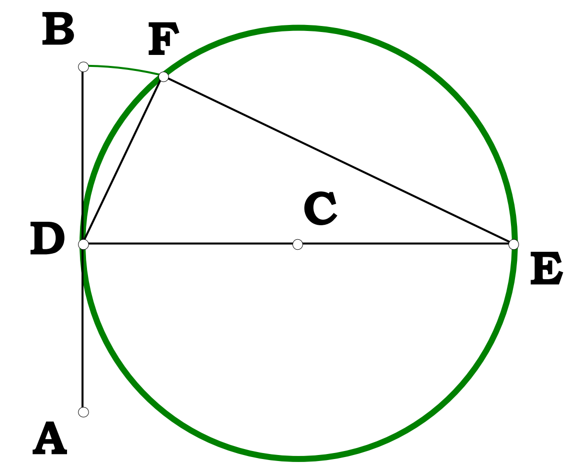
$AB := 1$

Given.

$N_1 := 2.052$

$N_2 := .62$

Given AB as unit, AD and DC, what is EF and DF?



042201

Descriptions.

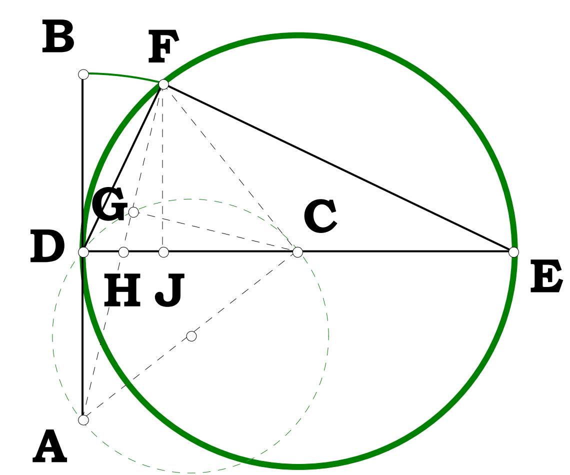
$$AD := \frac{AB}{N_1} \qquad CD := AB \cdot N_2 \qquad DE := 2CD \qquad AF := AB \qquad CF := CD$$

$$AC := \sqrt{AD^2 + CD^2} \qquad AG := \frac{AC^2 + AF^2 - CF^2}{2 \cdot AF} \qquad CG := \sqrt{AC^2 - AG^2}$$

$$DH := CD - \frac{CG \cdot \left(AD^2 + CD^2\right)}{CD \cdot CG + \sqrt{AD^2 + CD^2 - CG^2} \cdot AD} \qquad AH := AD \cdot \frac{\left(AD^2 + CD^2\right)}{\left(CD \cdot CG + \sqrt{AD^2 + CD^2 - CG^2} \cdot AD\right)}$$

$$FH := AF - AH \qquad HJ := \frac{DH \cdot FH}{AH} \qquad DJ := DH + HJ \qquad EJ := DE - DJ \qquad FJ := \frac{AD \cdot FH}{AH}$$

$$EF := \sqrt{FJ^2 + EJ^2} \qquad DF := \sqrt{DJ^2 + FJ^2}$$



Definitions.

$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0 \qquad (2AB) \cdot N_2 - DE = 0 \qquad AB \cdot \frac{\sqrt{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}}{N_1} - AC = 0$$

$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^{\,2}\right)}{N_1^{\,2}} - AG = 0 \qquad AB \cdot \frac{\sqrt{\left(2 \cdot N_1^{\,2} \cdot N_2 + N_1^{\,2} - 1\right) \cdot \left(2 \cdot N_1^{\,2} \cdot N_2 - N_1^{\,2} + 1\right)}}{2 \cdot N_1^{\,2}} - CG = 0$$

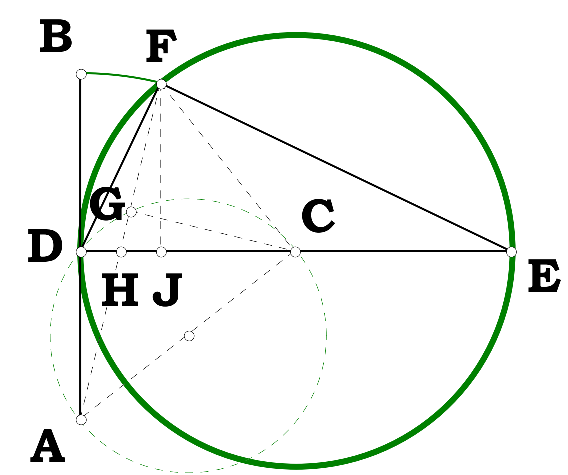
$$AB \cdot \frac{N_2 \cdot N_1^{\,3} + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}}}{N_1^{\,3} + N_1^{\,2} \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + N_1} - DH = 0 \qquad 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - FH = 0$$

$$\frac{-1}{2}\cdot AB\cdot\left(\frac{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}\cdot N_2^{\,2}+1}{N_1^{\,2}\cdot N_2+N_1^{\,4}\cdot N_2+N_1^{\,3}\cdot N_2^{\,2}\cdot\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}-\frac{N_1^{\,4}\cdot N_2^{\,2}+3\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+1}{N_1^{\,4}\cdot N_2^{\,3}+N_1^{\,2}\cdot N_2}\right)-HJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{4\cdot N_1^{\,3}\cdot N_2^{\,3}-N_1^{\,3}\cdot N_2+3\cdot N_1\cdot N_2+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-EJ=0$$



$$\frac{-1}{2}\cdot AB\cdot\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}+1-N_1^{\,2}+2\cdot N_2^{\,2}\cdot N_1^{\,2}\right)}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}-FJ=0$$

$$AB\cdot\frac{\sqrt{N_2\cdot\left[N_1\cdot N_2\cdot\left(4\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+3\right)+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right]}}{\sqrt{N_1^{\,3}\cdot N_2^{\,2}+N_1}}-EF=0$$

$$AB\cdot\frac{\sqrt{\left[N_2\cdot\left(N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right)\right]}}{\sqrt{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}}-DF=0$$

042301A

Unit.

$AB := 1$

Given.

$N := .36307 \qquad BC := N$

Counterpoint

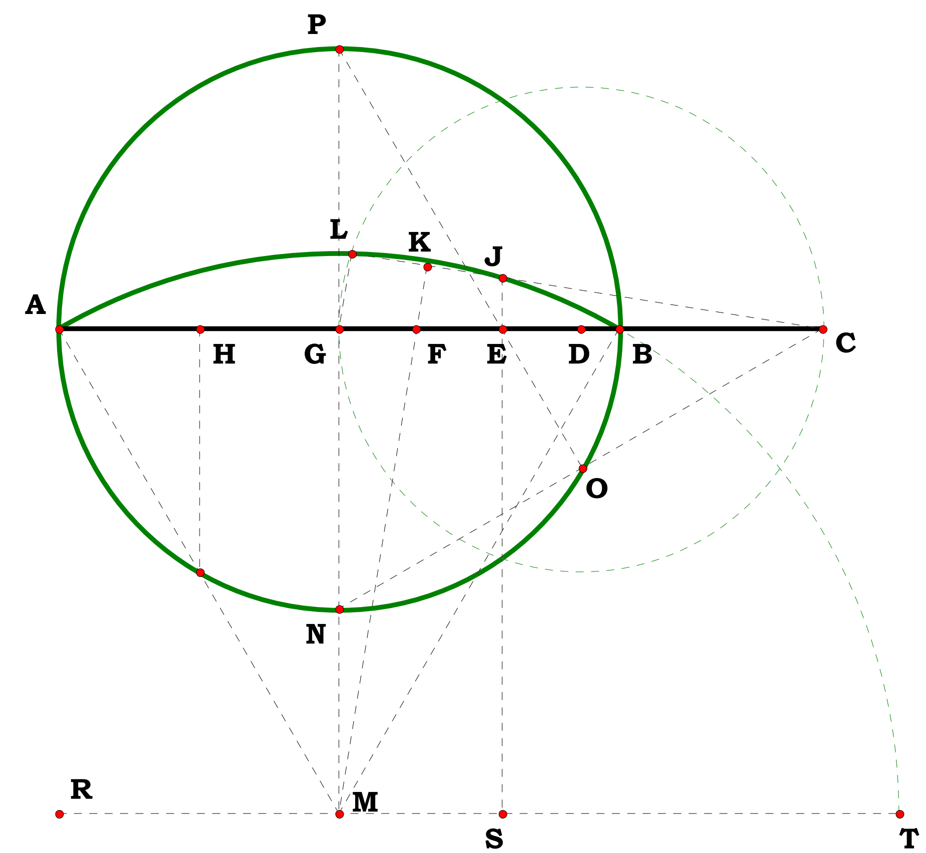
When I origionally did part of this, it was a mess. I have taken it upon myself to keep the figure and title.

$$AC := AB + BC \qquad AG := \frac{AB}{2} \qquad GM := \sqrt{3 \cdot AG^2} \qquad CG := AC - AG$$

$$CN := \sqrt{CG^2 + AG^2} \qquad NO := \frac{AG \cdot AB}{CN} \qquad CO := CN - NO$$

$$CE := \frac{CN \cdot CO}{CG} \qquad GE := CG - CE \qquad JS := \sqrt{(AB + GE) \cdot (AB - GE)}$$

$$EJ := JS - GM \qquad CJ := \sqrt{CE^2 + EJ^2} \qquad CL := \frac{CG \cdot CE}{CJ}$$



Definitions.

$$AC - (1 + N) = 0 \qquad AG - \frac{1}{2} = 0 \qquad GM - \frac{\sqrt{3}}{2} = 0$$

$$CG - \frac{2 \cdot N + 1}{2} \qquad CN - \frac{\sqrt{2 \cdot N^2 + 2 \cdot N + 1}}{\sqrt{2}} = 0$$

$$NO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N^2 + 2 \cdot N + 1}} = 0 \qquad CO - \frac{\sqrt{2} \cdot N \cdot (N + 1)}{\sqrt{2 \cdot N^2 + 2 \cdot N + 1}} = 0$$

$$CE - \frac{2 \cdot N \cdot (N + 1)}{2 \cdot N + 1} = 0 \qquad GE - \frac{1}{2 \cdot (2 \cdot N + 1)} = 0$$

$$JS - \frac{\sqrt{(4 \cdot N + 1) \cdot (4 \cdot N + 3)}}{2 \cdot (2 \cdot N + 1)} = 0 \qquad EJ - \frac{\sqrt{16 \cdot N^2 + 16 \cdot N + 3} - \sqrt{3} \cdot (2 \cdot N + 1)}{4 \cdot N + 2} = 0$$

$$CJ - \frac{\sqrt{2 \cdot N \cdot \left(2 \cdot N^2 + 3 \cdot N + 4\right) + 3 - \sqrt{[3 \cdot (4 \cdot N + 3) \cdot (4 \cdot N + 1)]}}}{\sqrt{4 \cdot N + 2}} = 0$$

$$CL - \frac{2 \cdot \sqrt{2} \cdot N \cdot (N + 1) \cdot \left(N + \frac{1}{2}\right)}{\sqrt{2 \cdot N + 1} \cdot \sqrt{2 \cdot N \cdot \left(2 \cdot N^2 + 3 \cdot N + 4\right) + 3 - \sqrt{[3 \cdot (4 \cdot N + 3) \cdot (4 \cdot N + 1)]}}} = 0$$

Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

042401A

Descriptions.

$BG := AG - AB \quad BF := \frac{BG}{2} \quad AF := AB + BF \quad AK := AF$

$FK := BF \quad AE := \frac{2AK^2 - FK^2}{2AF} \quad AJ := AE \quad JK := AK - AJ$

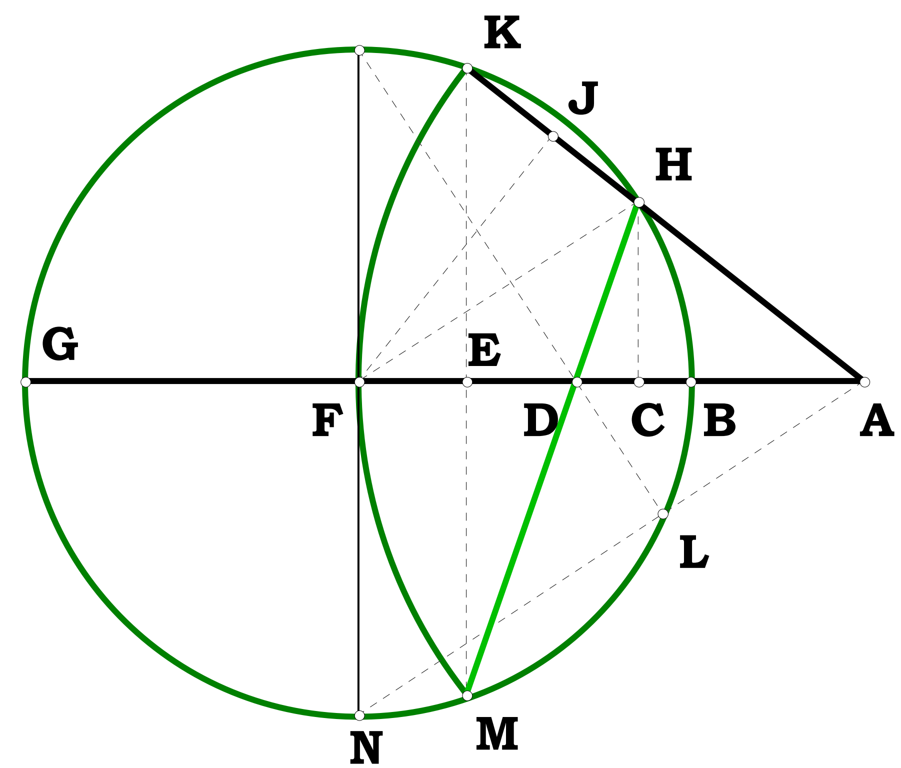
$HJ := JK \quad AH := AK - (JK + HJ) \quad AC := \frac{AE \cdot AH}{AK} \quad CE := AE - AC$

$BE := AE - AB \quad EG := BG - BE \quad EK := \sqrt{BE \cdot EG} \quad BC := AC - AB$

$CG := BG - BC \quad CH := \sqrt{BC \cdot CG} \quad DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE \quad HM := \sqrt{CE^2 + (EK + CH)^2}$

Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?



Definitions.

$$N - 1 - BG = 0 \qquad \frac{1}{2}\cdot N - \frac{1}{2} - BF = 0$$

$$\frac{1}{2} + \frac{1}{2}\cdot N - AF = 0 \qquad \frac{1}{4}\cdot\frac{\left(N^2 + 6\cdot N + 1\right)}{(1+N)} - AE = 0$$

$$\frac{1}{4}\cdot\frac{\left(1 - 2\cdot N + N^2\right)}{(1+N)} - JK = 0 \qquad 2\cdot\frac{N}{(1+N)} - AH = 0$$

$$\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^3}\cdot N - AC = 0 \qquad \frac{1}{4}\cdot\left(N^2 + 6\cdot N + 1\right)\cdot\frac{(N-1)^2}{(1+N)^3} - CE = 0$$

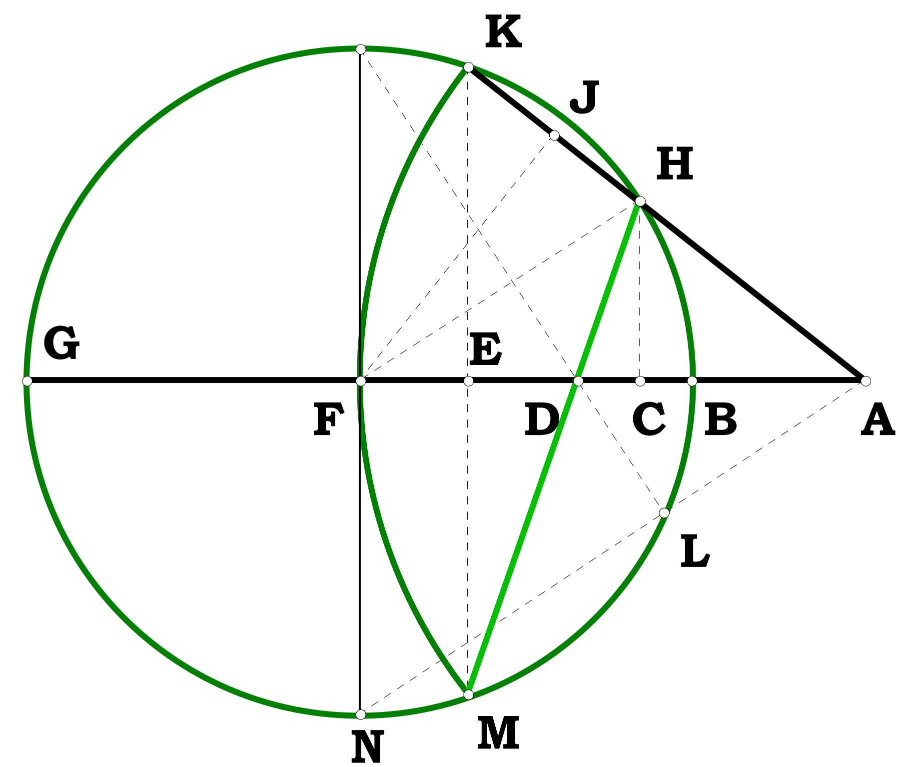
$$\frac{1}{4}\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} - BE = 0 \qquad \frac{1}{4}\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)} - EG = 0$$

$$(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3} - BC = 0 \qquad \frac{1}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} - EK = 0$$

$$N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3} - CG = 0 \qquad \sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3} - CH = 0$$

$$\frac{1}{4}\cdot\frac{(N-1)^2}{(1+N)} - DE = 0 \qquad \frac{1}{2}\cdot\frac{(N-1)^2}{(N+1)} - DF = 0$$

$$\frac{1}{2}\cdot(N-1)\cdot\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^2} - HM = 0$$



Unit.

$AB := 1$

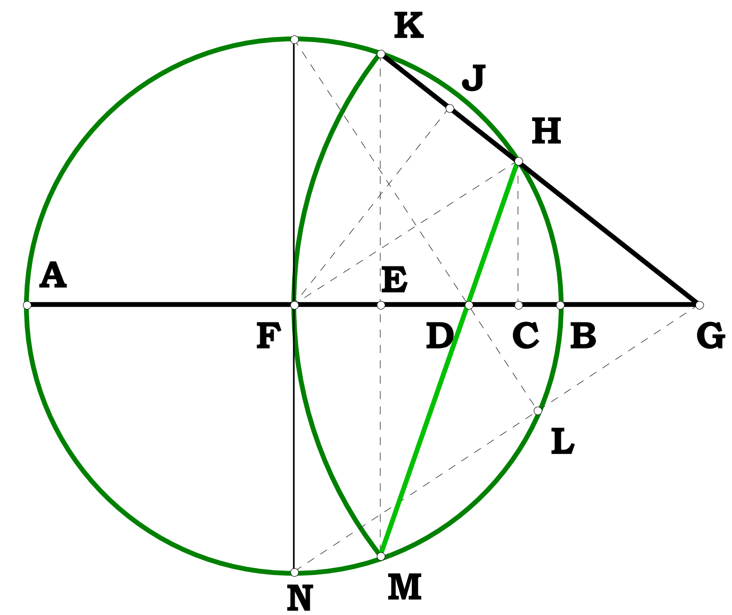
Given.

$N := 5 \qquad AG := N$

042401B

Descriptions.

Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?



$BG := AG - AB \qquad BF := \dfrac{AB}{2} \qquad GF := BG + BF \qquad GK := GF$

$FK := BF \qquad EG := \dfrac{2GK^2 - FK^2}{2GF} \qquad GJ := EG \qquad JK := GK - GJ \qquad HJ := JK$

$GH := GK - (JK + HJ) \qquad CG := \dfrac{EG \cdot GH}{GK} \qquad CE := EG - CG$

$BE := EG - BG \qquad AE := AB - BE \qquad EK := \sqrt{BE \cdot AE} \qquad AC := AG - CG$

$BC := AB - AC \qquad CH := \sqrt{BC \cdot AC} \qquad DE := \dfrac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE \qquad HM := \sqrt{CE^2 + (EK + CH)^2}$

Definitions.

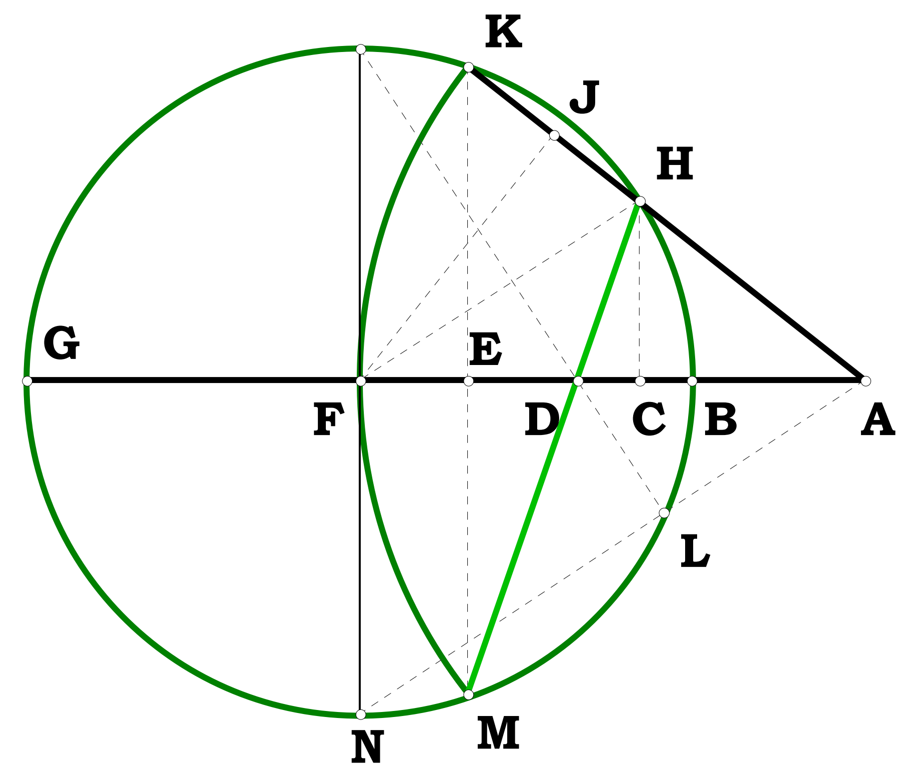
$BG - (N-1) = 0 \quad BF - \frac{1}{2} = 0 \quad GF - \frac{2\cdot N-1}{2} = 0 \quad GK - \frac{2\cdot N-1}{2} = 0$

$FK - \frac{1}{2} = 0 \quad EG - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0 \quad GJ - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0$

$JK - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad HJ - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad GH - \frac{2\cdot N\cdot(N-1)}{2\cdot N-1} = 0$

$CG - \frac{N\cdot(N-1)\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)}{(2\cdot N-1)^3} = 0 \quad CE - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)^3} = 0$

$BE - \frac{4\cdot N-3}{4\cdot(2\cdot N-1)} = 0 \quad AE - \frac{4\cdot N-1}{4\cdot(2\cdot N-1)} = 0 \quad EK - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)} = 0$



$AC - \frac{N^2\cdot(4\cdot N-3)}{(2\cdot N-1)^3} = 0 \quad BC - \frac{(4\cdot N-1)\cdot(N-1)^2}{(2\cdot N-1)^3} = 0$

$CH - \frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3} = 0$

Mathcad claims that this is solvable, however, it was spread over several pages and I do not know what to make of that; however, if it is right, then there you go for trisection.

$DE - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad DF - \frac{2}{4\cdot(2\cdot N-1)} = 0 \quad HM - \frac{\left(8\cdot N^2 - 8\cdot N + 1\right)}{2\cdot(2\cdot N-1)^2} = 0$

Unit.

$AB := 1$

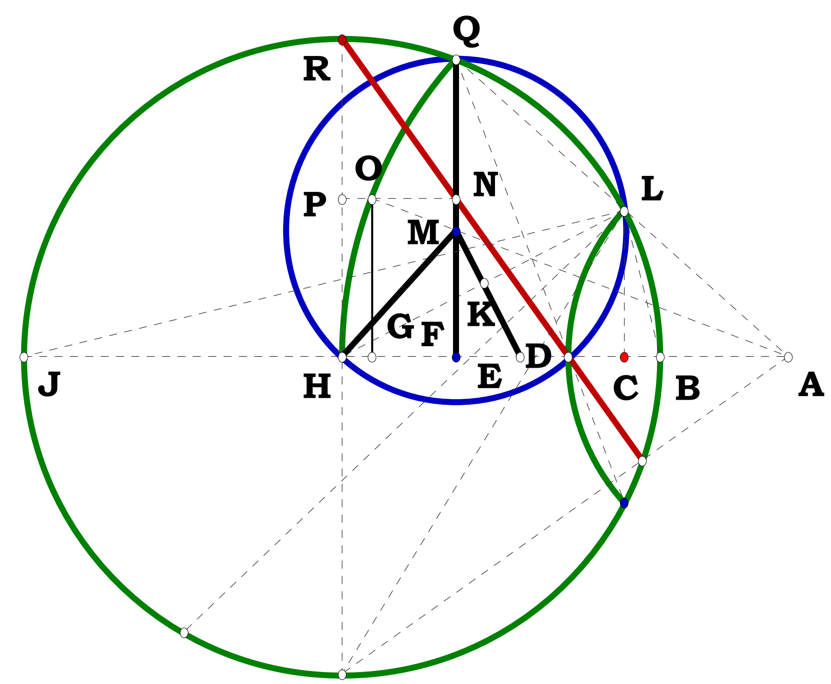
Given.

Descriptions.

$N := 5.768 \qquad AJ := N$

042501A

What is the Algebraic name of the circle HM? Does point N divide DR in half?



$BJ := AJ - AB \qquad BH := \dfrac{BJ}{2} \qquad HR := BH$

$HP := \dfrac{HR}{2} \qquad GO := HP \qquad AH := AB + BH$

$AO := AH \qquad AG := \sqrt{AO^2 - GO^2}$

$HQ := BH \qquad AQ := AH \qquad FH := \dfrac{HQ^2}{2 \cdot AH}$

$AF := AH - FH \qquad FM := \dfrac{GO \cdot AF}{AG}$

$HJ := BH \qquad FJ := FH + HJ \qquad BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ} \qquad MQ := FQ - FM \qquad HM := \sqrt{FH^2 + FM^2}$

$HM - MQ = 0 \qquad DH := \dfrac{HR^2}{AH} \qquad \dfrac{DH}{2} - FH = 0$

Definitions.

$BJ - (N - 1) = 0 \qquad BH - \dfrac{1}{2} \cdot (N - 1) = 0 \qquad HP - \dfrac{1}{4} \cdot (N - 1) = 0$

$AH - \dfrac{1}{2} \cdot (1 + N) = 0 \qquad AG - \dfrac{1}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} = 0 \qquad \dfrac{1}{4} \cdot \dfrac{(N - 1)^2}{(1 + N)} - FH = 0$

$AF - \dfrac{1}{4} \cdot \dfrac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} = 0 \qquad FM - \dfrac{1}{4} \cdot (N - 1) \cdot \dfrac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} = 0$

$$BF - \frac{1}{4}\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} = 0$$

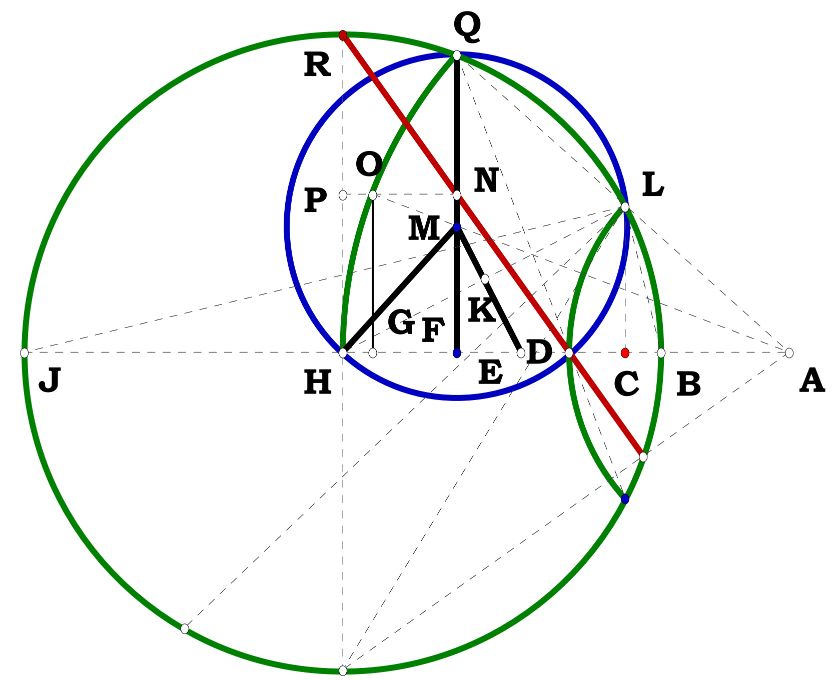
$$FJ - \frac{1}{4}\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)} = 0$$

$$FQ - \frac{1}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} = 0$$

$$MQ - \frac{1}{2}\cdot(1+N)\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$HM - \frac{1}{2}\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$DH - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \qquad FH - \frac{(N-1)^2}{4\cdot(N+1)} = 0$$



Unit.

$AB := 1$

Given.

Descriptions.

042501B

$N := 6 \qquad AJ := N$

What is the Algebraic name of the circle HM? Does point N divide DR in half?

$BJ := AJ - AB \qquad BH := \dfrac{AB}{2} \qquad HR := BH$

$HP := \dfrac{HR}{2} \qquad GO := HP \qquad JH := BJ + BH$

$JO := JH \qquad JG := \sqrt{JO^2 - GO^2}$

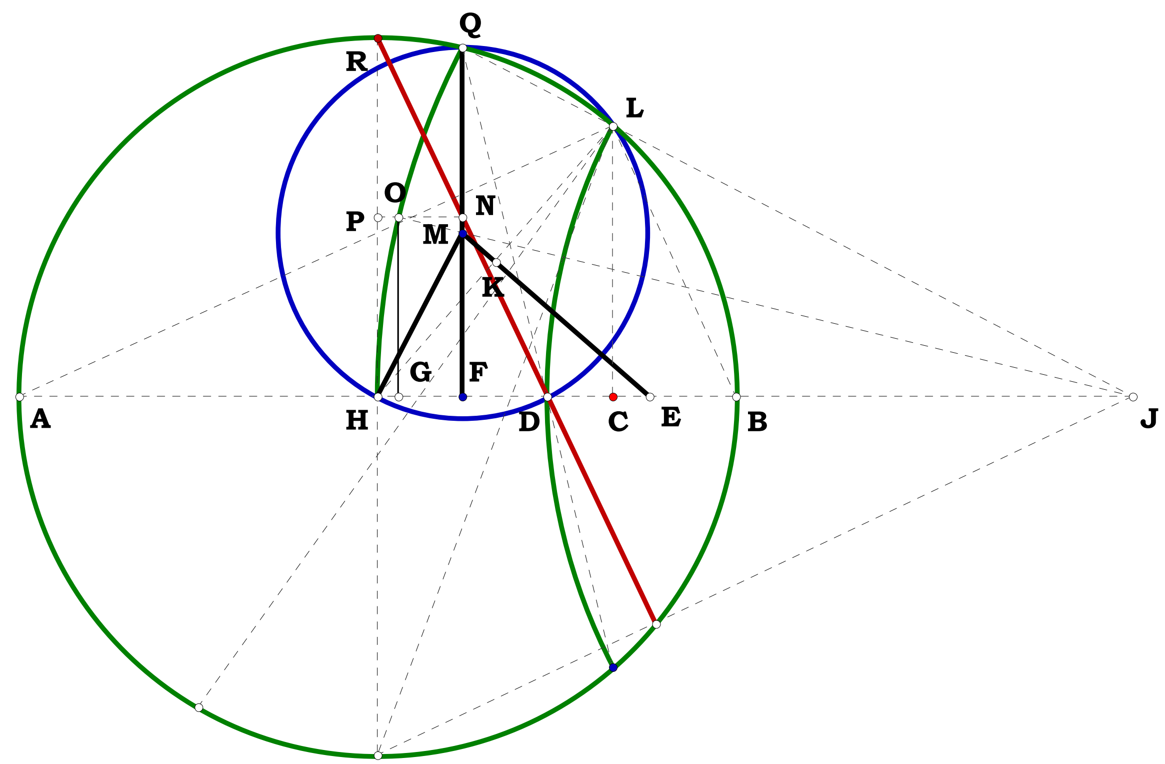
$HQ := BH \qquad JQ := JH \qquad FH := \dfrac{HQ^2}{2 \cdot JH}$

$AH := BH \qquad FJ := JH - FH \qquad FM := \dfrac{GO \cdot FJ}{JG}$

$AF := FH + AH \qquad BF := AB - AF \qquad FQ := \sqrt{BF \cdot AF}$

$MQ := FQ - FM \qquad HM := \sqrt{FH^2 + FM^2}$

$HM - MQ = 0 \qquad DH := \dfrac{HR^2}{JH} \qquad \dfrac{DH}{2} - FH = 0$



Definitions.

$BJ - (N - 1) = 0 \quad BH - \frac{1}{2} = 0 \quad HR - \frac{1}{2} = 0$

$HP - \frac{1}{4} = 0 \quad GO - \frac{1}{4} = 0 \quad JH - \frac{2 \cdot N - 1}{2} = 0$

$JO - \frac{2 \cdot N - 1}{2} = 0 \quad JG - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4} = 0$

$HQ - \frac{1}{2} = 0 \quad JQ - \frac{2 \cdot N - 1}{2} = 0 \quad FH - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0$

$AH - \frac{1}{2} = 0 \quad FJ - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0$

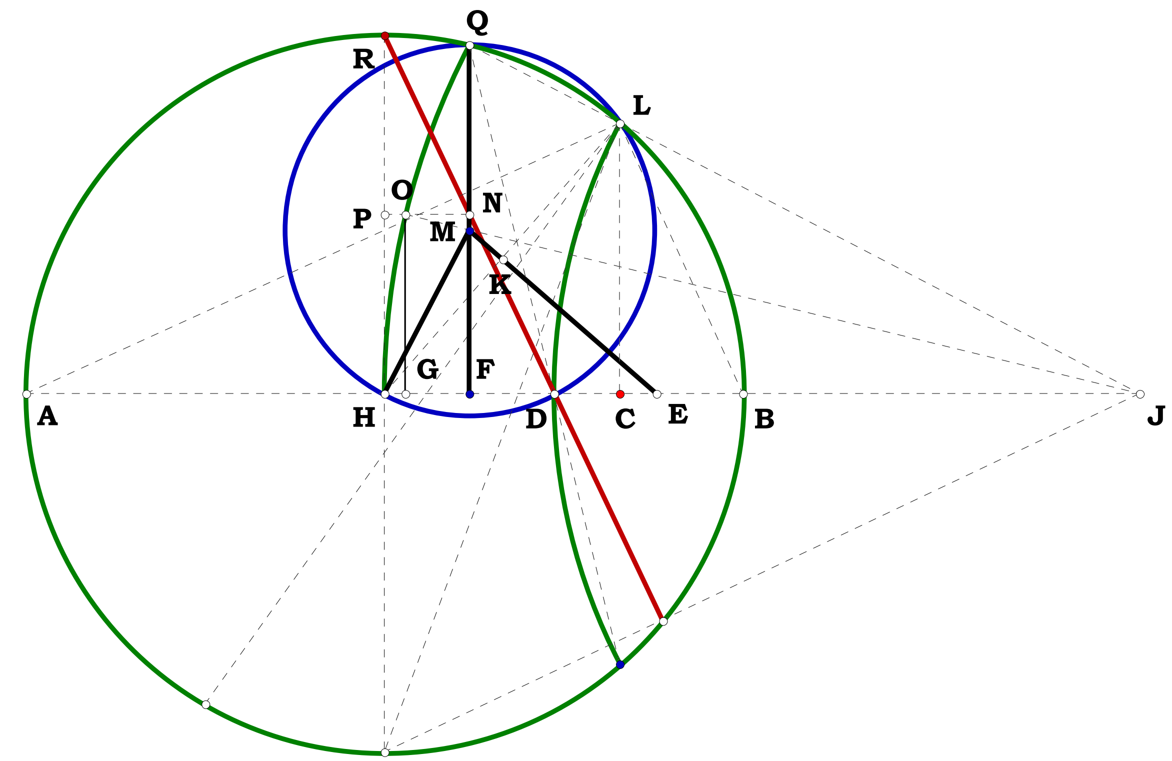
$FM - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} \cdot (2 \cdot N - 1)} = 0$

$AF - \frac{4 \cdot N - 1}{4 \cdot (2 \cdot N - 1)} = 0 \quad BF - \frac{4 \cdot N - 3}{4 \cdot (2 \cdot N - 1)} = 0$

$FQ - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4 \cdot (2 \cdot N - 1)} = 0 \quad MQ - \frac{2 \cdot N - 1}{2 \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}} = 0$

$HM - \frac{(2 \cdot N - 1)}{\sqrt{4 \cdot (4 \cdot N - 1) \cdot (4 \cdot N - 3)}} = 0 \quad DH - \frac{1}{2 \cdot (2 \cdot N - 1)} = 0$

$HM - MQ = 0 \quad \frac{DH}{2} - FH = 0$



Unit.

$AB := 1$

Given.

$N := 5 \quad AD := N$

Four Lines To A Point

Does the difference CJ and JT each have but one Algebraic name?

042901A

Descriptions.

$BD := AD - AB \quad BC := \frac{BD}{2} \quad CR := BC$

$CP := BC \quad CD := BC \quad AC := AB + BC \quad CG := \frac{CP^2}{AC}$

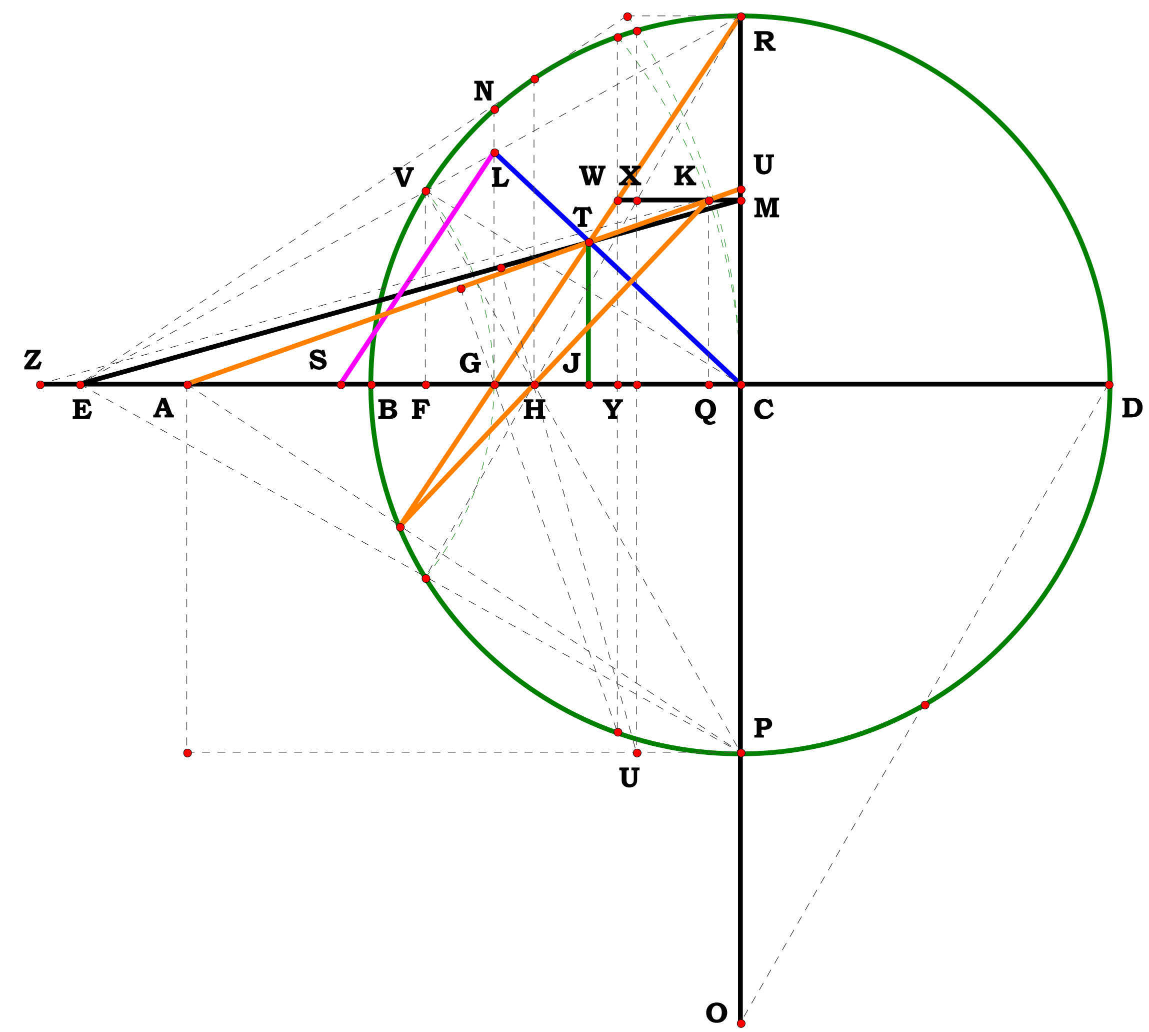
$AG := AC - CG \quad AV := AG \quad CV := BC \quad CF := \frac{CV^2 + AC^2 - AV^2}{2AC}$

$BF := BC - CF \quad DF := CF + CD \quad FV := \sqrt{BF \cdot DF} \quad CE := \frac{CF \cdot CR}{CR - FV}$

$BE := CE - BC \quad BG := BC - CG \quad EF := BE + BF \quad EG := BE + BG$

$GL := \frac{FV \cdot EG}{EF} \quad GS := \frac{CG \cdot GL}{CR} \quad CJ := \frac{CG^2}{GS + CG}$

$JT := \frac{GL \cdot CJ}{CG}$



$$\mathbf{CM} := \frac{\mathbf{CR}}{2} \qquad \mathbf{AK} := \mathbf{AC} \qquad \mathbf{KQ} := \mathbf{CM}$$

$$\mathbf{AQ} := \sqrt{\mathbf{AK}^2 - \mathbf{KQ}^2} \qquad \mathbf{CU} := \frac{\mathbf{KQ} \cdot \mathbf{AC}}{\mathbf{AQ}}$$

$$\mathbf{CZ} := \frac{\mathbf{CE} \cdot \mathbf{CU}}{\mathbf{CM}} \qquad \mathbf{AU} := \sqrt{\mathbf{AC}^2 + \mathbf{CU}^2}$$

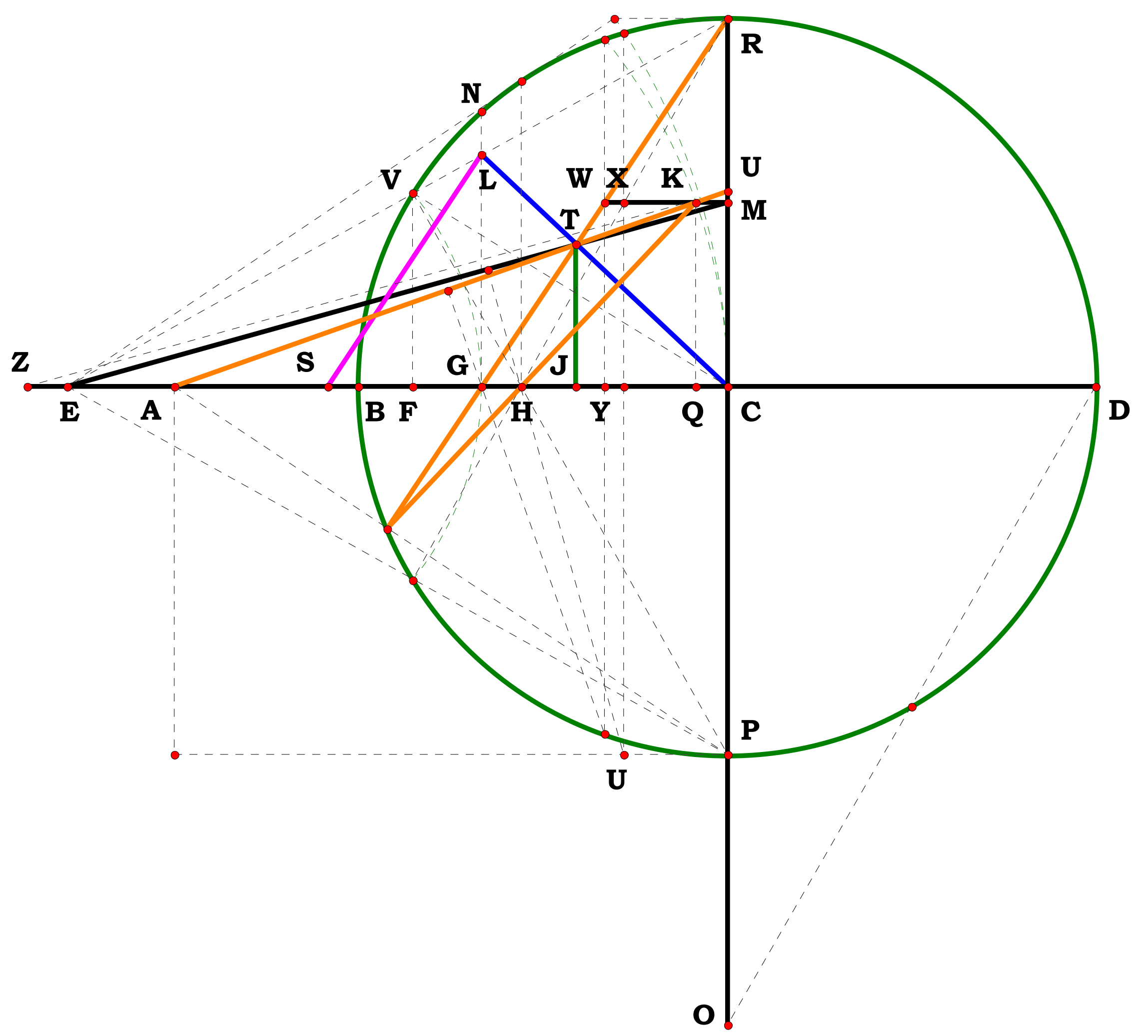
$$\mathbf{AZ} := \mathbf{CZ} - \mathbf{AC} \qquad \mathbf{AE} := \mathbf{CE} - \mathbf{AC}$$

$$\mathbf{AJ} := \frac{\mathbf{AC} \cdot \mathbf{AE}}{\mathbf{AZ}} \qquad \mathbf{JC} := \mathbf{AC} - \mathbf{AJ}$$

$$\mathbf{TJ} := \frac{\mathbf{CU} \cdot \mathbf{AJ}}{\mathbf{AC}}$$

$$\mathbf{JC} - \mathbf{CJ} = 0$$

$$\mathbf{JT} - \mathbf{TJ} = 0$$



$N - 5 = 0 \qquad AD - N = 0$

Definitions.

$BD - (N-1) = 0 \quad BC - \frac{N-1}{2} = 0 \quad CR - \frac{N-1}{2} = 0 \quad CP - \frac{N-1}{2} = 0$

$CD - \frac{N-1}{2} = 0 \quad AC - \frac{N+1}{2} = 0 \quad CG - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \quad AG - \frac{2\cdot N}{N+1} = 0$

$AV - \frac{2\cdot N}{N+1} = 0 \quad CV - \frac{N-1}{2} = 0 \quad CF - \frac{\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{2\cdot(N+1)^3} = 0$

$BF - \frac{(3\cdot N+1)\cdot(N-1)}{(N+1)^3} = 0 \qquad DF - \frac{N^2\cdot(N+3)\cdot(N-1)}{(N+1)^3} = 0$

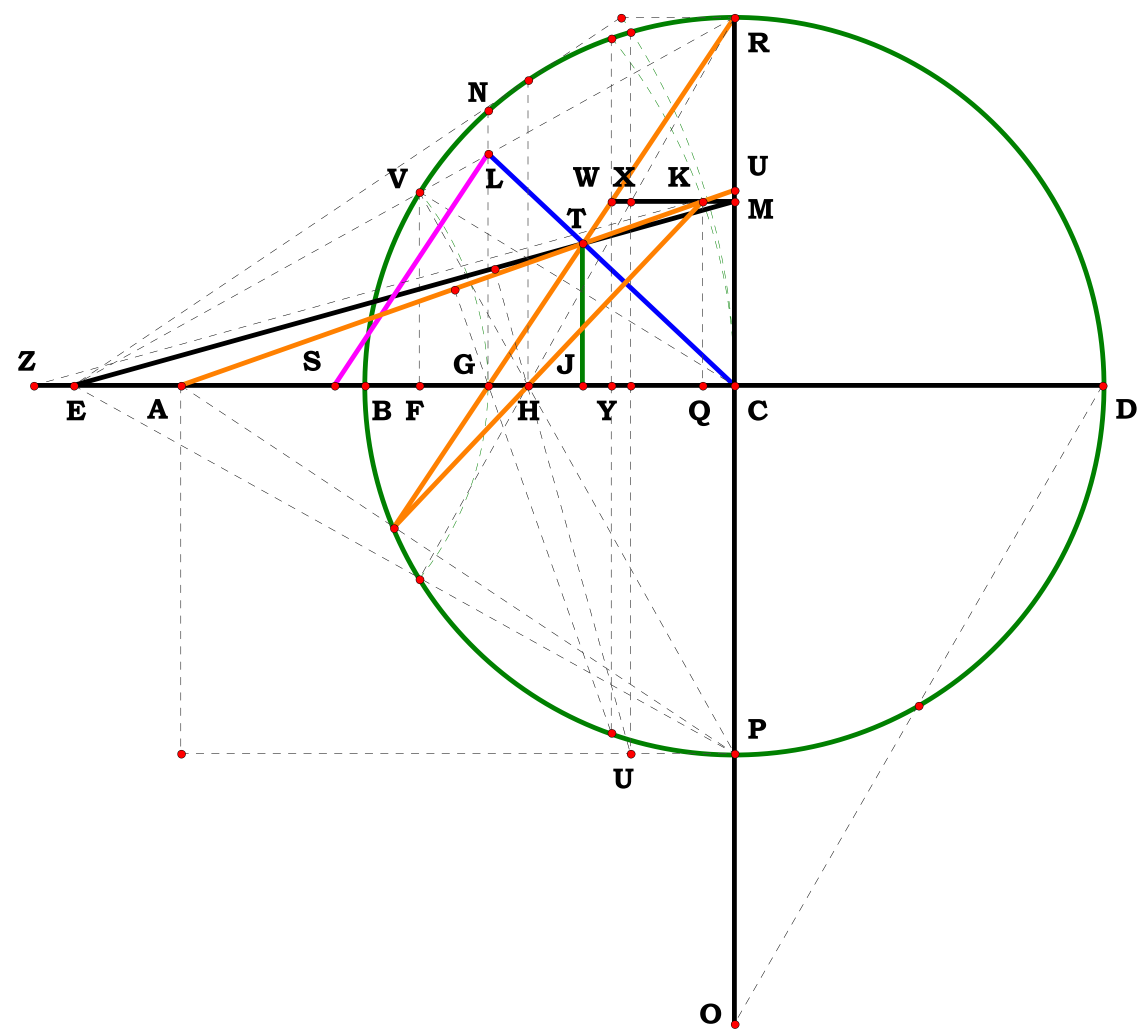
$$FV - \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} = 0$$

$$CE - \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]} = 0$$

$$BE - \frac{(N-1)\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-3\cdot N-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1} = 0 \qquad BG - \frac{N-1}{N+1} = 0$$

$$EF - \frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$

$$EG - \frac{N\cdot(N-1)^2\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$



$$GL - \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}} = 0$$

$$GS - \frac{N\cdot(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)^2\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}} = 0$$

$$CJ - \frac{N^4+2\cdot N^3-6\cdot N^2+2\cdot N+1}{14\cdot N\cdot(N+1)+2\cdot N^3+4\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+2} = 0$$

$$JT - \frac{N\cdot(N-1)\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{7\cdot N\cdot(N+1)+N^3+2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$

$$CM - \frac{N-1}{4} = 0 \qquad AK - \frac{N+1}{2} = 0$$

$$KQ - \frac{N-1}{4} = 0 \qquad AQ - \frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0$$

$$CU - \frac{(N+1)\cdot(N-1)}{2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$CZ - \frac{(N-1)^2\cdot(N+1)\cdot\left(N^2+4\cdot N+1\right)}{\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]} = 0$$

$$AU - \frac{(N+1)^2}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$AZ - \frac{\left[\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^3+3\cdot N^2+3\cdot N+1\right)-2\cdot N\cdot\left(N^3+5\cdot N^2+4\cdot N+5\right)-2\right]\cdot(N+1)}{4\cdot N\cdot(N+3)\cdot(3\cdot N+1)-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(2\cdot N^3+6\cdot N^2+6\cdot N+2\right)} = 0$$

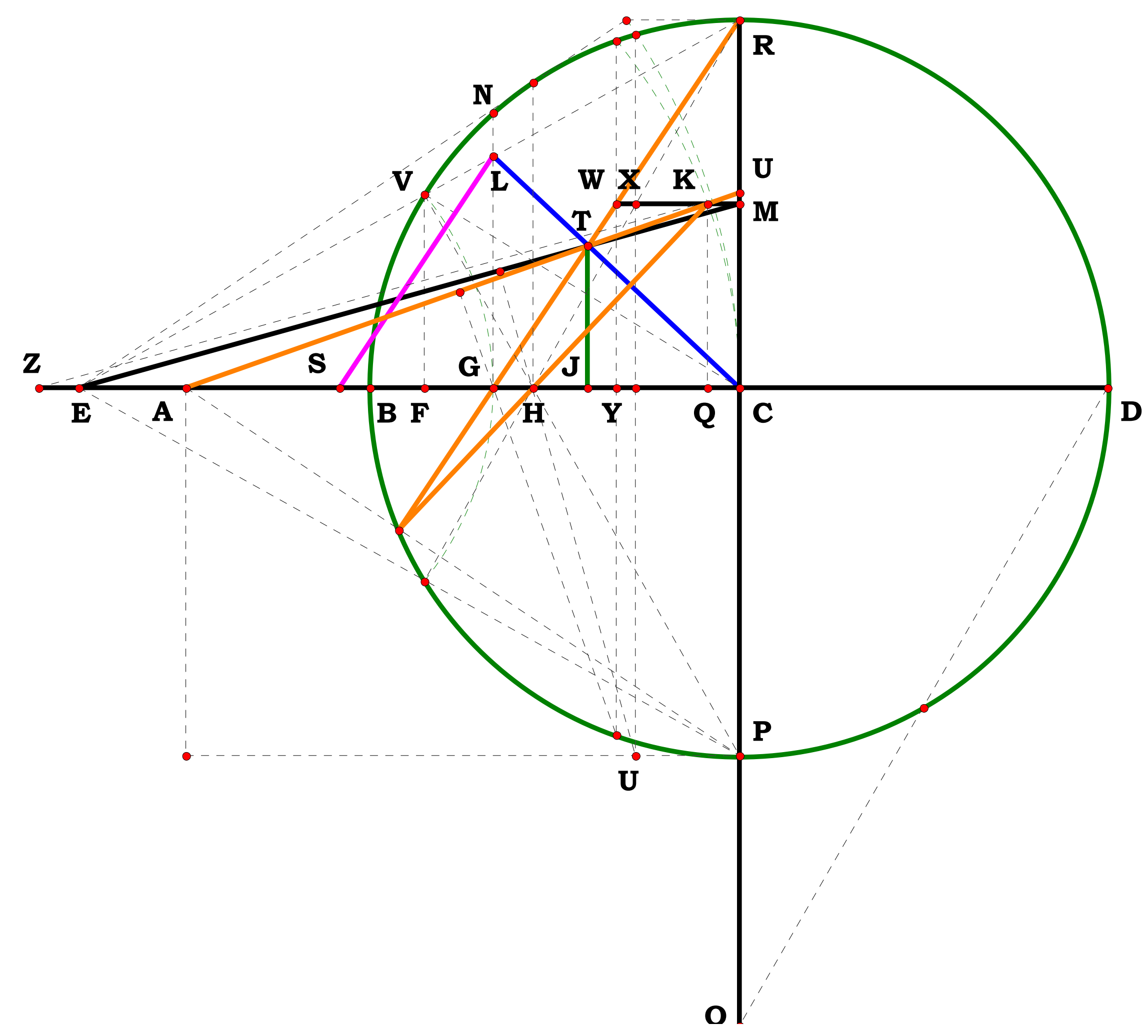
$$AE - \frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^2+N\right)-\left(N+6\cdot N^2+N^3\right)}{3\cdot N+3\cdot N^2+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$

$$AJ - \frac{3\cdot N-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[N\cdot(N+1)\cdot\left(N^4+14\cdot N^3+34\cdot N^2+14\cdot N+1\right)\right]+N^2\cdot\left(3\cdot N^5+28\cdot N^4+117\cdot N^3+216\cdot N^2+117\cdot N+28\right)}{2\cdot N\cdot\left(N^6+11\cdot N^5+41\cdot N^4+75\cdot N^3+75\cdot N^2+41\cdot N+11\right)-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[N^2\cdot\left(N^4+10\cdot N^3+35\cdot N^2+36\cdot N+35\right)+10\cdot N+1\right]+2} = 0$$

$$JC - \frac{18\cdot N-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^7+9\cdot N^6+15\cdot N^5-25\cdot N^4-25\cdot N^3+15\cdot N^2+9\cdot N+1\right)+48\cdot N^2-2\cdot N^3-132\cdot N^4-2\cdot N^5+48\cdot N^6+18\cdot N^7+2\cdot N^8+2}{44\cdot N+164\cdot N^2+300\cdot N^3+300\cdot N^4+164\cdot N^5+44\cdot N^6+4\cdot N^7-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(2\cdot N^6+20\cdot N^5+70\cdot N^4+72\cdot N^3+70\cdot N^2+20\cdot N+2\right)+4} = 0$$

$$TJ - \frac{99\cdot N^5-25\cdot N^2-89\cdot N^3-99\cdot N^4-3\cdot N+89\cdot N^6+25\cdot N^7+3\cdot N^8+\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(33\cdot N^3-14\cdot N^6-33\cdot N^5-N^7+14\cdot N^2+N\right)}{2\cdot(N+1)^5\cdot\left(N^2+6\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}-208\cdot N^2-488\cdot N^3-570\cdot N^4-488\cdot N^5-208\cdot N^6-40\cdot N^7-3\cdot N^8-40\cdot N-3} = 0$$

$$TJ - JT = 0 \qquad JT - \frac{N\cdot(N-1)\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{7\cdot N\cdot(N+1)+N^3+2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$



Unit.

$AB := 1$

Given.

$N := 1.22457 \quad AD := N$

Four Lines To A Point

Does the difference CJ and JT each have but one Algebraic name?

042901B

Descriptions.

$BD := AD - AB \quad BC := \frac{AB}{2} \quad CR := BC$

$CP := BC \quad AC := BC \quad CD := AD - AC \quad CG := \frac{CP^2}{CD}$

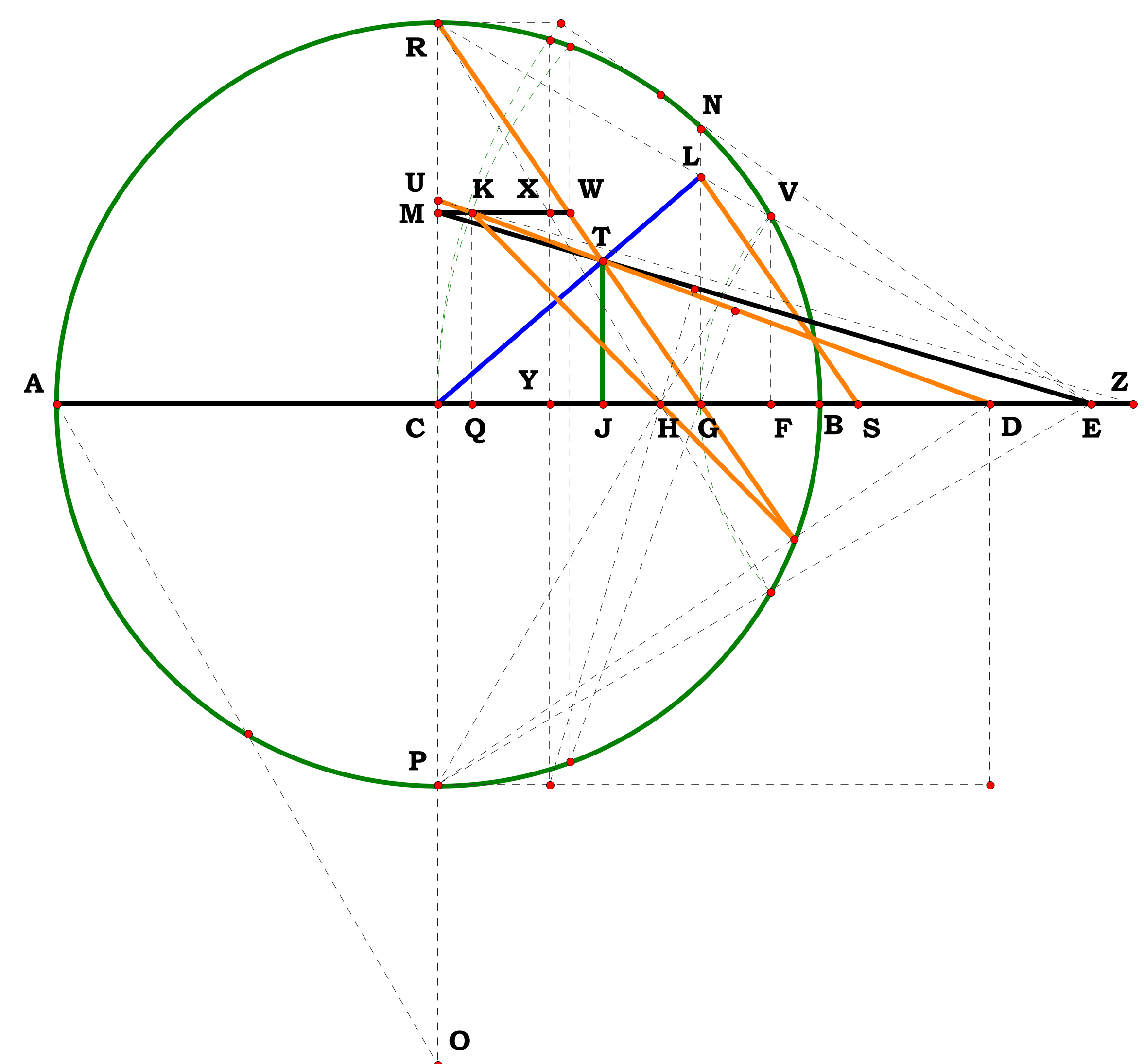
$DG := CD - CG \quad DV := DG \quad CV := BC \quad CF := \frac{CV^2 + CD^2 - DV^2}{2CD}$

$BF := BC - CF \quad AF := CF + AC \quad FV := \sqrt{BF \cdot AF} \quad CE := \frac{CF \cdot CR}{CR - FV}$

$BE := CE - BC \quad BG := BC - CG \quad EF := BE + BF \quad EG := BE + BG$

$GL := \frac{FV \cdot EG}{EF} \quad GS := \frac{CG \cdot GL}{CR} \quad CJ := \frac{CG^2}{GS + CG}$

$JT := \frac{GL \cdot CJ}{CG}$



$$CM := \frac{CR}{2} \qquad DK := CD \qquad KQ := CM$$

$$DQ := \sqrt{DK^2 - KQ^2} \qquad CU := \frac{KQ \cdot CD}{DQ}$$

$$CZ := \frac{CE \cdot CU}{CM} \qquad DU := \sqrt{CD^2 + CU^2}$$

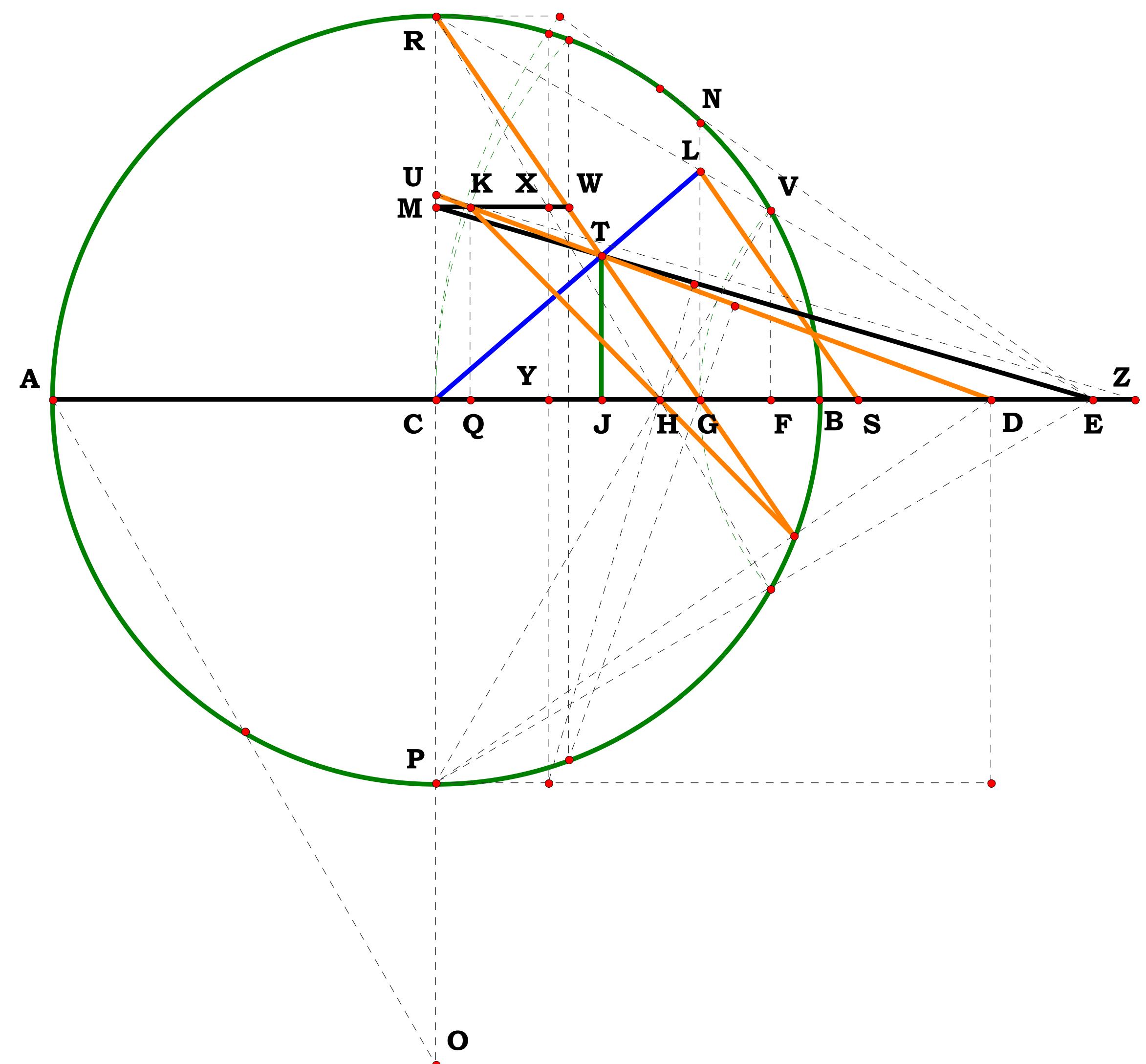
$$DZ := CZ - CD \qquad DE := CE - CD$$

$$DJ := \frac{CD \cdot DE}{DZ} \qquad JC := CD - DJ$$

$$TJ := \frac{CU \cdot DJ}{CD}$$

$$JC - CJ = 0$$

$$JT - TJ = 0$$



Definitions.

$$BD-(N-1)=0 \qquad BC-\frac{1}{2}=0 \qquad CR-\frac{1}{2}=0 \qquad CP-\frac{1}{2}=0 \qquad AC-\frac{1}{2}=0 \qquad CD-\frac{2\cdot N-1}{2}=0$$

$$CG-\frac{1}{2\cdot(2\cdot N-1)}=0 \qquad DG-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \qquad DV-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \qquad CV-\frac{1}{2}=0$$

$$CF-\frac{6\cdot N^2-6\cdot N+1}{2\cdot(2\cdot N-1)^3}=0 \qquad BF-\frac{(4\cdot N-1)\cdot(N-1)^2}{(2\cdot N-1)^3}=0 \qquad AF-\frac{N^2\cdot(4\cdot N-3)}{(2\cdot N-1)^3}$$

$$FV-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3}=0 \qquad CE-\frac{6\cdot N^2-6\cdot N+1}{\left(4\cdot N-4\cdot N^2\right)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}+2\cdot(2\cdot N-1)^3}=0$$

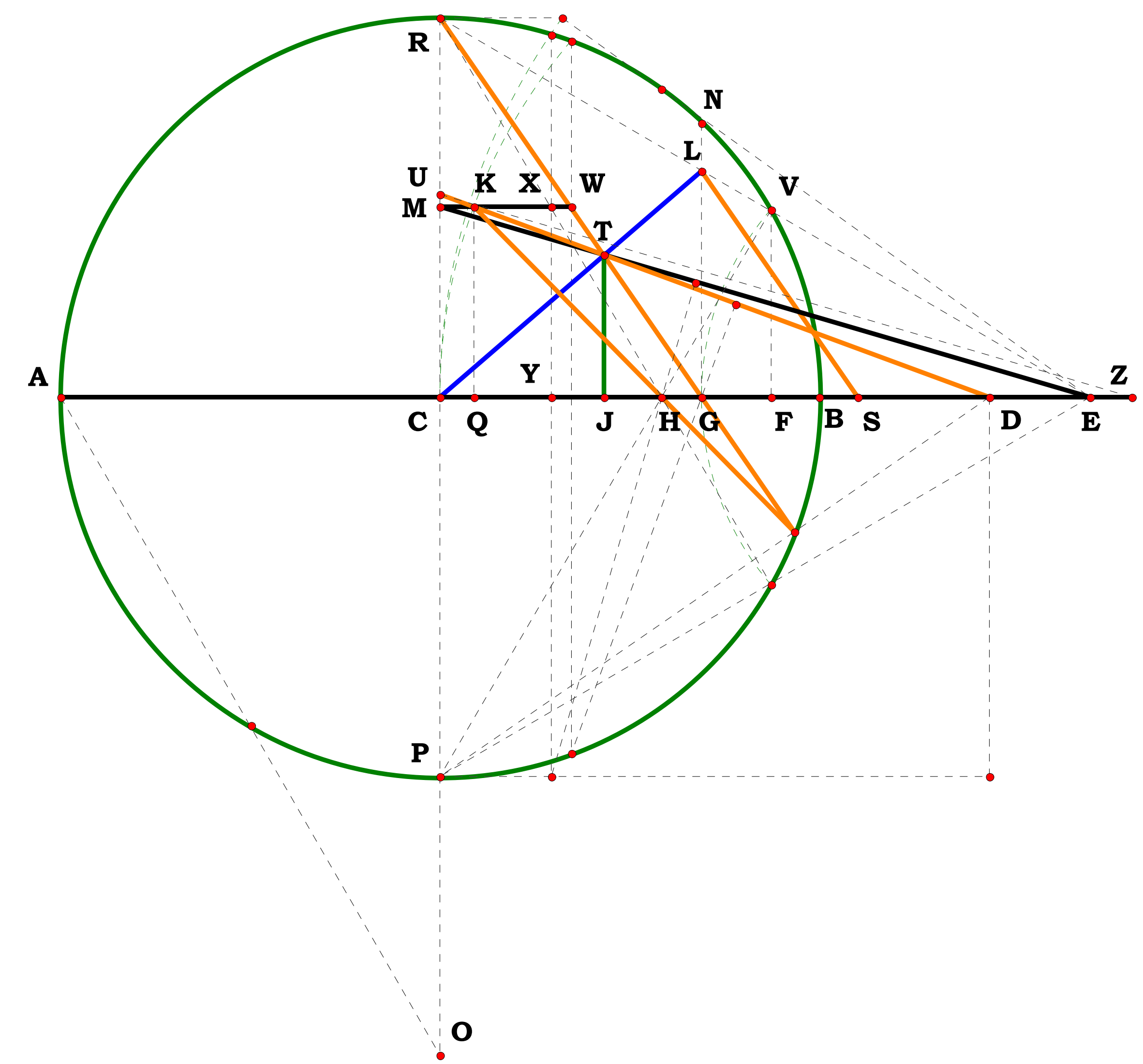
$$BE-\frac{(N-1)\cdot\left(5\cdot N+N\cdot\sqrt{16\cdot N^2-16\cdot N+3}-4\cdot N^2-1\right)}{\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+8\cdot N^3-12\cdot N^2+6\cdot N-1}=0 \qquad BG-\frac{N-1}{2\cdot N-1}=0$$

$$EF-\frac{\sqrt{16\cdot N^2-16\cdot N+3}\cdot N\cdot(N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)}{(2\cdot N-1)^3\cdot\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+(2\cdot N-1)^3\cdot\left[(2\cdot N-1)^3\right]}=0$$

$$EG-\frac{N\cdot(N-1)\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)\cdot\left[\left(8\cdot N^3-12\cdot N^2+6\cdot N-1\right)+\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}\right]}=0 \qquad GL-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0$$

$$GS-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)^2\cdot\left(6\cdot N^2-6\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0 \qquad CJ-\frac{\sqrt{16\cdot N^2-16\cdot N+3}\cdot\left(6\cdot N^2-6\cdot N+1\right)}{\left(4\cdot N^2-4\cdot N\right)\cdot\left(\sqrt{16\cdot N^2-16\cdot N+3}\right)^2+\left(32\cdot N^3-48\cdot N^2+20\cdot N-2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0$$

$$JT-\frac{N\cdot(N-1)\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{\left(2\cdot N^2-2\cdot N\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+(2\cdot N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0$$



$$CM-\frac{1}{4}=0 \qquad DK-\frac{2\cdot N-1}{2}=0 \qquad KQ-\frac{1}{4}=0$$

$$DQ-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4}=0 \qquad CU-\frac{2\cdot N-1}{2\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}=0$$

$$CZ-\frac{(2\cdot N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)}{(2\cdot N-1)^3\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}+(4\cdot N-1)\cdot(4\cdot N-3)\cdot\left(2\cdot N-2\cdot N^2\right)}=0$$

$$DU-\frac{(2\cdot N-1)^2}{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}=0$$

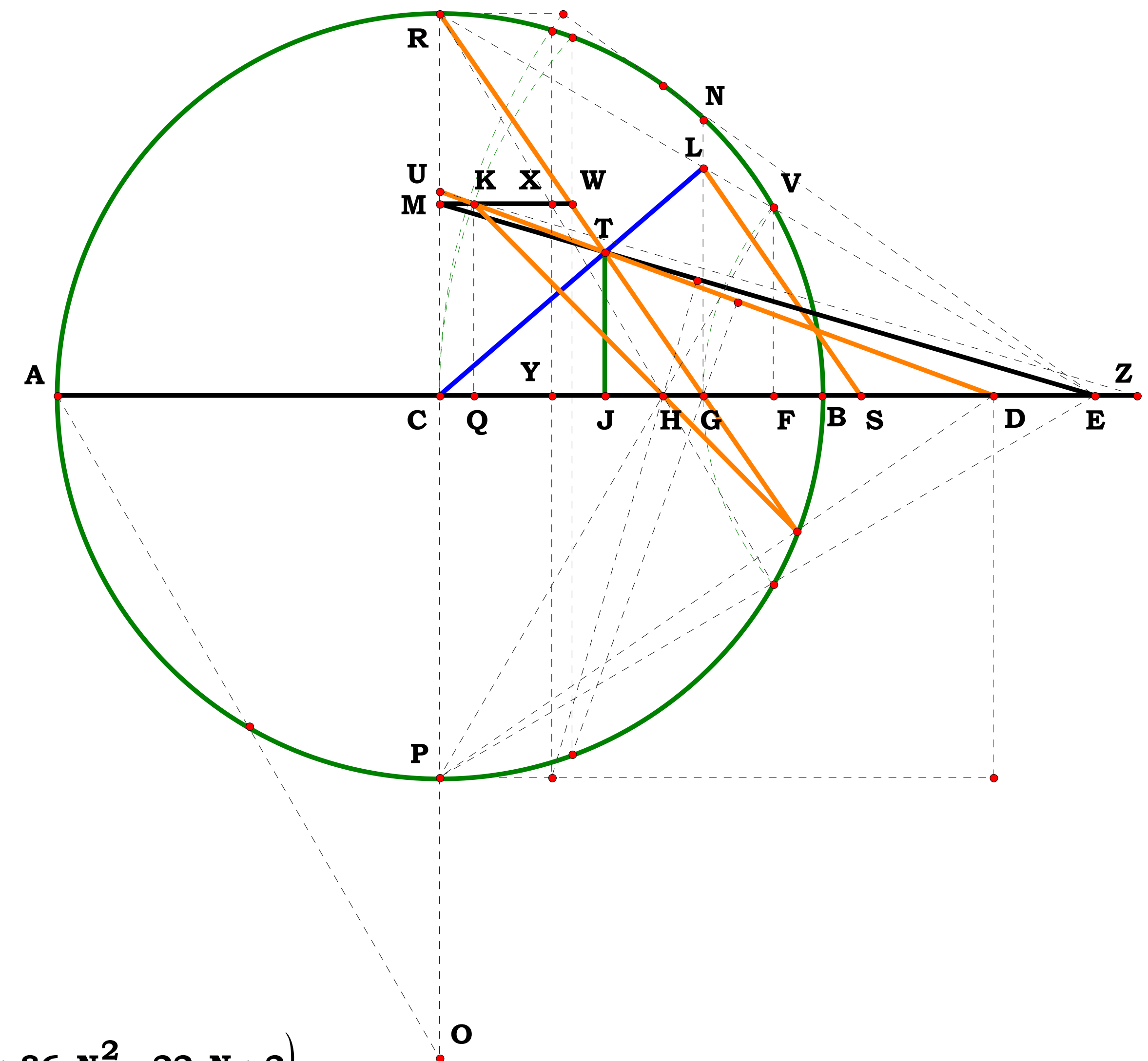
$$DZ-\frac{2\cdot N\cdot\left(32\cdot N^4-80\cdot N^3+82\cdot N^2-43\cdot N+11\right)-(2\cdot N-1)^4\cdot\sqrt{16\cdot N^2-16\cdot N+3}-2}{2\cdot(2\cdot N-1)^3\cdot\sqrt{16\cdot N^2-16\cdot N+3}-4\cdot N\cdot(N-1)\cdot(4\cdot N-3)\cdot(4\cdot N-1)}=0$$

$$DE-\frac{N\cdot(N-1)\cdot(2\cdot N-1)\cdot\sqrt{16\cdot N^2-16\cdot N+3}-N\cdot(N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}{\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+8\cdot N^3-12\cdot N^2+6\cdot N-1}=0$$

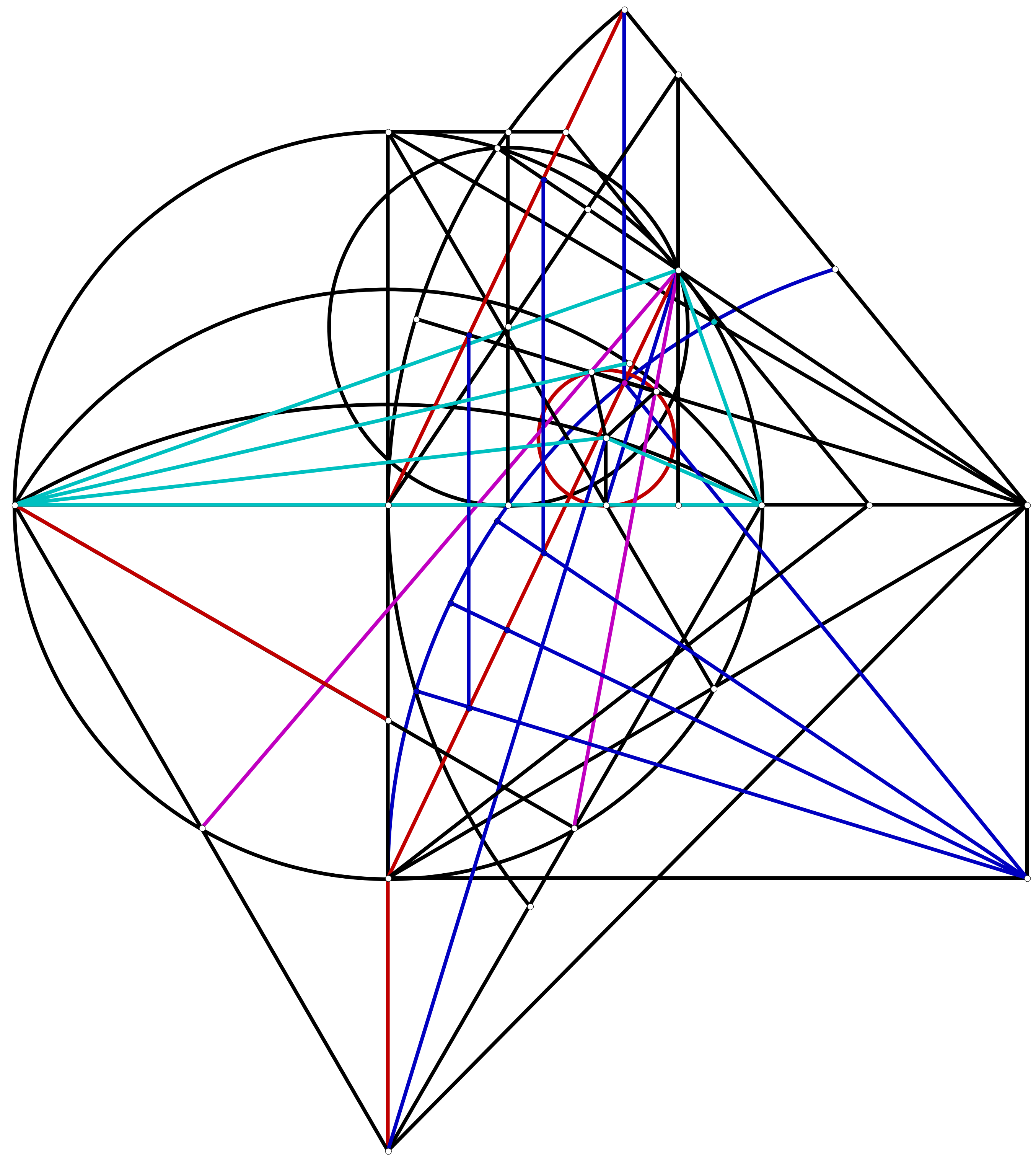
$$JC-\frac{\left(6\cdot N^2-6\cdot N+1\right)\cdot\left(16\cdot N^3-24\cdot N^2+10\cdot N-1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}-\left(6\cdot N^2-6\cdot N+1\right)\cdot\left(64\cdot N^4-128\cdot N^3+86\cdot N^2-22\cdot N+2\right)}{\left[8\cdot N\cdot(N-1)\cdot\left(32\cdot N^4-64\cdot N^3+53\cdot N^2-21\cdot N+4\right)+2\right]\cdot\sqrt{16\cdot N^2-16\cdot N+3}+-4\cdot\left(8\cdot N^2-8\cdot N+1\right)\cdot(2\cdot N-1)^5}=0$$

$$DJ-\frac{N\cdot(N-1)\cdot(2\cdot N-1)\cdot\sqrt{16\cdot N^2-16\cdot N+3}\cdot\left(64\cdot N^4-128\cdot N^3+82\cdot N^2-18\cdot N+1\right)-N\cdot(N-1)\cdot(4\cdot N-3)\cdot(4\cdot N-1)\cdot\left(32\cdot N^4-64\cdot N^3+42\cdot N^2-10\cdot N+1\right)}{\sqrt{16\cdot N^2-16\cdot N+3}\cdot\left(128\cdot N^6-384\cdot N^5+468\cdot N^4-296\cdot N^3+100\cdot N^2-16\cdot N+1\right)-(2\cdot N-1)^5\cdot\left(16\cdot N^2-16\cdot N+2\right)}=0$$

$$TJ-\frac{N\cdot(N-1)\cdot(4\cdot N-3)\cdot(4\cdot N-1)\cdot\left(32\cdot N^4-64\cdot N^3+42\cdot N^2-10\cdot N+1\right)-N\cdot(N-1)\cdot(2\cdot N-1)\cdot\left(64\cdot N^4-128\cdot N^3+82\cdot N^2-18\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}{2\cdot\left(8\cdot N^2-8\cdot N+1\right)\cdot(2\cdot N-1)^5\cdot\sqrt{16\cdot N^2-16\cdot N+3}-(4\cdot N-3)\cdot(4\cdot N-1)\cdot\left[4\cdot N\cdot(N-1)\cdot\left(32\cdot N^4-64\cdot N^3+53\cdot N^2-21\cdot N+4\right)+1\right]}=0$$



Just an Ilustration.



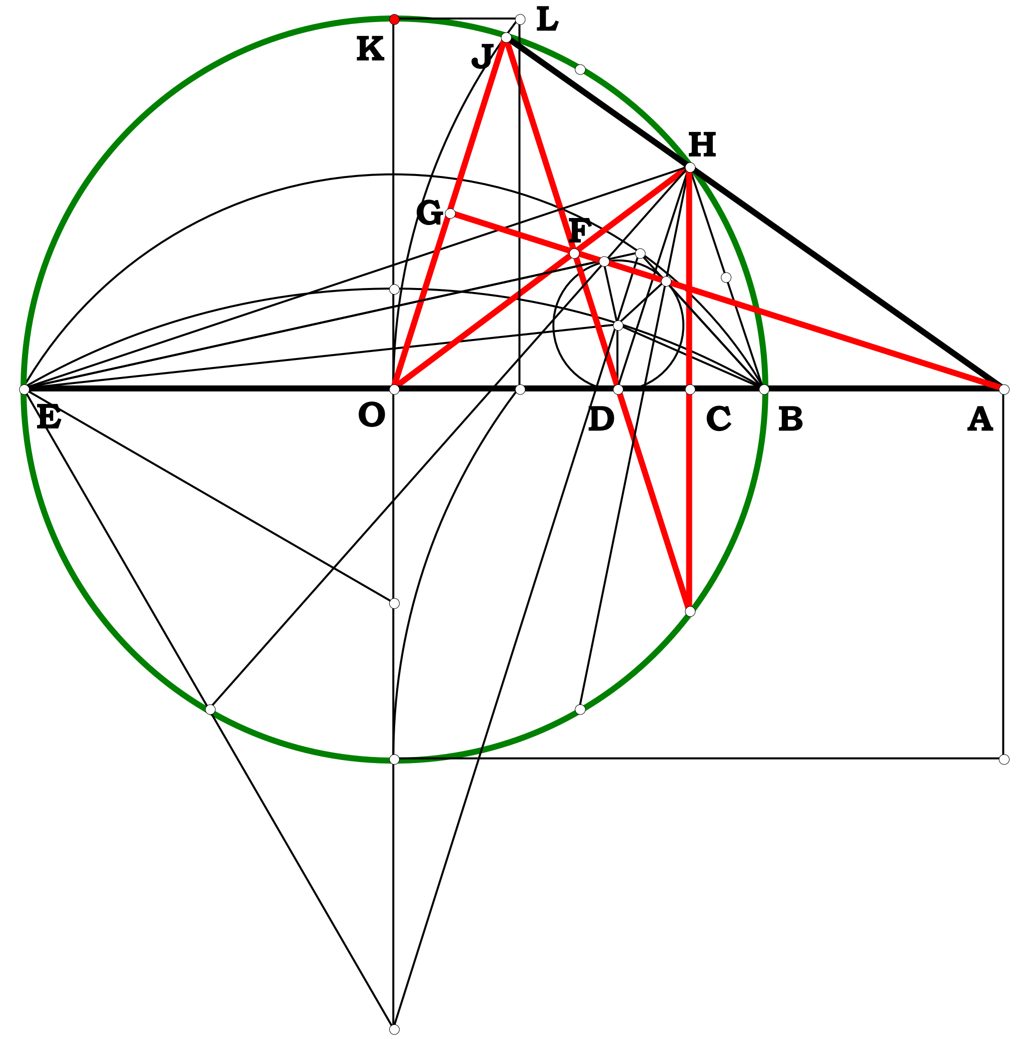
Unit.

$AB := 1$

Given.

$N := 4 \qquad AE := N$

Just some Algebraic Names



050601A

Descriptions.

$BE := AE - AB \qquad BO := \dfrac{BE}{2} \qquad AO := AB + BO \qquad AJ := AO \qquad JO := BO$

$GO := \dfrac{JO}{2} \qquad AG := \sqrt{AO^2 - GO^2} \qquad AP := \dfrac{AG^2}{AO} \qquad OP := AO - AP \qquad NO := 2 \cdot OP$

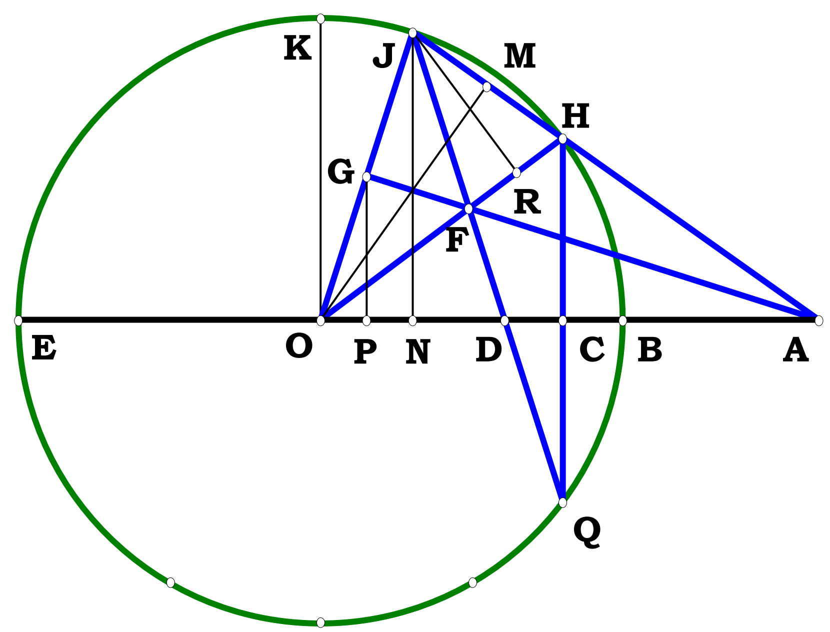
$AN := AO - NO \qquad JM := NO \qquad HO := BO \qquad HJ := 2 \cdot JM \qquad AH := AJ - HJ$

$AC := \dfrac{AN \cdot AH}{AJ} \qquad CH := \sqrt{AH^2 - AC^2} \qquad HQ := 2 \cdot CH \qquad CN := AN - AC$

$JN := \dfrac{CH \cdot AJ}{AH} \qquad CQ := CH \qquad JQ := \sqrt{(CQ + JN)^2 + CN^2} \qquad OR := \dfrac{JO^2 + HO^2 - HJ^2}{2 \cdot HO}$

$JR := \sqrt{JO^2 - OR^2} \qquad FO := \dfrac{JO \cdot GO}{OR} \qquad FJ := FO \qquad DQ := \dfrac{JQ \cdot CQ}{CQ + JN} \qquad DF := JQ - (DQ + FJ)$

$FH := HO - FO \qquad FG := \dfrac{JR \cdot GO}{OR} \qquad AF := AG - FG$



Definitions.

$AE - N = 0 \quad BE - (N-1) = 0 \quad BO - \dfrac{N-1}{2} = 0 \quad AO - \dfrac{N+1}{2} = 0 \quad AG - \dfrac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0$

$AP - \dfrac{(3\cdot N+1)\cdot(N+3)}{8\cdot(N+1)} = 0 \quad OP - \dfrac{(N-1)^2}{8\cdot(N+1)} = 0 \quad NO - \dfrac{(N-1)^2}{4\cdot(N+1)} = 0 \quad AN - \dfrac{N^2+6\cdot N+1}{4\cdot(N+1)} = 0$

$HJ - \dfrac{(N-1)^2}{2\cdot(N+1)} = 0 \quad AH - \dfrac{2\cdot N}{N+1} = 0 \quad AC - \dfrac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} = 0 \quad CH - \dfrac{N\cdot(N-1)\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{(N+1)^3} = 0$

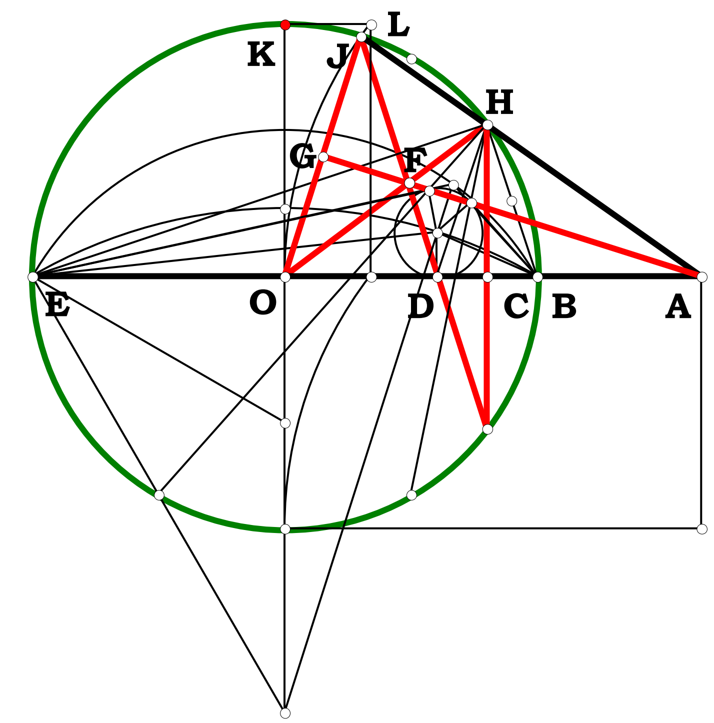
$HQ - \dfrac{2\cdot N\cdot(N-1)\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{(N+1)^3} = 0 \quad CN - \dfrac{(N-1)^2\cdot\left(N^2+6\cdot N+1\right)}{4\cdot(N+1)^3} = 0 \quad JN - \dfrac{\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot(N-1)}{4\cdot(N+1)} = 0$

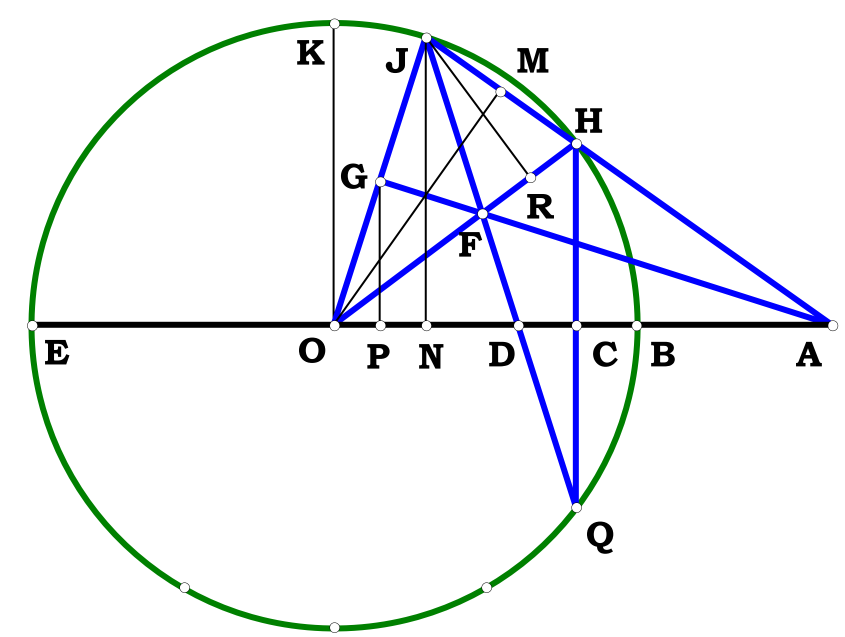
$JQ - \dfrac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} = 0 \quad OR - \dfrac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{4\cdot(N+1)^2} = 0 \quad JR - \dfrac{(N-1)^2\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{4\cdot(N+1)^2} = 0$

$FO - \dfrac{(N-1)\cdot(N+1)^2}{2\cdot\left(N^2+6\cdot N+1\right)} = 0 \quad DQ - \dfrac{2\cdot N\cdot(N-1)}{(N+1)^2} = 0 \quad DF - \dfrac{2\cdot N\cdot(N-1)}{N^2+6\cdot N+1} = 0 \quad FH - \dfrac{2\cdot N\cdot(N-1)}{N^2+6\cdot N+1} = 0$

$FG - \dfrac{(N-1)^2\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{4\cdot\left(N^2+6\cdot N+1\right)} = 0 \quad GO - \dfrac{N-1}{4} = 0$

$$AF - \frac{\left(2\cdot N - N^2 - 1\right)\cdot\sqrt{3\cdot N+1}\cdot\sqrt{N+3} + \left(N^2+6\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}}{4\cdot\left(N^2+6\cdot N+1\right)} = 0$$





Unit.

$AB := 1$

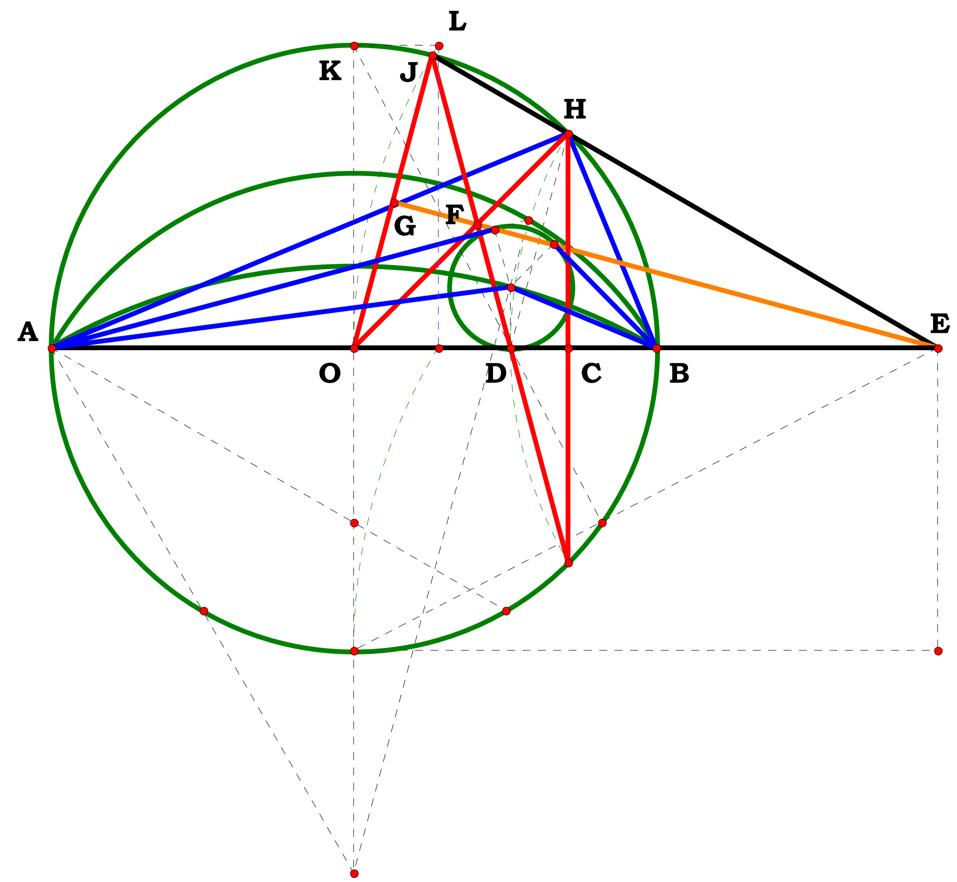
Given.

$N := 1.46595 \quad AE := N$

Just some Algebraic Names

050601B

Descriptions.



$BE := AE - AB \quad BO := \dfrac{AB}{2} \quad EO := BE + BO \quad EJ := EO$

$JO := BO \quad GO := \dfrac{JO}{2} \quad EG := \sqrt{EO^2 - GO^2}$

$EP := \dfrac{EG^2}{EO} \quad OP := EO - EP \quad NO := 2 \cdot OP$

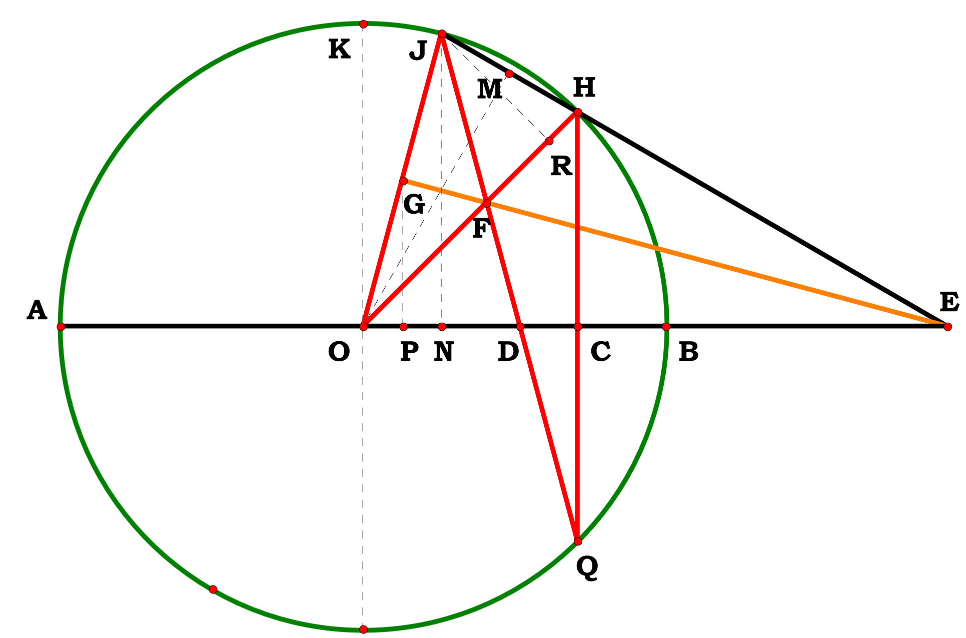
$EN := EO - NO \quad JM := NO \quad HO := BO \quad HJ := 2 \cdot JM \quad EH := EJ - HJ$

$EC := \dfrac{EN \cdot EH}{EJ} \quad CH := \sqrt{EH^2 - EC^2} \quad HQ := 2 \cdot CH \quad CN := EN - EC$

$JN := \dfrac{CH \cdot EJ}{EH} \quad CQ := CH \quad JQ := \sqrt{(CQ + JN)^2 + CN^2} \quad OR := \dfrac{JO^2 + HO^2 - HJ^2}{2 \cdot HO}$

$JR := \sqrt{JO^2 - OR^2} \quad FO := \dfrac{JO \cdot GO}{OR} \quad FJ := FO \quad DQ := \dfrac{JQ \cdot CQ}{CQ + JN}$

$DF := JQ - (DQ + FJ) \quad FH := HO - FO \quad FG := \dfrac{JR \cdot GO}{OR} \quad EF := EG - FG$



Definitions.

$$BE - (N-1) = 0 \quad BO - \frac{1}{2} = 0 \quad EO - \frac{2\cdot N - 1}{2} = 0 \quad EJ - \frac{2\cdot N-1}{2} = 0$$

$$JO - \frac{1}{2} = 0 \quad GO - \frac{1}{4} = 0 \quad EG - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4} = 0$$

$$EP - \frac{(4\cdot N-1)\cdot(4\cdot N-3)}{8\cdot(2\cdot N-1)} = 0 \quad OP - \frac{1}{8\cdot(2\cdot N-1)} = 0 \quad NO - \frac{1}{4\cdot(2\cdot N-1)} = 0$$

$$EN - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0 \quad JM - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad HO - \frac{1}{2} = 0 \quad HJ - \frac{1}{2\cdot(2\cdot N-1)} = 0$$

$$EH - \frac{2\cdot N\cdot(N-1)}{2\cdot N-1} = 0 \quad EC - \frac{N\cdot(N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}{(2\cdot N-1)^3} = 0 \quad CH - \frac{N\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot(N-1)}{(2\cdot N-1)^3} = 0$$

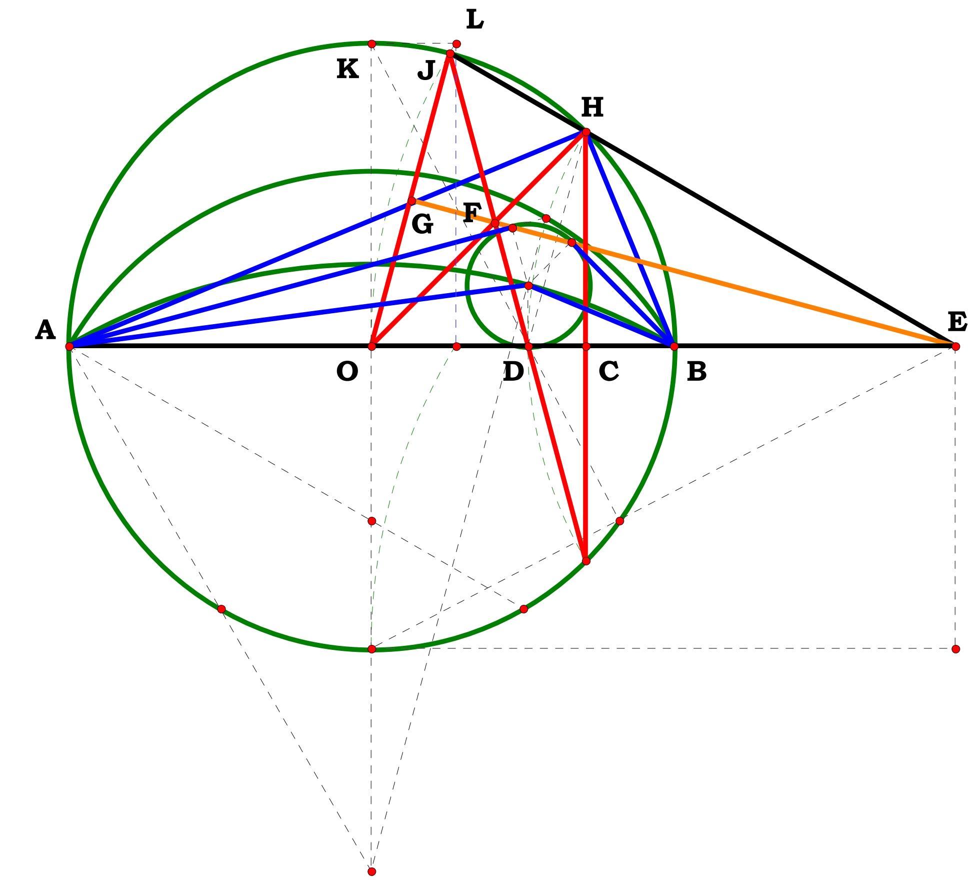
$$HQ - \frac{2\cdot N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3} = 0 \quad CN - \frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)^3} = 0 \quad JN - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)} = 0$$

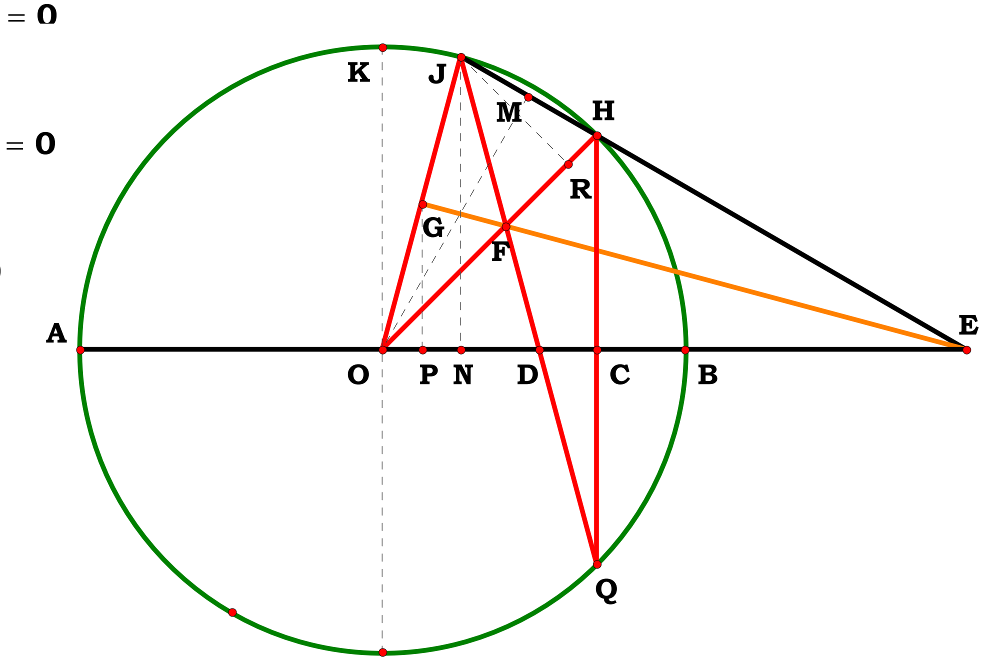
$$CQ - \frac{N\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot(N-1)}{(2\cdot N-1)^3} = 0 \quad JQ - \frac{\left(8\cdot N^2-8\cdot N+1\right)}{2\cdot(2\cdot N-1)^2} = 0 \quad OR - \frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)^2} = 0$$

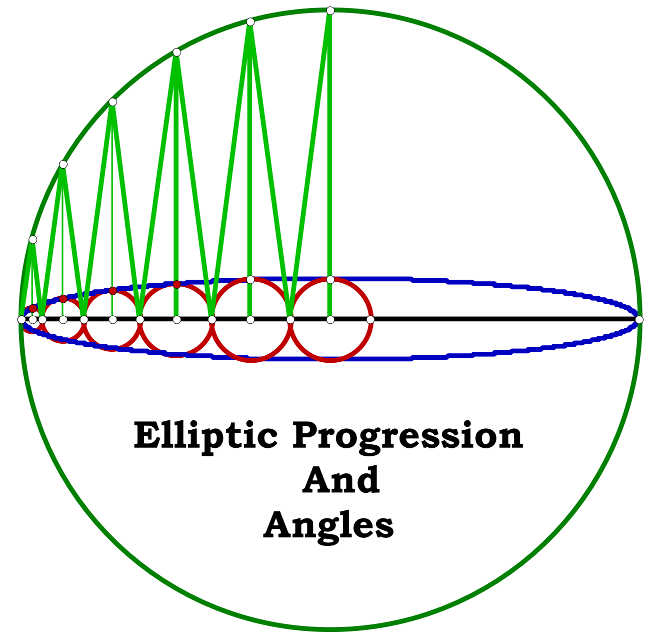
$$JR - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)^2} = 0 \quad FO - \frac{(2\cdot N-1)^2}{2\cdot\left(8\cdot N^2-8\cdot N+1\right)} = 0 \quad FJ - \frac{HO}{4\cdot\left(-HJ^2+HO^2+JO^2\right)} = 0$$

$$DQ - \frac{2\cdot N\cdot(N-1)}{(2\cdot N-1)^2} = 0 \quad DF - \frac{2\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1} = 0 \quad FH - \frac{2\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1} = 0$$

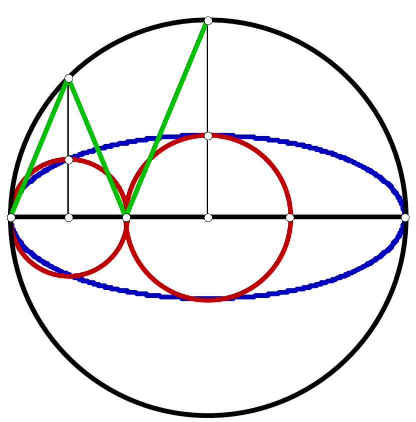
$$FG - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot\left(8\cdot N^2-8\cdot N+1\right)} = 0 \quad EF - \frac{2\cdot\sqrt{16\cdot N^2-16\cdot N+3\cdot N\cdot(N-1)}}{8\cdot N^2-8\cdot N+1} = 0$$



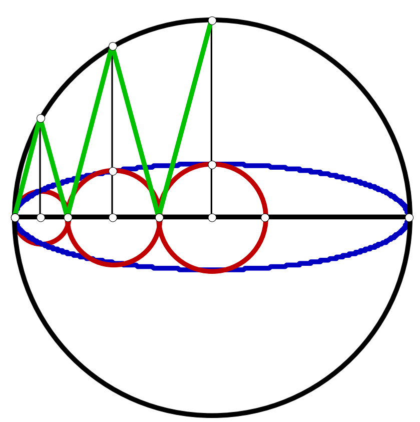




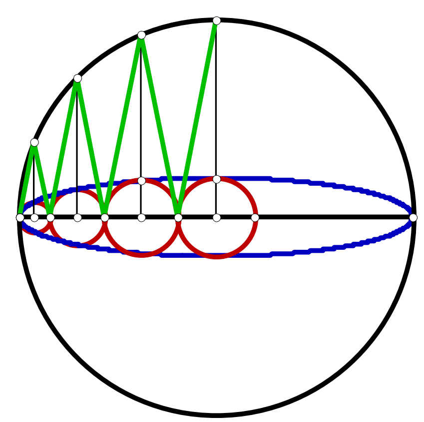
$$\frac{m\angle CAAW}{m\angle ARCN} = 2.000$$



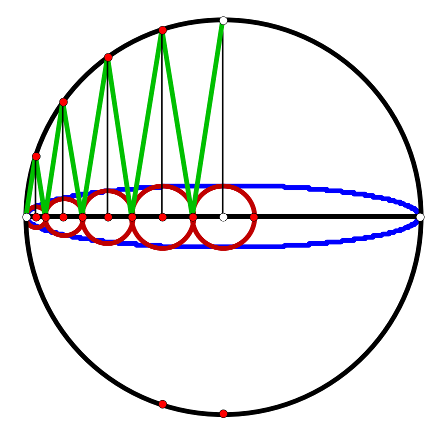
$$\frac{m\angle CAAW}{m\angle ARCN} = 3.000$$



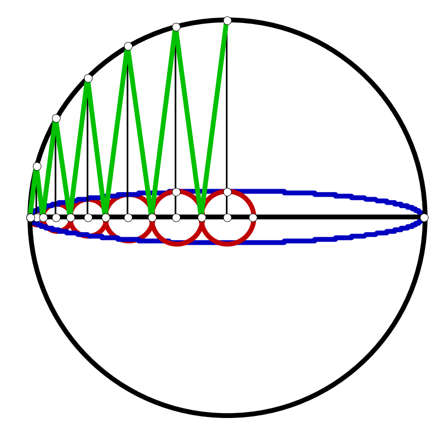
$$\frac{m\angle CAAW}{m\angle ARCN} = 4.000$$



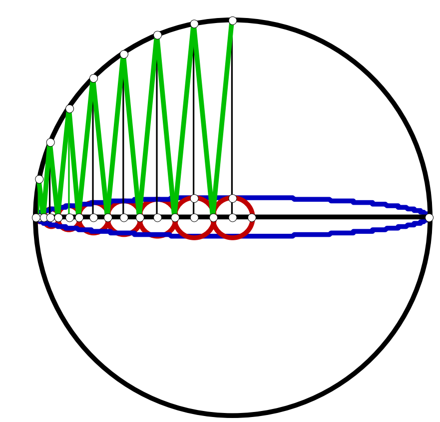
$$\frac{m\angle CAAW}{m\angle BCA} = 5.000$$



$$\frac{m\angle CAAW}{m\angle ARCN} = 6.000$$



$$\frac{m\angle CAAW}{m\angle ARCN} = 8.000$$



Unit.

$AB := 1$

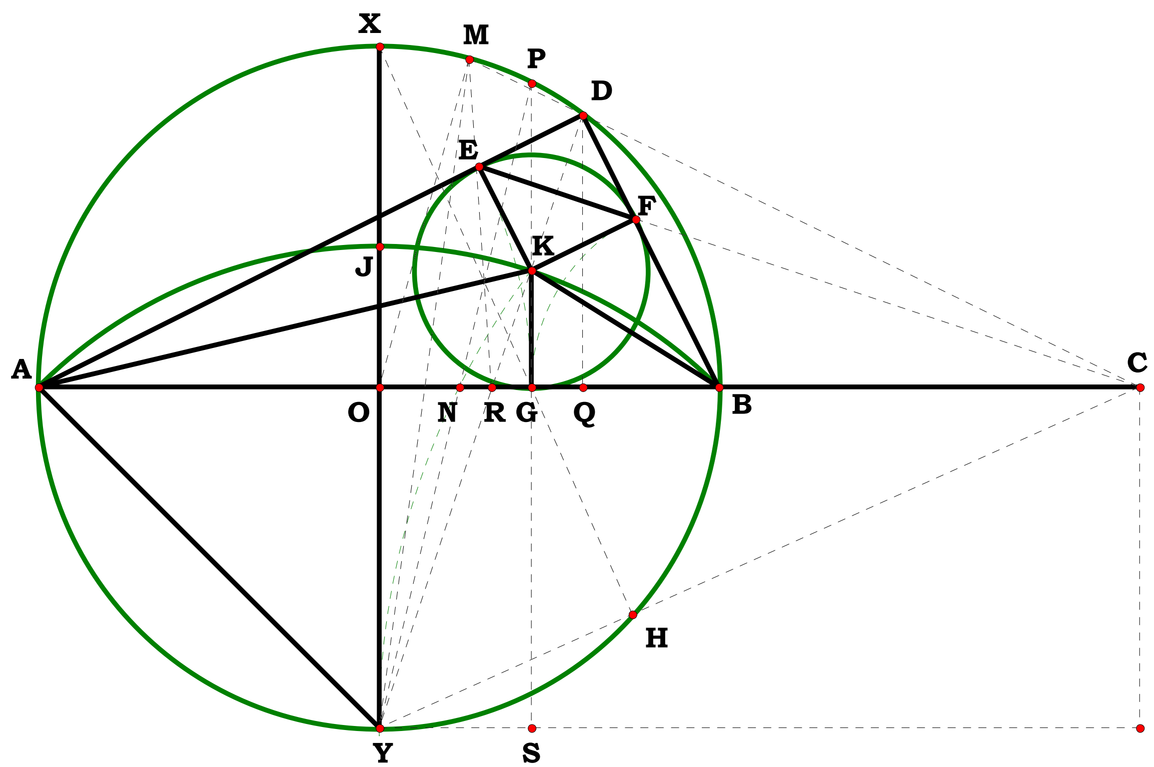
Given.

$N_1 := \frac{AB}{10}$

$N_2 := 8$

050701 EP

What is an Angle?



AC = 1.61803

N_2 = 8.00000

N_1 = 10.00000

$$\frac{\left(2\cdot\left(\frac{1}{N_1}\right)\cdot N_2-1\right)+\sqrt{1+2\cdot\sqrt{\left(\frac{1}{N_1}\right)\cdot N_2-\frac{1}{N_1}^2\cdot N_2^2}}}{2\cdot\left(2\cdot\left(\frac{1}{N_1}\right)\cdot N_2-1\right)} = 1.61803$$

$$AC-\frac{\left(2\cdot\left(\frac{1}{N_1}\right)\cdot N_2-1\right)+\sqrt{1+2\cdot\sqrt{\left(\frac{1}{N_1}\right)\cdot N_2-\frac{1}{N_1}^2\cdot N_2^2}}}{2\cdot\left(2\cdot\left(\frac{1}{N_1}\right)\cdot N_2-1\right)} = 0.00000$$

Descriptions.

$AQ := N_1 \cdot N_2 \quad BQ := AB - AQ \quad DQ := \sqrt{BQ \cdot AQ}$

$AO := \frac{AB}{2} \quad AY := \sqrt{2 \cdot AO^2} \quad OQ := AQ - AO$

$DY := \sqrt{(AO + DQ)^2 + OQ^2} \quad OR := \frac{OQ \cdot AO}{AO + DQ}$

$QR := OQ - OR \quad KS := \frac{AY \cdot (DQ + AO)}{DY}$

$GK := KS - AO \quad GR := \frac{QR \cdot GK}{DQ} \quad GO := GR + OR$

$CO := \frac{AO^2}{GO} \quad BC := CO - AO \quad AC := AB + BC$

Definitions.

$AQ - N_1 \cdot N_2 = 0 \quad BQ - \left(1 - N_1 \cdot N_2\right) = 0 \quad DQ - \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} = 0$

$AO - \frac{1}{2} = 0 \quad AY - \frac{1}{\sqrt{2}} = 0 \quad OQ - \frac{2 \cdot N_1 \cdot N_2 - 1}{2} = 0 \quad DY - \frac{\sqrt{4 \cdot \sqrt{-N_1 \cdot N_2 \cdot \left(N_1 \cdot N_2 - 1\right)} + 2}}{2} = 0 \quad OR - \frac{2 \cdot N_1 \cdot N_2 - 1}{2 \cdot \left(2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1\right)} = 0$

$QR - \frac{\sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)}{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} = 0 \quad KS - \frac{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}{2 \cdot \sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}} = 0 \quad GK - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1}{2} = 0$

$GR - \frac{\left(\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1\right) \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)}{2 \cdot \left(2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1\right)} \quad GO - \frac{N_1 \cdot N_2 - \frac{1}{2}}{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}} = 0 \quad CO - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}}{2 \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)} = 0$

$BC - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 2 \cdot N_1 \cdot N_2 + 1}{4 \cdot N_1 \cdot N_2 - 2} = 0 \quad AC - \frac{2 \cdot N_1 \cdot N_2 + \sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1}{2 \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)} = 0 \quad AC - \frac{N_2 + \sqrt{5} \cdot \sqrt{\sqrt{10 \cdot N_2 - N_2^2} + 5} - 5}{2 \cdot N_2 - 10} = 0$

Unit.

$AB := 1$

0507013
Descriptions.

$AC := \frac{AB}{2}$ $\quad BF := \sqrt{2 \cdot AC^2}$

$BG := \frac{BF}{2}$ $\quad CH := AC$ $\quad CG := BG$

$GH := CH - CG$ $\quad BH := \sqrt{BG^2 + GH^2}$

$BD := \frac{BH^2}{AB}$ $\quad CE := \frac{AB - (4 \cdot BD)}{2}$

Definitions.

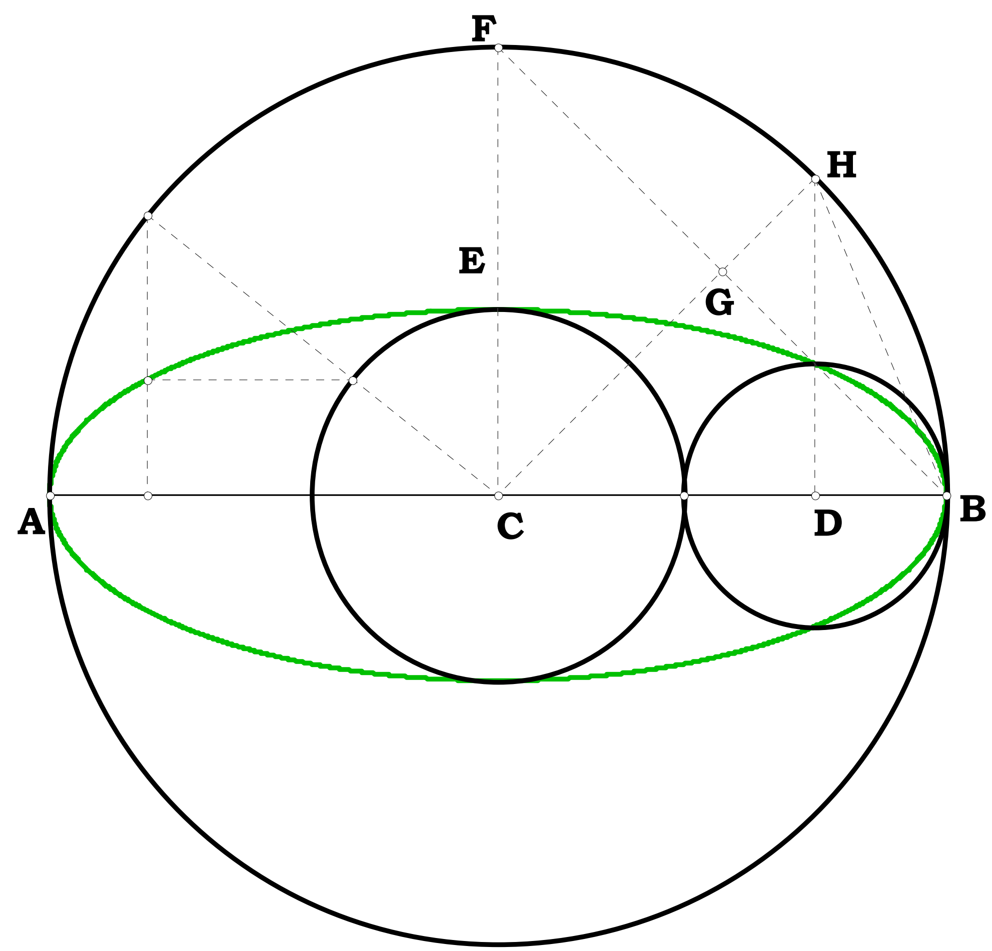
$AC - \frac{1}{2} = 0$ $\quad BF - \frac{\sqrt{2}}{2} = 0$

$BG - \frac{\sqrt{2}}{4} = 0$ $\quad CH - \frac{1}{2} = 0$ $\quad CG - \frac{\sqrt{2}}{4} = 0$

$GH - \frac{2-\sqrt{2}}{4} = 0$ $\quad BH - \frac{\sqrt{2-\sqrt{2}}}{2} = 0$

$BD - \frac{2-\sqrt{2}}{4} = 0$ $\quad CE - \frac{\sqrt{2}-1}{2} = 0$

Angles by Ellipse:



0507012

Unit.

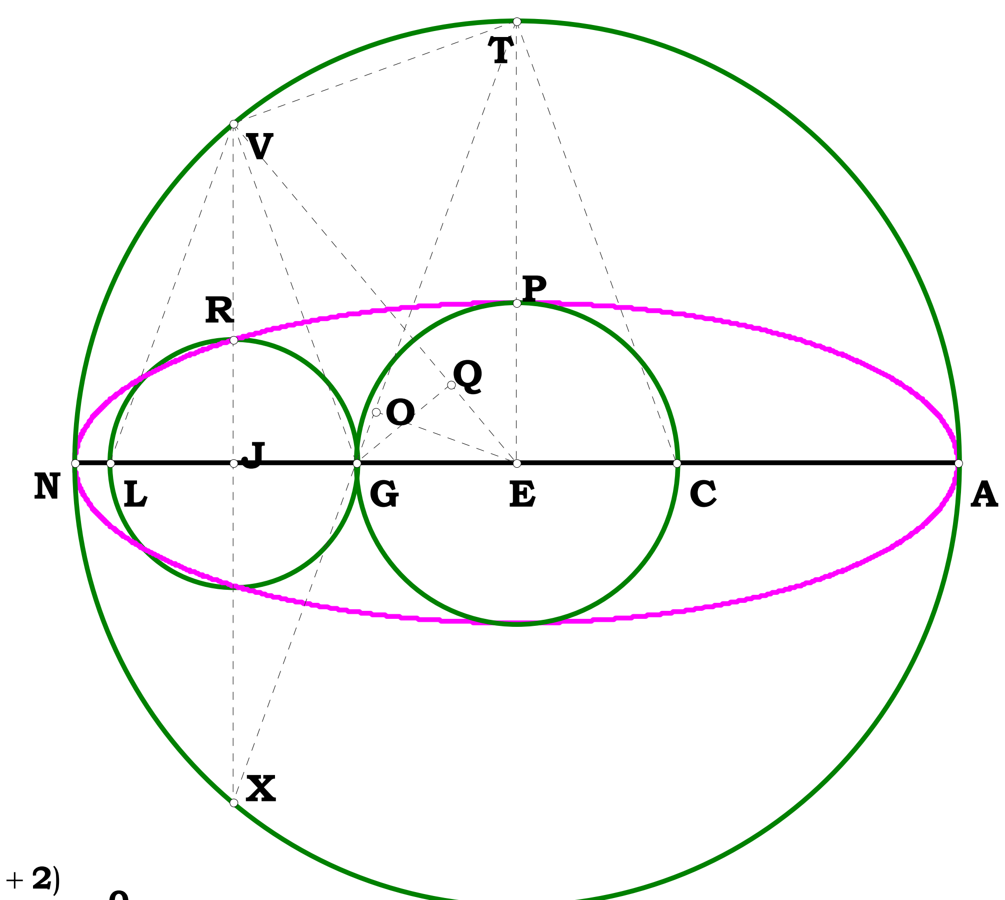
$AN := 1$

Given.

$N := 5.52$

Elliptic Progression Outtake Two

Angles TEV and EVJ equals CTG.



Descriptions.

$EN := \frac{AN}{2}$ $\quad ET := EN$ $\quad EV := EN$ $\quad EG := \frac{AN}{N}$ $\quad EP := EG$

$GT := \sqrt{EG^2 + ET^2}$ $\quad GO := \frac{EG^2}{GT}$ $\quad GX := GT - 2 \cdot GO$ $\quad JX := \frac{ET \cdot GX}{GT}$

$GJ := \frac{EG \cdot GX}{GT}$ $\quad \frac{ET}{EP} - \frac{JX}{GJ} = 0$ $\quad JV := JX$ $\quad EJ := EG + GJ$

$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$ $\quad EQ := \frac{EJ \cdot EG}{EV}$ $\quad GQ := \frac{JV \cdot EQ}{EJ}$

$GQ - GJ = 0$ $\quad \frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0$ $\quad TV - \frac{2 \cdot AN}{\sqrt{N^2 + 4}} = 0$

Definitions.

$EN - \frac{1}{2} = 0$ $\quad ET - \frac{1}{2} = 0$ $\quad EV - \frac{1}{2} = 0$ $\quad EG - \frac{1}{N} = 0$ $\quad EP - \frac{1}{N} = 0$

$GT - \frac{\sqrt{N^2 + 4}}{2 \cdot N} = 0$ $\quad GO - \frac{2}{N \cdot \sqrt{N^2 + 4}} = 0$ $\quad GX - \frac{(N-2) \cdot (N+2)}{2 \cdot N \cdot \sqrt{N^2 + 4}} = 0$ $\quad JX - \frac{(N-2) \cdot (N+2)}{2 \cdot \left(N^2 + 4\right)} = 0$

$GJ - \frac{(N-2) \cdot (N+2)}{N \cdot \left(N^2 + 4\right)} = 0$ $\quad \frac{ET}{EP} - \frac{JX}{GJ} = 0$ $\quad JV - \frac{(N-2) \cdot (N+2)}{2 \cdot \left(N^2 + 4\right)} = 0$ $\quad EJ - \frac{2 \cdot N}{N^2 + 4} = 0$

$TV - \frac{2}{\sqrt{N^2 + 4}} = 0$ $\quad EQ - \frac{4}{N^2 + 4} = 0$ $\quad GQ - \frac{(N-2) \cdot (N+2)}{N \cdot \left(N^2 + 4\right)} = 0$

Unit.

$AL := 1$

0507013

Descriptions.

$AE := \frac{AL}{2}$ $ER := AE$ $NR := ER$ $FN := \frac{NR}{2}$ $EN := AE$

$EF := \sqrt{EN^2 - FN^2}$ $DE := AE$ $DF := DE - EF$ $DN := \sqrt{DF^2 + FN^2}$

$AN := DN$ $AB := \frac{AN^2}{AL}$ $EL := AE$ $BL := AL - AB$ $BN := \sqrt{AN^2 - AB^2}$

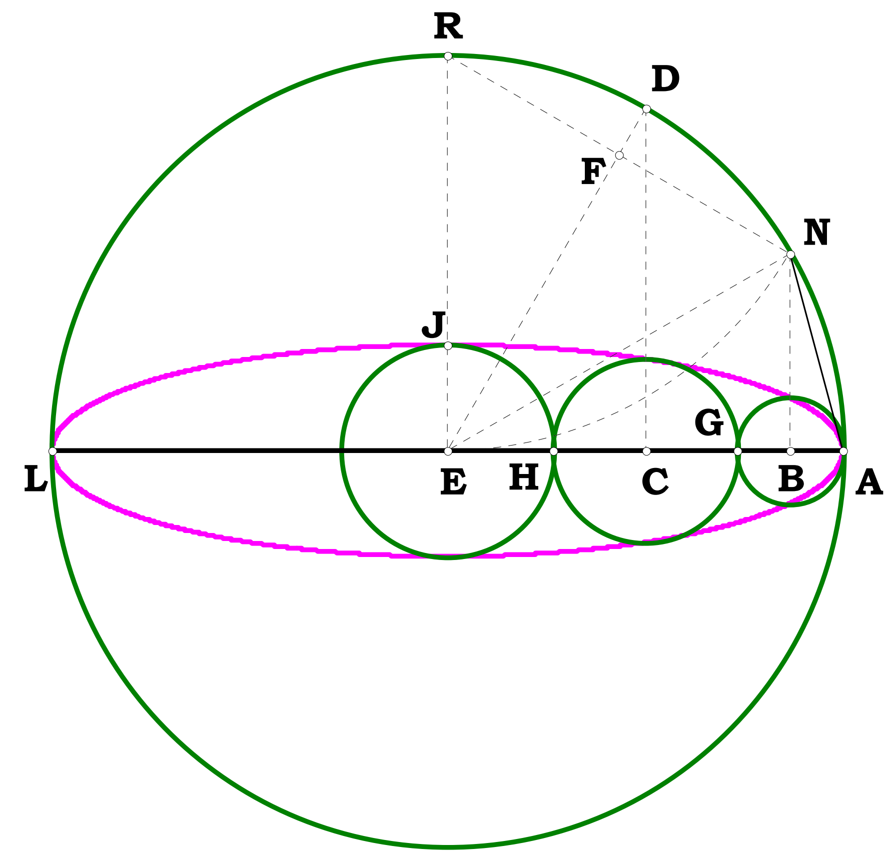
$EJ := \frac{BN \cdot EL}{BL}$ $GH := AE - (EJ + 2 \cdot AB)$

Definitions.

$FN - \frac{1}{4} = 0$ $EF - \frac{\sqrt{3}}{4} = 0$ $DF - \frac{2-\sqrt{3}}{4} = 0$ $AN - \frac{\sqrt{2}\cdot(\sqrt{3}-1)}{4} = 0$

$\frac{2-\sqrt{3}}{4} - AB = 0$ $\frac{\sqrt{3}+2}{4} - BL = 0$ $\frac{1}{4} - BN = 0$ $\frac{1}{2\cdot\sqrt{3}+4} - EJ = 0$ $\frac{\sqrt{3}}{2\cdot\sqrt{3}+4} - GH = 0$

Trisection:



Unit.

$AC := 1$

Given.

050701A

Descriptions.

$$CP := \sqrt{2AC^2} \quad AP := AC \quad EP := CP$$

$$CE := \sqrt{AC^2 + (AP + EP)^2}$$

$$EK := CE \quad EH := CE \quad AE := \sqrt{CE^2 - AC^2}$$

$$AH := EH - AE \quad AT := AH \quad EU := \frac{AT \cdot EK}{AC}$$

$$\frac{EK}{EU} = 5.027339 \quad 1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}} = 5.027339$$

Definitions.

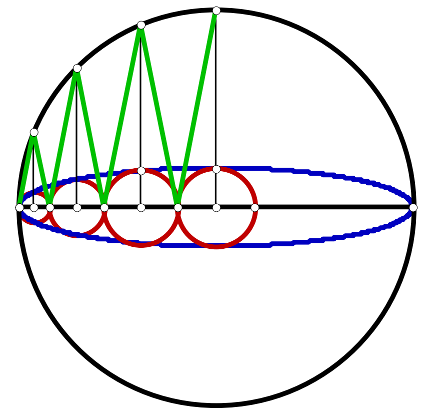
$$\frac{EK}{EU} - \left(1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}}\right) = 0$$

$$CP - \sqrt{2} = 0 \quad EP - \sqrt{2} = 0 \quad CE - \sqrt{2} \cdot \sqrt{2 + \sqrt{2}} = 0$$

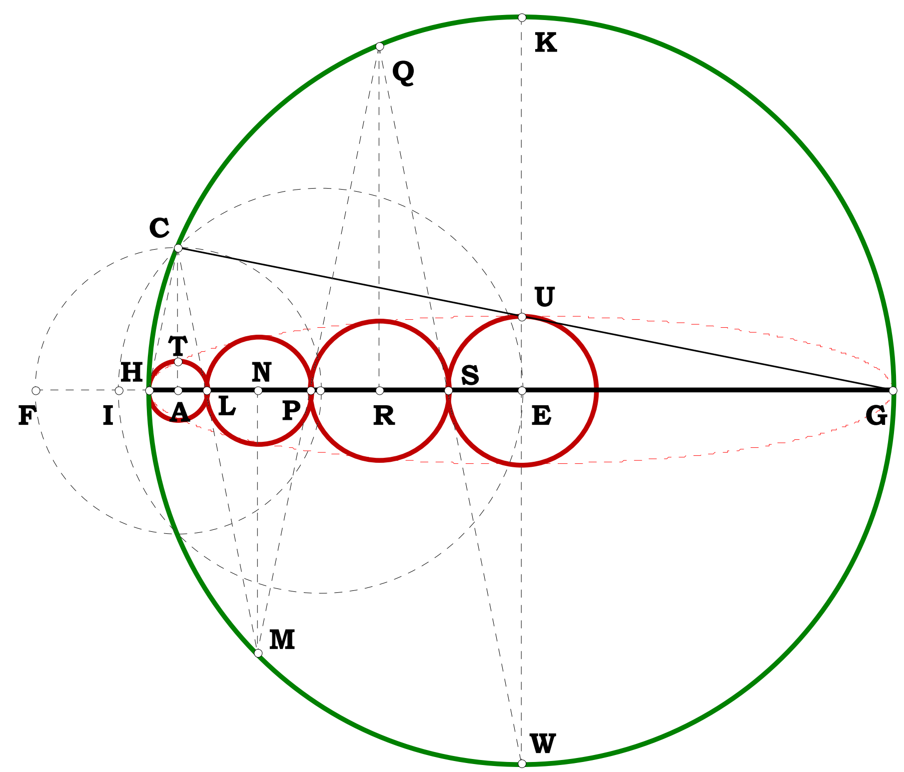
$$AE - \left(\sqrt{2} + 1\right) = 0 \quad AH - \left(\sqrt{2} \cdot \sqrt{2 + \sqrt{2}} - 1 - \sqrt{2}\right) = 0$$

$$EU - \left(2 - \sqrt{\sqrt{2} + 2}\right) \cdot \left(\sqrt{2} + 2\right) = 0$$

$$\frac{m\angle CAAW}{m\angle ARCN} = 4.000$$



An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle, from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles.



Unit.

$AL := 1$

0507013B

Descriptions.

$AE := \frac{AL}{2} \quad AR := \sqrt{2 \cdot AE^2}$

$AM := \frac{AR}{2} \quad EO := AE \quad EM := AM$

$MO := EO - EM \quad AO := \sqrt{AM^2 + MO^2}$

$AY := \frac{AO}{2} \quad EY := \sqrt{AE^2 - AY^2}$

$EN := AE \quad NY := EN - EY \quad AN := \sqrt{AY^2 + NY^2}$

$AP := \frac{AN^2}{AL} \quad NP := \sqrt{AN^2 - AP^2} \quad ES := \frac{NP \cdot AE}{AL - AP}$

Definitions.

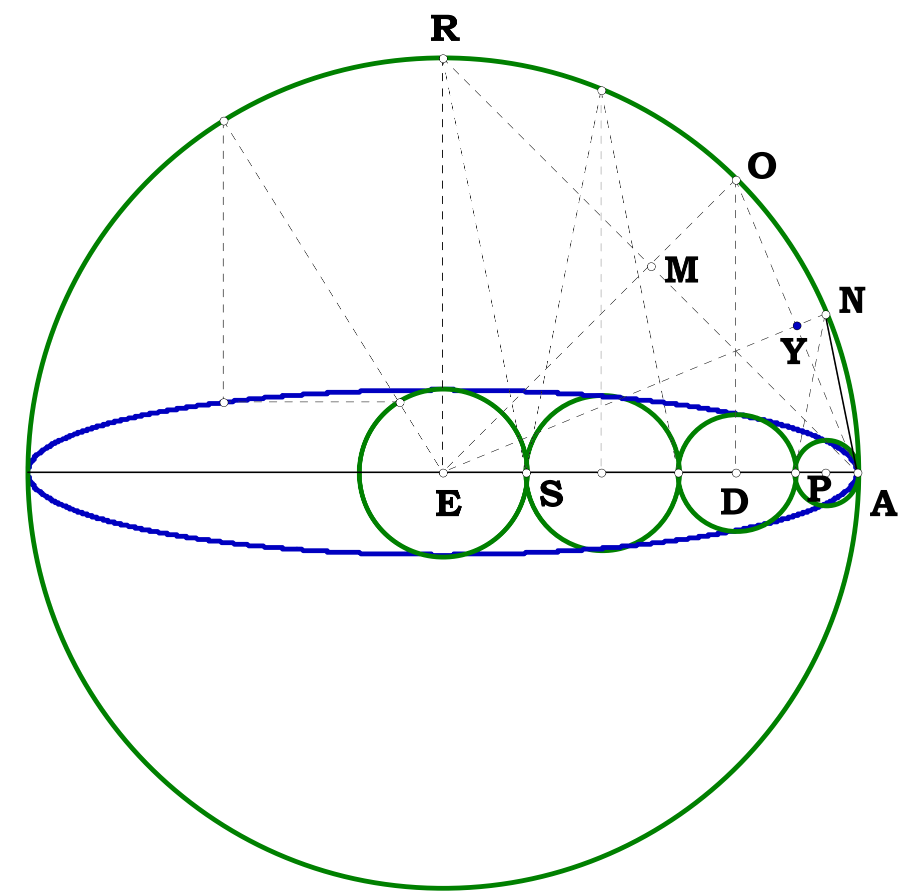
$AR - \frac{\sqrt{2}}{2} = 0 \quad AM - \frac{\sqrt{2}}{4} = 0 \quad MO - \frac{2-\sqrt{2}}{4} = 0 \quad AO - \frac{\sqrt{2-\sqrt{2}}}{2} = 0$

$AY - \frac{\sqrt{2-\sqrt{2}}}{4} = 0 \quad EY - \frac{\sqrt{\sqrt{2}+2}}{4} = 0 \quad NY - \frac{2-\sqrt{\sqrt{2}+2}}{4} = 0$

$AN - \frac{\sqrt{2-\sqrt{\sqrt{2}+2}}}{2} = 0 \quad AP - \frac{2-\sqrt{2+\sqrt{2}}}{4} = 0 \quad NP - \frac{\sqrt{2-\sqrt{2}}}{4} = 0$

$ES - \frac{\sqrt{2-\sqrt{2}}}{2 \cdot \left(\sqrt{\sqrt{2}+2}+2\right)} = 0$

Quadsection:



Unit.

$AG := 1$

0507014

Descriptions.

$AE := \frac{AG}{2}$ $\quad AC := \frac{AE}{2}$ $\quad CG := AG - AC$ $\quad CJ := \sqrt{AC \cdot CG}$

$EL := AE$ $\quad CE := AC$ $\quad JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

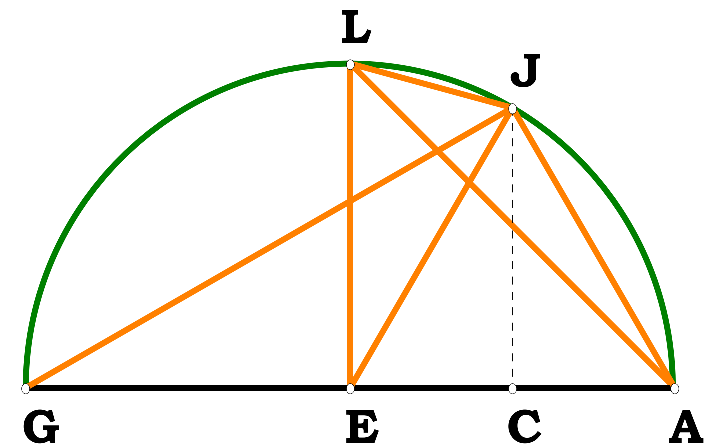
$AJ := \sqrt{AC^2 + CJ^2}$ $\quad GJ := \sqrt{CG^2 + CJ^2}$ $\quad AL := \sqrt{AE^2 + EL^2}$

Definitions.

$AE - \frac{1}{2} = 0$ $\quad AC - \frac{1}{4} = 0$ $\quad CG - \left(1 - \frac{1}{4}\right) = 0$ $\quad CJ - \frac{1}{4} \cdot \sqrt{3} = 0$

$JL - \left(\frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}\right) = 0$ $\quad AJ - \frac{1}{2} = 0$ $\quad GJ - \frac{1}{2} \cdot \sqrt{3} = 0$ $\quad AL - \frac{1}{2} \cdot \sqrt{2} = 0$

Outtake Four: Some Names



0507013

Descriptions.

Unit.

$AL := 1$

Alternate Method: Pentasection Or Irrational Rationals

Irrational **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

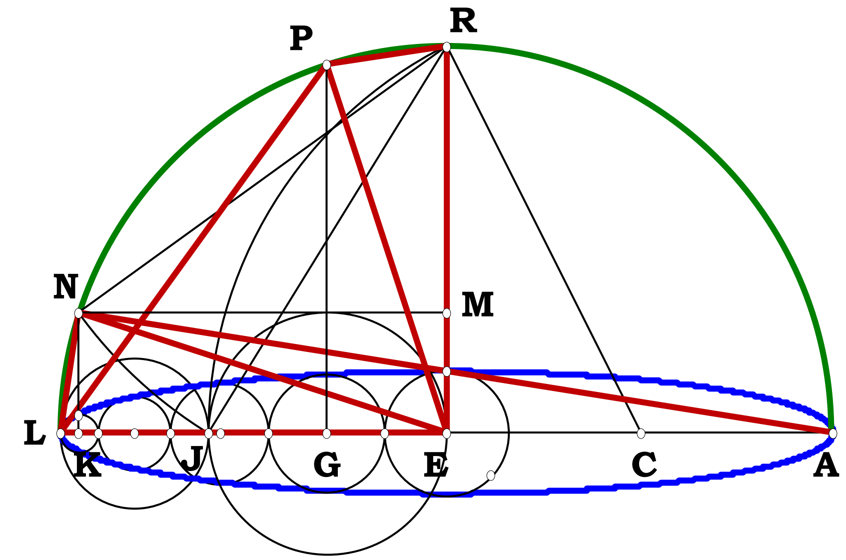
$AE := \frac{AL}{2}$ $\quad AC := \frac{AE}{2}$ $\quad CE := AC$ $\quad ER := AE$ $\quad CR := \sqrt{CE^2 + ER^2}$ $\quad CJ := CR$

$EJ := CJ - CE$ $\quad JR := \sqrt{EJ^2 + ER^2}$ $\quad NR := JR$ $\quad EN := AE$ $\quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM$ $\quad EK := \sqrt{EN^2 - KN^2}$ $\quad EL := AE$ $\quad KL := EL - EK$ $\quad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2}$ $\quad GL := EL - EG$ $\quad AG := AE + EG$ $\quad GP := \sqrt{AG \cdot GL}$

$PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$ $\quad PR - LN = 0$ $\quad AN := \sqrt{AL^2 - LN^2}$



Definitions:

$AE - \frac{1}{2} = 0$ $\quad AC - \frac{1}{4} = 0$ $\quad CR - \frac{1}{4} \cdot \sqrt{5} = 0$ $\quad EJ - \left(\frac{\sqrt{5}}{4} - \frac{1}{4}\right) = 0$

$JR - \frac{\sqrt{2} \cdot \sqrt{5 - \sqrt{5}}}{4} = 0$ $\quad EM - \left(\frac{\sqrt{5}}{8} - \frac{1}{8}\right) = 0$ $\quad EK - \frac{1}{2} \cdot \sqrt{\frac{\sqrt{5}}{8} + \frac{5}{8}} = 0$

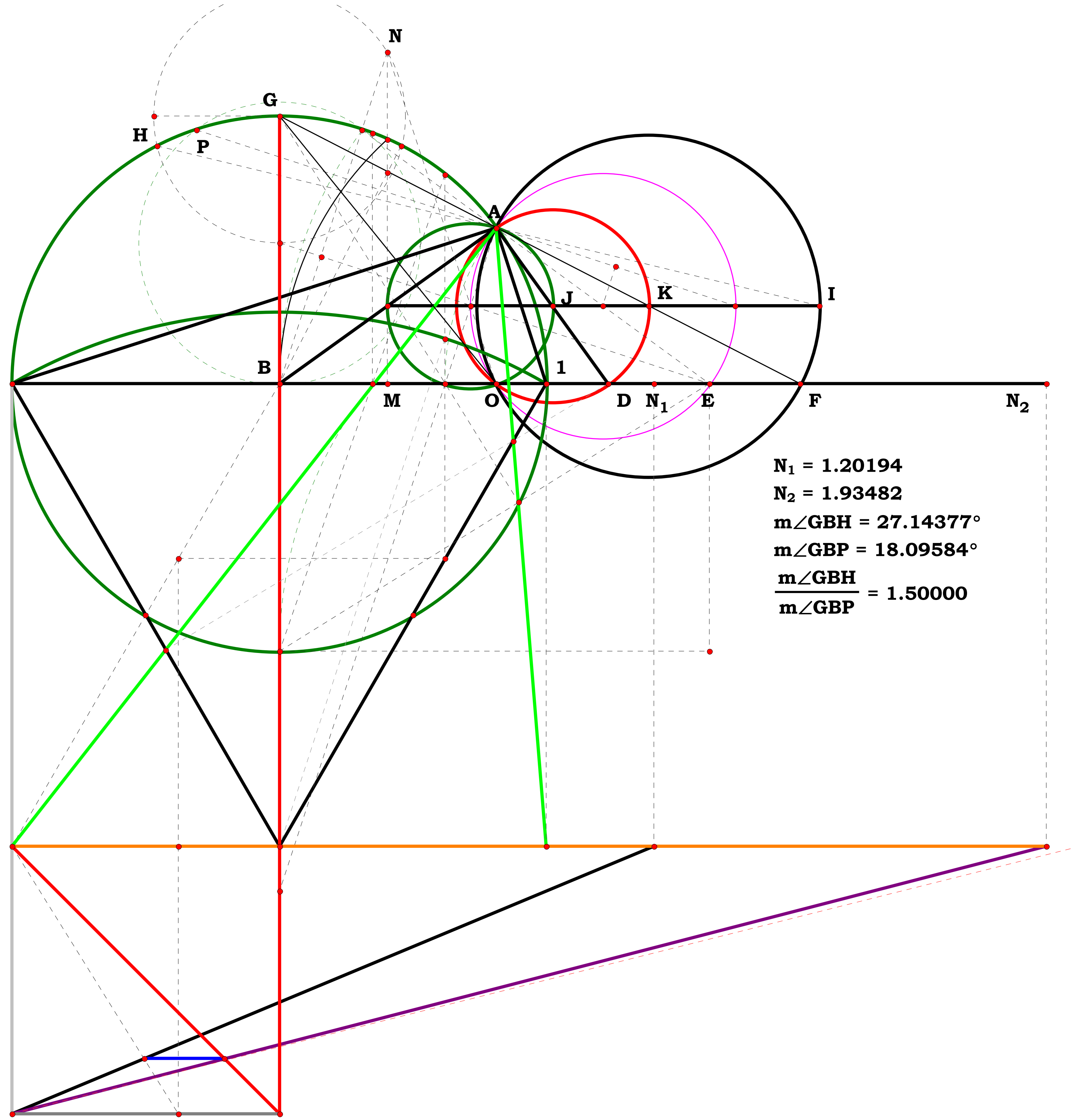
$KL - \left(\frac{1}{2} - \frac{\sqrt{2} \cdot \sqrt{\sqrt{5} + 5}}{8}\right) = 0$ $\quad LN - \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0$ $\quad EG - \left(\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5}\right) = 0$

$GL - \left(\frac{5}{8} - \frac{1}{8} \cdot \sqrt{5}\right) = 0$ $\quad AG - \left(\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5}\right) = 0$ $\quad GP - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} = 0$

$PR - \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0$ $\quad AN - \frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0$

051101 Round Tuits

I had intended to someday write this figure up, it is chock full of fixed intersections which are part of the figure, you just have to find them. Here it is, some eighteen years later. I know that there are a lot of circles in it, but in order to writhe this figure up, I need a round tuit.



Unit.

$AB := 1$

Given.

$N_1 := 1.20194$

$N_2 := 1.93482$

Descriptions.

$AS := \dfrac{N_1 + N_2}{2 \cdot N_2}$ $\quad AS = 0.810608$ $\quad ST := \sqrt{AS \cdot (AB - AS)}$ $\quad AO := \dfrac{AB}{2}$

$OQ := AO \quad OP := AO \quad OS := AS - AO \quad PS := \sqrt{OS^2 + OP^2} \quad CO := \dfrac{OP^2}{OS}$

$AC := CO + AO \quad CW := CO \quad OW := AO \quad OX := \dfrac{CO^2 + OW^2 - CW^2}{2 \cdot CO}$

$FW := 2 \cdot OX \quad CF := CW - FW \quad CI := \dfrac{CF}{2} \quad FO := AO \quad EO := \dfrac{CO^2 + FO^2 - CF^2}{2 \cdot CO}$

$BO := AO \quad BE := BO - EO \quad AE := AO + EO \quad EF := \sqrt{AE \cdot BE} \quad CE := CF$

$DF := \dfrac{FO \cdot EF}{EO} \quad FH := \dfrac{DF}{2} \quad DE := \dfrac{EF \cdot DF}{FO} \quad DO := EO + DE \quad GH := \dfrac{DO}{2}$

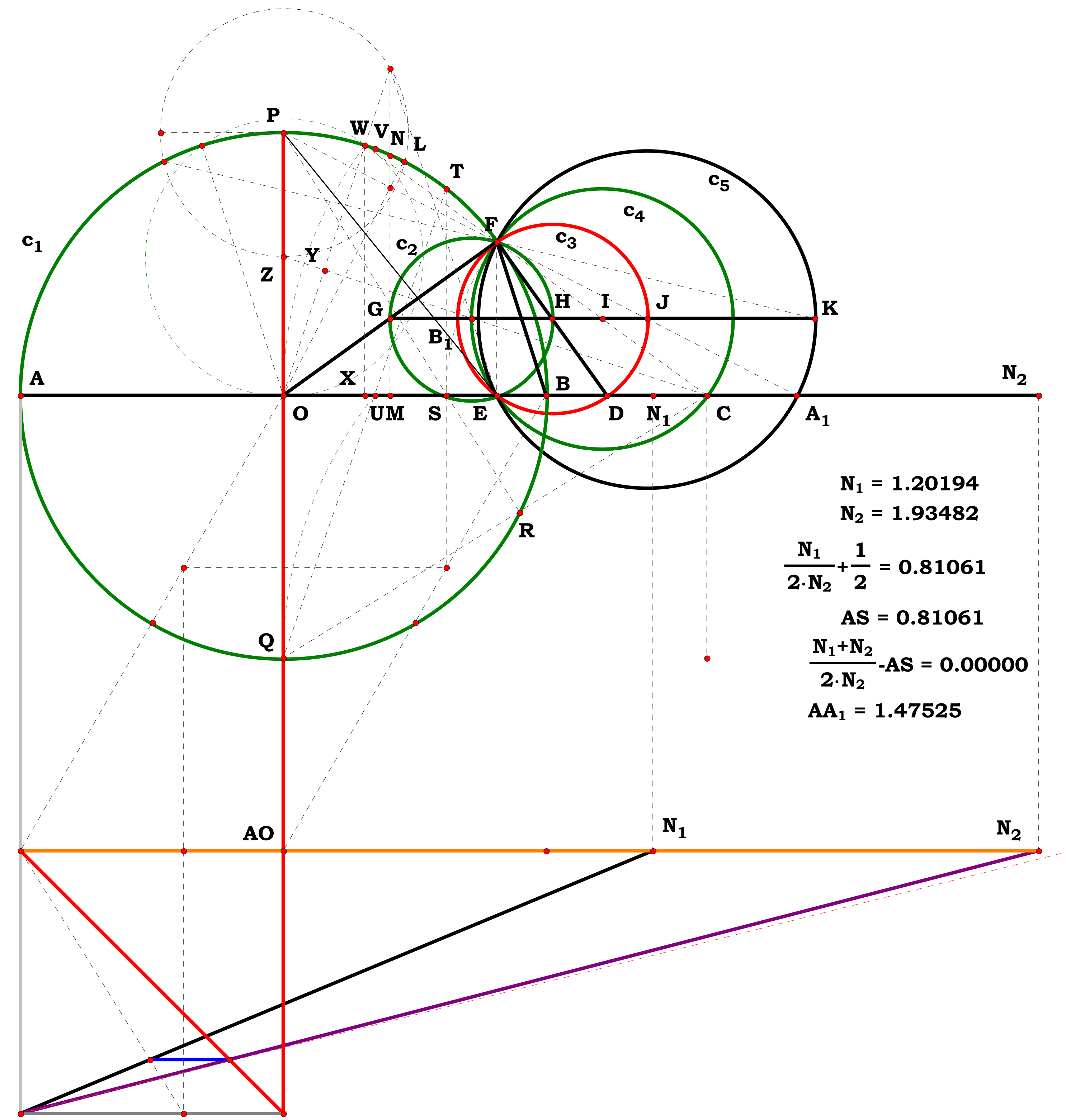
$GB_1 := \dfrac{GH}{2} \quad AA_1 := \dfrac{EO \cdot OP}{OP - EF} + AO \quad AA_1 = 1.475247 \quad OA_1 := AA_1 - AO$

$PA_1 := \sqrt{OA_1{}^2 + OP^2} \quad EA_1 := OA_1 - EO \quad FA_1 := \dfrac{PA_1 \cdot EF}{OP} \quad FJ := \dfrac{FA_1}{2}$

$c_1 := AO \quad c_2 := GB_1 \quad c_3 := FH \quad c_4 := CI \quad c_5 := FJ$

$c_1 = 0.5 \quad c_2 = 0.153949 \quad c_3 = 0.179725$

$c_4 = 0.247133 \quad c_5 = 0.319863$



$c_1 = 0.5 \quad c_2 = 0.153949 \quad c_3 = 0.179725$

$c_4 = 0.247133 \quad c_5 = 0.319863$

Definitions.

$$AS - \frac{N_1 + N_2}{2 \cdot N_2} = 0 \qquad ST - \frac{\sqrt{(N_1 + N_2) \cdot (N_2 - N_1)}}{2 \cdot N_2} = 0 \qquad AO - \frac{1}{2} = 0$$

$$OQ - \frac{1}{2} = 0 \qquad OP - \frac{1}{2} = 0 \qquad OS - \frac{N_1}{2 \cdot N_2} = 0 \qquad PS - \frac{\sqrt{N_1^{\,2} + N_2^{\,2}}}{2 \cdot N_2} = 0$$

$$CO - \frac{N_2}{2 \cdot N_1} = 0 \qquad AC - \frac{N_1 + N_2}{2 \cdot N_1} = 0 \qquad CW - \frac{N_2}{2 \cdot N_1} = 0$$

$$OW - \frac{1}{2} = 0 \qquad OX - \frac{N_1}{4 \cdot N_2} = 0 \qquad FW - \frac{N_1}{2 \cdot N_2} = 0$$

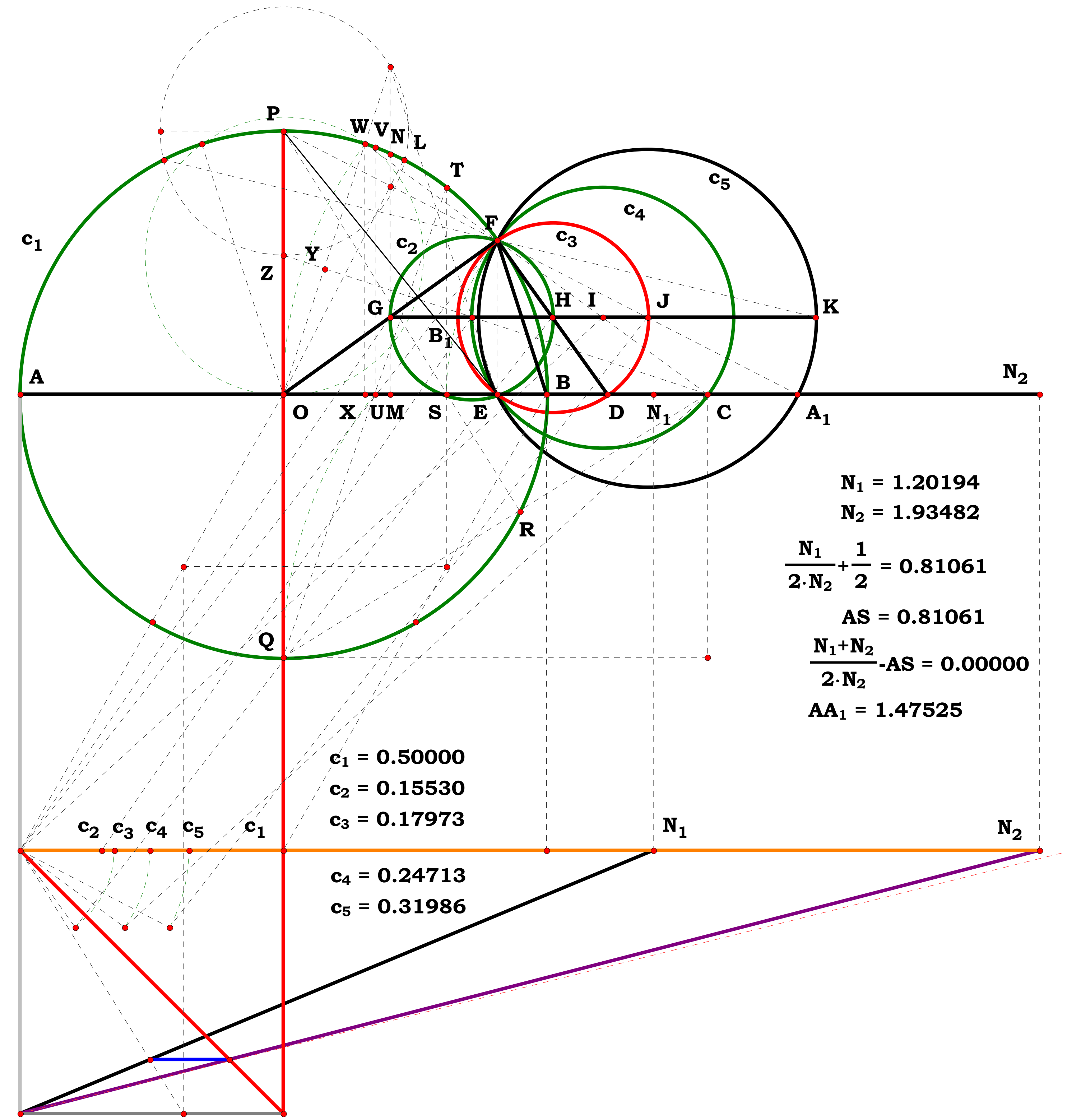
$$CF - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{2 \cdot N_1 \cdot N_2} = 0 \qquad CI - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$$

$$FO - \frac{1}{2} = 0 \qquad EO - \frac{N_1 \cdot \left(3 \cdot N_2^{\,2} - N_1^{\,2}\right)}{4 \cdot N_2^{\,3}} = 0 \qquad BO - \frac{1}{2} = 0$$

$$BE - \frac{(N_1 + 2 \cdot N_2) \cdot (N_1 - N_2)^2}{4 \cdot N_2^{\,3}} = 0 \qquad AE - \frac{(2 \cdot N_2 - N_1) \cdot (N_1 + N_2)^2}{4 \cdot N_2^{\,3}} = 0$$

$$EF - \frac{(N_1 + N_2) \cdot (N_2 - N_1) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)}}{4 \cdot N_2^{\,3}} = 0$$

$$CE - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{2 \cdot N_1 \cdot N_2} = 0$$



$$DF - \frac{(N_1 + N_2) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)} \cdot (N_1 - N_2)}{2 \cdot N_1 \cdot \left(N_1^{\,2} - 3 \cdot N_2^{\,2}\right)} = 0 \qquad FH - \frac{(N_1 + N_2) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)} \cdot (N_1 - N_2)}{4 \cdot N_1 \cdot \left(N_1^{\,2} - 3 \cdot N_2^{\,2}\right)} = 0$$

$$DE - \frac{(N_1 + N_2)^2 \cdot (N_1 - N_2)^2 \cdot (N_1 - 2 \cdot N_2) \cdot (N_1 + 2 \cdot N_2)}{4 \cdot N_1 \cdot N_2^{\,3} \cdot \left(N_1^{\,2} - 3 \cdot N_2^{\,2}\right)} = 0$$

$$DO-\left[\frac{N_2^{\,3}}{N_1\cdot\left(3\cdot N_2^{\,2}-N_1^{\,2}\right)}\right]=0 \qquad GH-\frac{N_2^{\,3}}{2\cdot N_1\cdot\left(3\cdot N_2^{\,2}-N_1^{\,2}\right)}=0$$

$$GB_1-\frac{N_2^{\,3}}{4\cdot N_1\cdot\left(3\cdot N_2^{\,2}-N_1^{\,2}\right)}=0$$

$$AA_1-\frac{\left(N_1+N_2\right)\cdot\left[N_1\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}-N_2\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}-\left(N_1-2\cdot N_2\right)\cdot\left(N_1+N_2\right)\right]}{2\cdot\left(N_1^{\,2}\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}-N_2^{\,2}\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}+2\cdot N_2^{\,3}\right)}=0$$

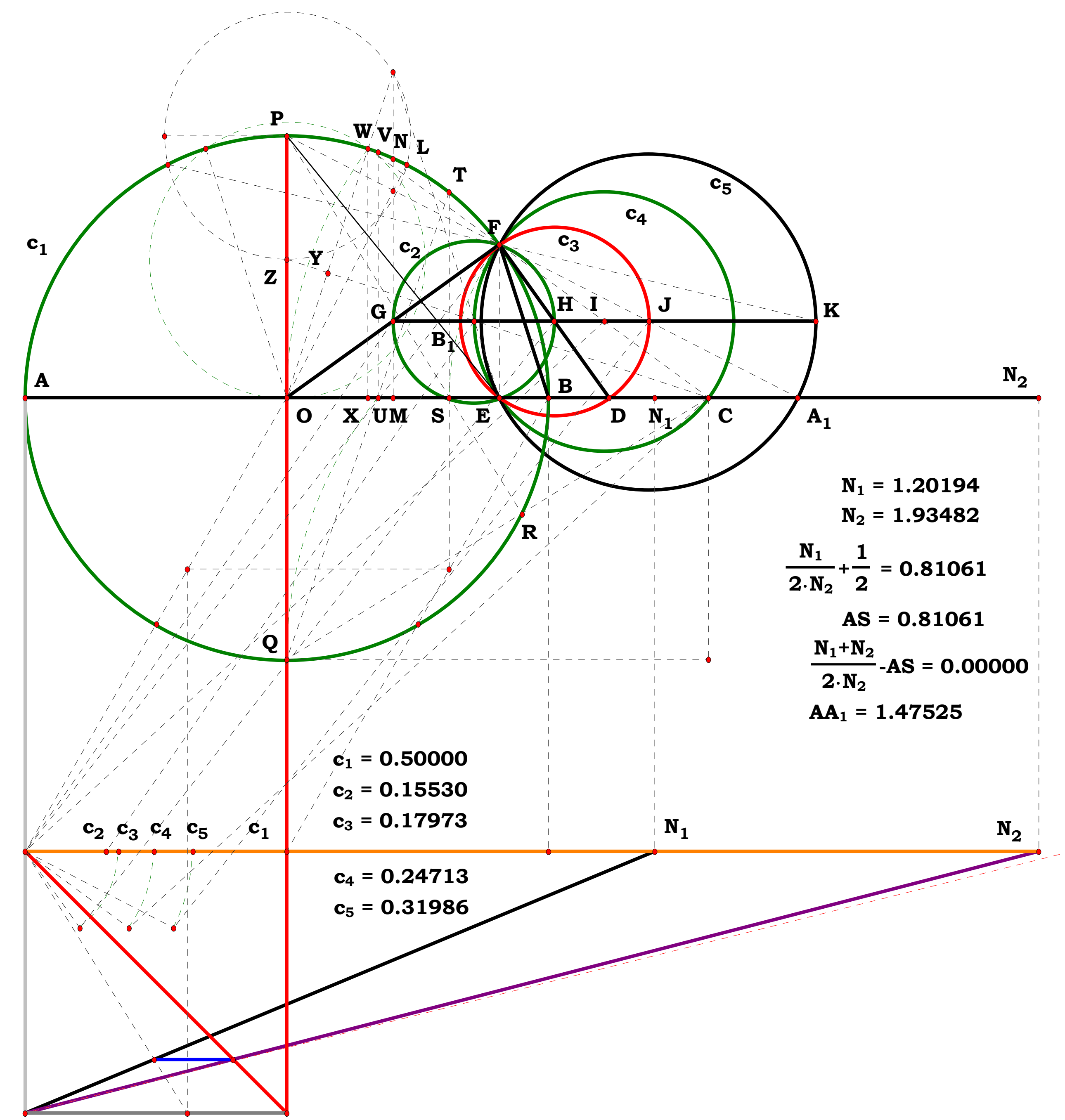
$$OA_1-\frac{N_1\cdot\left(3\cdot N_2^{\,2}-N_1^{\,2}\right)}{2\cdot\left(N_1^{\,2}\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}-N_2^{\,2}\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}+2\cdot N_2^{\,3}\right)}=0$$

$$PA_1-\frac{\sqrt{N_2^{\,3}\cdot\left[\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot\left(N_1-N_2\right)\cdot\left(N_1+N_2\right)+2\cdot N_2^{\,3}\right]}}{\sqrt{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,6}-9\cdot N_1^{\,2}\cdot N_2^{\,4}+8\cdot N_2^{\,6}+4\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot N_2^{\,3}\cdot\left(N_1-N_2\right)\cdot\left(N_1+N_2\right)}}=0$$

$$EA_1-\frac{\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot N_1\cdot\left(N_1-N_2\right)\cdot\left(N_1+N_2\right)\cdot\left(N_1^{\,2}-3\cdot N_2^{\,2}\right)}{4\cdot N_2^{\,3}\cdot\left[\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot\left(N_1-N_2\right)\cdot\left(N_1+N_2\right)+2\cdot N_2^{\,3}\right]}=0$$

$$FA_1-\frac{\left(N_1+N_2\right)\cdot\sqrt{N_2^{\,3}\cdot\left[2\cdot N_2^{\,3}+\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot\left(N_1-N_2\right)\right]}\cdot\sqrt{-\left(N_1-2\cdot N_2\right)\cdot\left(N_1+2\cdot N_2\right)\cdot\left(N_2-N_1\right)}}{2\cdot N_2^{\,3}\cdot\sqrt{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,6}-9\cdot N_1^{\,2}\cdot N_2^{\,4}+8\cdot N_2^{\,6}+4\cdot N_2^{\,3}\cdot\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot\left(N_1-N_2\right)}}=0$$

$$FJ-\frac{\left(N_1+N_2\right)\cdot\sqrt{N_2^{\,3}\cdot\left[2\cdot N_2^{\,3}+\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot\left(N_1-N_2\right)\right]}\cdot\sqrt{-\left(N_1-2\cdot N_2\right)\cdot\left(N_1+2\cdot N_2\right)\cdot\left(N_2-N_1\right)}}{4\cdot N_2^{\,3}\cdot\sqrt{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,6}-9\cdot N_1^{\,2}\cdot N_2^{\,4}+8\cdot N_2^{\,6}+4\cdot N_2^{\,3}\cdot\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^{\,2}-N_1^{\,2}}\cdot\left(N_1-N_2\right)}}=0$$



051301

Descriptions.

Unit.

$BG := 1$

Given.

$N := 9 \quad AG := N$

$AB := AG - BG \quad BF := \frac{BG}{2} \quad AF := AB + BF$

$AN := AF \quad AK := AN \quad FK := BF$

$AE := \frac{AK^2 + AF^2 - FK^2}{2 \cdot AF} \quad AI := AE \quad IK := AK - AI$

$HI := IK \quad AH := AK - (HI + IK) \quad AC := \frac{AE \cdot AH}{AK}$

$BC := AC - AB \quad BE := AE - AB$

Definitions.

$N - 1 - AB = 0 \quad \frac{1}{2} - BF = 0 \quad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0$

$\frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0 \quad \frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0$

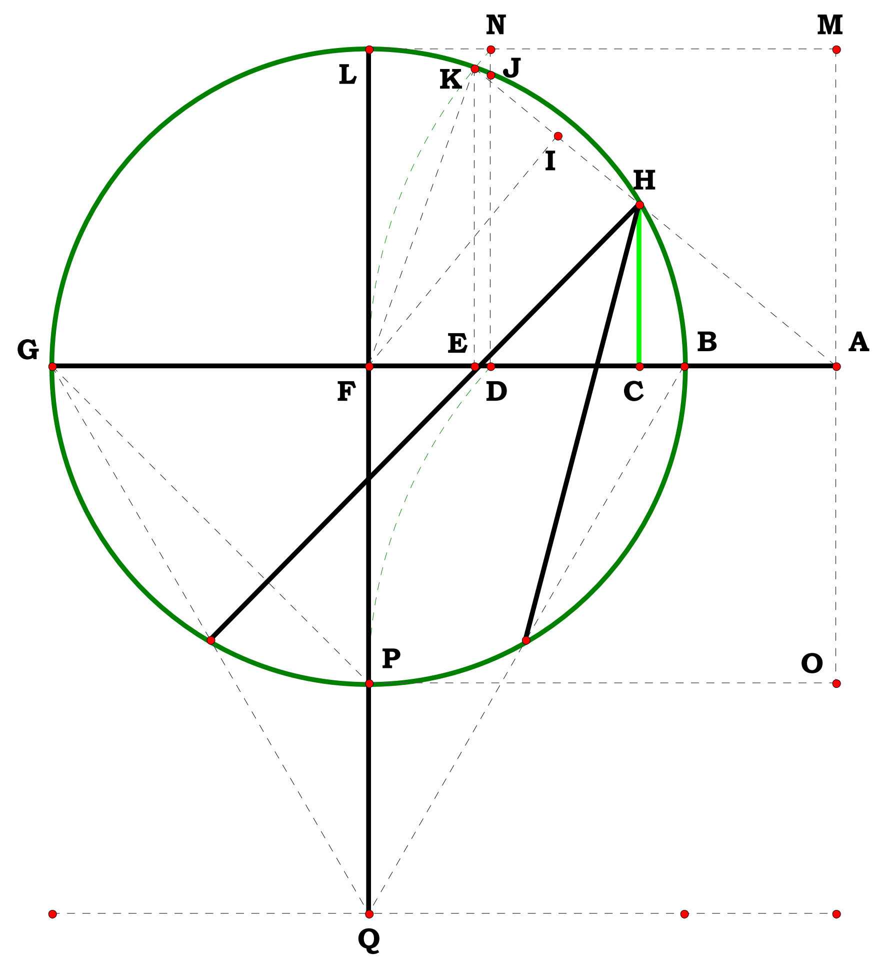
$2 \cdot N \cdot \frac{(N-1)}{(2 \cdot N - 1)} - AH = 0 \quad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N-1) - AC = 0$

$(N-1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \quad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

$\frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

On Trisection

For any given trisection what is the Algebraic names of BC and BE taking BG as unit?



Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

051401

Descriptions.

$BG := AG - AB \quad BO := \frac{BG}{2} \quad NO := BO \quad AO := AB + BO \quad AM := AO$

$AF := \frac{AM^2 + AO^2 - NO^2}{2 \cdot AO} \quad AK := AF \quad KM := AM - AF \quad JK := KM$

$AJ := AM - (JK + KM) \quad AD := \frac{AF \cdot AJ}{AM} \quad BD := AD - AB \quad DG := BG - BD$

$DJ := \sqrt{BD \cdot DG} \quad OT := DJ \quad OP := BO \quad DO := BO - BD \quad PT := OP - OT$

$OU := \frac{DO \cdot OP}{PT} \quad BU := OU - BO \quad AU := BU - AB \quad DU := AU + AD$

$PU := \sqrt{OU^2 + OP^2} \quad JU := \frac{PU \cdot DU}{OU} \quad JP := PU - JU \quad AP := \sqrt{AO^2 + OP^2}$

$AS := \frac{AO^2}{AP} \quad PS := AP - AS \quad HS := PS \quad AH := AP - (PS + HS) \quad CH := \frac{OP \cdot AH}{AP}$

$AC := \frac{AO \cdot AH}{AP} \quad BC := AC - AB \quad CU := BU + BC \quad HU := \sqrt{CU^2 + CH^2}$

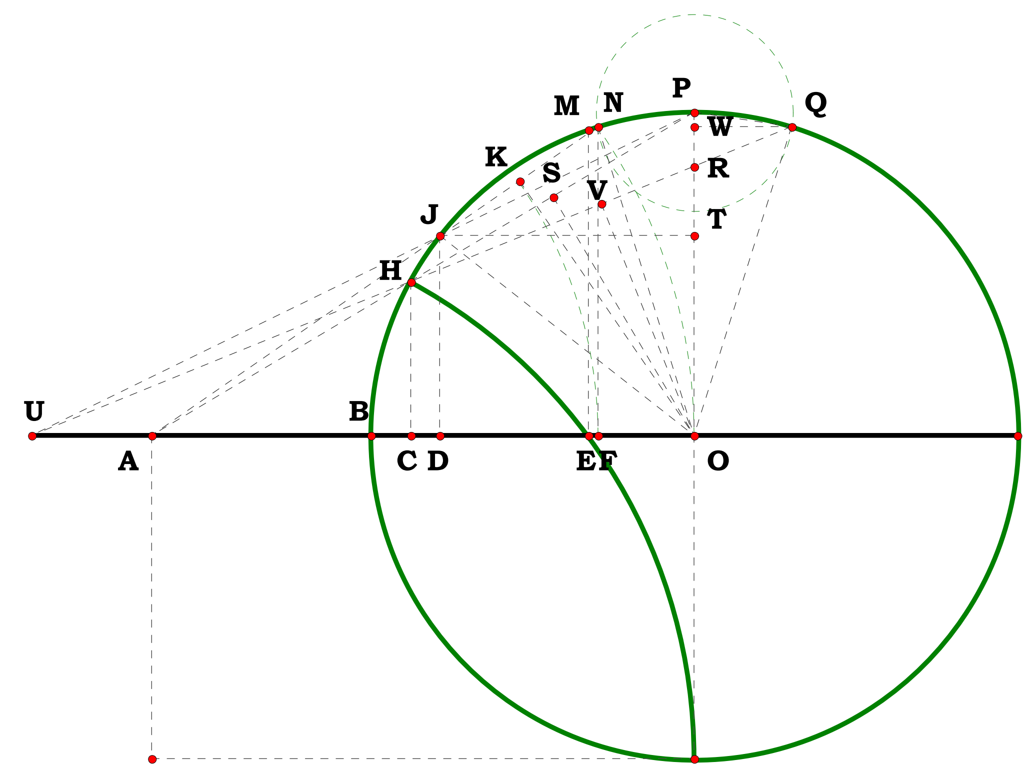
$UV := \frac{CU \cdot OU}{HU} \quad HV := UV - HU \quad QV := HV \quad QU := HU + (HV + QV)$

$OR := \frac{CH \cdot OU}{CU} \quad RU := \frac{HU \cdot OU}{CU} \quad QR := QU - RU \quad RW := \frac{CH \cdot QR}{HU}$

$QW := \frac{CU \cdot QR}{HU} \quad PW := OP - (OR + RW) \quad PQ := \sqrt{PW^2 + QW^2} \quad GU := BO + OU$

For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?

On Trisection



Definitions.

$$BG-(N-1)=0 \quad BO-\frac{N-1}{2}=0 \quad NO-\frac{N-1}{2}=0 \quad AO-\frac{N+1}{2}=0 \quad AM-\frac{N+1}{2}=0$$

$$AF-\frac{N^2+6\cdot N+1}{4\cdot(N+1)}=0 \quad AK-\frac{N^2+6\cdot N+1}{4\cdot(N+1)}=0 \quad KM-\frac{(N-1)^2}{4\cdot(N+1)}=0 \quad JK-\frac{(N-1)^2}{4\cdot(N+1)}=0$$

$$AJ-\frac{2\cdot N}{N+1}=0 \quad AD-\frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3}=0 \quad BD-\frac{(3\cdot N+1)\cdot(N-1)}{(N+1)^3}=0 \quad DG-\frac{N^2\cdot(N+3)\cdot(N-1)}{(N+1)^3}=0$$

$$DJ-\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}=0 \quad OT-\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}=0 \quad OP-\frac{N-1}{2}=0$$

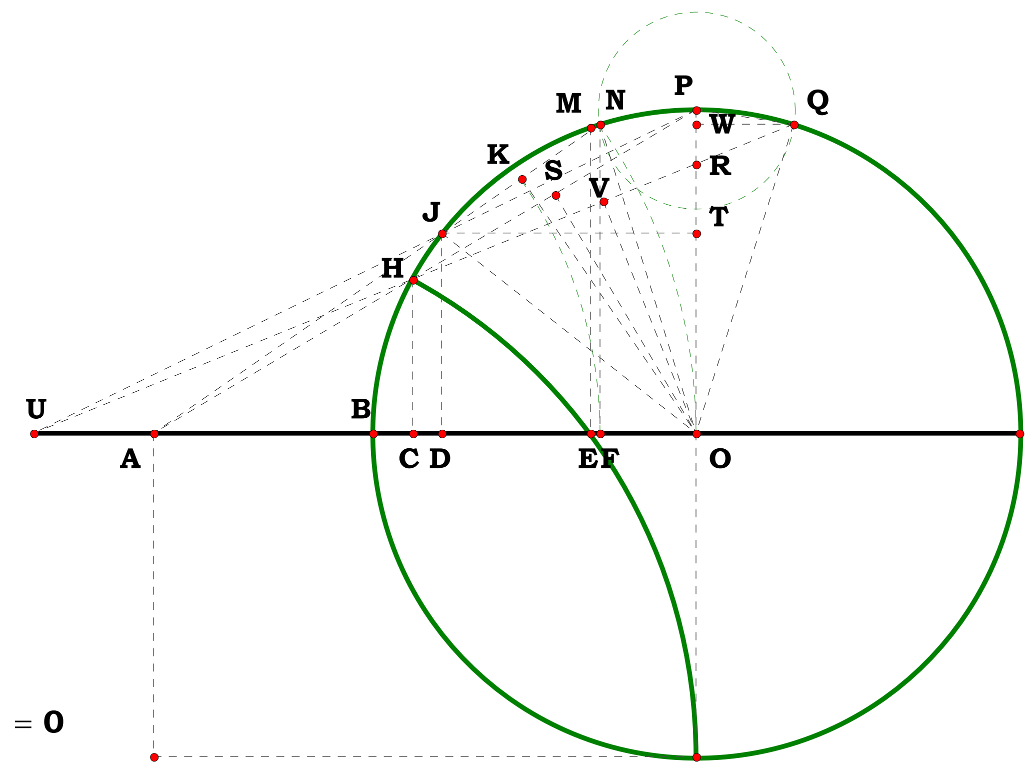
$$DO-\frac{\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{2\cdot(N+1)^3}=0 \quad PT-\frac{(N-1)\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}{2\cdot(N+1)^3}=0$$

$$OU-\frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}=0 \quad BU-\frac{(N-1)\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-3\cdot N-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}=0$$

$$AU-\frac{N\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-6\cdot N-N^2+\sqrt{3\cdot N^2+10\cdot N+3}-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}=0 \quad DU-\frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}=0$$

$$PU-\frac{\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}}{\sqrt{2\cdot\left[N^6+6\cdot N^5+3\cdot N\cdot\left(9\cdot N^3+20\cdot N^2+9\cdot N+2\right)+1\right]-8\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}}}=0$$

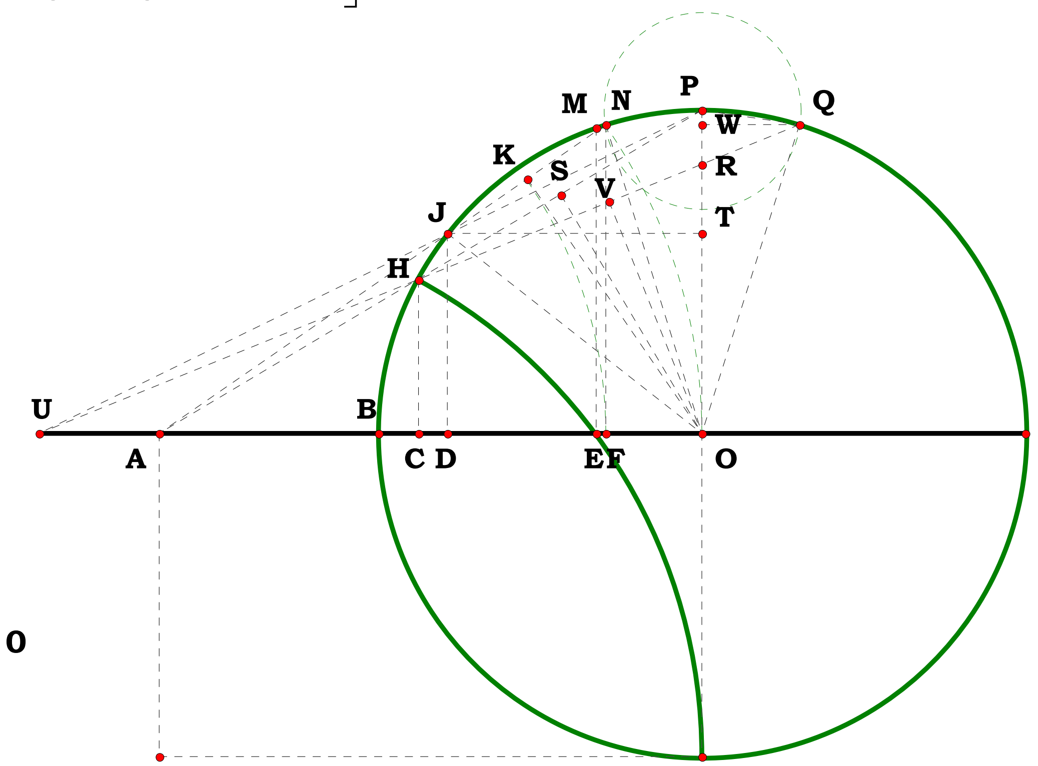
$$JU-\frac{\sqrt{2}\cdot N\cdot\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}{(N+1)^3\cdot\sqrt{6\cdot N^5+N^6+3\cdot N\cdot\left(9\cdot N^3+20\cdot N^2+9\cdot N+2\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1}\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}=0$$



$$JP - \left[\frac{\sqrt{2}\cdot\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left(3\cdot N - 2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}{2\cdot(N+1)^3\cdot\sqrt{\left(N^6+6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}}}\right] = 0$$

$$AP - \frac{\sqrt{N^2+1}}{\sqrt{2}} = 0 \qquad AS - \frac{\sqrt{2}\cdot(N+1)^2}{4\cdot\sqrt{N^2+1}} = 0 \qquad PS - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0 \qquad HS - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0$$

$$AH - \frac{\sqrt{2}\cdot N}{\sqrt{N^2+1}} = 0 \qquad CH - \frac{N\cdot(N-1)}{N^2+1} = 0 \qquad AC - \frac{N\cdot(N+1)}{N^2+1} = 0 \qquad BC - \frac{N-1}{N^2+1} = 0$$



$$CU - \frac{N\cdot(N-1)^2\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-2\cdot N+\sqrt{3\cdot N^2+10\cdot N+3}\right)}{\left(N^2+1\right)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$

$$HU - \frac{2\cdot N\cdot(N-1)\cdot\sqrt{(N+1)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}}{\sqrt{\left(N^2+1\right)\cdot\left[N^6+\left(6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}} = 0$$

$$UV - \frac{(N-1)^3\cdot\sqrt{\left(N^2+1\right)\cdot\left[\left(N^6+6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}\cdot\left[(N+1)\cdot\sqrt{3\cdot N^2+10\cdot N+3}-2\cdot N\right]\cdot\left(N^2+4\cdot N+1\right)}{4\cdot\sqrt{(N+1)\cdot\left[(N+1)^3-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}\cdot\left(N^2+1\right)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{10\cdot N+3\cdot N^2+3}+3\cdot N^2+N^3+1\right)^2} = 0$$

$$HV - (UV - HU) = 0$$

$$QV - HV = 0 \qquad QU - [HU + (HV + QV)] = 0$$

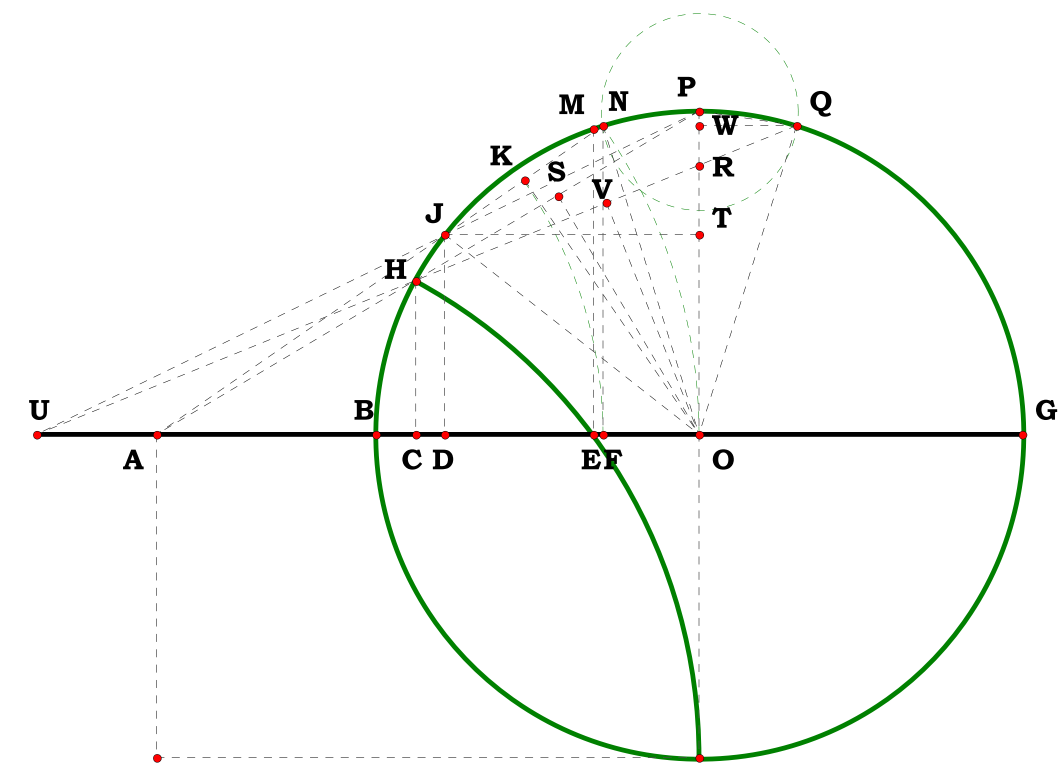
$$OR - \frac{CH \cdot OU}{CU} = 0 \qquad RU - \frac{HU \cdot OU}{CU} = 0$$

$$QR - (QU - RU) = 0 \qquad RW - \frac{CH \cdot QR}{HU} = 0 \qquad QW - \frac{CU \cdot QR}{HU} = 0$$

$$PW - [OP - (OR + RW)] = 0$$

$$PQ - \frac{(N-1) \cdot \sqrt{14 \cdot N + 32 \cdot N^2 + 14 \cdot N^3 + 2 \cdot N^4 - (N+1) \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} \cdot \left(N^2 + 6 \cdot N + 1\right) + 2}}{2 \cdot \sqrt{(N+1) \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]}} = 0$$

$$GU - \left[\frac{N \cdot (N-1) \cdot \left(3 \cdot N + N^2 - \sqrt{3 \cdot N^2 + 10 \cdot N + 3}\right)}{3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1}\right] = 0$$



Unit.

$AB := 1$

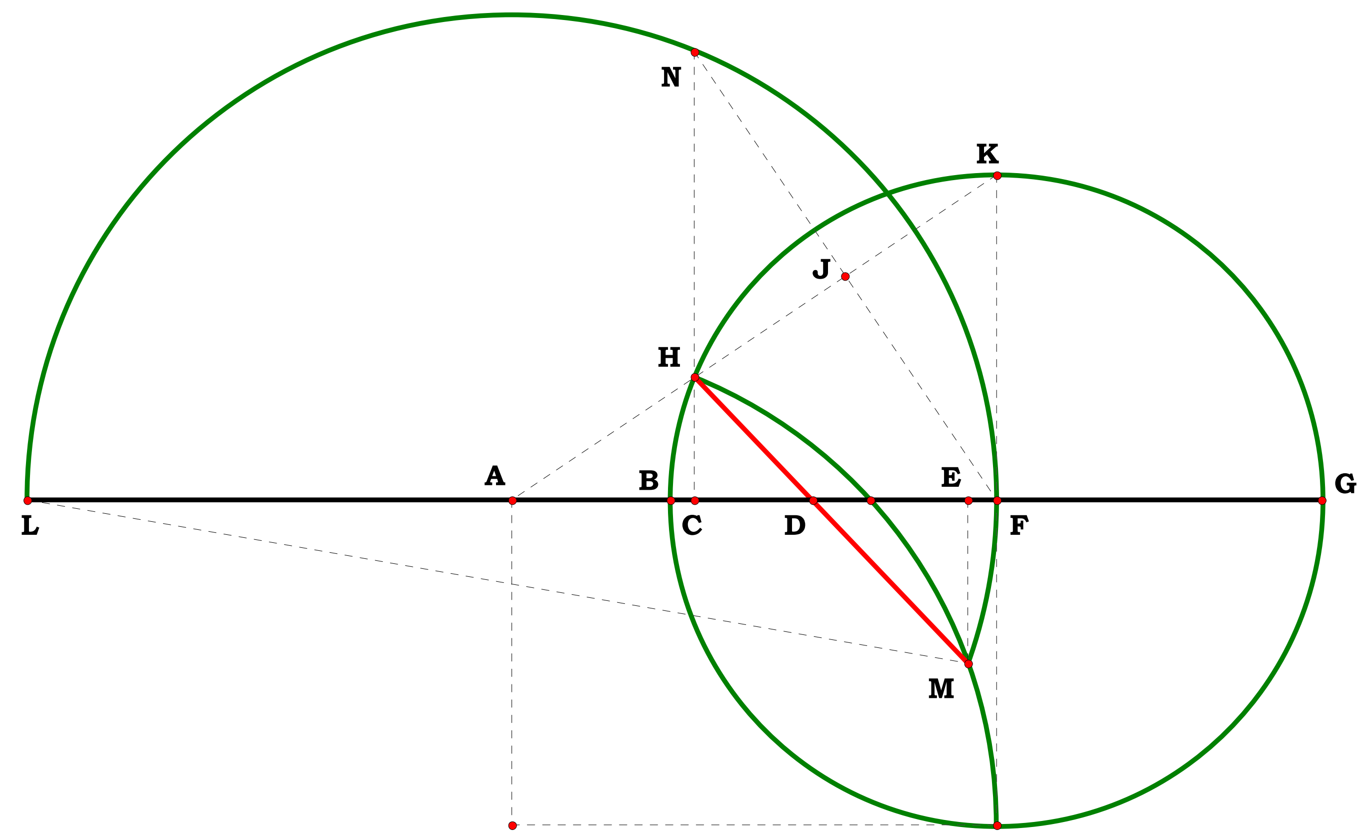
Given.

$N := 5 \quad AG := N$

052201A

Segment DF And HM

Given AB and AG, what is HM and DF?



Descriptions.

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FK := BF \quad AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2} \quad AJ := \frac{AF^2}{AK} \quad JK := AK - AJ \quad HJ := JK$

$AH := AK - (JK + HJ) \quad AC := \frac{AF \cdot AH}{AK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot AF$

$EF := \frac{FL - \sqrt{FL^2 - 4 \cdot EM^2}}{2} \quad AE := AF - EF \quad CE := AE - AC \quad CH := \frac{FK \cdot AH}{AK}$

$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$

Definitions.

$BG - (N-1) = 0 \quad BF - \frac{N-1}{2} = 0 \quad FK - \frac{N-1}{2} = 0 \quad AF - \frac{N+1}{2} = 0$

$AK - \frac{\sqrt{N^2+1}}{\sqrt{2}} = 0 \quad AJ - \frac{\sqrt{2}\cdot(N+1)^2}{4\cdot\sqrt{N^2+1}} = 0 \quad JK - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0 \quad HJ - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0 \quad AH - \frac{\sqrt{2}\cdot N}{\sqrt{N^2+1}} = 0 \quad AC - \frac{N\cdot(N+1)}{N^2+1} = 0$

$EM - \frac{N-1}{4} = 0 \quad FL - (N+1) \quad EF - \frac{2\cdot N - \sqrt{(N+3)\cdot(3\cdot N+1)} + 2}{4} = 0 \quad AE - \frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0 \quad CE - \frac{\left(N^2+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} - 4\cdot N\cdot(N+1)}{4\cdot\left(N^2+1\right)} = 0$

$CH - \frac{N\cdot(N-1)}{N^2+1} = 0 \quad HM - \frac{\sqrt{(N+1)\cdot\left(3\cdot N - 2\cdot N\cdot\sqrt{3\cdot N^2 + 10\cdot N + 3} + 3\cdot N^2 + N^3 + 1\right)}}{2\cdot\sqrt{N^2+1}} = 0 \quad DE - \frac{\left(N^2+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} - \left(4\cdot N^2 + 4\cdot N\right)}{4\cdot\left(N^2 + 4\cdot N + 1\right)} = 0$

$DF - \frac{3\cdot N - 2\cdot N\cdot\sqrt{3\cdot N^2 + 10\cdot N + 3} + 3\cdot N^2 + N^3 + 1}{2\cdot\left(N^2 + 4\cdot N + 1\right)} = 0$

Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

052201B

Descriptions.

$BG := AG - AB \quad BF := \frac{AB}{2} \quad FK := BF \quad GF := AG - BF$

$GK := \sqrt{GF^2 + FK^2} \quad GJ := \frac{GF^2}{GK} \quad JK := GK - GJ \quad HJ := JK$

$GH := GK - (JK + HJ) \quad CG := \frac{GF \cdot GH}{GK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot GF$

$EF := \frac{FL - \sqrt{FL^2 - 4 \cdot EM^2}}{2} \quad GE := GF - EF \quad CE := GE - CG \quad CH := \frac{FK \cdot GH}{GK}$

$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$

Definitions.

$BG - (N-1) = 0 \quad BF - \frac{1}{2} = 0 \quad FK - \frac{1}{2} = 0 \quad GF - \frac{2 \cdot N - 1}{2} = 0$

$GK - \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{\sqrt{2}} = 0 \quad GJ - \frac{\sqrt{2} \cdot (2 \cdot N - 1)^2}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad JK - \frac{\sqrt{2}}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0$

$HJ - \frac{\sqrt{2}}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad GH - \frac{\sqrt{2} \cdot N \cdot (N-1)}{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad CG - \frac{N \cdot (N-1) \cdot (2 \cdot N - 1)}{2 \cdot N^2 - 2 \cdot N + 1} = 0$

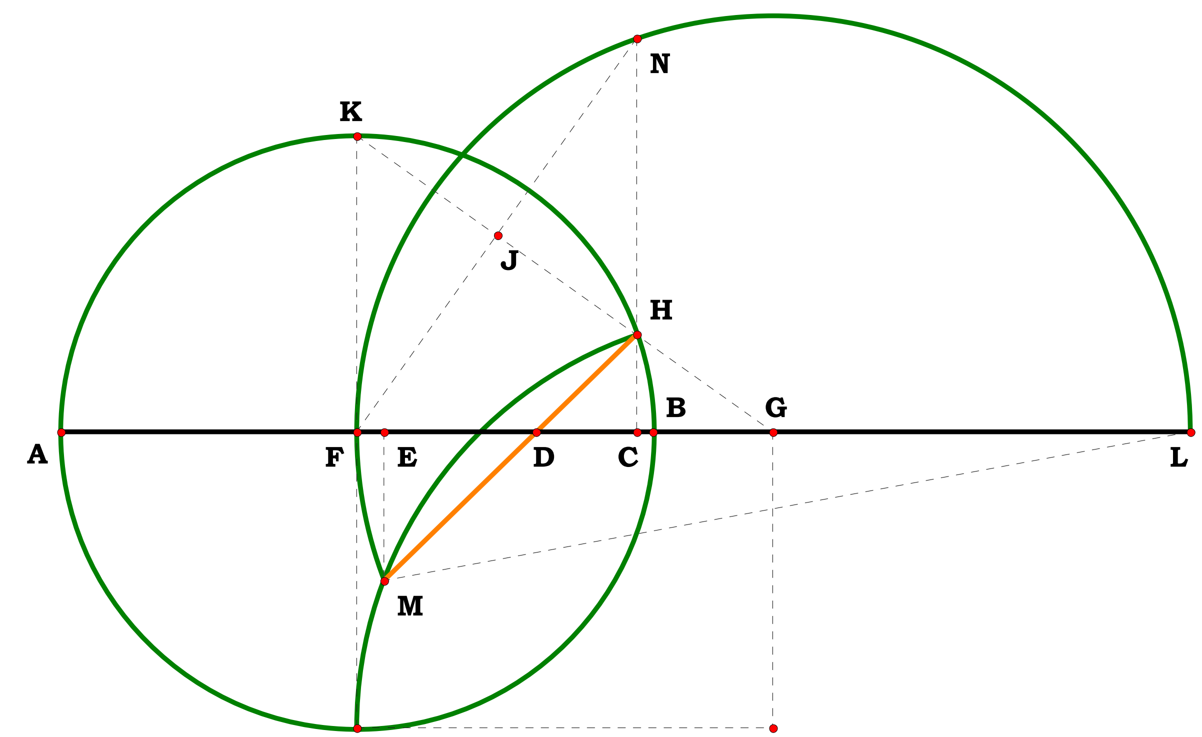
$EM - \frac{1}{4} = 0 \quad FL - (2 \cdot N - 1) = 0 \quad EF - \frac{4 \cdot N - \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} - 2}{4} = 0 \quad GE - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4} = 0 \quad CE - \frac{\left(2 \cdot N^2 - 2 \cdot N + 1\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} - 4 \cdot N \cdot (2 \cdot N - 1) \cdot (N-1)}{4 \cdot \left(2 \cdot N^2 - 2 \cdot N + 1\right)} = 0$

$CH - \frac{N \cdot (N-1)}{2 \cdot N^2 - 2 \cdot N + 1} = 0 \quad HM - \frac{\sqrt{(2 \cdot N - 1) \cdot \left[\left(2 \cdot N - 2 \cdot N^2\right) \cdot \sqrt{16 \cdot N^2 - 16 \cdot N + 3} + (2 \cdot N - 1)^3\right]}}{2 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad DE - \frac{4 \cdot N \cdot (2 \cdot N - 1) \cdot (1 - N) - \left(2 \cdot N - 2 \cdot N^2 - 1\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4 \cdot \left(6 \cdot N^2 - 6 \cdot N + 1\right)} = 0$

$DF - \frac{\left(2 \cdot N - 2 \cdot N^2\right) \cdot \sqrt{16 \cdot N^2 - 16 \cdot N + 3} + (2 \cdot N - 1)^3}{2 \cdot \left(6 \cdot N^2 - 6 \cdot N + 1\right)} = 0$

Segment DF And HM

Given AB and AG, what is HM and DF?



Unit.

$AB := 1$

Given.

$N := 5 \qquad AH := N$

Point of Intersection

052701A

Descriptions.

$BH := AH - AB \qquad BG := \frac{BH}{2} \qquad GN := BG \qquad GM := BG$

$GP := BG \qquad AG := AB + BG \qquad AM := AG \qquad FG := \frac{GM^2 + AG^2 - AM^2}{2 \cdot AG}$

$AF := AG - FG \qquad AL := AF \qquad LM := AM - AL \qquad KL := LM$

$AK := AM - (LM + KL) \qquad AC := \frac{AF \cdot AK}{AM} \qquad CK := \sqrt{AK^2 - AC^2}$

$CG := AG - AC \qquad DG := \frac{CG \cdot GP}{(GP + CK)}$

Definitions.

$BH - (N-1) = 0 \qquad BG - \frac{(N-1)}{2} = 0 \qquad GN - \frac{(N-1)}{2} = 0 \qquad GM - \frac{(N-1)}{2} = 0$

$GP - \frac{(N-1)}{2} = 0 \qquad AG - \frac{N+1}{2} = 0 \qquad AM - \frac{N+1}{2} = 0 \qquad FG - \frac{(N-1)^2}{4 \cdot (N+1)} = 0$

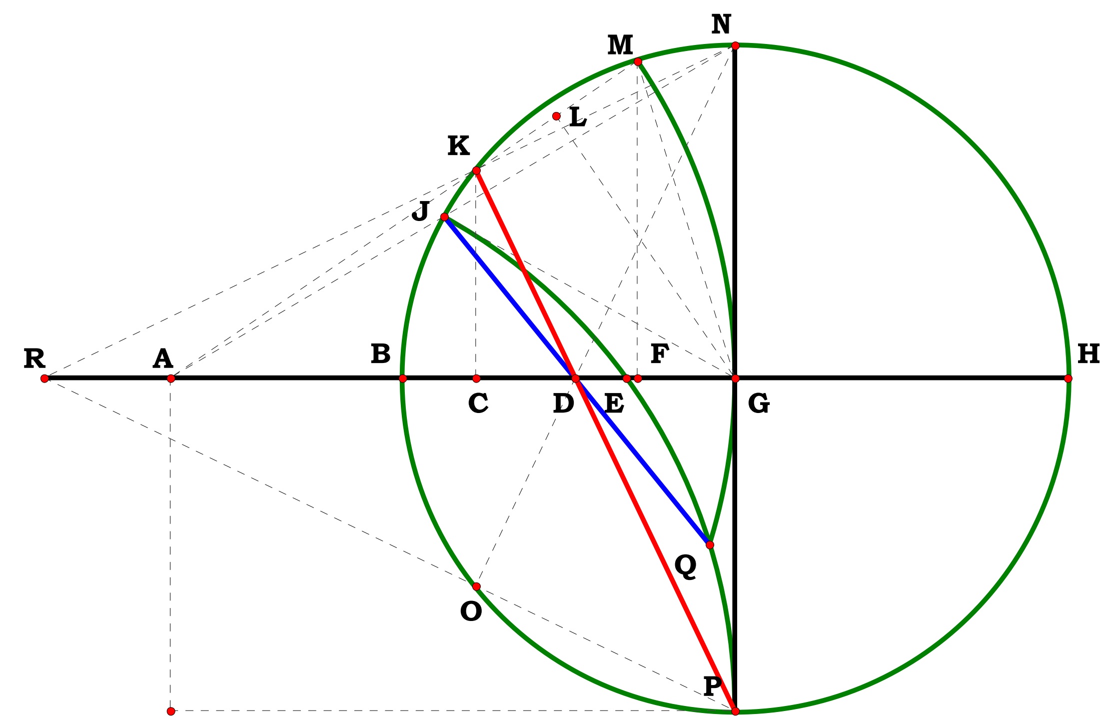
$AF - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad AL - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad LM - \frac{(N-1)^2}{4 \cdot (N+1)} = 0$

$KL - \frac{(N-1)^2}{4 \cdot (N+1)} = 0 \qquad AK - \frac{2 \cdot N}{N+1} = 0 \qquad AC - \frac{N \cdot \left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} = 0$

$CK - \frac{N \cdot (N-1) \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}}{(N+1)^3} = 0 \qquad CG - \frac{\left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{2 \cdot (N+1)^3} = 0$

$$DG - \frac{(N-1)^2 \cdot \left(N^2 + 4 \cdot N + 1\right)}{2 \cdot \left[(N+1)^3 + 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}\right]} = 0$$

Do KP and JQ intersect at D?



This is the other way to look at the figure I work on for trisection.

$m\angle ABC = 9.59307°$

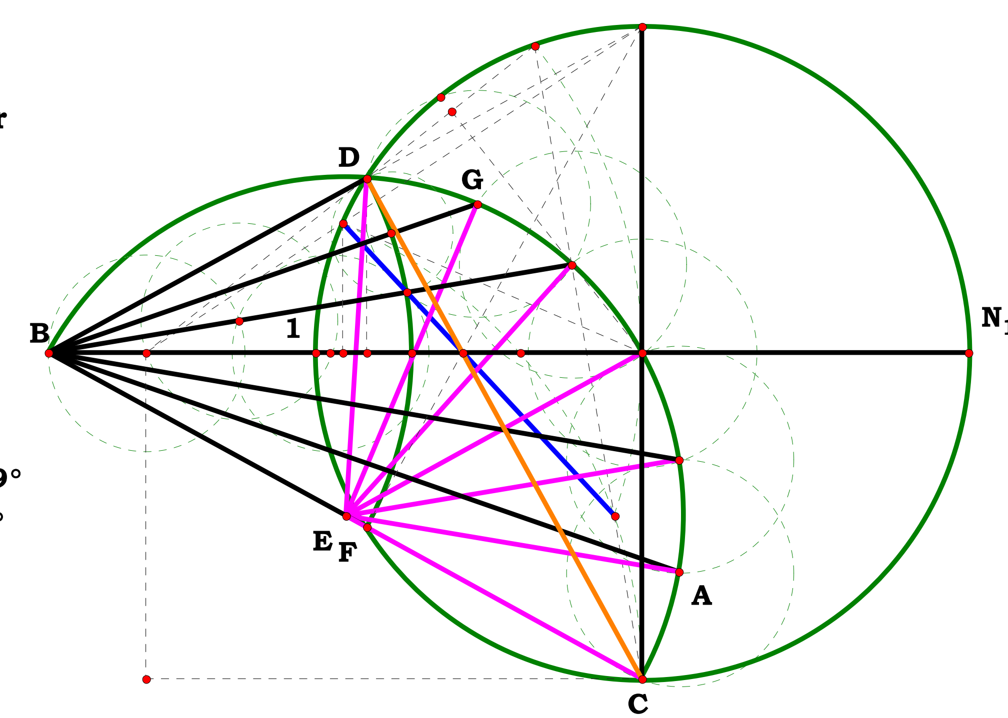
$m\angle DBF = 57.55844°$

$\frac{m\angle DBF}{m\angle ABC} = 6.00000$

$m\angle DEC = 115.11689°$

$m\angle DEG = 19.18615°$

$\frac{m\angle DEC}{m\angle DEG} = 6.00000$



052701A

Unit.

$AB := 1$

Given.

$N := 1.26810 \quad AH := N$

Point of Intersection

Descriptions.

$BH := AH - AB \quad BG := \dfrac{AB}{2} \quad GN := BG \quad GM := BG$

$GP := BG \quad GH := AH - BG \quad HM := GH \quad FG := \dfrac{GM^2 + GH^2 - HM^2}{2 \cdot GH}$

$FH := GH - FG \quad HL := FH \quad LM := HM - HL \quad KL := LM$

$HK := HM - (LM + KL) \quad CH := \dfrac{FH \cdot HK}{HM} \quad CK := \sqrt{HK^2 - CH^2}$

$CG := GH - CH \quad DG := \dfrac{CG \cdot GP}{(GP + CK)}$

Definitions.

$BH - (N-1) = 0 \quad BG - \dfrac{1}{2} = 0 \quad GN - \dfrac{1}{2} = 0 \quad GM - \dfrac{1}{2} = 0$

$GP - \dfrac{1}{2} = 0 \quad GH - \dfrac{2 \cdot N - 1}{2} = 0 \quad HM - \dfrac{2 \cdot N - 1}{2} = 0$

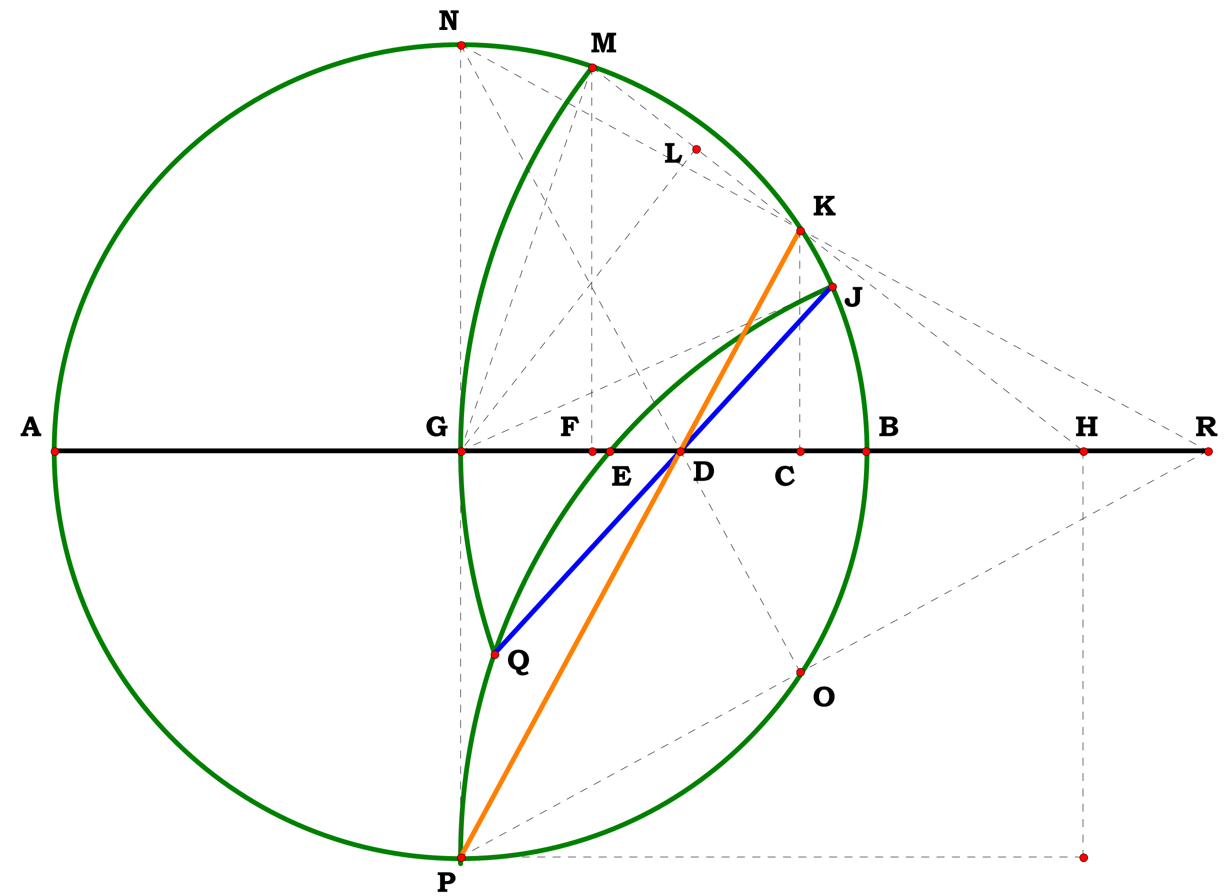
$FG - \dfrac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad FH - \dfrac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0 \quad HL - \dfrac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0$

$LM - \dfrac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad KL - \dfrac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad HK - \dfrac{2 \cdot N \cdot (N-1)}{2 \cdot N - 1} = 0$

$CH - \dfrac{N \cdot (N-1) \cdot \left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} = 0 \quad CK - \dfrac{N \cdot (N-1) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{(2 \cdot N - 1)^3} = 0$

$CG - \dfrac{6 \cdot N^2 - 6 \cdot N + 1}{2 \cdot (2 \cdot N - 1)^3} = 0 \quad DG - \dfrac{6 \cdot N^2 - 6 \cdot N + 1}{2 \cdot \left[\left(2 \cdot N^2 - 2 \cdot N\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} + (2 \cdot N - 1)^3\right]} = 0$

Do KP and JQ intersect at D?



No, I am not drawing this the other way.

$m\angle ABC = 9.59307°$

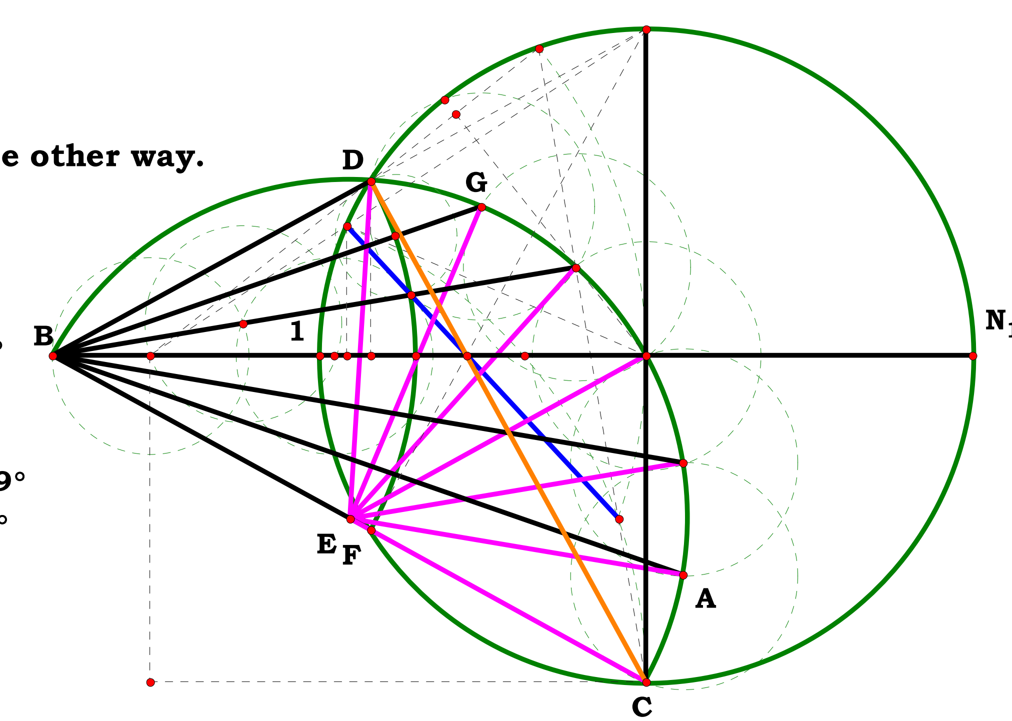
$m\angle DBF = 57.55844°$

$\dfrac{m\angle DBF}{m\angle ABC} = 6.00000$

$m\angle DEC = 115.11689°$

$m\angle DEG = 19.18615°$

$\dfrac{m\angle DEC}{m\angle DEG} = 6.00000$



Unit.

$AB := 1$

Given.

$N_1 := 5 \quad AJ := N_1$

What is AV and ST?

Given any angle BHP, the unit which defines it will lay between W and V.

0530011A

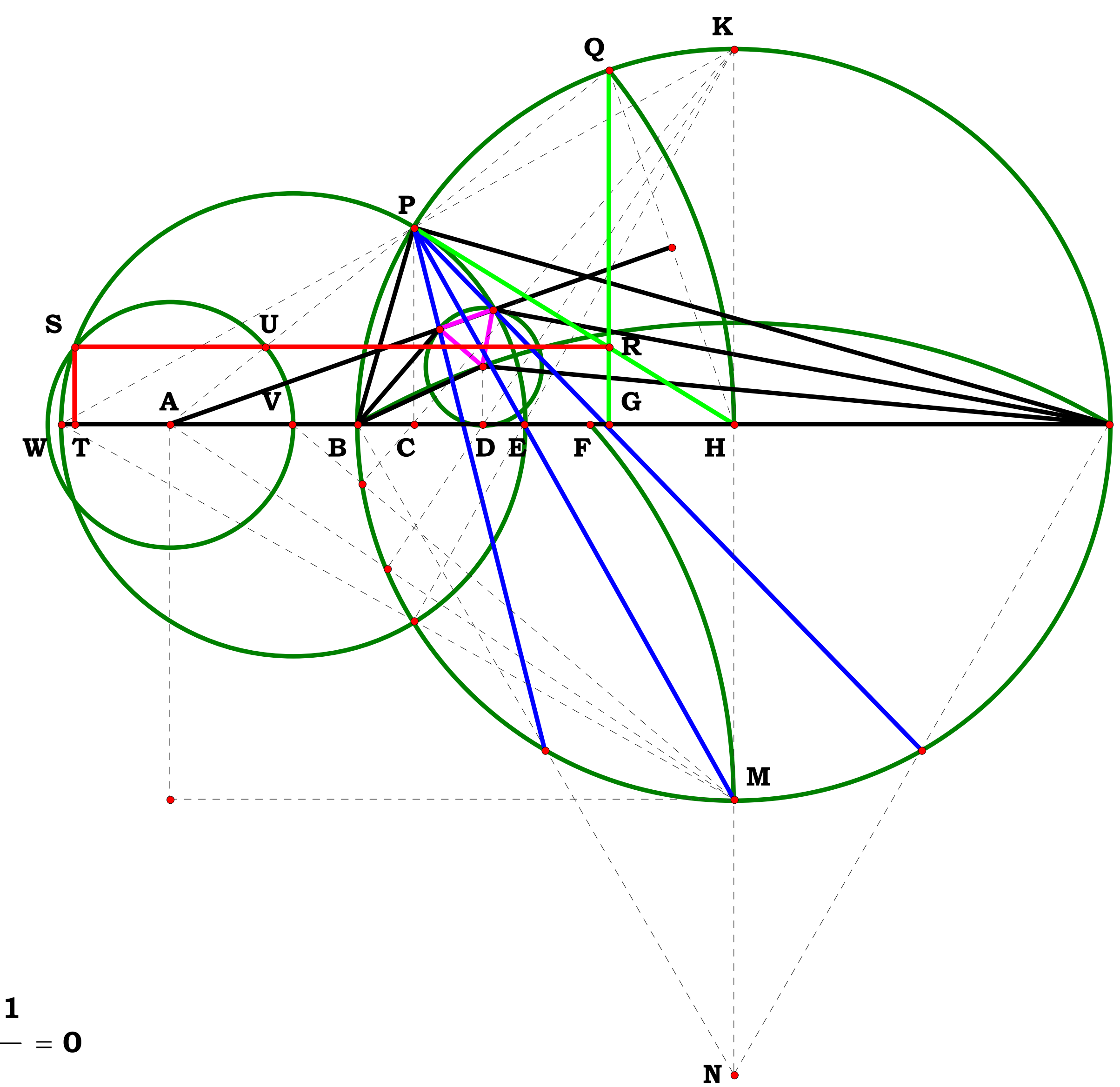
Descriptions.

$BJ := AJ - AB \quad BH := \frac{BJ}{2} \quad AH := AB + BH \quad AQ := AH$

$PQ := \frac{BH^2}{AQ} \quad HM := BH \quad AP := AQ - PQ \quad AC := \frac{AP^2 + AH^2 - BH^2}{2 \cdot AH}$

$CP := \sqrt{AP^2 - AC^2} \quad HK := BH \quad CH := AH - AC \quad CK := \sqrt{CH^2 + HK^2}$

$HV := \frac{HK \cdot HM}{CH} \quad AV := AH - HV \quad ST := \frac{CP \cdot AV}{AP}$



Definitions.

$AJ - N_1 = 0 \quad BJ - (N_1 - 1) = 0 \quad BH - \frac{N_1 - 1}{2} = 0 \quad AH - \frac{N_1 + 1}{2} = 0$

$AQ - \frac{N_1 + 1}{2} = 0 \quad PQ - \frac{(N_1 - 1)^2}{2 \cdot (N_1 + 1)} = 0 \quad HM - \frac{N_1 - 1}{2} = 0 \quad AP - \frac{2 \cdot N_1}{N_1 + 1} = 0$

$AC - \frac{N_1 \cdot (N_1^2 + 6 \cdot N_1 + 1)}{(N_1 + 1)^3} = 0 \quad CP - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)}}{(N_1 + 1)^3} = 0 \quad HK - \frac{N_1 - 1}{2} = 0$

$CH - \frac{(N_1^2 + 4 \cdot N_1 + 1) \cdot (N_1 - 1)^2}{2 \cdot (N_1 + 1)^3} = 0 \quad CK - \frac{(N_1 - 1) \cdot \sqrt{N_1^6 + 6 \cdot N_1^5 + 9 \cdot N_1^4 + 9 \cdot N_1^2 + 6 \cdot N_1 + 1}}{\sqrt{2} \cdot (N_1 + 1)^3} = 0$

$HV - \frac{(N_1 + 1)^3}{2 \cdot (N_1^2 + 4 \cdot N_1 + 1)} = 0 \quad AV - \frac{N_1 \cdot (N_1 + 1)}{N_1^2 + 4 \cdot N_1 + 1} = 0 \quad ST - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)}}{2 \cdot (N_1 + 1) \cdot (N_1^2 + 4 \cdot N_1 + 1)} = 0 \quad AV - \frac{(AB + BJ) \cdot (AB + BJ + AB)}{AB^2 + 2 \cdot AB \cdot BJ + 4 \cdot AB + BJ^2 + 4 \cdot BJ + AB} = 0$

Unit.

$AB := 1$

Given.

$N_1 := 1.47027 \quad AJ := N_1$

What is AV and ST?

Given any angle BHP, the unit which defines it will lay between W and V.

0530011B

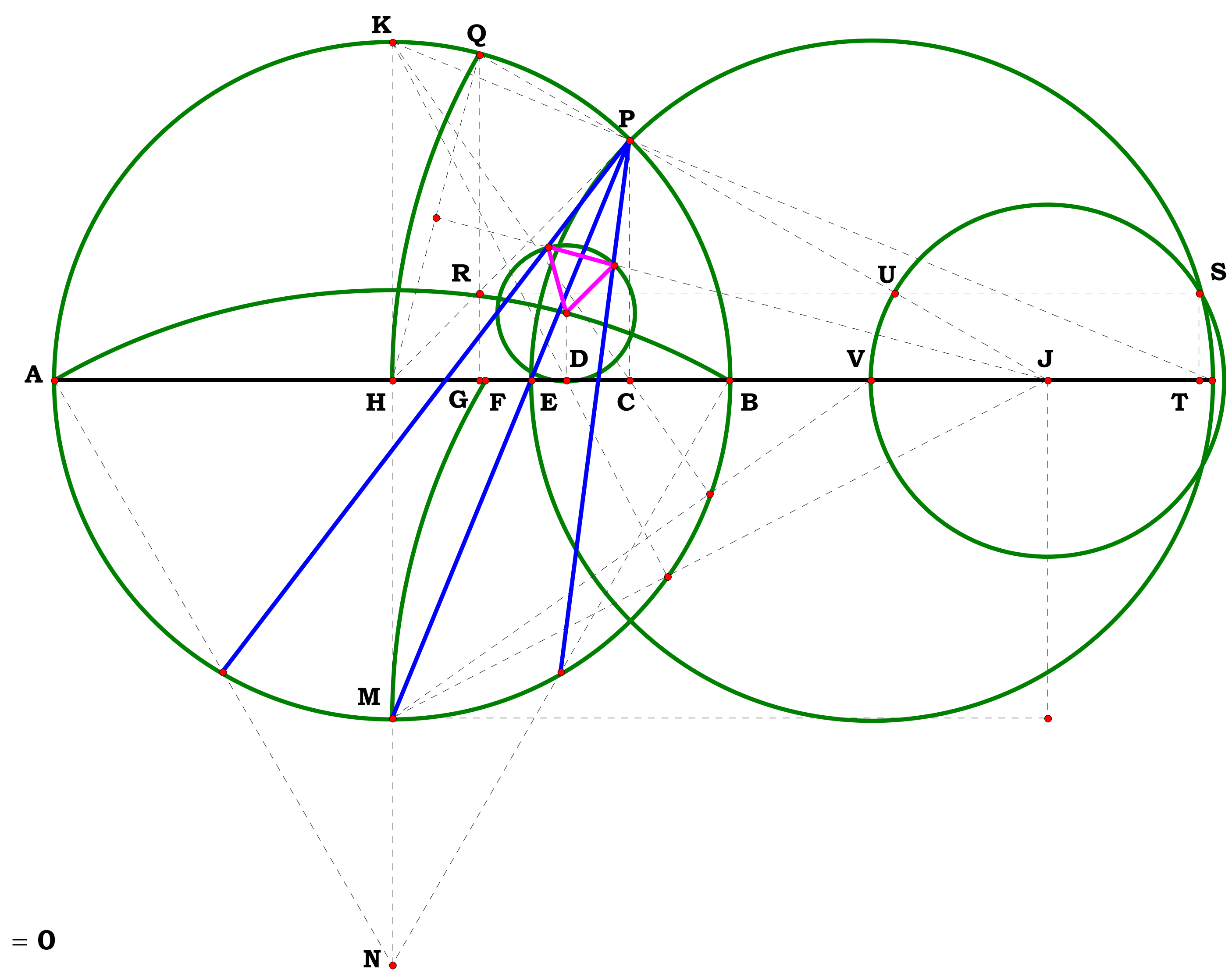
Descriptions.

$BJ := AJ - AB \quad BH := \frac{AB}{2} \quad JH := AJ - BH \quad JQ := JH$

$PQ := \frac{BH^2}{JQ} \quad HM := BH \quad JP := JQ - PQ \quad CJ := \frac{JP^2 + JH^2 - BH^2}{2 \cdot JH}$

$CP := \sqrt{JP^2 - CJ^2} \quad HK := BH \quad CH := JH - CJ \quad CK := \sqrt{CH^2 + HK^2}$

$HV := \frac{HK \cdot HM}{CH} \quad JV := JH - HV \quad ST := \frac{CP \cdot JV}{JP}$



Definitions.

$AJ - N_1 = 0 \quad BJ - (N_1 - 1) = 0 \quad BH - \frac{1}{2} = 0 \quad JH - \frac{2 \cdot N_1 - 1}{2}$

$JQ - \frac{2 \cdot N_1 - 1}{2} = 0 \quad PQ - \frac{1}{2 \cdot (2 \cdot N_1 - 1)} = 0 \quad HM - \frac{1}{2} = 0 \quad JP - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0$

$CJ - \frac{N_1 \cdot (N_1 - 1) \cdot (8 \cdot N_1^2 - 8 \cdot N_1 + 1)}{(2 \cdot N_1 - 1)^3} = 0 \quad CP - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(4 \cdot N_1 - 1) \cdot (4 \cdot N_1 - 3)}}{(2 \cdot N_1 - 1)^3} = 0$

$HK - \frac{1}{2} = 0 \quad CH - \frac{6 \cdot N_1^2 - 6 \cdot N_1 + 1}{2 \cdot (2 \cdot N_1 - 1)^3} = 0 \quad CK - \frac{\sqrt{2 \cdot N_1 \cdot (N_1 - 1) \cdot \left[16 \cdot N_1^3 \cdot (N_1 - 2) + 37 \cdot N_1^2 - 21 \cdot N_1 + 6\right] + 1}}{\sqrt{2} \cdot (2 \cdot N_1 - 1)^3} = 0$

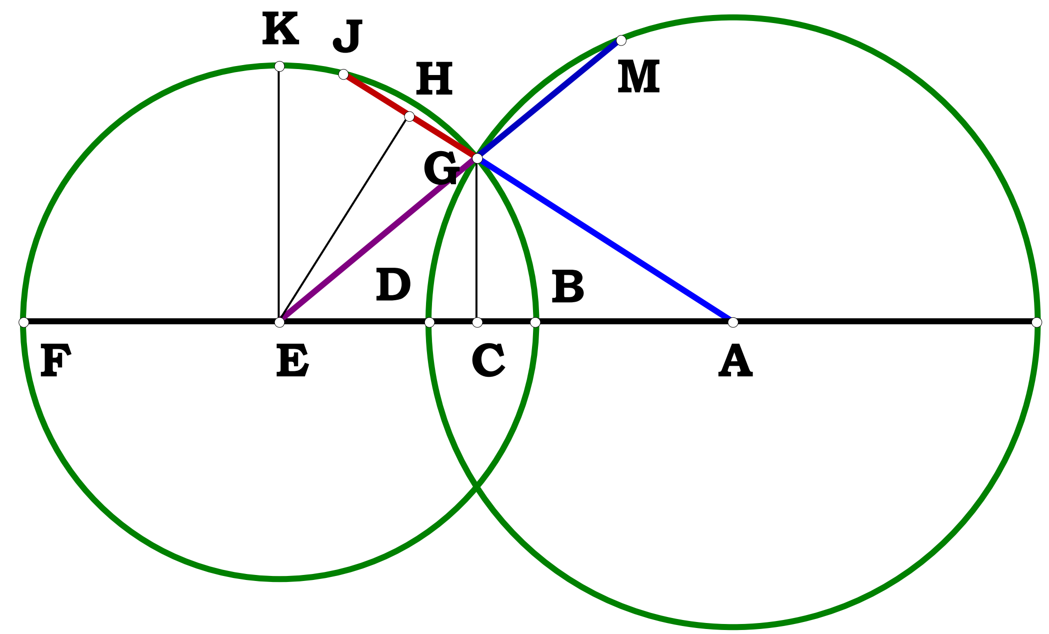
$HV - \frac{(2 \cdot N_1 - 1)^3}{2 \cdot (6 \cdot N_1^2 - 6 \cdot N_1 + 1)} = 0 \quad JV - \frac{N_1 \cdot (2 \cdot N_1 - 1) \cdot (N_1 - 1)}{6 \cdot N_1^2 - 6 \cdot N_1 + 1} = 0 \quad ST - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(4 \cdot N_1 - 1) \cdot (4 \cdot N_1 - 3)}}{2 \cdot (2 \cdot N_1 - 1) \cdot (6 \cdot N_1^2 - 6 \cdot N_1 + 1)} = 0$

Unit.
Given.
Descriptions.
Definitions.

060101

A Small Extrapolation

Given AE, AG, and EG, what is the Algebraic name of the segment GJ?



$S_1 := 6.00604 \qquad S_2 := 4.02167 \qquad S_3 := 3.38667$

$AE := S_1 \qquad AG := S_2 \qquad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2AE}$$

$$AH := \frac{AC \cdot AE}{AG} \qquad GH := AH - AG$$

$$HJ := GH \qquad GJ := GH + HJ$$

Some Algebraic Names:

$$\frac{S_2^{\,2} + S_1^{\,2} - S_3^{\,2}}{2S_1} - AC = 0 \qquad \frac{S_1^{\,2} + S_2^{\,2} - S_3^{\,2}}{2S_2} - AH = 0 \qquad \frac{S_1^{\,2} - S_2^{\,2} - S_3^{\,2}}{2S_2} - GH = 0$$

$$\frac{S_1^{\,2} - S_2^{\,2} - S_3^{\,2}}{S_2} - GJ = 0$$

Unit.

$AB := 1$

Given.

$N := 5.727 \quad AG := N$

Units From Both Sides

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.

060201A

Descriptions.

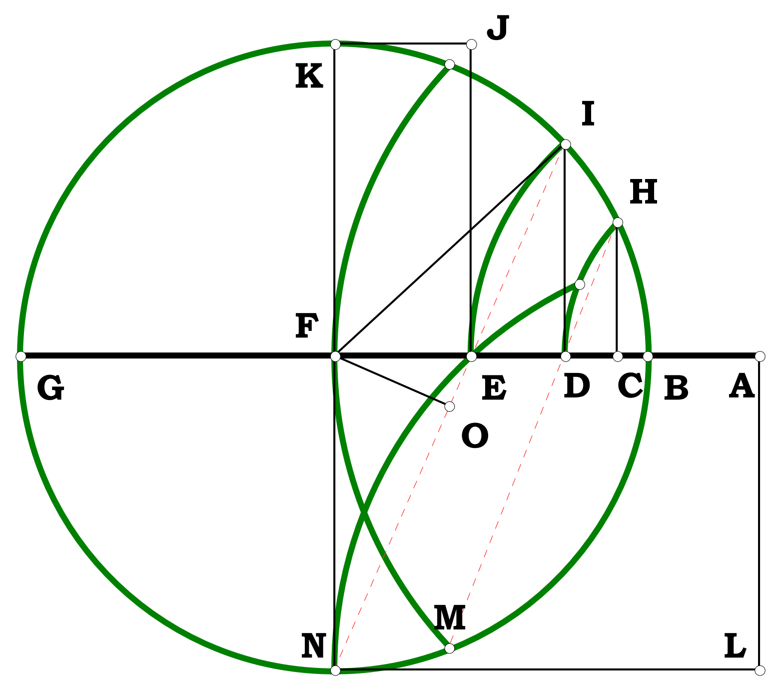
$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

$FN := BF \qquad EF := BF - BE \qquad EN := \sqrt{FN^2 + EF^2}$

$NO := \frac{FN^2}{EN} \qquad NI := 2 \cdot NO \qquad EI := NI - EN$

$DE := \frac{EF \cdot EI}{EN}$



Definitions.

$BG - (N-1) = 0 \qquad BF - \frac{N-1}{2} = 0 \qquad AF - \frac{N+1}{2} = 0$

$AE - \sqrt{N} = 0 \qquad BE - \left(\sqrt{N} - 1\right) = 0 \qquad FN - \frac{N-1}{2} = 0$

$EF - \frac{\left(\sqrt{N}-1\right)^2}{2} = 0 \qquad EN - \frac{\sqrt{2} \cdot \sqrt{N+1} \cdot \left(\sqrt{N}-1\right)}{2} = 0 \qquad NO - \frac{\sqrt{2} \cdot \left(\sqrt{N}-1\right) \cdot \left(\sqrt{N}+1\right)^2}{4 \cdot \sqrt{N+1}} = 0$

$NI - \frac{\sqrt{2} \cdot \left(\sqrt{N}-1\right) \cdot \left(\sqrt{N}+1\right)^2}{2 \cdot \sqrt{N+1}} = 0 \qquad EI - \frac{\sqrt{2} \cdot \sqrt{N} \cdot \left(\sqrt{N}-1\right)}{\sqrt{N+1}} = 0 \qquad DE - \frac{\sqrt{N} \cdot \left(\sqrt{N}-1\right)^2}{N+1} = 0$

060201B

Unit.

$BG := 1$

Given.

$N := 2$

Units From Both Sides

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.

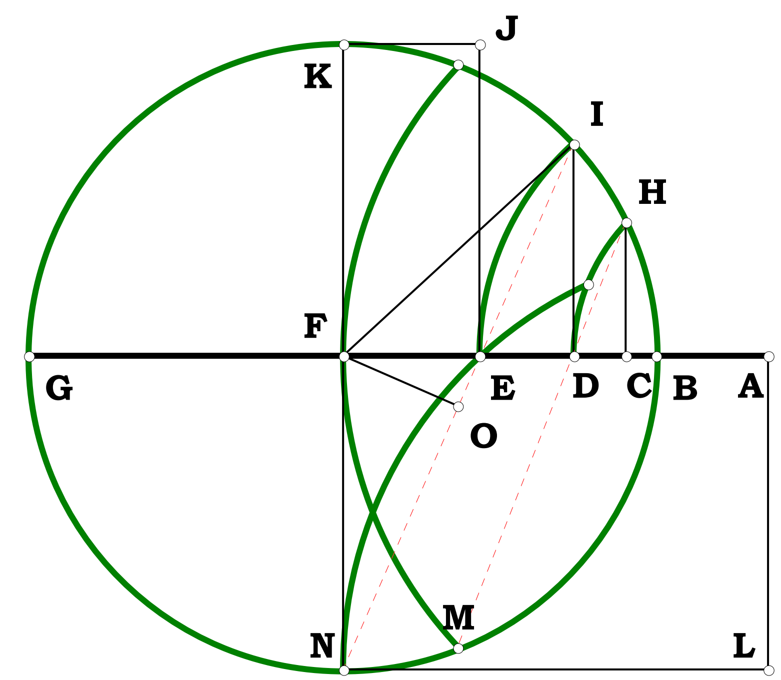
Descriptions.

$BF := \frac{BG}{2}$ $\quad BE := \frac{BF}{N}$ $\quad EF := BF - BE$

$FN := BF$ $\quad EN := \sqrt{EF^2 + FN^2}$ $\quad NP := \frac{EN}{2}$

$LN := \frac{EN \cdot NP}{EF}$ $\quad AF := LN$ $\quad AB := AF - BF$

$AB = 0.125$



Definitions.

$$\frac{BG}{2} - BF = 0 \qquad \frac{BG}{(2 \cdot N)} - BE = 0 \qquad \frac{BG}{2} \cdot \frac{(N-1)}{N} - EF = 0$$

$$\frac{BG}{2} \cdot \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{N} - EN = 0 \qquad \frac{BG}{4} \cdot \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{N} - NP = 0$$

$$\frac{BG}{4} \cdot \frac{\left(2 \cdot N^2 - 2 \cdot N + 1\right)}{[N \cdot (N-1)]} - LN = 0 \qquad \frac{BG}{4 \cdot N \cdot (N-1)} - AB = 0$$

060201C

Unit.

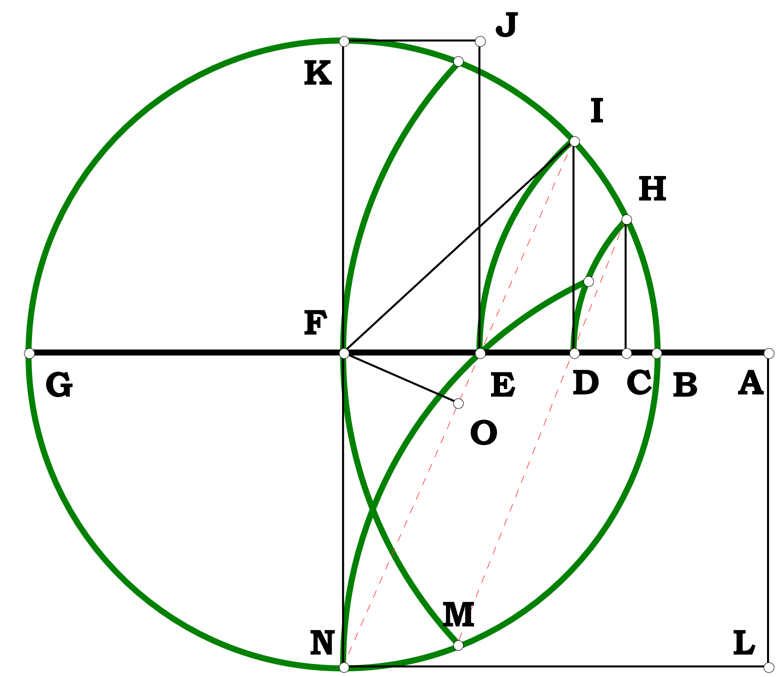
$BG := 1$

Given.

$N := 2$

Units From Both Sides

Start with AB as unit and find. . . . then start with . .
. . as unit and find AB.



Descriptions.

$BF := \frac{BG}{2}$ $\quad BD := \frac{BF}{N}$ $\quad DG := BG - BD$

$DI := \sqrt{BD \cdot DG}$ $\quad DF := BF - BD$ $\quad FN := BF$

$EF := \frac{DF \cdot FN}{FN + DI}$ $\quad EN := \sqrt{EF^2 + FN^2}$ $\quad NP := \frac{EN}{2}$

$LN := \frac{EN \cdot NP}{EF}$ $\quad AF := LN$ $\quad AB := AF - BF$

$AB = 0.5$

Definitions.

$BD - \frac{1}{2} \cdot \frac{BG}{N} = 0$ $\quad DG - \frac{1}{2} \cdot BG \cdot \frac{(2 \cdot N - 1)}{N} = 0$ $\quad DI - \frac{1}{(2 \cdot N)} \cdot BG \cdot \sqrt{2 \cdot N - 1} = 0$

$DF - \frac{1}{2} \cdot BG \cdot \frac{(N-1)}{N} = 0$ $\quad EF - \frac{1}{2} \cdot BG \cdot \frac{(N-1)}{\left(N + \sqrt{2 \cdot N - 1}\right)} = 0$ $\quad NP - \frac{1}{4} \cdot BG \cdot \sqrt{2} \cdot \sqrt{\frac{N}{\left(N + \sqrt{2 \cdot N - 1}\right)}} = 0$

$EN - \frac{1}{2} \cdot BG \cdot \sqrt{2} \cdot \sqrt{\frac{N}{\left(N + \sqrt{2 \cdot N - 1}\right)}} = 0$ $\quad LN - \frac{1}{2} \cdot BG \cdot \frac{N}{(N-1)} = 0$ $\quad AB - \frac{1}{2} \cdot \frac{BG}{(N-1)} = 0$

Unit.

$BE := 1$

Given.

$N := 4$

060301

Descriptions.

$BD := \frac{BE}{2}$ $\quad BC := \frac{BE}{N}$ $\quad CE := BE - BC$

$CG := \sqrt{BC \cdot CE}$ $\quad CD := BD - BC$ $\quad AC := \frac{CG^2}{CD}$

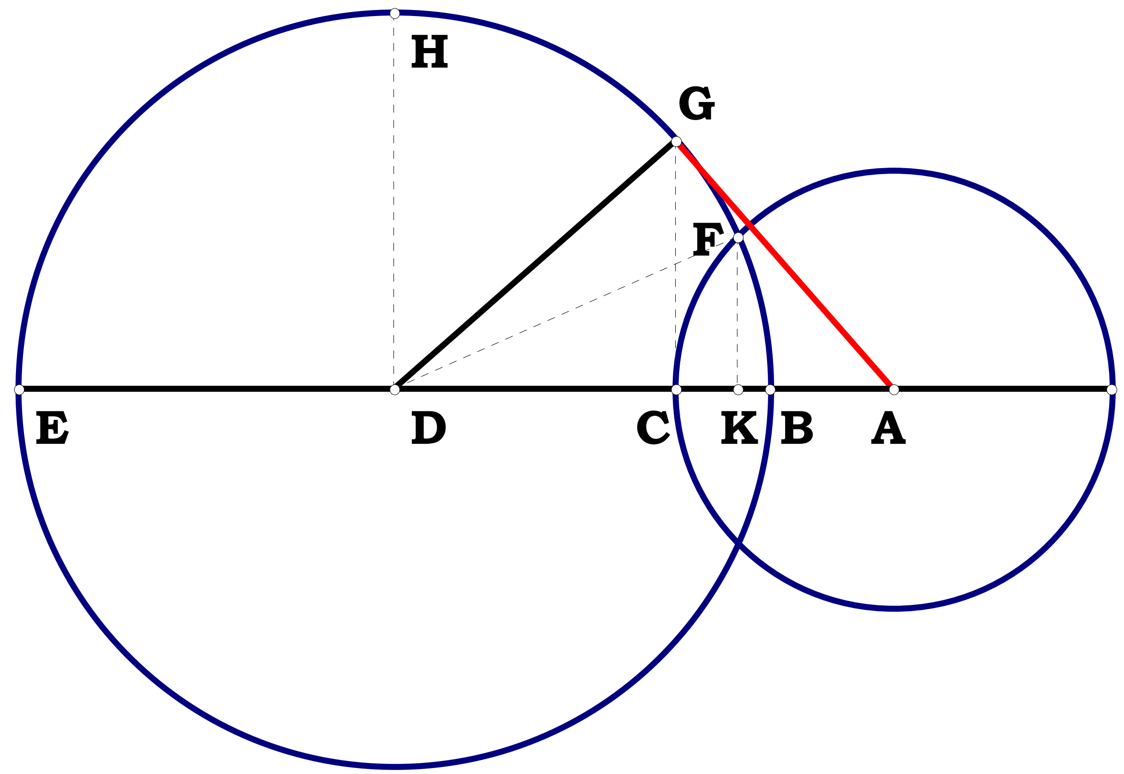
$AF := AC$ $\quad AD := AC + CD$ $\quad DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2AD}$ $\quad BK := BD - DK$

$CK := BC - BK$

Isolating A Problem

If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?



Definitions.

$\frac{N-1}{N} - CE = 0$ $\quad \frac{\sqrt{N-1}}{N} - CG = 0$ $\quad \frac{N-2}{2 \cdot N} - CD = 0$

$\frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0$ $\quad \frac{N}{2 \cdot (N-2)} - AD = 0$ $\quad \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$\frac{3 \cdot N - 2}{N^3} - BK = 0$ $\quad \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0$ $\quad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

Unit.

$BE := 1$

Given.

$N := 4$

060301

Descriptions.

$$BD := \frac{BE}{2} \qquad BC := \frac{BD}{N} \qquad AC := \frac{1}{2} \cdot \frac{BE}{N} \cdot \frac{(2 \cdot N - 1)}{(N - 1)}$$

$$CM := \frac{1}{4} \cdot BE \cdot (2 \cdot N - 1) \cdot \frac{(N - 1)}{N^3} \qquad AM := AC - CM$$

$$AF := AC \qquad FM := \sqrt{AF^2 - AM^2} \qquad AD := \frac{1}{2} \cdot BE \cdot \frac{N}{(N - 1)}$$

$$FK := \frac{AD^2 - AC^2 - BD^2}{AC} \qquad FK = 0.375$$

Definitions.

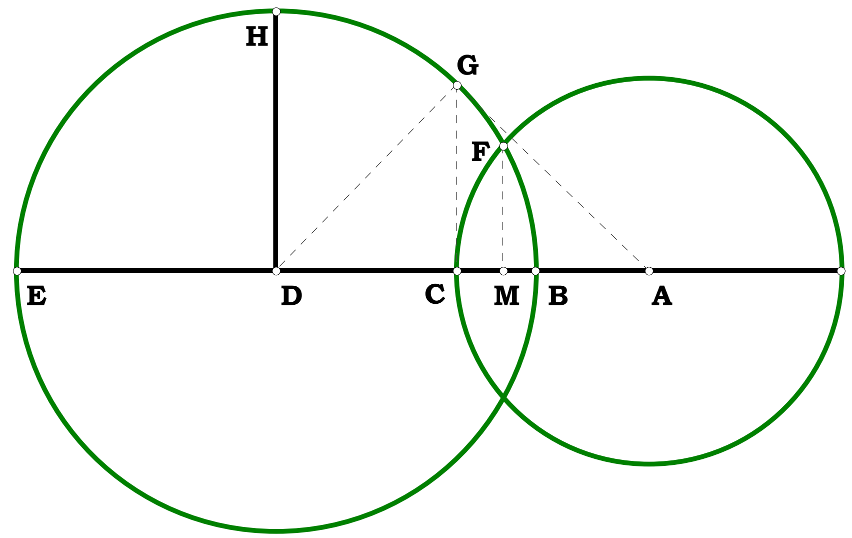
$$BD - \frac{1}{2} = 0 \qquad BC - \frac{1}{2 \cdot N} = 0 \qquad AC - \frac{2 \cdot N - 1}{2 \cdot N \cdot (N - 1)} = 0$$

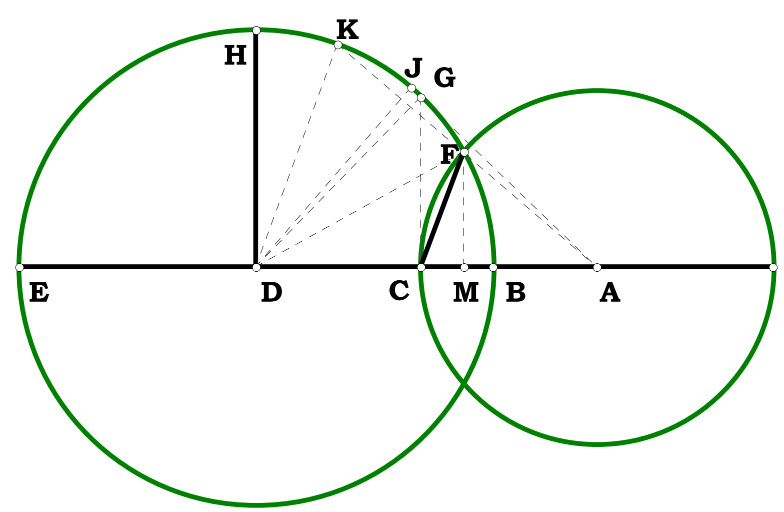
$$CM - \frac{(N - 1) \cdot (2 \cdot N - 1)}{4 \cdot N^3} = 0 \qquad AM - \frac{(2 \cdot N - 1) \cdot \left(N^2 + 2 \cdot N - 1\right)}{4 \cdot N^3 \cdot (N - 1)} = 0$$

$$AF - \frac{2 \cdot N - 1}{2 \cdot N \cdot (N - 1)} = 0 \qquad FM - \frac{\sqrt{(N + 1) \cdot (3 \cdot N - 1)} \cdot (2 \cdot N - 1)}{4 \cdot N^3} = 0$$

$$AD - \frac{N}{2 \cdot (N - 1)} = 0 \qquad FK - \frac{N - 1}{2 \cdot N} = 0$$

For any point C on BD, FCG is 1/3 of the angle FDH, will the Algebraic Name for DM remain constant if one 'steps back to it' from D?





061001

Given.

$N_1 := 5$

$N_2 := 3$

Descriptions.

For any $N_1 \cdot N_2$ what is DG?

$AF := N_1 \cdot N_2 \quad BF := AF - N_2 \quad BE := \frac{BF}{2}$

$EK := BE \quad AE := N_2 + BE \quad DE := \frac{EK^2}{AE}$

$EF := BE \quad FM := BF \quad EM := \sqrt{FM^2 - EF^2}$

$GM := FM \quad GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2}$

$EQ := MQ - EM \quad DG := EQ$

Definitions.

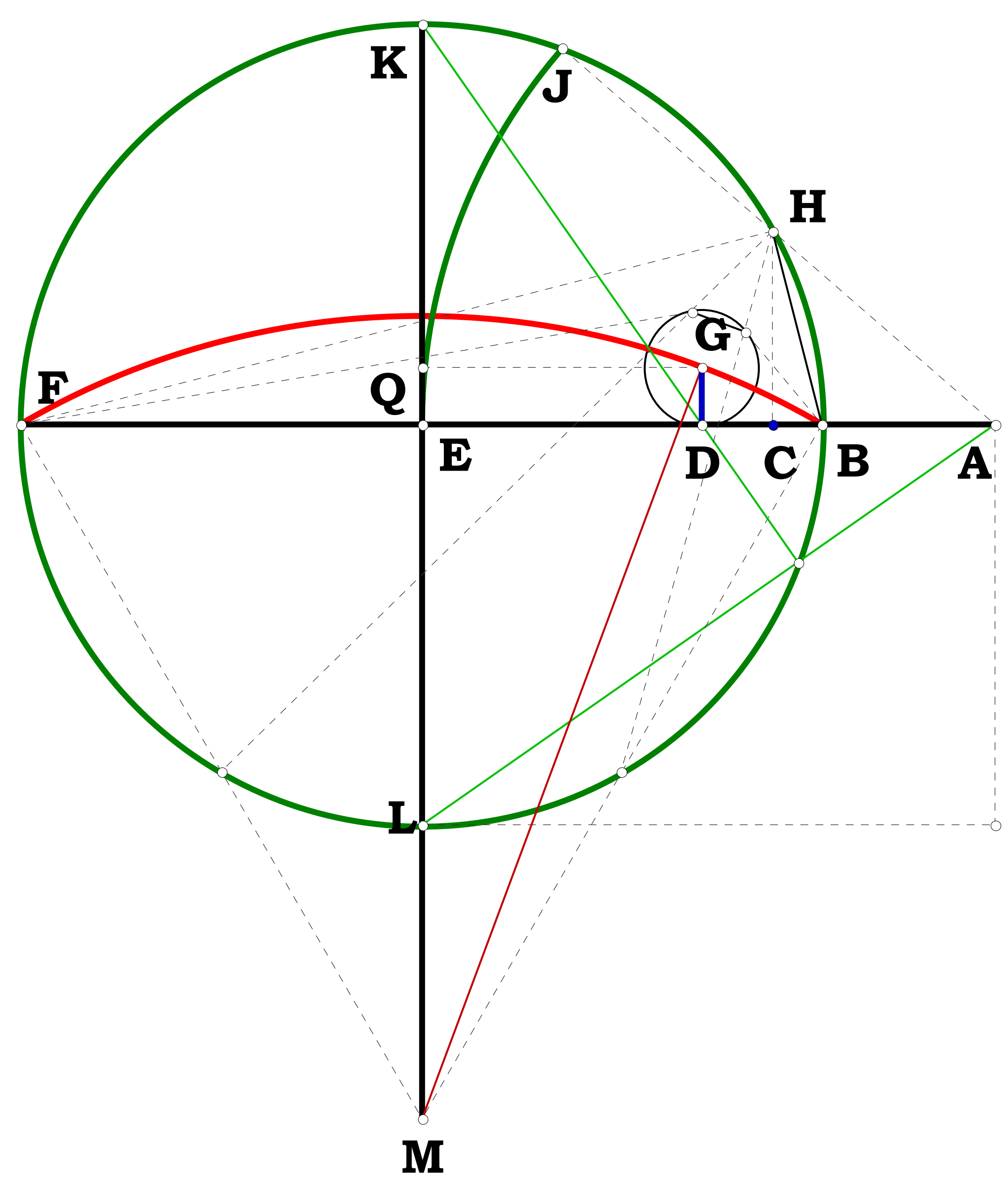
$$N_2 \cdot (N_1 - 1) - BF = 0 \qquad \frac{N_2 \cdot (N_1 - 1)}{2} - BE = 0$$

$$\frac{1}{2} \cdot N_2 \cdot (N_1 + 1) - AE = 0 \qquad \frac{1}{2} \cdot N_2 \cdot \frac{(N_1 - 1)^2}{(N_1 + 1)} - DE = 0$$

$$\frac{1}{2} \cdot \sqrt{3} \cdot N_2 \cdot (N_1 - 1) - EM = 0 \qquad \frac{N_2 \cdot \left[\sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)} \cdot (N_1 - 1)\right]}{2 \cdot (N_1 + 1)} - MQ = 0$$

$$\frac{N_2 \cdot (N_1 - 1) \cdot \left[\sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)} - \sqrt{3} - \sqrt{3} \cdot N_1\right]}{2 \cdot (N_1 + 1)} - DG = 0$$

For Any $N_1 \cdot N_2$



082601

Unit.

$AF := 1$

Given.

$N_1 := 4$

$N_2 := 3$

Descriptions.

$$AC := \frac{AF}{2} \qquad CJ := AC \qquad BC := \frac{AC}{N_1}$$

$$AE := \frac{AF}{N_2} \qquad EF := AF - AE \qquad EH := \sqrt{AE \cdot EF}$$

$$EG := \frac{BC \cdot EH}{CJ} \qquad CE := AE - AC \qquad CG := \sqrt{CE^2 + EG^2}$$

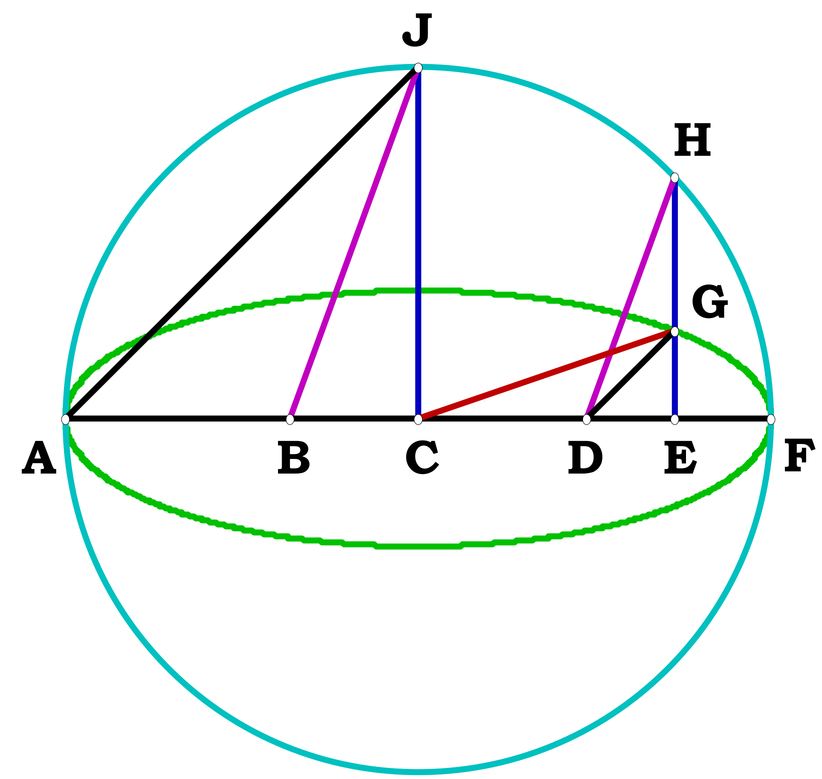
Definitions.

$$BC - \frac{1}{(2 \cdot N_1)} = 0 \qquad AE - \frac{1}{N_2} = 0 \qquad EF - \left(1 - \frac{1}{N_2}\right) = 0$$

$$EH - \frac{\sqrt{N_2 - 1}}{N_2} = 0 \qquad EG - \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2} = 0$$

$$CE - \left(\frac{1}{N_2} - \frac{1}{2}\right) = 0 \qquad CG - \frac{1}{2} \cdot \frac{\sqrt{(N_2 - 2)^2 \cdot N_1{}^2 + 4 \cdot (N_2 - 1)}}{(N_1 \cdot N_2)} = 0$$

Elipse By Parallels



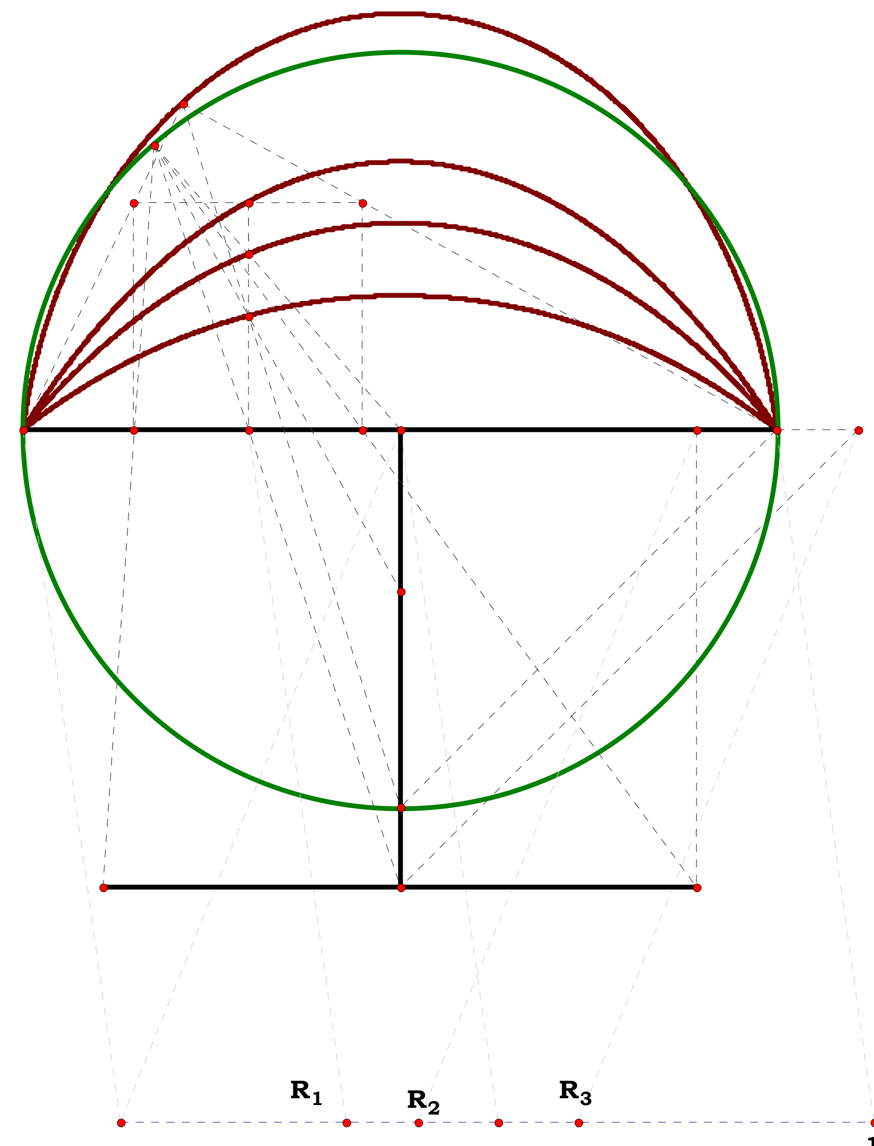
Four Curves and Procrastination.

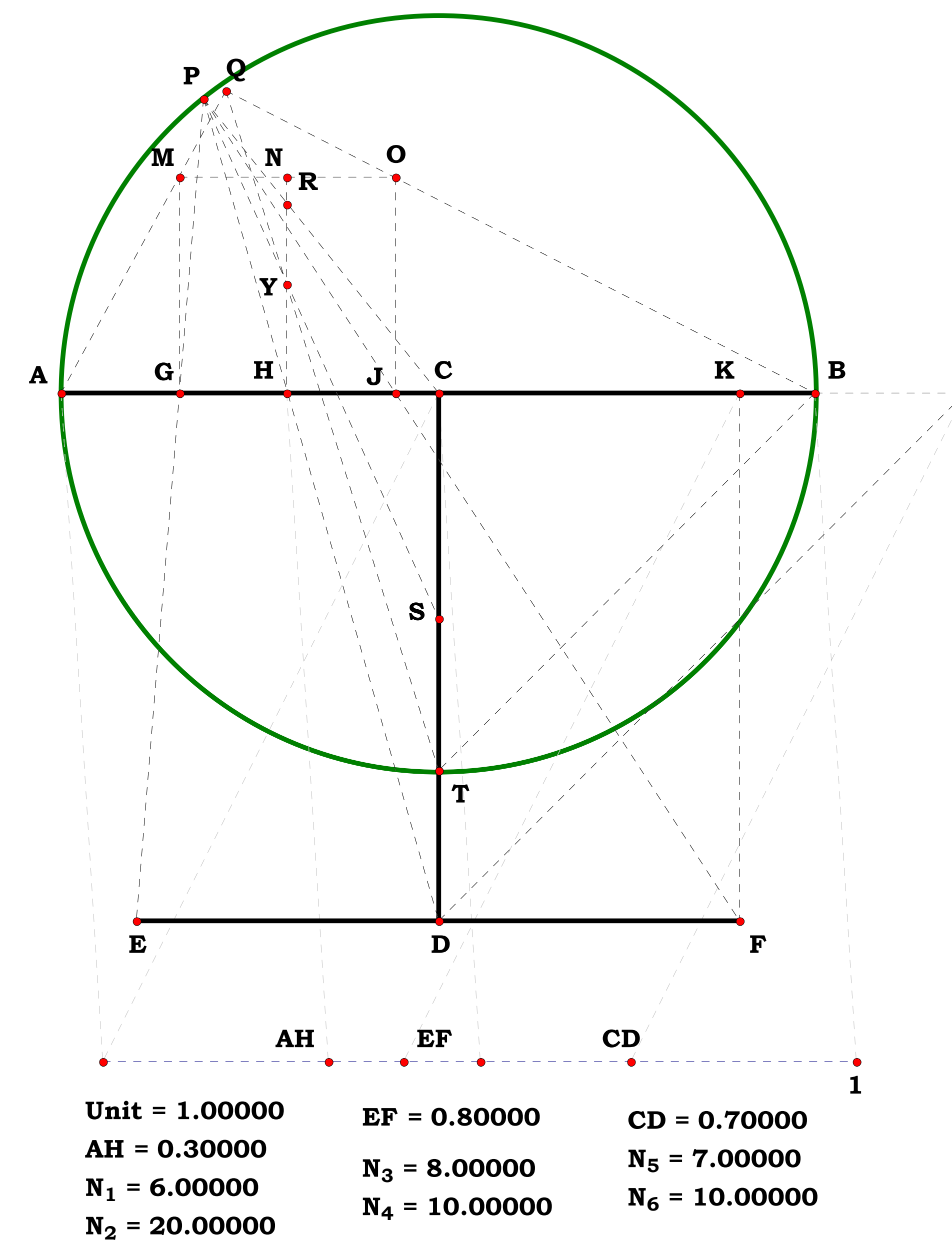
102201

I had sketched this out over 18 years ago and have put off writing it up for some very good reasons, the most prominate is because of the way I wanted to write it up. Normally I aim for between one and four variables for this novel; this one I need six. For the work Basic Analog Mathematics that number of variables is not unusual, but here it is. For BAM, I set my limit on 8, which is twice that I set for this work.

I am so motivated to write this up that when I got to it in this Delian Quest revision, I put this project aside and did the projects OTOH (On The Other Hand), Alice Innocent Plays and Conducts Bach, and Sergio Vosh Goshen Inkscapes Durer, which took me over a month to do. I was curious about the state of ABC notation and Vector Graphics which have been on my mind for some 20 years now.

I also made some new Windows 95 and 98 virtual boxes which have all the software, and more, that I started these projects with.





I am not even going to go through all of this, just the major portion.

Unit.

$AB := 1$

Given.

$N_1 := 6 \quad N_2 := 20$

$N_3 := 8 \quad N_4 := 10$

$N_5 := 7 \quad N_6 := 11$

Descriptions.

$$AH := \frac{N_1}{N_2} \qquad EF := \frac{N_3}{N_4} \qquad CD := \frac{N_5}{N_6} \qquad AC := \frac{AB}{2}$$

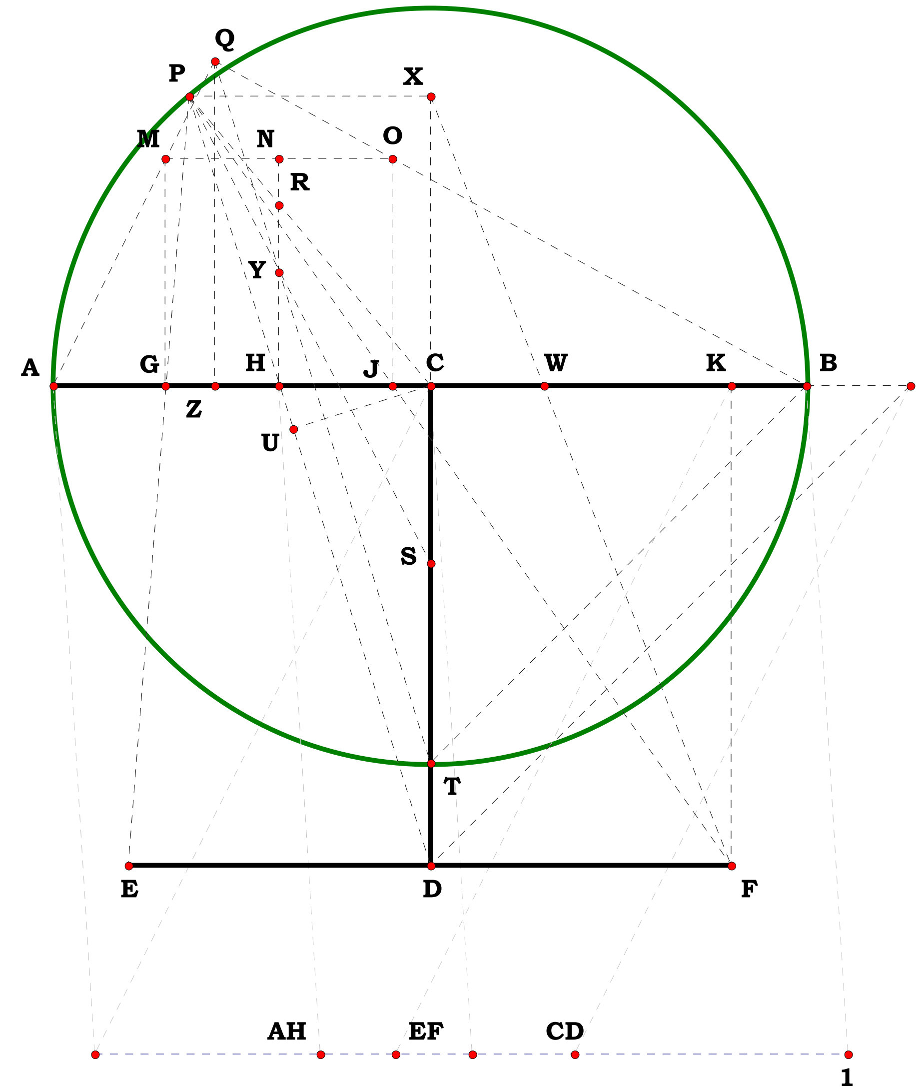
$$CP := AC \qquad CH := \sqrt{(AC - AH)^2} \qquad DH := \sqrt{CD^2 + CH^2}$$

$$CU := \frac{CD \cdot CH}{DH} \qquad PU := \sqrt{CP^2 - CU^2} \qquad HU := \frac{CH^2}{DH}$$

$$HP := PU - HU \qquad DP := DH + HP \qquad DX := \frac{CD \cdot DP}{DH}$$

$$DF := \frac{EF}{2} \qquad CX := DX - CD \qquad CW := \frac{DF \cdot CX}{DX}$$

$$GJ := 2 \cdot CW \qquad \text{Etc.}$$



Unit = 1.00000
AH = 0.30000
N_1 = 6.00000
N_2 = 20.00000

EF = 0.80000
N_3 = 8.00000
N_4 = 10.00000

CD = 0.63636
N_5 = 7.00000
N_6 = 11.00000

Definitions.

$$AH - \frac{N_1}{N_2} = 0 \qquad EF - \frac{N_3}{N_4} = 0 \qquad CD - \frac{N_5}{N_6} = 0 \qquad AC - \frac{1}{2}$$

$$CP - \frac{1}{2} = 0 \qquad CH - \frac{\sqrt{(2 \cdot N_1 - N_2)^2}}{2 \cdot N_2} = 0 \qquad DH - \frac{\sqrt{\left(4 \cdot N_5^2 + N_6^2\right) \cdot N_2^2 + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}}{2 \cdot N_2 \cdot N_6} = 0 \qquad CU - \frac{N_5 \cdot \sqrt{(N_2 - 2 \cdot N_1)^2}}{\sqrt{N_2^2 \cdot \left(4 \cdot N_5^2 + N_6^2\right) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

$$PU - \frac{\sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}}{2 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2}} = 0 \qquad HU - \frac{N_6 \cdot (2 \cdot N_1 - N_2)^2}{2 \cdot N_2 \cdot \sqrt{N_2^2 \cdot \left(4 \cdot N_5^2 + N_6^2\right) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

$$HP-\frac{N_2\cdot\sqrt{(2\cdot N_1-N_2)^2\cdot N_6^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}-N_6\cdot(2\cdot N_1-N_2)^2}{2\cdot N_2\cdot\sqrt{N_2^2\cdot\left(4\cdot N_5^2+N_6^2\right)+4\cdot N_1\cdot N_6^2\cdot(N_1-N_2)}}=0$$

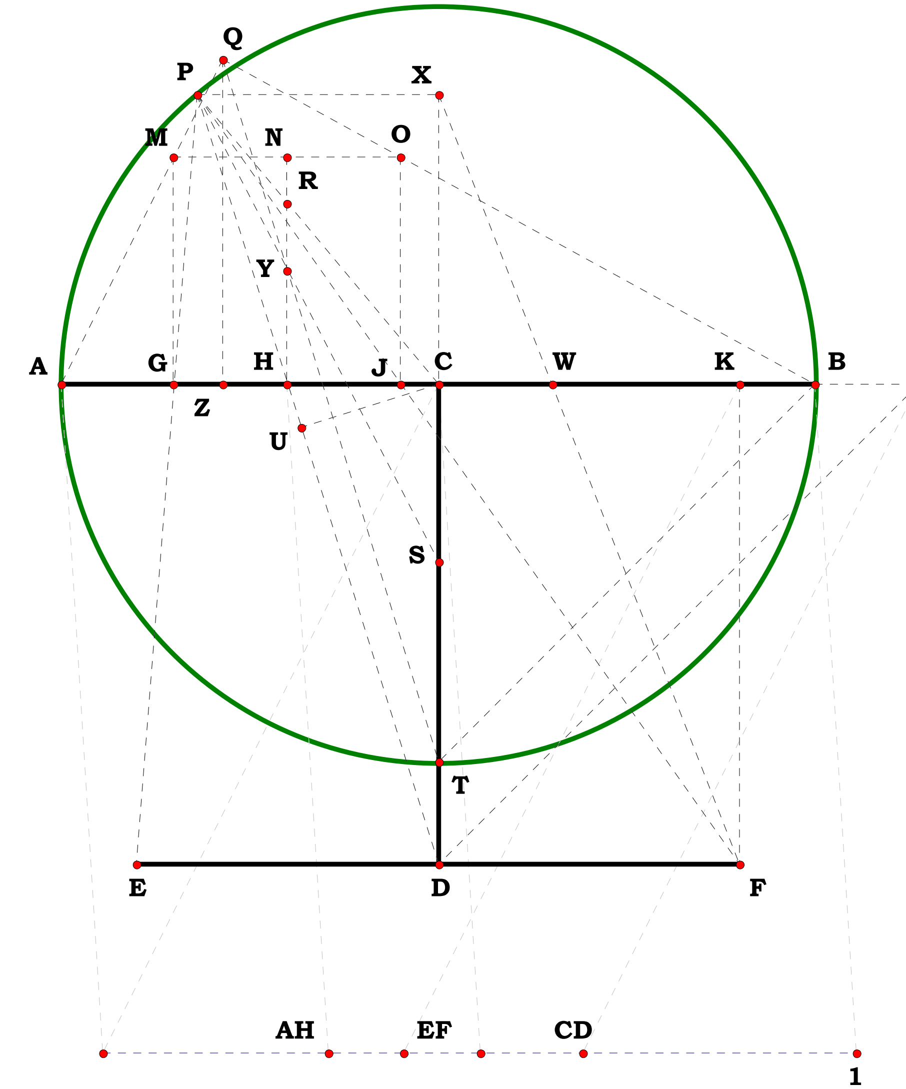
$$DP-\frac{4\cdot N_2\cdot N_5^2+N_6\cdot\sqrt{N_6^2\cdot(2\cdot N_1-N_2)^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}}{2\cdot N_6\cdot\sqrt{(2\cdot N_1-N_2)^2\cdot N_6^2+4\cdot N_2^2\cdot N_5^2}}=0$$

$$DX-\frac{N_2\cdot N_5\cdot\left[4\cdot N_2\cdot N_5^2+N_6\cdot\sqrt{N_6^2\cdot(2\cdot N_1-N_2)^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}\right]}{N_6\cdot\left[(2\cdot N_1-N_2)^2\cdot N_6^2+4\cdot N_2^2\cdot N_5^2\right]}=0$$

$$DF-\frac{N_3}{2\cdot N_4}=0 \qquad CX-\frac{\left[\sqrt{N_6^2\cdot(2\cdot N_1-N_2)^2-16\cdot N_5^2\cdot N_1\cdot(N_1-N_2)}\cdot N_2-N_6\cdot(2\cdot N_1-N_2)^2\right]\cdot N_5}{(2\cdot N_1-N_2)^2\cdot N_6^2+4\cdot N_2^2\cdot N_5^2}=0$$

$$CW-\frac{N_3\cdot N_6\cdot\left[\sqrt{N_6^2\cdot(N_2-2\cdot N_1)^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}\cdot N_2-N_6\cdot(2\cdot N_1-N_2)^2\right]}{2\cdot N_2\cdot N_4\cdot\left[4\cdot N_2\cdot N_5^2+N_6\cdot\sqrt{N_6^2\cdot(N_2-2\cdot N_1)^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}\right]}=0$$

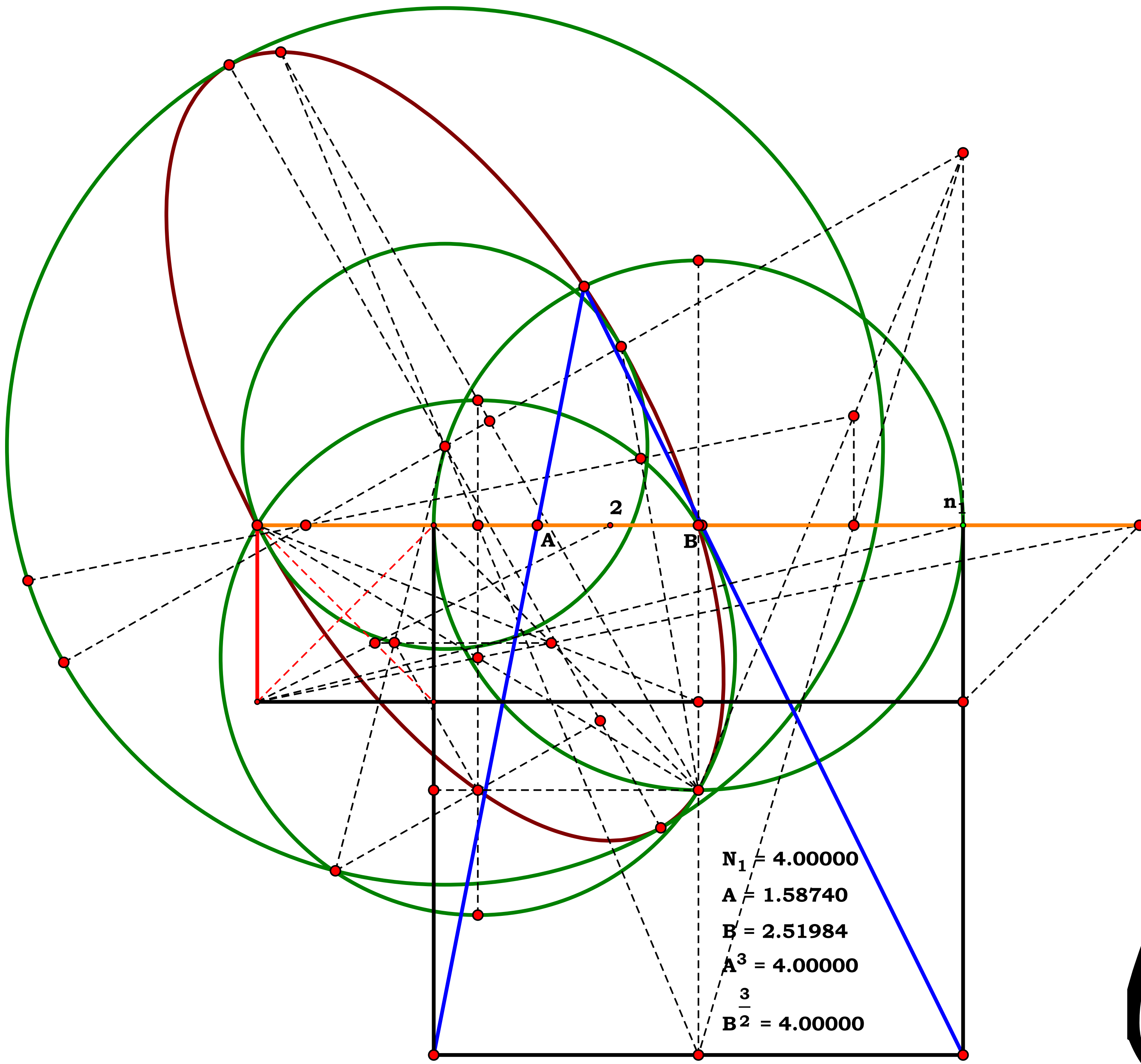
$$GJ-\frac{N_3\cdot N_6\cdot\left[\sqrt{N_6^2\cdot(N_2-2\cdot N_1)^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}\cdot N_2-N_6\cdot(2\cdot N_1-N_2)^2\right]}{N_2\cdot N_4\cdot\left[4\cdot N_2\cdot N_5^2+N_6\cdot\sqrt{N_6^2\cdot(N_2-2\cdot N_1)^2-16\cdot N_1\cdot N_5^2\cdot(N_1-N_2)}\right]}=0$$



Unit = 1.00000
AH = 0.30000
N_1 = 6.00000
N_2 = 20.00000

EF = 0.80000
N_3 = 8.00000
N_4 = 10.00000

CD = 0.63636
N_5 = 7.00000
N_6 = 11.00000



The Delian Quest 2000

John Clark

Just Another Proof Of Paper

Unit.

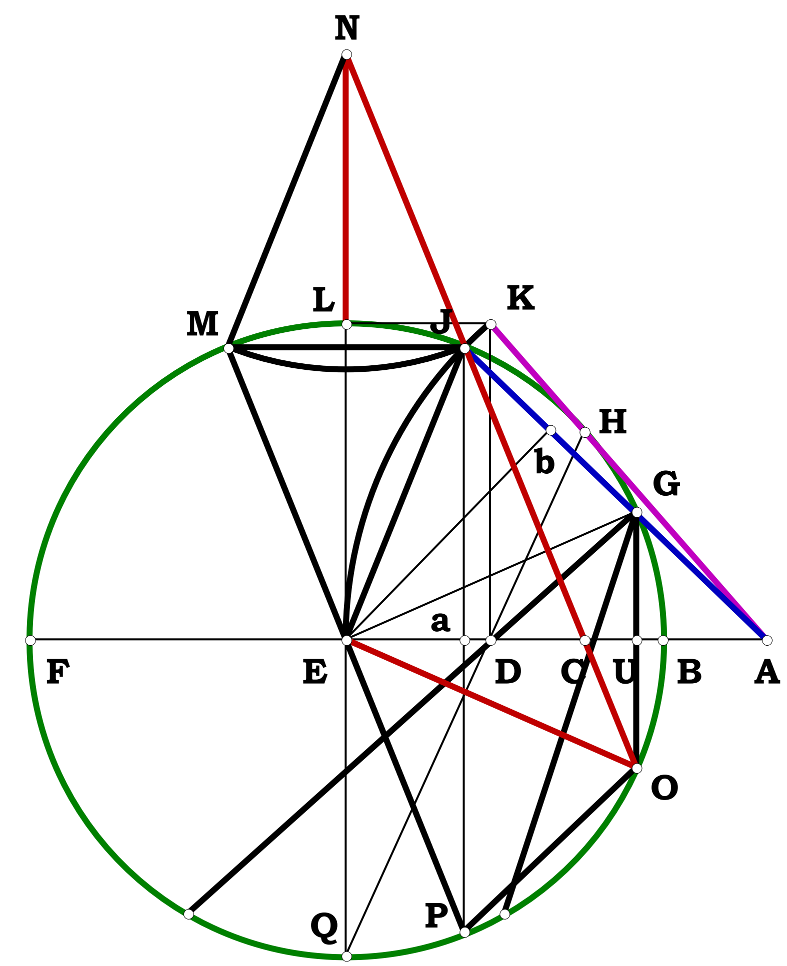
$AB := 1$

Given.

$N := 7 \qquad AF := N$

010202

Descriptions.



$BF := AF - AB \qquad BE := \dfrac{BF}{2} \qquad AE := BE + AB$

$AJ := AE \qquad EJ := BE \qquad Ea := \dfrac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \qquad GJ := 2 \cdot Gb \qquad AG := AJ - GJ$

$Aa := AE - Ea \qquad AU := \dfrac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \qquad GU := \dfrac{Ja \cdot AG}{AJ}$

$Ua := Aa - AU \qquad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \dfrac{JO \cdot Ea}{Ua} \qquad JN - BE = 0$

From 4/29/94 $\quad OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \qquad NO := JO + JN \qquad EU := Ua + Ea$

$EN := \sqrt{NO^2 - EU^2} \qquad EL := BE \qquad LN := EN - EL$

Definitions.

$$N-1-BF=0 \qquad \frac{N-1}{2}-BE=0 \qquad \frac{N+1}{2}-AE=0$$

$$\frac{(N-1)^2}{4\cdot(N+1)}-Ea=0 \qquad \frac{(N-1)^2}{2\cdot(N+1)}-GJ=0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)}-Aa=0$$

$$\frac{2\cdot N}{N+1}-AG=0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3}-AU=0 \qquad \frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)}-Ja=0$$

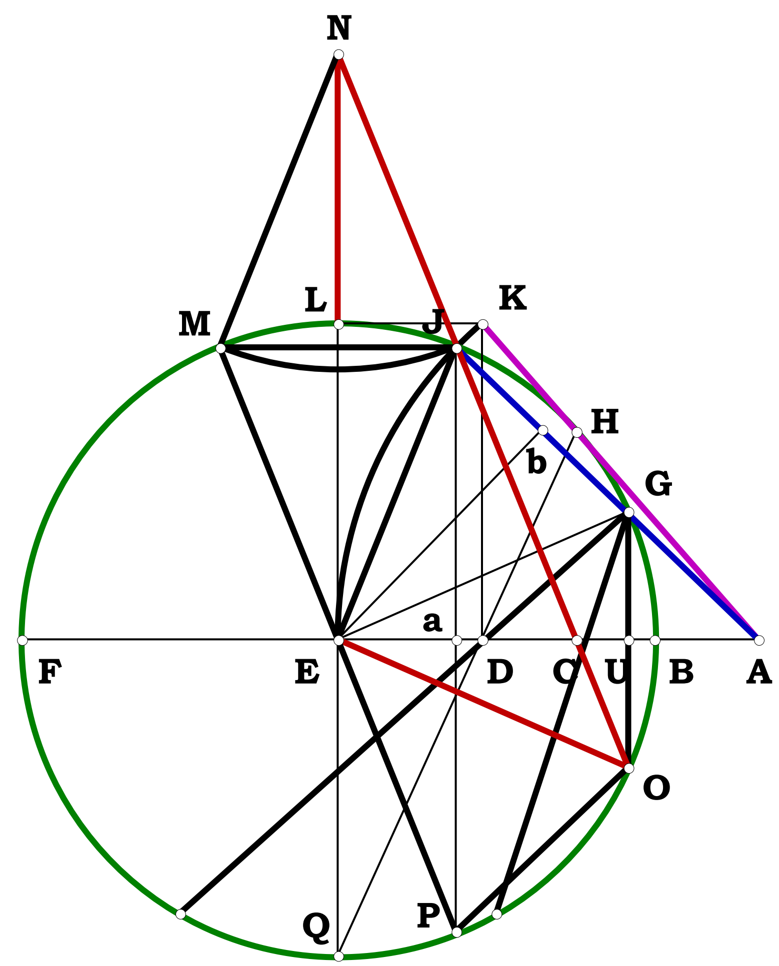
$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}-GU=0 \qquad \frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3}-Ua=0$$

$$\frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2}-JO=0 \qquad \frac{N-1}{2}-JN=0 \qquad \frac{(N-1)^2}{2\cdot(N+1)}-OP=0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2}-NO=0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3}-EU=0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3}-EN=0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3}-\frac{N-1}{2}-LN=0$$



062002A

Unit.

$AC := 1$

Given.

$N_1 := 4 \quad AF := N_1$

$N_2 := 3 \quad CE := N_2$

Easy Power-Line

For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.

Descriptions.

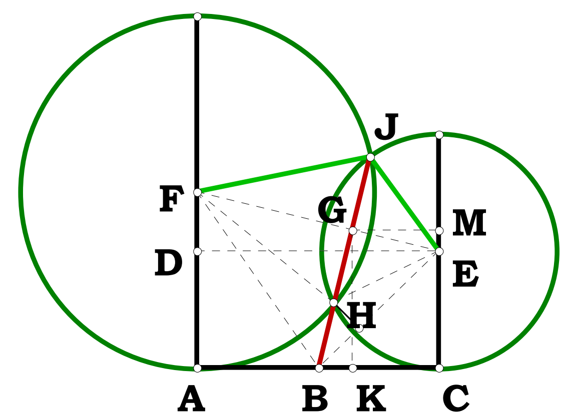
$FH := AF \quad EH := CE \quad AD := CE \quad DF := AF - AD$

$EJ := CE \quad FJ := AF \quad DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

$EG := \dfrac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF} \quad EM := \dfrac{DF \cdot EG}{EF} \quad CM := CE + EM$

$GM := \dfrac{DE \cdot EG}{EF} \quad CK := GM \quad GK := CM \quad BK := \dfrac{DF \cdot GK}{DE}$

$BC := CK + BK \quad BC - \dfrac{AC}{2} = 0$



Definitions.

$FH - N_1 = 0 \quad EH - N_2 = 0 \quad AD - N_2 = 0$

$DF - (N_1 - N_2) = 0 \quad EJ - N_2 = 0 \quad FJ - N_1 = 0$

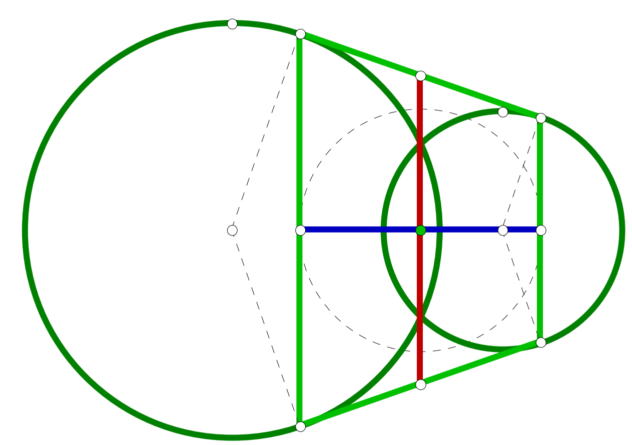
$DE - 1 = 0 \quad EF - \sqrt{(N_1 - N_2)^2 + 1} = 0$

$EG - \dfrac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot \sqrt{(N_1 - N_2)^2 + 1}} = 0 \quad EM - \dfrac{[1 - 2 \cdot N_2 \cdot (N_1 - N_2)] \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

$CM - \dfrac{N_1 + N_2}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0 \quad GM - \dfrac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

$CK - \dfrac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0 \quad GK - \dfrac{N_1 + N_2}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

$BK - \dfrac{(N_1 + N_2) \cdot (N_1 - N_2)}{2 \cdot (N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + 1)} = 0 \quad BC - \dfrac{1}{2} = 0$



062002B

Unit.

$AB := 1$

Given.

$N_1 := 7 \qquad N_2 := 19$

$N_3 := 15 \qquad N_4 := 17$

On the Other Hand

Have you ever pondered a figure in terms not of the particulars but of the concept of parallel lines? What does parallel mean? Between any two objects there is one, and only one, difference. If this is not true, what does it mean for existence itself? Is it not a self-referential fallacy? And since this is obvious, what does it say for so called non-Euclidean Geometers? If one cannot master the first prnciple of reasoning, can one ever know when they are speaking and thinking gibberish?

Descriptions.

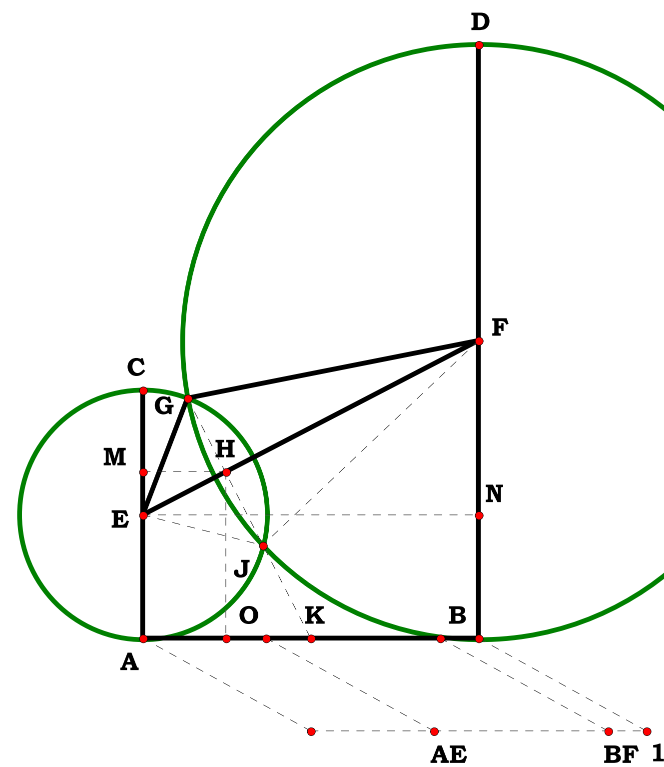
$$AE := \frac{N_1}{N_2} \qquad BF := \frac{N_3}{N_4} \qquad BN := AE$$

$$FN := BF - BN \qquad EF := \sqrt{AB^2 + FN^2}$$

$$EH := \frac{EF^2 + AE^2 - BF^2}{2 \cdot EF} \qquad AO := \frac{AB \cdot EH}{EF}$$

$$EM := \frac{FN \cdot EH}{EF} \qquad HO := AE + EM$$

$$KO := \frac{FN \cdot HO}{AB} \qquad AB - (AO + KO) = 0.5$$



Between any two objects there is one, and only one, difference such that the operations equitable to both, become half the difference between them and this is called their power.

Unit = 1.00000
AB = 1.00000
AE = 0.36842
N_1 = 7.00000
N_2 = 19.00000

BF = 0.88235
N_3 = 15.00000
N_4 = 17.00000

Definitions.

$$AE - \frac{N_1}{N_2} = 0 \qquad BF - \frac{N_3}{N_4} = 0 \qquad BN - \frac{N_1}{N_2} = 0$$

$$FN - \left(\frac{N_3}{N_4} - \frac{N_1}{N_2}\right) = 0 \qquad FN - \frac{N_2 \cdot N_3 - N_1 \cdot N_4}{N_2 \cdot N_4} = 0$$

$$EF - \frac{\sqrt{\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)}}{N_2 \cdot N_4} = 0$$

$$EH - \frac{\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot N_4}{2 \cdot N_2 \cdot N_4 \cdot \sqrt{N_4^2 \cdot \left(N_1^2 + N_2^2\right) + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)}} = 0$$

$$AO - \frac{N_4 \cdot \left[\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4 - 2 \cdot N_1 \cdot N_2 \cdot N_3\right]}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$EM - \frac{\left(N_2 \cdot N_3 - N_1 \cdot N_4\right) \cdot \left[\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4 - 2 \cdot N_1 \cdot N_2 \cdot N_3\right]}{2 \cdot N_2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$HO - \frac{N_2 \cdot N_4 \cdot \left(N_1 \cdot N_4 + N_2 \cdot N_3\right)}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$KO - \frac{N_2^2 \cdot N_3^2 - \left(N_1 \cdot N_4\right)^2}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0 \qquad AB - \frac{1}{2} = 0.5$$

071902

Descriptions.

Unit.

$AX := 1$

Given.

$N_1 := 9 \quad N_2 := 12$

$N_3 := 4 \quad N_4 := 11$

On Linear Division

If G were at A, then one would have the simple textbook method, however, if one took any point G on the perpendicular to AB, the results would be the same.

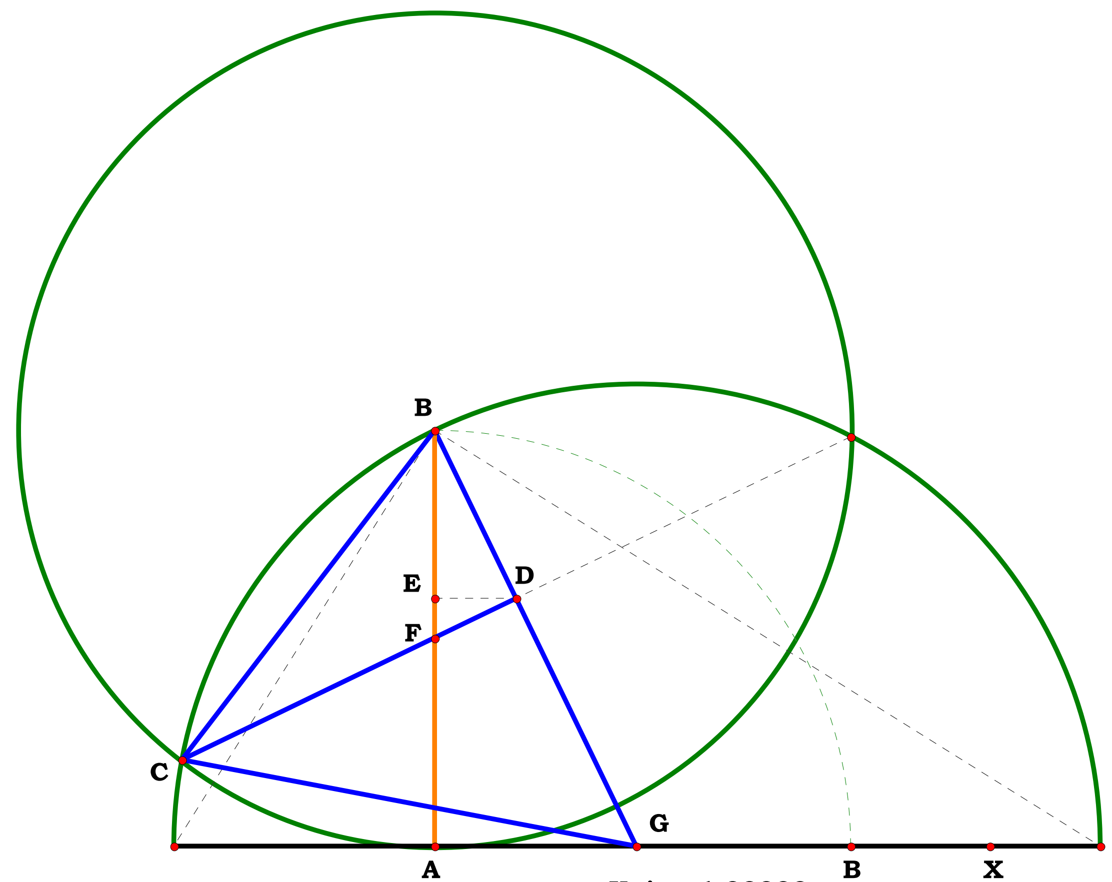
$$AB := \frac{N_1}{N_2} \qquad AG := \frac{N_3}{N_4} \qquad BG := \sqrt{AB^2 + AG^2}$$

I do not memorize equations; every time I have to use an equation from Pythagorus Revisited, I have to look it up.

$$BD := \frac{BG^2 + AB^2 - BG^2}{2 \cdot BG} \quad \text{Or again;} \quad BD_1 := \frac{AB^2}{2 \cdot BG}$$

$$BE := \frac{AB \cdot BD}{BG} \qquad DE := \frac{AG \cdot BE}{AB} \qquad EF := \frac{AG \cdot DE}{AB}$$

$$BF := BE + EF \qquad BF - \frac{AB}{2} = 0$$



Unit = 1.00000
x_X = 1.00000
AB = 0.75000
N_1 = 9.00000
N_2 = 12.00000

AG = 0.36364
N_3 = 4.00000
N_4 = 11.00000

Definitions.

$$AB - \frac{N_1}{N_2} = 0 \qquad AG - \frac{N_3}{N_4} = 0$$

$$BG - \frac{\sqrt{N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2}}{N_2 \cdot N_4} = 0 \qquad BD - \frac{N_1^2 \cdot N_4}{2 \cdot N_2 \cdot \sqrt{N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2}} = 0$$

$$BE - \frac{N_1^3 \cdot N_4^2}{2 \cdot N_2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0 \qquad DE - \frac{N_1^2 \cdot N_3 \cdot N_4}{2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0$$

$$EF - \frac{N_1 \cdot N_2 \cdot N_3^2}{2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0 \qquad BF - \frac{N_1}{2 \cdot N_2} = 0$$

090902

Unit. $AB := 1$

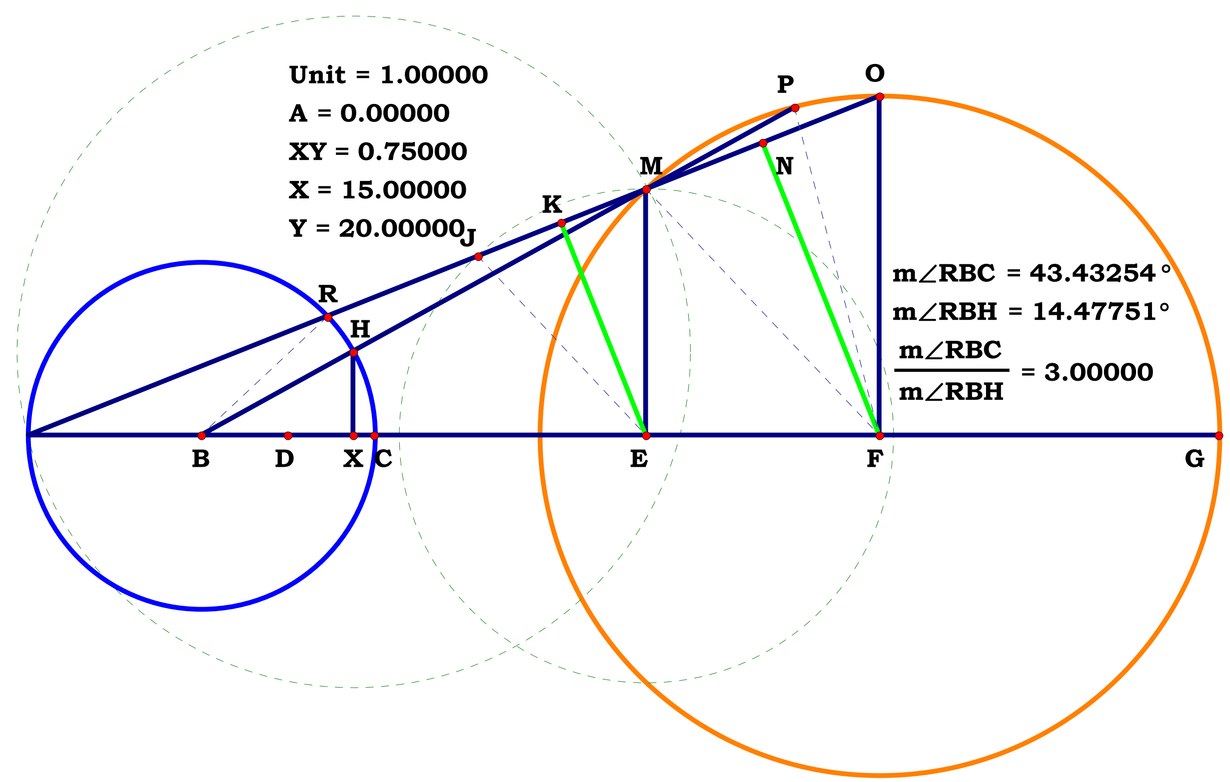
Given.

$X := 15$

$Y := 20$

Trisection Illusion

Basically, one is simply adding one half of angle CBH to it.



Description.

$BC := AB \quad CD := \frac{BC}{2} \quad DX := CD \cdot \frac{X}{Y} \quad BX := CD + DX$

$AX := AB + CD + DX \quad HX := \sqrt{AX \cdot (2 \cdot AB - AX)}$

$HM := \sqrt{AX^2 + HX^2} \quad BM := AB + HM \quad EM := HX \cdot \frac{BM}{AB}$

$BE := BX \cdot \frac{EM}{HX} \quad AE := AB + BE \quad AM := \sqrt{AE^2 + EM^2}$

$AK := \frac{AE^2}{AM} \quad KM := AM - AK \quad AJ := AM - 2 \cdot KM$

$AF := \frac{AE \cdot AM}{AJ} \quad FO := AF \cdot \frac{EM}{AE} \quad FO = 1.957661$

Definitions.

$BC - 1 = 0 \quad CD - \frac{1}{2} = 0 \quad DX - \frac{X}{2 \cdot Y} = 0 \quad BX - \frac{X + Y}{2 \cdot Y} = 0$

$AX - \frac{X + 3 \cdot Y}{2 \cdot Y} = 0 \quad HX - \frac{\sqrt{(Y - X) \cdot (X + 3 \cdot Y)}}{2 \cdot Y} = 0 \quad HM - \frac{\sqrt{X + 3 \cdot Y}}{\sqrt{Y}} = 0$

$BM - \frac{\sqrt{Y} + \sqrt{X + 3 \cdot Y}}{\sqrt{Y}} = 0 \quad EM - \frac{\sqrt{(Y - X) \cdot (X + 3 \cdot Y)} \cdot \left(\sqrt{Y} + \sqrt{X + 3 \cdot Y}\right)}{2 \cdot Y^{\frac{3}{2}}} = 0 \quad BE - \frac{(X + Y) \cdot \left(\sqrt{Y} + \sqrt{X + 3 \cdot Y}\right)}{2 \cdot Y^{\frac{3}{2}}} = 0 \quad AE - \frac{X \cdot \sqrt{Y} + \sqrt{X + 3 \cdot Y} \cdot (X + Y) + 3 \cdot Y^{\frac{3}{2}}}{2 \cdot Y^{\frac{3}{2}}} = 0$

$$AM - \frac{\sqrt{\left(X^2 + 4 \cdot X \cdot Y + 3 \cdot Y^2\right) \cdot \sqrt{X + 3 \cdot Y} + 4 \cdot X \cdot Y^{\frac{3}{2}} - X \cdot (X + 3 \cdot Y)^{\frac{3}{2}} + Y \cdot (X + 3 \cdot Y)^{\frac{3}{2}} + 12 \cdot Y^{\frac{5}{2}}}}{\sqrt{2} \cdot \sqrt{Y^{\frac{5}{2}}}} = 0$$

$$AK-\frac{\frac{\sqrt{2}}{4}\cdot\sqrt{Y^{\frac{5}{2}}}\cdot\left(X\cdot\sqrt{Y}+X\cdot\sqrt{X+3\cdot Y}+Y\cdot\sqrt{X+3\cdot Y}+3\cdot Y^{\frac{3}{2}}\right)^{2}}{Y^{3}\cdot\sqrt{4\cdot X\cdot Y^{\frac{3}{2}}-X\cdot(X+3\cdot Y)^{\frac{3}{2}}+Y\cdot(X+3\cdot Y)^{\frac{3}{2}}+12\cdot Y^{\frac{5}{2}}+\sqrt{X+3\cdot Y}\cdot\left(X^{2}+4\cdot X\cdot Y+3\cdot Y^{2}\right)}}=0$$

$$KM-\left[\frac{\left(\sqrt{Y}\right)^{5}\cdot(Y-X)\cdot\left[\sqrt{2}\cdot X^{2}+12\cdot\sqrt{2}\cdot Y^{2}+7\cdot\sqrt{2}\cdot X\cdot Y+2\cdot\sqrt{2}\cdot\sqrt{Y}\cdot(X+3\cdot Y)^{\frac{3}{2}}\right]}{4\cdot Y^{3}\cdot\sqrt{Y^{\frac{5}{2}}}\cdot\sqrt{4\cdot X\cdot Y^{\frac{3}{2}}-X\cdot(X+3\cdot Y)^{\frac{3}{2}}+Y\cdot(X+3\cdot Y)^{\frac{3}{2}}+12\cdot Y^{\frac{5}{2}}+X^{2}\cdot\sqrt{X+3\cdot Y}+3\cdot Y^{2}\cdot\sqrt{X+3\cdot Y}+4\cdot X\cdot Y\cdot\sqrt{X+3\cdot Y}}}\right]=0$$

$$AJ-\frac{\sqrt{2}\cdot\left[9\cdot X\cdot Y^{2}+6\cdot X^{2}\cdot Y+X^{3}-Y^{\frac{3}{2}}\cdot(X+3\cdot Y)^{\frac{3}{2}}+3\cdot Y^{\frac{5}{2}}\cdot\sqrt{X+3\cdot Y}+X\cdot\sqrt{Y}\cdot(X+3\cdot Y)^{\frac{3}{2}}+4\cdot X\cdot Y^{\frac{3}{2}}\cdot\sqrt{X+3\cdot Y}+X^{2}\cdot\sqrt{Y}\cdot\sqrt{X+3\cdot Y}\right]}{2\cdot\sqrt{Y}\cdot\sqrt{Y^{\frac{5}{2}}}\cdot\sqrt{4\cdot X\cdot Y^{\frac{3}{2}}-X\cdot(X+3\cdot Y)^{\frac{3}{2}}+Y\cdot(X+3\cdot Y)^{\frac{3}{2}}+12\cdot Y^{\frac{5}{2}}+X^{2}\cdot\sqrt{X+3\cdot Y}+3\cdot Y^{2}\cdot\sqrt{X+3\cdot Y}+4\cdot X\cdot Y\cdot\sqrt{X+3\cdot Y}}}=0$$

$$AF-\frac{\left(X\cdot\sqrt{Y}+X\cdot\sqrt{X+3\cdot Y}+Y\cdot\sqrt{X+3\cdot Y}+3\cdot Y^{\frac{3}{2}}\right)\cdot\left[4\cdot X\cdot Y^{\frac{3}{2}}-X\cdot(X+3\cdot Y)^{\frac{3}{2}}+Y\cdot(X+3\cdot Y)^{\frac{3}{2}}+12\cdot Y^{\frac{5}{2}}+X^{2}\cdot\sqrt{X+3\cdot Y}+3\cdot Y^{2}\cdot\sqrt{X+3\cdot Y}+4\cdot X\cdot Y\cdot\sqrt{X+3\cdot Y}\right]}{2\cdot Y\cdot\left[9\cdot X\cdot Y^{2}+6\cdot X^{2}\cdot Y+X^{3}-Y^{\frac{3}{2}}\cdot(X+3\cdot Y)^{\frac{3}{2}}+3\cdot Y^{\frac{5}{2}}\cdot\sqrt{X+3\cdot Y}+X\cdot\sqrt{Y}\cdot(X+3\cdot Y)^{\frac{3}{2}}+4\cdot X\cdot Y^{\frac{3}{2}}\cdot\sqrt{X+3\cdot Y}+X^{2}\cdot\sqrt{Y}\cdot\sqrt{X+3\cdot Y}\right]}=0$$

$$FO-\frac{\sqrt{(Y-X)\cdot(X+3\cdot Y)}\cdot\left(\sqrt{Y}+\sqrt{X+3\cdot Y}\right)\cdot\left[4\cdot X\cdot Y^{\frac{3}{2}}-X\cdot(X+3\cdot Y)^{\frac{3}{2}}+Y\cdot(X+3\cdot Y)^{\frac{3}{2}}+12\cdot Y^{\frac{5}{2}}+X^{2}\cdot\sqrt{X+3\cdot Y}+3\cdot Y^{2}\cdot\sqrt{X+3\cdot Y}+4\cdot X\cdot Y\cdot\sqrt{X+3\cdot Y}\right]}{2\cdot Y\cdot\left[9\cdot X\cdot Y^{2}+6\cdot X^{2}\cdot Y+X^{3}-Y^{\frac{3}{2}}\cdot(X+3\cdot Y)^{\frac{3}{2}}+3\cdot Y^{\frac{5}{2}}\cdot\sqrt{X+3\cdot Y}+X\cdot\sqrt{Y}\cdot(X+3\cdot Y)^{\frac{3}{2}}+4\cdot X\cdot Y^{\frac{3}{2}}\cdot\sqrt{X+3\cdot Y}+X^{2}\cdot\sqrt{Y}\cdot\sqrt{X+3\cdot Y}\right]}=0$$

092102

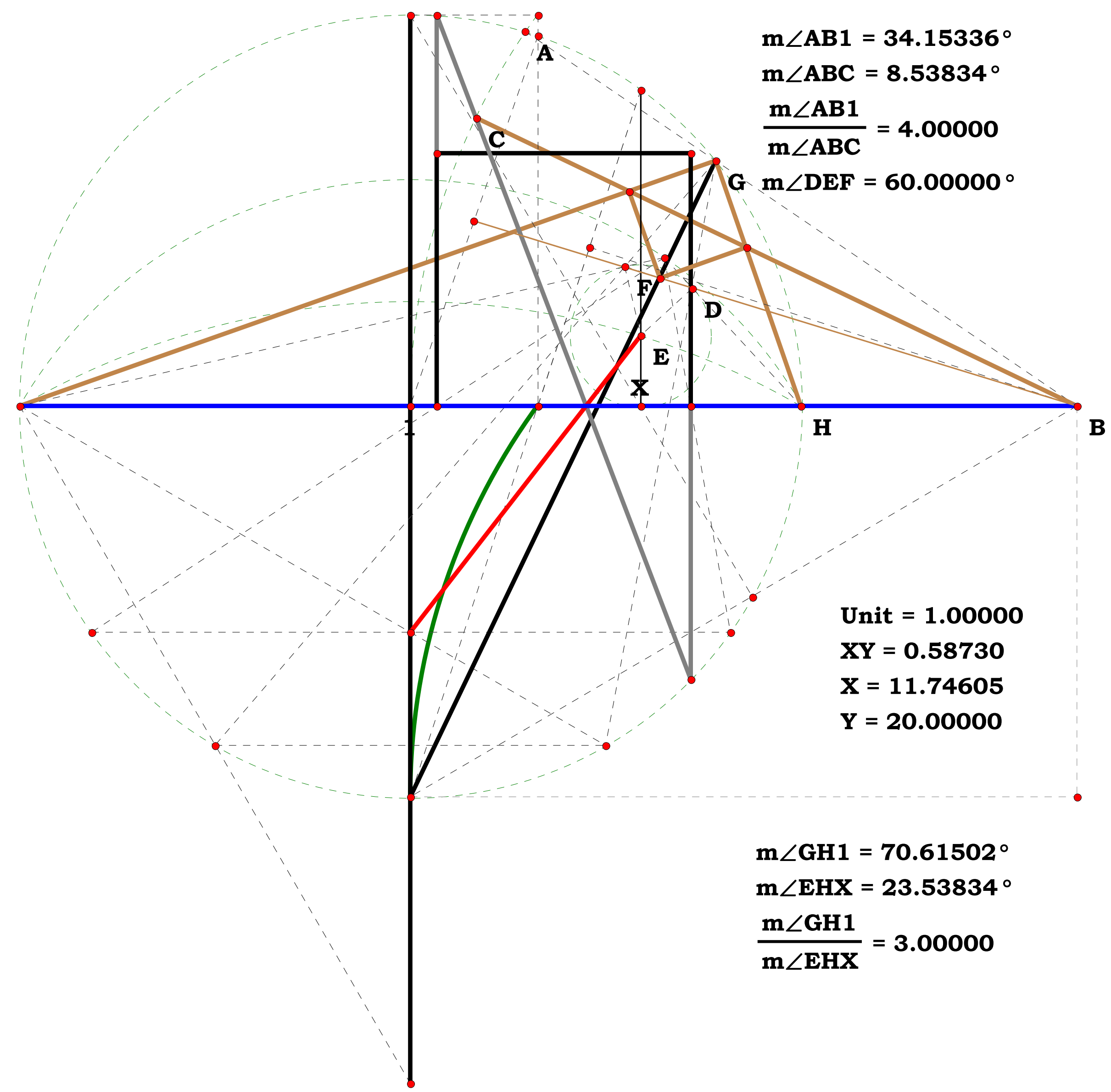
Descriptions.
Definitions.

Unit.
Given.

Project 092102

Name the segement in red.

And various other structures.



m∠AB1 = 34.15336°
m∠ABC = 8.53834°

$\frac{m\angle AB1}{m\angle ABC}$ = 4.00000

m∠DEF = 60.00000°

Unit = 1.00000
XY = 0.58730
X = 11.74605
Y = 20.00000

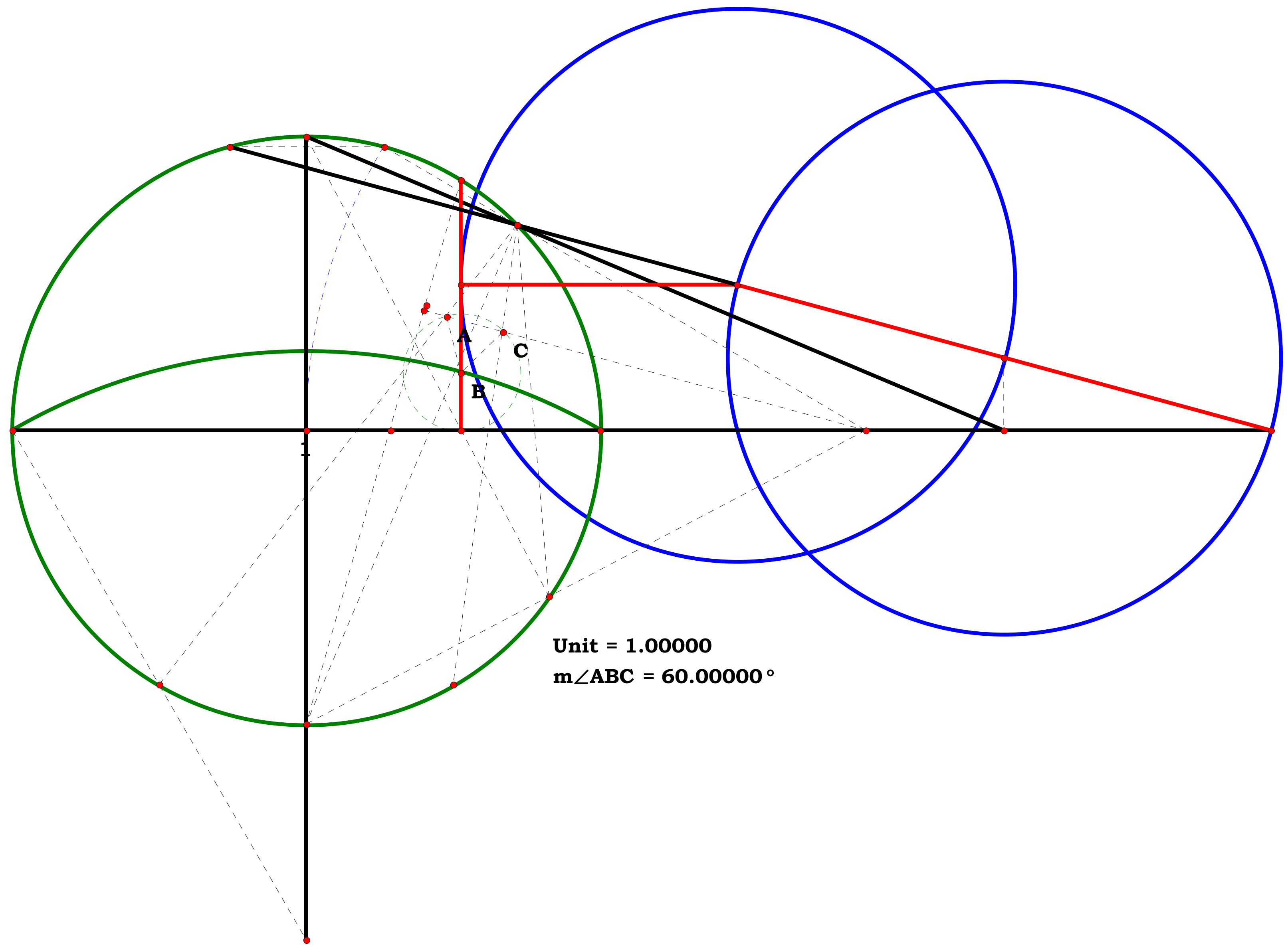
m∠GH1 = 70.61502°
m∠EHX = 23.53834°

$\frac{m\angle GH1}{m\angle EHX}$ = 3.00000

Unit.
Given.

Pair of Blue Circles

Descriptions.
Definitions.

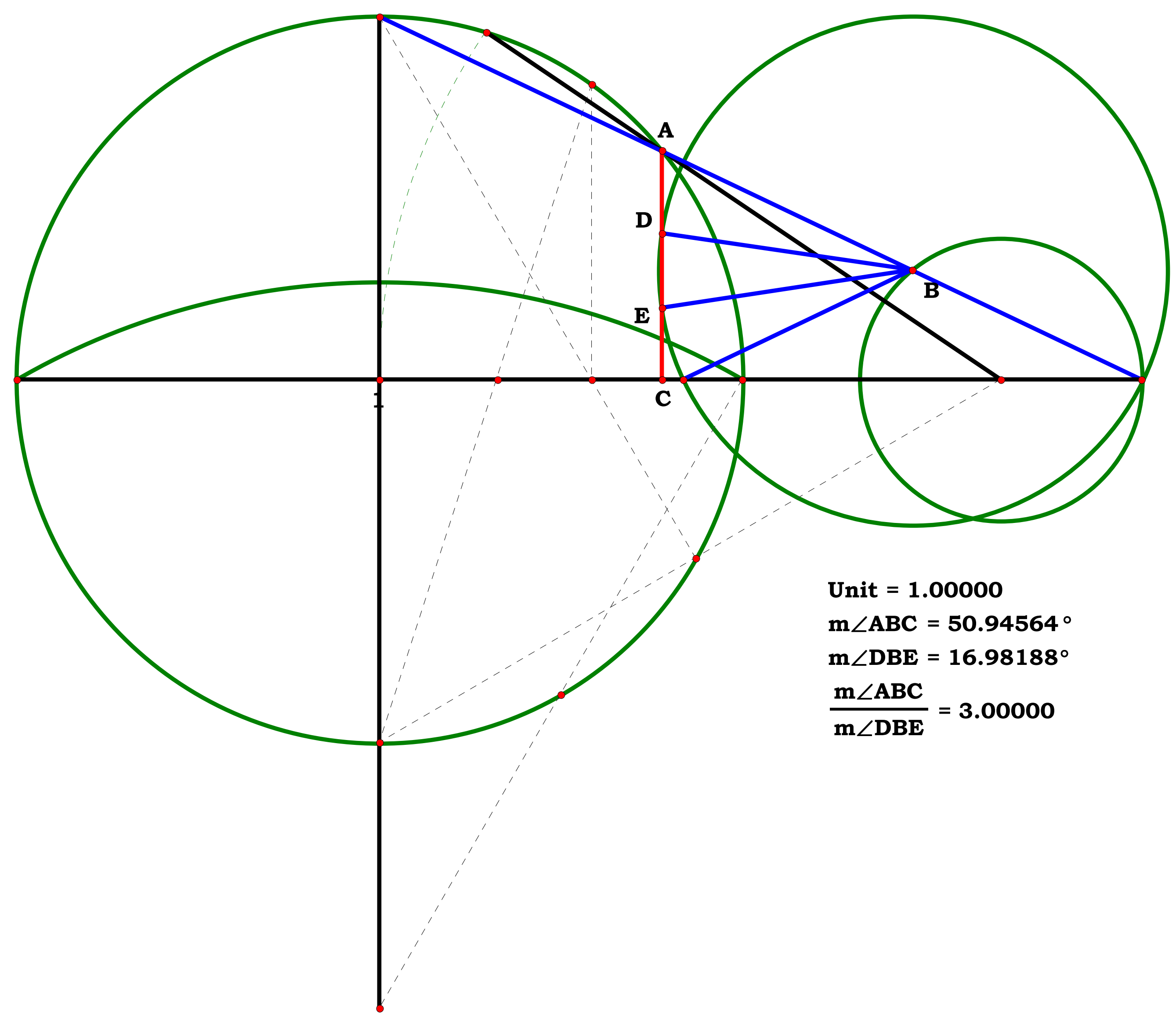


093002

Unit.
Given.

Trisection by Pole

Descriptions.
Definitions.



Unit = 1.00000
m∠ABC = 50.94564°
m∠DBE = 16.98188°

$\frac{m\angle ABC}{m\angle DBE} = 3.00000$

Parcing project 100402

Unit.
Given.

Descriptions.
Definitions.

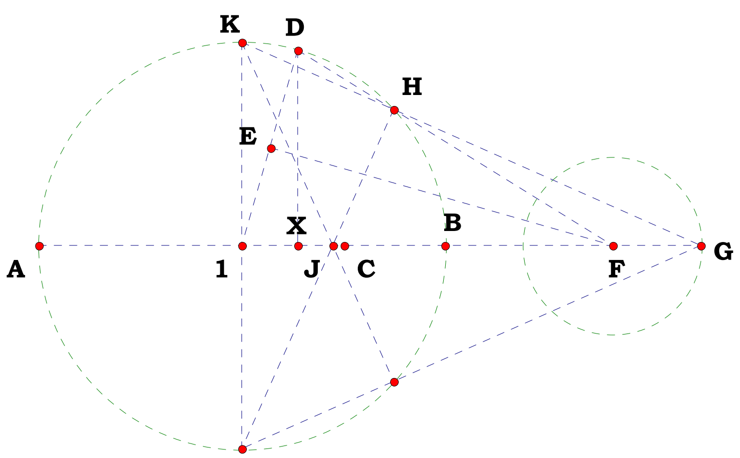


Plate A

Unit = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000

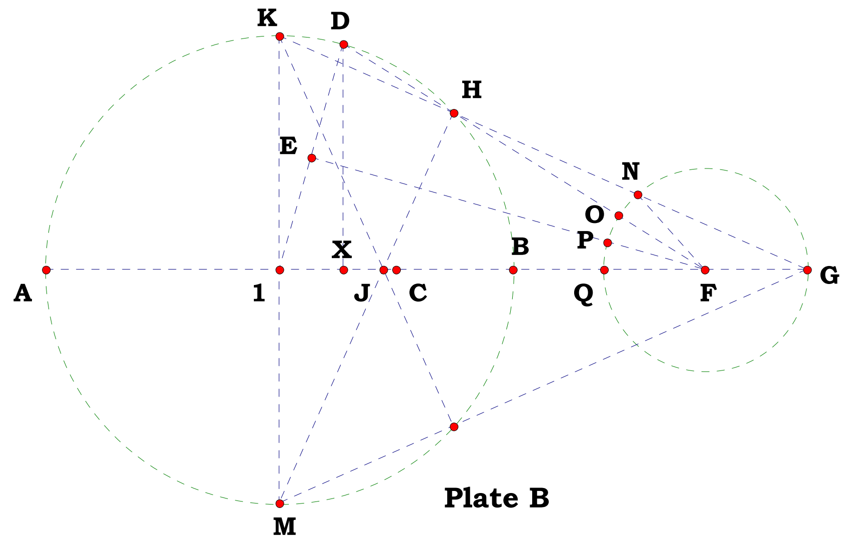


Plate B

Unit = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000

$m\angle NFQ = 47.88604°$
$m\angle NFO = 15.96201°$
$m\angle OFP = 15.96201°$
$m\angle PFQ = 15.96201°$
$\frac{m\angle NFQ}{m\angle NFO} = 3.00000$

Adding 042398

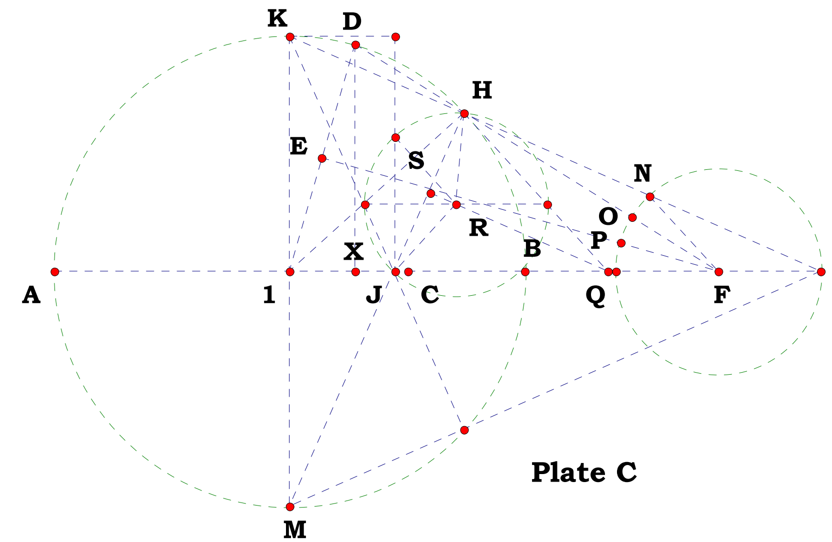


Plate C

Unit = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000

$m\angle NFQ = 47.88604°$
$m\angle NFO = 15.96201°$
$m\angle OFP = 15.96201°$
$m\angle PFQ = 15.96201°$
$\frac{m\angle NFQ}{m\angle NFO} = 3.00000$

$m\angle HRJ = 143.65813°$
$m\angle HRS = 47.88604°$
$\frac{m\angle HRJ}{m\angle HRS} = 3.00000$
$\frac{m\angle HRJ}{m\angle NFO} = 9.00000$

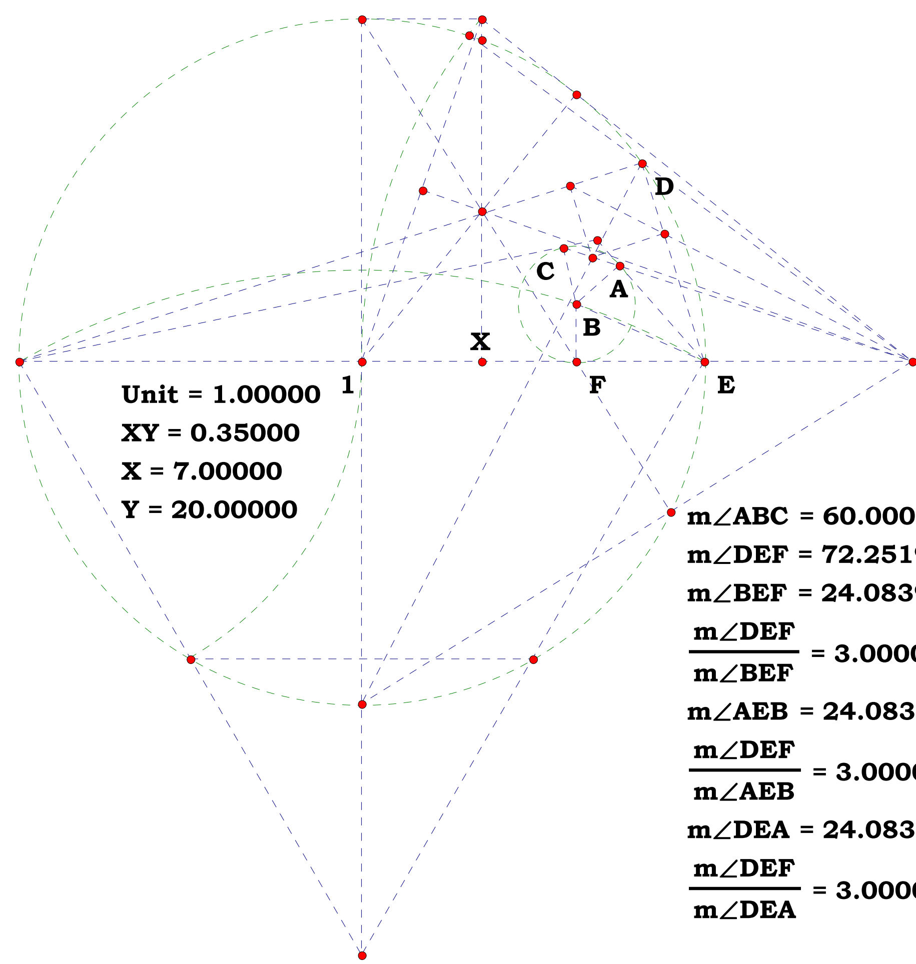
One can add all the developments if one likes and see how they all relate.

Unit.
Given.

Parcing project 101402

Square, rectangle and complements.

Descriptions.
Definitions.



m∠ABC = 60.00000°
m∠DEF = 72.25197°
m∠BEF = 24.08399°

$$\frac{m\angle DEF}{m\angle BEF} = 3.00000$$

m∠AEB = 24.08399°

$$\frac{m\angle DEF}{m\angle AEB} = 3.00000$$

m∠DEA = 24.08399°

$$\frac{m\angle DEF}{m\angle DEA} = 3.00000$$

111402

Unit.
Given.

Parcing project for 111402

Descriptions.
Definitions.

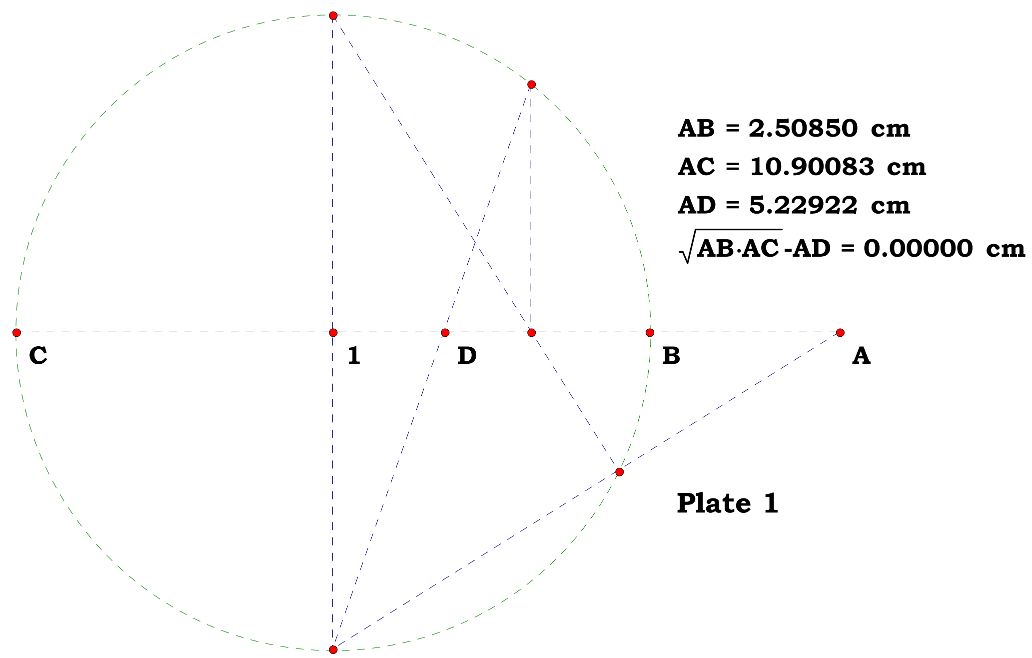


Plate 1

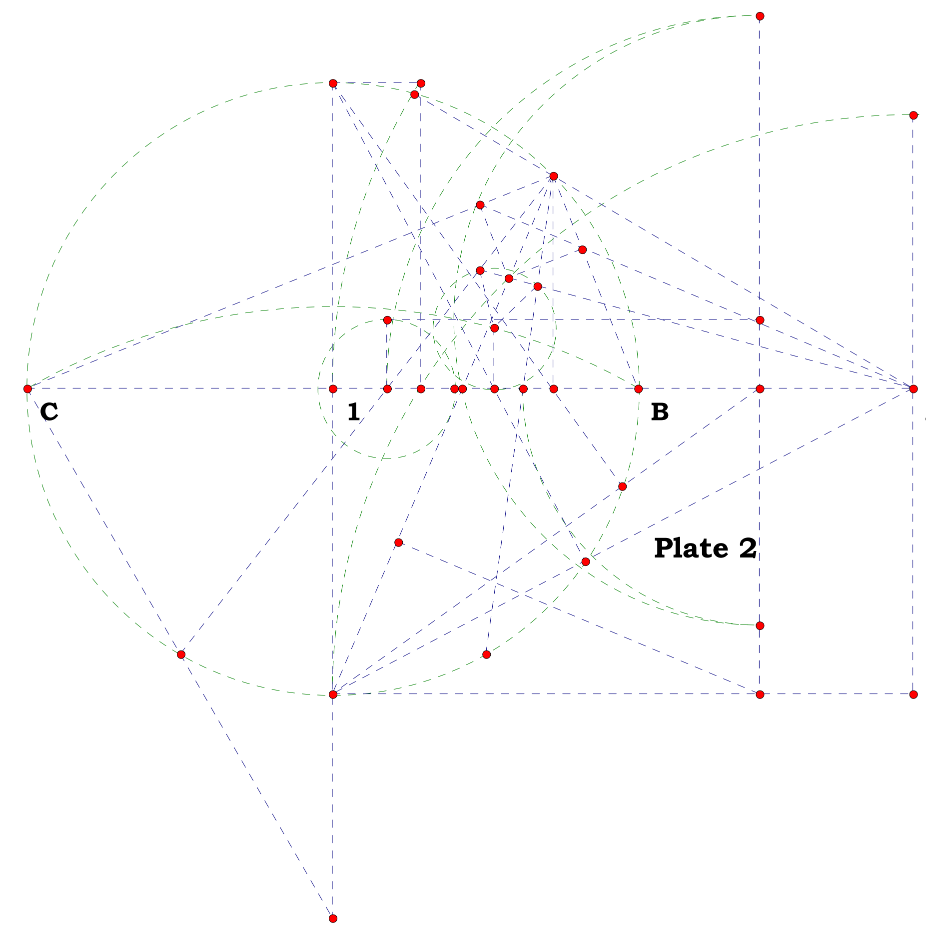
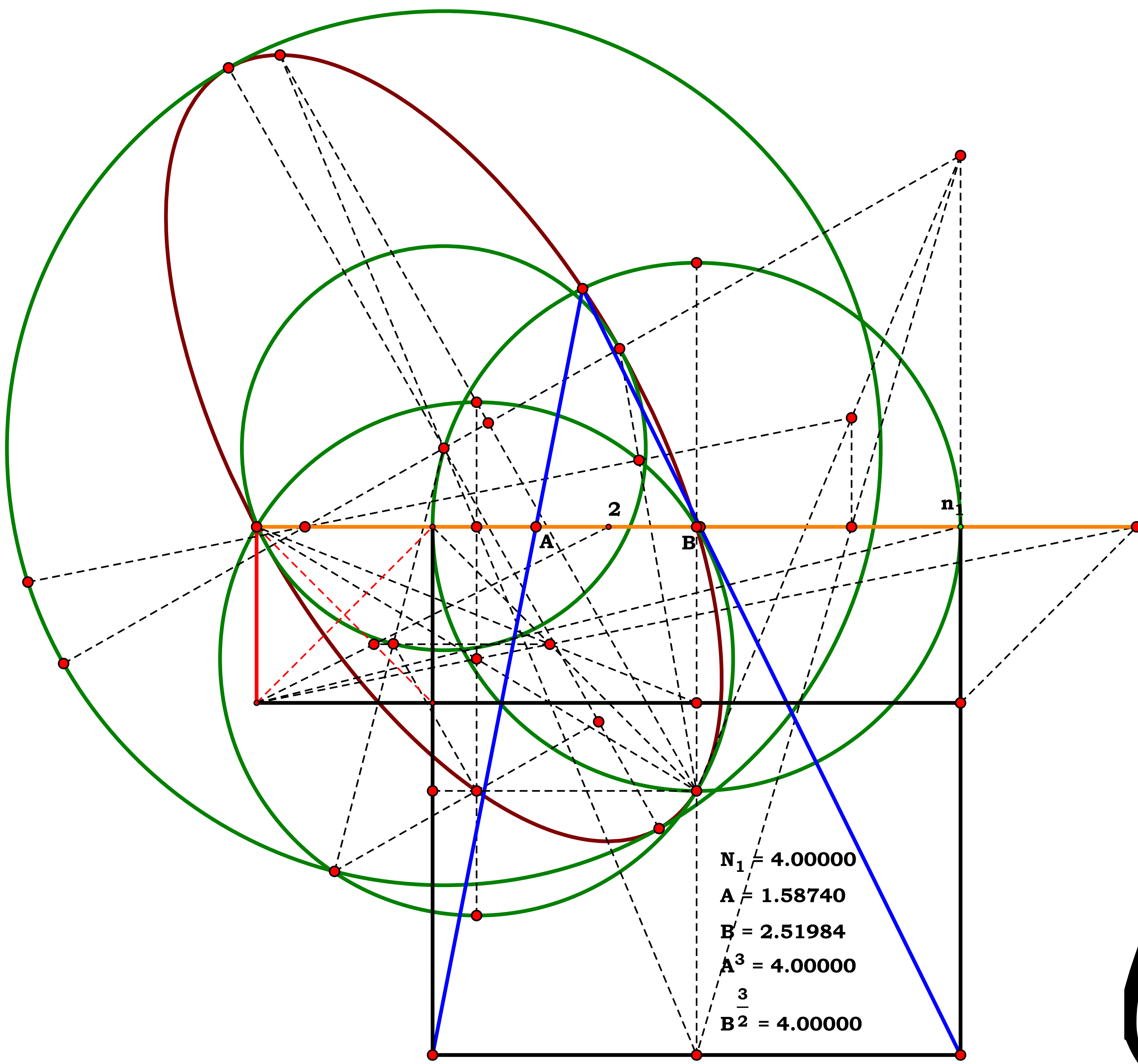


Plate 2



The Delian Quest 2003

John Clark

021603

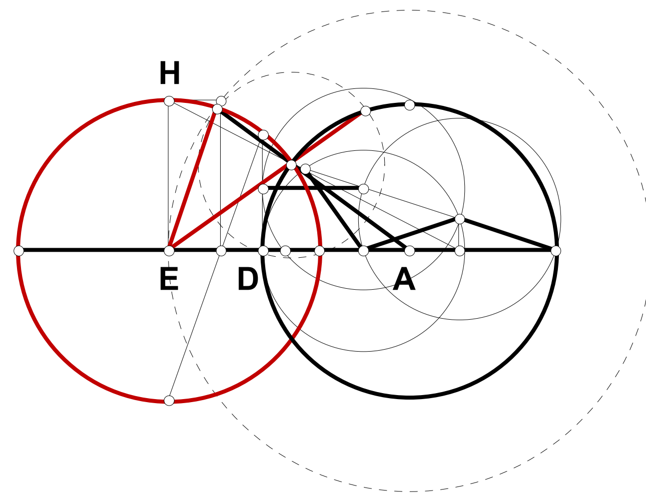
Descriptions.

Unit.

$AD := 1$

Given.

$N := 1.5$



From AD project a trisection to EH. Make AD the unit.

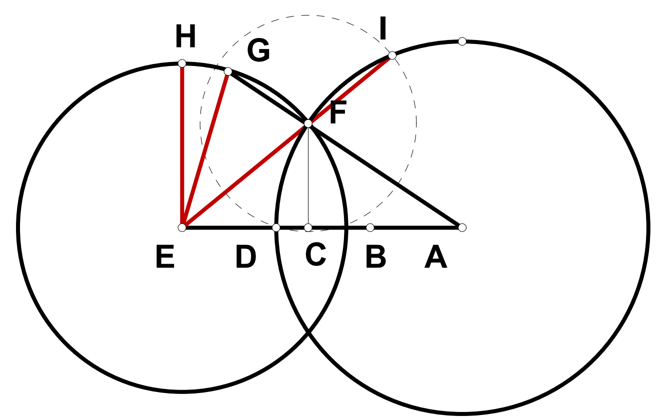
$AB := \frac{AD}{2}$

$BD := AD - AB \qquad BC := \frac{BD}{N}$

$CD := BD - BC \qquad AC := AB + BC$

$AF := AD \qquad CF := \sqrt{AF^2 - AC^2}$

$FI := \sqrt{CD^2 + CF^2}$



$DI := 2 \cdot CF \qquad AI := AD$

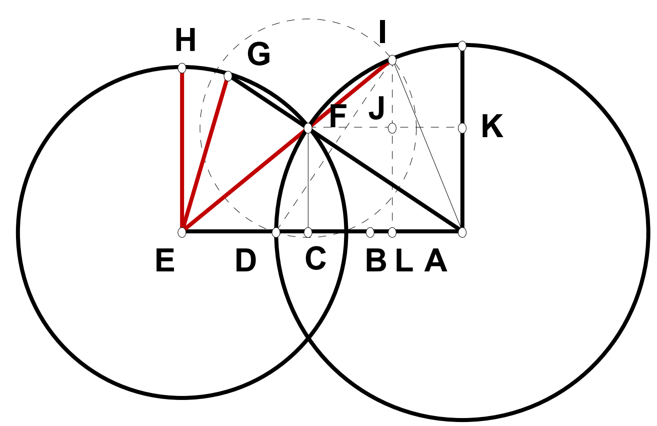
$AL := \frac{\left(2AD^2 - DI^2\right)}{2 \cdot AD}$

$FJ := AC - AL \qquad IL := \sqrt{AI^2 - AL^2}$

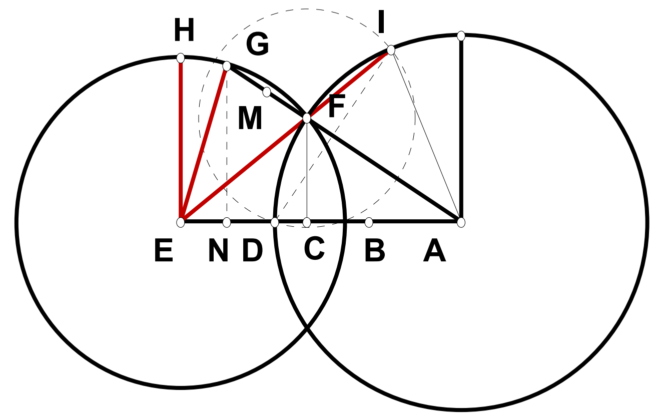
$IJ := IL - CF \qquad EL := \frac{FJ \cdot IL}{IJ}$

$AE := AL + EL \qquad EI := \frac{FI \cdot IL}{IJ}$

$EF := EI - FI$



$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF}$$



$$AG := AF + FG$$

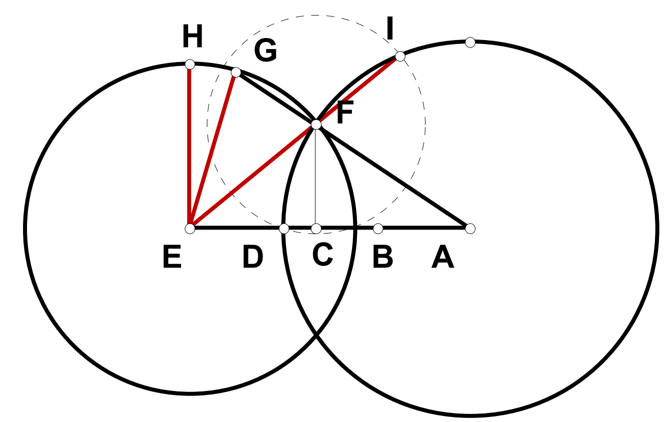
$$AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \qquad FM := \frac{FG}{2}$$

$$FM - EN = 0$$

Definitions. In the following definitions, one should remember, N is not something which is a part of the figure, it is an assertion you make of the figure.

$$AB - \frac{1}{2} = 0 \qquad BD - (N-1) = 0 \qquad BC - \frac{N-1}{N} = 0 \qquad CD - \frac{(N-1)^2}{N} = 0$$

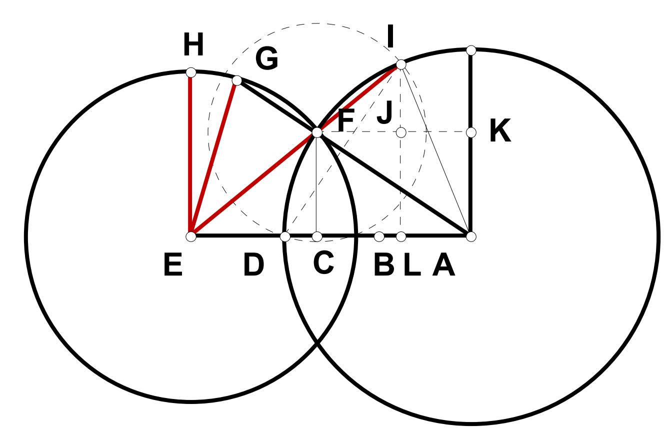


$$AC - \frac{3 \cdot N - 2}{2 \cdot N} = 0 \qquad AF - 1 = 0 \qquad CF - \frac{\sqrt{(2-N)\cdot(5\cdot N-2)}}{2\cdot N} = 0$$

$$FI - \frac{\sqrt{4\cdot N^3 - 16\cdot N^2 + 19\cdot N - 4}}{2\cdot\sqrt{N}} = 0 \qquad DI - \frac{\sqrt{(2-N)\cdot(5\cdot N-2)}}{N} = 0$$

$$AI - 1 = 0 \qquad AL - \frac{7\cdot N^2 - 12\cdot N + 4}{2\cdot N^2} = 0 \qquad FJ - \frac{(2\cdot N-1)\cdot(2-N)}{N^2} = 0$$

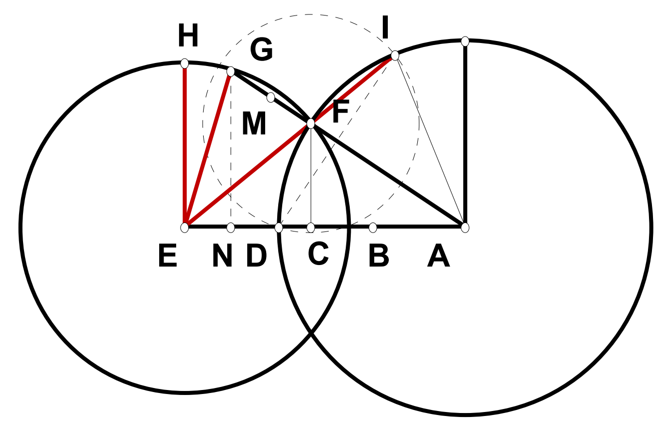
$$IL - \frac{\sqrt{(5\cdot N-2)\cdot(2-N)}\cdot(3\cdot N-2)}{2\cdot N^2} = 0 \qquad IJ - \frac{\sqrt{12\cdot N - 5\cdot N^2 - 4}\cdot(N-1)}{N^2} = 0$$



$$EL - \frac{(2-N)\cdot(2\cdot N-1)\cdot(3\cdot N-2)\cdot\sqrt{(2-N)\cdot(5\cdot N-2)}}{2\cdot N^2\cdot(N-1)\cdot\sqrt{12\cdot N - 5\cdot N^2 - 4}} = 0 \qquad AE - \frac{N}{2\cdot(N-1)} = 0$$

$$EI - \frac{(3\cdot N-2)\cdot\sqrt{-(N-2)\cdot(5\cdot N-2)}\cdot\sqrt{4\cdot N^3 - 16\cdot N^2 + 19\cdot N - 4}}{4\cdot\sqrt{N}\cdot\left(\sqrt{N}-1\right)\cdot\left(\sqrt{N}+1\right)\cdot\sqrt{12\cdot N - 5\cdot N^2 - 4}} = 0 \qquad EF - \frac{\sqrt{N}\cdot\sqrt{4\cdot N^3 - 16\cdot N^2 + 19\cdot N - 4}}{4\cdot\left(\sqrt{N}-1\right)\cdot\left(\sqrt{N}+1\right)} = 0$$

$$FG - \frac{\left(16 \cdot N^3 - 4 \cdot N^4 - 31 \cdot N^2 + 36 \cdot N - 16\right)}{16 \cdot (N-1)^2} = 0$$



$$AG - \frac{N \cdot (1 - 2 \cdot N) \cdot \left(2 \cdot N^2 - 7 \cdot N + 4\right)}{16 \cdot (N-1)^2} = 0$$

$$AN - \frac{(1 - 2 \cdot N) \cdot (3 \cdot N - 2) \cdot \left(2 \cdot N^2 - 7 \cdot N + 4\right)}{32 \cdot (N-1)^2} = 0$$

$$EN - \frac{12 \cdot N^4 - 56 \cdot N^3 + 93 \cdot N^2 - 58 \cdot N + 8}{32 \cdot (N-1)^2} = 0$$

$$FM - \frac{16 \cdot N^3 - 4 \cdot N^4 - 31 \cdot N^2 + 36 \cdot N - 16}{32 \cdot (N-1)^2} = 0$$

$$FM - EN = 0$$

$$\frac{(2 \cdot N - 3) \cdot (1 - 2 \cdot N) \cdot \left(2 \cdot N^2 - 5 \cdot N + 4\right)}{16 \cdot (N-1)^2} = 0 \qquad 16 \cdot (N-1)^2 = 4 \qquad (2 \cdot N - 3) \cdot (1 - 2 \cdot N) \cdot \left(2 \cdot N^2 - 5 \cdot N + 4\right) = 0$$

$$36 \cdot N^3 - 8 \cdot N^4 - 62 \cdot N^2 + 47 \cdot N - 12$$

$$36 \cdot N^3 - 8 \cdot N^4 - 62 \cdot N^2 + 47 \cdot N - 12 = 0$$

Solve for N.

$$\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{5}{4} + \frac{\sqrt{7} \cdot i}{4} \\ \frac{5}{4} - \frac{\sqrt{7} \cdot i}{4} \end{pmatrix}$$

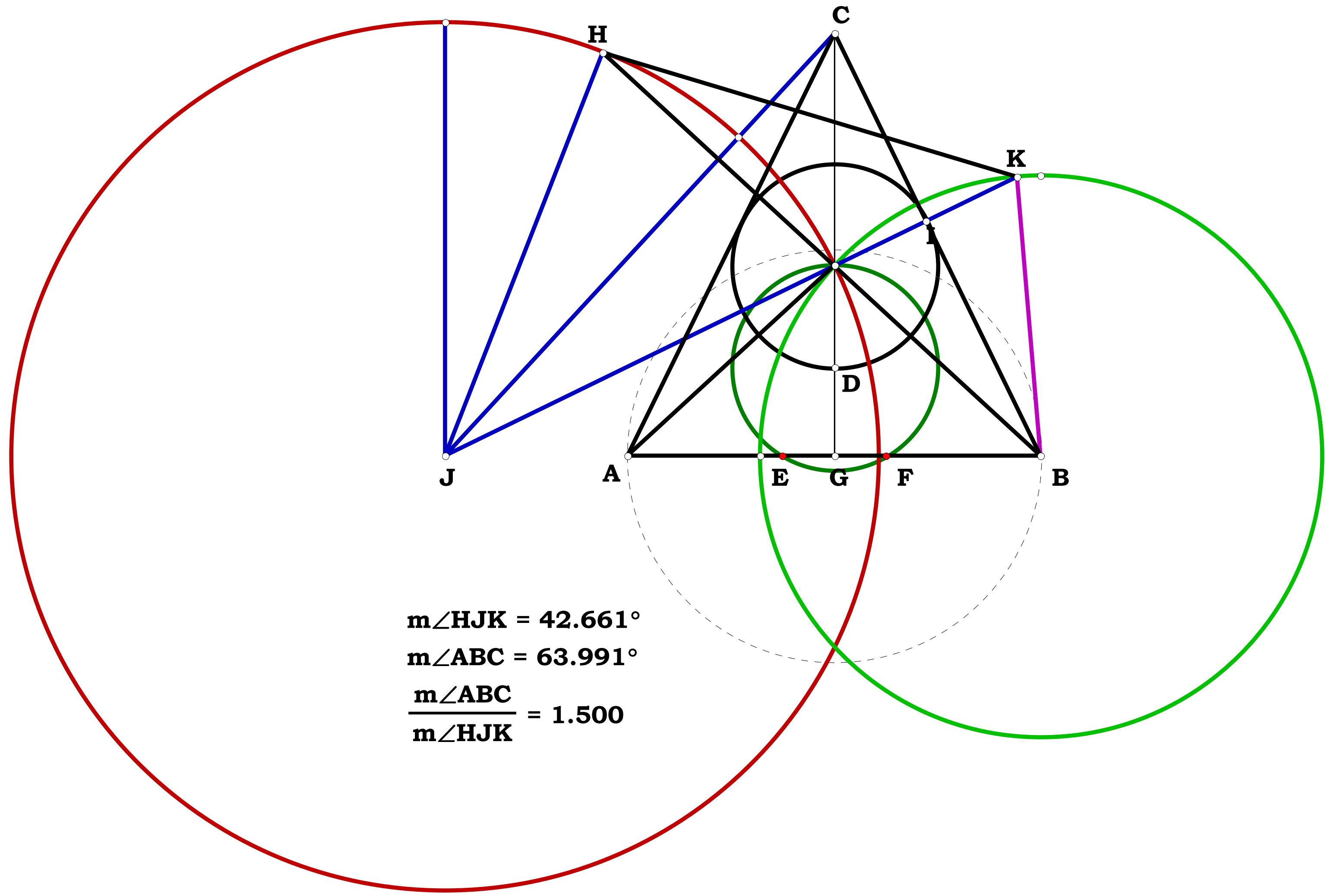
Two of these solutions are not even part of the grammar. If one finds anyone who defends them, as them, show me how complete induction and deduction of a unit cannot produce anything more than a recursion of that unit.

Unit.
Given.

Twin Circles in an Equilateral Triangle

021903

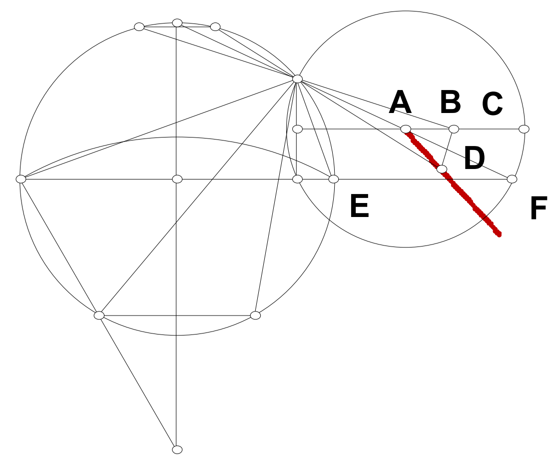
Descriptions.
Definitions.



Fair Pencil Construction

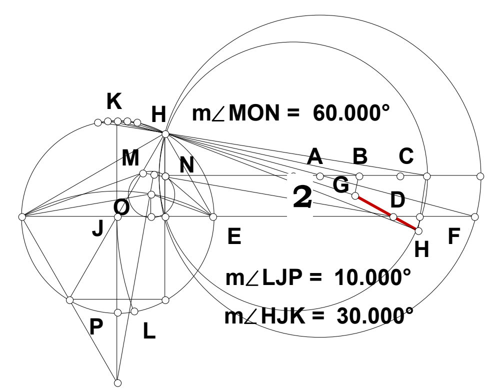
022803

If D were between EF, then it would be the point sought for angle trisection. When it is moved between A and C the locus AD is formed. This locus is not straight, but it is fairly straight.

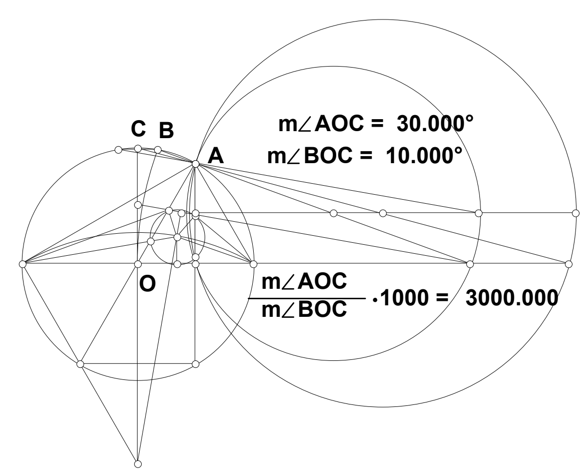


Even if a large segment is used from two points on AD, say GH for an intercept, this drawing program claims that I have attained a trisection for any angle.

If a very small segment is taken, as below, tolerance is beyond the drawing program. What does it look like using Algebra? Trisection to within millionths, in some circumstances, may be tolerable.



As the different points of intersection is not actually viewable on the finer figure, the rougher figure will be used for drawings, but the equations will refer to the finer.



3

Unit.

$AC := 1$

Given.

$N_1 := 2$

$N_2 := 4$

Descriptions.

$AO := \dfrac{AC}{2}$

$AB := \dfrac{AC}{N_2} \qquad BC := AC - AB$

$BE := \sqrt{AB \cdot BC} \qquad DE := \dfrac{BE}{2}$

$BO := AO - AB \qquad GO := AO$

$DL := \dfrac{BO \cdot DE}{GO - BE} \qquad EL := \sqrt{DL^2 + DE^2}$

$LN := EL \qquad LM := \dfrac{LN}{N_1} \qquad DM := DL + LM \qquad EM := \sqrt{DE^2 + DM^2} \qquad EP := \dfrac{EM \cdot BE}{DE}$

$BP := \dfrac{DM \cdot BE}{DE} \qquad PO := BP + BO$

$EH := \dfrac{AO^2 + EP^2 - PO^2}{-EP}$

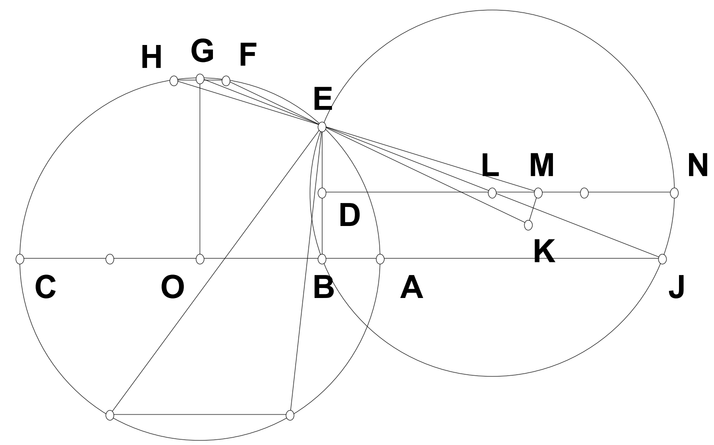
$HP := EP + EH \qquad PR := \dfrac{BP \cdot HP}{EP}$

$OR := PR - PO \qquad BQ := BO - OR$

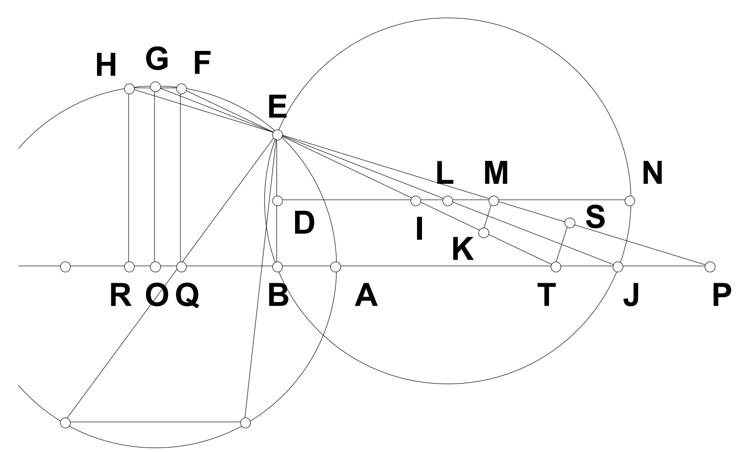
$HR := \dfrac{BE \cdot HP}{EP} \qquad BT := \dfrac{BQ \cdot BE}{HR - BE}$

$ET := \sqrt{BE^2 + BT^2} \qquad PT := BP - BT \qquad PS := \dfrac{PT^2 + EP^2 - ET^2}{2 \cdot EP} \qquad ES := EP - PS \qquad ST := \sqrt{ET^2 - ES^2}$

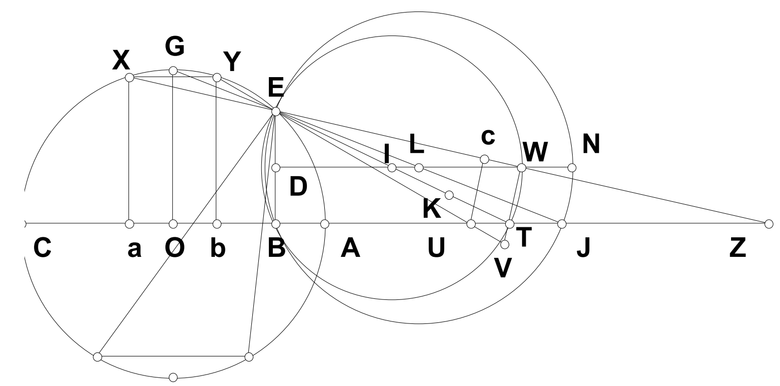
$KM := \dfrac{ST \cdot EM}{ES} \qquad EI := \dfrac{ET}{2} \qquad EK := \dfrac{ET \cdot EM}{ES} \qquad OT := BT + BO$



4



5



6

$IW := E]DI := \sqrt{EI^2 - DE^2} \qquad DW := DI + IW$

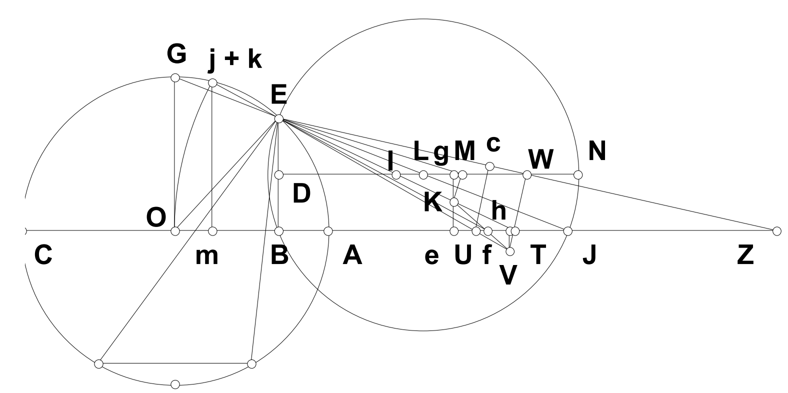
$EW := \sqrt{DE^2 + DW^2} \qquad EZ := \frac{EW \cdot BE}{DE} \qquad BZ := \frac{DW \cdot BE}{DE}$

$OZ := BZ + BO \qquad TW := \sqrt{ET^2 - EW^2}$

$EX := \frac{AO^2 + EZ^2 - OZ^2}{-EZ} \qquad XZ := EZ + EX$

$Za := \frac{BZ \cdot XZ}{EZ} \qquad Oa := Za - OZ \qquad Bb := BO - Oa$

$Xa := \frac{BE \cdot XZ}{EZ} \qquad BU := \frac{Bb \cdot BE}{Xa - BE} \qquad EU := \sqrt{BE^2 +]UZ := BZ - BU \qquad Zc := \frac{UZ^2 + EZ^2 - EU^2}{2 \cdot EZ}$



7

$Uc := \sqrt{UZ^2 - Zc^2} \qquad VW := \frac{Uc \cdot EW}{EZ - Zc} \qquad OU := BU + BO \qquad IM := DM - DI \qquad IK := EK - EI$

$Ig := \frac{IK^2 + IM^2 - KM^2}{2 \cdot IM} \qquad EV := \frac{EU \cdot EW}{EZ - Zc} \qquad TU := OT - OU \quad UV := EV - EU$

$TV := VW - TW \qquad Uh := \frac{UV^2 + TU^2 - TV^2}{2 \cdot TU} \qquad Kg := \sqrt{IK^2 - Ig^2}$

$Ke := DE - Kg \qquad Vh := \sqrt{UV^2 - Uh^2} \qquad Dg := DI + Ig \qquad Bh := BU + Uh$

$eh := Bh - Dg \qquad ef := \frac{eh \cdot Ke}{(Ke + Vh)} \qquad Bf := Dg + ef \qquad Of := Bf + BO$

$Om := \frac{AO^2}{2 \cdot Of} \quad Ej := \frac{AO^2}{Of} \qquad Om - \frac{Ej}{2} = 0 \qquad Ef := \sqrt{BE^2 + Bf^2} \qquad Ek := \frac{AO^2 + Ef^2 - Of^2}{-Ef}$

$Om - \frac{Ek}{2} = -2.639125 \times 10^{-12} \qquad \frac{Ek}{Ej} = 1$

And so the Trisection is accurate to within a few decimal places. 30 degree angle shown.

Unit.

The Gravitating Answer.

030503

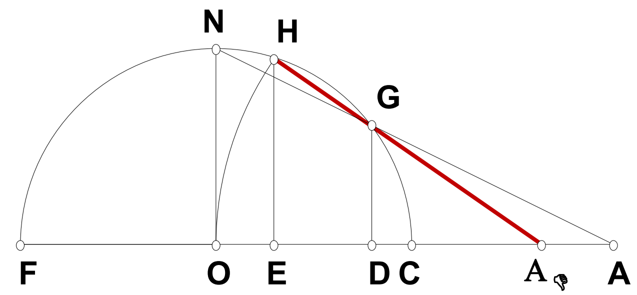
Descriptions.

Given.

$\Delta := 22$

$\delta := 0 .. \Delta$

$N := 2$



How many iterations to go beyond 15 decimal places precision in trisection? The itteration is from AO where GH determines a new A.

$CF := 1 \quad CO := \frac{CF}{2}$

$CD := \frac{CO}{N} \quad DO := CO - CD$

$DF := DO + C \; DG := \sqrt{CD \cdot DF}$

$AD_0 := \frac{DO \cdot DG}{CO - DG} \quad AO_0 := AD_0 + DO \quad EO_0 := \frac{CO^2}{2 \cdot (AD_0 + DO)} \quad DE_0 := DO - EO_0$

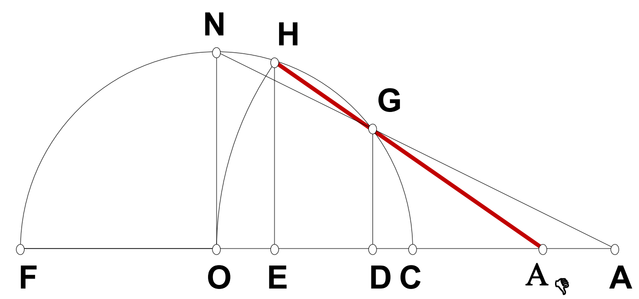
$EH_0 := \sqrt{(CO + EO_0) \cdot (CO - EO_0)}$

$$\begin{pmatrix} AD_{\delta+1} \\ AO_{\delta+1} \\ EO_{\delta+1} \\ DE_{\delta+1} \\ EF_{\delta+1} \\ EH_{\delta+1} \end{pmatrix} := \begin{bmatrix} \dfrac{DE_\delta \cdot DG}{EH_\delta - DG} \\ \dfrac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \\ \dfrac{CO^2}{2 \cdot \left(\dfrac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ DO - \dfrac{CO^2}{2 \cdot \left(\dfrac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ CO + \dfrac{CO^2}{2 \cdot \left(\dfrac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ \sqrt{\left[CO - \dfrac{CO^2}{2 \cdot \left(\dfrac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right] \cdot \left[CO + \dfrac{CO^2}{2 \cdot \left(\dfrac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right]} \end{bmatrix}$$

$$AG := \sqrt{DG^2 + (AD_\Delta)^2}$$

$$GH := \frac{CO^2 + AG^2 - (AO_\Delta)^2}{-AG}$$

$$EO_\Delta - \frac{GH}{2} = 0$$



The displayed precision is for 15 decimal places. Trisection is beyond that. Since the physical world is quantitized, physical trisection is possible.

$EO_\delta - \frac{GH}{2} =$	$\delta =$
-0.019836790725685	0
-0.004498504834066	1
-0.001019750164038	2
-0.000231158851096	3
-0.000052399460482	4
-0.000011877993414	5
-0.000002692522516	6
-0.000000610345304	7
-0.000000138354048	8
-0.000000031362317	9
-0.000000007109260	10
-0.000000001611539	11
-0.000000000365306	12
...	...

One can see how rapidly each recursion increases precision. And so, for any required precision, one can trisect an angle grearter than that, relatively rapidly--espectially if one combine yesterdays plate with today's.

030803

Descriptions.

$DO := \frac{DF}{2}$ $\quad Dx := \frac{DO}{N}$

$Ox := DO - Dx$ $\quad Fx := DF - Dx$

$Gx := \sqrt{Dx \cdot Fx}$ $\quad AO := \frac{Ox \cdot DO}{DO - Gx}$

$HO := DO$ $\quad LO := \frac{HO}{2}$

$Ex := \frac{Ox \cdot Gx}{Gx + DO}$ $\quad Ob := \frac{LO^2}{AO}$

$Lb := \sqrt{LO^2 - Ob^2}$ $\quad ab := \frac{Ex \cdot Lb}{Gx}$

$Aa := AO + ab - Ob$

$Ax := AO - Ox$ $\quad AE := Ax + Ex$

$Kc := \frac{Lb \cdot AE}{Aa}$ $\quad Cc := \frac{AO \cdot Kc}{DO}$

$Ec := \frac{ab \cdot AE}{Aa}$ $\quad CE := Cc + Ec$ $\quad AC := AE - CE$

$AB := \frac{AC}{2}$

Unit.

$DF := 1$

Given.

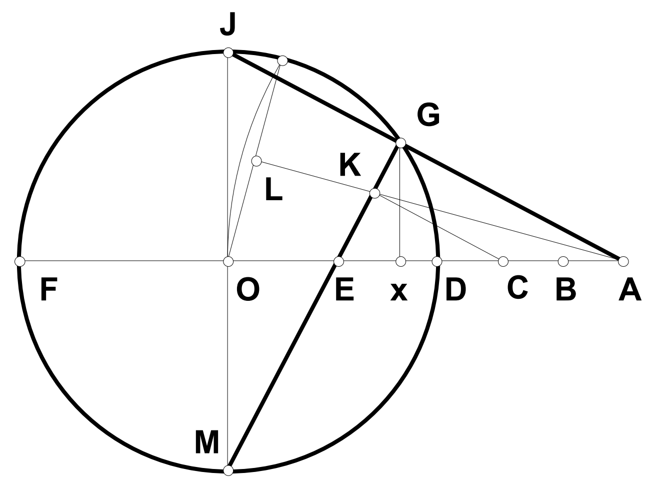
$N := 2$

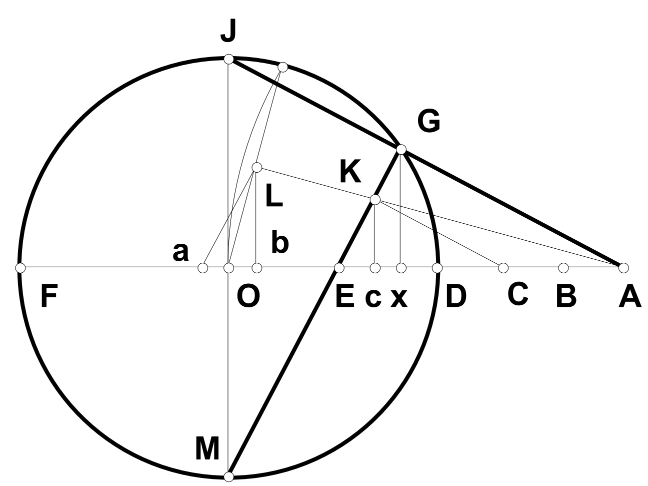
$\Delta := 22$

$\delta := 0 .. \Delta$

Nothing Saved

Is anything saved by starting from a much more precise point for itteration demonstrated in 0305?



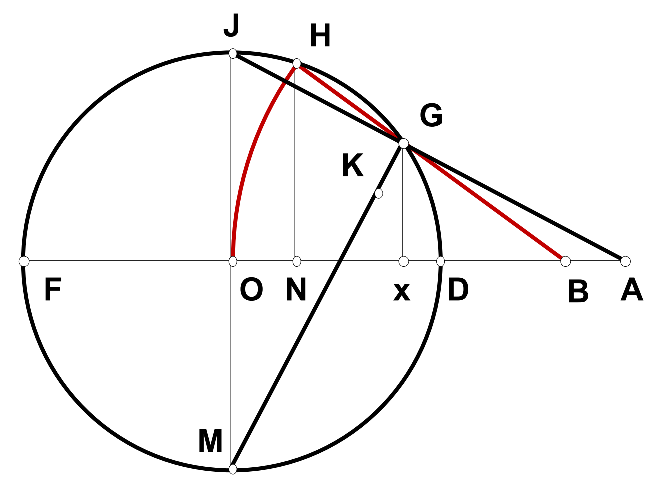


$$BO_0 := AO - AB$$

$$NO_0 := \frac{DO^2}{2\,BO_0}$$

$$Nx_0 := Ox - NO_0$$

$$HN_0 := \sqrt{(DO + NO_0)\cdot(DO - NO_0)}$$



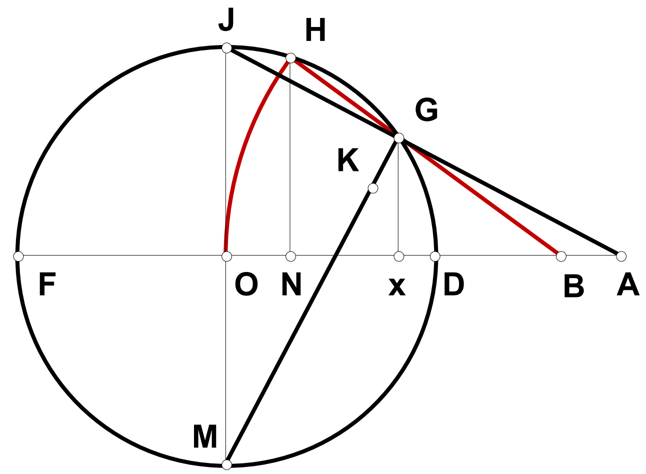
$$\begin{pmatrix} BO_{\delta+1} \\ NO_{\delta+1} \\ Nx_{\delta+1} \\ HN_{\delta+1} \end{pmatrix} := \begin{bmatrix} \dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta \\[2ex] \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\[2ex] Ox - \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\[2ex] \sqrt{\left[DO + \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]\cdot\left[DO - \dfrac{DO^2}{2\left(\dfrac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]} \end{bmatrix}$$

$$Bx_{\Delta} := BO_{\Delta} - Ox$$

$$BG := \sqrt{Gx^2 + (Bx_{\Delta})^2}$$

$$GH := \frac{DO^2 + BG^2 - (BO_{\Delta})^2}{-BG}$$

$$NO_{\Delta} - \frac{GH}{2} = 0$$



$NO_{\delta} - \frac{GH}{2} =$

-0.000733072475044
-0.000166174134206
-0.000037668619212
-0.000008538782769
-0.000001935584914
-0.000000438761479
-0.000000099459153
-0.000000022545559
-0.000000005110663
-0.000000001158493
-0.000000000262609
-0.000000000059529
-0.000000000013494
...

$\delta =$

0
1
2
3
4
5
6
7
8
9
10
11
12
...

Although one starts off in a more precise spot, not much in the way of steps for 15 decimal place precision is saved. The steps are a waste of time.

Trisection and Square Roots

031503

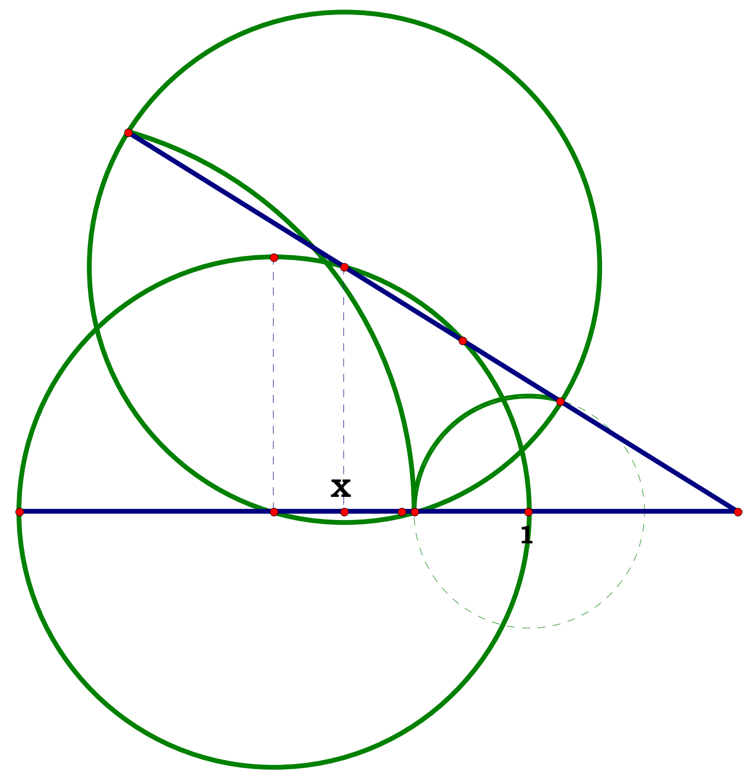
Descriptions.

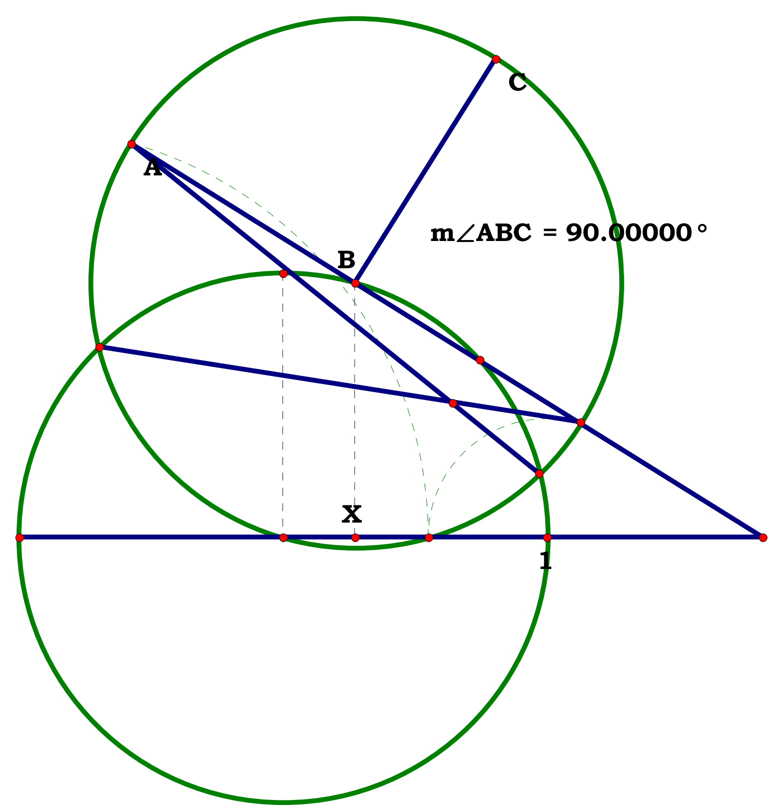
This figure may appear to many to be very counter intuitive, so I will show the construction step by step. I think way back then is the only other time I drew this. I have lost count on how many ways one can actually draw a figure demonstrating trisection. Why the so called intellectuals still claim it is impossible is way beyond me and my talent.

X will range between the center and first half of that segment giving one 30 working degrees.

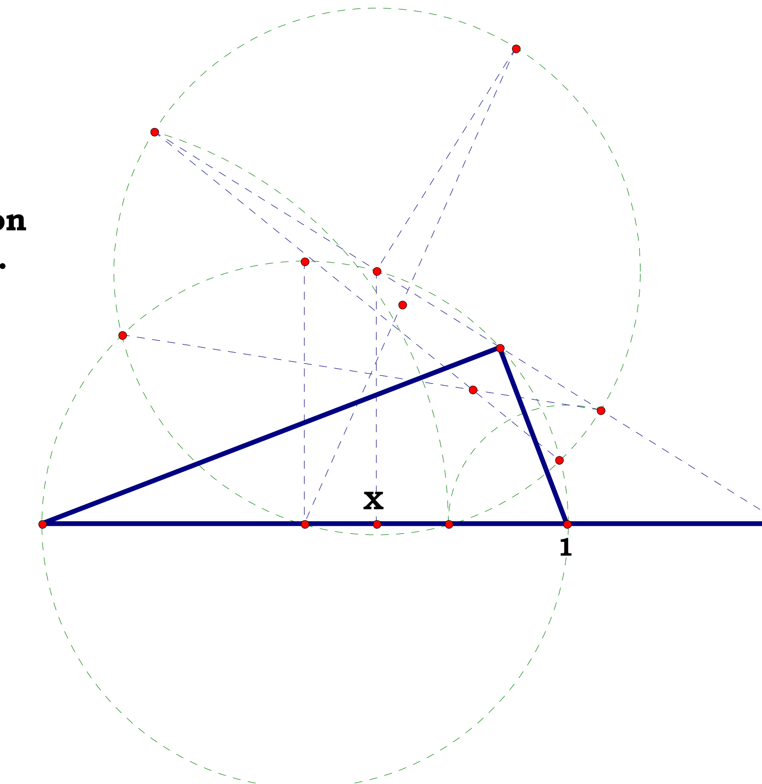
From that point I will draw the second circle to the center of the first which will produce a second point of intersection with the diameter. From that point I will draw two more circles to produce the line that terminates on the base like. I immediately have my square root point of the figure and I have the point

I will construct two lines with opposing endpoints which will produce a point which is suspended in the air it seems.

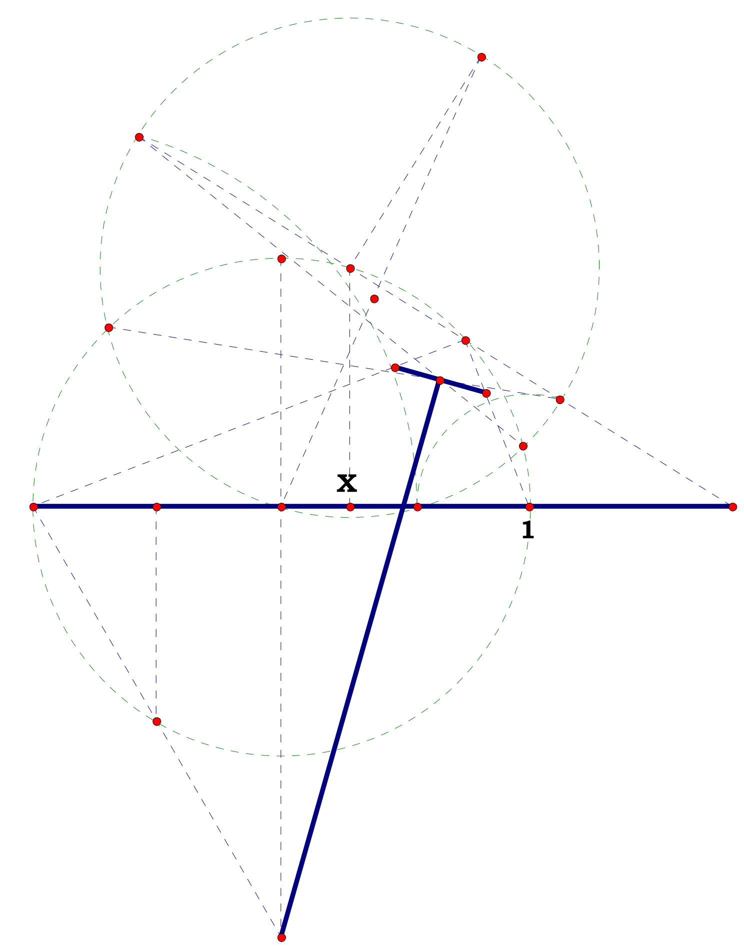




I will use the second point of intersetion of top line to construct a right triangle. One might say a tent for my hanging point.

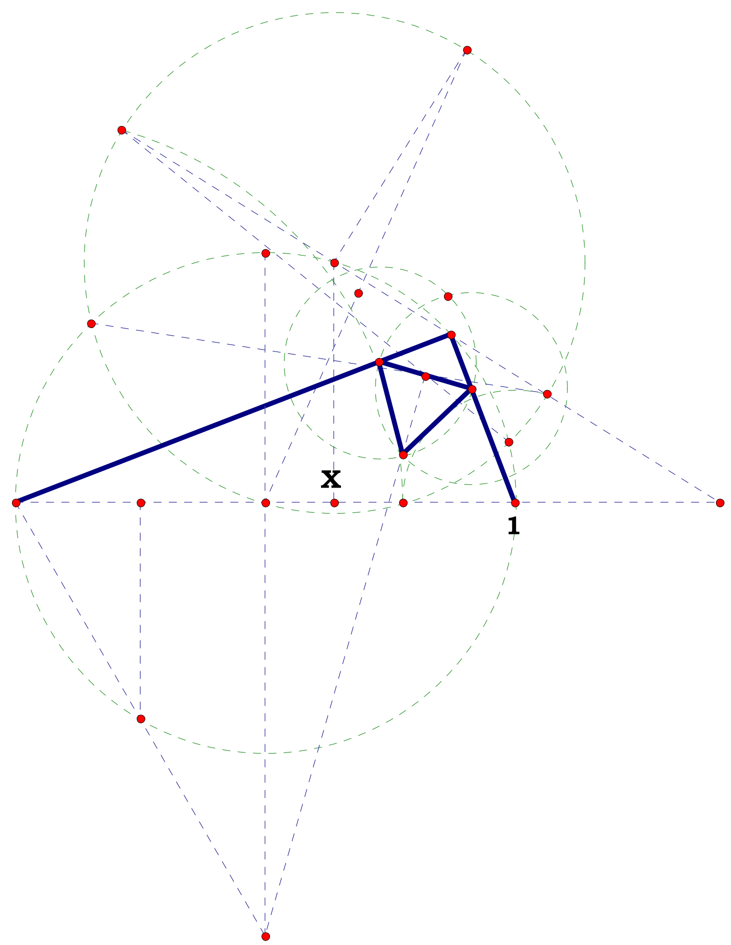


From the point we draw a 60 degree arc from I am going to make a segment with our floating point and form a perpendicular limited by our tent so that we have a very skinny long T.



Descriptions.

With that skinny long T, we are going to make an equilateral triangle.

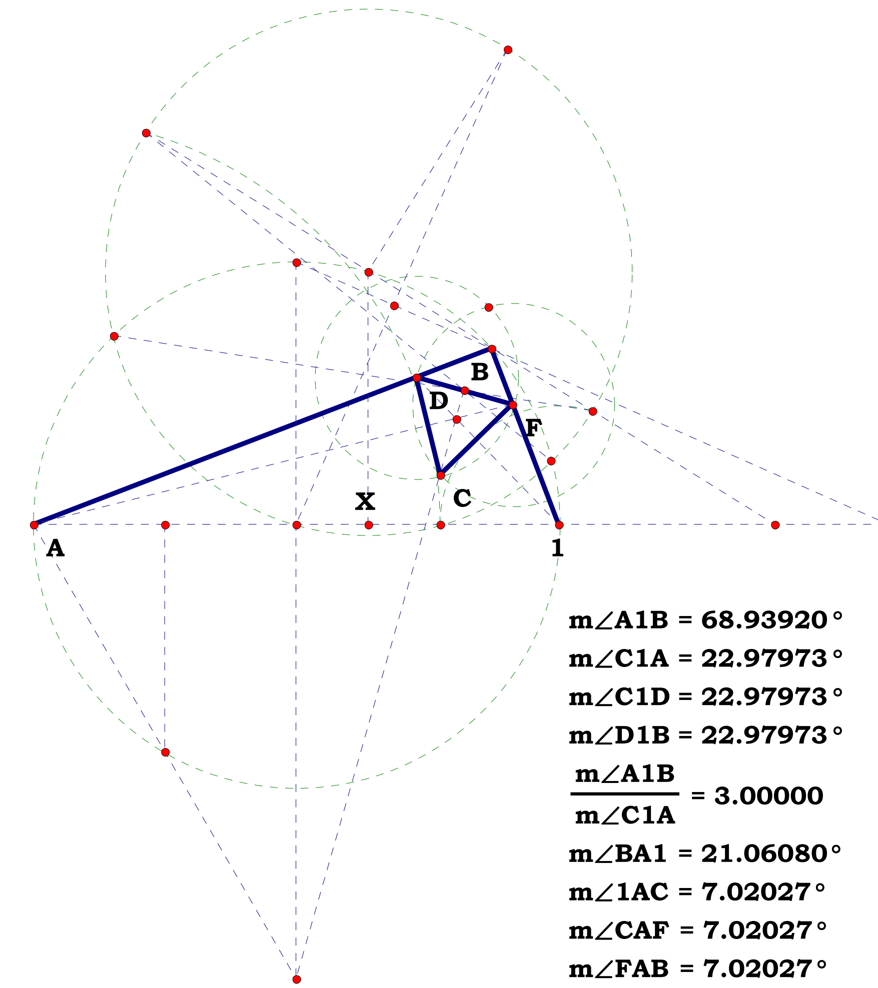


That equilateral triangle is the triangle in a figure that demonstrates angle trisection.

And as you have seen for yourself, it just fell out of the sky.

I will not make another plate with all the dressing. One may say, this is just lunch to go.

And so, angle trisection is actually a well documented part of geometry, all one has to do is find the write ups at the end of their instrument of choice.



$m\angle A1B = 68.93920°$

$m\angle C1A = 22.97973°$

$m\angle C1D = 22.97973°$

$m\angle D1B = 22.97973°$

$\dfrac{m\angle A1B}{m\angle C1A} = 3.00000$

$m\angle BA1 = 21.06080°$

$m\angle 1AC = 7.02027°$

$m\angle CAF = 7.02027°$

$m\angle FAB = 7.02027°$

$\dfrac{m\angle BA1}{m\angle 1AC} = 3.00000$

031503

Unit.

$BO := 1$

Given.

$Y := 20$

$X := 11$

Descriptions.

$AB := 2 \cdot BO \quad NO := \dfrac{X}{2 \cdot Y} \quad EO := BO \quad AO := BO$

$FO := \dfrac{EO}{2} \quad GO := \dfrac{NO}{2} \quad FG := \sqrt{FO^2 - GO^2}$

$CO := \dfrac{FO^2}{GO} \quad CO = 1.818182 \quad AC := AO + CO$

$OY := 2 \cdot NO \quad HP := AB \quad OP := \sqrt{AB^2 - BO^2}$

$OP = 1.732051 \quad MP := \sqrt{AB^2 - OY^2} \quad MP = 1.922888$

$MO := MP - OP \quad MO = 0.190838 \quad HJ := 2 \cdot MO$

$CY := CO - OY \quad \dfrac{AC}{CY} = 2.222222 \quad AY := AO + OY$

$\dfrac{AC}{OY} = 5.123967 \quad \dfrac{AB}{AY} = 1.290323$ etc.

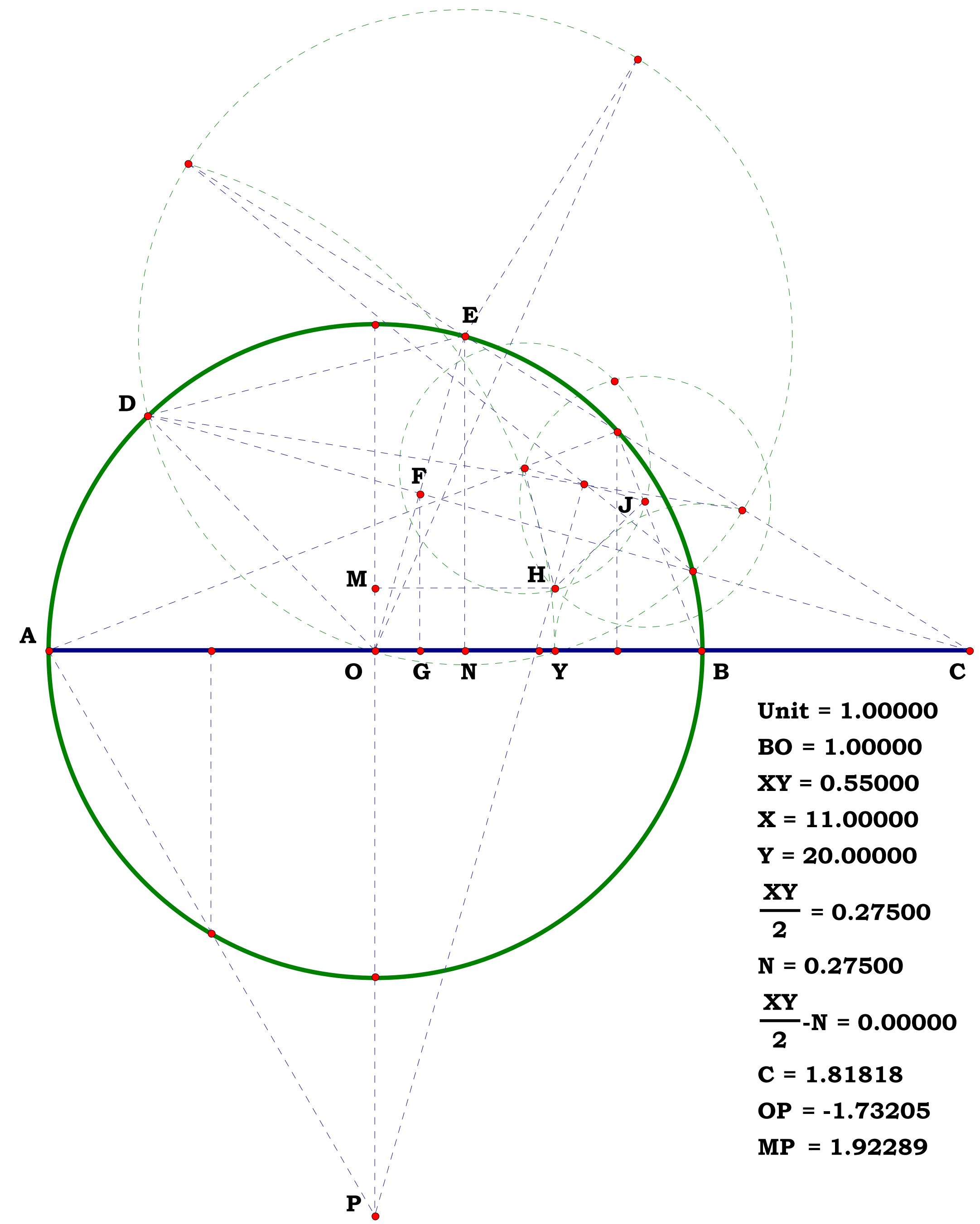
Definitions.

$AB - 2 = 0 \quad NO - \dfrac{X}{2 \cdot Y} = 0 \quad EO - 1 = 0 \quad AO - 1 = 0 \quad FO - \dfrac{1}{2} = 0$

$GO - \dfrac{X}{4 \cdot Y} = 0 \quad FG - \dfrac{\sqrt{4 \cdot Y^2 - X^2}}{4 \cdot Y} = 0 \quad CO - \dfrac{Y}{X} = 0 \quad AC - \dfrac{X + Y}{X} = 0$

$OY - \dfrac{X}{Y} = 0 \quad HP - 2 = 0 \quad OP - \sqrt{3} = 0 \quad MP - \dfrac{\sqrt{4 \cdot Y^2 - X^2}}{Y} = 0 \quad MO - \dfrac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0 \quad HJ - 2 \cdot \dfrac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0$

$CY - \dfrac{Y^2 - X^2}{X \cdot Y} = 0 \quad AY - \dfrac{X + Y}{Y} = 0 \quad \dfrac{AC}{CY} - \dfrac{Y}{(Y - X)} = 0 \quad \dfrac{AC}{OY} - \dfrac{Y \cdot (X + Y)}{X^2} = 0 \quad \dfrac{AB}{AY} - \dfrac{2 \cdot Y}{X + Y} = 0$



Unit = 1.00000

BO = 1.00000

XY = 0.55000

X = 11.00000

Y = 20.00000

$\dfrac{XY}{2} = 0.27500$

N = 0.27500

$\dfrac{XY}{2}$-N = 0.00000

C = 1.81818

OP = -1.73205

MP = 1.92289

032303

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 8$

Figure in a Figure.

Descriptions.

$AN := \frac{X}{Y}$ $\quad AD := AB$ $\quad DN := \sqrt{AD^2 + AN^2}$ $\quad CK := \frac{AN \cdot 2 \cdot AD}{DN}$

$KO := \frac{(AN \cdot CK)}{DN}$ $\quad CO := \frac{AD \cdot KO}{AN}$ $\quad AP := CO$ $\quad AO := AB - KO$

$AK := \sqrt{3}$ $\quad HN := \frac{2 \cdot AB \cdot AO}{AK + AO}$ $\quad HN = 0.589644$

$AG := \frac{CO \cdot AK}{AK + AO}$ $\quad AG = 0.48633$ $\quad$ etc.

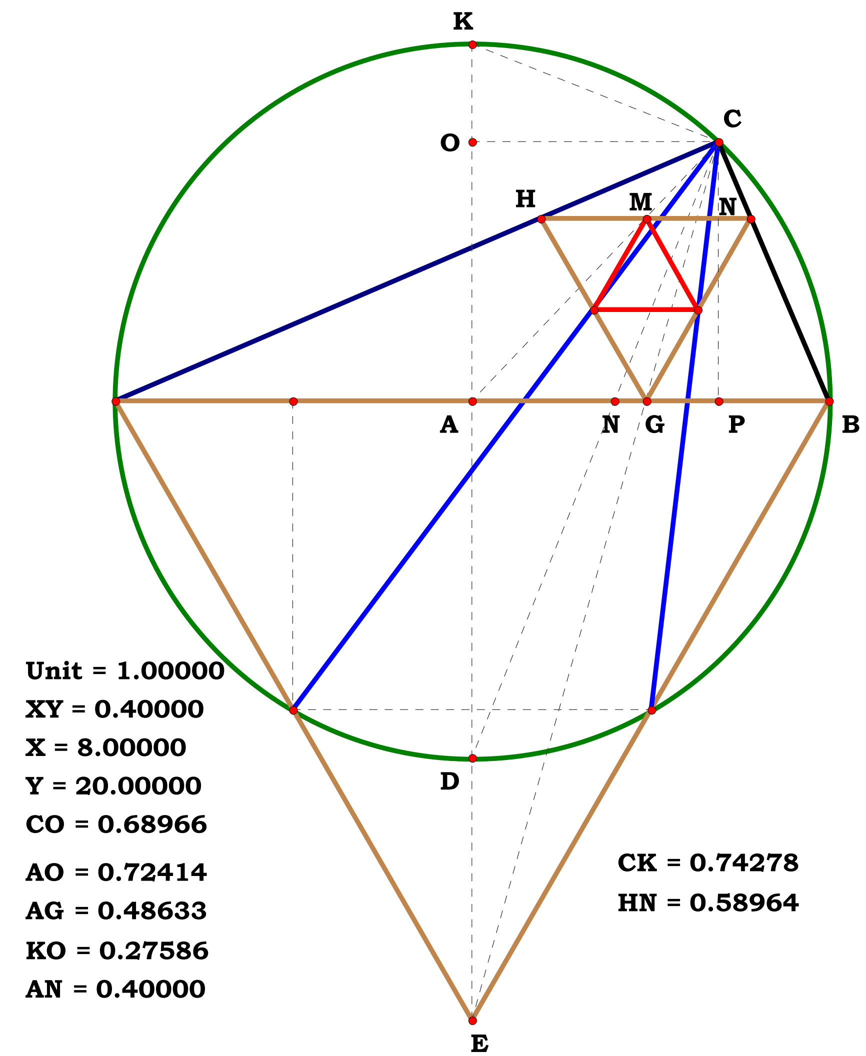
Definitions.

$AN - \frac{X}{Y} = 0$ $\quad AD - 1 = 0$ $\quad DN - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$ $\quad CK - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$

$KO - \frac{2 \cdot X^2}{X^2 + Y^2} = 0$ $\quad CO - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $\quad AP - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$

$AO - \frac{Y^2 - X^2}{X^2 + Y^2} = 0$ $\quad AK - \sqrt{3} = 0$ $\quad HN - \frac{2 \cdot \left(Y^2 - X^2\right)}{\left(X^2 + Y^2\right) \cdot \left(\sqrt{3} - 1\right) + 2 \cdot Y^2} = 0$

$AG - \frac{2 \cdot \sqrt{3} \cdot X \cdot Y}{\left(X^2 + Y^2\right) \cdot \left(\sqrt{3} - 1\right) + 2 \cdot Y^2} = 0$ $\quad$ etc.



032403

Unit.

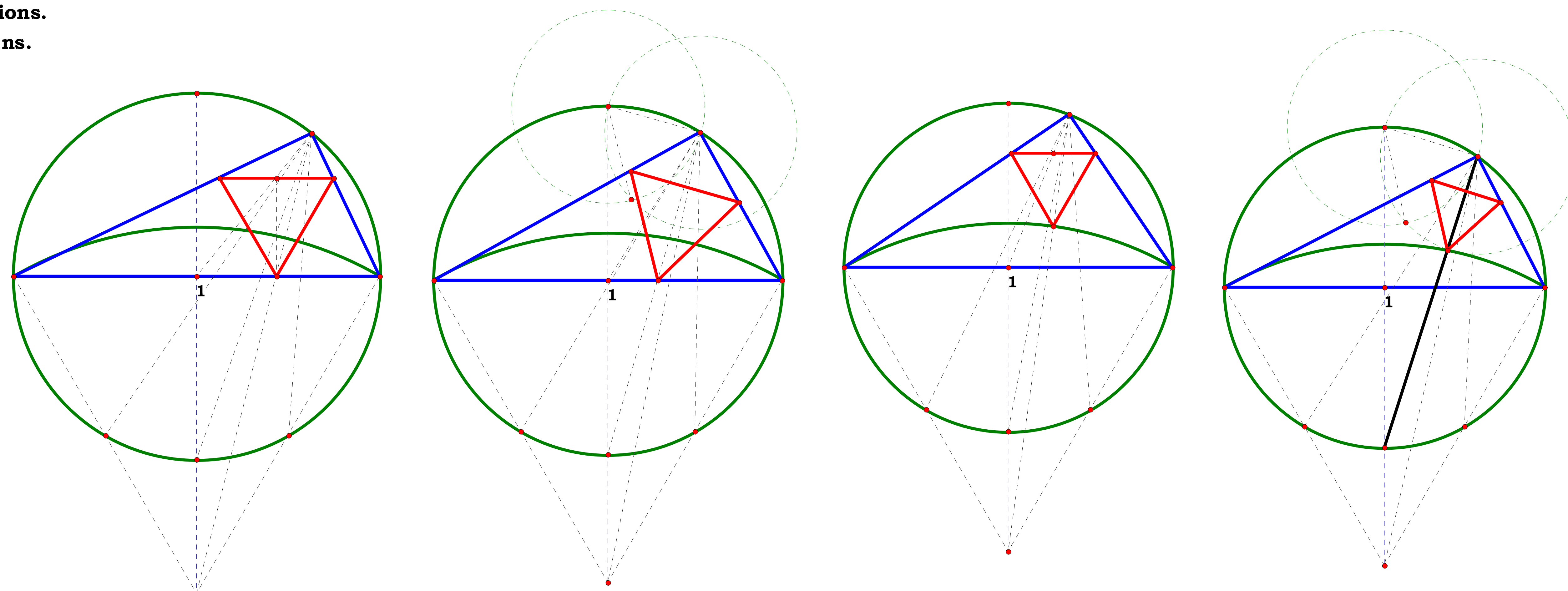
Given.

Four Siblings

When looking for the equalateral triangle which produces trisection in a right angle, we may come to meet its four lazy siblings.

Descriptions.

Definitions.



Unit.
Given.

PacMan
Animation writeup

040903

Animate

Descriptions.

Definitions.

So called Fractal Geometry is concerned with the recursion of the perceptible, however it does not compare much with the recursion of an intelligible. And so, one may find, in the recursion a hidden message, from the impossible to packman, it is all just playing one tune, binary recursion is binary recursion.

$m\angle FAK = 11.534°$

$m\angle ABC = 62.301°$

$m\angle AGJ = 34.601°$

$\frac{m\angle AGJ}{m\angle FAK} = 3.000$

$m\angle MKN = 11.534°$

$m\angle DBE = 20.767°$

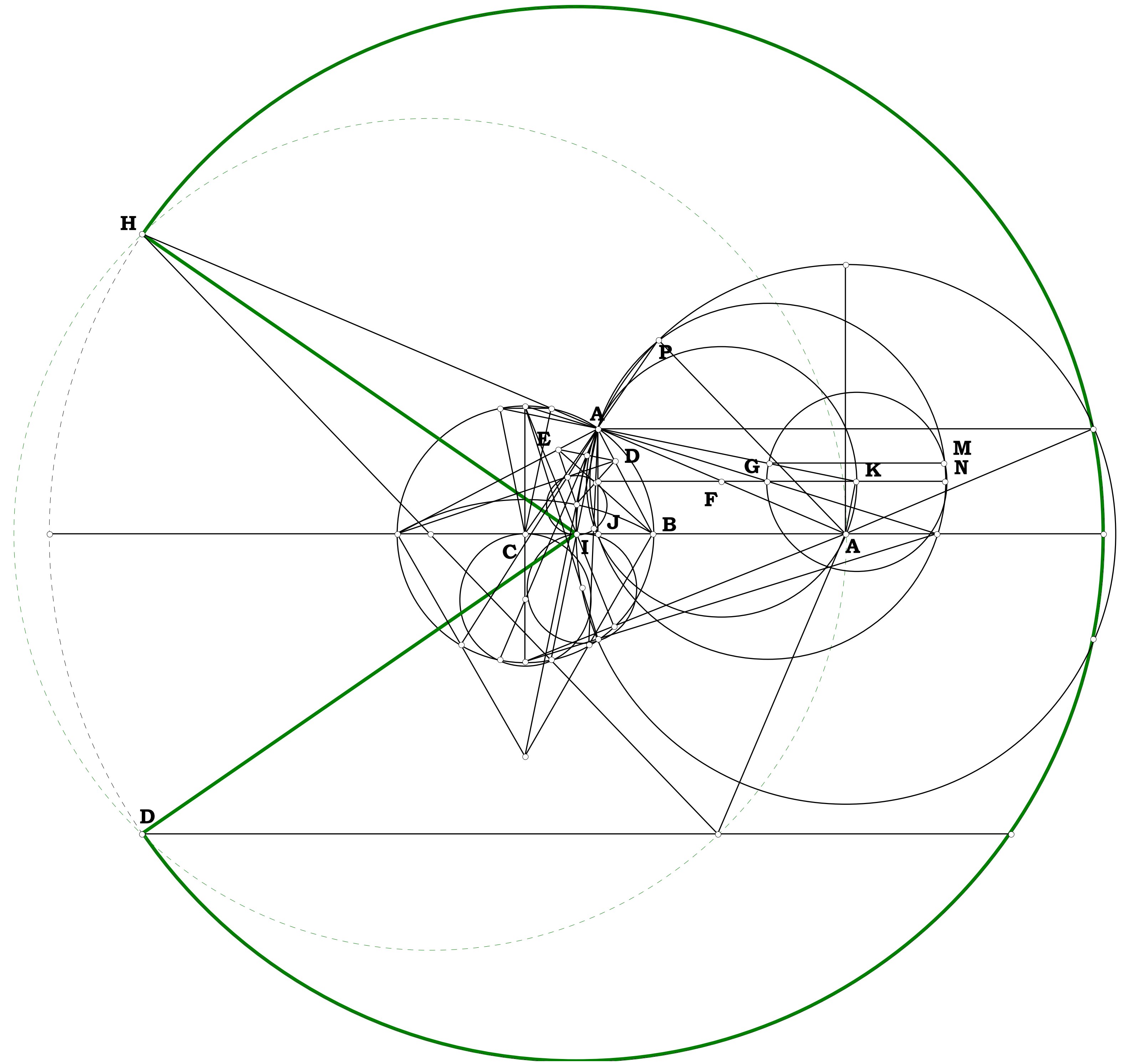
$\frac{m\angle ABC}{m\angle DBE} = 3.000$

$m\angle HID = 69.203°$

$m\angle PAA = 23.068°$

$\frac{m\angle HID}{m\angle PAA} = 3.000$

Eaten by HID
or PakMan?



Unit.
Given.
Descriptions.
Definitions.

An Indeterminate Problem Reduced To An Equation

Page 5 of *A Treatise on Algebraic Geometry* by Rev. Dionysius Lardner, 1831

072903

Given the base AB, and the sum of the sides (AC and BC) of a triangle, to find the vertex (C).
Let AB = a, AC = y, and CB = x, and the the excess of the sum of the sides above the base be d.

$\therefore\ y + x = a + b.$

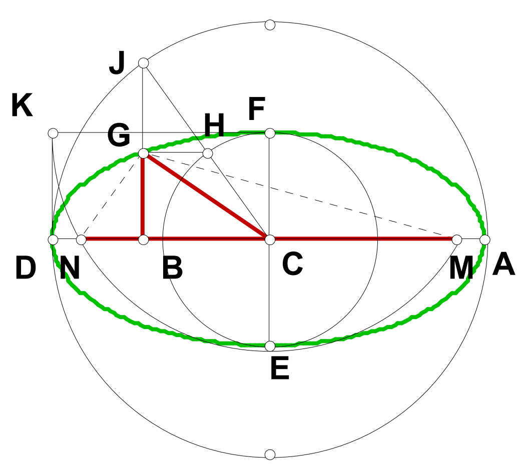
Any values of y and x, which fulfill the conditions of this equation, represent the sides of the triangle, whose vertex solves the problem.

Lardner was wholly unaware that the problem actually descibes an ellipse which means that it is not indeterminate at all.

Perhaps this problem is indeterminate is because the author did not have a clue that a point (C) is not a magnitude. How does one find a non-magnitude from magnitues? Only by establishing a co-ordinate system.

09/11/97 The Ellipse

Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.



$$N_1 := 3 \quad N_2 := 4$$

$$AD := 1 \quad EF := \frac{AD}{N_1} \quad AB := \frac{AD}{N_2}$$

$$BD := AD - AB \quad BJ := \sqrt{AB \cdot BD}$$

$$AC := \frac{AD}{2} \quad BC := AC - AB \quad CH := \frac{EF}{2}$$

$$CJ := AC \quad BG := \frac{BJ \cdot CH}{CJ}$$

$$CG := \sqrt{BG^2 + BC^2} \quad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$$

$$\frac{\sqrt{4 \cdot N_2 - 4 + N_2^{\,2} \cdot N_1^{\,2} - 4 \cdot N_2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,2}}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{\sqrt{N_2 - 1}}{\left(N_2 \cdot N_1\right)} - BG = 0 \qquad \frac{\sqrt{\left(N_1^{\,2} - 1\right)}}{N_1} - MN = 0$$

120103A

Unit.

Given. $N := 1.454$

$AD := 3.073$

Descriptions.

$DH := AD \quad BD := \dfrac{AD}{N} \quad BH := \sqrt{BD^2 + DH^2} \quad HI := \dfrac{BH}{2}$

$EH := \dfrac{BH \cdot HI}{DH} \quad DE := DH - EH \quad DF := 2 \cdot DE \quad FH := DH - DF$

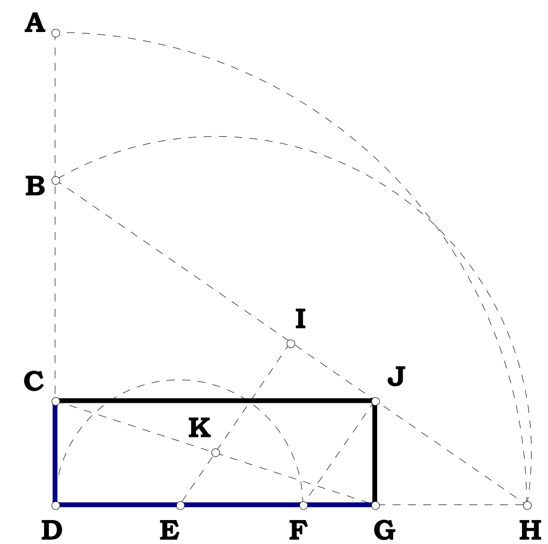
$HJ := \dfrac{DH \cdot FH}{BH} \quad GJ := \dfrac{BD \cdot HJ}{BH} \quad GH := \dfrac{DH \cdot HJ}{BH} \quad DG := DH - GH$

$CD := GJ \quad BC := BD - CD \quad \dfrac{DG}{BC} - \dfrac{BC}{GH} = 0 \quad \dfrac{BC}{GH} - \dfrac{GH}{GJ} = 0$

$\left(\dfrac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)} - \dfrac{DG}{BC} = 0 \quad \dfrac{AD}{N^3 + N} - GJ = 0 \quad \dfrac{AD \cdot N^2}{N^2 + 1} - DG = 0$

$DG + DG^{\left(\frac{1}{3}\right)} \cdot GJ^{\left(\frac{2}{3}\right)} - AD = 0 \quad \dfrac{DG \cdot N^2 + DG}{N^2} - AD = 0$

$N - \left(\dfrac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)} = 0 \quad \dfrac{AD}{N} = 2.11348 \quad \dfrac{DG + DG^{\left(\frac{1}{3}\right)} \cdot GJ^{\left(\frac{2}{3}\right)}}{\left(\dfrac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)}} = 2.11348 \quad DG^{\left(\frac{2}{3}\right)} \cdot GJ^{\left(\frac{1}{3}\right)} + GJ = 2.11348$



AD = 2.45833 in.

AB = 0.76777 in.

DB = 1.69056 in.

$\dfrac{AD}{DB} = 1.45415$

BD = 1.69056 in.

EH = 1.81046 in.

HJ = 0.95793 in.

DG = 1.66903 in.

GJ = 0.54280 in.

$\dfrac{DG}{GJ} = 3.07487$

120103B

Unit.

Given. $AD := 2.458$

$AB := .768$

Descriptions.

$DH := AD \quad BD := AD - AB$

$BH := \sqrt{BD^2 + DH^2} \quad HI := \frac{BH}{2} \quad EH := \frac{BH \cdot HI}{DH} \quad DE := DH - EH$

$DF := 2 \cdot DE \quad FH := DH - DF \quad HJ := \frac{DH \cdot FH}{BH} \quad GJ := \frac{BD \cdot HJ}{BH}$

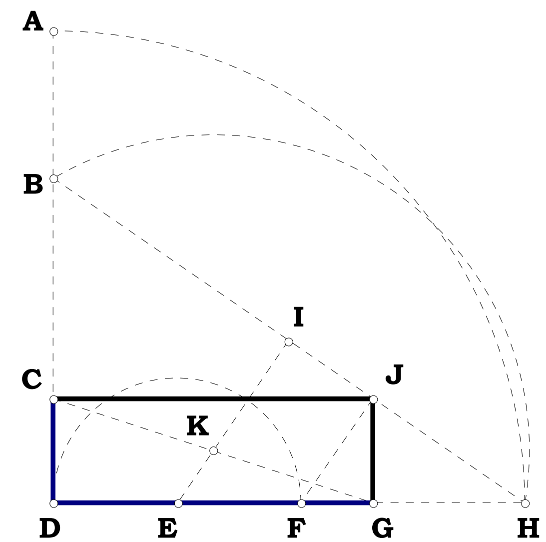
$GH := \frac{DH \cdot HJ}{BH} \quad DG := DH - GH \quad CD := GJ \quad BC := BD - CD$

$\frac{DG}{BC} - \frac{BC}{GH} = 0 \quad \frac{BC}{GH} - \frac{GH}{GJ} = 0 \quad \left(\frac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)} - \frac{DG}{BC} = 0$

$\frac{AD^3}{\left(2 \cdot AD^2 - 2 \cdot AD \cdot AB + AB^2\right)} - DG = 0 \quad \left(\frac{AD}{BD}\right)^3 - \frac{DG}{CD} = 0$

$\frac{(AD - AB)^3}{AB^2 - 2 \cdot AB \cdot AD + 2 \cdot AD^2} - CD = 0 \quad \frac{AD^3}{(AD - AB)^3} = 3.076703$

$\frac{DG}{CD} = 3.076703 \quad \left(\frac{AD}{AD - AB}\right)^3 - \frac{DG}{CD} = 0$



AD = 2.45833 in.
AB = 0.76777 in.
DB = 1.69056 in.
$\frac{AD}{DB} = 1.45415$
BD = 1.69056 in.
EH = 1.81046 in.
HJ = 0.95793 in.
DG = 1.66903 in.
GJ = 0.54280 in.
$\frac{DG}{GJ} = 3.07487$

120103C

Unit.

Given. $X := 13$

$Y := 20$

Descriptions.

$AB := \frac{Y}{Y}$ $AC := \frac{X}{Y}$ $BC := \sqrt{AB^2 + AC^2}$ $BC = 1.192686$

$BD := \frac{BC}{2}$ $DE := \frac{AC \cdot BD}{AB}$ $BE := \frac{BC \cdot BD}{AB}$ $BE = 0.71125$

$AE := AB - BE$ $AF := 2 \cdot AE$ $BF := AB - AF$ $BG := \frac{AB \cdot BF}{BC}$

$BG = 0.354242$ $BH := \frac{AB \cdot BG}{BC}$ $BH = 0.297012$

$GH := \frac{AC \cdot BG}{BC}$ $GH = 0.193058$ $AH := AB - BH$

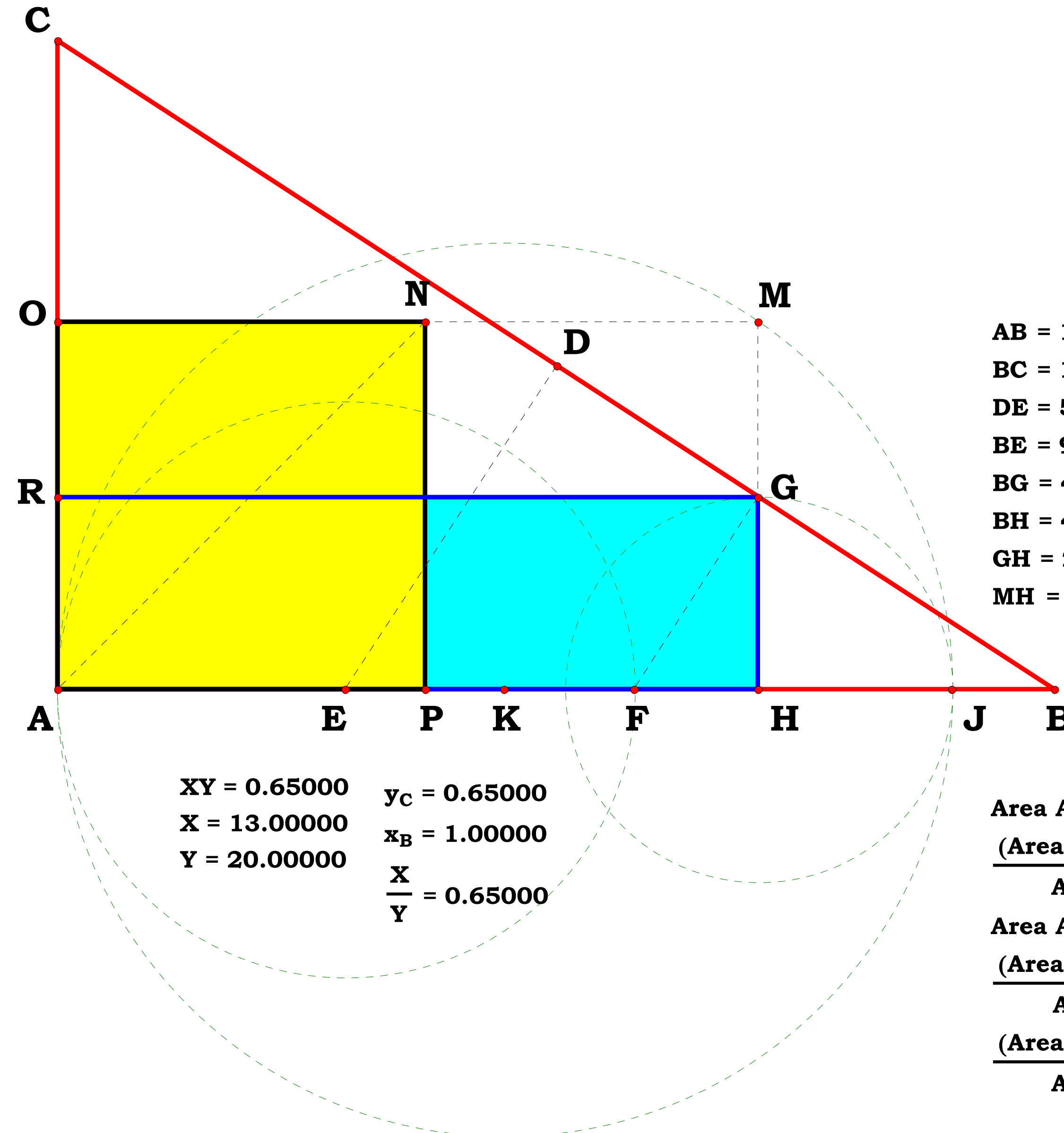
$AJ := AH + GH$ $HM := \sqrt{AH \cdot GH}$ $HM = 0.368398$

$AO := HM$ $AP := HM$ $NP := HM$ $AR := GH$ $GR := AH$

$APNO := AP^2$ $APNO = 0.135717$ $AHGR := AH \cdot GH$ $AHGR = 0.135717$

$APNO - AHGR = 0$ $CR := AC - AR$ $\frac{CR}{AH} = 0.65$ $\frac{GH}{BH} = 0.65$

$\left(\frac{GH}{AH}\right)^{\frac{1}{3}} - \frac{GH}{BH} = 0$ $\left(\frac{GH}{AP}\right)^2 - \frac{GH}{AH} = 0$



$\frac{BC}{AB} = 1.19269$

$\frac{DE}{AB} = 0.38762$

$\frac{BE}{AB} = 0.71125$

$\frac{BG}{AB} = 0.35424$

$\frac{BH}{AB} = 0.29701$

$\frac{GH}{AB} = 0.19306$

$\frac{MH}{AB} = 0.36840$

AB = 13.95400 cm
BC = 16.64274 cm
DE = 5.40889 cm
BE = 9.92478 cm
BG = 4.94310 cm
BH = 4.14451 cm
GH = 2.69393 cm
MH = 5.14063 cm

XY = 0.65000
X = 13.00000
Y = 20.00000

$y_C = 0.65000$

$x_B = 1.00000$

$\frac{X}{Y} = 0.65000$

Area APNO = 26.42609 cm^2

$\frac{(\text{Area APNO})}{AB^2} = 0.13572$

Area AHGR = 26.42609 cm^2

$\frac{(\text{Area AHGR})}{AB^2} = 0.13572$

$\frac{(\text{Area APNO})}{AB^2} - \frac{(\text{Area AHGR})}{AB^2} = 0.00000$

Definitions.

$AB - 1 = 0$ $\quad AC - \frac{X}{Y} = 0$ $\quad BC - \frac{\sqrt{X^2+Y^2}}{Y} = 0$ $\quad BD - \frac{\sqrt{X^2+Y^2}}{2\cdot Y} = 0$

$DE - \frac{X\cdot\sqrt{X^2+Y^2}}{2\cdot Y^2} = 0$ $\quad BE - \frac{X^2+Y^2}{2\cdot Y^2} = 0$ $\quad AE - \frac{(Y-X)\cdot(X+Y)}{2\cdot Y^2} = 0$

$AF - \frac{(Y-X)\cdot(X+Y)}{Y^2} = 0$ $\quad BF - \frac{X^2}{Y^2} = 0$ $\quad BG - \frac{X^2}{Y\cdot\sqrt{X^2+Y^2}} = 0$

$BH - \frac{X^2}{X^2+Y^2} = 0$ $\quad GH - \frac{X^3}{Y\cdot\left(X^2+Y^2\right)} = 0$ $\quad AH - \frac{Y^2}{X^2+Y^2} = 0$

$AJ - \frac{(X+Y)\cdot\left(X^2-X\cdot Y+Y^2\right)}{Y\cdot\left(X^2+Y^2\right)} = 0$ $\quad HM - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0$

$AO - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0$ $\quad AP - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0$ $\quad NP - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0$

$AR - \frac{X^3}{Y\cdot\left(X^2+Y^2\right)} = 0$ $\quad GR - \frac{Y^2}{X^2+Y^2} = 0$ $\quad APNO - \frac{X^3\cdot Y}{\left(X^2+Y^2\right)^2} = 0$

$AHGR - \frac{X^3\cdot Y}{\left(X^2+Y^2\right)^2} = 0$ $\quad CR - \frac{X\cdot Y}{X^2+Y^2} = 0$ $\quad \frac{CR}{AH} - \frac{X}{Y} = 0$

$\frac{GH}{BH} - \frac{X}{Y} = 0$ $\quad \left(\frac{GH}{AH}\right)^{\frac{1}{3}} - \frac{X}{Y} = 0$ $\quad \left(\frac{GH}{AP}\right)^2 - \frac{X^3}{Y^3} = 0$

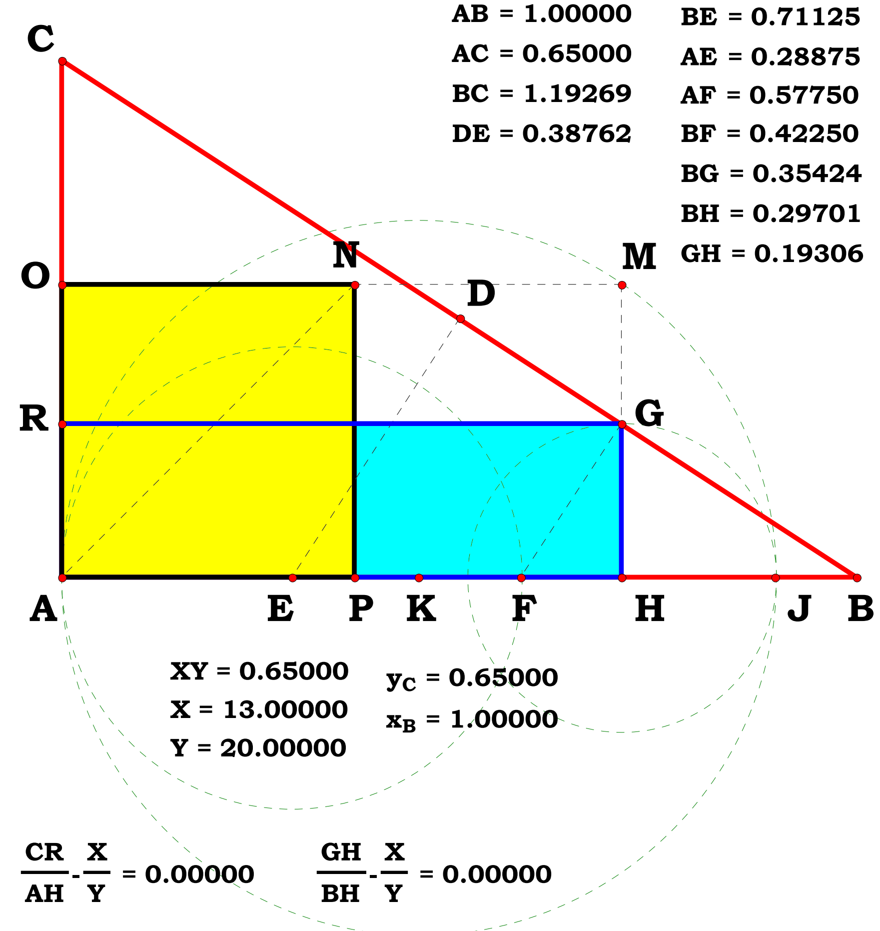
AB = 10.52500 cm
BC = 12.55302 cm
DE = 4.07973 cm
BE = 7.48591 cm
AE = 3.03909 cm

AF = 6.07819 cm
BF = 4.44681 cm
BG = 3.72840 cm
GH = 2.03194 cm
BH = 3.12605 cm

AH = 7.39895 cm
AJ = 9.43088 cm
HM = 3.87739 cm
AO = 3.87739 cm
AP = 3.87739 cm

NP = 3.87739 cm
AR = 2.03194 cm
GR = 7.39895 cm
Area APNO = 15.03418 cm^2
Area AHGR = 15.03418 cm^2

CR = 4.80931 cm
BH = 3.12605 cm
$\frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.36840$



AB = 1.00000
AC = 0.65000
BC = 1.19269
DE = 0.38762

BE = 0.71125
AE = 0.28875
AF = 0.57750
BF = 0.42250
BG = 0.35424
BH = 0.29701
GH = 0.19306

AH = 0.70299
AJ = 0.89605
HM = 0.36840
AO = 0.36840
AP = 0.36840
NP = 0.36840

AR = 0.19306
GR = 0.70299
APNO = 0.13572
AHGR = 0.13572
CR = 0.45694
BH = 0.29701

XY = 0.65000
X = 13.00000
Y = 20.00000

y_C = 0.65000
x_B = 1.00000

$\frac{CR}{AH} - \frac{X}{Y} = 0.00000$ $\quad \frac{GH}{BH} - \frac{X}{Y} = 0.00000$

$\frac{GH}{AH}^{\frac{1}{3}} - \frac{X}{Y} = 0.00000$ $\quad \frac{GH}{AP}^{2} - \frac{X^3}{Y^3} = 0.00000$

$DE - \frac{X\cdot\sqrt{X^2+Y^2}}{2\cdot Y^2} = 0.00000$

$BE - \frac{X^2+Y^2}{2\cdot Y^2} = 0.00000$

$AE - \frac{(Y-X)\cdot(X+Y)}{2\cdot Y^2} = 0.00000$

$AF - \frac{(Y-X)\cdot(X+Y)}{Y^2} = 0.00000$

$BF - \frac{X^2}{Y^2} = 0.00000$

$BG - \frac{X^2}{Y\cdot\sqrt{X^2+Y^2}} = 0.00000$

$BH - \frac{X^2}{X^2+Y^2} = 0.00000$

$GH - \frac{X^3}{Y\cdot(X^2+Y^2)} = 0.00000$

$AH - \frac{Y^2}{X^2+Y^2} = 0.00000$

$AJ - \frac{(X+Y)\cdot((X^2-X\cdot Y)+Y^2)}{Y\cdot(X^2+Y^2)} = 0.00000$

$AB - \frac{Y}{Y} = 0.00000$

$AC - \frac{X}{Y} = 0.00000$

$BC - \frac{\sqrt{X^2+Y^2}}{Y} = 0.00000$

$AO - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$HM - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$AP - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$NP - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$AR - \frac{X^3}{Y\cdot(X^2+Y^2)} = 0.00000$

$GR - \frac{Y^2}{X^2+Y^2} = 0.00000$

$APNO - \frac{X^3\cdot Y}{(X^2+Y^2)^2} = 0.00000$

$AHGR - \frac{X^3\cdot Y}{(X^2+Y^2)^2} = 0.00000$

$CR - \frac{X\cdot Y}{X^2+Y^2} = 0.00000$

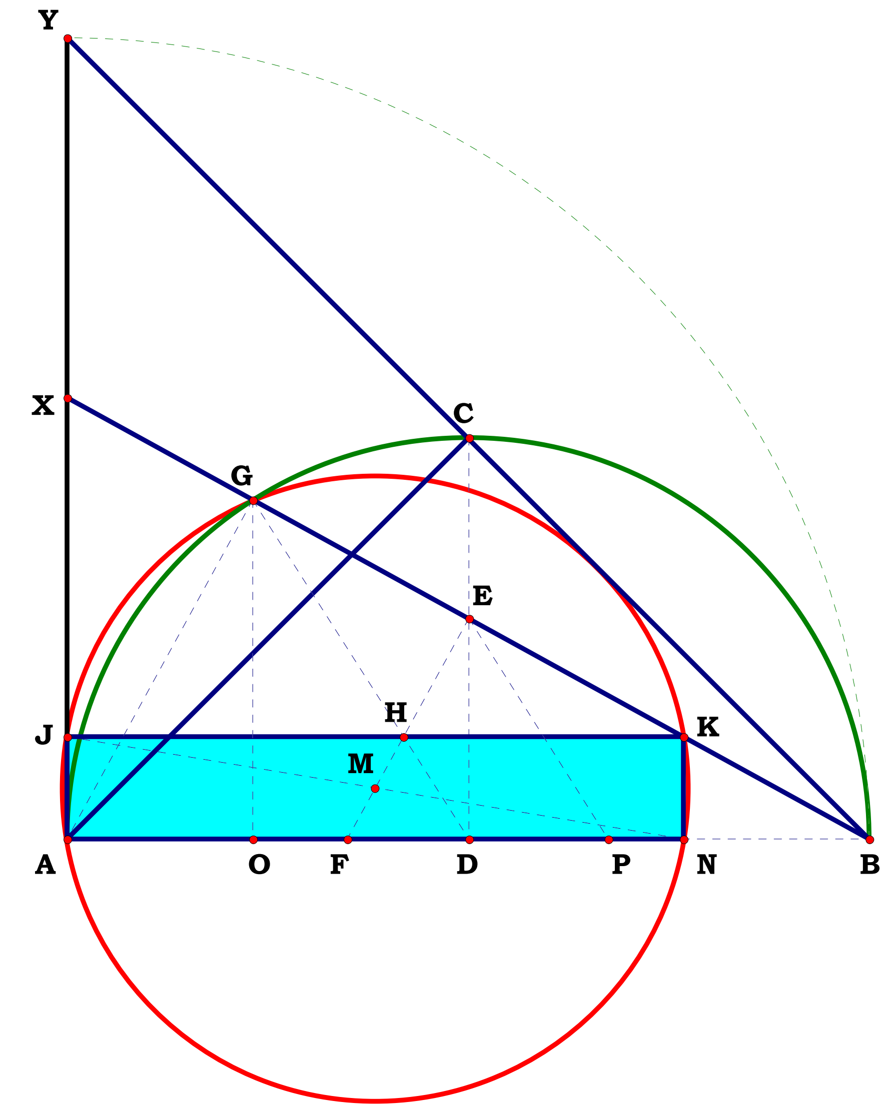
120103D

Unit.

Given. $X := 11$

$Y := 20$

Unit = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000



Descriptions.

$AB := \frac{Y}{Y} \quad AX := \frac{X}{Y} \quad BX := \sqrt{AB^2 + AX^2} \quad BE := \frac{BX}{2} \quad BF := \frac{BX \cdot BE}{AB}$

$AG := \frac{AX \cdot AB}{BX} \quad GO := \frac{AG^2}{AX} \quad AO := \frac{AX \cdot AG}{BX} \quad BD := \frac{AB}{2} \quad DF := BF - BD$

$DO := BD - AO \quad DE := \frac{AX \cdot BD}{AB} \quad DP := \frac{DO \cdot DE}{GO} \quad AJ := \frac{DE \cdot DF}{DP + DF}$

$BN := \frac{AB \cdot AJ}{AX} \quad AN := AB - BN \quad JN := \sqrt{AJ^2 + AN^2} \quad JM := \frac{JN}{2}$

Definitions.

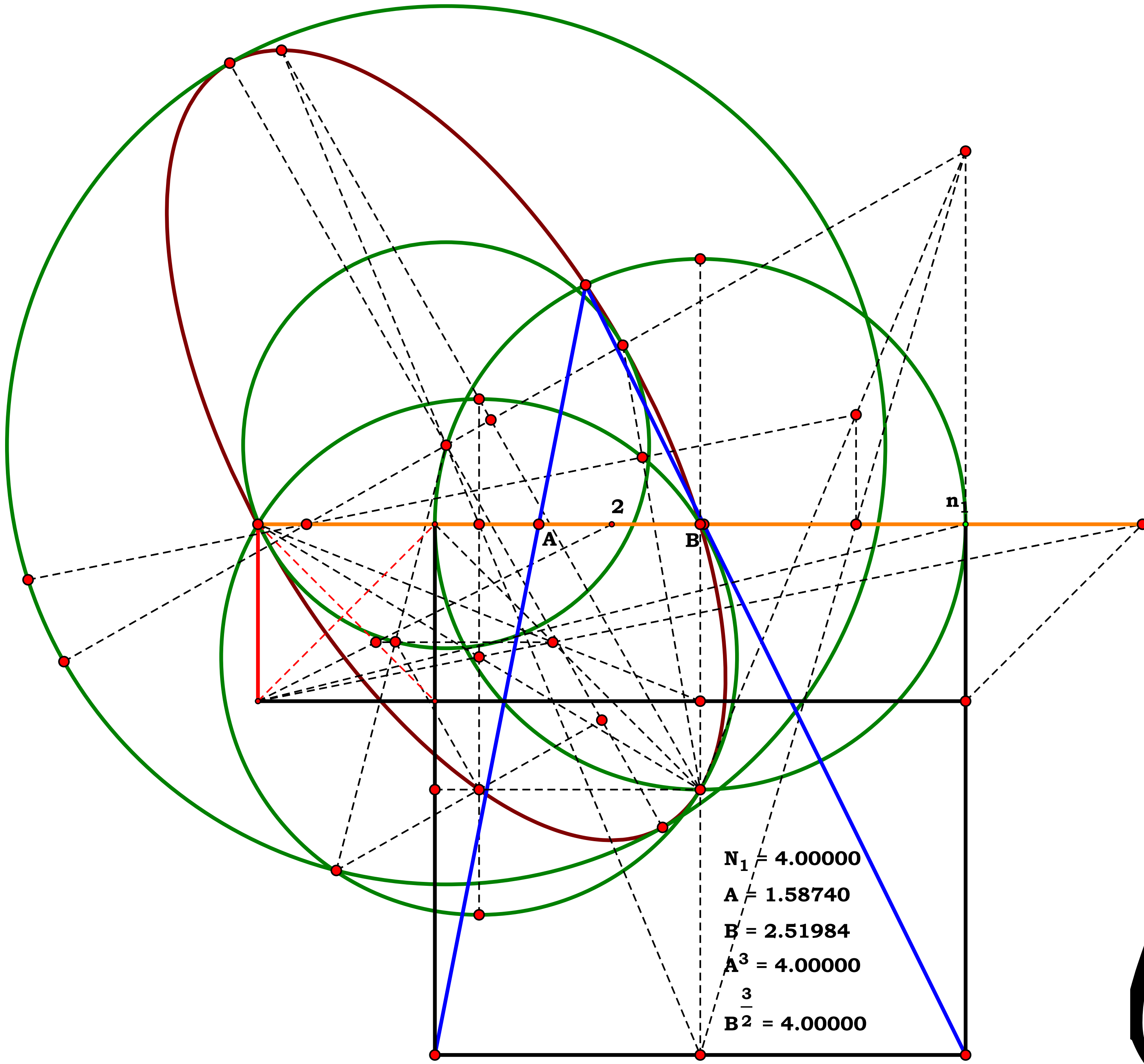
$AB - \frac{Y}{Y} = 0 \quad AX - \frac{X}{Y} = 0 \quad BX - \sqrt{\left(\frac{Y}{Y}\right)^2 + \left(\frac{X}{Y}\right)^2} = 0 \quad BE - \frac{\sqrt{\left(\frac{Y}{Y}\right)^2 + \left(\frac{X}{Y}\right)^2}}{2} = 0$

$BF - \frac{X^2 + Y^2}{2 \cdot Y^2} = 0 \quad AG - \frac{X}{Y \cdot \sqrt{\frac{X^2}{Y^2} + 1}} = 0 \quad GO - \frac{X \cdot Y}{X^2 + Y^2} = 0 \quad AO - \frac{X^2}{X^2 + Y^2} = 0$

$BD - \frac{1}{2} = 0 \quad DF - \frac{X^2}{2 \cdot Y^2} = 0 \quad DO - \frac{(Y - X) \cdot (X + Y)}{2 \cdot \left(X^2 + Y^2\right)} = 0 \quad DE - \frac{X}{2 \cdot Y} = 0$

$DP - \frac{(Y - X) \cdot (X + Y)}{4 \cdot Y^2} = 0 \quad AJ - \frac{X^3}{Y \cdot \left(X^2 + Y^2\right)} = 0 \quad BN - \frac{X^2}{X^2 + Y^2} = 0 \quad AN - \frac{Y^2}{X^2 + Y^2} = 0$

$JN - \sqrt{\frac{X^4 - X^2 \cdot Y^2 + Y^4}{Y^2 \cdot \left(X^2 + Y^2\right)}} = 0 \quad JM - \frac{\sqrt{\frac{X^4 - X^2 \cdot Y^2 + Y^4}{Y^2 \cdot \left(X^2 + Y^2\right)}}}{2} = 0$



The Delian Quest 2004

John Clark

Unit.

$AD := 1$

Given.

02

031604A

Descriptions.

$AK := AD \quad AJ := AD \quad AH := \dfrac{\sqrt{2 \cdot AD^2}}{2}$

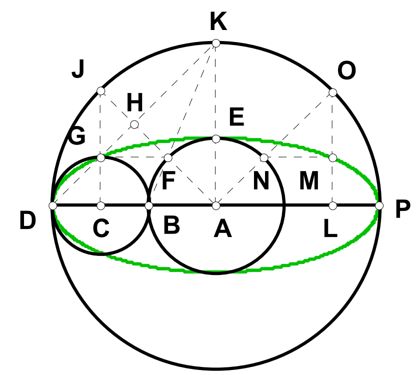
$HJ := AJ - AH \quad AC := AH$

$CD := AD - AC \quad FJ := 2 \cdot CD$

$AF := AJ - FJ \quad AB := AF$

$DK := \sqrt{2 \cdot AK^2} \quad AK + AB - DK = 0$

$\dfrac{AB}{CD} = 1.414214 \qquad \dfrac{DK}{AD} = 1.414214 \; \sqrt{2} = 1.414214$



Unit.

$AE := 1$

Given.

03

031604B

Descriptions.

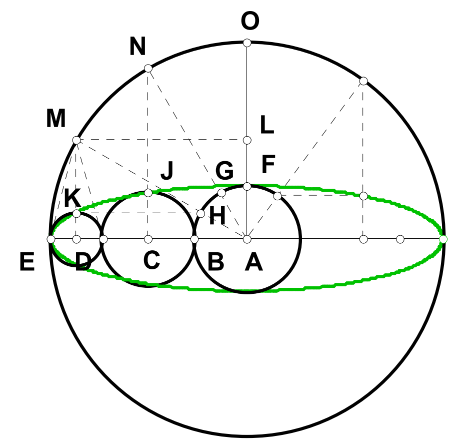
$AO := AE \quad AL := \frac{AO}{2}$

$AM := AE \quad ML := \sqrt{AM^2 - AL^2}$

$AD := ML \quad DE := AE - AD$

$DK := DE \quad DM := AL \quad AH := \frac{AM \cdot DK}{DM}$

$\frac{AH}{DE} = 2$



Unit.

$AE := 1 \quad AG := AE$

Given.

$N := 3 \quad AC := N$

Given AE, AG, AC find the ellipse

031704

Descriptions.

$DE := AE \cdot 2 \quad CJ := AE$

$CE := \sqrt{AE^2 + AC^2}$

$CG := AE \quad FG := \dfrac{DE \cdot CG}{CE}$

Definitions.

$JL := FG \quad JM := \dfrac{JL^2}{2 \cdot CJ}$

$CM := CJ - JM \quad LM := \sqrt{JL^2 - JM^2} \quad NO := AC$

$CN := \dfrac{CM \cdot NO}{LM} \quad AO := CN \quad EO := AO + AE \quad OP := \dfrac{AE^2}{LM}$

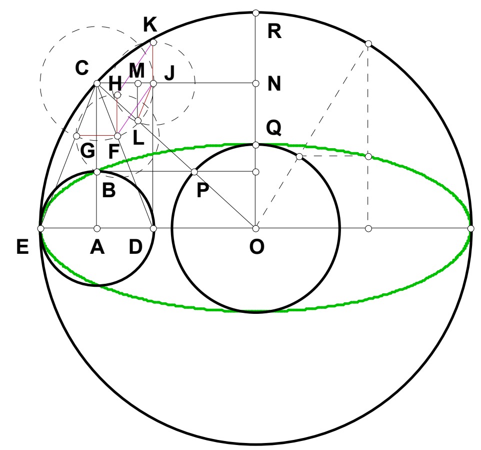
$Major := EO \quad Minor := OP$

$Major = 5$

$Minor = 1.666667$

$Major - \dfrac{N^2 + 1}{2} = 0 \quad Minor - \dfrac{N^2 + 1}{2 \cdot N} = 0$

$\dfrac{Major}{Minor} - N = 0$



On given any AE, AC find the diameter of the Circle.

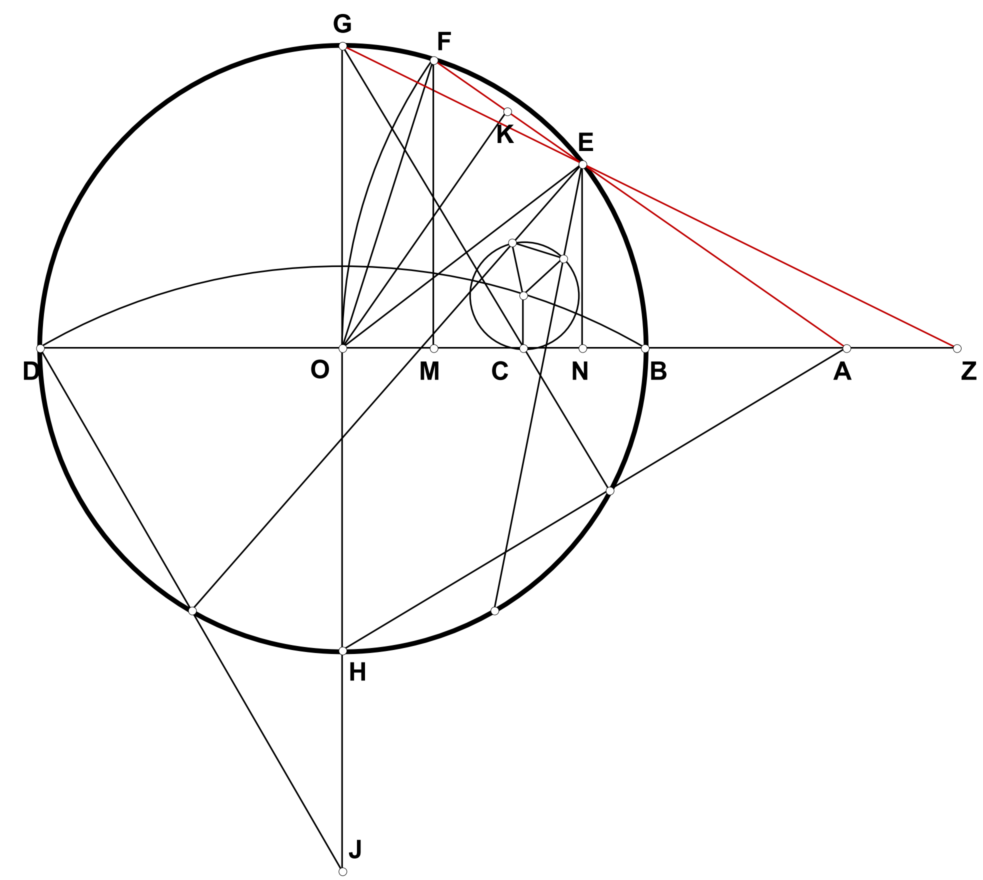
032004

Unit.

$AB := 1$

Given.

$N := 4$



Descriptions.

$AD := N \qquad BD := AD - AB \qquad BO := \dfrac{BD}{2} \qquad AO := AB + BO$

$FO := BO \qquad AF := AO \qquad FK := \dfrac{FO^2}{2 \cdot AF} \qquad EF := 2 \cdot FK$

$MO := FK \qquad AM := AO - MO \qquad FM := \sqrt{AF^2 - AM^2} \qquad GO := BO$

$AE := AF - EF \qquad EN := \dfrac{FM \cdot AE}{AF} \qquad AN := \dfrac{AM \cdot AE}{AF} \qquad NO := AO - AN$

$ZO := \dfrac{NO \cdot GO}{GO - EN} \qquad AZ := ZO - AO \qquad ZN := ZO - NO \qquad GZ := \sqrt{ZO^2 + GO^2}$

$EG := \dfrac{GZ \cdot NO}{ZO} \qquad EG = 1.323879$

Definitions.

$AD - N = 0 \qquad BD - (N-1) = 0 \qquad BO - \dfrac{N-1}{2} = 0 \qquad AO - \dfrac{N+1}{2} = 0 \qquad FO - \dfrac{N-1}{2} = 0 \qquad AF - \dfrac{N+1}{2} = 0 \qquad FK - \dfrac{(N-1)^2}{4 \cdot (N+1)} = 0 \qquad EF - \dfrac{(N-1)^2}{2 \cdot (N+1)} = 0$

$MO - \dfrac{(N-1)^2}{4 \cdot (N+1)} = 0 \qquad AM - \dfrac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad FM - \dfrac{\sqrt{(N+3) \cdot (3 \cdot N + 1)} \cdot (N-1)}{4 \cdot (N+1)} = 0 \qquad GO - \dfrac{N-1}{2} = 0 \qquad AE - \dfrac{2 \cdot N}{N+1} = 0 \qquad EN - \dfrac{N \cdot (N-1) \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}}{(N+1)^3} = 0$

$AN - \dfrac{N \cdot \left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} = 0 \qquad NO - \dfrac{\left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{2 \cdot (N+1)^3} = 0 \qquad ZO - \dfrac{(N-1)^2 \cdot \left(N^2 + 4 \cdot N + 1\right)}{2 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]} = 0 \qquad AZ - \dfrac{N \cdot \left[(N+1) \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} - \left(N^2 + 6 \cdot N + 1\right)\right]}{3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1} = 0$

$ZN - \dfrac{\sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{(N+1)^3 \cdot \left(3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1\right)} = 0 \qquad GZ - \dfrac{\sqrt{(N-1)^2 \cdot (N+1)^3 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{[(N+3) \cdot (3 \cdot N + 1)]} + 3 \cdot N^2 + N^3 + 1\right]}}{\sqrt{2 \cdot \left[N^6 + \left[27 \cdot \left(N^4 + N^2\right) + 6 \cdot \left(N^5 + N\right) + 60 \cdot N^3 + 1\right] - 4 \cdot N \cdot (N+1)^3 \cdot \sqrt{[(N+3) \cdot (3 \cdot N + 1)]}\right]}} = 0$

$$EG - \frac{\sqrt{2} \cdot \sqrt{(N-1)^2 \cdot (N+1)^3 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]} \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]}{2 \cdot (N+1)^3 \cdot \sqrt{27 \cdot \left(N^4 + N^2\right) + \left[6 \cdot \left(N^5 + N\right) + 60 \cdot N^3 + N^6 - 4 \cdot N \cdot (N+1)^3 \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 1\right]}} = 0$$

032304

Descriptions.

Unit.

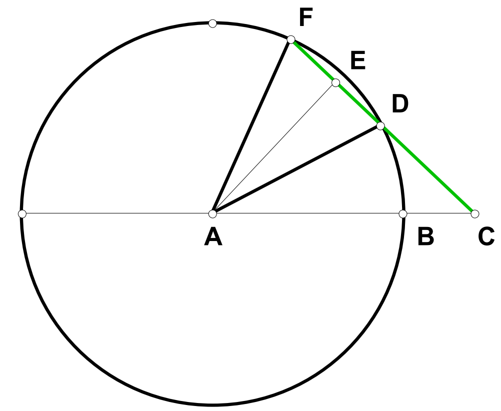
Given.

$N_1 := 1.708$ $\quad AC := N_1$

$N_2 := 1.649$ $\quad CF := N_2$

$N_3 := 1.24$ $\quad AF := N_3$

Given AC, AB and either point of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C?



$$DF_1 := \frac{{N_3}^2 + {N_2}^2 - {N_1}^2}{N_2}$$

$CD := CF - DF_1$ $\qquad N_4 := CD$

$$DF_2 := \frac{{N_3}^2 + {N_4}^2 - {N_1}^2}{N_4}$$

$DF_1 = 0.812333$ $\quad DF_2 = -0.812333$ $\quad DF_1 + DF_2 = 0$

0405043

Descriptions.

Unit.

$CE := 1$

Given.

$N := 4$

Definitions.

All this extra work and I have lost accuracy from 022803!

$CO := \frac{CE}{2} \quad CD := \frac{CE}{N}$

$DO := CO - CD$

$DN := \sqrt{(CO + DO)\cdot(CO - DO)}$

$AD := \frac{DO \cdot DN}{CO - DN} \quad DM := \frac{DN}{2}$

$AN := \sqrt{AD^2 + DN^2} \quad HL := \frac{AN}{2} \quad LA := \frac{HL}{2} \quad Ok := \sqrt{CO^2 - DO^2} \quad Ck := CO - Ok$

$Fm := \frac{Ck \cdot HL}{CO} \quad Jm := \frac{DO \cdot HL}{CO}$

$JF := \sqrt{Fm^2 + Jm^2} \quad Fn := \frac{JF}{2}$

$Fo := \frac{Fm \cdot Fn}{JF} \quad Lo := HL - Fo$

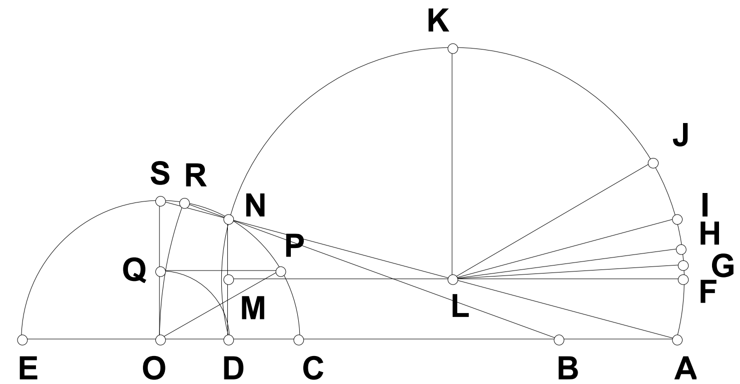
$no := \frac{Jm}{2} \quad Ln := \sqrt{no^2 + Lo^2}$

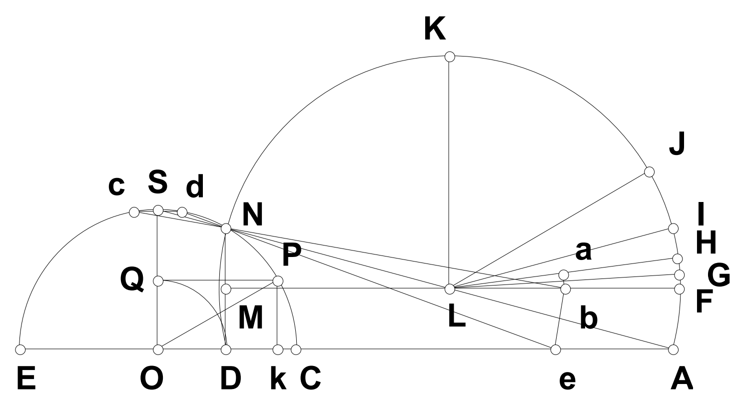
$Iq := \frac{no \cdot HL}{Ln} \quad Lq := \frac{Lo \cdot HL}{Ln}$

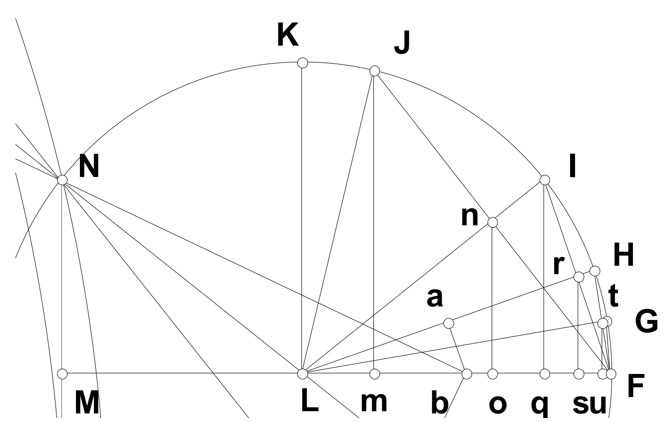
$Fq := HL - Lq \quad FI := \sqrt{Iq^2 + Fq^2} \quad Fr := \frac{FI}{2}$

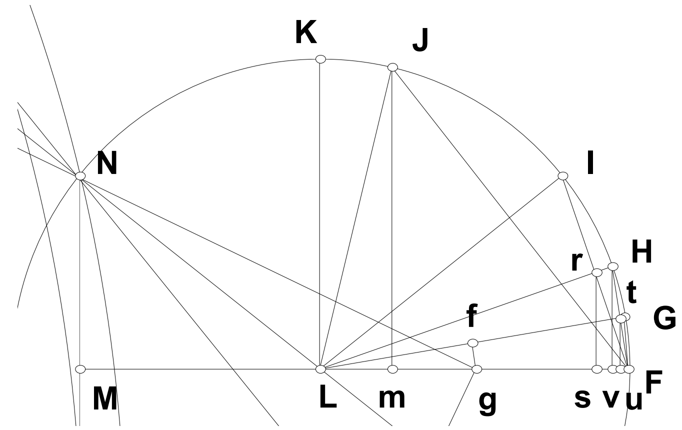
$Fs := \frac{Fq \cdot Fr}{FI} \quad Ls := HL - Fs$

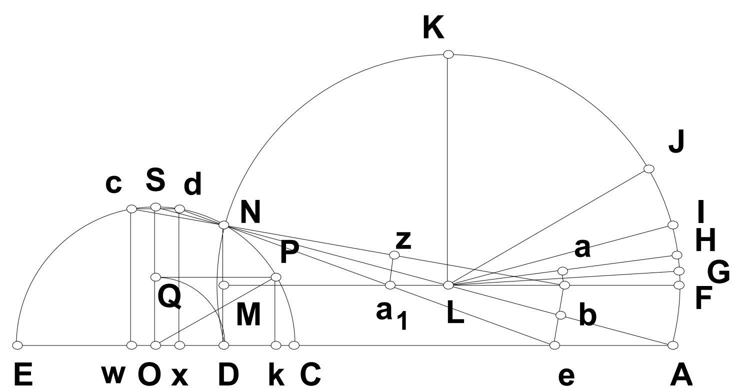
$Lr := \sqrt{HL^2 - Fr^2} \quad Lb := \frac{HL^2}{2 \cdot Lr} \quad rs := \frac{Iq}{2}$











$$Lv := \frac{Ls \cdot HL}{Lr} \qquad Hv := \frac{rs \cdot HL}{Lr}$$

$$Lv := \frac{Ls \cdot HL}{Lr} \qquad Fv := HL - Lv$$

$$FH := \sqrt{Hv^2 + Fv^2} \qquad Ft := \frac{FH}{2}$$

$$Lf := \frac{HL}{2} \qquad Fu := \frac{Fv \cdot Ft}{FH} \qquad tu := \frac{Hv}{2}$$

$$Lu := HL - Fu \qquad Lt := \sqrt{Lu^2 + tu^2}$$

$$Lg := \frac{Lt \cdot Lf}{Lu} \qquad MN := \frac{DN}{2} \qquad NL := \frac{AN}{2}$$

$$ML := \sqrt{NL^2 - MN^2} \qquad Mb := ML + Lb$$

$$Nb := \sqrt{MN^2 + Mb^2}$$

$$Nc := \frac{CO^2 + (2 \cdot Nb)^2 - (2 \cdot Mb + DO)^2}{-2 \cdot Nb}$$

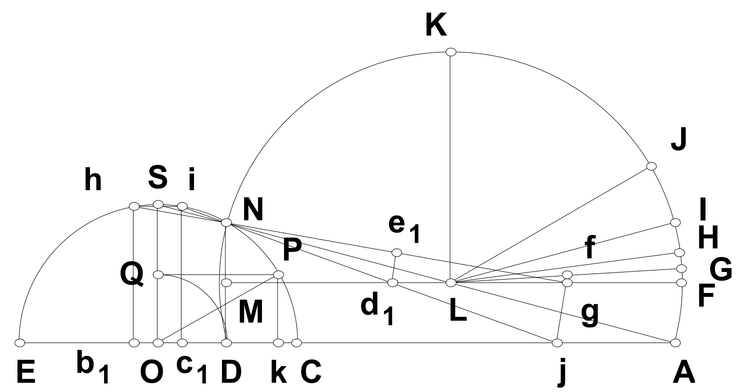
$$cw := \frac{MN \cdot (Nb + Nc)}{Nb} + MN \qquad Dw := \frac{2 \cdot Mb \cdot (2 \cdot Nb + Nc)}{2 \cdot Nb} - 2 \cdot Mb$$

$$Ow := Dw - DO \qquad Dx := Dw - 2 \cdot Ow \qquad Ma_1 := \frac{Dx \cdot DN}{2 \cdot (cw - DN)}$$

$$Na_1 := \sqrt{MN^2 + \left(Ma_1\right)^2} \qquad ba_1 := Mb - Ma_1$$

$$Nz := \frac{\left(Na_1\right)^2 + Nb^2 - \left(ba_1\right)^2}{2 \cdot Nb} \qquad za_1 := \sqrt{\left(Na_1\right)^2 - Nz^2} \qquad be := \frac{za_1 \cdot Nb}{Nz}$$

$$\mathbf{Mg} := \mathbf{ML} + \mathbf{Lg} \qquad \mathbf{Ng} := \sqrt{\mathbf{MN}^2 + \mathbf{Mg}^2}$$



$$\mathbf{Nh} := \frac{\mathbf{CO}^2 + (2\cdot\mathbf{Ng})^2 - (2\cdot\mathbf{Mg} + \mathbf{DO})^2}{-2\cdot\mathbf{Ng}}$$

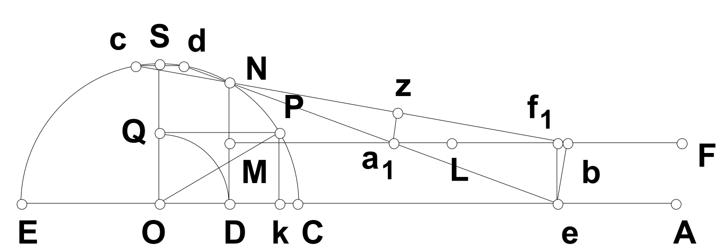
$$\mathbf{hb_1} := \frac{\mathbf{MN}\cdot(\mathbf{Ng} + \mathbf{Nh})}{\mathbf{Ng}} + \mathbf{MN} \qquad \mathbf{Db_1} := \frac{2\cdot\mathbf{Mg}\cdot(2\cdot\mathbf{Ng} + \mathbf{Nh})}{2\cdot\mathbf{Ng}} - 2\cdot\mathbf{Mg}$$

$$\mathbf{Ob_1} := \mathbf{Db_1} - \mathbf{DO} \qquad \mathbf{Dc_1} := \mathbf{Db_1} - 2\cdot\mathbf{Ob_1} \qquad \mathbf{Md_1} := \frac{\mathbf{Dc_1}\cdot\mathbf{DN}}{2\cdot\left(\mathbf{hb_1} - \mathbf{DN}\right)}$$

$$\mathbf{Nd_1} := \sqrt{\mathbf{MN}^2 + \left(\mathbf{Md_1}\right)^2} \qquad \mathbf{gd_1} := \mathbf{Mg} - \mathbf{Md_1} \qquad \mathbf{Ne_1} := \frac{\left(\mathbf{Nd_1}\right)^2 + \mathbf{Ng}^2 - \left(\mathbf{gd_1}\right)^2}{2\cdot\mathbf{Ng}}$$

$$\mathbf{ed_1} := \sqrt{\left(\mathbf{Nd_1}\right)^2 - \left(\mathbf{Ne_1}\right)^2} \qquad \mathbf{gj} := \frac{\mathbf{ed_1}\cdot\mathbf{Ng}}{\mathbf{Ne_1}} \qquad \mathbf{Ne} := \frac{\mathbf{Na_1}\cdot\mathbf{Nb}}{\mathbf{Nz}}$$

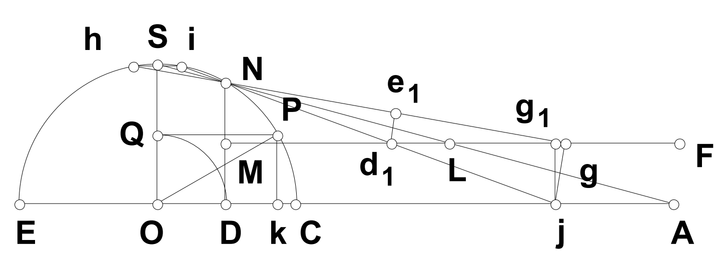
$$\mathbf{ea_1} := \mathbf{Ne} - \mathbf{Na_1}$$



$$\mathbf{bf_1} := \frac{\mathbf{be}^2 + \mathbf{ba_1}^2 - \mathbf{ea_1}^2}{2\cdot\mathbf{ba_1}}$$

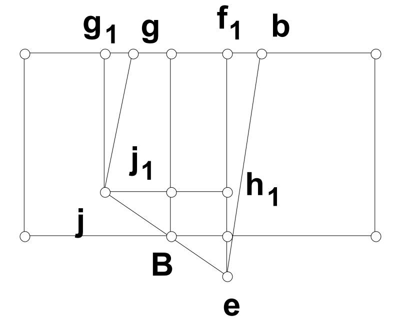
$$\mathbf{ef_1} := \sqrt{\mathbf{be}^2 - \mathbf{bf_1}^2}$$

$$\mathbf{Nj} := \frac{\mathbf{Nd_1}\cdot\mathbf{Ng}}{\mathbf{Ne_1}} \qquad \mathbf{jd_1} := \mathbf{Nj} - \mathbf{Nd_1}$$



$$\mathbf{gg_1} := \frac{\mathbf{gj}^2 + \mathbf{gd_1}^2 - \mathbf{jd_1}^2}{2\cdot\mathbf{gd_1}}$$

$$\mathbf{jg_1} := \sqrt{\mathbf{gj}^2 - \mathbf{gg_1}^2}$$



$$Mg_1 := Mg - gg_1 \qquad Mf_1 := Mb - bf_1$$

$$jh_1 := Mf_1 - Mg_1 \qquad eh_1 := ef_1 - jg_1$$

$$ej := \sqrt{jh_1{}^2 + eh_1{}^2} \qquad Bj_1 := DM - jg_1$$

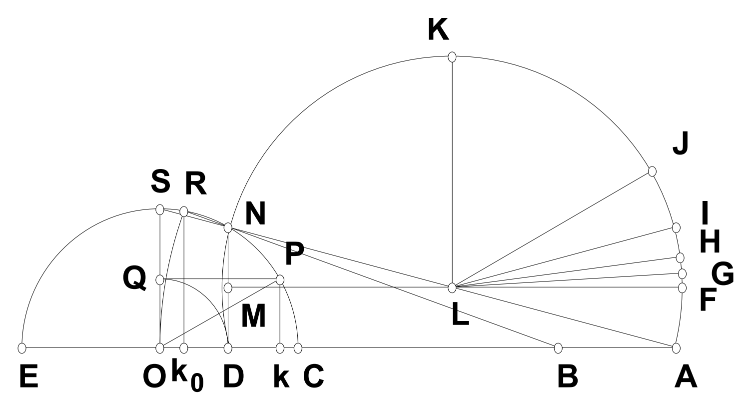
$$jj_1 := \frac{jh_1 \cdot Bj_1}{eh_1} \qquad BD := Mg_1 + jj_1 \qquad BO := BD + DO$$

$$BN := \sqrt{DN^2 + BD^2}$$

$$NR := \frac{CO^2 + BN^2 - BO^2}{-BN}$$

$$Bk_0 := \frac{BD \cdot (BN + NR)}{BN}$$

$$Ok_0 := BO - Bk_0$$



$$NR - 2 \cdot Ok_0 = -0.000000000019581$$

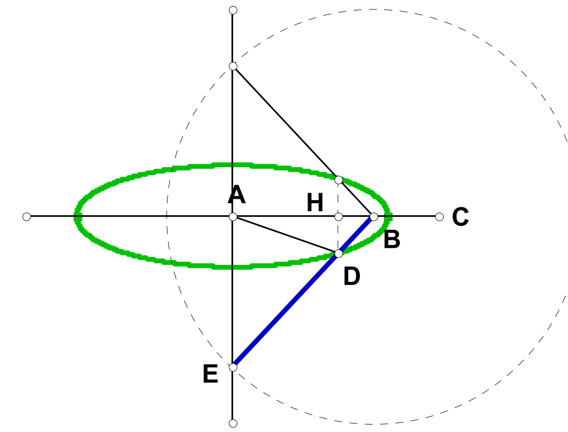
Compared to 022803

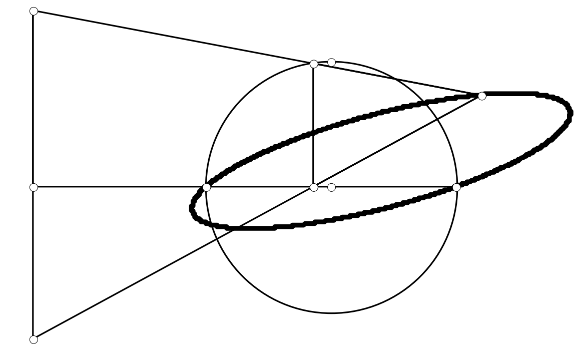
$$\frac{19591}{2639} = 7.423645$$

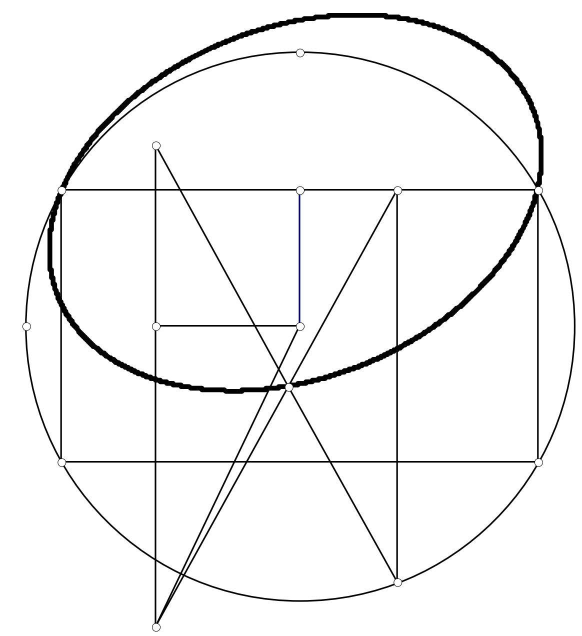
$$Om - \frac{Ek}{2} = -0.000000000002639$$

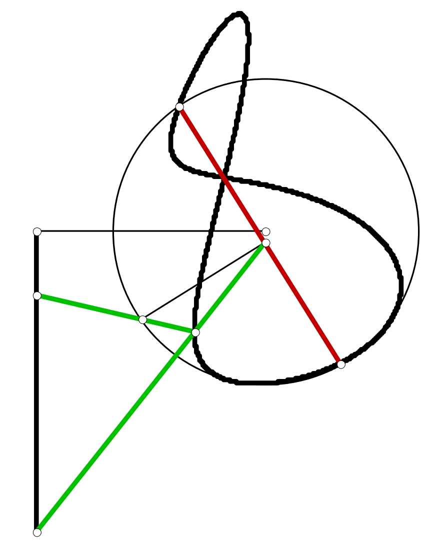
The Ellipse

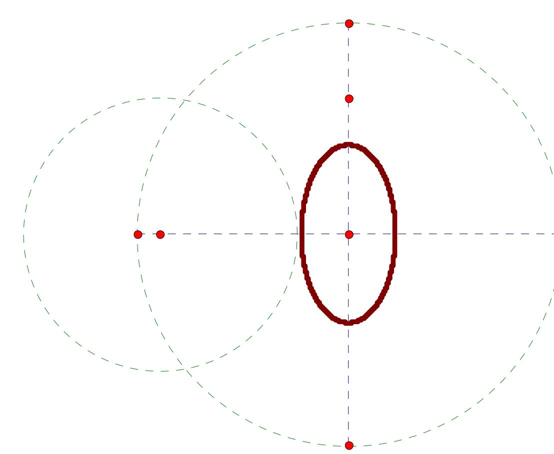
041904A











I once worked on a project I called Eloi, which was about the different ways to construct an ellipse, the different equations one would have to use to make that figure, and the different ways one could solve for those ellipses. And this entertained me until I started thinking about the figure 8 locus. One can draw the figure eight as a locus between a straight line and a cirlce.

All of this applies to science and mechanics when one is writing up equations to the motion of objects; How do you comprehend what you are seeing?

This plate series is called the Straight Line Ellipse because it reduces the ellipse to a single linear action between X and Y axies. In other words, we draw what some would claim to be a trig function, which is actually a linear function, one just doess not see it. When every possible grammar is effected by complete induction and deduction of a unit, one should keep the unit in mind instead of obfuscating it with particular names which factually do not apply. One is always in danger of claiming that there are many different mathematics, yet the same single unit makes them all; this amounts to a thing is different from itself, and if one is stupid, it produces the modern mathematician.

As one can see by the last figure, the major and minor axis do not determine the resulting figure, meaning the shape is independent of the axis, but when it is shaped, or created is. The resulting figure, a photon, or a burst of energy, or one can say a pulse, is the product of a simple tic, toc, tic of two objects, an oscilation. At a certain point they interact, and release an elliptical signiture, or temporarily existing third object. This means one can spend a lot of time claiming that mechanics is wave mechanics, or quantum mechanics, or that these names both miss the point; how you write an object up does not create a new grammar, unless you are really stupid. Grammar is not a theory, and if you are teaching theory, you may as well elect a Pope.

What is more important, we start to get back to fundamentals in linguistic fact, between any two limits is one, and only one relative difference, which is not only called a unit, but even observed by Plato. We can obfuscate it, like all of the other ways we can produce an ellipse, but those figures do not express the fundamental ellipse.

041904B

Descriptions.

$BE := AC \quad BD := \dfrac{BE}{R_1}$

$AB := \dfrac{AC}{R_2} \quad AE := \sqrt{BE^2 - AB^2}$

$BH := \dfrac{AB \cdot BD}{BE} \quad DH := \dfrac{AE \cdot BD}{BE}$

$AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$

Definitions.

$$AD - \frac{\sqrt{R_1^{\,2} - 2 \cdot R_1 + R_2^{\,2}}}{R_1 \cdot R_2} = 0$$

Unit.

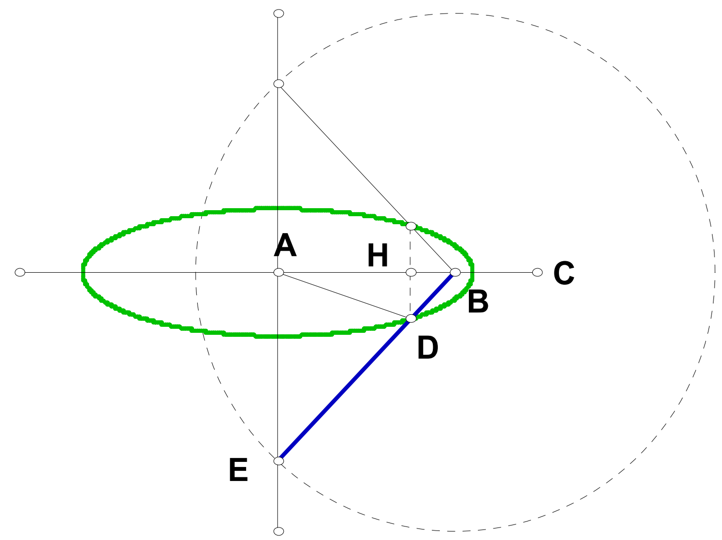
$AC := 1$

Given.

$R_1 := 3$

$R_2 := 2$

Straight Line Ellipse: Cardinal



041904C

Descriptions.

$$AE := \sqrt{BE^2 - AB^2}$$

$$BH := \frac{AB \cdot BD}{BE} \qquad DH := \frac{AE \cdot BD}{BE}$$

$$AH := AB - BH \qquad AD := \sqrt{AH^2 + DH^2}$$

Unit.

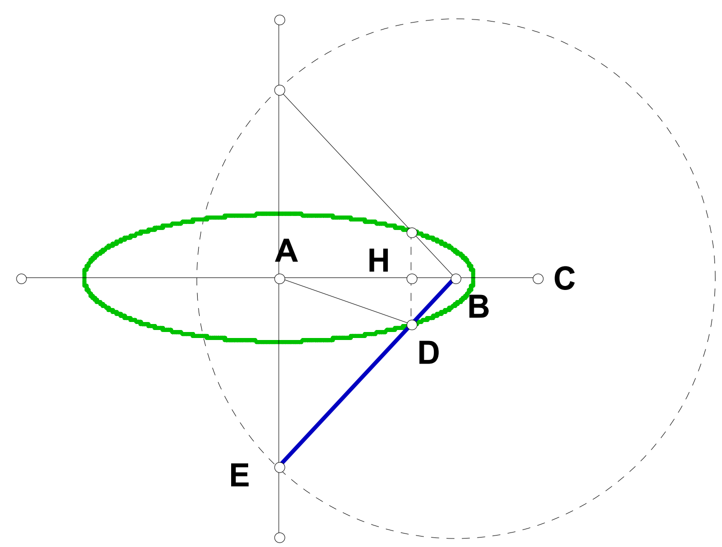
$AC := 1 \qquad BE := AC$

Given.

$N_1 := .5 \qquad AB := N_1$

$N_2 := .3 \qquad BD := N_2$

Straight Line Ellipse: Ordinal



Definitions.

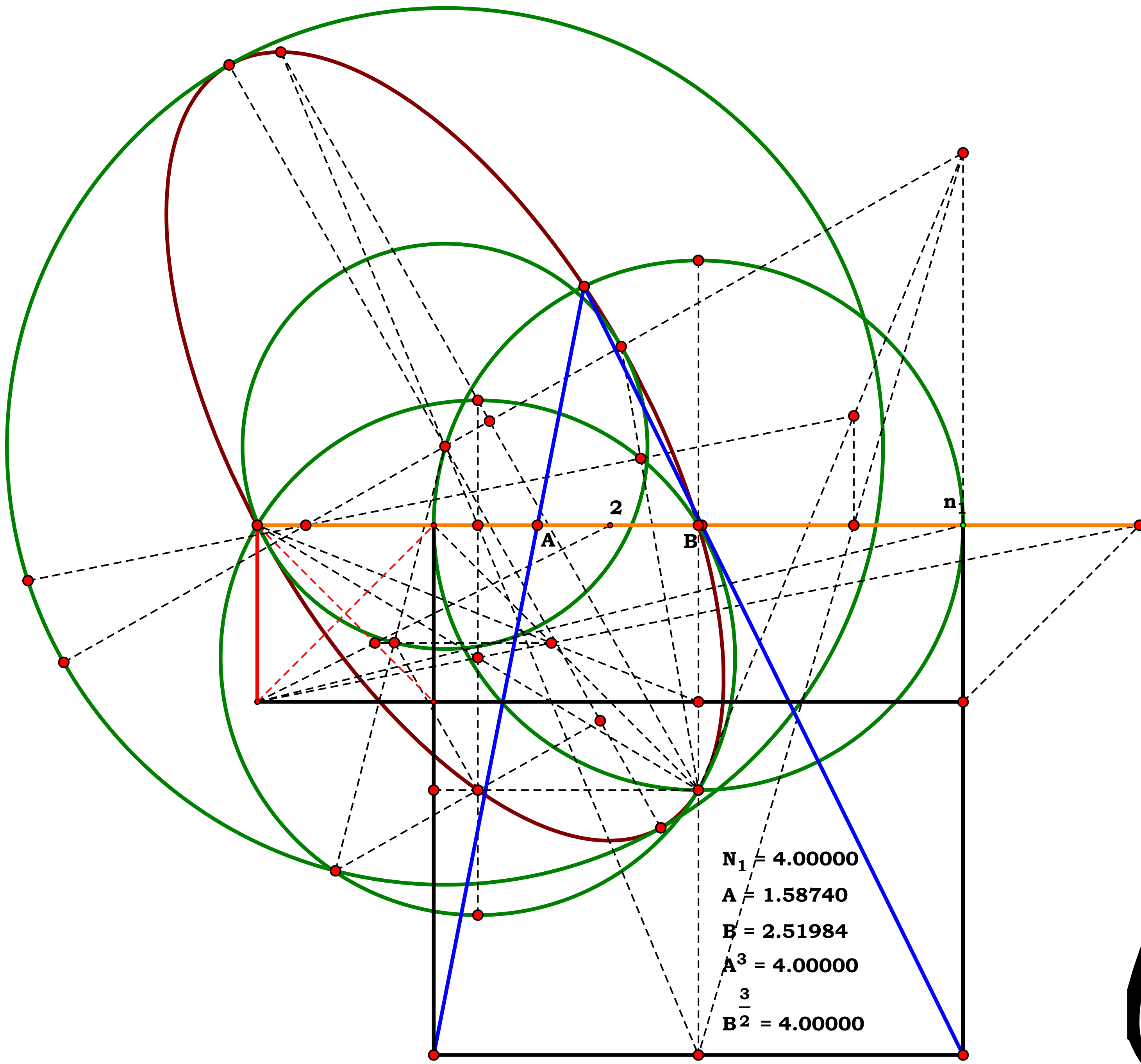
$$AE - \sqrt{(1 - N_1) \cdot (N_1 + 1)} = 0$$

$$BH - N_1 \cdot N_2 = 0$$

$$DH - \sqrt{(1 - N_1) \cdot (N_1 + 1)} \cdot N_2 = 0$$

$$AH - (N_1 - N_1 \cdot N_2) = 0$$

$$AD - \sqrt{N_1^{\,2} - 2 \cdot N_1^{\,2} \cdot N_2 + N_2^{\,2}} = 0$$



The Delian Quest 2005

John Clark

031405

Descriptions.

$AD := N_1 \qquad DV := AC$

$AV := \sqrt{AD^2 + DV^2} \qquad AF := 2 \cdot AC$

$VX := AF - AD \qquad AY := \dfrac{AV \cdot AC}{AC - VX}$

$AG := \dfrac{AD \cdot AY}{AV} \qquad BC := \dfrac{AC \cdot AC}{AC + AD}$

$CG := AG - AC \qquad BG := BC + CG$

$BE := \dfrac{BG}{2} \qquad CE := BE - BC \qquad ES := BE$

$CS := \sqrt{ES^2 - CE^2} \qquad CR := \dfrac{DV \cdot AC}{AD}$

$EU := \dfrac{ES \cdot CR}{CS} \qquad EZ := EU$

Definitions.

$$EZ - \frac{\sqrt{N_1{}^2 - 1}}{(N_1 - 1) \cdot (N_1 + 1)} = 0$$

$$BG - \frac{2 \cdot N_1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$$

Unit.

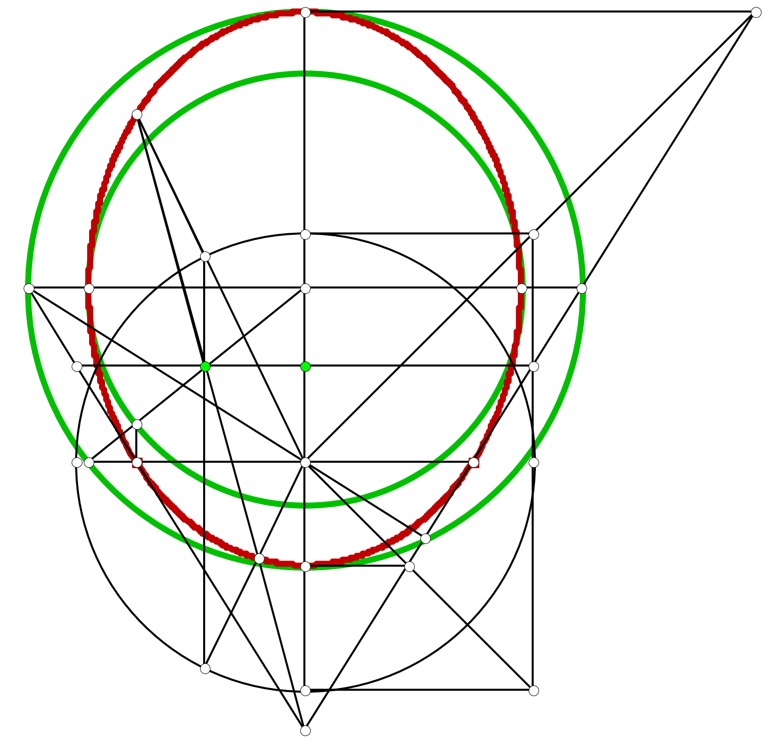
$AC := 1$

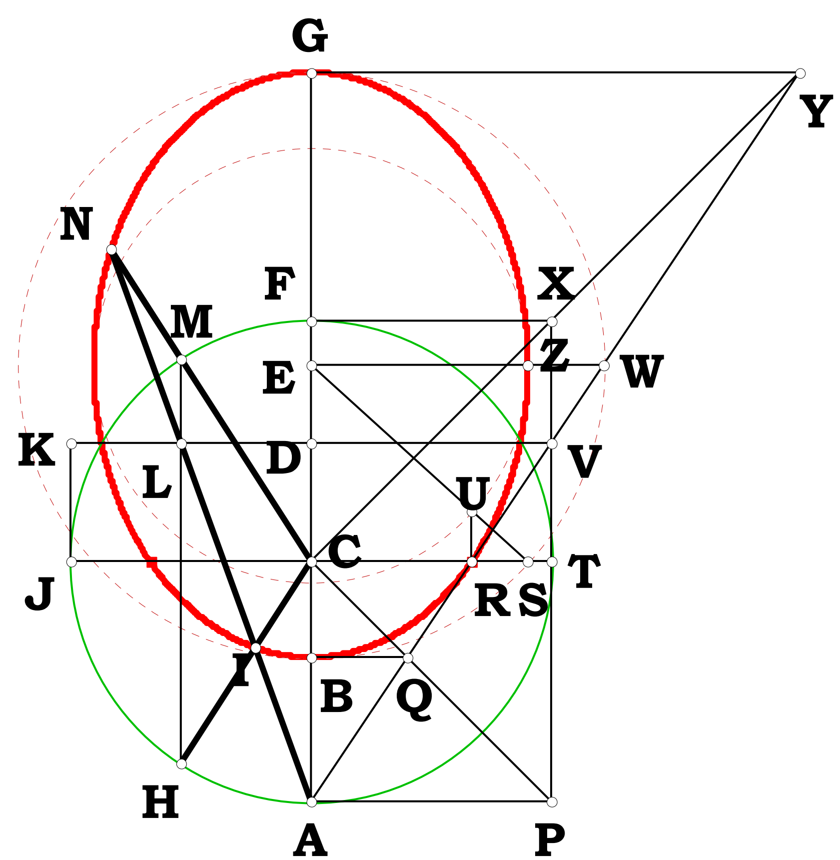
Given.

$N_1 := 3$

Another Ellipse

The locus formed by N and I as determined by L provides an ellipse. Provide an Algebraic name for the Major and Minor Axis.

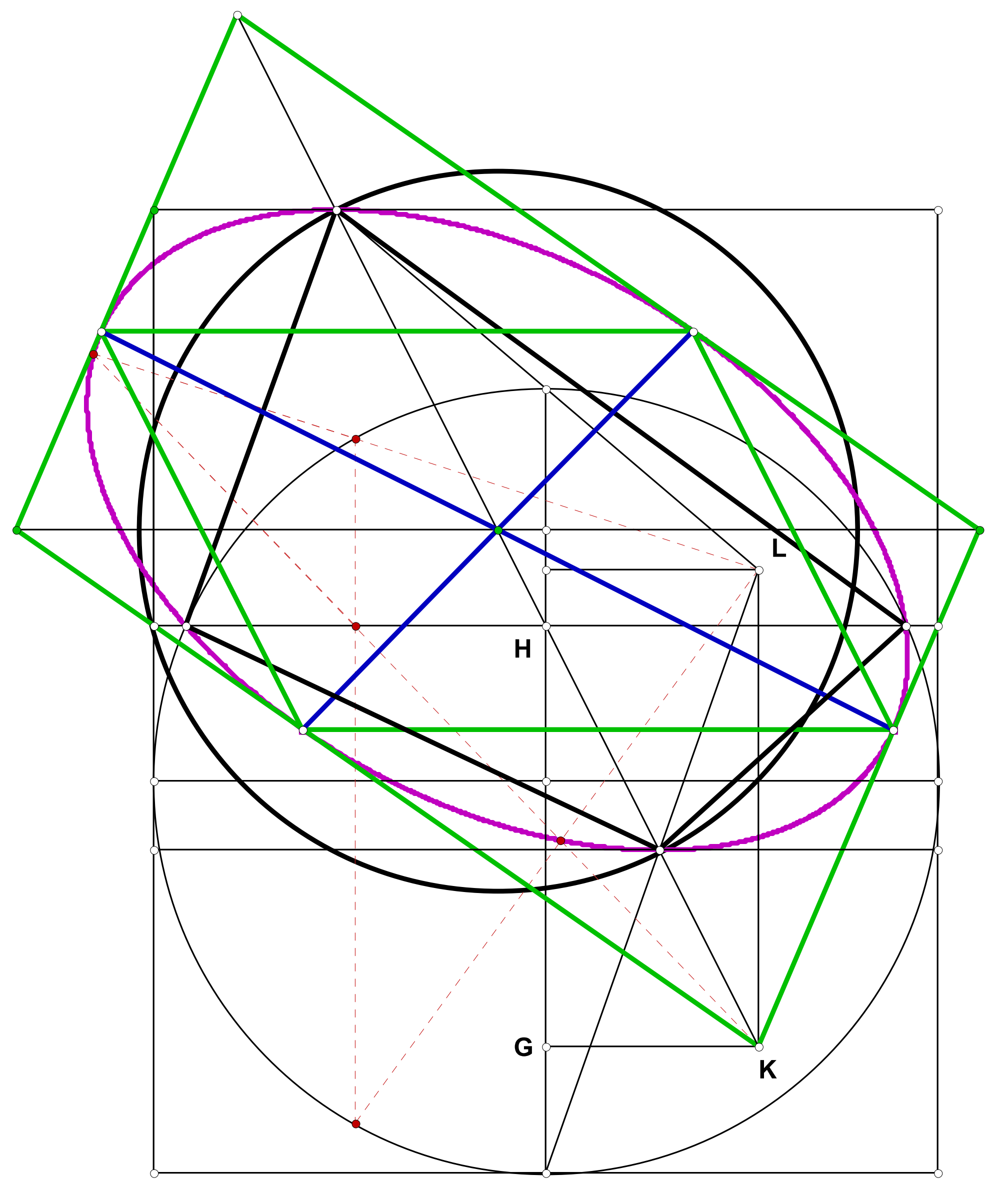




Unit.
Given.

Parcing project for 031605

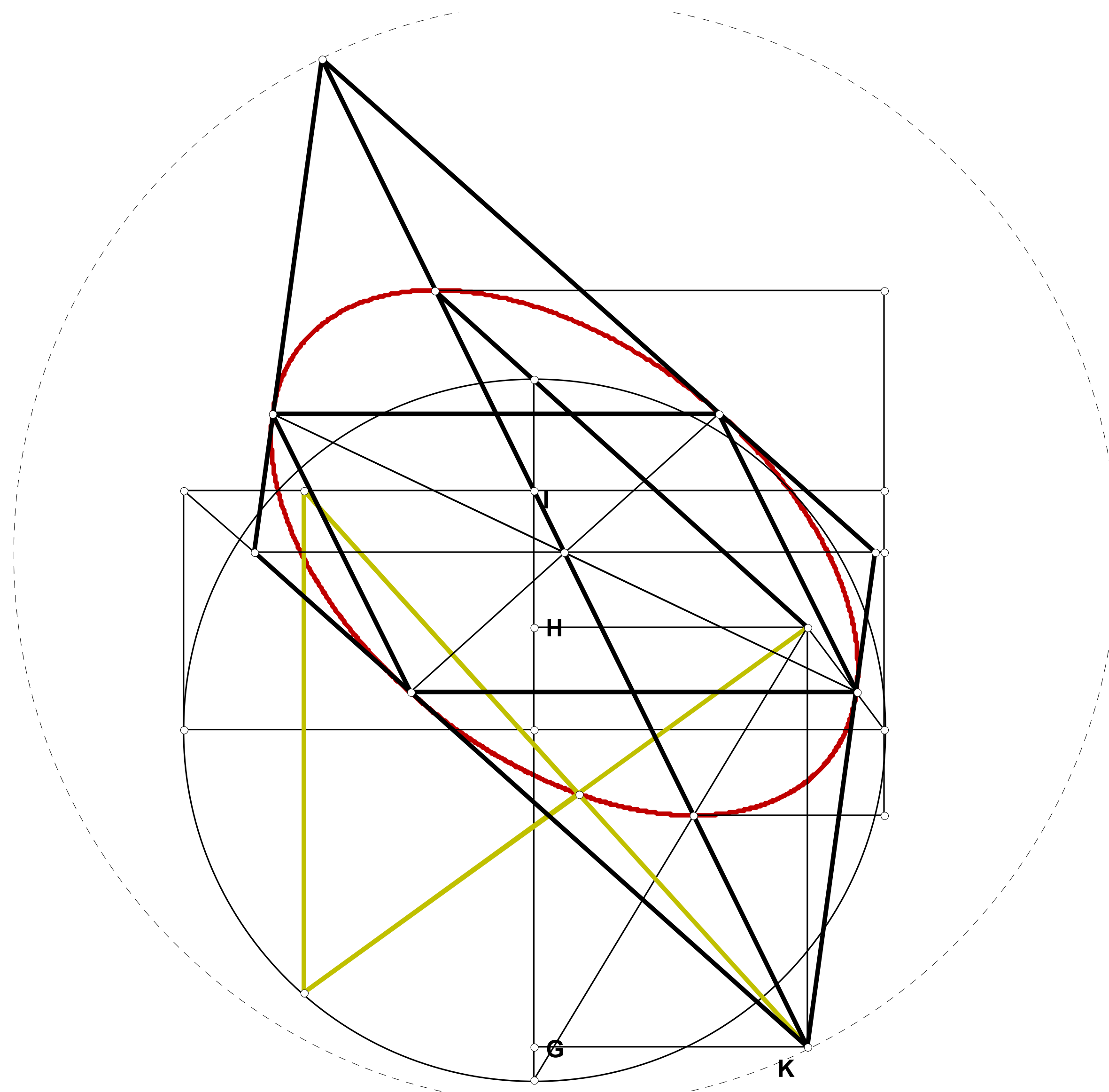
Descriptions.
Definitions.



Unit.
Given.

Parcing project for 031705

Descriptions.
Definitions.



0932305

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 2 \quad BE := N_1$

$N_2 := .5 \quad BD := N_2$

An Ellipse

$MN := 2 \cdot AB$

$DE := BE - BD \qquad AG := AB$

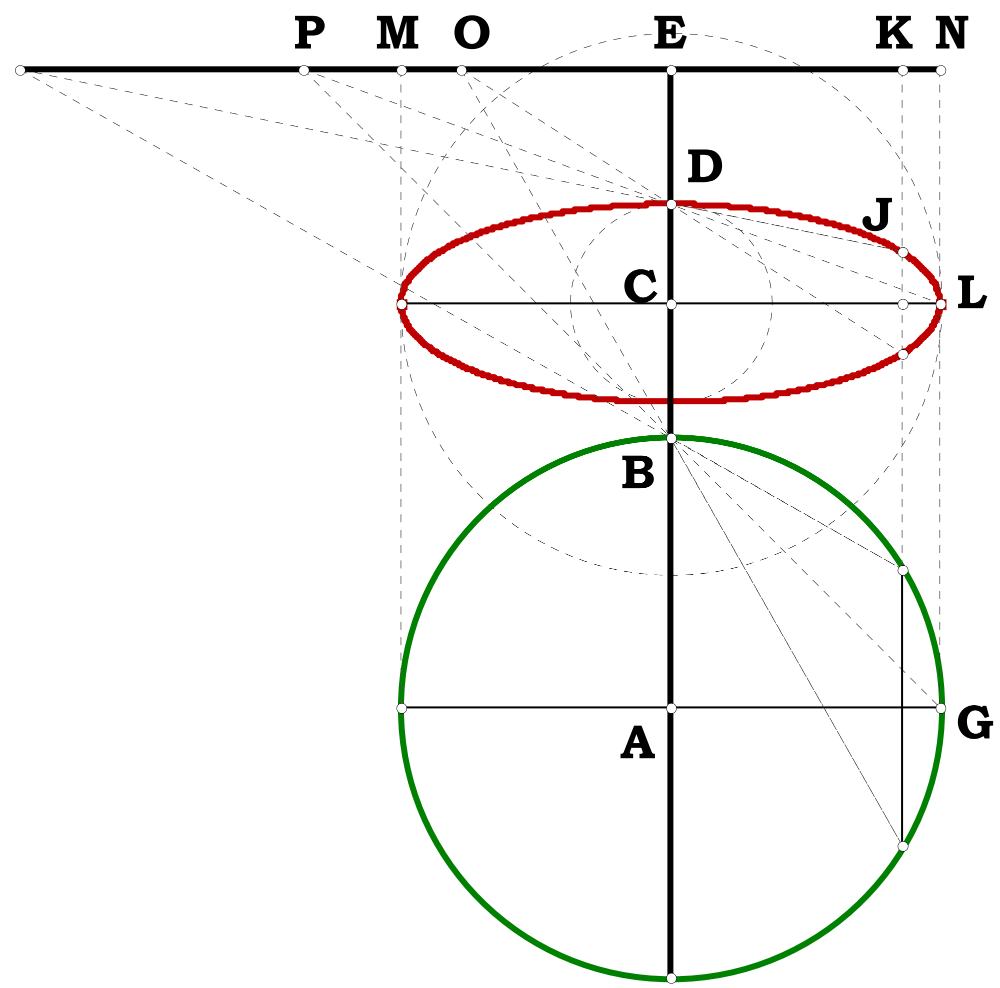
$EP := \dfrac{AG \cdot BE}{AB} \qquad EN := AB$

$NL := \dfrac{DE \cdot (EP + EN)}{EP}$

$CE := NL \qquad CD := CE - DE$

Definitions.

$$\frac{(N_1 - N_2)}{N_1} - CD = 0$$

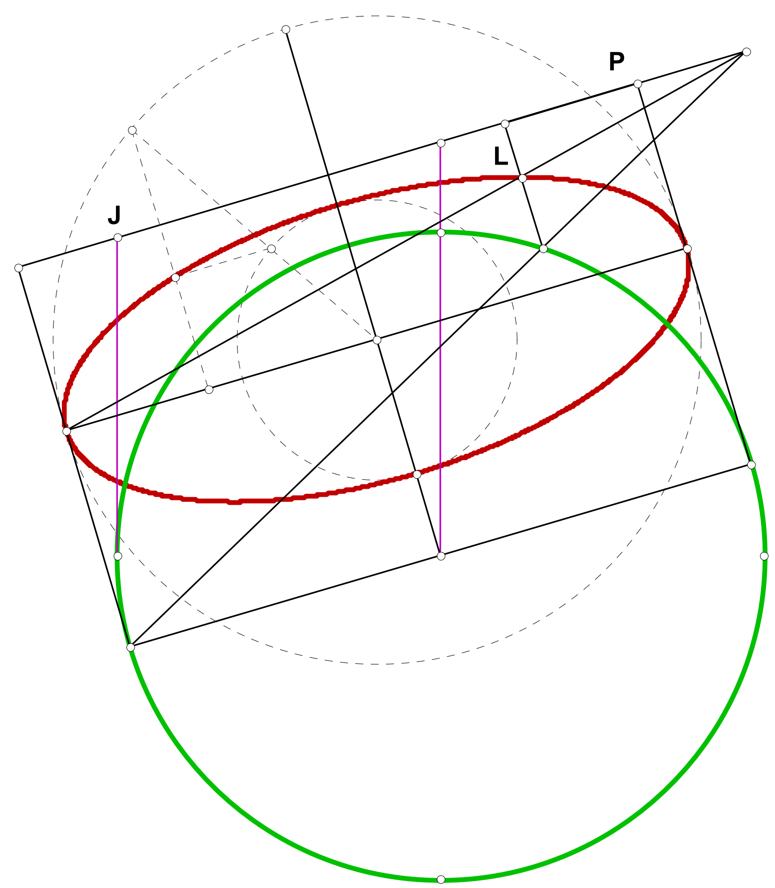


Unit.
Given.

Parcing project for 032405

Descriptions.
Definitions.

032405a.gsp



032905

Descriptions.

Unit.

Given.

$N_1 := 1.9167 \qquad AC := N_1$

$N_2 := .3244 \qquad CD := N_2$

$N_3 := .437 \qquad GI := N_3$

Elipse Projected From a Perpendicular.

Let AC be some perpendicular on some line GH.

$CE := CD \qquad CG := \sqrt{CE \cdot AC} \qquad GH := 2 \cdot CG$

$CI := CG - GI \qquad AD := AC - CD \qquad AI := \sqrt{AC^2 + CI^2} \qquad AM := \frac{AD}{2}$

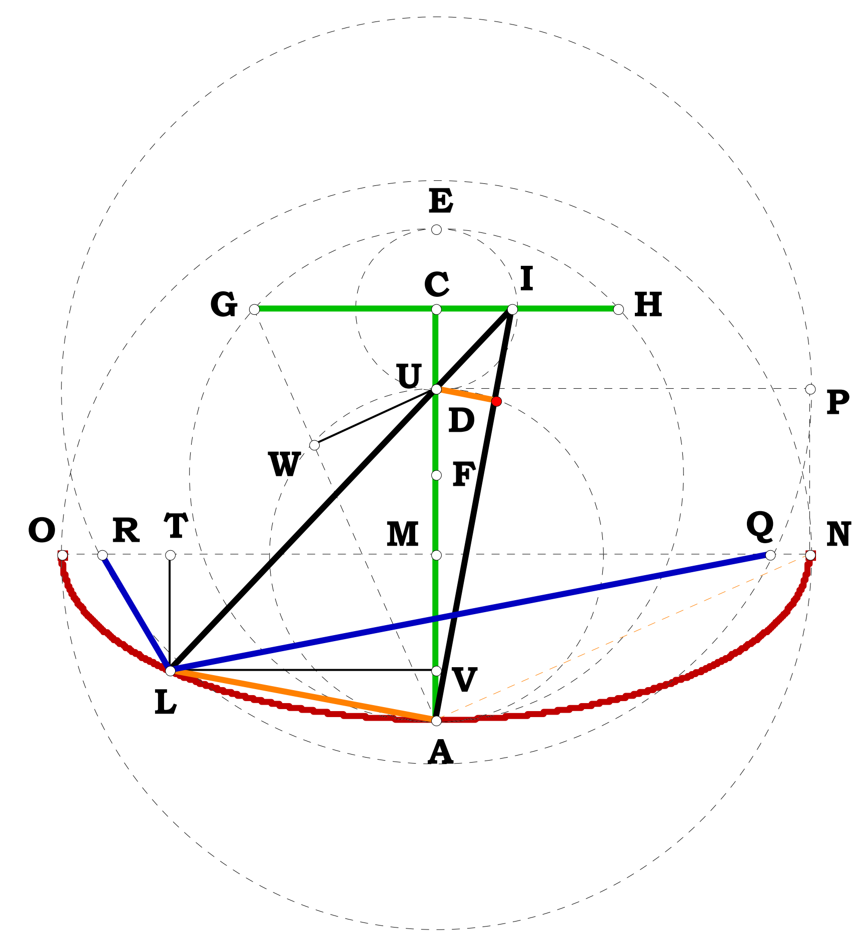
$DM := AM \qquad DU := \frac{CI \cdot AD}{AI} \qquad AU := \frac{AC \cdot AD}{AI} \qquad IU := AI - AU \qquad AL := \frac{DU \cdot AI}{IU}$

$IL := \sqrt{AI^2 + AL^2} \qquad DI := \sqrt{DU^2 + IU^2} \qquad DL := IL - DI \qquad DV := \frac{CD \cdot DL}{DI}$

$MV := DV - DM \qquad AV := AM - MV \qquad LV := \sqrt{AL^2 - AV^2} \qquad MT := LV$

$AG := \sqrt{CG^2 + AC^2} \qquad DW := \frac{CG \cdot AD}{AG} \qquad AW := \frac{AC \cdot AD}{AG} \qquad GW := AG - AW$

$AN := \frac{DW \cdot AG}{GW} \qquad MN := \sqrt{AN^2 - AM^2} \qquad ON := 2 \cdot MN$



Definitions.

$$ON - \sqrt{(N_2 - N_1)^2 \cdot \frac{N_1}{N_2}} = 0$$

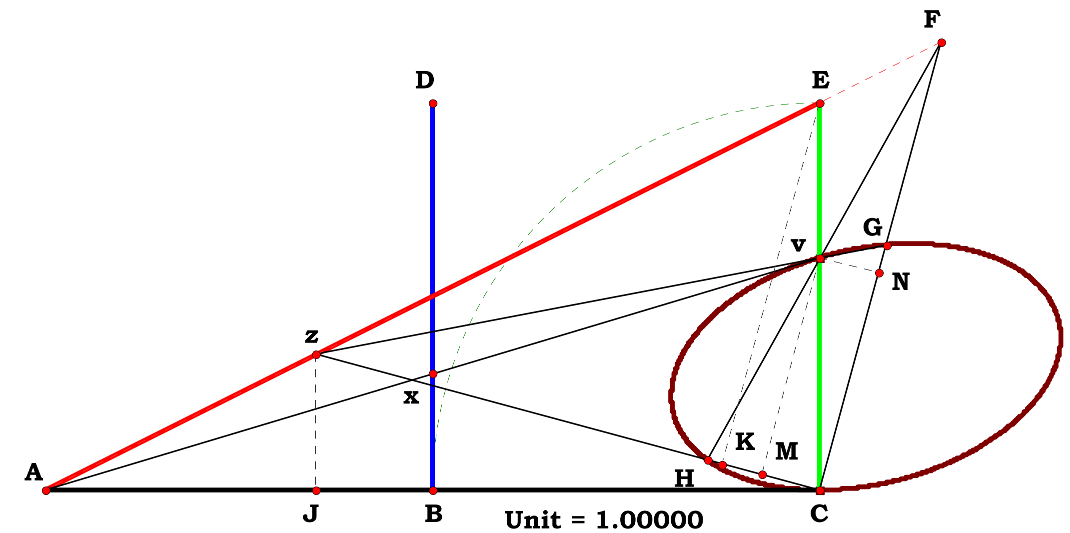
032905B

Parcing project for 032905b, using just straight lines.

Unit. $BC := 1$

Given. $z := 7 \quad x := 6 \quad v := 12$

$y := 20 \quad w := 20 \quad u := 20$



z/y = 0.35000
z = 7.00000
y = 20.00000

x/w = 0.30000
x = 6.00000
w = 20.00000

v/C = 0.60000
v = 12.00000
u = 20.00000

Descriptions.

$CE := BC \quad BD := BC \quad Cv := \frac{CE \cdot v}{u} \quad Bx := \frac{BD \cdot x}{w} \quad vx := \sqrt{BC^2 + (Cv - Bx)^2} \quad AC := \frac{BC \cdot Cv}{Cv - Bx}$

$AE := \sqrt{AC^2 + CE^2} \quad Az := AE \cdot \frac{z}{y} \quad AJ := \frac{AC \cdot Az}{AE} \quad Jz := \frac{CE \cdot AJ}{AC} \quad CJ := AC - AJ$

$Cz := \sqrt{Jz^2 + CJ^2} \quad Ez := AE - Az \quad Kz := \frac{Cz^2 + Ez^2 - CE^2}{2 \cdot Cz} \quad EK := \sqrt{Ez^2 - Kz^2}$

$CF := EK \cdot \frac{Cz}{Kz} \quad Fz := \frac{Ez \cdot Cz}{Kz} \quad CK := Cz - Kz \quad CM := \frac{CK \cdot Cv}{CE} \quad Mv := \frac{EK \cdot Cv}{CE} \quad CN := Mv$

$Nv := CM \quad FN := CF - CN \quad CH := \frac{Nv \cdot CF}{FN} \quad Mz := Cz - CM \quad vz := \sqrt{Mv^2 + Mz^2} \quad Gv := \frac{vz \cdot Nv}{Mz}$

Definitions.

$CE - 1 = 0 \quad BD - 1 = 0 \quad Cv - \frac{v}{u} = 0 \quad Bx - \frac{x}{w} = 0 \quad vx - \frac{\sqrt{u^2 \cdot w^2 + u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + v^2 \cdot w^2}}{u \cdot w} = 0 \quad AC - \frac{v \cdot w}{(v \cdot w - u \cdot x)} = 0 \quad AE - \frac{\sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2}}{(v \cdot w - u \cdot x)} = 0$

$Az - \frac{z \cdot \sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2}}{y \cdot (v \cdot w - u \cdot x)} = 0 \quad AJ - \frac{v \cdot w \cdot z}{y \cdot (v \cdot w - u \cdot x)} = 0 \quad Jz - \frac{z}{y} = 0 \quad CJ - \frac{v \cdot w \cdot (y - z)}{y \cdot (v \cdot w - u \cdot x)} = 0 \quad Ez - \frac{\sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2} \cdot (y - z)}{y \cdot (v \cdot w - u \cdot x)} = 0$

$$Cz - \frac{\sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}{y \cdot (v \cdot w - u \cdot x)} = 0$$

$$Kz - \frac{2 \cdot u^2 \cdot x^2 \cdot y \cdot z - 2 \cdot u^2 \cdot x^2 \cdot z^2 - 4 \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z + 4 \cdot u \cdot v \cdot w \cdot x \cdot z^2 - 2 \cdot v^2 \cdot w^2 \cdot y^2 + 6 \cdot v^2 \cdot w^2 \cdot y \cdot z - 4 \cdot v^2 \cdot w^2 \cdot z^2}{2 \cdot y \cdot (u \cdot x - v \cdot w) \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$EK - \frac{v \cdot w \cdot (y - z)}{\sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0 \quad CF - \frac{v \cdot w \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}{v^2 \cdot w^2 \cdot y - u^2 \cdot x^2 \cdot z - 2 \cdot v^2 \cdot w^2 \cdot z + 2 \cdot u \cdot v \cdot w \cdot x \cdot z} = 0$$

$$Fz-\frac{\sqrt{u^2\cdot x^2-2\cdot u\cdot v\cdot w\cdot x+2\cdot v^2\cdot w^2}\cdot\left(u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2\right)}{y\cdot(u\cdot x-v\cdot w)\cdot\left(u^2\cdot x^2\cdot z-v^2\cdot w^2\cdot y+2\cdot v^2\cdot w^2\cdot z-2\cdot u\cdot v\cdot w\cdot x\cdot z\right)}=0$$

$$CK-\frac{z\cdot(v\cdot w-u\cdot x)}{\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}=0$$

$$CM-\frac{v\cdot z\cdot(v\cdot w-u\cdot x)}{u\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}=0$$

$$Nv-\frac{v\cdot z\cdot(v\cdot w-u\cdot x)}{u\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}=0$$

$$Mv-\frac{v^2\cdot w\cdot(y-z)}{u\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}=0$$

$$CN-\frac{v^2\cdot w\cdot(y-z)}{u\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}=0$$

$$FN-\left[\frac{\begin{array}{l}v\cdot w\cdot\left(u^3\cdot x^2-2\cdot u^2\cdot v\cdot w\cdot x-u^2\cdot v\cdot x^2+2\cdot u\cdot v^2\cdot w^2+2\cdot u\cdot v^2\cdot w\cdot x-2\cdot v^3\cdot w^2\right)\cdot z^2\ \ldots\\ +\ v\cdot w\cdot\left(y\cdot u^2\cdot v\cdot x^2-2\cdot y\cdot u\cdot v^2\cdot w^2-2\cdot y\cdot u\cdot v^2\cdot w\cdot x+3\cdot y\cdot v^3\cdot w^2\right)\cdot z-v\cdot w\cdot\left(v^3\cdot w^2\cdot y^2-u\cdot v^2\cdot w^2\cdot y^2\right)\end{array}}{u\cdot\left(v^2\cdot w^2\cdot y-u^2\cdot x^2\cdot z-2\cdot v^2\cdot w^2\cdot z+2\cdot u\cdot v\cdot w\cdot x\cdot z\right)\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}\right]=0$$

$$CH-\frac{v\cdot z\cdot(u\cdot x-v\cdot w)\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}{\left(w^2\cdot y^2-3\cdot w^2\cdot y\cdot z+2\cdot w^2\cdot z^2\right)\cdot v^3+\left(2\cdot u\cdot w^2\cdot y\cdot z-u\cdot w^2\cdot y^2-2\cdot u\cdot w^2\cdot z^2+2\cdot u\cdot x\cdot w\cdot y\cdot z-2\cdot u\cdot x\cdot w\cdot z^2\right)\cdot v^2+\left(u^2\cdot x^2\cdot z^2-y\cdot u^2\cdot x^2\cdot z+2\cdot w\cdot u^2\cdot x\cdot z^2\right)\cdot v-u^3\cdot x^2\cdot z^2}=0$$

$$Mz-\left[\frac{u^3\cdot x^2\cdot z^2-2\cdot u^2\cdot v\cdot w\cdot x\cdot z^2-u^2\cdot v\cdot x^2\cdot y\cdot z+u\cdot v^2\cdot w^2\cdot y^2-2\cdot u\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot u\cdot v^2\cdot w^2\cdot z^2+2\cdot u\cdot v^2\cdot w\cdot x\cdot y\cdot z-v^3\cdot w^2\cdot y\cdot z}{u\cdot y\cdot(v\cdot w-u\cdot x)\cdot\sqrt{u^2\cdot x^2\cdot z^2-2\cdot u\cdot v\cdot w\cdot x\cdot z^2+v^2\cdot w^2\cdot y^2-2\cdot v^2\cdot w^2\cdot y\cdot z+2\cdot v^2\cdot w^2\cdot z^2}}\right]=0$$

$$vz-\left[\frac{\sqrt{\left(u^2\cdot v^2\cdot y^2-2\cdot u^2\cdot v^2\cdot y\cdot z+2\cdot u^2\cdot v^2\cdot z^2-2\cdot u\cdot v^3\cdot y\cdot z+v^4\cdot y^2\right)\cdot w^2+\left(4\cdot x\cdot u^2\cdot v^2\cdot y\cdot z-2\cdot x\cdot u^3\cdot v\cdot z^2-2\cdot x\cdot u\cdot v^3\cdot y^2\right)\cdot w+u^4\cdot x^2\cdot z^2-2\cdot u^3\cdot v\cdot x^2\cdot y\cdot z+u^2\cdot v^2\cdot x^2\cdot y^2}}{u\cdot y\cdot(v\cdot w-u\cdot x)}\right]=0$$

$$Gv-\frac{v\cdot z\cdot(v\cdot w-u\cdot x)\cdot\sqrt{\begin{array}{l}\left(u^2\cdot v^2\cdot y^2-2\cdot u^2\cdot v^2\cdot y\cdot z+2\cdot u^2\cdot v^2\cdot z^2-2\cdot u\cdot v^3\cdot y\cdot z+v^4\cdot y^2\right)\cdot w^2\ \ldots\\ +\ \left(4\cdot x\cdot u^2\cdot v^2\cdot y\cdot z-2\cdot x\cdot u^3\cdot v\cdot z^2-2\cdot x\cdot u\cdot v^3\cdot y^2\right)\cdot w+u^4\cdot x^2\cdot z^2-2\cdot u^3\cdot v\cdot x^2\cdot y\cdot z+u^2\cdot v^2\cdot x^2\cdot y^2\end{array}}}{u\cdot\left[\left(u^3\cdot x^2-2\cdot u^2\cdot v\cdot w\cdot x+2\cdot u\cdot v^2\cdot w^2\right)\cdot z^2+\left(2\cdot y\cdot u\cdot v^2\cdot w\cdot x-2\cdot y\cdot u\cdot v^2\cdot w^2-y\cdot u^2\cdot v\cdot x^2-y\cdot v^3\cdot w^2\right)\cdot z+u\cdot v^2\cdot w^2\cdot y^2\right]}=0$$

Unit.

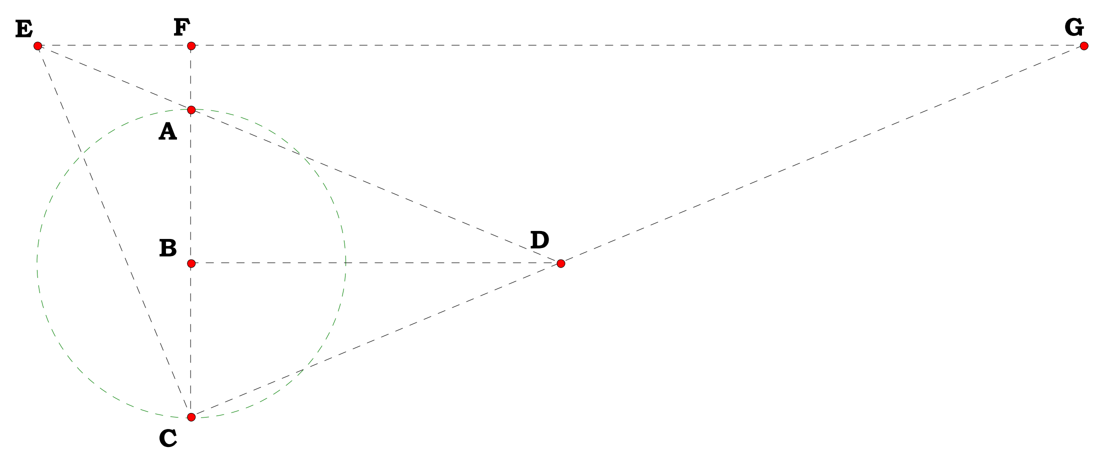
Given.

Writeup project for 032905e

Descriptions.

Definitions.

Another way to do cube roots, one which the ancients were looking for, is accomplished by crossing an isosceles triangle with a right triangle. I put off investigating it for some future date. What it does say is that cube roots are not impossible, but perhaps difficult. I found this while examining the figure of the preceeding ellipse.



EF = 2.04153 cm

AF = 0.84843 cm

FG = 11.82053 cm

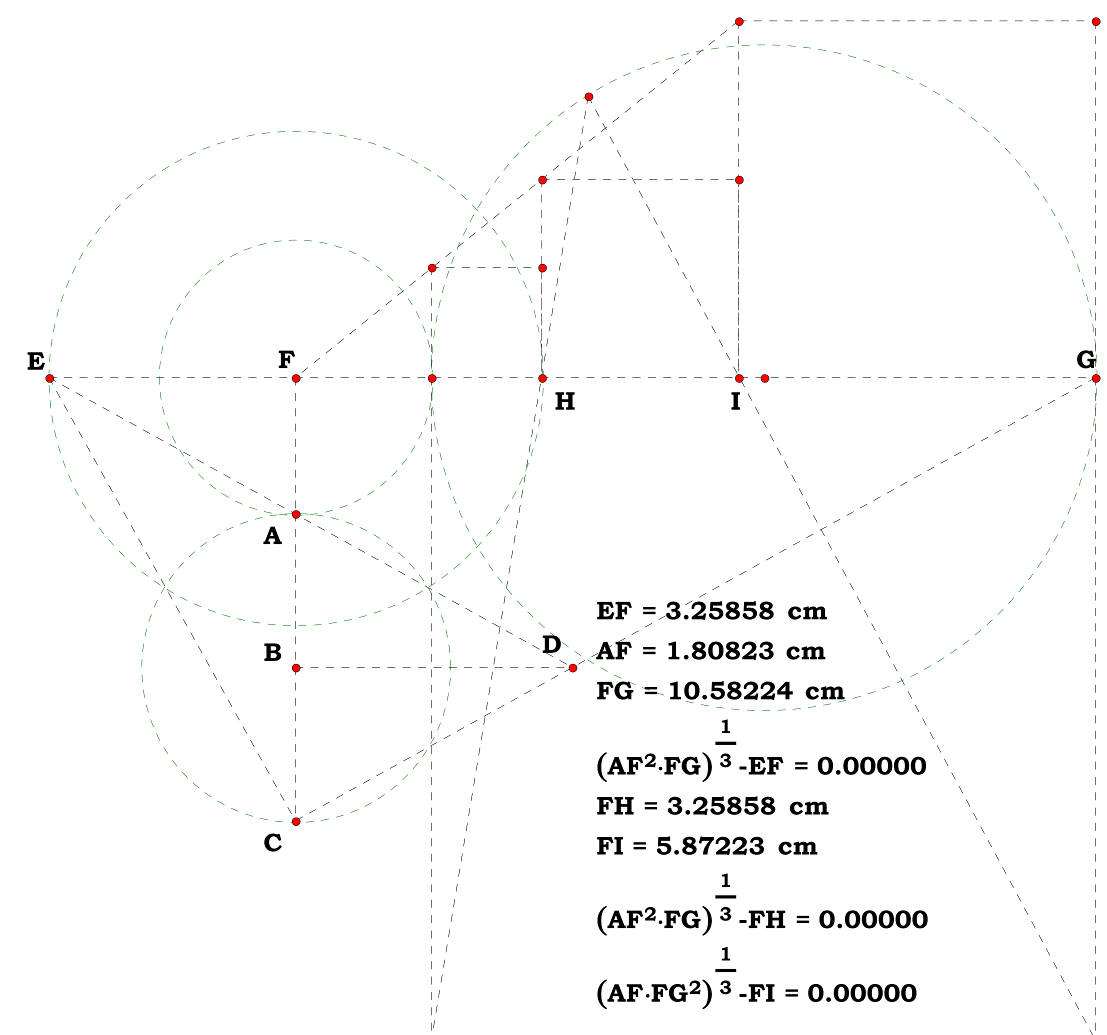
$(AF^2 \cdot FG)^{\frac{1}{3}} - EF = 0.00000$

What can be said, however, is that the figure can be used to prove my original construction which would be much, much shorter than the original. However, my interest was no longer on the Delian Quest, but it was cooking to arrive at BAM, and this took time but once it did happen, what unfolded is a work which I probably have no time to finish, it consists of thousands of pages and covers all of how to use it for even logical operations.

The Delian Quest, essentially conquered, is making me comprehend that it is only the door to a much bigger place. The Delian Quest contains the search, the climax, and the after glow, but the results is contained in the volumes of BAM.

One has to remark, though, it is a very beautiful figure. Very simple, straightforward, and reasonable.

Filling in the rest of the figure, we see it.



EF = 3.25858 cm

AF = 1.80823 cm

FG = 10.58224 cm

$(AF^2 \cdot FG)^{\frac{1}{3}} - EF = 0.00000$

FH = 3.25858 cm

FI = 5.87223 cm

$(AF^2 \cdot FG)^{\frac{1}{3}} - FH = 0.00000$

$(AF \cdot FG^2)^{\frac{1}{3}} - FI = 0.00000$

It might be said that a elliptial construct proved the head of the figure, and another proved its hands.

Unit.

$AB := 1$

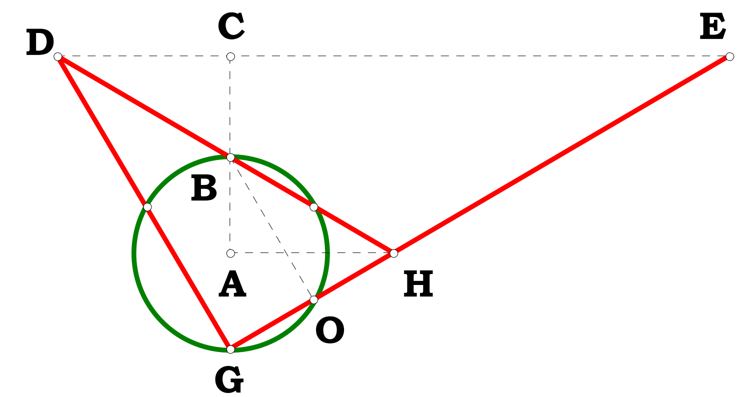
Given.

$N_1 := 2 \qquad AH := N_1$

033005c

N^3 and More

Descriptions.



$BH := \sqrt{AB^2 + AH^2} \qquad GO := \frac{AB \cdot 2 \cdot AB}{BH} \qquad HO := BH - GO$

$DH := \frac{BH \cdot BH}{HO} \qquad AC := \frac{AB \cdot DH}{BH} \qquad BO := \sqrt{BH^2 - HO^2}$

$DG := \frac{BO \cdot BH}{HO} \qquad DE := \frac{BH \cdot DG}{AB} \qquad CD := \frac{DG^2}{DE}$

$BC := AC - AB \qquad CE := DE - CD \qquad N_1^{\,3} - \frac{CE}{BC} = 0$

Definitions.

$BH - \sqrt{N_1^{\,2} + 1} = 0 \qquad GO - \frac{2}{\sqrt{N_1^{\,2} + 1}} = 0 \qquad HO - \frac{(N_1 - 1)\cdot(N_1 + 1)}{\sqrt{N_1^{\,2} + 1}} = 0$

$DH - \frac{\left(\sqrt{N_1^{\,2} + 1}\right)^3}{(N_1 - 1)\cdot(N_1 + 1)} = 0 \qquad AC - \frac{N_1^{\,2} + 1}{(N_1 - 1)\cdot(N_1 + 1)} = 0 \qquad BO - \frac{2 \cdot N_1}{\sqrt{N_1^{\,2} + 1}} = 0$

$DG - \frac{2 \cdot N_1 \cdot \sqrt{N_1^{\,2} + 1}}{(N_1 - 1)\cdot(N_1 + 1)} = 0 \qquad DE - \frac{2 \cdot N_1 \cdot \left(N_1^{\,2} + 1\right)}{(N_1 - 1)\cdot(N_1 + 1)} = 0 \qquad CD - \frac{2 \cdot N_1}{(N_1 - 1)\cdot(N_1 + 1)} = 0$

$BC - \frac{2}{(N_1 - 1)\cdot(N_1 + 1)} = 0 \qquad CE - \frac{2 \cdot N_1^{\,3}}{(N_1 - 1)\cdot(N_1 + 1)} = 0 \qquad N_1^{\,3} - N_1^{\,3} = 0$

Given.

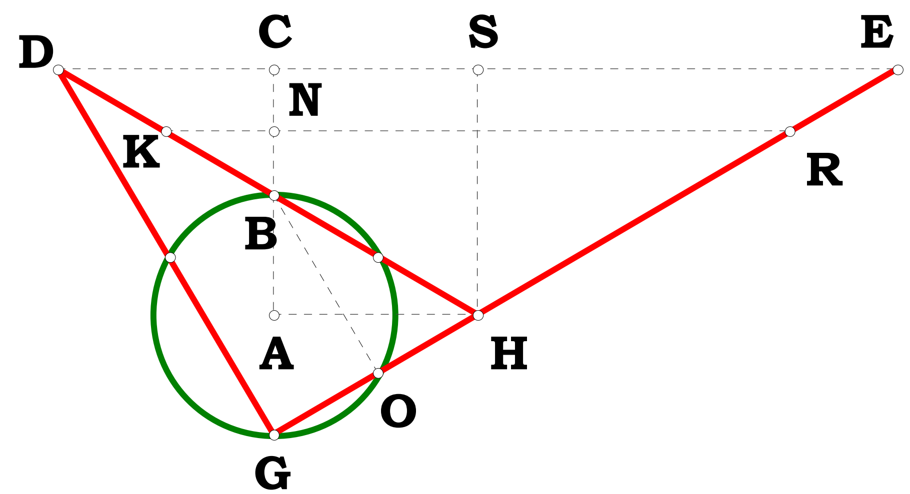
$N_2 := 4$

Descriptions.

$BN := \frac{BC}{N_2}$ $\quad$ $KN := \frac{CD}{N_2}$

$CG := AC + AB$ $\quad$ $GN := 2 \cdot AB + \frac{BC}{N_2}$

$NR := \frac{CE \cdot GN}{CG}$



Definitions.

$$\left[N_1^{\,3} \cdot N_2 - N_1 \cdot (N_2 - 1)\right] - \frac{NR}{BN} = 0$$

$$N_1^{\,3} \cdot N_2 - (N_2 - 1) \cdot N_1 = 26 \qquad \frac{NR}{BN} = 26$$

$$BN - \frac{2}{N_2 \cdot (N_1 - 1) \cdot (N_1 + 1)} = 0 \qquad KN - \frac{2 \cdot N_1}{N_2 \cdot (N_1 - 1) \cdot (N_1 + 1)} = 0$$

$$CG - \frac{2 \cdot N_1^{\,2}}{(N_1 - 1) \cdot (N_1 + 1)} = 0 \qquad GN - \left[\frac{2 \cdot \left(N_2 \cdot N_1^{\,2} - N_2 + 1\right)}{N_2 \cdot (N_1 - 1) \cdot (N_1 + 1)}\right] = 0$$

$$NR - \frac{2 \cdot N_1 \cdot \left(N_2 \cdot N_1^{\,2} - N_2 + 1\right)}{N_2 \cdot (N_1 - 1) \cdot (N_1 + 1)} = 0$$

033105

Unit.

$AB := 1$

Given.

$N_1 := .15$

$N_2 := .6$

Descriptions.

$CD := N_1 \quad DE := 2 \cdot CD \quad AC := N_2 \quad BC := AB - N_2$

$FG := DE + AB \quad AE := AC + CD \quad BD := BC + CD$

$AH := AE \quad BH := BD$

$$AJ := \frac{AB^2 + AH^2 - BH^2}{2 \cdot AB} \qquad HJ := \sqrt{AH^2 - AJ^2}$$

Definitions.

$AE = 0.75 \quad BD = 0.55$

$AJ = 0.63 \quad HJ = 0.40694$

$CD - N_1 = 0 \quad DE - 2 \cdot N_1 = 0 \quad AC - N_2 = 0 \quad BC - (1 - N_2) = 0$

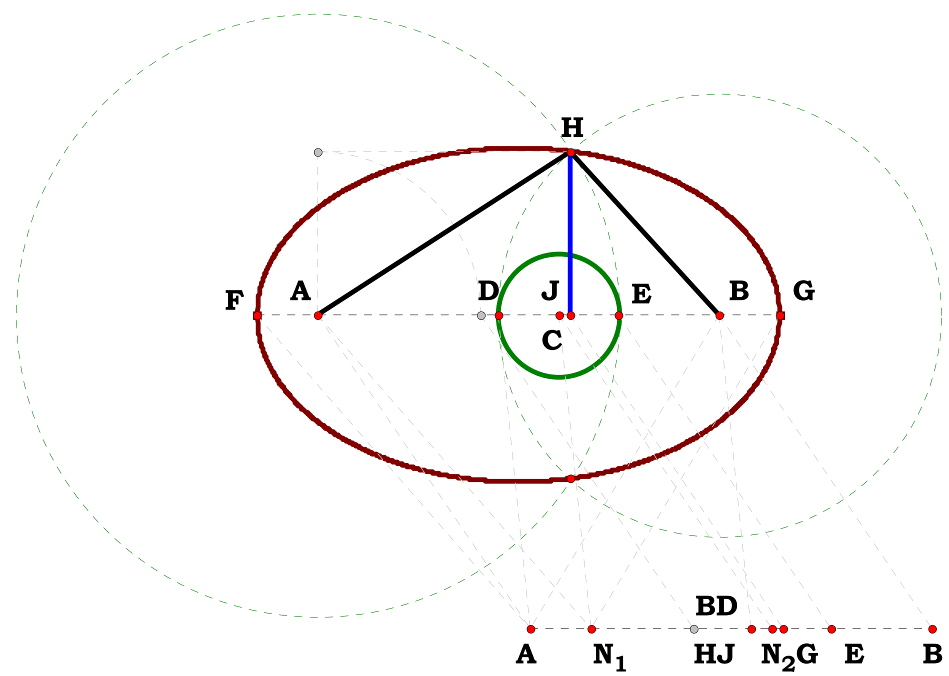
$FG - (2 \cdot N_1 + 1) = 0 \quad AE - (N_2 + N_1) = 0 \quad BD - (N_1 - N_2 + 1) = 0$

$AH - (N_2 + N_1) = 0 \quad BH - (N_1 - N_2 + 1) = 0$

$$AJ - (N_2 - N_1 + 2 \cdot N_1 \cdot N_2) = 0 \qquad HJ - \sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - N_2) \cdot (N_1 + 1)} = 0$$

Just Another Ellipse

Given the difference between the foci and difference between the proportional radii, etc., etc.



Unit = 1.00000
AB = 1.00000
N_1 = 0.15000
X = 3.00000
Y = 20.00000

N_2 = 0.60000
X = 12.00000
Y = 20.00000

AE = 0.75000
BD = 0.55000
AJ = 0.63000
HJ = 0.40694

Unit.

$CD := 1$

Given.

$N_1 := .278 \quad EF := N_1$

$N_2 := 3.095$

Mixing methods of naming

041205A

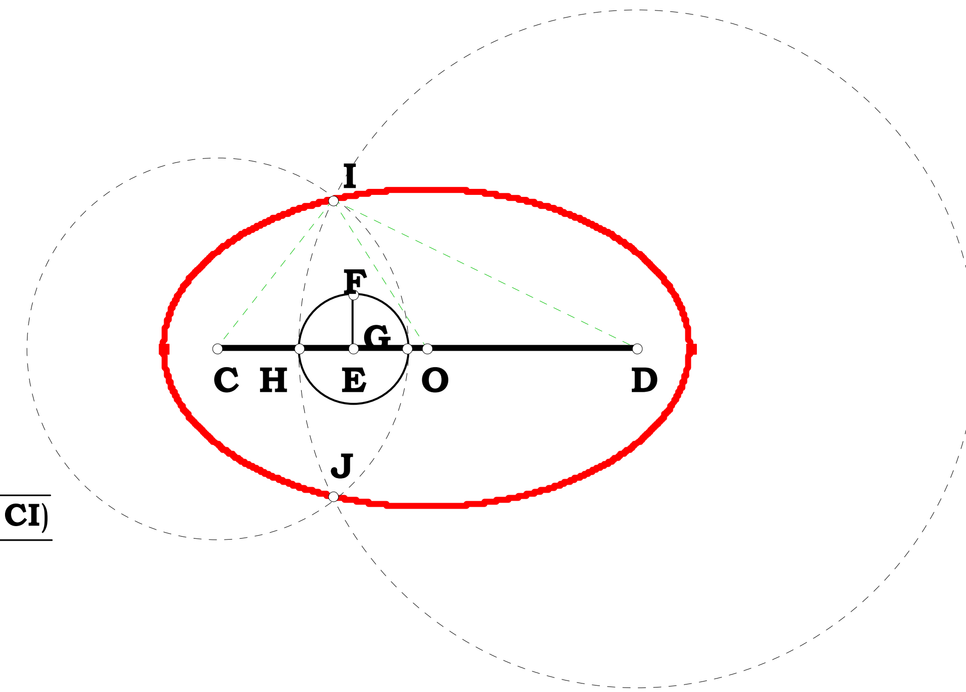
Descriptions.

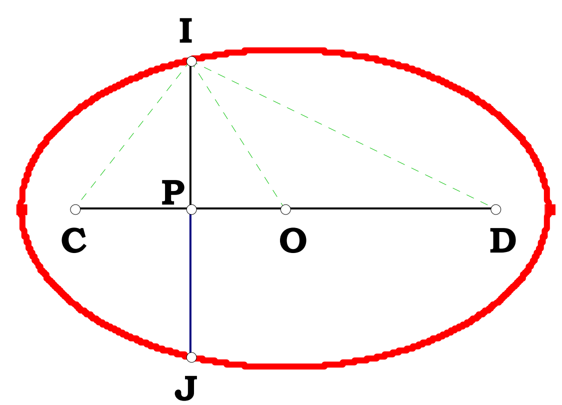
$$CO := \frac{CD}{2} \qquad CE := \frac{CD}{N_2} \qquad CG := CE + EF$$

$$DH := CD - CG + 2 \cdot EF \qquad CI := CG \qquad DI := DH$$

$$IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$





Definitions.

$$2 \cdot \frac{\sqrt{N_1 \cdot (1 + N_1) \cdot (N_2 - 1)}}{N_2} - IP = 0$$

041205B

Descriptions.

Unit.

$CD := 1$

Given.

$N_1 := .278$

$N_2 := .3$

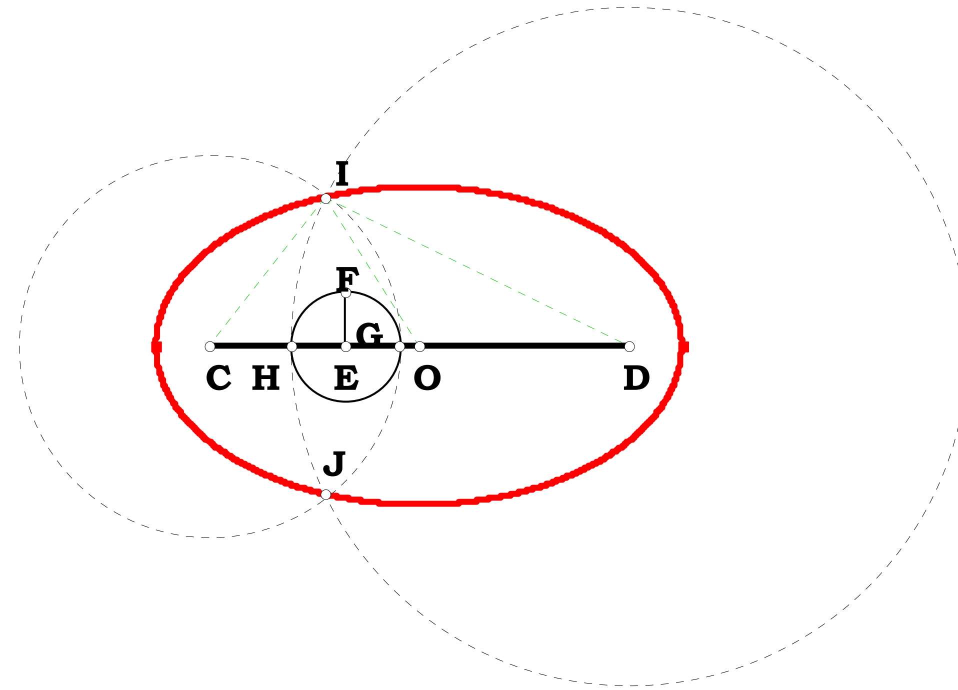
A trip to IO

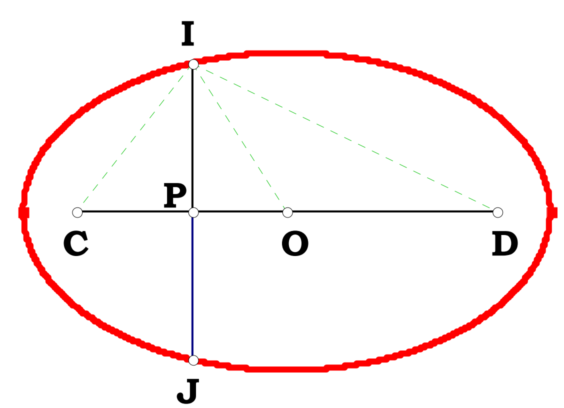
$$CO := \frac{CD}{2} \qquad EF := N_1$$

$$CE := N_2 \qquad CG := CE + EF \qquad DH := CD - CG + 2 \cdot EF$$

$$CI := CG \qquad DI := DH \qquad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$





Definitions.

$$2 \cdot \sqrt{\left[-N_1 \cdot N_2 \cdot \left(N_1 + 1\right) \cdot \left(N_2 - 1\right)\right]} - IP = 0$$

041305b

Unit.

$DH := 1$

Given.

$N_1 := 36 \qquad FH := N_1$

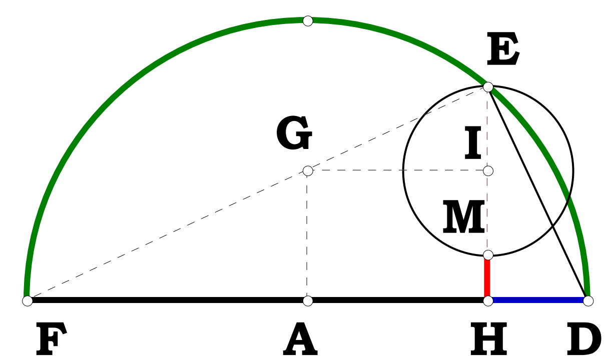
$\left(\sqrt{N_1}\right)^3$

Descriptions.

$DF := DH + FH \qquad AD := \dfrac{DF}{2} \qquad EH := \sqrt{DH \cdot FH}$

$HI := \dfrac{EH \cdot AD}{FH} \qquad HM := EH - 2 \cdot (EH - HI)$

$DE := \sqrt{DH^2 + EH^2} \qquad EF := \sqrt{FH^2 + EH^2}$



Definitions.

$\dfrac{FH}{HM} = 216 \qquad \left(\dfrac{FH}{DH}\right)^{1.5} = 216 \qquad \dfrac{FH}{HM} - \left(\dfrac{DH}{HM}\right)^3 = 0 \qquad \dfrac{FH}{HM} - \left(\dfrac{EF}{DE}\right)^3 = 0$

$DF - \left(1 + N_1\right) = 0 \qquad AD - \dfrac{1 + N_1}{2} = 0 \qquad EH - \sqrt{N_1} = 0$

$HI - \dfrac{N_1 + 1}{2 \cdot \sqrt{N_1}} = 0 \qquad HM - \dfrac{1}{\sqrt{N_1}} = 0$

$DE - \sqrt{N_1 + 1} = 0 \qquad EF - \sqrt{\left[N_1 \cdot \left(N_1 + 1\right)\right]} = 0$

$N_1^{\frac{3}{2}} = 216 \qquad \left(N_1\right)^{1.5} = 216 \qquad N_1^{\frac{3}{2}} - \left(\sqrt{N_1}\right)^3 = 0 \qquad N_1^{\frac{3}{2}} - \left[\dfrac{\sqrt{N_1 \cdot \left(N_1 + 1\right)}}{\sqrt{N_1 + 1}}\right]^3 = 0$ **Mathcad 15 will not reduce this last one.**

One can see the taxing of logic in terms of precision on the computer.

$N_1^{\frac{3}{2}} - \left(\dfrac{FH}{HM}\right) = 3.694822 \times 10^{-13} \qquad \left(N_1\right)^{1.5} - \left[\left(\dfrac{FH}{DH}\right)^{1.5}\right] = 0 \qquad N_1^{\frac{3}{2}} - \left(\dfrac{DH}{HM}\right)^3 = 1.13687 \times 10^{-12} \qquad N_1^{\frac{3}{2}} - \left(\dfrac{EF}{DE}\right)^3 = 0$

Unit.
Given.

Another procrastinated writeup

041405

Descriptions.
Definitions.

AO = 1.167 in.
BO = 1.759 in.
CO = 2.649 in.
DO = 3.991 in.

$\frac{BO}{AO} = 1.507$

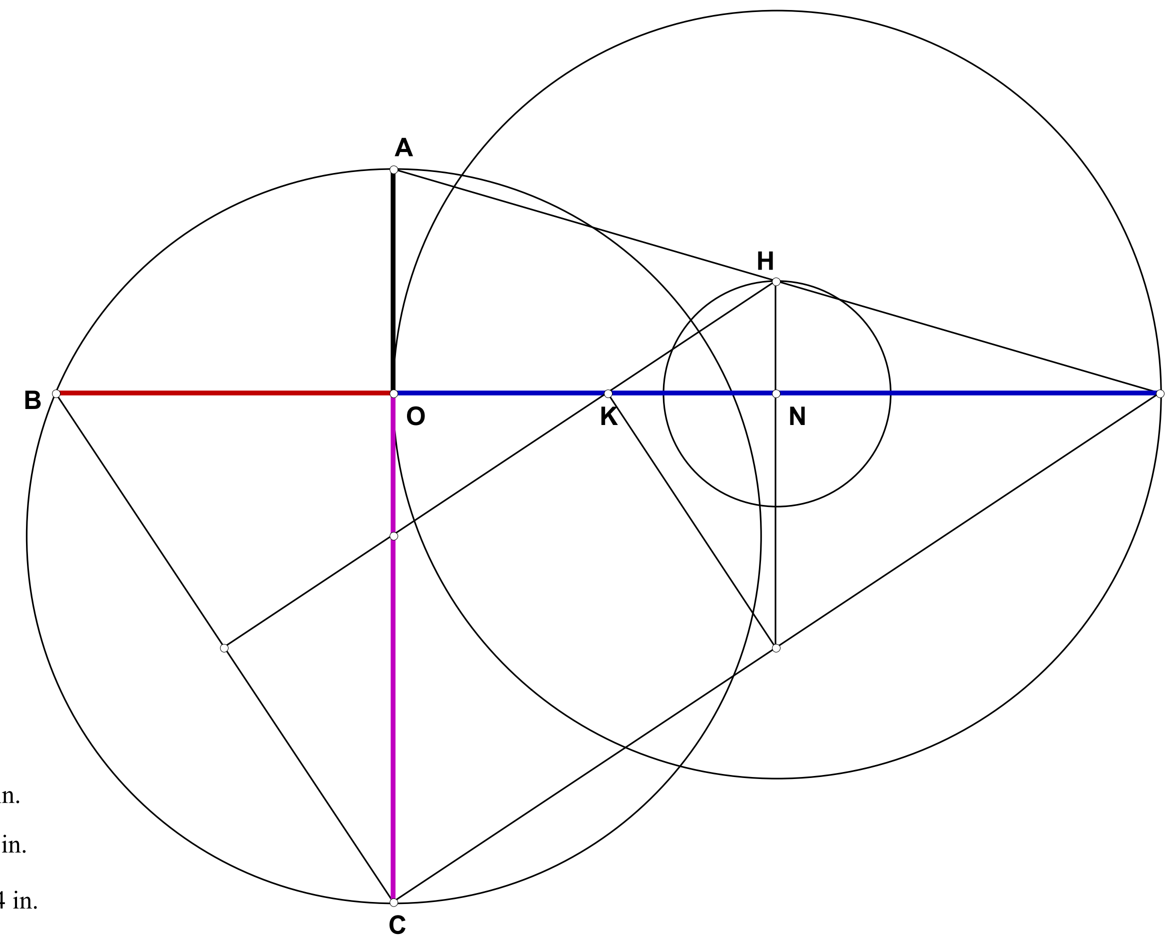
$\frac{CO}{AO} = 2.270$

$\frac{CO}{AO}^{1.5} = 3.419$

$\frac{DO}{AO} = 3.419$

$\frac{BO}{AO}^{3} = 3.419$

$\frac{DO}{AO}^{\frac{1}{3}} - \frac{BO}{AO} = 0.000$



NK = 0.879 in.

ND = 1.996 in.

HN = 0.584 in.

$\frac{NK}{HN} = 1.507$

$\frac{ND}{HN} = 3.419$

$\frac{NK}{HN}^{3} = 3.419$

$\frac{ND}{HN} - \frac{NK}{HN}^{3} = 0.000$

042205

Unit.

Given.

$N_1 := 2.604 \quad DE := N_1$

$N_2 := 2.234 \quad FI := N_2$

Given the major axis and the difference between the two foci, whatis the minor axis?

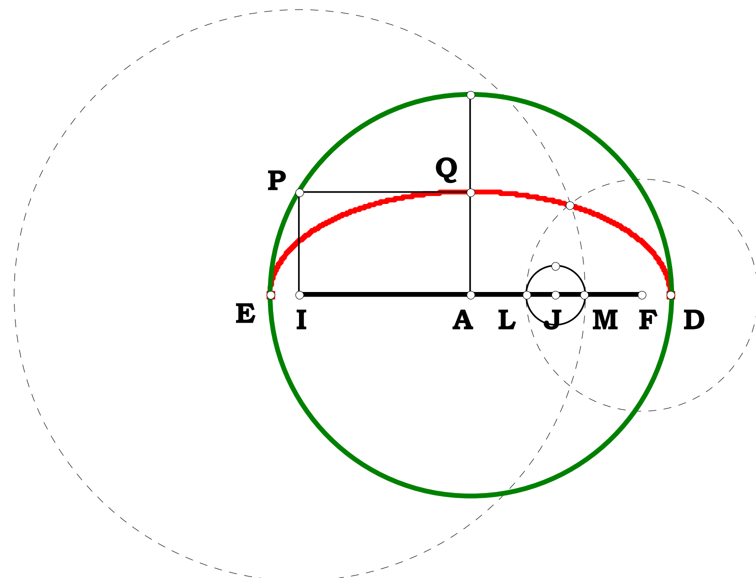
Descriptions.

$EI := \dfrac{DE - FI}{2} \qquad DI := DE - EI$

$EP := \sqrt{EI \cdot DI} \qquad AQ := EP$

Definitions.

$$AQ - \frac{1}{2}\cdot\sqrt{(N_1 + N_2)\cdot(N_1 - N_2)} = 0$$



ED = 2.083 in.

FI = 1.787 in.

AQ = 0.535 in.

$AQ - \dfrac{\sqrt{(ED+FI)\cdot(ED-FI)}}{2} = 0.000$ in.

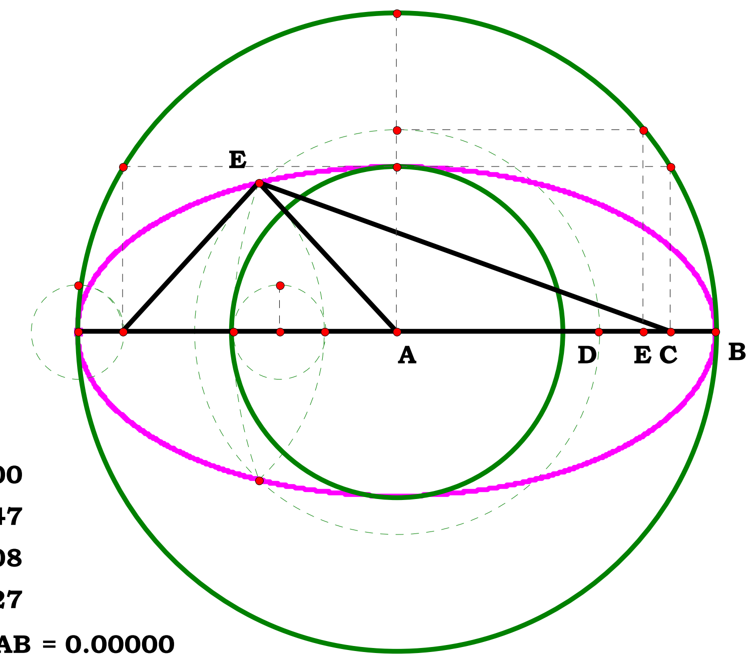
Sum of Area

042305A

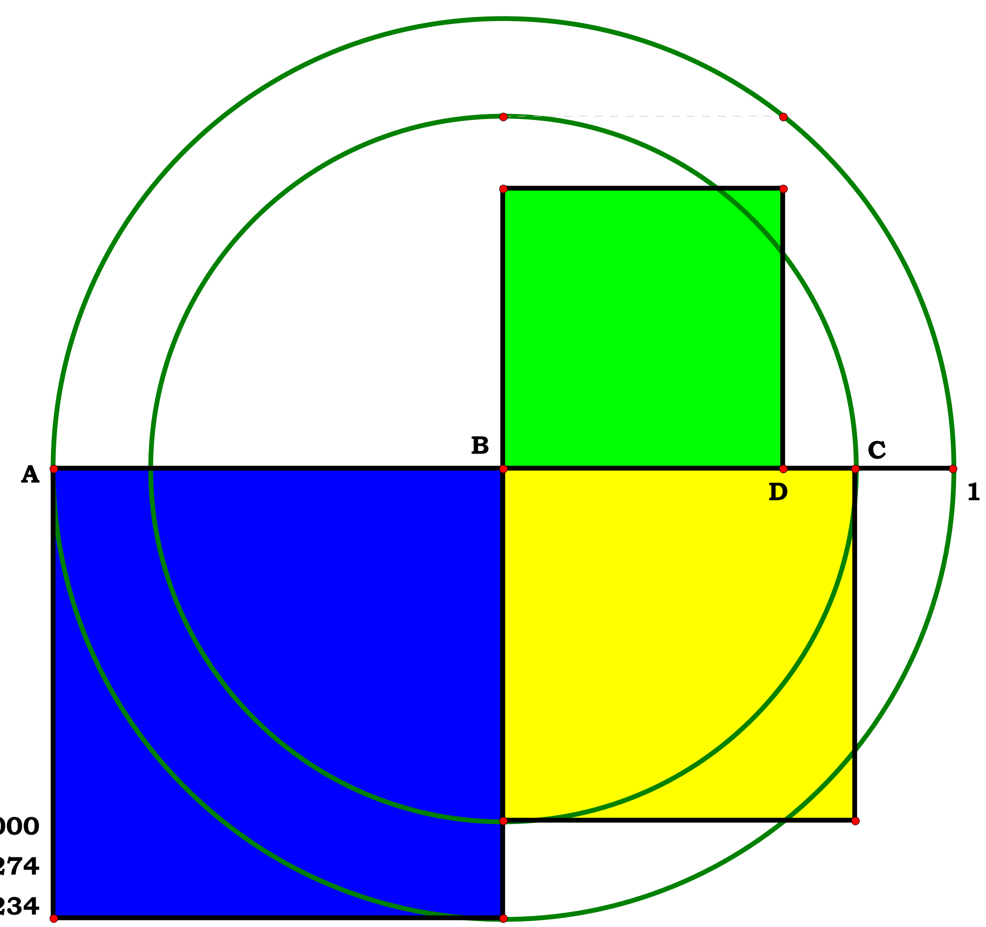
Descriptions.

It seems that the bottom two figures came to mind while I was examining an ellipse. This consist of three different applications. This write-will be Plate A.

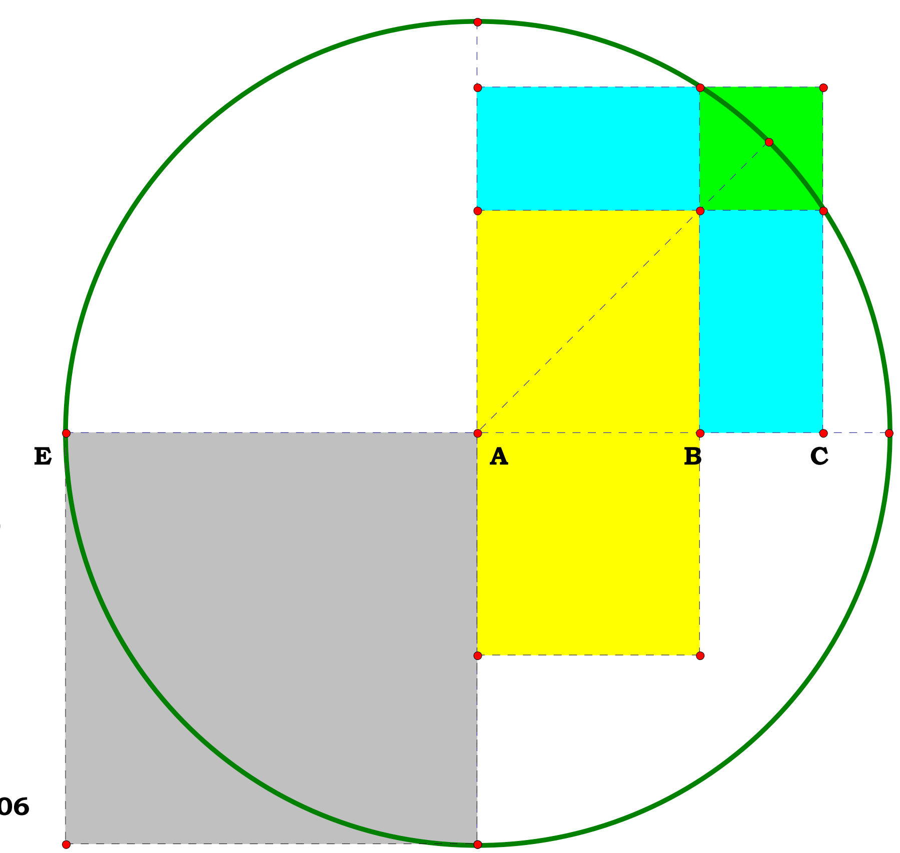
Is there a perfect way to write-up an ellipse? This whole ellipse is wholly determined by the ratio AB to AC.



AB = 1.00000
AC = 0.85647
AD = 0.63408
AE = 0.77327
$\sqrt{AD^2+AE^2}-AB = 0.00000$



AB = 1.00000
AC = 0.78274
AD = 0.62234
$AB^2-(AC^2+AD^2) = 0.00000$



Unit = 1.00000
XY = 0.54037
X = 10.80742
Y = 20.00000
AB = 0.54037
AC = 0.84143
AD = 1.00000
AC-AB = 0.30106
BC = 0.30106
$AD^2-(2\cdot AB^2+2\cdot AB\cdot BC+BC^2) = 0.00000$

042305A

Descriptions of ellipse

Unit.

$AB := 1$

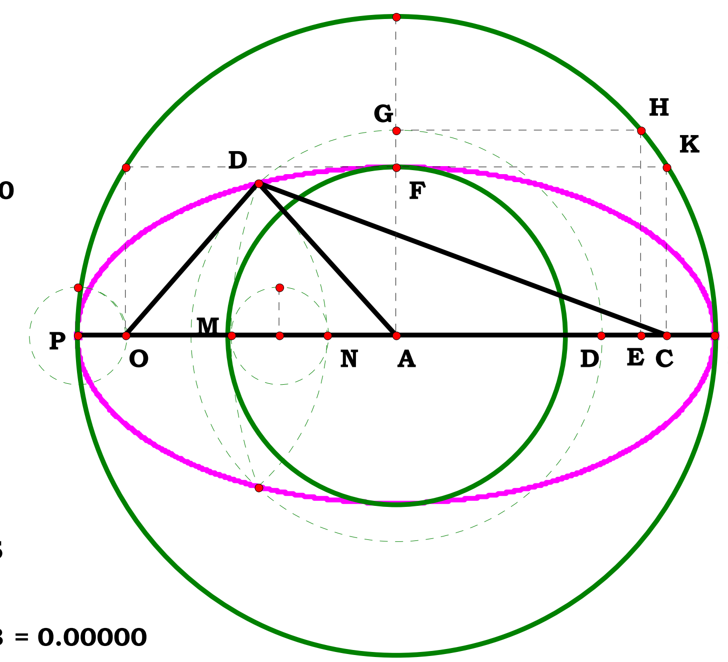
Given.

$Y := 20$

$X := 17$

Plate A.

Unit = 1.00000
XY = 0.85000
X = 17.00000
Y = 20.00000
AB = 1.00000
AC = 0.85000
AD = 0.64114
AE = 0.76743
AF = 0.52678
CM = 1.36545
NO = 0.63455
$\sqrt{AD^2+AE^2}$-AB = 0.00000



$AC := \dfrac{X}{Y}$ $\quad AC = 0.85 \quad BC := AB - AC \quad BP := 2 \cdot AB$

$AF := \sqrt{(BP - BC) \cdot BC} \quad AF = 0.526783 \quad CO := 2 \cdot AC$

$NM := 2 \cdot BC$

Descriptions AE, either CM or NO has to be given.

Given NO: $\quad NO := .63455$

$CM := CO - NO + NM \quad DO := NO \quad CE := CM$

$AD := \dfrac{\sqrt{2 \cdot DO^2 - CO^2 + 2 \cdot CE^2}}{2}$ **(Pythagoras Revisted)** $AD = 0.641135$

$DP := AB + AD \quad BD := BP - DP \quad AE := \sqrt{DP \cdot BD} \quad AE = 0.767428$

$\sqrt{AD^2 + AE^2} - AB = 0$

Definitions. **The equation for AD is indifferent as to which side, DO or CD, of the elliptical triangle given.**

$AC - \dfrac{X}{Y} = 0 \quad BC - \dfrac{Y - X}{Y} = 0 \quad BP - 2 = 0 \quad AF - \dfrac{\sqrt{Y^2 - X^2}}{Y} = 0 \quad CO - \dfrac{2 \cdot X}{Y} = 0$

$NM - \dfrac{2 \cdot (Y - X)}{Y} = 0 \quad CM - (2 - NO) = 0 \quad DO - NO = 0 \quad CE - (2 - NO) = 0 \quad AD - \dfrac{\sqrt{NO \cdot Y^2 \cdot (NO - 2) - X^2 + 2 \cdot Y^2}}{Y} = 0$

$DP - \dfrac{Y + \sqrt{NO^2 \cdot Y^2 - 2 \cdot NO \cdot Y^2 - X^2 + 2 \cdot Y^2}}{Y} = 0 \quad BD - \dfrac{Y - \sqrt{NO^2 \cdot Y^2 - 2 \cdot NO \cdot Y^2 - X^2 + 2 \cdot Y^2}}{Y} = 0 \quad AE - \dfrac{\sqrt{Y^2 + \left(2 \cdot NO \cdot Y^2 - NO^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y} = 0$

$$\sqrt{\left[\frac{\sqrt{NO \cdot Y^2 \cdot (NO - 2) - X^2 + 2 \cdot Y^2}}{Y}\right]^2 + \left[\frac{\sqrt{Y^2 + \left(2 \cdot NO \cdot Y^2 - NO^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y}\right]^2} = 1 \qquad \sqrt{\left[\frac{\sqrt{CM \cdot Y^2 \cdot (CM - 2) - X^2 + 2 \cdot Y^2}}{Y}\right]^2 + \left[\frac{\sqrt{Y^2 + \left(2 \cdot CM \cdot Y^2 - CM^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y}\right]^2} = 1$$

042305B

Descriptions.

Unit.

$AB := 1$

$BE := AB$

Given.

$Y := 20$

$X := 12$

Sum of Area

Plate B.

$$BC := \frac{X}{Y} \quad AE := 2 \cdot AB \quad MN := AE \quad FN := AB + BC$$

$$FG := \sqrt{(MN - FN) \cdot FN} \qquad BD := FG$$

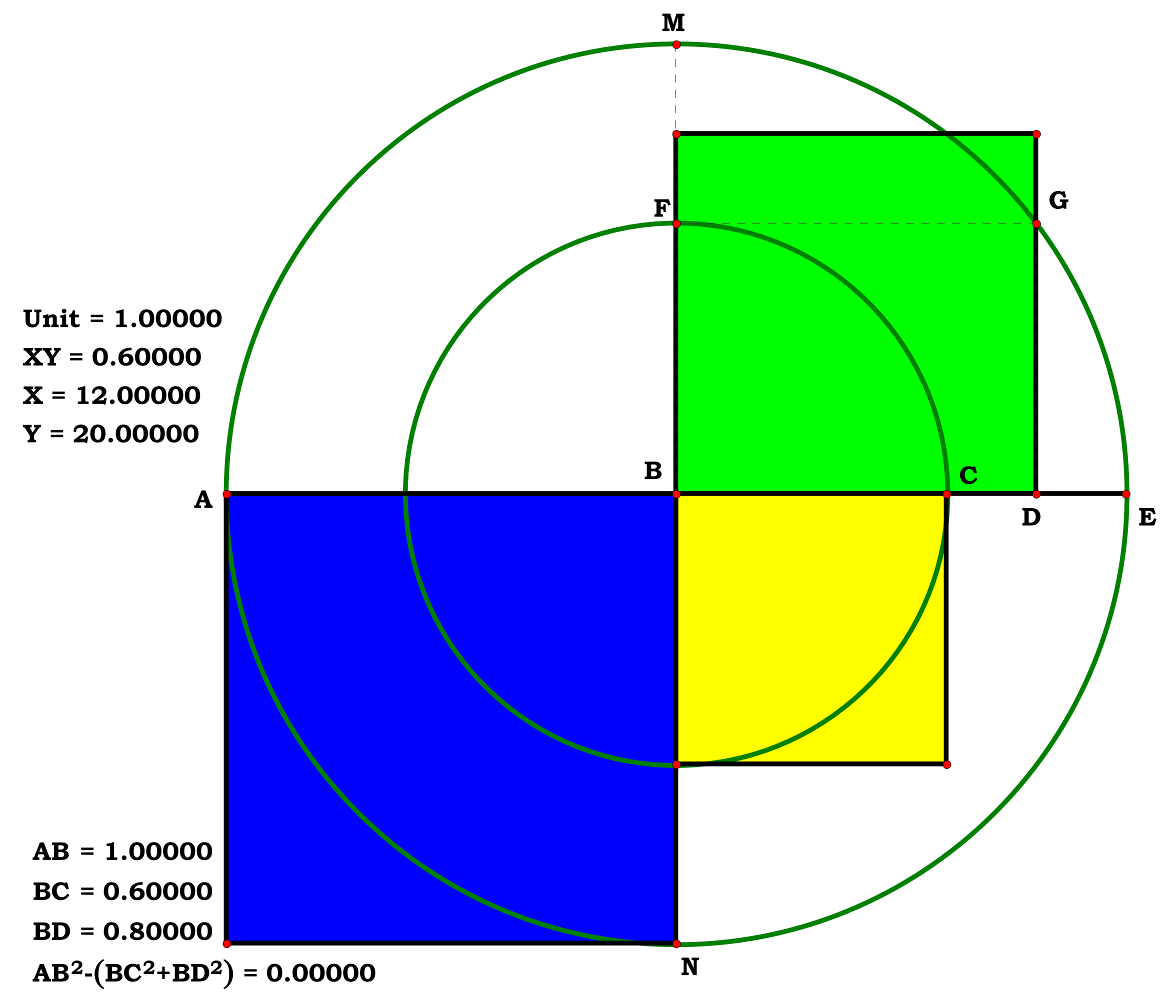
$$AB^2 - \left(BC^2 + BD^2\right) = 0$$

Definitions.

$$BC - \frac{X}{Y} = 0 \quad AE - 2 = 0 \quad MN - 2 = 0$$

$$FN - \frac{X+Y}{Y} = 0 \quad FG - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$$

$$BD - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$$



Unit.

$AD := 1$

$AE := AD$

Given.

$Y := 20$

$X := 12$

Sum of Area

Plate C.

Descriptions.

$AB := \dfrac{X}{Y} \quad AB = 0.6 \quad BE := AB + AE \quad DE := 2 \cdot AD$

$BD := DE - BE \quad AC := \sqrt{BE \cdot BD} \quad BC := AC - AB$

$AD^2 - \left(2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2\right) = 0$

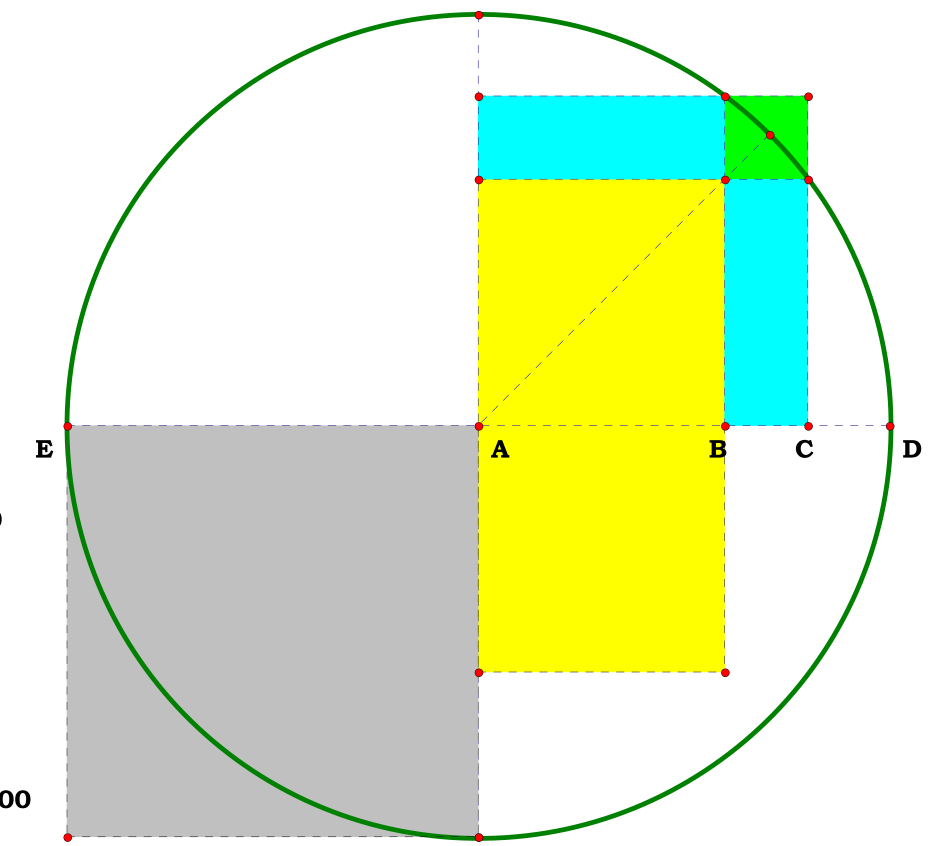
Definitions.

$AB - \dfrac{X}{Y} = 0 \quad BE - \dfrac{X + Y}{Y} = 0 \quad DE - 2 = 0$

$BD - \dfrac{Y - X}{Y} = 0 \quad AC - \dfrac{\sqrt{Y^2 - X^2}}{Y} = 0$

$BC - \dfrac{\sqrt{Y^2 - X^2} - X}{Y} = 0$

$AD^2 - \left(2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2\right) = 0$



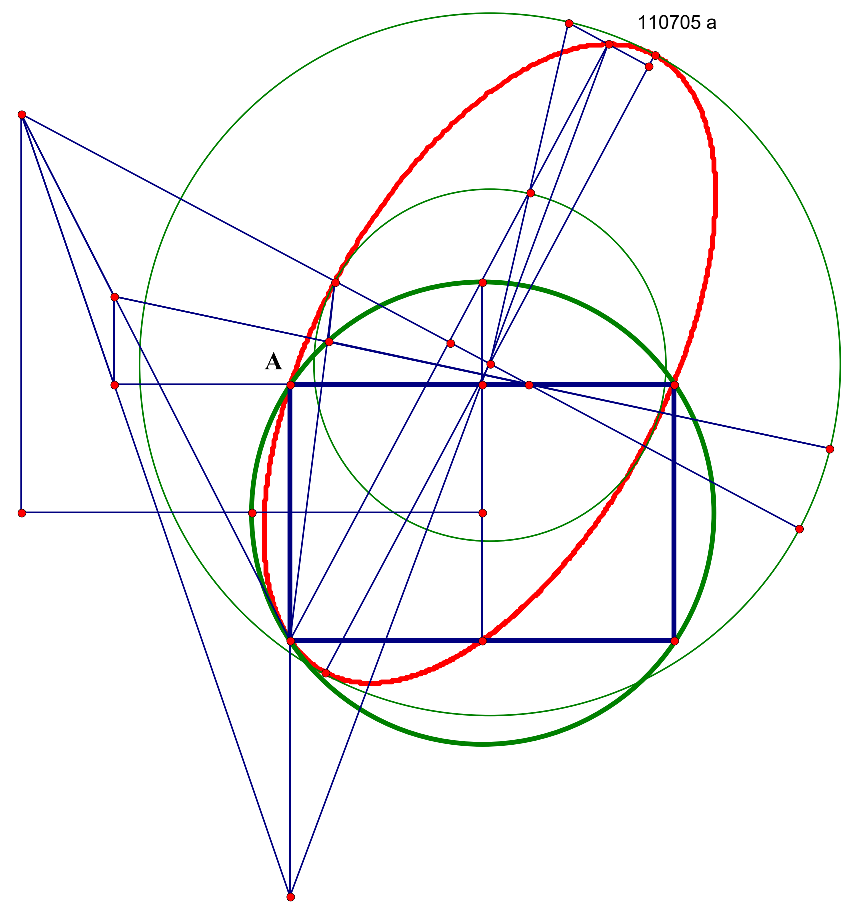
Unit = 1.00000
XY = 0.60000
X = 12.00000
Y = 20.00000
AB = 0.60000
AC = 0.80000
AD = 1.00000
AC-AB = 0.20000
BC = 0.20000
AD^2-($2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2$) = 0.00000

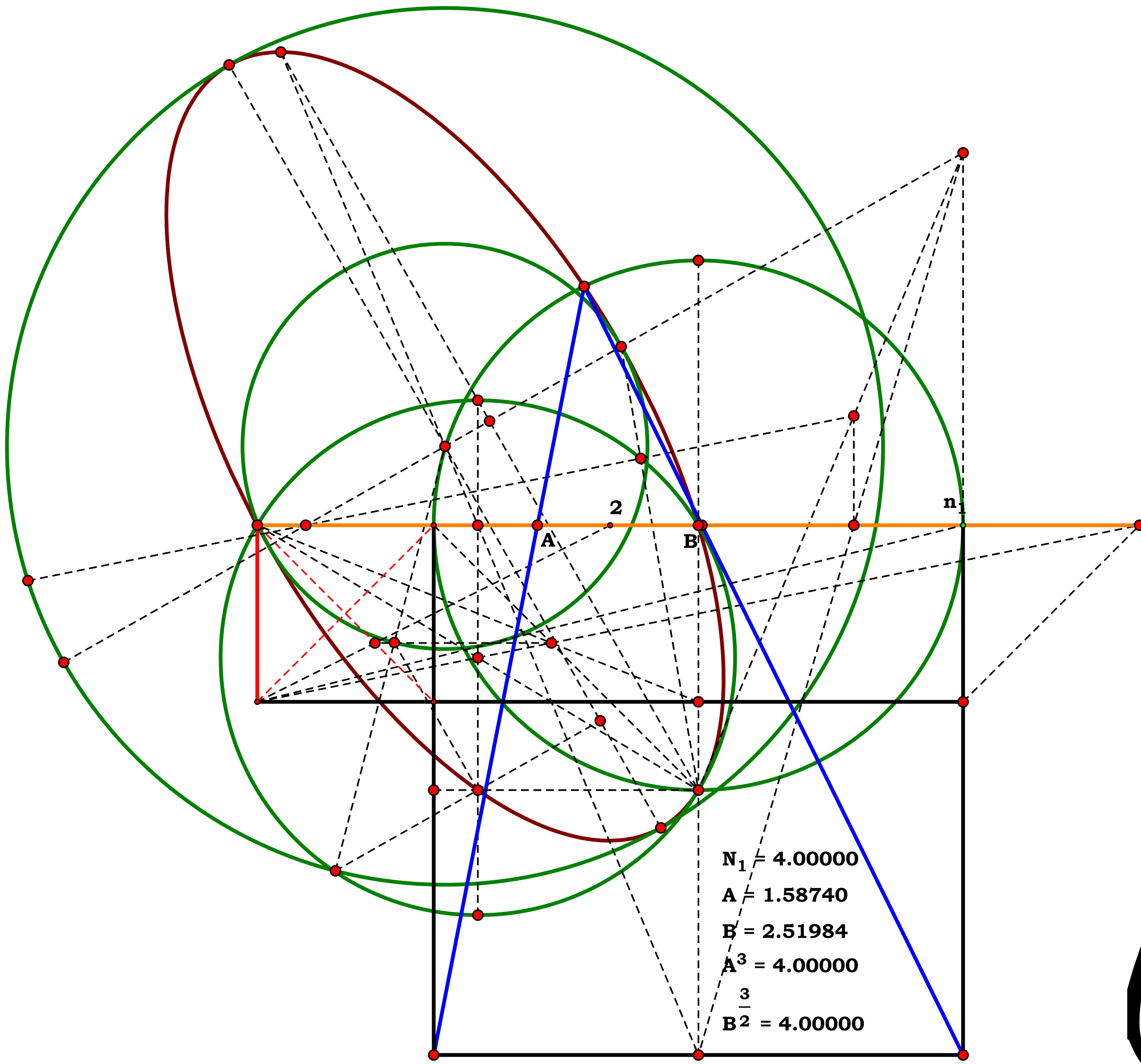
Unit.
Given.

Procrastinated Writeup for 110705

And by looking at it, maybe another twenty years it get done.

Descriptions.
Definitions.





The Delian Quest 2006

John Clark

Unit. $AB := 1$

Given.

$x := 9 \quad z := 15$

$w := 20 \quad y := 10$

Parcing project for 012306

Descriptions.

$$Bx := \frac{x}{w} \quad Ax := AB - Bx \quad Bz := \frac{z}{y} \quad Hx := \sqrt{Ax\cdot(2\cdot AB - Ax)}$$

$$Gx := \frac{Bx\cdot Hx}{Bz + Hx} \quad DG := \frac{Hx\cdot(Bx - Gx)}{Bx} \quad BF := \frac{Bx\cdot Bz}{(Bz - Hx)}$$

$$Fx := |BF - Bx| \quad EF := \frac{Bz\cdot Fx}{Bx}$$

Definitions.

$$Bx - \frac{x}{w} = 0 \quad Ax - \left(\frac{w-x}{w}\right) = 0 \quad Bz - \frac{z}{y} = 0$$

$$Hx - \frac{\sqrt{(w+x)\cdot(w-x)}}{w} = 0$$

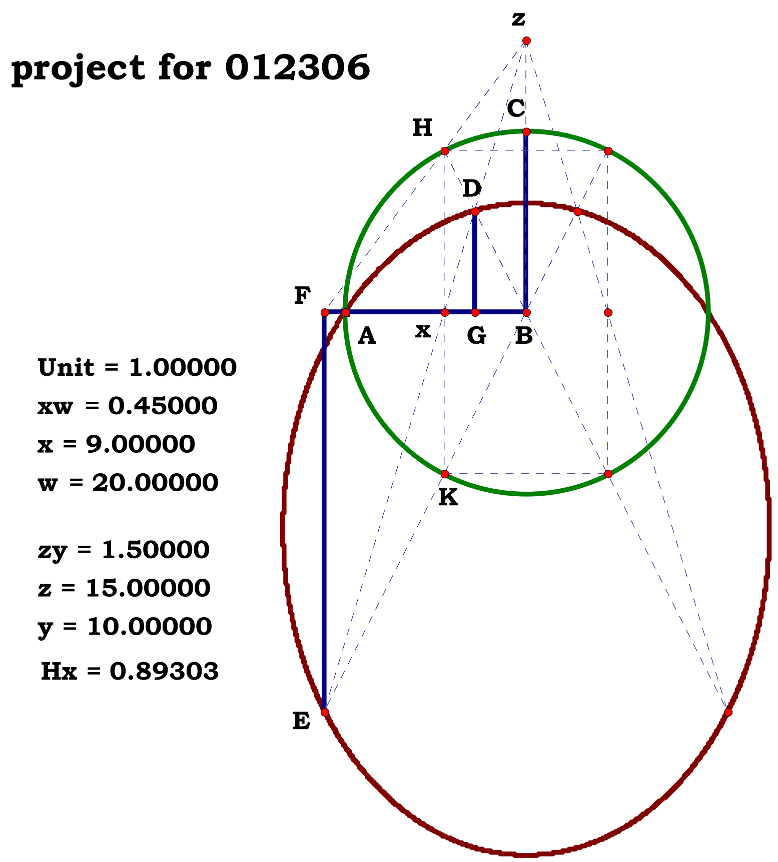
$$Gx - \frac{x\cdot\sqrt{(w-x)\cdot(w+x)}\cdot y}{w\cdot\left[y\cdot\sqrt{(w-x)\cdot(w+x)} + w\cdot z\right]} = 0$$

$$DG - \frac{\sqrt{(w-x)\cdot(w+x)}\cdot z}{y\cdot\sqrt{(w-x)\cdot(w+x)} + w\cdot z} = 0$$

$$BF - \frac{x\cdot z}{w\cdot z - y\cdot\sqrt{(w-x)\cdot(w+x)}} = 0$$

$$Fx - \frac{\sqrt{w^2 - x^2}\cdot x\cdot y}{\left|w^2\cdot z - w\cdot y\cdot\sqrt{w^2 - x^2}\right|} = 0$$

$$EF - \frac{w\cdot z\cdot\sqrt{w^2 - x^2}}{\left|w^2\cdot z - w\cdot y\cdot\sqrt{w^2 - x^2}\right|} = 0$$



$\frac{w\text{-}x}{w}\text{-}Ax = 0.00000$

$\frac{z}{y}\text{-}Bz = 0.00000$

$\frac{\sqrt{(w+x)\cdot(w-x)}}{w}\text{-}Hx = 0.00000$

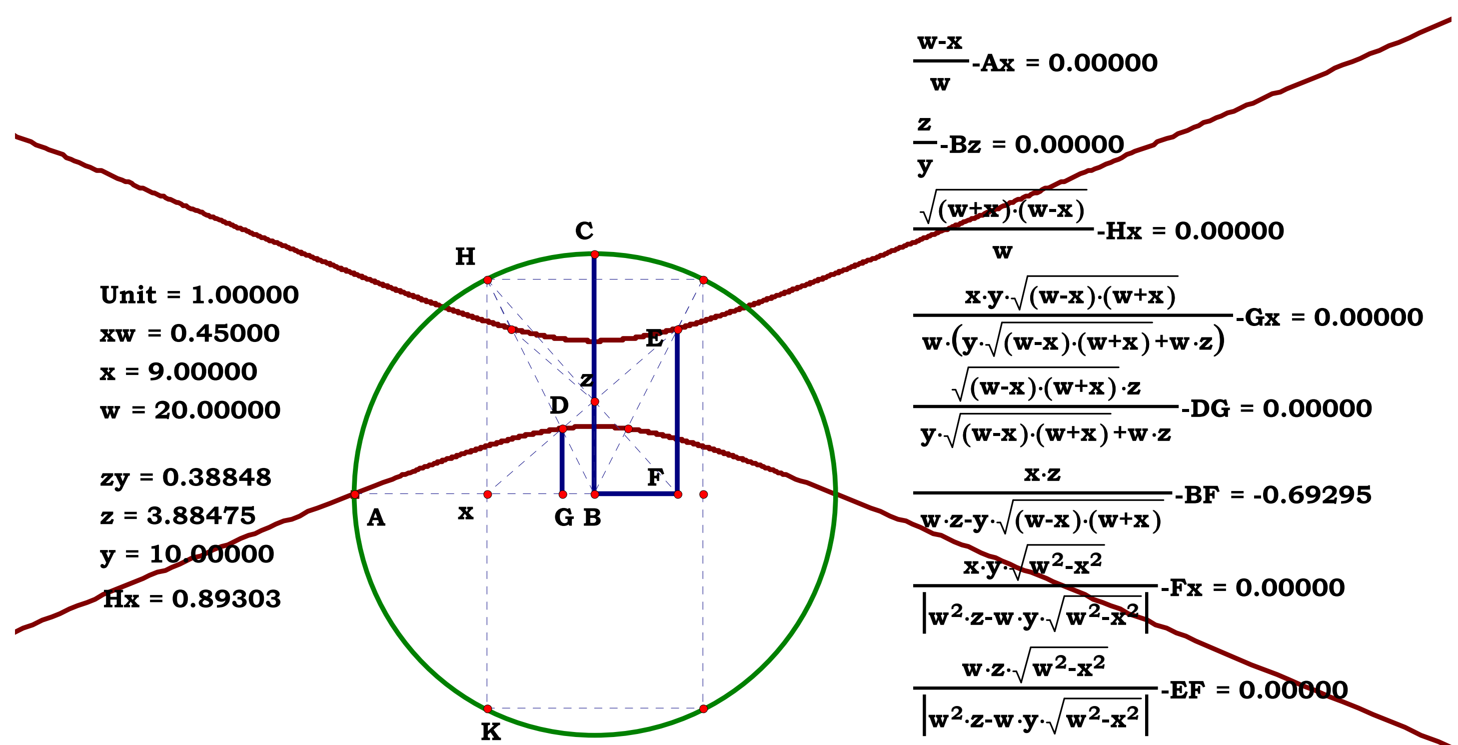
$\frac{x\cdot y\cdot\sqrt{(w-x)\cdot(w+x)}}{w\cdot\left(y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z\right)}\text{-}Gx = 0.00000$

$\frac{\sqrt{(w-x)\cdot(w+x)}\cdot z}{y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z}\text{-}DG = 0.00000$

$\frac{x\cdot z}{w\cdot z\text{-}y\cdot\sqrt{(w-x)\cdot(w+x)}}\text{-}BF = 0.00000$

$\frac{x\cdot y\cdot\sqrt{w^2\text{-}x^2}}{\left|w^2\cdot z\text{-}w\cdot y\cdot\sqrt{w^2\text{-}x^2}\right|}\text{-}Fx = 0.00000$

$\frac{w\cdot z\cdot\sqrt{w^2\text{-}x^2}}{\left|w^2\cdot z\text{-}w\cdot y\cdot\sqrt{w^2\text{-}x^2}\right|}\text{-}EF = 0.00000$



$\frac{w\text{-}x}{w}\text{-}Ax = 0.00000$

$\frac{z}{y}\text{-}Bz = 0.00000$

$\frac{\sqrt{(w+x)\cdot(w-x)}}{w}\text{-}Hx = 0.00000$

$\frac{x\cdot y\cdot\sqrt{(w-x)\cdot(w+x)}}{w\cdot\left(y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z\right)}\text{-}Gx = 0.00000$

$\frac{\sqrt{(w-x)\cdot(w+x)}\cdot z}{y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z}\text{-}DG = 0.00000$

$\frac{x\cdot z}{w\cdot z\text{-}y\cdot\sqrt{(w-x)\cdot(w+x)}}\text{-}BF = -0.69295$

$\frac{x\cdot y\cdot\sqrt{w^2\text{-}x^2}}{\left|w^2\cdot z\text{-}w\cdot y\cdot\sqrt{w^2\text{-}x^2}\right|}\text{-}Fx = 0.00000$

$\frac{w\cdot z\cdot\sqrt{w^2\text{-}x^2}}{\left|w^2\cdot z\text{-}w\cdot y\cdot\sqrt{w^2\text{-}x^2}\right|}\text{-}EF = 0.00000$

012906

Descriptions.

Unit.

Given.

$N := 7.75095$

$CD := 5.55625$

Given AC what is CK?

This may be the first time I tried to get an equation in terms of N, so that given CK one could project AC. However, the version of Mathcad at that time could not reduce the equations, and I am not interested now to try.

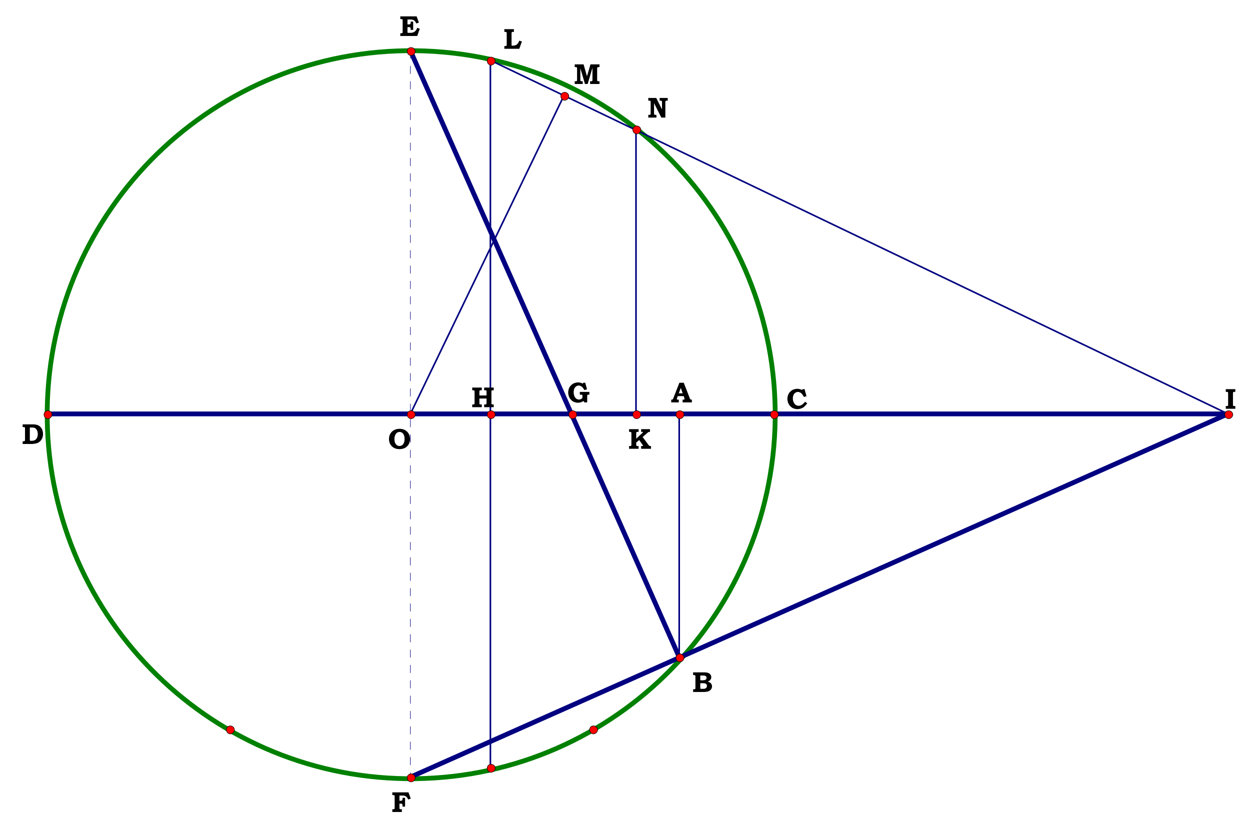
$AC := \frac{CD}{N}$ $\quad AD := CD - AC$ $\quad AB := \sqrt{AC \cdot AD}$ $\quad EO := \frac{CD}{2}$ $\quad CO := EO$

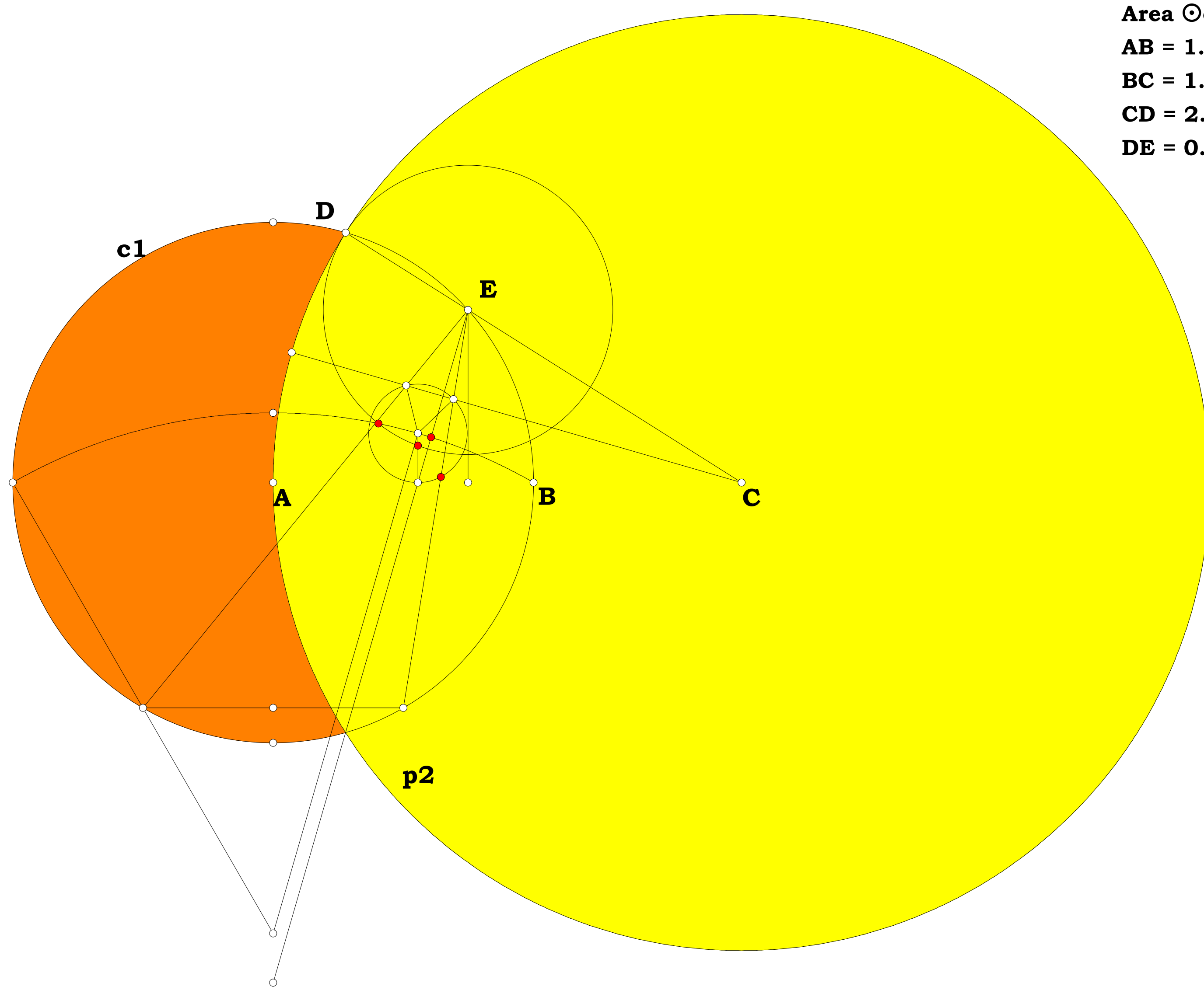
$AO := CO - AC$ $\quad GO := \frac{AO \cdot EO}{EO + AB}$ $\quad HO := \frac{GO}{2}$ $\quad AG := AO - GO$ $\quad FO := EO$

$IO := \frac{AB \cdot FO}{AG}$ $\quad IL := IO$ $\quad CH := CO - HO$ $\quad DO := EO$ $\quad DH := DO + HO$

$HI := IO - HO$ $\quad IK := \frac{HI \cdot (IL - 2 \cdot HO)}{IL}$ $\quad KO := IO - IK$

$CK := CO - KO$





Area p1 = 2.33369 in^2

Area p2 = 24.41091 in^2

Area ⊙c1 = 7.54768 in^2

AB = 1.55000 in.

BC = 1.23751 in.

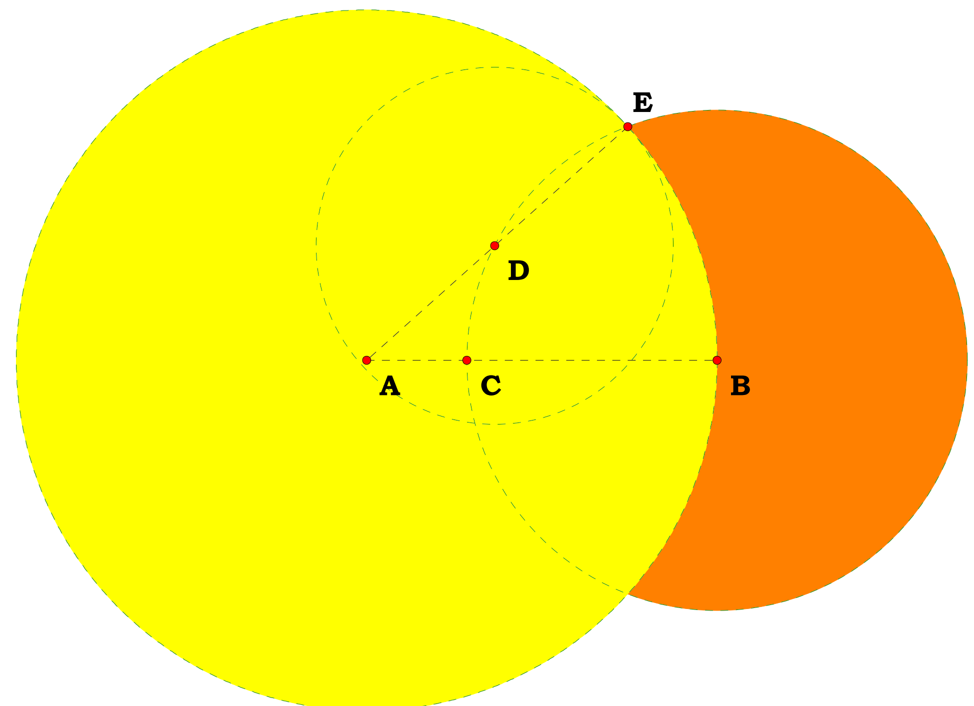
CD = 2.78751 in.

DE = 0.86188 in.

$$\sqrt{(\text{Area p1})\cdot(\text{Area p2})}-(\text{Area}\ \odot\text{c1}) = 0.00000\ \text{in}^2$$

$$\sqrt{\text{CD}\cdot\text{DE}}-\text{AB} = 0.00000\ \text{in.}$$

$$\frac{(\text{Area p1})\cdot(\text{Area p2})}{\text{AB}} = 36.75317\ \text{in}^3$$



Area ⊙AB = 67.50610 cm²

Area ⊙BC = 34.39919 cm²

Area ⊙DE = 17.52885 cm²

$\sqrt{(\text{Area } \odot AB)\cdot(\text{Area } \odot DE)} - (\text{Area } \odot BC) = 0.00000 \text{ cm}^2$

AB = 4.63550 cm

CB = 3.30902 cm

DE = 2.36212 cm

$\sqrt{AB \cdot DE} - CB = 0.00000 \text{ cm}$

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 9$

102606A

Angles and Area Plate A

While exploring trisection, I found something else actually worth exploring. An entirely different way to think of an angle. An angle is takes the so called Pythagorean down to its elements, and it is wholly independent of any particular angle.

Descriptions.

$$AN := \frac{X}{Y} \qquad AK := AB \qquad AF := AB \qquad KN := \sqrt{AN^2 + AK^2}$$

$$FG := \frac{AN \cdot 2 \cdot AB}{KN} \qquad FG = 0.820729$$

$$GH := \frac{AB \cdot FG}{KN} \qquad GH = 0.748441 \qquad GO := GH - AN$$

$$FH := \frac{AN \cdot FG}{KN} \qquad AH := AF - FH \qquad AH = 0.663202$$

$$CN := \frac{GO \cdot AB}{FH} \qquad AC := CN + AN \qquad AC = 1.336111$$

$$AJ := GH \qquad CJ := AC - AJ \qquad CE := CJ \qquad CE = 0.58767$$

$\sqrt{AC \cdot AJ} = 1$ **Circles are to each other as their radii.**

Definitions.

$$AN - \frac{X}{Y} = 0 \qquad AK - 1 = 0 \qquad AF - 1 = 0$$

$$KN - \frac{\sqrt{X^2 + Y^2}}{Y} = 0 \qquad FG - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$$

$$GH - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0 \qquad GO - \frac{X \cdot Y^2 - X^3}{Y \cdot \left(X^2 + Y^2\right)} = 0 \qquad FH - \frac{2 \cdot X^2}{X^2 + Y^2} = 0 \qquad AH - \frac{Y^2 - X^2}{X^2 + Y^2} = 0 \qquad CN - \frac{Y^2 - X^2}{2 \cdot X \cdot Y} = 0$$

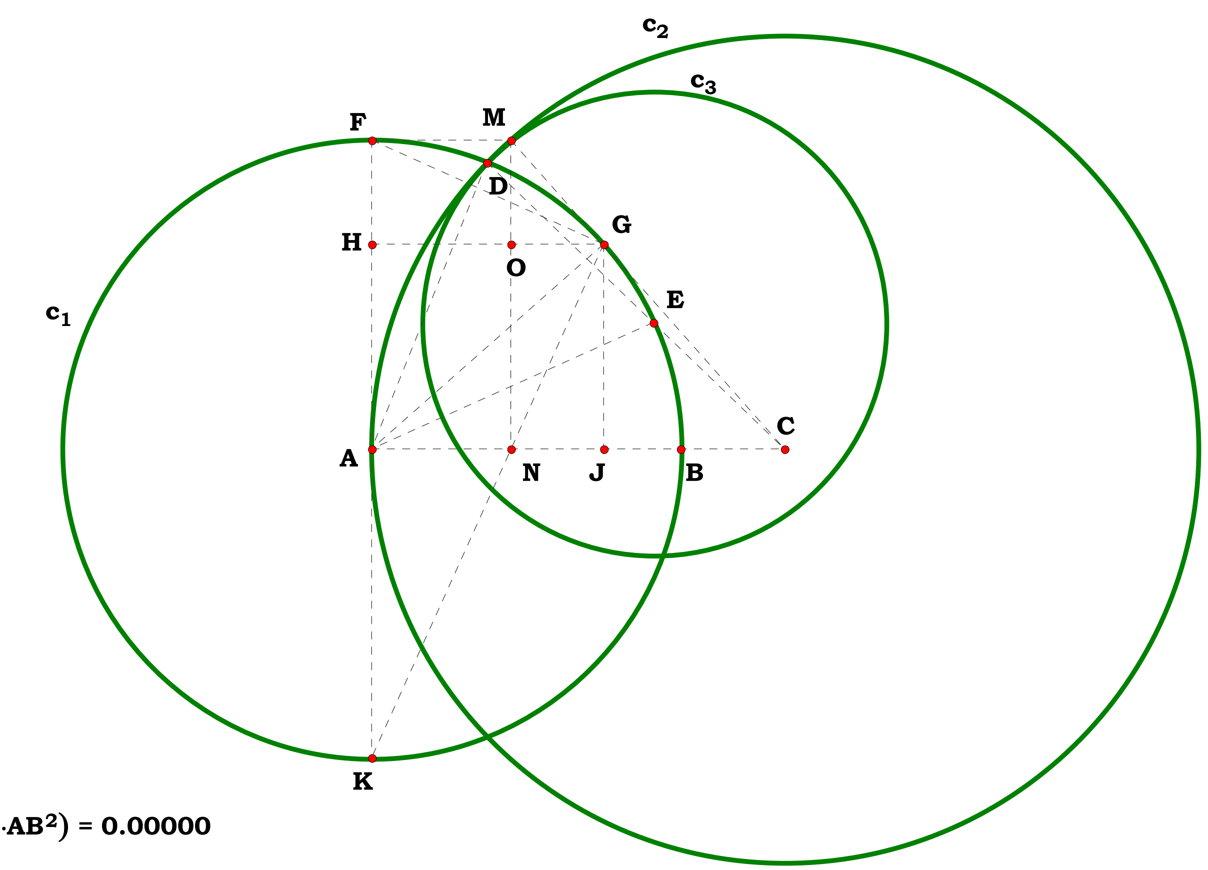
$$AC - \frac{X^2 + Y^2}{2 \cdot X \cdot Y} = 0 \qquad AJ - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0 \qquad CJ - \frac{(X - Y)^2 \cdot (X + Y)^2}{2 \cdot X \cdot Y \cdot \left(X^2 + Y^2\right)} = 0 \qquad CE - \frac{(X - Y)^2 \cdot (X + Y)^2}{2 \cdot X \cdot Y \cdot \left(X^2 + Y^2\right)} = 0$$

Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000
AB = 1.00000
AN = 0.45000
FG = 0.82073
GH = 0.74844
AJ = 0.74844
AH = 0.66320
CE = 0.58767
AC = 1.33611
$\sqrt{AC \cdot AJ} = 1.00000$

$\frac{X}{Y} = 0.45000$

$AN - \frac{X}{Y} = 0.00000$

$\pi \cdot AB^2 = 3.14159$
$\pi \cdot AC^2 = 5.60835$
$\pi \cdot AJ^2 = 1.75981$
$\sqrt{(\pi \cdot AC^2) \cdot (\pi \cdot AJ^2)} - (\pi \cdot AB^2) = 0.00000$



$$\pi \cdot AB^2 = 3.141593$$

$$\pi \cdot AC^2 = 5.608349$$

$$\pi \cdot AJ^2 = 1.759806$$

$$\sqrt{\pi \cdot AJ^2 \cdot \pi \cdot AC^2} - \pi \cdot AB^2 = 0$$

102606B

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 9$

Angles and Area Plate B

Descriptions.

$AC := \frac{X}{Y} \quad BC := AB - AC$

$BH := \frac{BC^2}{2 \cdot AB}$ **(Pythagoras Revisited)** $BH = 0.15125$

$BD := 2 \cdot BH \quad AD := AB - BD \quad FG := BD$

$\sqrt{AB \cdot BD} - BC = 0$

Definitions.

$AC - \frac{X}{Y} = 0 \quad BC - \frac{Y - X}{Y} = 0 \quad BH - \frac{(X - Y)^2}{2 \cdot Y^2} = 0$

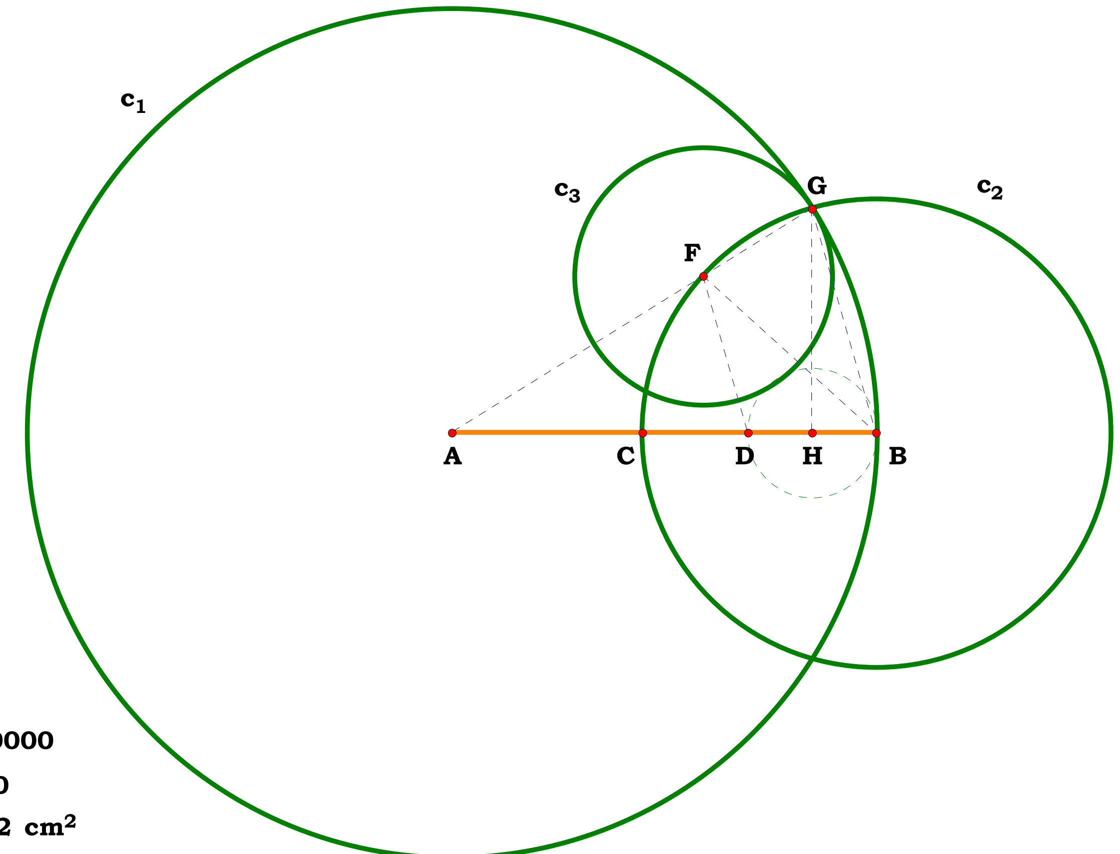
$BD - \frac{(X - Y)^2}{Y^2} = 0 \quad AD - \frac{X \cdot (2 \cdot Y - X)}{Y^2} = 0$

$FG - \frac{(X - Y)^2}{Y^2} = 0$

Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000

AB = 1.00000
AC = 0.45000
AD = 0.69750
BC = 0.55000
BD = 0.30250
BH = 0.15125

$\sqrt{AB \cdot BD}$-BC = 0.00000
$\sqrt{BD}$-BC = 0.00000
Area c_1 = 99.02532 cm^2
Area c_2 = 29.95516 cm^2
Area c_3 = 9.06144 cm^2
$\sqrt{(\text{Area } c_1) \cdot (\text{Area } c_3)}$-(Area c_2) = 0.00000 cm^2



110706 Sketchbook A

Going around in a circle

Unit.

Given.

$N_1 := 3.12208$ $\quad AD := N_1$

$N_2 := 5.31813$ $\quad AC := N_2$

What is the tangent to AD from C? Although no one in their right mind would do this for a circle, it is essential as an introduction to finding the inverse ellilpse for any external point to it.

Descriptions.

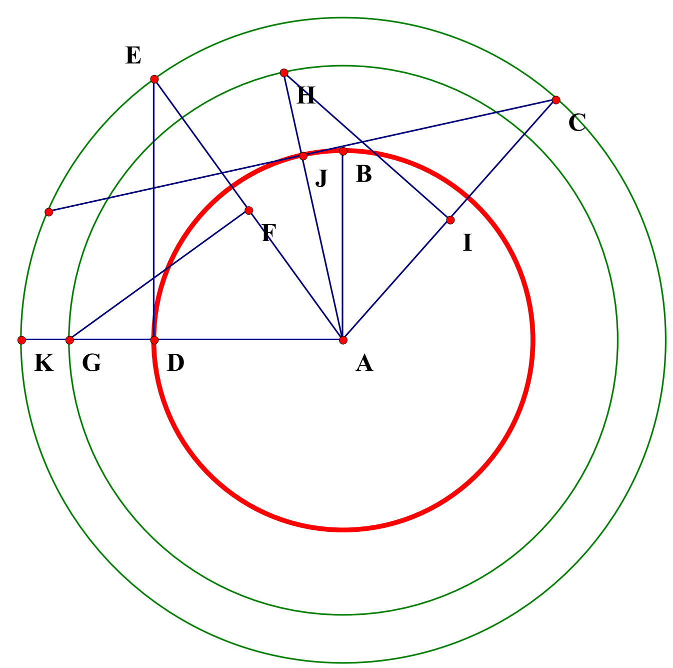
$AE := AC$

$DE := \sqrt{AE^2 - AD^2} \quad AF := \frac{AE}{2}$

$AG := \frac{AE \cdot AF}{AD} \quad AH := AG \quad AI := \frac{AC}{2}$

$AJ := \frac{AI \cdot AC}{AH} \quad AJ - N_1 = 0$

$CJ := \sqrt{AC^2 - AJ^2} \quad CJ = 4.305244$



Definitions.

$AE - N_2 = 0$

$DE - \sqrt{N_2^{\,2} - N_1^{\,2}} = 0 \qquad AF - \frac{N_2}{2} = 0$

$AG - \frac{N_2^{\,2}}{2 \cdot N_1} = 0 \qquad AH - \frac{N_2^{\,2}}{2 \cdot N_1} = 0 \qquad AI - \frac{N_2}{2} = 0$

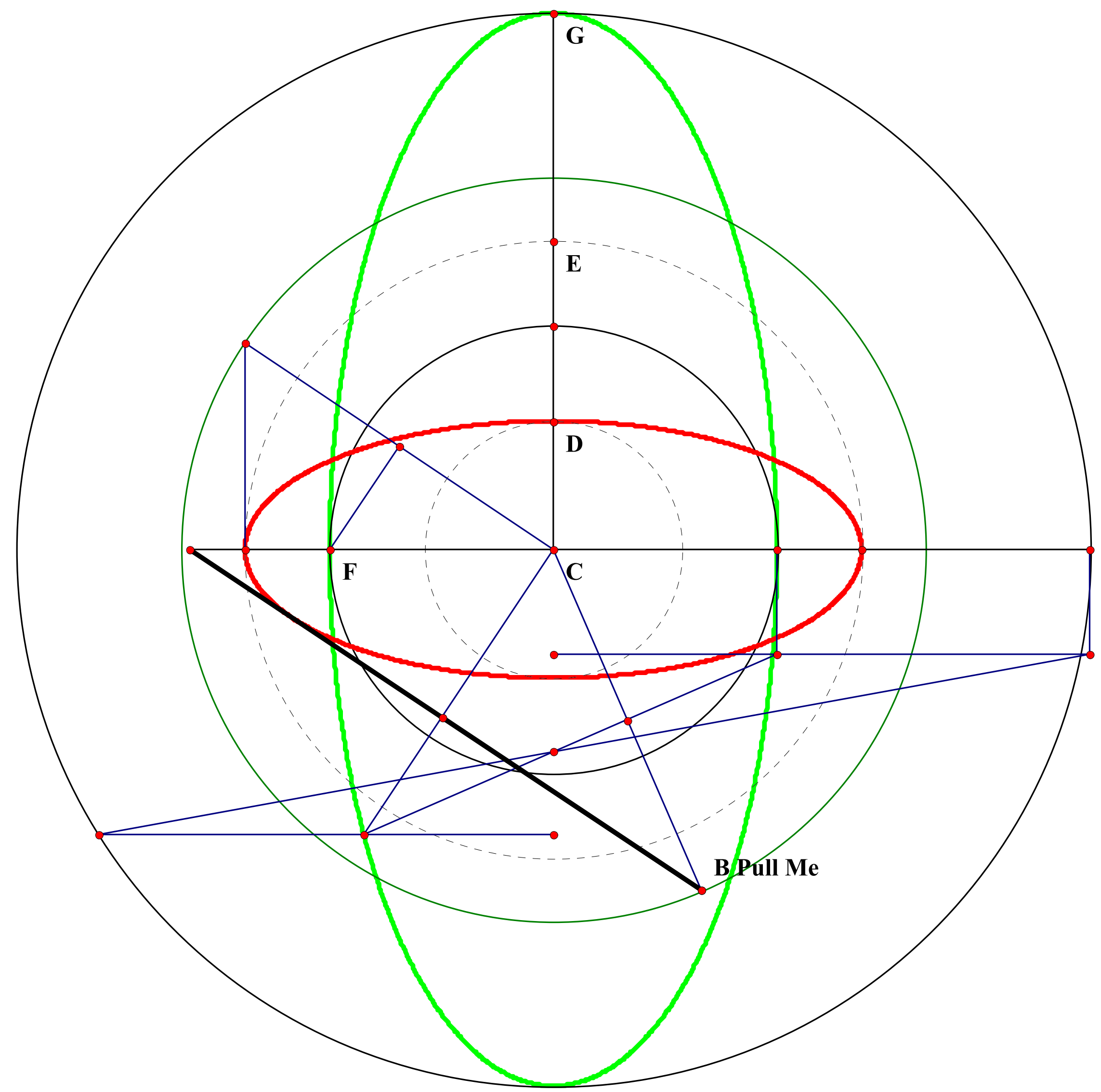
$AJ - N_1 = 0 \qquad CJ - \sqrt{N_2^{\,2} - N_1^{\,2}} = 0$

110706 Sketchbook B

Descriptions.
Definitions.

Unit.
Given.

From any point B construct a tangent to any ellipse (red).



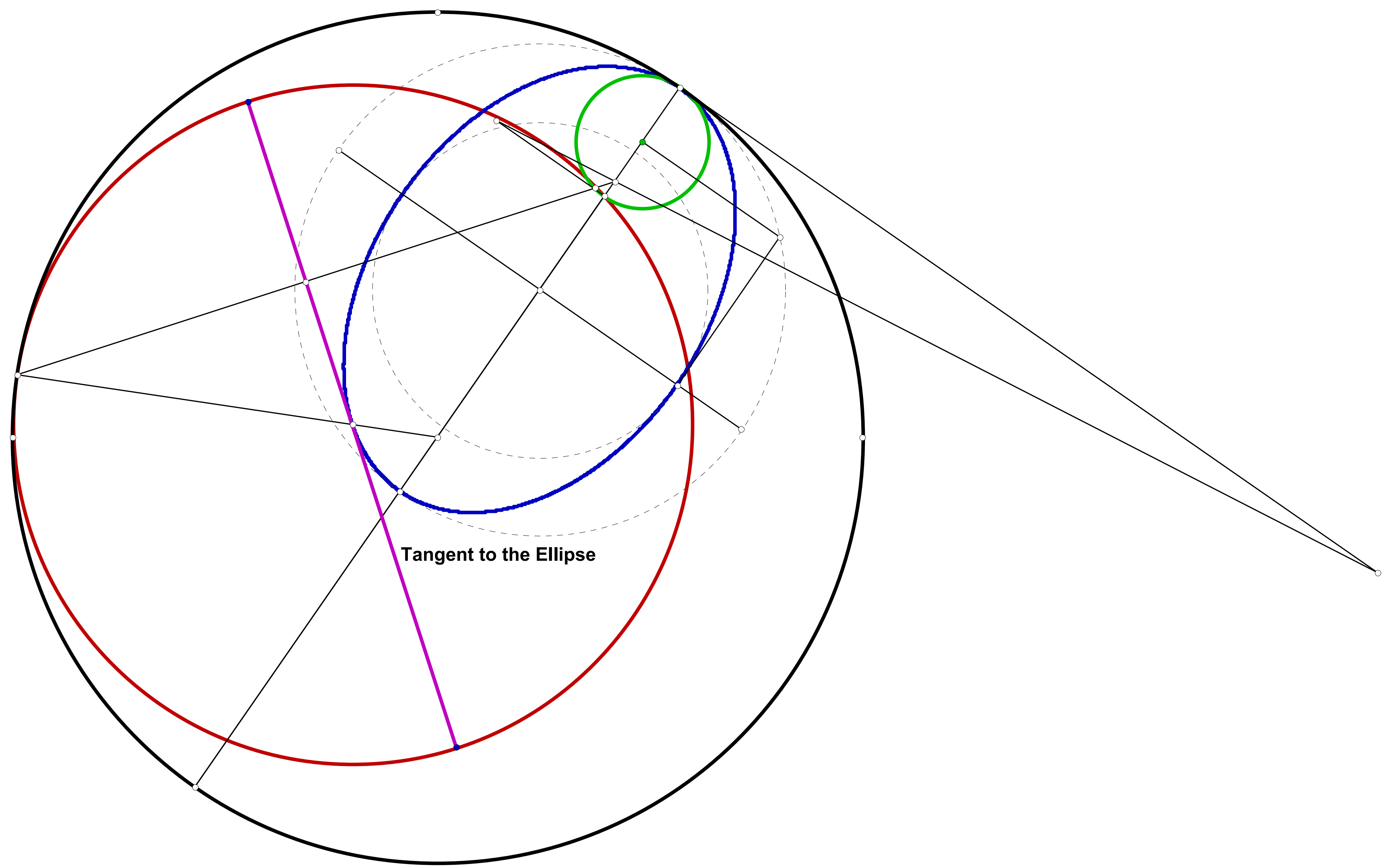
Project the inverse
ellipse (green) and solve.

$$\frac{CE}{CD} - \frac{CG}{CF} = 0.00000$$

Unit.
Given.

110706 Sketchbook C

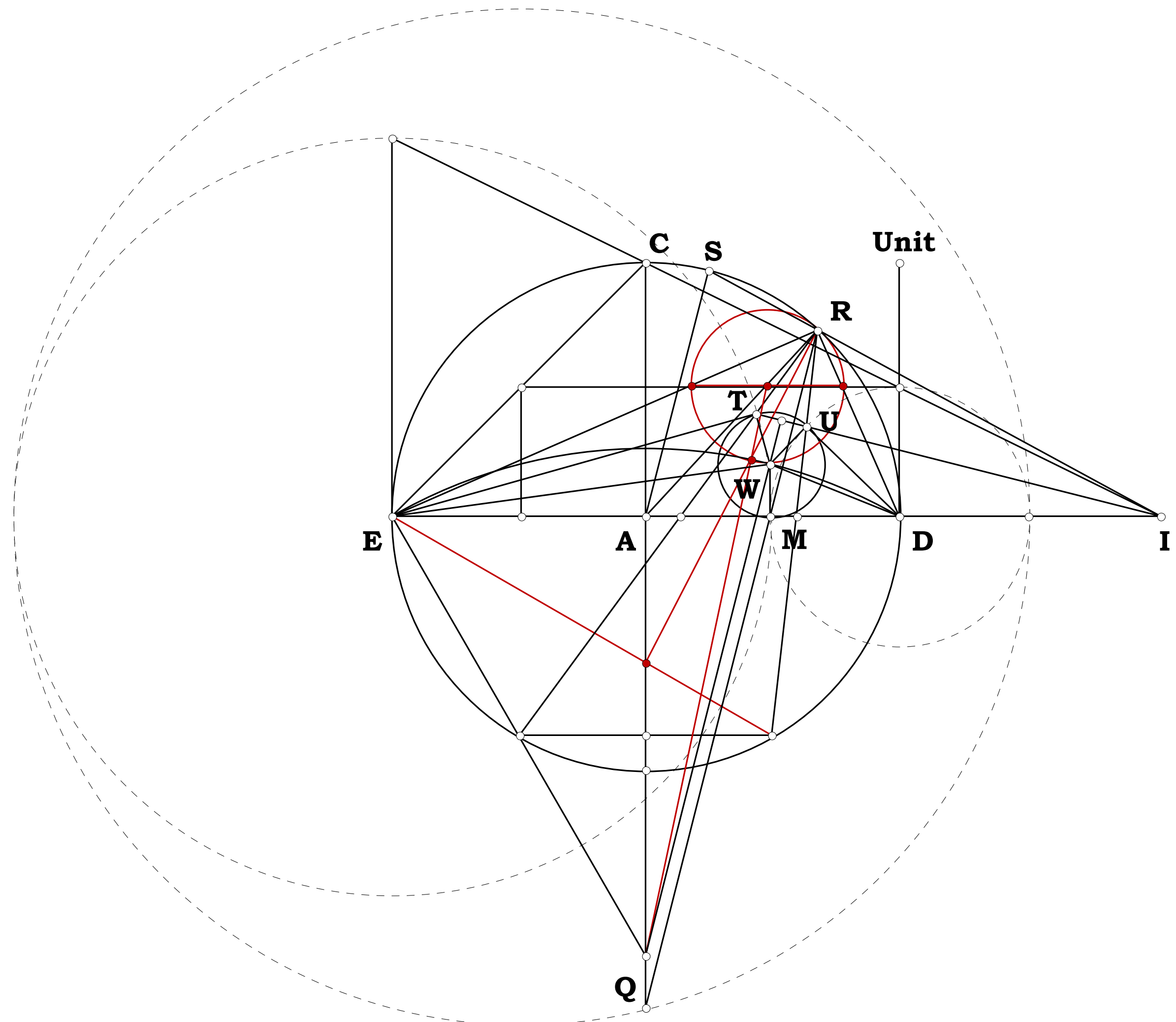
Descriptions.
Definitions.



Unit.
Given.

110706 Sketchbook D

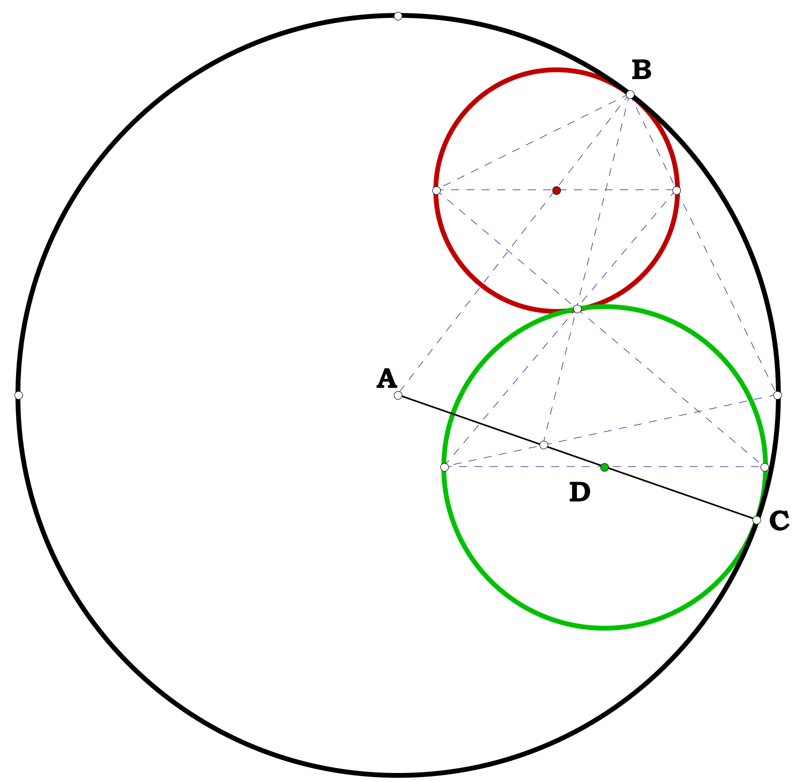
Descriptions.
Definitions.



Unit.
Given.

110706 Sketchbook E

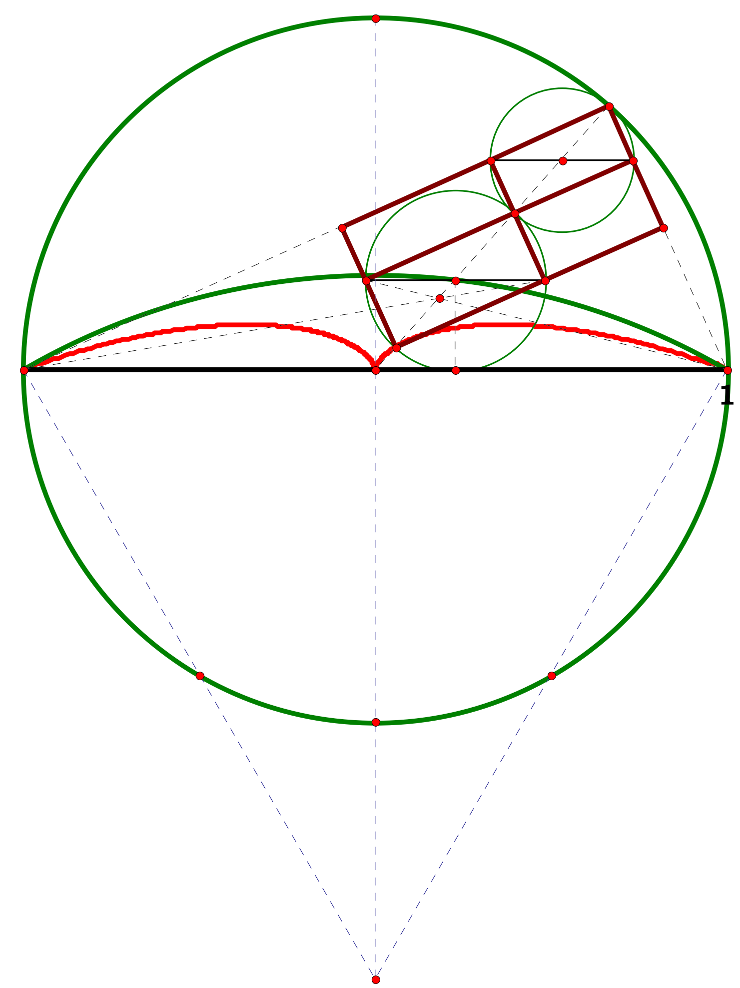
Descriptions.
Definitions.



Unit.
Given.

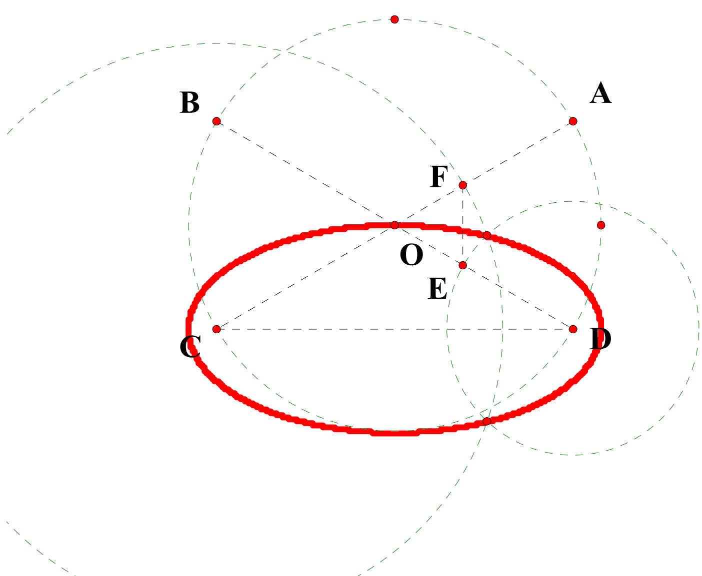
110706 Sketchbook F

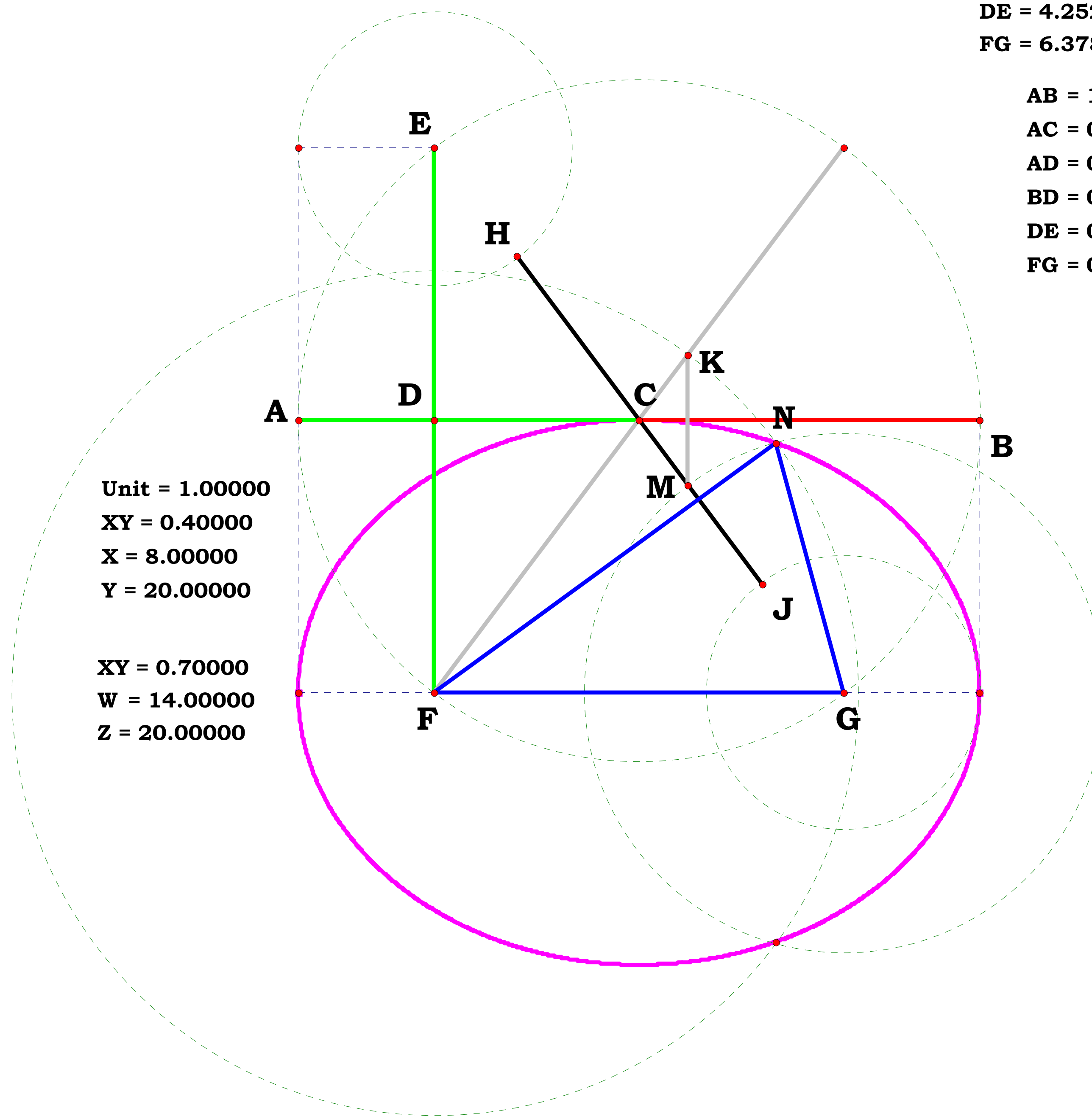
Descriptions.
Definitions.



Really? Is this as far as you got on this project?
2008-0611

There are, as one can notice, a great deal of work left for some future time to do, this was one of them. These sketches are usually only complete to show what the finished project is aiming at, or to indicate the need for parsing to construct many individual projects.





Unit = 1.00000
XY = 0.40000
X = 8.00000
Y = 20.00000

XY = 0.70000
W = 14.00000
Z = 20.00000

AB = 10.63083 cm
AC = 5.31542 cm
AD = 2.12617 cm
BD = 8.50467 cm
DE = 4.25233 cm
FG = 6.37850 cm

HJ = 6.37850 cm
HM = 4.46495 cm
FK = 6.59112 cm
FN = 6.59112 cm
GM = 4.03972 cm
GN = 4.03972 cm

AB = 1.00000
AC = 0.50000
AD = 0.20000
BD = 0.80000
DE = 0.40000
FG = 0.60000

HM = 0.42000
FK = 0.62000
FN = 0.62000
GM = 0.38000
GN = 0.38000

$$AC-\frac{1}{2} = 0.00000$$

$$AD-\frac{X}{2\cdot Y} = 0.00000$$

$$BD-\frac{2\cdot Y-X}{2\cdot Y} = 0.00000$$

$$DE-\frac{\sqrt{X\cdot(2\cdot Y-X)}}{2\cdot Y} = 0.00000$$

$$FG-\frac{Y-X}{Y} = 0.00000$$

$$HM-\frac{W\cdot(Y-X)}{Y\cdot Z} = 0.00000$$

$$FK-\frac{(2\cdot W\cdot Y-2\cdot W\cdot X)+X\cdot Z}{2\cdot Y\cdot Z} = 0.00000$$

$$FN-\frac{(2\cdot W\cdot Y-2\cdot W\cdot X)+X\cdot Z}{2\cdot Y\cdot Z} = 0.00000$$

$$GM-\frac{(2\cdot W\cdot X-2\cdot W\cdot Y-X\cdot Z)+2\cdot Y\cdot Z}{2\cdot Y\cdot Z} = 0.00000$$

$$GN-\frac{(2\cdot W\cdot X-2\cdot W\cdot Y-X\cdot Z)+2\cdot Y\cdot Z}{2\cdot Y\cdot Z} = 0.00000$$

110806R

Unit. $AB := 1$

Given. $X := 8$ $W := 14$

$Y := 20$ $Z := 20$

Descriptions.

$$AC := \frac{AB}{2} \quad AD := \frac{X}{2 \cdot Y} \quad BD := AB - AD \quad DE := \sqrt{AD \cdot BD}$$

$$FG := AB - 2 \cdot AD \quad HJ := FG \quad HM := \frac{HJ \cdot W}{Z} \quad FK := HM + AD$$

$$FN := FK \quad GM := AB - FK \quad GN := GM$$

Definitions.

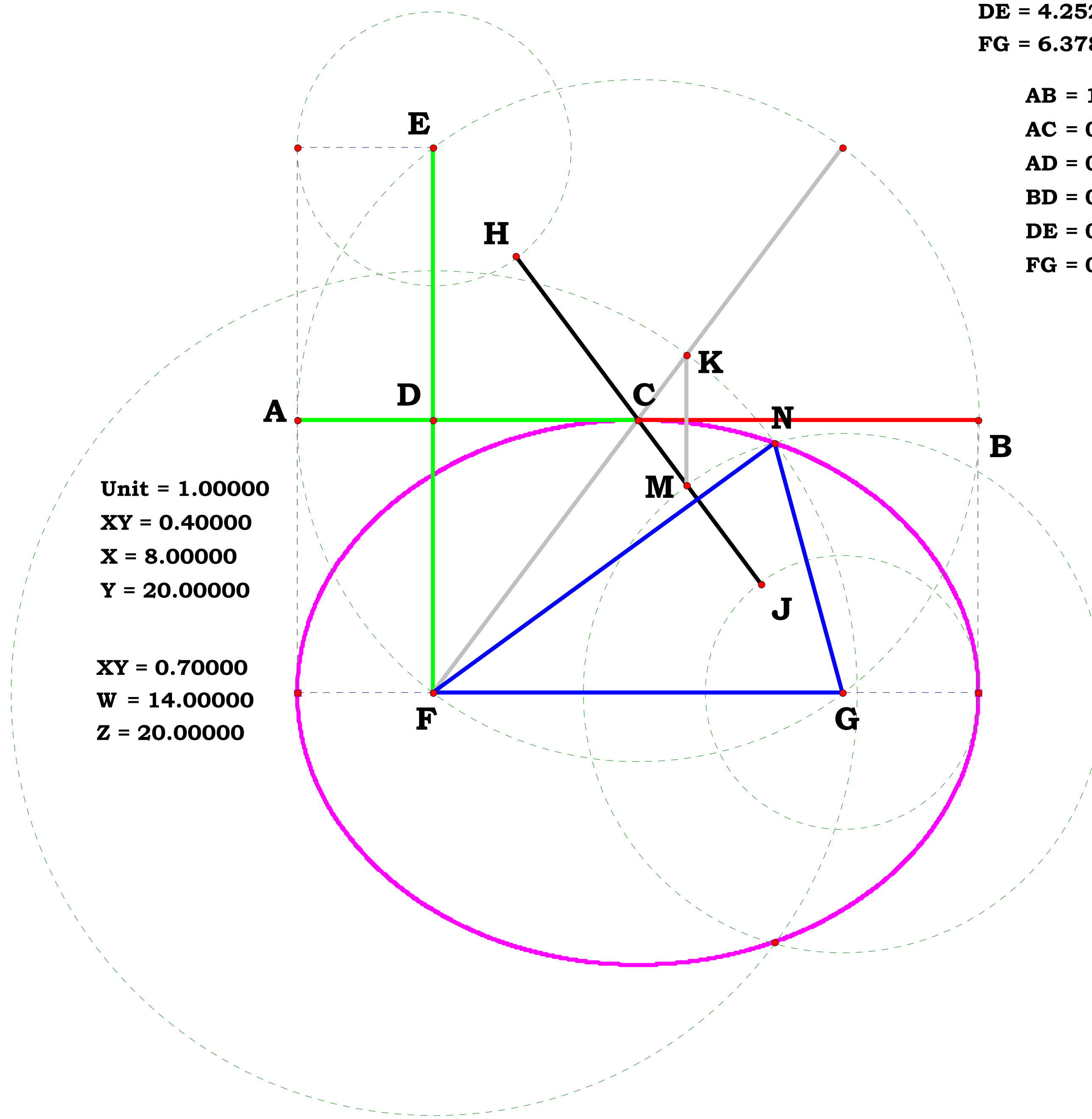
$$AC - \frac{1}{2} = 0 \quad AD - \frac{X}{2 \cdot Y} = 0 \quad BD - \frac{2 \cdot Y - X}{2 \cdot Y} = 0$$

$$DE - \frac{\sqrt{X \cdot (2 \cdot Y - X)}}{2 \cdot Y} = 0 \quad FG - \frac{Y - X}{Y} = 0 \quad HJ - \frac{Y - X}{Y} = 0$$

$$HM - \frac{W \cdot (Y - X)}{Y \cdot Z} = 0 \quad FK - \frac{2 \cdot W \cdot Y - 2 \cdot W \cdot X + X \cdot Z}{2 \cdot Y \cdot Z} = 0$$

$$FN - \frac{2 \cdot W \cdot Y - 2 \cdot W \cdot X + X \cdot Z}{2 \cdot Y \cdot Z} = 0$$

$$GM - \frac{2 \cdot W \cdot X - 2 \cdot W \cdot Y - X \cdot Z + 2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0$$



AB = 10.63083 cm
AC = 5.31542 cm
AD = 2.12617 cm
BD = 8.50467 cm
DE = 4.25233 cm
FG = 6.37850 cm

HJ = 6.37850 cm
HM = 4.46495 cm
FK = 6.59112 cm
FN = 6.59112 cm
GM = 4.03972 cm
GN = 4.03972 cm

AB = 1.00000
AC = 0.50000
AD = 0.20000
BD = 0.80000
DE = 0.40000
FG = 0.60000

HM = 0.42000
FK = 0.62000
FN = 0.62000
GM = 0.38000
GN = 0.38000

$AC - \frac{1}{2} = 0.00000$

$AD - \frac{X}{2 \cdot Y} = 0.00000$

$BD - \frac{2 \cdot Y - X}{2 \cdot Y} = 0.00000$

$DE - \frac{\sqrt{X \cdot (2 \cdot Y - X)}}{2 \cdot Y} = 0.00000$

$FG - \frac{Y - X}{Y} = 0.00000$

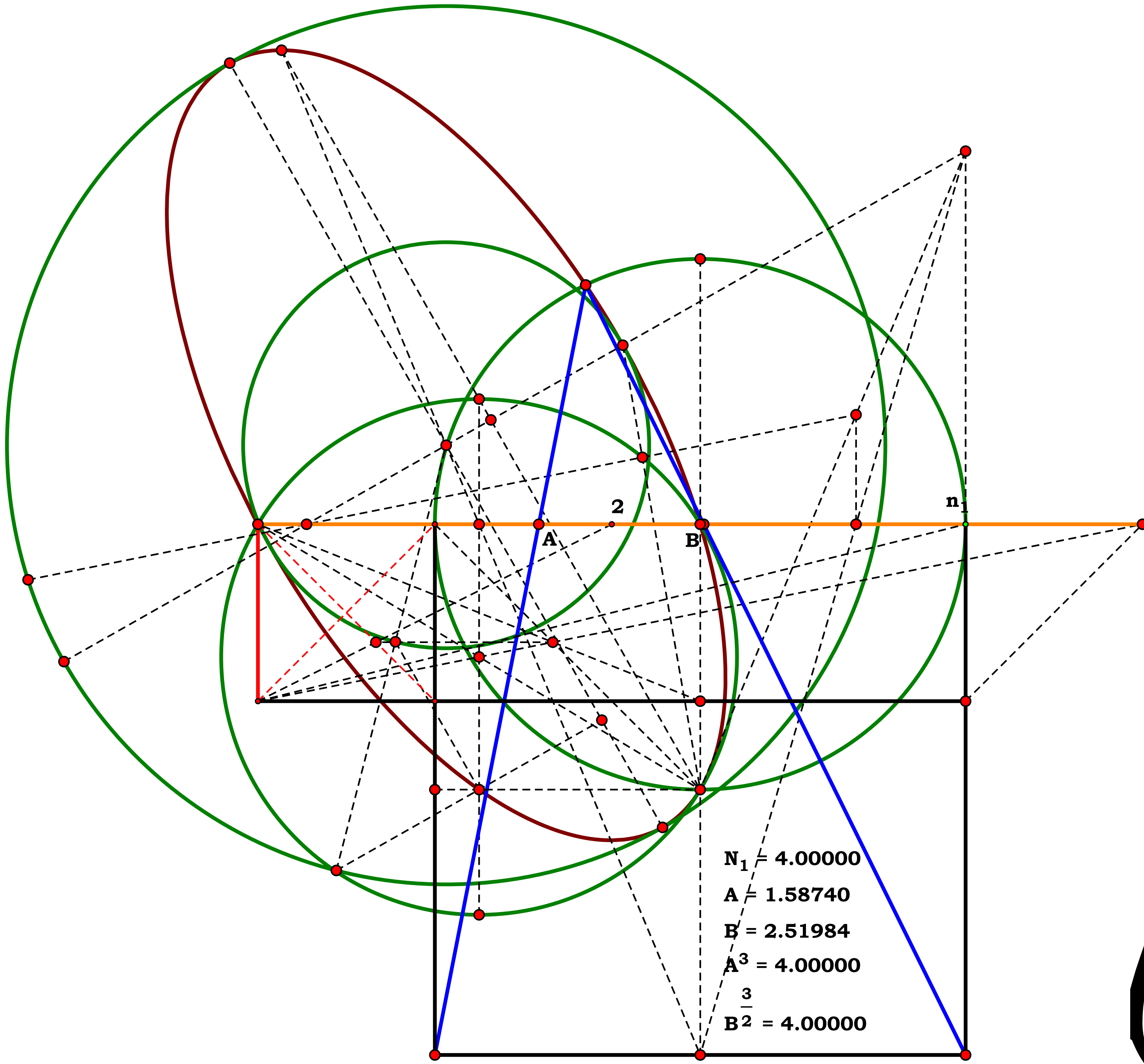
$HM - \frac{W \cdot (Y - X)}{Y \cdot Z} = 0.00000$

$FK - \frac{(2 \cdot W \cdot Y - 2 \cdot W \cdot X) + X \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

$FN - \frac{(2 \cdot W \cdot Y - 2 \cdot W \cdot X) + X \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

$GM - \frac{(2 \cdot W \cdot X - 2 \cdot W \cdot Y - X \cdot Z) + 2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

$GN - \frac{(2 \cdot W \cdot X - 2 \cdot W \cdot Y - X \cdot Z) + 2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$



The Delian Quest 2007

John Clark

Unit.

$DF := 1$

Given.

$N_1 := .49 \qquad AI := N_1$

060807A

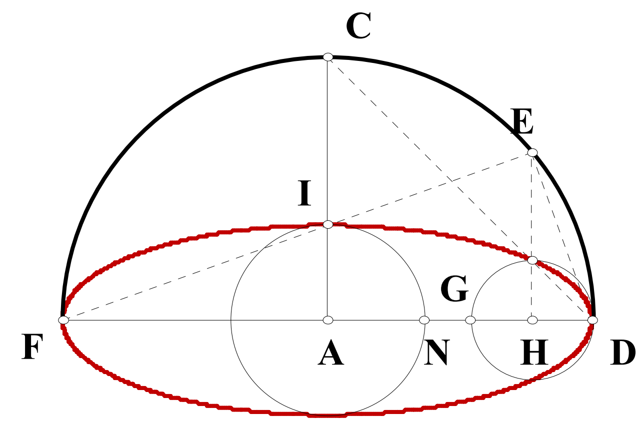
Descriptions.

$AF := \frac{DF}{2} \qquad AN := AI \qquad AD := AF$

$FI := \sqrt{AF^2 + AI^2} \qquad DE := \frac{AI \cdot DF}{FI}$

$DH := \frac{DE^2}{DF} \qquad GH := DH$

$GN := AD - (AN + GH + DH)$



Definitions.

$\frac{1}{2} - AF = 0 \qquad \frac{1}{2} \cdot \sqrt{1 + 4 \cdot N_1^{\,2}} - FI = 0$

$2 \cdot \frac{N_1}{\sqrt{1 + 4N_1^{\,2}}} - DE = 0 \qquad 4 \cdot \frac{N_1^{\,2}}{\left(1 + 4 \cdot N_1^{\,2}\right)} - DH = 0$

$$\frac{\left(2 \cdot N_1 + 1\right) \cdot \left(1 - 4 \cdot N_1 - 4 \cdot N_1^{\,2}\right)}{2 \cdot \left(4 \cdot N_1^{\,2} + 1\right)} - GN = 0$$

060807B

Unit.

Given.

Equation for an Ellipse

$N_1 := 3.38667$ $\quad AB := N_1$

$N_2 := 2.05199$ $\quad AI := N_2$

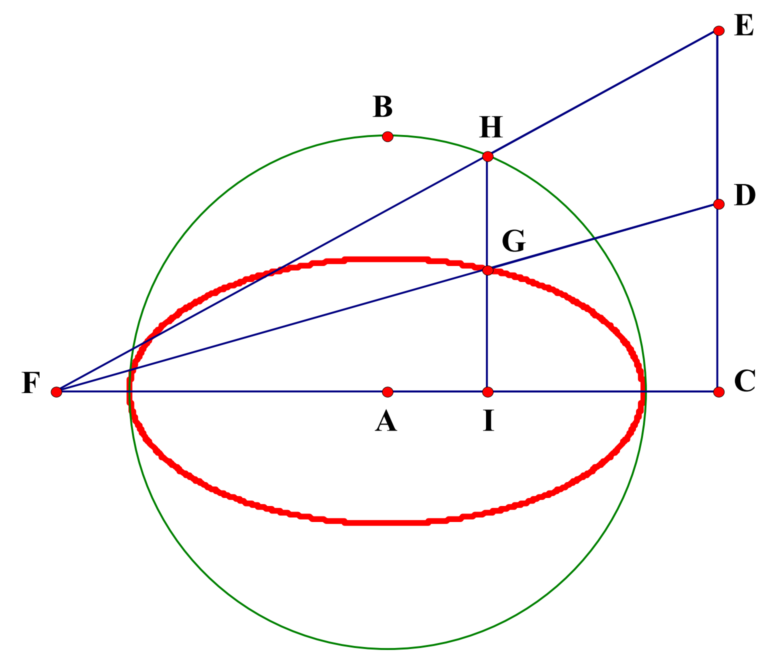
$N_3 := 3.57039$ $\quad CE := N_3$

$N_4 := .94919$ $\quad CD := N_4$

Descriptions.

$$HI := \sqrt{(AB + AI)\cdot(AB - AI)}$$

$$GI := \frac{CD \cdot HI}{CE} \qquad AG := \sqrt{AI^2 + GI^2}$$



Definitions.

$$\frac{N_4 \cdot \sqrt{\left(N_1 + N_2\right)\cdot\left(N_1 - N_2\right)}}{N_3} - GI = 0$$

$$\frac{1}{N_3} \cdot \sqrt{N_2^{\,2} \cdot N_3^{\,2} + N_4^{\,2} \cdot N_1^{\,2} - N_4^{\,2} \cdot N_2^{\,2}} - AG = 0$$

Unit. **Procrastinated Write up for 061307**

Given.

The blue circle is always tangent to both and its center is always on the ellipse, Therefore, it always resides in a cresant and, it switches between inclusion and exclusion at the intersection of all three for each of the other circles. This means that it is aways excluded from an area included in the other two.

Descriptions.

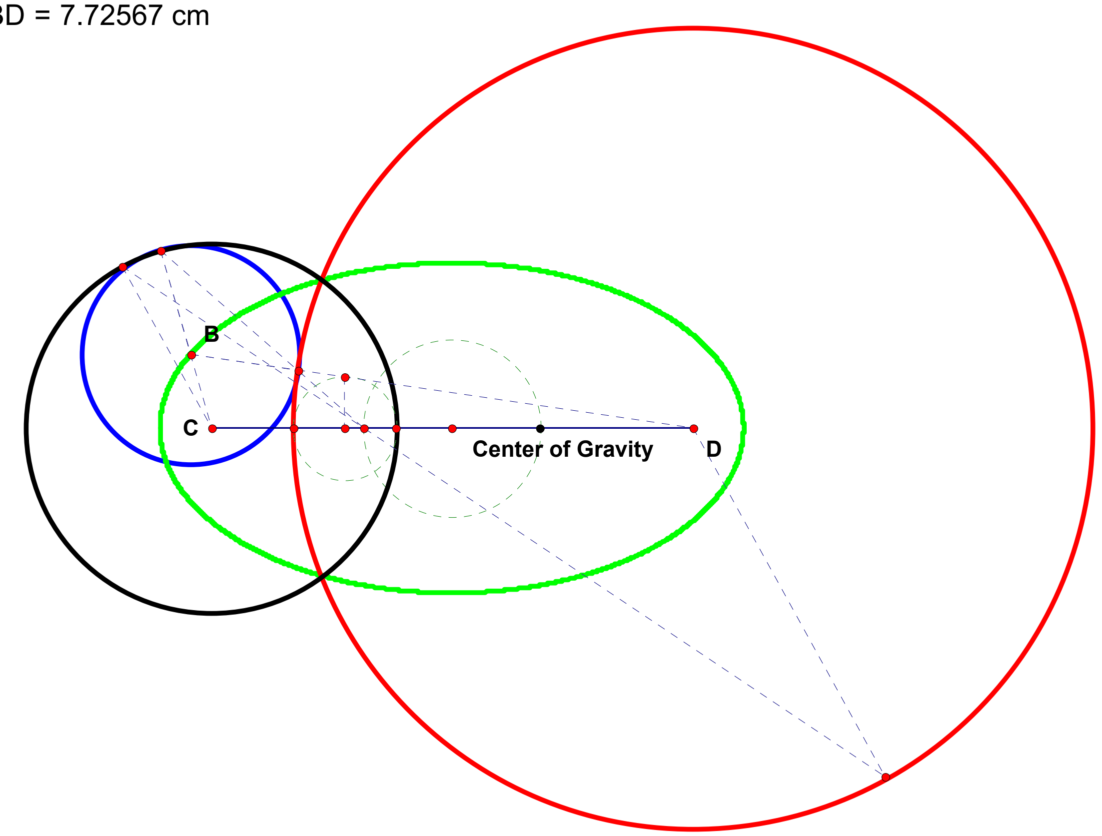
Definitions.

Animate Point

BC = 1.00975 cm

BD = 6.71592 cm

BC+BD = 7.72567 cm



062007A

Unit.

$AB := 1$

Given.

$X := 7 \quad Z := 12$

$W := 20 \quad Y := 19$

Descriptions.

$$AC := 2 \cdot AB \quad BC := AB \quad BX := BC \cdot \frac{X}{W} \quad AX := AB + BX$$

$$BZ := BC \cdot \frac{Z}{Y} \quad AZ := AB + BZ \quad EZ := \sqrt{AZ \cdot (AC - AZ)}$$

$$GZ := BX \cdot \frac{EZ}{BC} \quad BJ := \frac{BC^2}{2 \cdot BZ} \quad BK := 2 \cdot BJ$$

$$KZ := BK - BZ \quad GK := \sqrt{GZ^2 + KZ^2}$$

Definitions.

$$AC - 2 = 0 \quad BC - 1 \quad BX - \frac{X}{W} = 0 \quad AX - \frac{W + X}{W} = 0$$

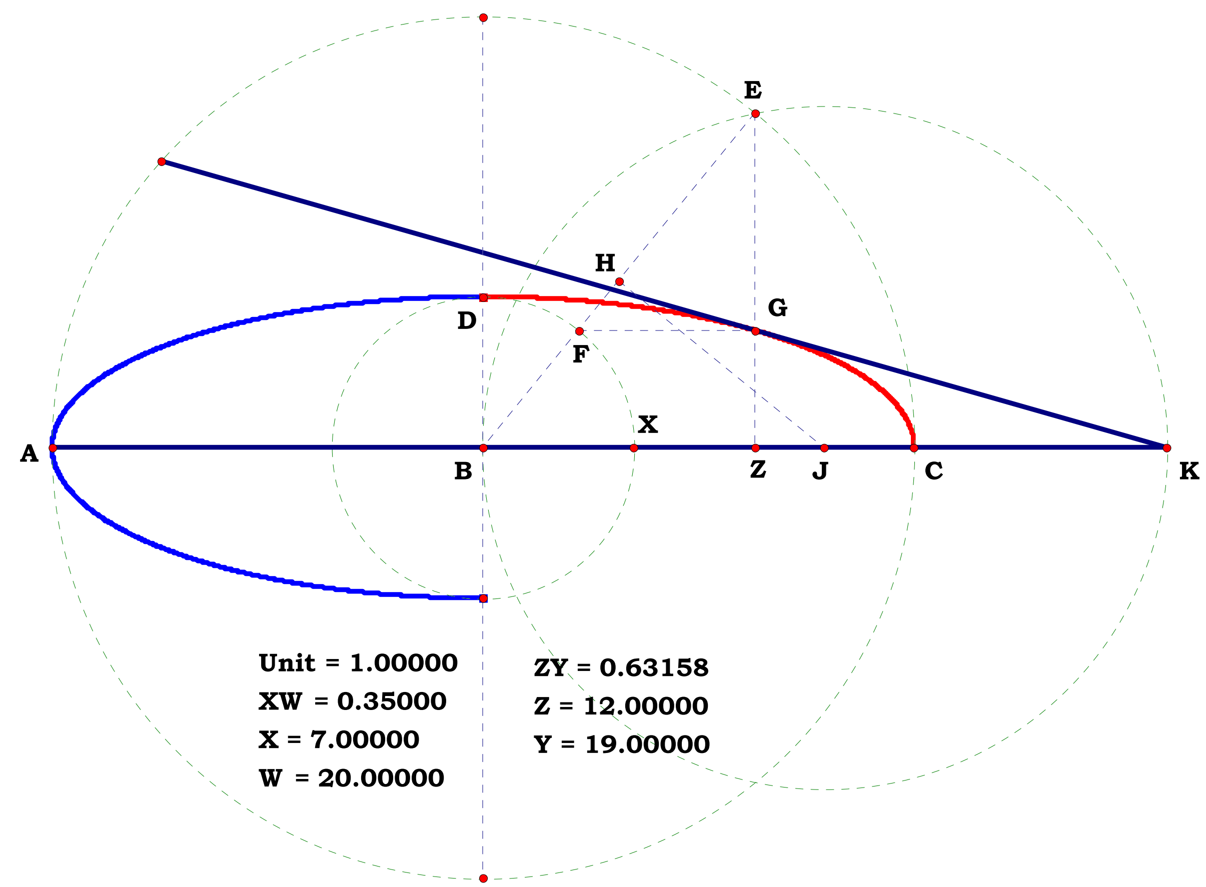
$$BZ - \frac{Z}{Y} = 0 \quad AZ - \frac{Y + Z}{Y} = 0 \quad EZ - \frac{\sqrt{(Y - Z) \cdot (Y + Z)}}{Y} = 0$$

$$GZ - \frac{X \cdot \sqrt{(Y - Z) \cdot (Y + Z)}}{W \cdot Y} = 0 \quad BJ - \frac{Y}{2 \cdot Z} = 0 \quad BK - \frac{Y}{Z} = 0$$

$$KZ - \frac{(Y - Z) \cdot (Y + Z)}{Y \cdot Z} = 0$$

$$GK - \frac{\sqrt{(Y - Z) \cdot (Y + Z) \cdot \left(W^2 \cdot Y^2 - W^2 \cdot Z^2 + X^2 \cdot Z^2\right)}}{W \cdot Y \cdot Z} = 0$$

Tangent from Major Axis



$$\frac{(Y-Z)\cdot(Y+Z)}{Y\cdot Z} - KZ = 0.00000$$

$$\frac{X\cdot\sqrt{(Y-Z)\cdot(Y+Z)}}{W\cdot Y} - GZ = 0.00000$$

$$\frac{\sqrt{(Y-Z)\cdot(Y+Z)\cdot((W^2\cdot Y^2-W^2\cdot Z^2)+X^2\cdot Z^2)}}{W\cdot Y\cdot Z} - GK = 0.00000$$

062007B

Descriptions.

Unit.

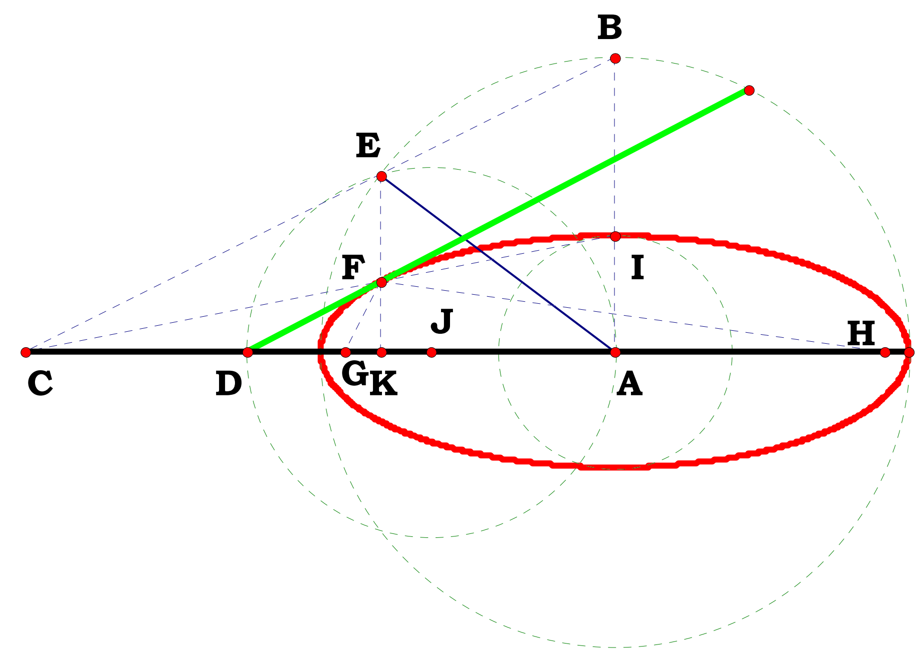
$AB := 1$

Given.

$N_1 := .39368 \qquad AI := N_1$

$N_2 := 1.25170 \qquad AD := N_2$

Tangent from Major Axis



$AJ := \dfrac{AD}{2}$ From 080193

$$EK := \frac{AB \cdot \sqrt{(2 \cdot AJ - AB) \cdot (2 \cdot AJ + AB)}}{2 \cdot AJ}$$

$AK := \sqrt{AB^2 - EK^2} \qquad DK := AD - AK \qquad FK := \dfrac{AI \cdot EK}{AB}$

$DF := \sqrt{DK^2 + FK^2} \qquad AG := \sqrt{AB^2 - AI^2} \qquad AH := AG \qquad HK := AH + AK$

$GK := AG - AK \qquad FG := \sqrt{FK^2 + GK^2} \qquad FM := FG \qquad FH := \sqrt{HK^2 + FK^2}$

$HM := FH - FM \qquad HO := \dfrac{HK \cdot HM}{FH} \qquad GH := 2 \cdot AH \qquad GO := GH - HO$

$MO := \dfrac{FK \cdot HM}{FH} \qquad \dfrac{GO}{MO} - \dfrac{DK}{FK} = 0$

Definitions.

$$N_1 \cdot \frac{\sqrt{(N_2 - 1) \cdot (N_2 + 1)}}{N_2} - FK = 0$$

062007C

Unit.

$AB := 1$

Given.

$X := 20 \qquad Z := 10$

$W := 6 \qquad Y := 4$

Descriptions.

$$BE := AB \qquad BX := \frac{X}{W} \qquad AX := AB + BX \qquad BZ := \frac{Z}{Y}$$

$$XZ := BX - BZ \qquad CD := 2 \cdot BX \qquad CF := XZ$$

$$DF := CD - CF \qquad FG := \sqrt{CF \cdot DF} \qquad BN := \frac{BE \cdot BX}{2 \cdot FG}$$

$$BP := 2 \cdot BN \qquad FH := BE \cdot \frac{FG}{BX} \qquad MP := BP - FH$$

$$BR := BZ \cdot \frac{BP}{MP} \qquad PR := \sqrt{BP^2 + BR^2}$$

Definitions.

$$BE - 1 = 0 \qquad BX - \frac{X}{W} = 0 \qquad AX - \frac{W + X}{W} = 0 \qquad BZ - \frac{Z}{Y} = 0$$

$$XZ - \frac{X \cdot Y - W \cdot Z}{W \cdot Y} = 0 \qquad CD - 2 \cdot \frac{X}{W} = 0 \qquad CF - \frac{X \cdot Y - W \cdot Z}{W \cdot Y} = 0$$

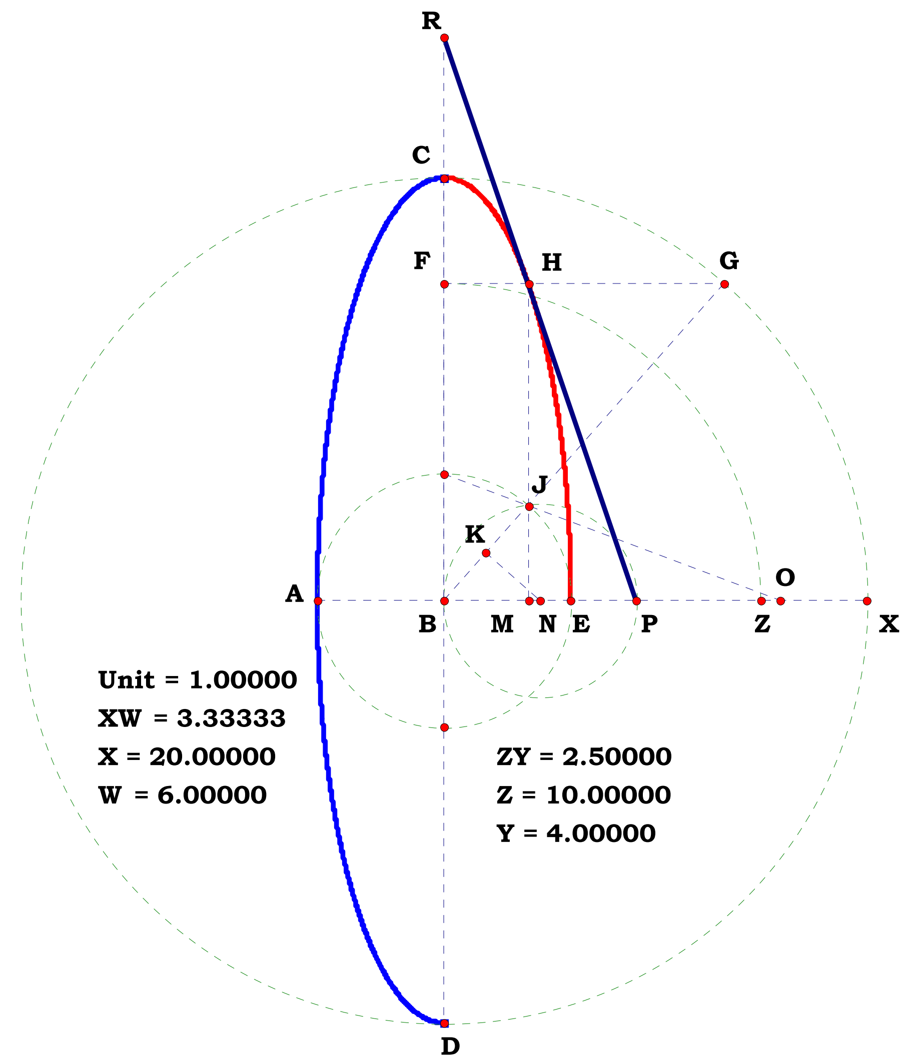
$$DF - \frac{W \cdot Z + X \cdot Y}{W \cdot Y} = 0 \qquad FG - \frac{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}}{W \cdot Y} = 0$$

$$BN - \frac{X \cdot Y}{2 \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0 \qquad BP - \frac{X \cdot Y}{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0$$

$$FH - \frac{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}}{X \cdot Y} = 0 \qquad MP - \frac{W^2 \cdot Z^2}{X \cdot Y \cdot \sqrt{X^2 \cdot Y^2 - W^2 \cdot Z^2}} = 0$$

$$BR - \frac{X^2 \cdot Y \cdot \sqrt{X^2 \cdot Y^2 - W^2 \cdot Z^2}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0 \qquad PR - \frac{X \cdot Y \cdot \sqrt{\left(W^4 \cdot Z^2 - W^2 \cdot X^2 \cdot Z^2 + X^4 \cdot Y^2\right)}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0$$

Tangent from Minor Axis



$$\frac{X \cdot Y}{\sqrt{(W \cdot Z+X \cdot Y) \cdot (X \cdot Y-W \cdot Z)}}-BP = 0.00000$$

$$\frac{X^2 \cdot Y \cdot \sqrt{X^2 \cdot Y^2-W^2 \cdot Z^2}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z+X \cdot Y) \cdot (X \cdot Y-W \cdot Z)}}-BR = 0.00000$$

$$\frac{X \cdot Y \cdot \sqrt{(W^4 \cdot Z^2-W^2 \cdot X^2 \cdot Z^2)+X^4 \cdot Y^2}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z+X \cdot Y) \cdot (X \cdot Y-W \cdot Z)}}-PR = 0.00000$$

062007D

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := .40636 \qquad AI := N_1$

$N_2 := .60804 \qquad AD := N_2$

$$AJ := \frac{AD}{2} \qquad PK := \frac{AI \cdot \sqrt{(2 \cdot AJ - AI) \cdot (2 \cdot AJ + AI)}}{2 \cdot AJ} \qquad AK := \sqrt{AI^2 - PK^2}$$

$$DK := AD - AK \qquad FK := \frac{PK \cdot AB}{AI} \qquad DF := \sqrt{DK^2 + FK^2} \qquad AG := \sqrt{AB^2 - AI^2}$$

$$AH := AG \qquad HR := AH + FK \qquad GR := AG - FK \qquad FG := \sqrt{AK^2 + GR^2} \qquad FM := FG$$

$$FH := \sqrt{HR^2 + AK^2} \qquad HM := FH - FM \qquad HO := \frac{HR \cdot HM}{FH} \qquad GH := 2 \cdot AH$$

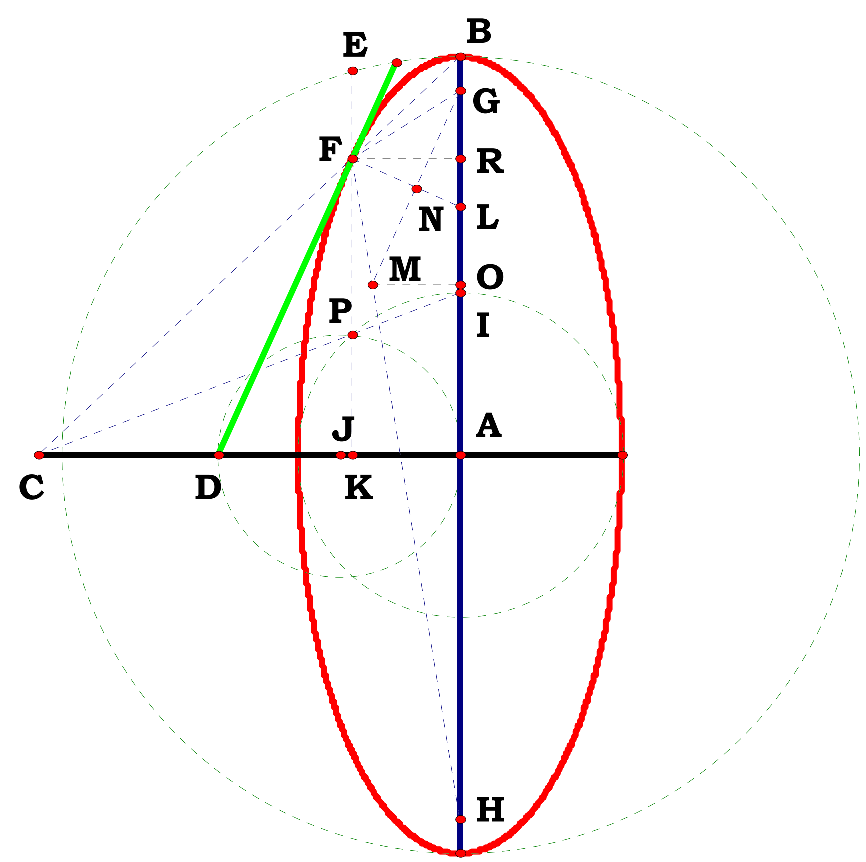
$$GO := GH - HO \qquad MO := \frac{AK \cdot HM}{FH} \qquad \frac{GO}{MO} - \frac{FK}{DK} = 0$$

Definitions.

$$\frac{\sqrt{\left(N_2 - N_1\right) \cdot \left(N_2 + N_1\right)}}{N_2} - FK = 0$$

$$Major := N_1 \cdot \frac{\sqrt{\left(N_2 - 1\right) \cdot \left(N_2 + 1\right)}}{N_2}$$

Tangent from Minor Axis



062407A

Unit.

$AB := 1$

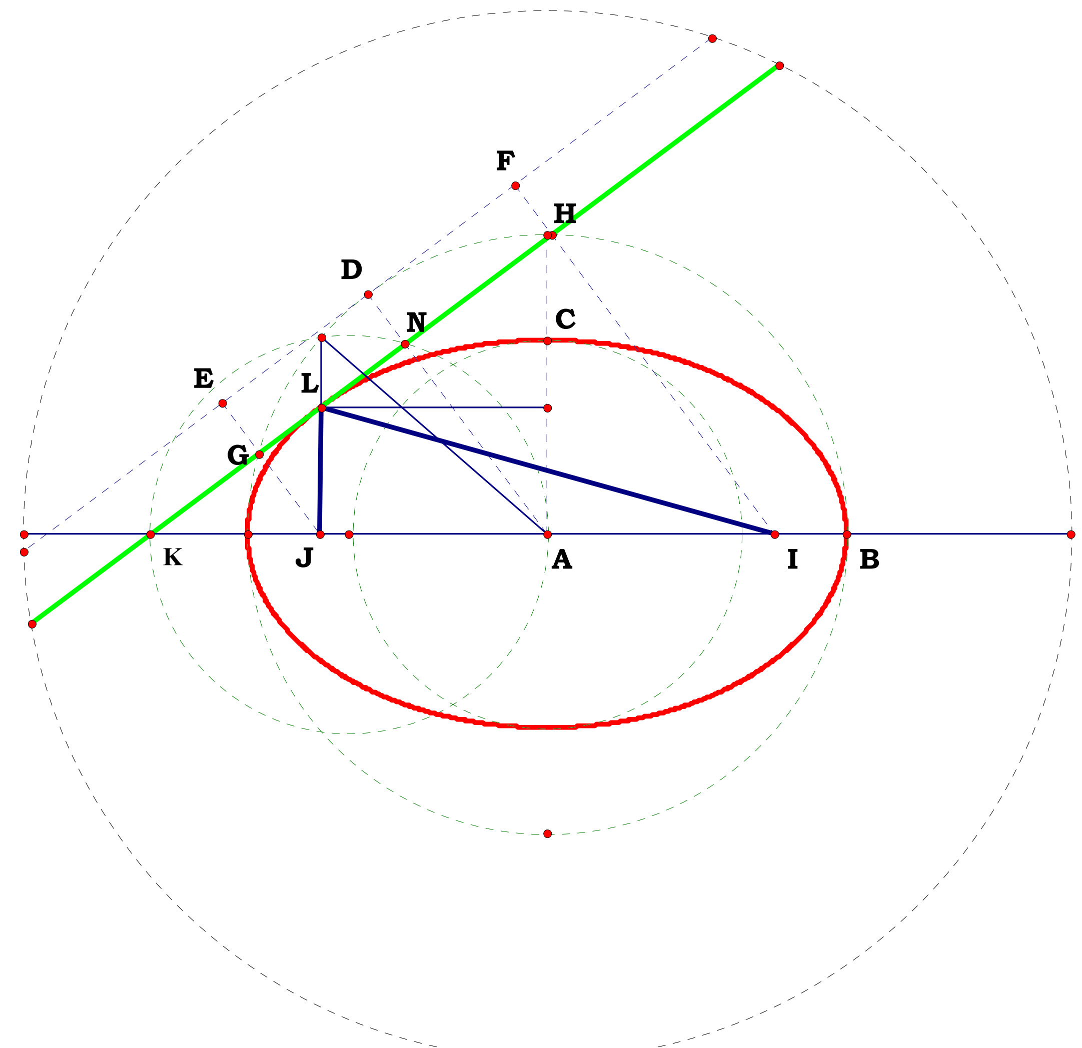
Given.

$N_1 := .64776$ $\quad AC := N_1$

$N_2 := .5444$ $\quad EJ := N_2$

Found on the Internet

Found the construction, now I explore it with Algebra.



AB = 3.95817 cm

AC = 2.56393 cm

EJ = 2.15482 cm

JL = 1.68103 cm

IL = 6.23530 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AC}{AB} = 0.64776$

$\frac{EJ}{AB} = 0.54440$

$\frac{JL}{AB} = 0.42470$

$\frac{IL}{AB} = 1.57530$

Descriptions.

$AJ := \sqrt{AB^2 - AC^2}$ $\quad FI := (AB - EJ) + AB$ $\quad DE := \sqrt{AJ^2 - (AB - EJ)^2}$

$AN := \frac{\sqrt{DE^2 \cdot (DE + AB) \cdot (-DE + AB)}}{DE}$ $\quad DN := AB - AN$ $\quad GJ := EJ - DN$

$HI := FI - DN$ $\quad GL := \frac{GJ \cdot 2 \cdot DE}{GJ + HI}$ $\quad HL := 2 \cdot DE - GL$

$JL := \sqrt{GL^2 + GJ^2}$ $\quad IL := \sqrt{HI^2 + HL^2}$ $\quad (JL + IL) - 2 \cdot AB = 0$

Definitions:

$AJ - \sqrt{\left(1 - N_1^{\,2}\right)} = 0$ $\quad FI - \left(2 - N_2\right) = 0$ $\quad DE - \sqrt{\left(2 \cdot N_2 - N_2^{\,2} - N_1^{\,2}\right)} = 0$

$\sqrt{\left(N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1\right)} - AN = 0$ $\quad \left(\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - N_2 + 1\right) - HI = 0$

$1 - \sqrt{\left(N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1\right)} - DN = 0$ $\quad N_2 + \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - 1 - GJ = 0$

$$\frac{\sqrt{2 \cdot N_2 - N_2^{\,2} - N_1^{\,2}} \cdot \left(N_2 + \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - 1\right)}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - GL = 0 \qquad \frac{\sqrt{2 \cdot N_2 - N_2^{\,2} - N_1^{\,2}} \cdot \left(\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - N_2 + 1\right)}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - HL = 0$$

$$\frac{\sqrt{N_1^{\,2} - 4 \cdot N_2 + 2 \cdot N_2^{\,2} + 2 \cdot \left(N_2 - 1\right) \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} + 2}}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - JL = 0 \qquad \frac{\sqrt{N_1^{\,2} - 4 \cdot N_2 + 2 \cdot N_2^{\,2} - 2 \cdot \left(N_2 - 1\right) \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} + 2}}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - IL = 0$$

062407B

Unit.

$AB := 1$

Given.

$W := 20 \quad Y := 20$

$X := 13 \quad Z := 8$

Found on the Internet

Writeup A and its figure were rather a bit bad and awkward.

Descriptions.

$$AX := \frac{X}{W} \quad AZ := \frac{Z}{Y} \quad BX := AB - AX \quad AC := 2 \cdot AB$$

$$BH := BX \quad CZ := AC - AZ \quad MZ := \sqrt{AZ \cdot CZ}$$

$$GZ := BX \cdot \frac{MZ}{AB} \quad BZ := AB - AZ \quad BN := \frac{AB}{2} \quad BE := AB \cdot \frac{BN}{BZ}$$

$$BO := 2 \cdot BE \quad OZ := BO - BZ \quad GO := \sqrt{OZ^2 + GZ^2}$$

Definitions.

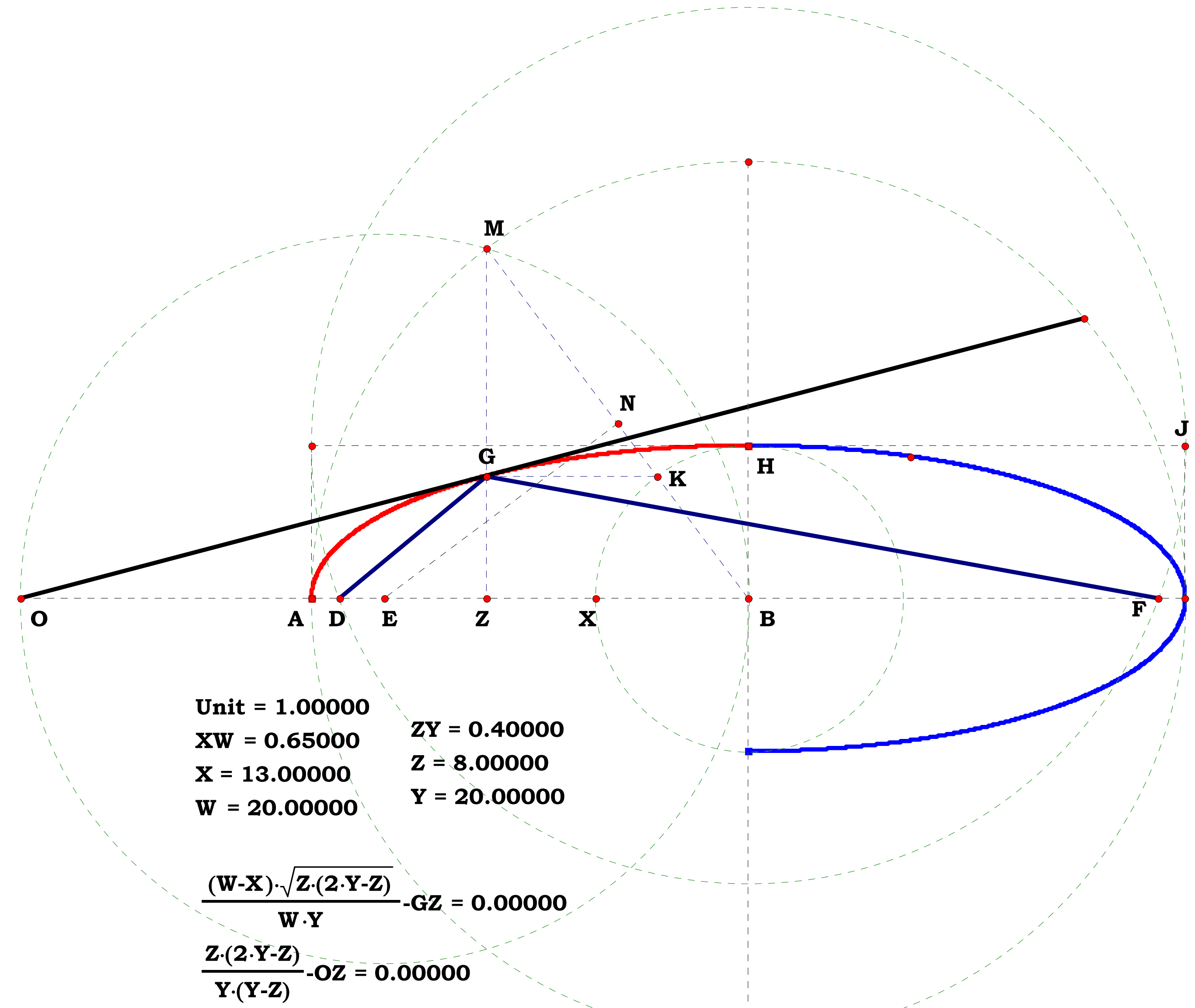
$$AX - \frac{X}{W} = 0 \quad AZ - \frac{Z}{Y} = 0 \quad BX - \frac{W-X}{W} = 0 \quad AC - 2 = 0$$

$$BH - \frac{W-X}{W} = 0 \quad CZ - \frac{2 \cdot Y - Z}{Y} = 0 \quad MZ - \frac{\sqrt{Z \cdot (2 \cdot Y - Z)}}{Y} = 0$$

$$GZ - \frac{(W-X) \cdot \sqrt{Z \cdot (2 \cdot Y - Z)}}{W \cdot Y} = 0 \quad BZ - \frac{Y-Z}{Y} = 0 \quad BN - \frac{1}{2} = 0$$

$$BE - \frac{Y}{2 \cdot (Y-Z)} = 0 \quad BO - \frac{Y}{Y-Z} = 0 \quad OZ - \frac{Z \cdot (2 \cdot Y - Z)}{Y \cdot (Y-Z)} = 0$$

$$GO - \frac{\sqrt{Z \cdot (2 \cdot Y - Z) \cdot \left[\left(X^2 - 2 \cdot W \cdot X\right) \cdot Z^2 + \left(4 \cdot W \cdot X \cdot Y - 2 \cdot X^2 \cdot Y\right) \cdot Z + Y^2 \cdot (W-X)^2\right]}}{W \cdot Y \cdot (Y-Z)} = 0$$



$$\frac{(W-X)\cdot\sqrt{Z\cdot(2\cdot Y-Z)}}{W\cdot Y} - GZ = 0.00000$$

$$\frac{Z\cdot(2\cdot Y-Z)}{Y\cdot(Y-Z)} - OZ = 0.00000$$

$$\frac{\sqrt{Z\cdot(2\cdot Y-Z)\cdot((X^2-2\cdot W\cdot X)\cdot Z^2+(4\cdot W\cdot X\cdot Y-2\cdot X^2\cdot Y)\cdot Z+Y^2\cdot(W-X)^2)}}{W\cdot Y\cdot(Y-Z)} - GO = 0.00000$$

Unit.

$AB := 1$

Given.

$X := 8 \qquad Y := 20$

$W := 20 \qquad Z := 9$

062407C

Descriptions.

$$AX := \frac{X}{W} \qquad AZ := \frac{Z}{Y} \qquad CZ := \sqrt{AZ \cdot (2AB - AZ)}$$

$$BX := AB - AX \qquad BZ := AB - AZ \qquad FZ := \frac{CZ^2}{BZ}$$

$$DZ := BX \cdot \frac{CZ}{AB} \qquad DF := \sqrt{FZ^2 + DZ^2}$$

Definitions.

$$AX - \frac{X}{W} = 0 \qquad AZ - \frac{Z}{Y} = 0 \qquad CZ - \frac{\sqrt{Z \cdot (2 \cdot Y - Z)}}{Y} = 0$$

$$BX - \frac{W - X}{W} = 0 \qquad BZ - \frac{Y - Z}{Y} = 0 \qquad FZ - \frac{Z \cdot (2 \cdot Y - Z)}{Y \cdot (Y - Z)} = 0$$

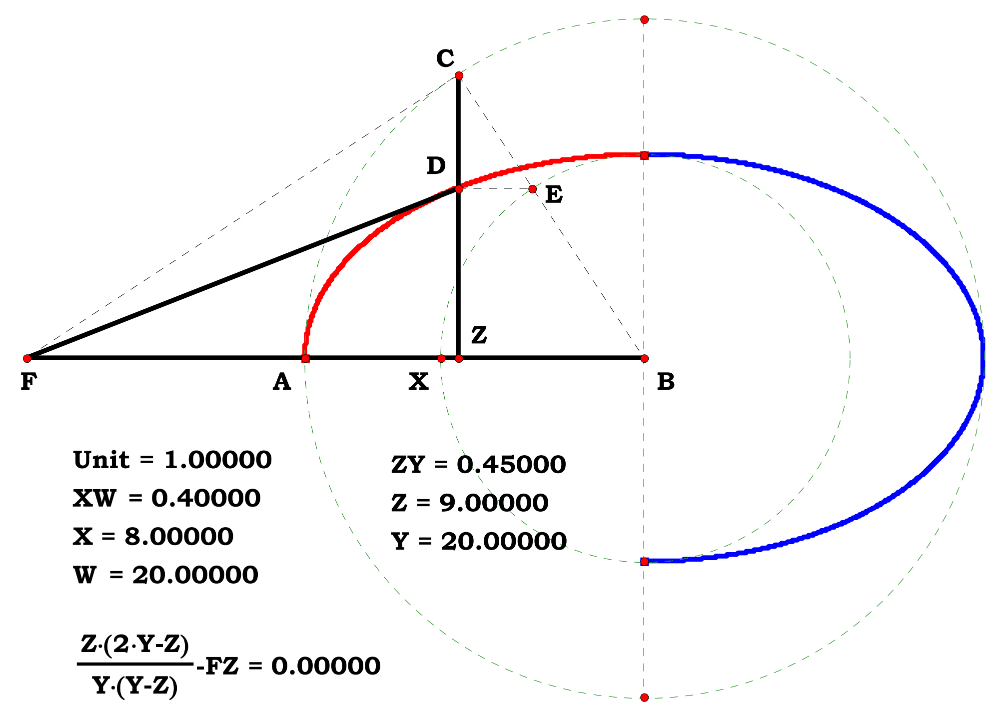
$$DZ - \frac{(W - X) \cdot \sqrt{Z \cdot (2 \cdot Y - Z)}}{W \cdot Y} = 0$$

$$DF - \frac{\sqrt{Z \cdot (2 \cdot Y - Z) \cdot \left[Y^2 \cdot W^2 + \left(4 \cdot X \cdot Y \cdot Z - 2 \cdot X \cdot Y^2 - 2 \cdot X \cdot Z^2\right) \cdot W + X^2 \cdot (Y - Z)^2\right]}}{W \cdot Y \cdot (Y - Z)} = 0$$

Found on the Internet

Found the construction, now I explore it with Algebra.

The raw construction was found on the internet. I added the structures required for writing it up in my usual algebraic method.



Unit = 1.00000
XW = 0.40000
X = 8.00000
W = 20.00000

ZY = 0.45000
Z = 9.00000
Y = 20.00000

$$\frac{Z \cdot (2 \cdot Y - Z)}{Y \cdot (Y - Z)} - FZ = 0.00000$$

$$\frac{(W - X) \cdot \sqrt{Z \cdot (2 \cdot Y - Z)}}{W \cdot Y} - DZ = 0.00000$$

$$\frac{\sqrt{Z \cdot (2 \cdot Y - Z) \cdot (Y^2 \cdot W^2 + (4 \cdot X \cdot Y \cdot Z - 2 \cdot X \cdot Y^2 - 2 \cdot X \cdot Z^2) \cdot W + X^2 \cdot (Y - Z)^2)}}{W \cdot Y \cdot (Y - Z)} - DF = 0.00000$$

Unit.

Given.

Parcing project for 062507

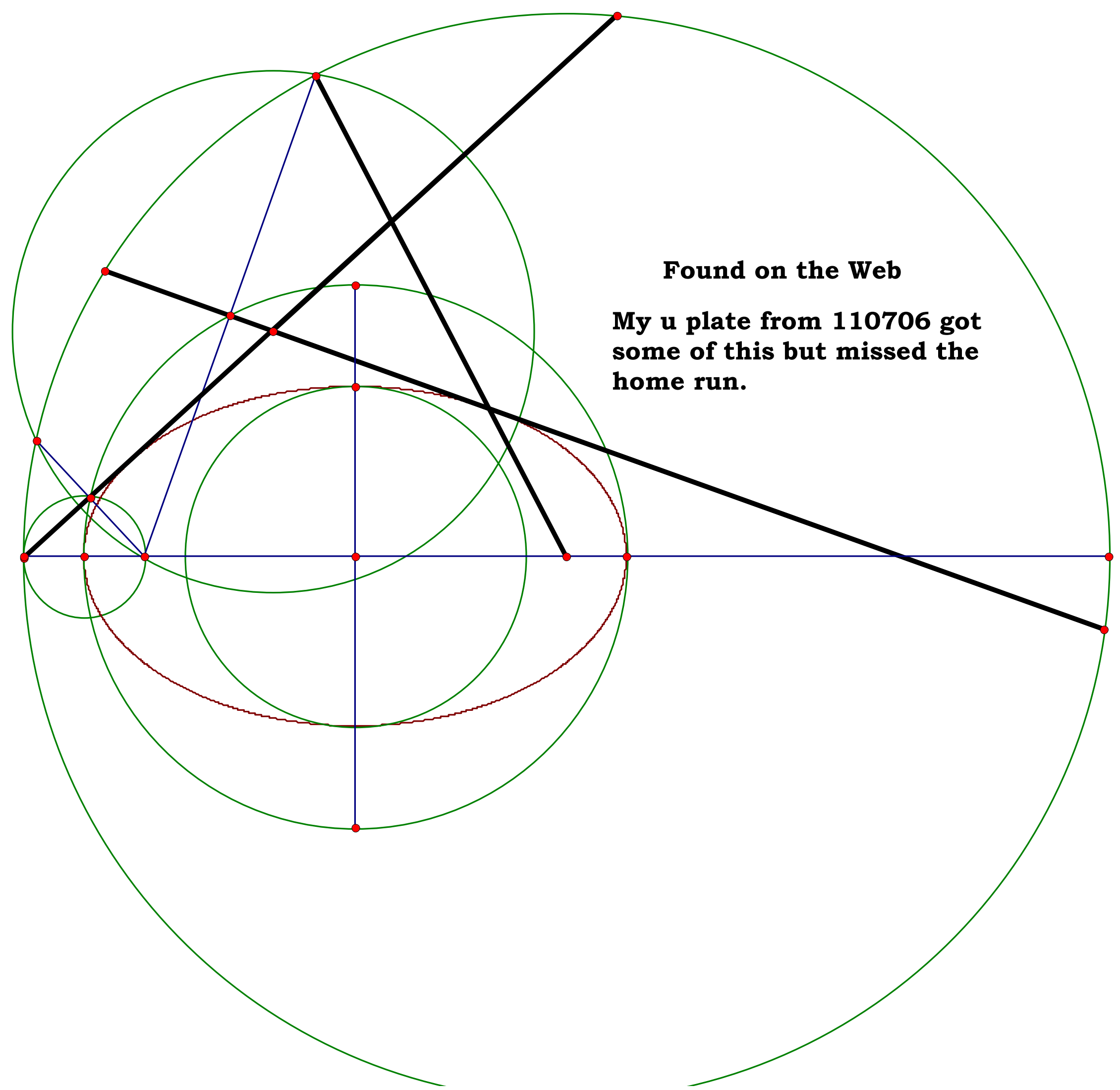
This is the second plate on the web I found for tangents, and it is quite good.

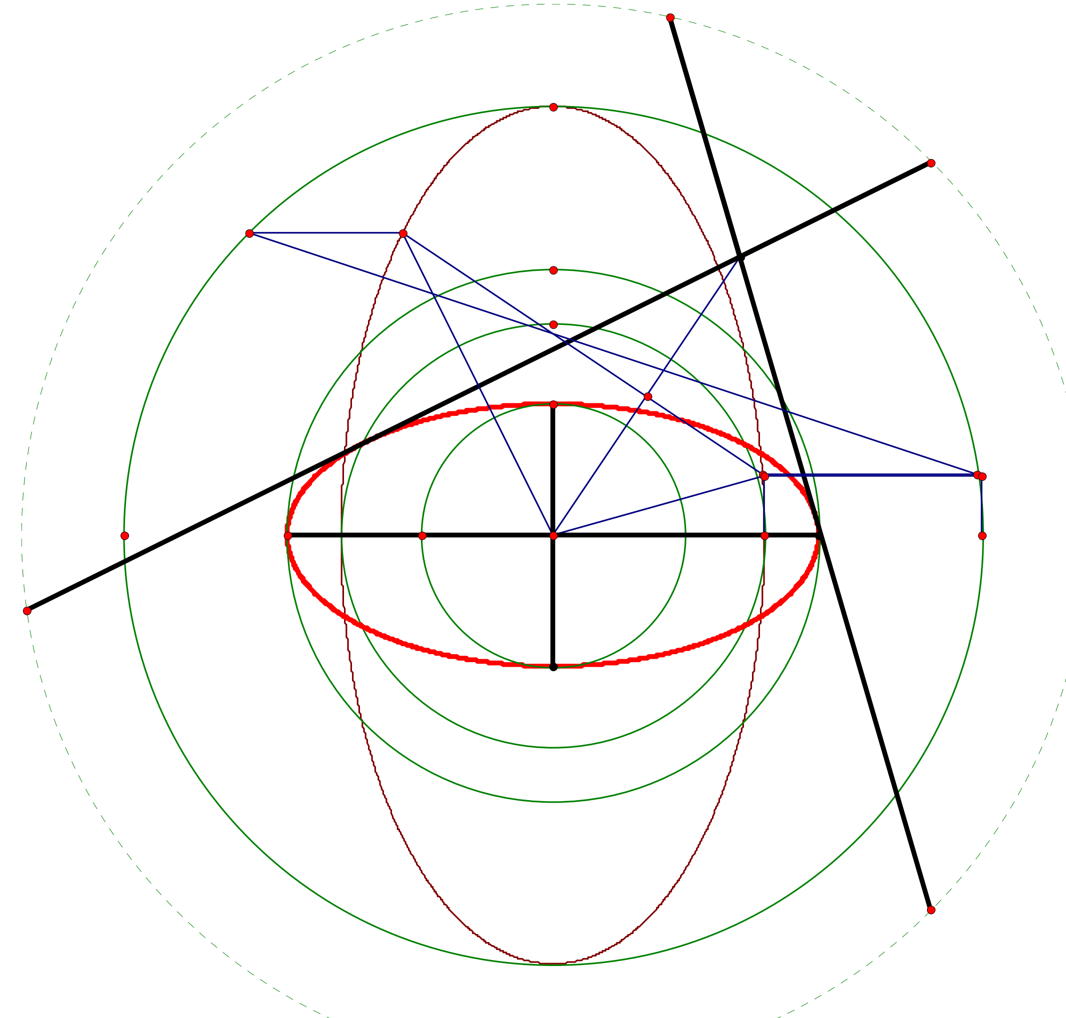
I did find a cleaner plate for 110706 here

I call it the inverse ellipse method. I should at least example the stps in construction.

Descriptions.

Definitions.





110706b

Unit.
Given.

Parcing project for 072707

Descriptions.
Definitions.

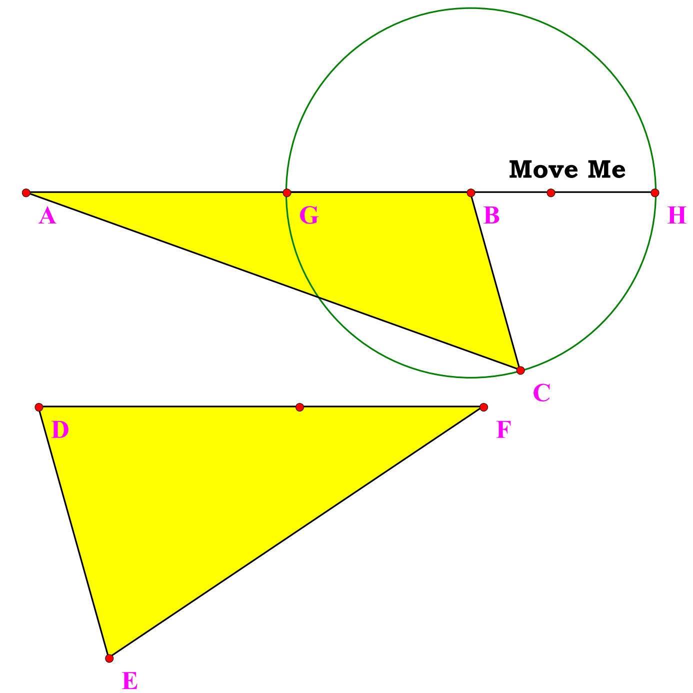
Area $\triangle ABC = 6.89734\ cm^2$
Area $\triangle DEF = 9.76888\ cm^2$

$$\frac{(\text{Area } \triangle DEF)}{(\text{Area } \triangle ABC)} = 1.41633$$

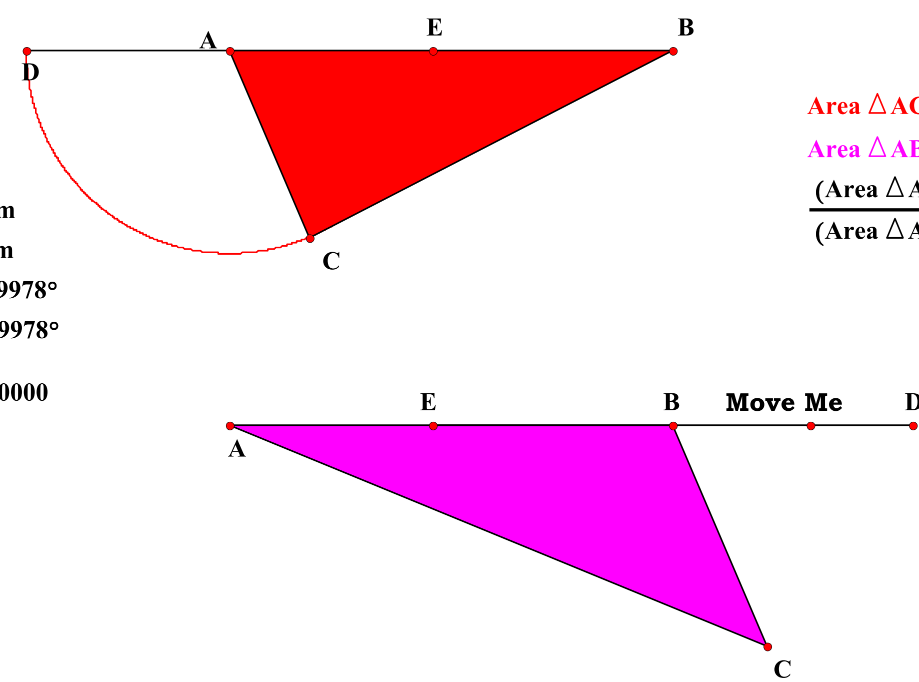
$AB = 5.88433$ cm
$BG = 2.43524$ cm

$$\frac{AB}{BG} = 2.41633$$

$m\angle FDE = 74.29322°$
$m\angle HBC = 74.29322°$



A series of plates exploring the relationship between an angle common to two figures which maintain a constant ratio in area.



$AB = 5.86317$ cm
$BE = 3.17743$ cm
$m\angle BAC = 66.89978°$
$m\angle DBC = 66.89978°$

$$\frac{m\angle BAC}{m\angle DBC} = 1.00000$$

Area $\triangle ACB = 7.24215\ cm^2$
Area $\triangle ABC = 8.56804\ cm^2$

$$\frac{(\text{Area } \triangle ACB)}{(\text{Area } \triangle ABC)} = 0.84525$$

My Name is John.

Hello. My name is John and I am going to explain how to multiply and divide a line by a line in Geometry. Now, if you are going to ask me if I am a geometer, I have to reply by myth. Explanation by myth is one the ancient Greek's methods of teaching by discourse.

Once upon a time, God created man; They created him male and female, in the image of God. Or one can say, male and female created They him, which is rather awkward, but it does have that ancient New England flair to it. At any rate, once upon a time is not this time. It came to pass as men multiplied on the earth that men started to work for a living and not being god's themselves needed a way to designate each other and so individuals, which are not, by definition man started calling each other by their craft. That is where we got Mr. Smith and Mr. Clark, etc. A vestige of this remains today. Not being man, we tend to think of each other by an assigned craft. I work in a factory, but my name is Clark. The conflict of course is why I spend the entirety of my wages in therapy.

Now this works to my advantage. I have learned that individuals calling themselves geometers (I am personally hoping for the day I become part of man) cannot multiply and divide a line by a line. So, I guess one could say, that a geometer is someone who cannot do the math, which is really a sign for some serious expenditure on therapy—and, if those in mind—field knew what they were doing, the outlay would be advantageous. Too bad they cannot define a man. Now a non-Euclidean Geometer is someone who not only cannot do the math, they demand, as part of their initiation rights, that one will never be able to do the math. So, in due respect to non-Euclidean Geometers, please stop reading and go back to your scribbling—and contradicting yourself. Doing geometry inside of or on the outside of a tennis ball, or a Frisbee, makes me think that one has spent way to many days on the court, spiking one's tea, and certainly missing the ball.

Now, if your like me, a factory worker, and someone were to give you two lines and say,

Hey, you (He is hairy and has a club). Here are two lines, show me how to multiply one by the other, and after that, show me how to divide one by the other.

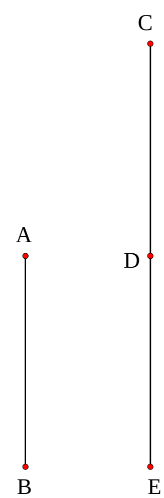
I would look at the man, think for a moment and draw a blank. What the heck does he mean? Then I would say, I am sorry, but I don't understand

what you mean. The man would leave off and I would go get another cup of coffee.

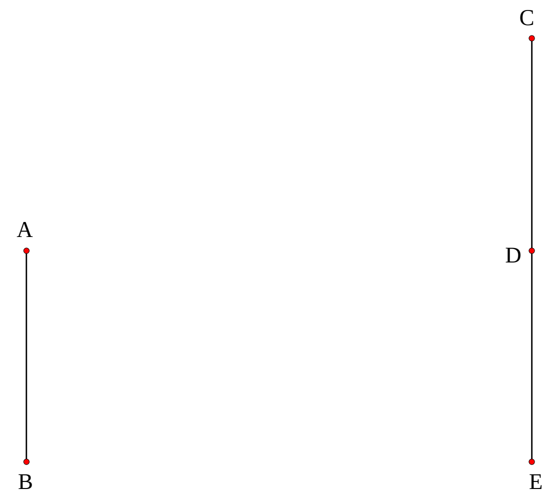
If I were a bit strange, I would consult Euclid's ***Elements*** and find to my dismay, the chap could do the math, but seems to have left this off for some reason, probably because it was too easy (so who don't lie for a friend?). Now, I happen to have in my possession a number of unpublished manuscripts which does have the answer in them and they are full of doing the math. I acquired them from the God's (and for those of you interested, the Delian Problem does have a solution—and it has something to do with Plato under extending himself). If it should be discovered that I am stealing a bit of fire, and giving it to man, please don't tell where you got it from. I have learned from first hand experience, you don't want to mess with Them—they be giants—really, really, big giants.

Now I am not going to explain this exactly as it was explained to me, as I have a poor memory. Please bear with me.

If I were given two lines, and asked to compare them, I would look at them and say;

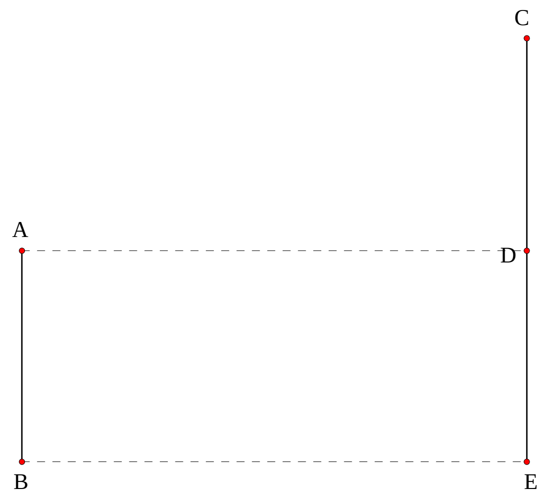


well, AB is shorter than CE. I mean, what can you do with two lines anyway. Reminds me of when I was a kid asking my mother what could I do with seven cents, realizing early on I was three cents short of a dime. If I were Euclid I would subtract one from the other and find that CE — AB = CD, or if you're a top down programmer, CE — AB = DE. If I move CE off a ways,

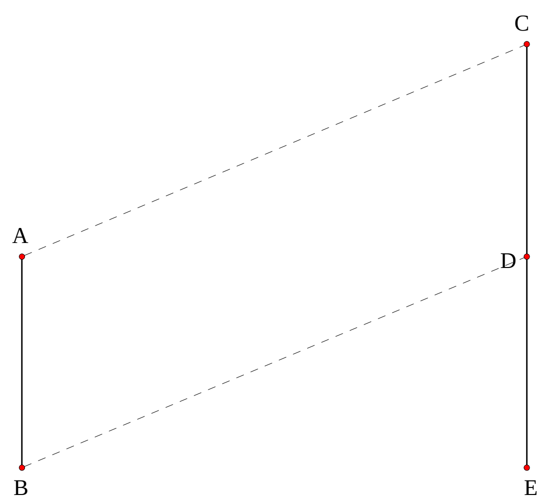


I would say that CE — AB = CD, or DE which ever you choose. Non-Euclidean Geometers, like Einstein, claim that this equality, this simultaneity, is not true and that at some point of moving AB and CE apart, as if it were part of the equation, does mysterious things to these segments. It amounts to a thief's logic—moving CE off sufficiently will make AB infinitely greater than CE 'cause we exact a kind of tribute on it and subtract that tribute as we go. It amounts to constructing a square say, of 25 square inches or so, and claiming if we repeat it enough, well, it just plain disappears—we wore it out. While on the other hand, there are those who claim that if I assert a point an infinite number of times, I can create a line. You know, like waving a knife in the air an infinite number of times an making a salad. This is the kind of mentality that makes credit card lenders rich. As I said, non-Euclidean Geometers are really crooked bankers in disguise—or really lousy cooks. A basic fact of abstraction, when you really know that a boundary is not the difference (a point is that which has not part), a form is in fact absolute, you know you can never attribute difference to that form, the form is applied as a boundary to any given difference—material. The cut is not the cutted! Wow, that was trashy!

Now if I had AB, and wanted to construct CE from it.



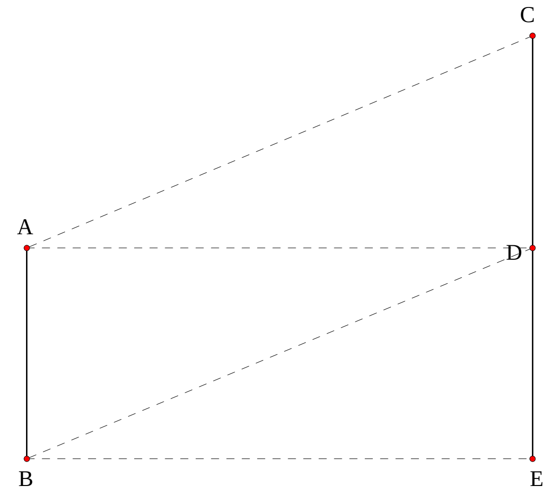
I could transfer one segment at a time



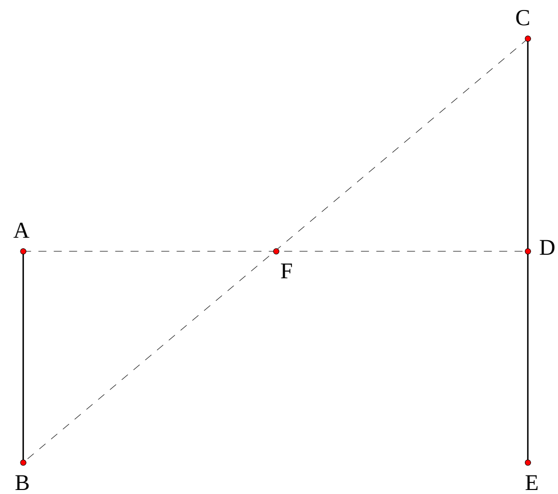
using parallel lines, but this is not multiplication, it is multiple processes, or simply addition. Parallel lines gives us the ability to do multiple additions, which is again not multiplication. One sign of that is that we have to assert each unit point in constructing CE. We have to assert each unit point just to do the parallels. Duh!

One of the things our ancient quibbling buddies, the Greeks, did tell us is that in order to multiply and divide, we have to have a unit. This is just part of plain simple Arithmetic. And they also said that when dealing with numbers in multiplication and division we were dealing with square and oblong (rectangular) numbers. Keep these ideas in mind. A square, an oblong, and a unit. Euclid drew a number of them. We will have need of them. For the moment let us learn what they did say about ratio,

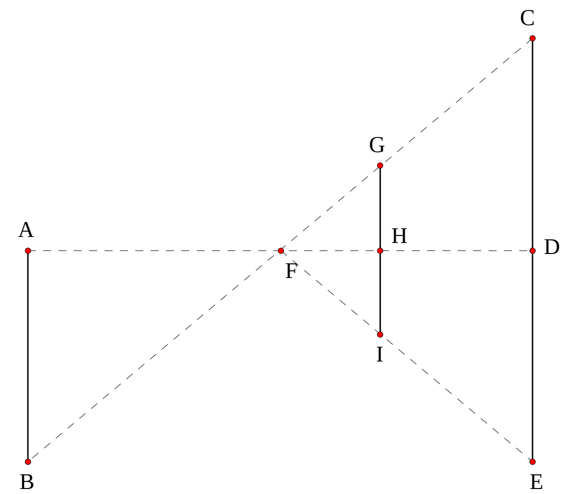
which we will also need. Now, if in constructing CE, we stayed up too late;—



and made a mistake in drawing—or were simply dyslexic;



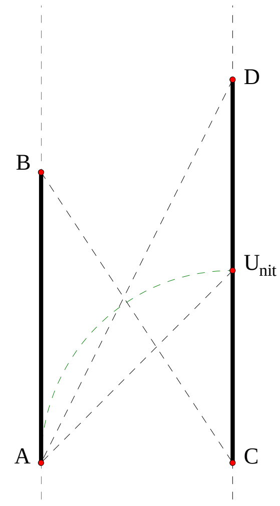
we would discover the ratio. As AB is to CD, so AF is to DF. And by George—(if you remember, he too was a hairy fellow and curious), One learns how to take any multiple and divide another segment of any length by the same multiple. From multiple addition, we have a kind of multiple division, but it is not division, it is still just a plain ratio, of anther segment.



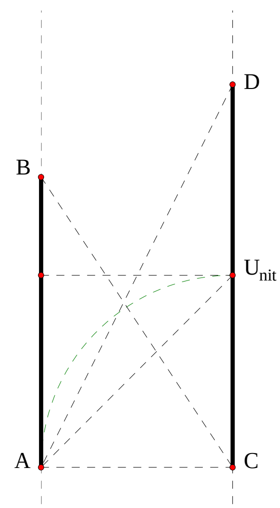
Now, as AB is to GH, so to DE is to HI, etc., etc. This is all fine and good, but, we still have not really learned how to multiply and divide. That is because these ratio's work regardless of the notion of unit, or square. Unless you are a crooked banker or a non-Euclidean Geometer, or a bad cook, this relationship is always true. There is one, and only one, difference between two points.

We are building our ideas up, one standard at a time. Intellectually, we fail, at the point we cannot abstract and use a standard—or what Plato called ***form*** because a boundary is not a difference and by definition (not a difference) always true. The divergence of language itself, starts with the inability to establish a standard even for a name. Many linguists call it the "growth" of language when meaning changes, but then they are non-Euclidean Geometers at heart also. What do they say of a government that has got its constitution saying exactly the opposite of what is written? If you want to reduce them to rubble, ask them outright, *Why can one word be or not be predicated of another*? Or again, if definition is conventional, and meaning can never be conventional, what in the heck does meaning have to do with definition? or even language? They will either get a funny look on their face mumbling to themselves, or start babbling non-sense to you. I have some books by the gods on that topic also. It is really simple, . . . but not here, not now.

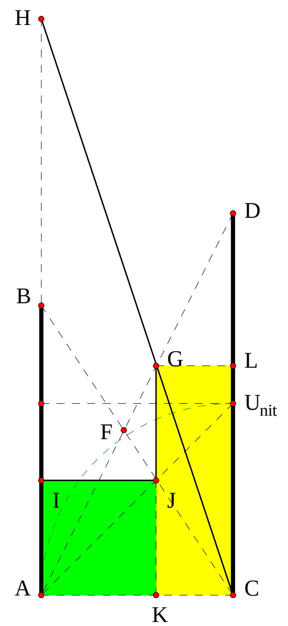
Multiplication and division rely on a standard in unit. So lets add that and see where we go.



At the outset the figure is very shy and unassuming. If you saw it laying in the street, you would hardly be pressed to pick it up. We have placed our segments the difference of our chosen unit apart, and we do have a square. No offence to Descartes who tried to find what I am doing, we don't have a number line, but a lined number. First time I ever seen a studious use of cross hairs actually miss the target.

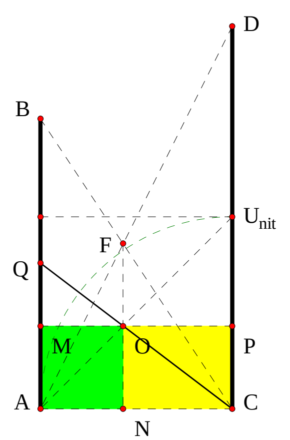


It don't look like much, but it can not only multiply and divide, one can use it to do much in the way of exponential manipulation as well. Let us take a closer look as to what the figure tells us.



$$\frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} - \frac{(\text{Area IJKA})}{(\text{Area KGLC})} = 0.00000$$

This is how we perform multiplication. Given AC as our unit, AB × CD = AH. In order to see this using the Arithmetic Grammar system, We divide AC by AC and get 1, our Unit. We then divide AB by AC which gives AB in terms of our unit. We then divide CE by AC and acquire that in units, and again for AH. We will find that by using the notion of Unit, Square and Oblong Numbers, which is incorporated in the idea of ratio, we can Multiply. And we can do what no binary calculator will ever do, we do it exactly. What about division?



$$\frac{(\text{Area MANO})}{(\text{Area ONCP})} - \frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} = 0.00000$$

Wouldn't you know it, there is a triplicate ratio in the figure! Right under our pencil. Didn't Euclid write that it was the hardest thing to do in geometry? Well, I have never taken geometry in school and set out to comprehend the triplicate ratio, guess I got somewhere. Going through our steps as before, we find that AB ÷ CD = AQ. Each of these steps is proven individually in Euclid. I suspect he was like Plato and wanted to see if his readers were smart enough to add and subtract ideas. And again, no binary computer will ever be up to Geometry, as Geometry is exact.

One can do a whole lot with this figure, through various projections. One can do a lot in the way of exponential manipulation. Try that with cross hairs! Some of the methods one will find in those unpublished books I was talking about. I don't know how long the gods will let me work on them, in fact, if it were not for Them, I would have been killed over thirty years ago. Imagine that, I am a walking contradiction, a living dead man. At any rate, I hope you have fun playing with the figure.

Now this is not the place to show the solution to the Delian Problem. My god, if one is just learning the simple four, by adding multiplication and division to our list of addition and subtraction, it may be too difficult realize a revolution in Euclidean Geometry based upon a standard long ago recognized but left unemployed—just like these. I will put the idea in the Geometer's Sketchpad file.

I hope I have made it clear that through multiple addition and subtraction, one leads into the understanding of ratio, just like Euclid did, but it is still a step away from multiplication and division. Those depend upon a respect for, and understanding of a standard in definition. We learn to add, and subtract. These teach us ratio—it is part of them. We learn about the units which is taught by them also. This then leads to multiplication and division and our primary four are thus established.

I do have some food for thought though. Using the facts of conventions in language, can you count the ways non-Euclidean geometries commit self-referential errors in simple logic? Apparently not, they are popular. Maybe it has something to do with linguist waving their knife in the air constructing sentences. What is prediction? Maybe I will read it to you sometime. The solution was once written on a Temple "Know Thyself." I will say this, as a sense system, the human mind is suppose to abstract form and create things with it. To deny form as the foundation for thought is simply a sign of dysfunction. I know, look at me.

Multiplication And Division of Lines

1. An unit is that by virtue of which each of the things that exist is called one.
Euclid's Elements

312

The Basic figures in this little thing are written up in my work Threee Pieces of Paper, or The Delian Quest. This is not a formal presentation, is a presentation of craft basics.

John Clark

Printed via import to MS Word.

Following the Yellow Brick Road

Multiplication

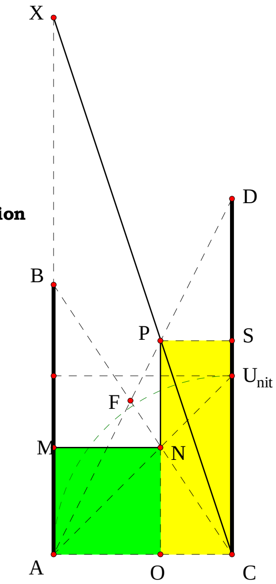
Unit = 1.00000

AB = 1.51095

CD = 1.99270

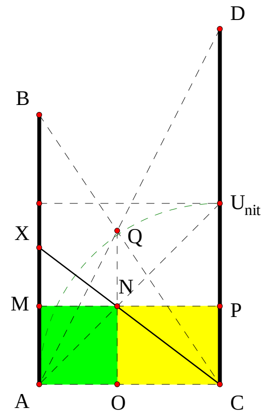
AX = 3.01087

AB·CD-AX = 0.00000



$$\frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} - \frac{(\text{Area MNOA})}{(\text{Area OPSC})} = 0.00000$$

In Ancient Geometry, one calls the Green Square Numbers, and one calls the yellow Oblong Numbers. Today we just do swatches.



Division

Unit = 1.00000

AB = 1.48921

CD = 1.96403

AX = 0.75824

$$\frac{AB}{CD} - AX = 0.00000$$

$$\frac{(\text{Area MAON})}{(\text{Area NOCP})} - \frac{(\text{Area } \triangle BAC)}{(\text{Area } \triangle ADC)} = 0.00000$$

Introduction

Maybe I am too dogmatic, but I think one should have geometry teach one something of basic math. One should be able to add, subtract, multiply and divide with lines. These can provide proofs and constructible.

The figures can be modified in various ways to produce various results. I present a few here. The main figure is composed of the notion of common unit, and that multiplication and division works with square numbers, which is distinct from squaring a number. The square thus constructed provides the properties needed for multiplication and division.

I once read, in an Algebra book, that exponential notation had nothing to do with Geometry, that it was a pure mental abstract. What am I, then, to do with all the figures I have come up with that display the principles?

I would also like to see how the four basic operations of Math hold up in "non-Euclidean" Geometries. In fact, as part of their presentation, I think the four basic operations of mathematics should be a requirement. Perhaps by teaching the remaining two in geometry, something about reality and standards of thought will be learned.

The material in this little flyer is not new to me, it is part of four works I am currently engaged in, The Delian Quest, which is essentially completed, it needs some lipstick and a dress, Three Pieces of Paper, Eloi, and something with a puny Latin name.

Oh, and no, I have never studied geometry in an institution-I have never seen ideas survive in an institution. I have and probably will be again, be institutionalized at my own request.

Contents

Link to Introdution

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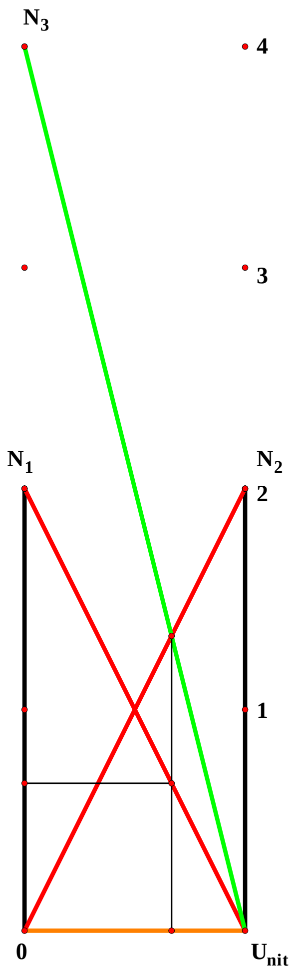


$N_1 = 2.00000$

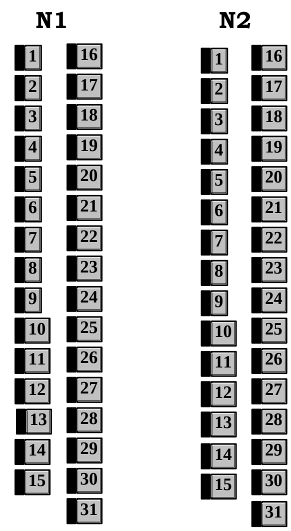
$N_2 = 2.00000$

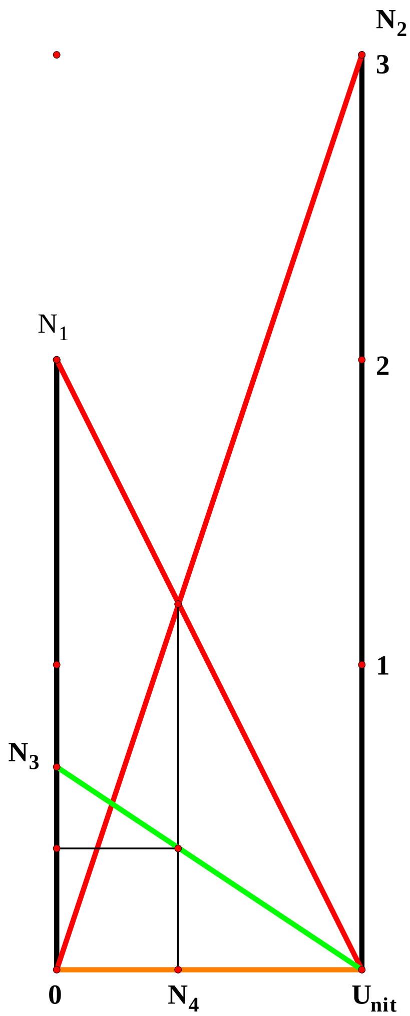
$N_1 \cdot N_2 = 4.00000$

$(N_1 \cdot N_2) - N_3 = 0.00000$

$N_3 = 4.00000$

Multiply N1 by N2





$N_1 = 2.00000$

$N_2 = 3.00000$

$$\frac{N_1}{N_2} = 0.66667$$

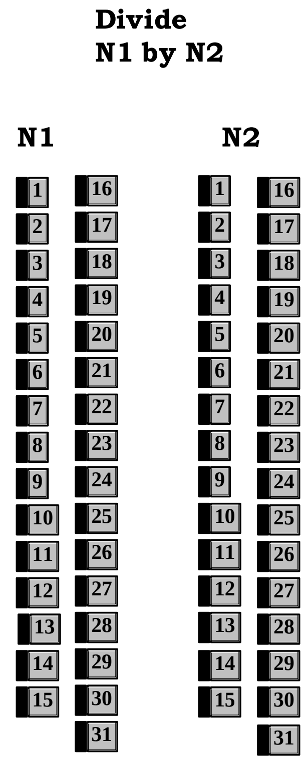
$N_3 = 0.66667$

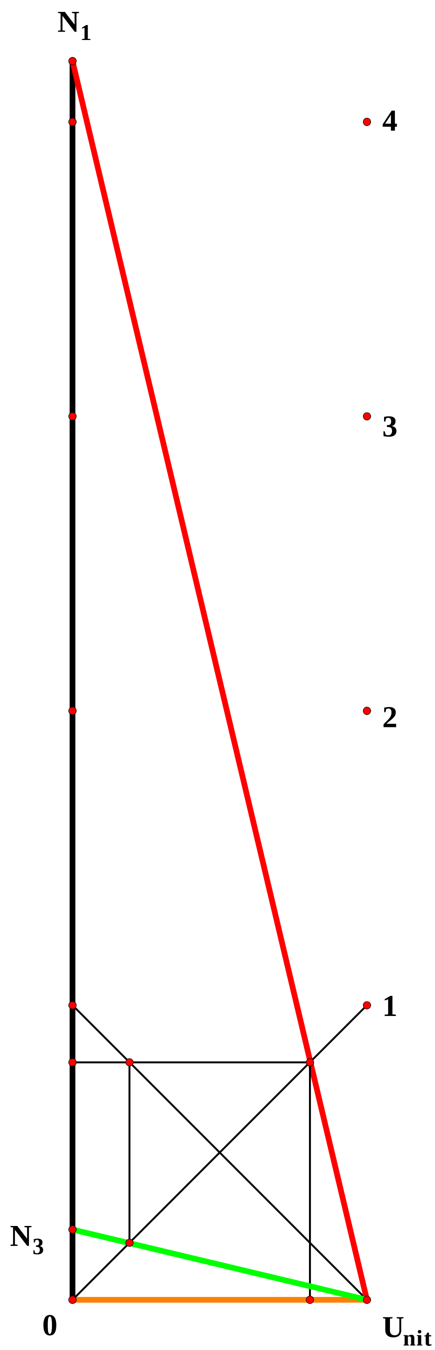
$$\frac{U_{nit}}{0N_4} = 2.50000$$

$$\frac{N_1+N_2}{N_1} = 2.50000$$

$$\frac{U_{nit}}{U_{nit}N_4} = 1.66667$$

$$\frac{N_1+N_2}{N_2} = 1.66667$$





Find the Recprocal N1

$N_1 = 4.20930$

$\frac{1}{N_1} = 0.23757$

$N_3 = 0.23757$

$\frac{1}{N_1} - N_3 = 0.00000$

N1

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

$N_1 = 3.00000$

$N_2 = 2.00000$

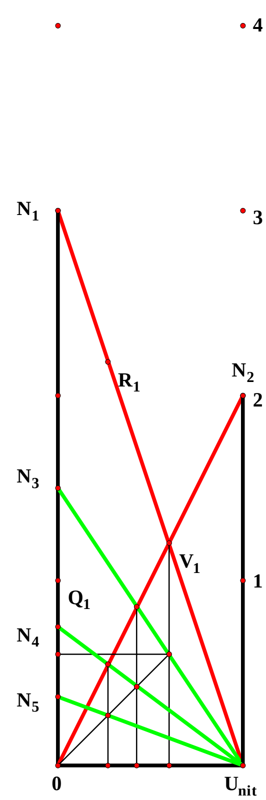
$\dfrac{N_1}{N_2} = 1.50000$

$N_3 = 1.50000$

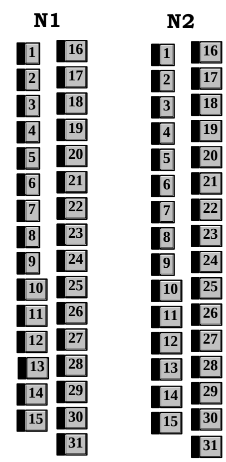
$N_4 = 0.75000 \qquad \dfrac{N_1}{N_2^{\,2}} - N_4 = 0.00000$

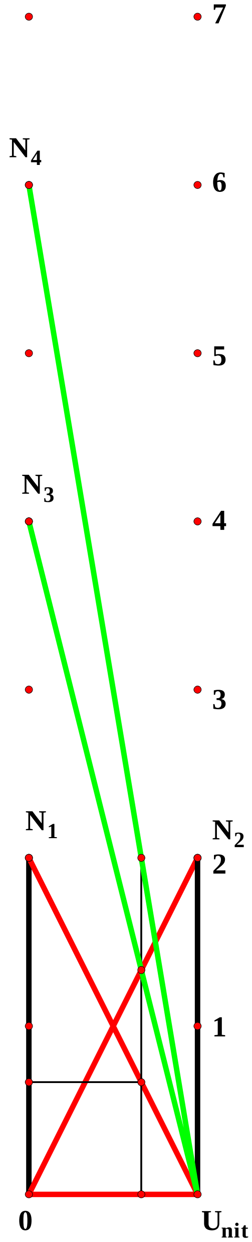
$N_5 = 0.37500 \qquad \dfrac{N_1}{N_2^{\,3}} - N_5 = 0.00000$

etc.



Divide N1 by a Power of N2





$N_1 = 2.00000$

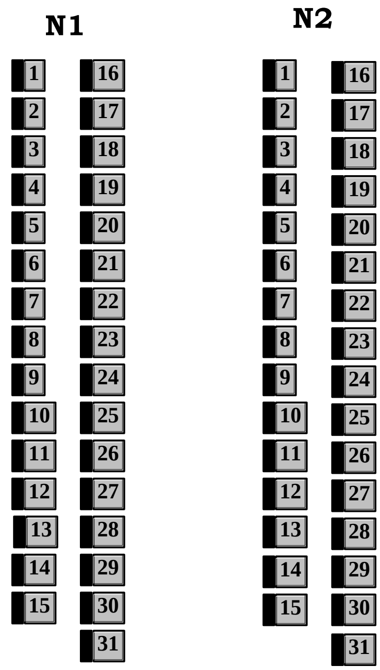
$N_2 = 2.00000$

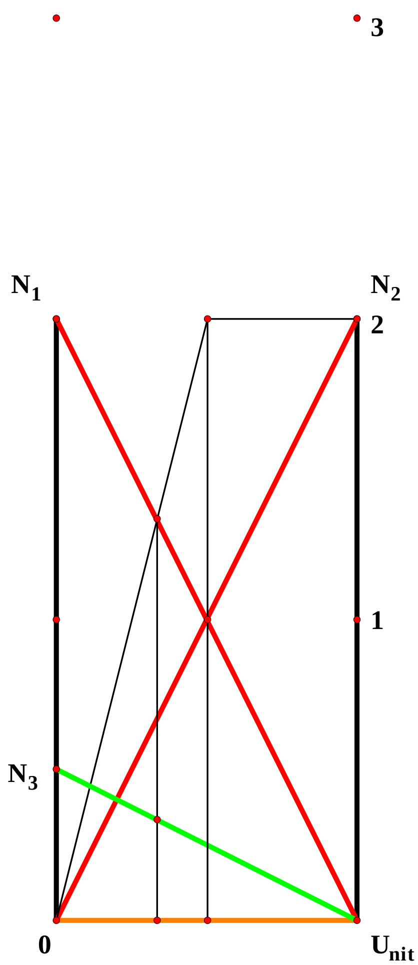
$N_1 \cdot N_2 = 4.00000$

$N_3 = 4.00000$

$N_4 = 6.00000$

$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$



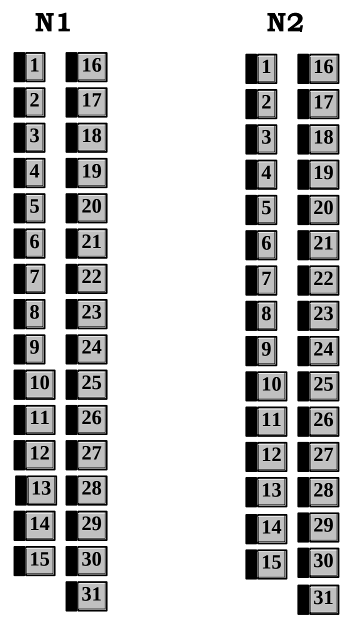


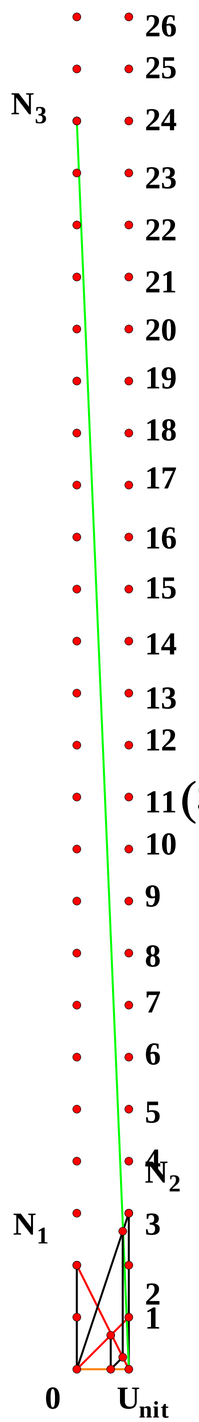
$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 0.50000$

$$\frac{N_1^2}{(N_2+N_1)\cdot N_2} - N_3 = 0.00000$$



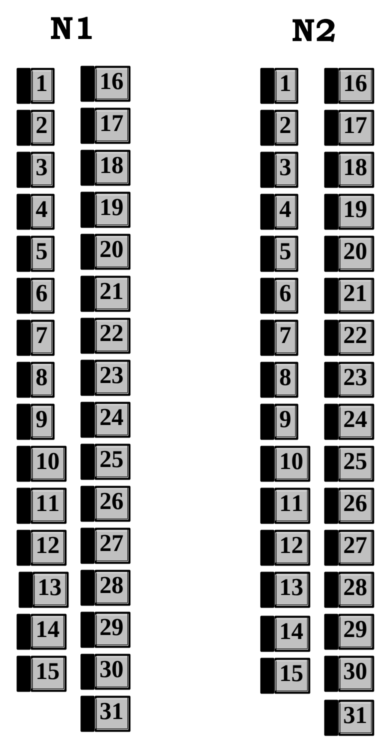


$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 24.00000$

$(2\cdot N_1\cdot N_2+N_1^2\cdot N_2)-N_3 = 0.00000$



$\frac{BC}{BV_1} = 0.81650$ $N_3^{-0.5} = 0.81650$

$N_3^{0} = 1.00000$

$\frac{BC}{BU_1} = 1.22474$ $N_3^{0.5} = 1.22474$

$\frac{BC}{BP_1} = 1.50000$

$\frac{BC}{BR_1} = 2.25000$ $N_3^{2} - \frac{BC}{BR_1} = 0.00000$

$\frac{BC}{BS_1} = 3.37500$ $N_3^{3} - \frac{BC}{BS_1} = 0.00000$

$\frac{BC}{BT_1} = 5.06250$ $N_3^{4} - \frac{BC}{BT_1} = 0.00000$

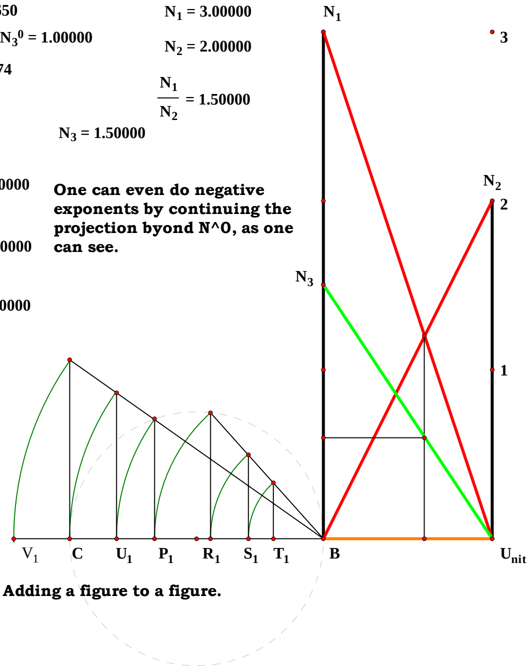
$N_1 = 3.00000$

$N_2 = 2.00000$

$\frac{N_1}{N_2} = 1.50000$

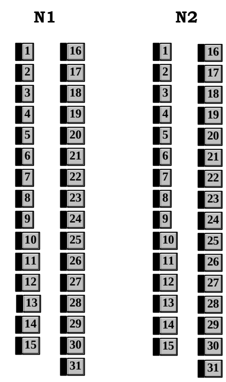
$N_3 = 1.50000$

One can even do negative exponents by continuing the projection byond N^0, as one can see.



Adding a figure to a figure.

I tossed this together, so it is not perfect--it just looks good



How to find a Proportional Unit for a given pair of Magnitudes

$\mathbf{Unit_2 = 1.00000}$

$\mathbf{N1_2 = 1.39420}$

$\mathbf{N2_2 = 1.94380}$

$\mathbf{N3_2 = 0.71726}$

$\mathbf{N4_2 = 2.71004}$

$\mathbf{\dfrac{N1_2}{N2_2} - N3_2 = 0.00000}$

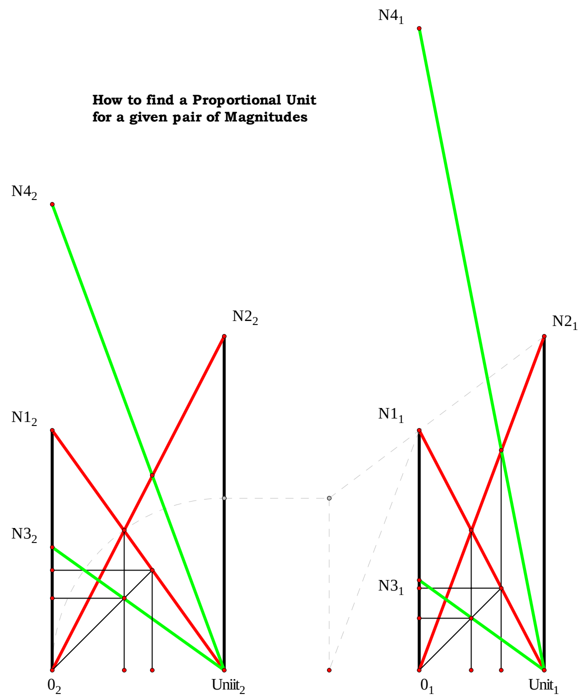
$\mathbf{N1_2 \cdot N2_2 - N4_2 = 0.00000}$

$\mathbf{\dfrac{N4_2}{N2_2} = 1.39420}$

$\mathbf{\dfrac{N2_2}{N1_2} = 1.39420}$

$\mathbf{\dfrac{N1_2}{Unit_2} = 1.39420}$

$\mathbf{\dfrac{Unit_2}{N3_2} = 1.39420}$



$\mathbf{Unit_1 = 1.00000}$

$\mathbf{N1_1 = 1.91861}$

$\mathbf{N2_1 = 2.67492}$

$\mathbf{N3_1 = 0.71726}$

$\mathbf{N4_1 = 5.13213}$

$\mathbf{\dfrac{N1_1}{N2_1} - N3_1 = 0.00000}$

$\mathbf{N1_1 \cdot N2_1 - N4_1 = 0.00000}$

$\mathbf{\dfrac{N4_1}{N2_1} = 1.91861}$

$\mathbf{\dfrac{N2_1}{N1_1} = 1.39420}$

$\mathbf{\dfrac{N1_1}{Unit_1} = 1.91861}$

$\mathbf{\dfrac{Unit_1}{N3_1} = 1.39420}$

Unit = 1.00000

$\mathbf{N_1 = 2.00000}$

$\mathbf{N_2 = 3.00000}$ $\qquad \mathbf{\frac{N_1}{N_2} - N_3 = 0.00000}$

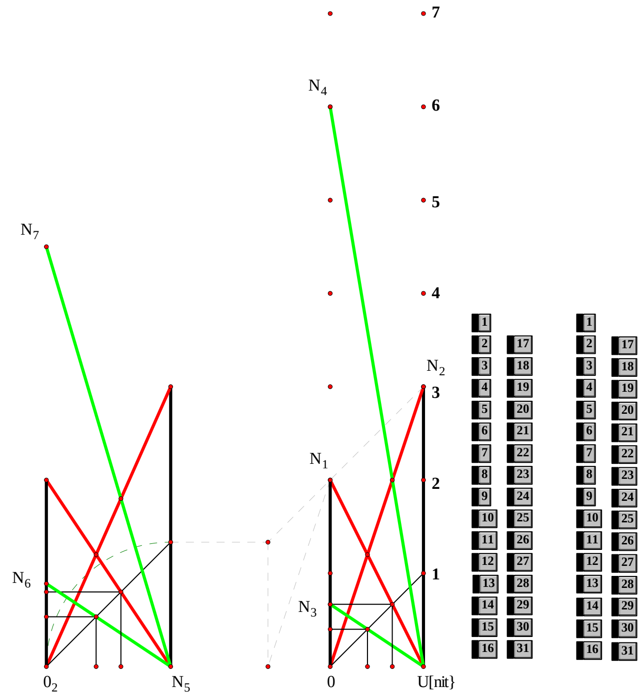
$\mathbf{N_3 = 0.66667}$

$\mathbf{N_4 = 6.00000}$ $\qquad \mathbf{N_1 \cdot N_2 - N_4 = 0.00000}$

$\mathbf{N_5 = 1.33333}$ $\qquad \mathbf{\frac{N_1^2}{N_2} - N_5 = 0.00000}$

$\mathbf{N_6 = 0.88889}$ $\qquad \mathbf{\frac{N_1^3}{N_2^2} - N_6 = 0.00000}$

$\mathbf{N_7 = 4.50000}$ $\qquad \mathbf{\frac{N_2^2}{N_1} - N_7 = 0.00000}$



$\text{Unit} = 1.00000$

$N_1 = 2.00000$

$N_2 = 2.30952$

$N_3 = 0.86598$

$N_4 = 4.61905$

$\frac{N_1}{N_2} - N_3 = 0.00000$

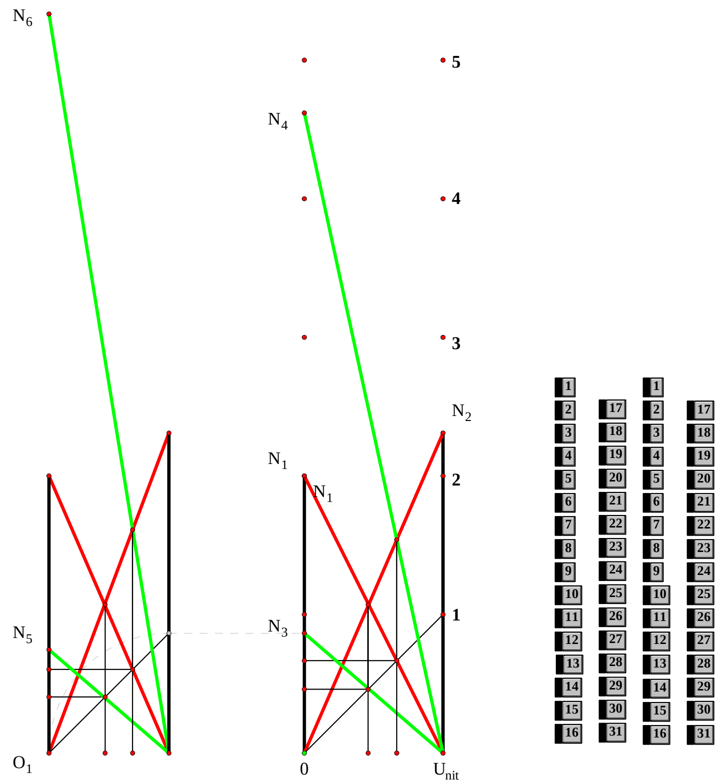
$N_1 \cdot N_2 - N_4 = 0.00000$

$N_5 = 0.74992$

$\frac{N_1^2}{N_2^2} - N_5 = 0.00000$

$N_6 = 5.33390$

$N_2^2 - N_6 = 0.00000$



Unit = 1.00000

$\mathbf{N_1 = 2.00000}$

$\mathbf{N_2 = 2.43939}$

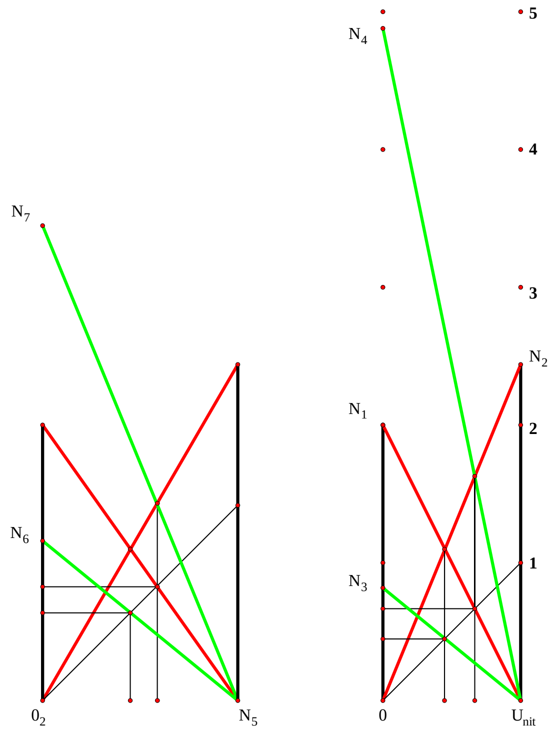
$\mathbf{N_3 = 0.81988}$ $\qquad \mathbf{\frac{N_1}{N_2} - N_3 = 0.00000}$

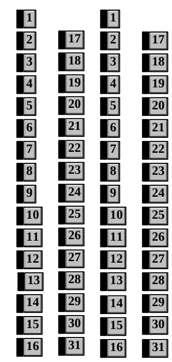
$\mathbf{N_4 = 4.87879}$ $\qquad \mathbf{N_1 \cdot N_2 - N_4 = 0.00000}$

$\mathbf{N_5 = 1.41421}$ $\qquad \mathbf{\sqrt{2} - N_5 = 0.00000}$

$\mathbf{N_6 = 1.15948}$ $\qquad \mathbf{\frac{\sqrt{2} \cdot N_1}{N_2} - N_6 = 0.00000}$

$\mathbf{N_7 = 3.44982}$ $\qquad \mathbf{\frac{N_1 \cdot N_2}{\sqrt{2}} - N_7 = 0.00000}$





Unit = 1.00000

$N_1 = 2.00000$

$N_2 = 2.00000 \qquad \frac{N_1}{N_2} - N_3 = 0.00000$

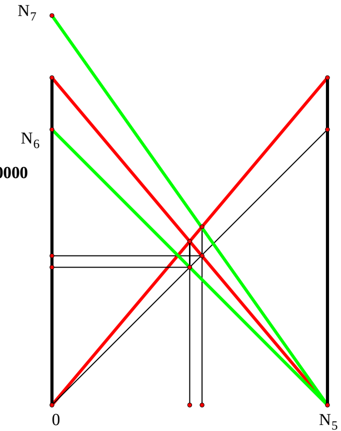
$N_3 = 1.00000$

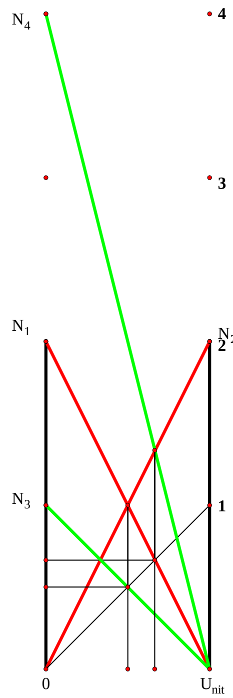
$N_4 = 4.00000 \qquad N_1 \cdot N_2 - N_4 = 0.00000$

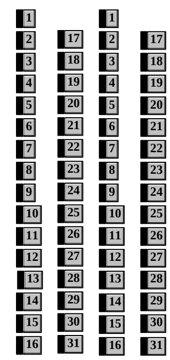
$N_5 = 1.68179 \qquad N_5 - 2^{0.75} = 0.00000$

$N_6 = 1.68179 \qquad \left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$

$N_7 = 2.37841 \qquad \frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$







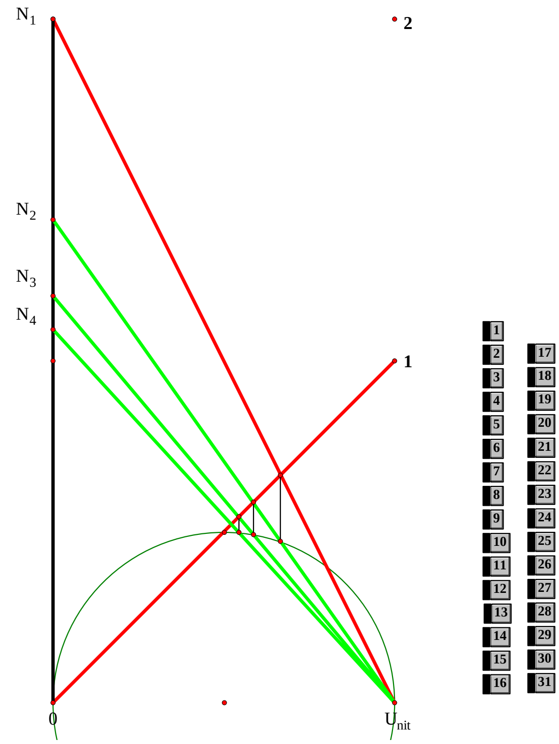
Unit = 1.00000

N_1 = 2.00000

N_2 = 1.41421 **$N_1^{0.5}$-N_2 = 0.00000**

N_3 = 1.18921 **$N_1^{0.25}$-N_3 = 0.00000**

N_4 = 1.09051 **$N_1^{0.125}$-N_4 = 0.00000**



Unit = 1.00000

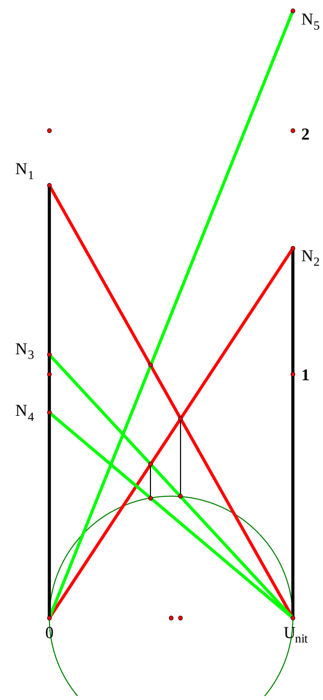
$N_1 = 1.77542$

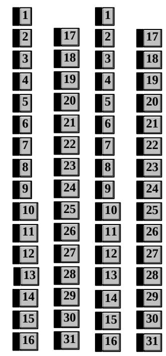
$N_2 = 1.51695$

$N_3 = 1.08185 \qquad \dfrac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$

$N_4 = 0.84450 \qquad \dfrac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$

$N_5 = 2.48947 \qquad N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$





Unit = 1.00000

$\mathbf{N_1 = 1.00000}$

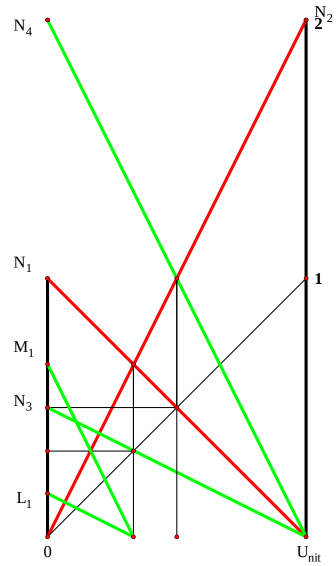
$\mathbf{N_2 = 2.00000}$ $\quad \dfrac{N_1}{N_2} - N_3 = 0.00000$

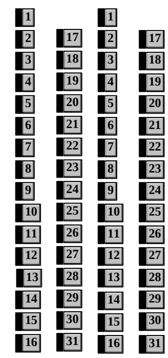
$\mathbf{N_3 = 0.50000}$

$\mathbf{N_4 = 2.00000}$ $\quad N_1 \cdot N_2 - N_4 = 0.00000$

$\mathbf{L_1 = 0.16667}$ $\quad \dfrac{N_1^2}{\left(N_1 + N_2\right) \cdot N_2} - L_1 = 0.00000$

$\mathbf{M_1 = 0.66667}$ $\quad \dfrac{N_1^2 \cdot N_2}{N_1 + N_2} - M_1 = 0.00000$





$\mathbf{Unit = 1.00000}$

$\mathbf{N_1 = 2.00000}$

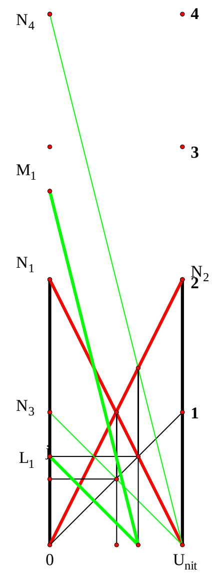
$\mathbf{N_2 = 2.00000}$ $\quad \mathbf{\frac{N_1}{N_2} - N_3 = 0.00000}$

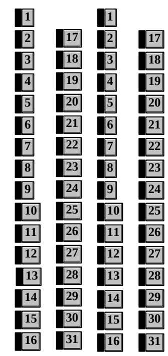
$\mathbf{N_3 = 1.00000}$

$\mathbf{N_4 = 4.00000}$ $\quad \mathbf{N_1 \cdot N_2 - N_4 = 0.00000}$

$\mathbf{L_1 = 0.66667}$ $\quad \mathbf{\frac{N_1^2}{N_2 \cdot (N_1 + 1)} - L_1 = 0.00000}$

$\mathbf{M_1 = 2.66667}$ $\quad \mathbf{\frac{N_1^2 \cdot N_2}{N_1 + 1} - M_1 = 0.00000}$





$\mathbf{U_1 = 1.00000}$

$\mathbf{N_1 = 2.00000}$

$\mathbf{N_2 = 2.00000}$

$\mathbf{N_3 = 1.00000}$ $\quad \mathbf{\frac{N_1}{N_2} = 1.00000}$

$\mathbf{N_4 = 4.00000}$ $\quad \mathbf{N_1 \cdot N_2 - N_4 = 0.00000}$

$\mathbf{U[1] / U_2 = 0.25000}$

$\mathbf{N_5 = 2.00000}$

$\mathbf{N_6 = 2.00000}$

$\mathbf{N_7 = 0.25000}$ $\quad \mathbf{\frac{N_1^2}{N_2^4} - N_7 = 0.00000}$

$\mathbf{N_8 = 16.00000}$ $\quad \mathbf{N_2^4 - N_8 = 0.00000}$

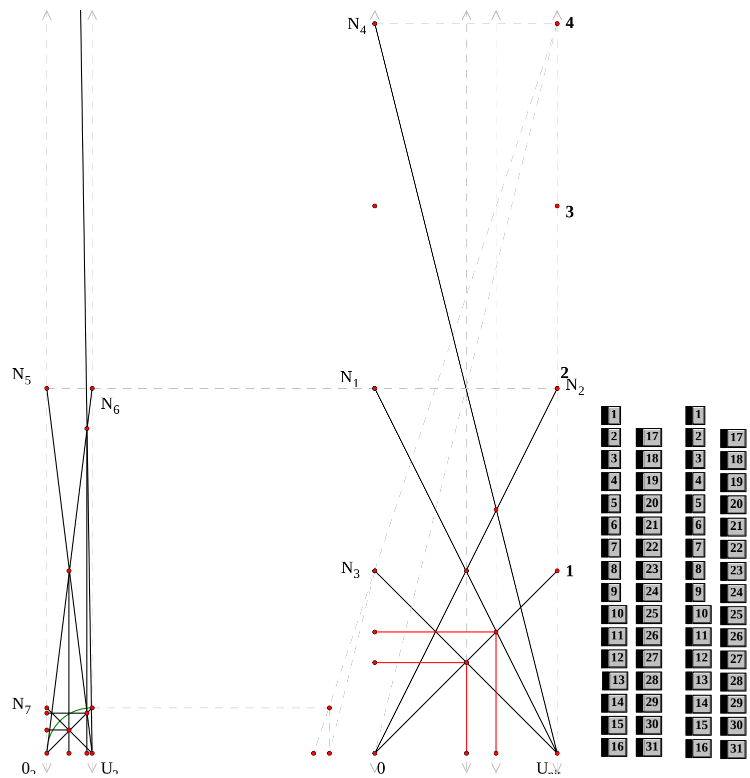
$\mathbf{U_2 = 1.00000}$

$\mathbf{N_{25} = 8.00000}$ $\quad \mathbf{N_2^3 - N_{25} = 0.00000}$

$\mathbf{N_{26} = 8.00000}$ $\quad \mathbf{\frac{N_2^4}{N_1} - N_{26} = 0.00000}$

$\mathbf{N_{27} = 1.00000}$ $\quad \mathbf{\frac{N_1}{N_2} - N_{27} = 0.00000}$

$\mathbf{N_8 = 64.00000}$ $\quad \mathbf{\frac{N_2^7}{N_1} - N_8 = 0.00000}$



$\mathbf{Unit = 1.00000}$

$\mathbf{N_1 = 1.72159}$

$$\frac{N_1}{N_2} - N_3 = 0.00000$$

$\mathbf{N_2 = 2.12500}$

$$N_1 \cdot N_2 - N_4 = 0.00000$$

$\mathbf{N_3 = 0.81016}$

$$\frac{N_1}{N_2} = 0.81016$$

$\mathbf{N_4 = 3.65838}$

$\mathbf{N_5 = 0.57386}$

$\mathbf{N_6 = 1.14773}$

$\mathbf{N_7 = 0.70833}$

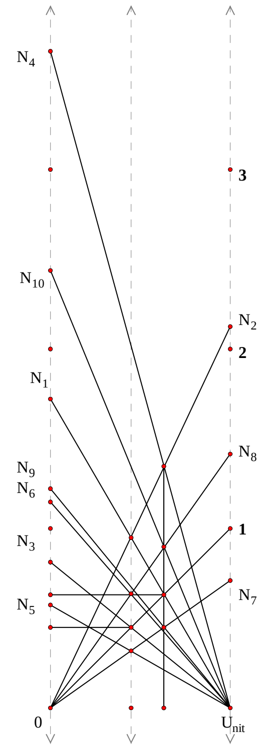
$\mathbf{N_8 = 1.41667}$

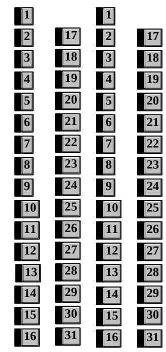
$\mathbf{N_9 = 1.21946}$

$$N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$$

$\mathbf{N_{10} = 2.43892}$

$$N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$$





Unit = 1.00000

$N_1 = 1.37056$

$N_2 = 1.73096$

$N_3 = 0.79179$ $\quad \frac{N_1}{N_2} = 0.79179$

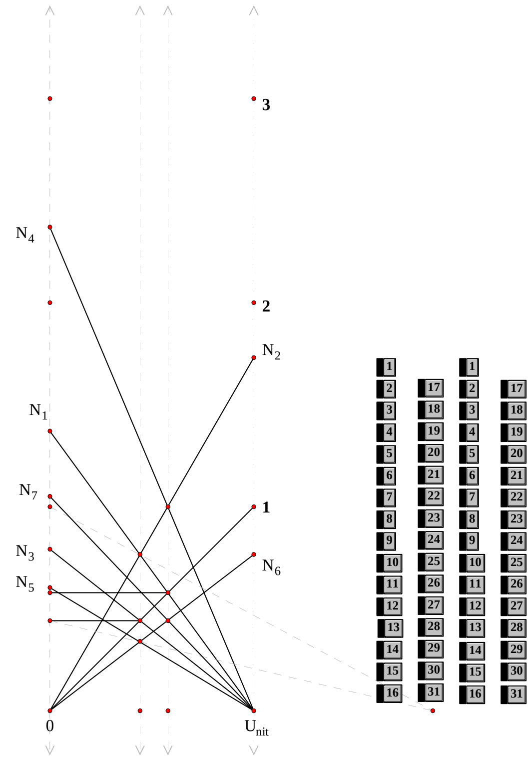
$N_4 = 2.37239$ $\quad N_1 \cdot N_2 = 2.37239$

$$\frac{N_1}{N_2} = 0.79179$$

$N_5 = 0.60565$ $\quad N_1 \cdot \left(\frac{N_1}{N_1+N_2} \right) - N_5 = 0.00000$

$N_6 = 0.76491$ $\quad N_1 \cdot \left(\frac{N_2}{N_1+N_2} \right) - N_6 = 0.00000$

$N_7 = 1.04835$ $\quad N_1 \cdot N_2 \cdot \left(\frac{N_1}{N_1+N_2} \right) - N_7 = 0.00000$



Unit = 1.00000

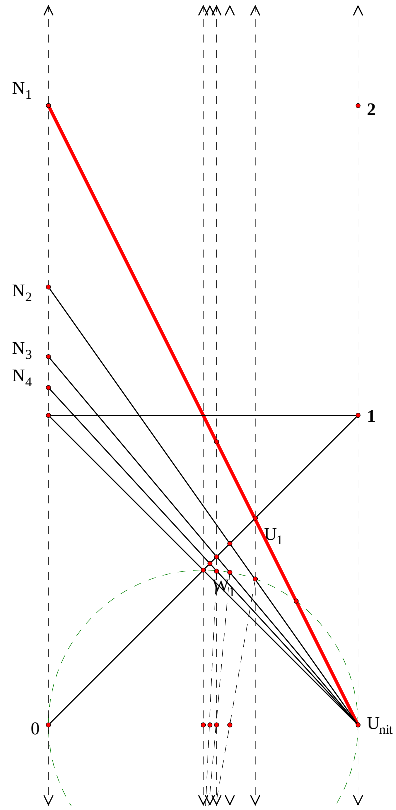
$N_1 = 2.00000$

$N_2 = 1.41421$ $\qquad$ **$N_1^{\frac{1}{2}} - N_2 = 0.00000$**

$N_3 = 1.18921$ $\qquad$ **$N_1^{\frac{1}{4}} - N_3 = 0.00000$**

$N_4 = 1.09051$ $\qquad$ **$N_1^{\frac{1}{8}} - N_4 = 0.00000$**

etc.



1
2 17
3 18
4 19
5 20
6 21
7 22
8 23
9 24
10 25
11 26
12 27
13 28
14 29
15 30
16 31

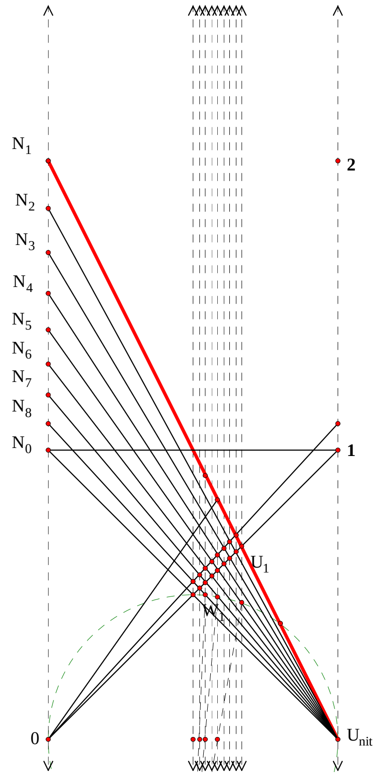
Unit = 1.00000

Eight root series.

One can project as long as they like.

$N_1 = 2.00000$	$N_1^{\frac{8}{8}} - N_1 = 0.00000$
$N_2 = 1.83401$	$N_1^{\frac{7}{8}} - N_2 = 0.00000$
$N_3 = 1.68179$	$N_1^{\frac{6}{8}} - N_3 = 0.00000$
$N_4 = 1.54221$	$N_1^{\frac{5}{8}} - N_4 = 0.00000$
$N_5 = 1.41421$	$N_1^{\frac{4}{8}} - N_5 = 0.00000$
$N_6 = 1.29684$	$N_1^{\frac{3}{8}} - N_6 = 0.00000$
$N_7 = 1.18921$	$N_1^{\frac{2}{8}} - N_7 = 0.00000$
$N_8 = 1.09051$	$N_1^{\frac{1}{8}} - N_8 = 0.00000$
$N_0 = 1.00000$	$N_1^{\frac{0}{8}} - N_0 = 0.00000$

etc.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

The computational speed by straight edge and compass outdoes long hand by factors. The computational accuracy exceeds that of any binary computer. The understanding as to what numbers mean cannot be outdone. Yet, instead of improving Euclid, they made a mess of it.

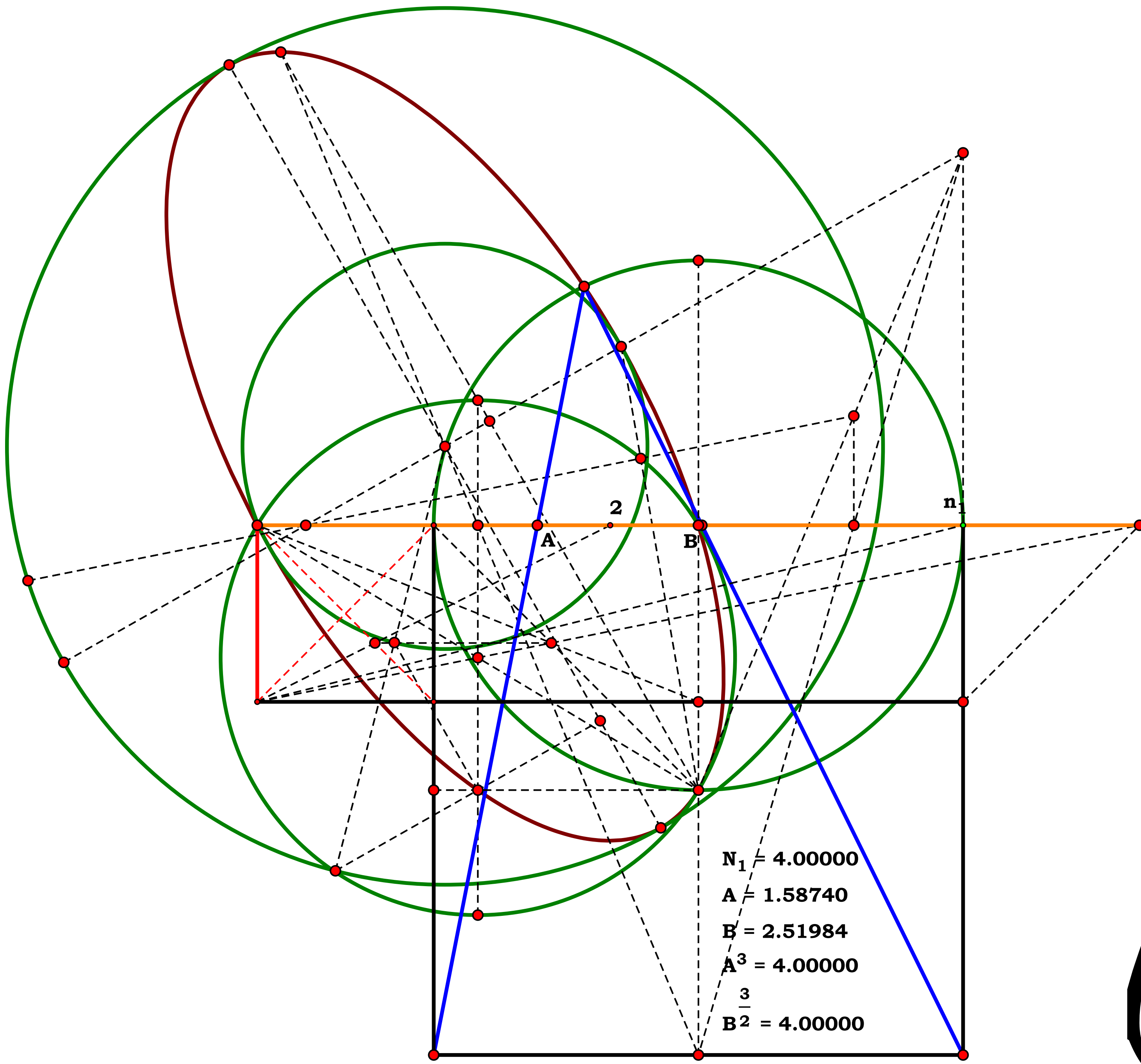
What led me to this solution was not Euclid, it was my own geometry play-especially doing the formula's and solution to a power line In order to solve for the power line, I actually had to know how to divide a square by a line. That coupled with the feeling that one should know the basic mathematical operations in geometry, as a starter made me break down and simply do it.

Geometry is still undefined. It is undefined because, as we know, a set can be constructed in only two ways, by enumeration and by definition. By saying that Euclidean Geometry only uses two tools, the straight edge and compass, we have enumerated its set. To define it, one would have to say, Geometry is that language by which we speak where there is one, and only one difference between two points.

This change not only defines Euclidean Geometry, but we find that it has been short changed for a long time. A straight edge does indeed give us one and only one difference between two points, and so does a compass, these are the unit and universe of discourse in the subject. However, there is yet one more tool, that tool that gives us every ratio in-between the unit and the universe, the ellipes. There is indeed one and only one difference between the two points called the foci of an ellipse.

If one can accept that, one can then understand my solution to the Delian Problem. A figure that gives one every aspect of an ellipse and one simply has to lay it down. Accepting that definition also takes something that is implied in Euclidean Geometry and makes it explicit, the ability to add, to do the math.

I hope you have fun.



The Delian Quest 2008

John Clark

Unit.

$BE := 1$

Given.

$N_1 := 3.86292 \quad AB := N_1$

$N_2 := .74482 \quad BD := N_2$

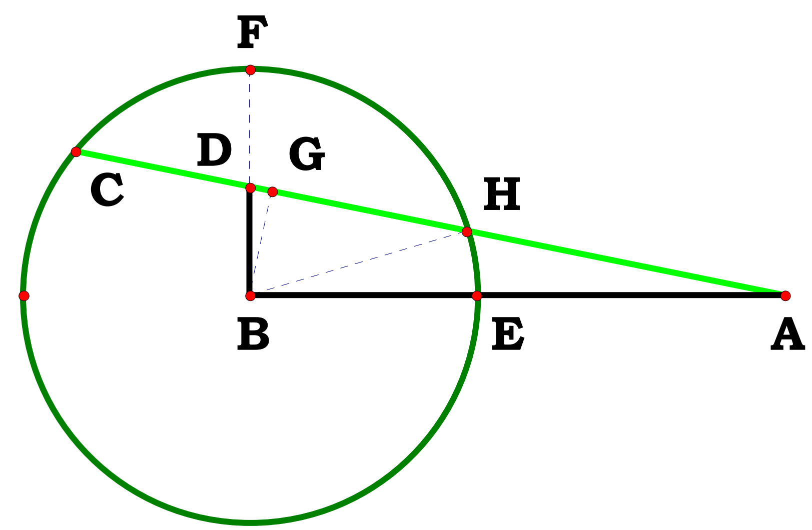
052108

Descriptions.

$AD := \sqrt{AB^2 + BD^2} \qquad DG := \dfrac{BD^2}{AD}$

$BH := BE \qquad BG := \dfrac{AB \cdot BD}{AD}$

$GH := \sqrt{BH^2 - BG^2} \qquad AC := AD + GH - DG$



Definitions.

$$\frac{\sqrt{AB^2 \cdot BE^2 + BD^2 \cdot BE^2 - AB^2 \cdot BD^2} + AB^2}{\sqrt{AB^2 + BD^2}} - AC = 0$$

$$AC - \frac{{N_1}^2 + \sqrt{{N_1}^2 - {N_1}^2 \cdot {N_2}^2 + {N_2}^2}}{\sqrt{{N_1}^2 + {N_2}^2}} = 0$$

052208

Unit.

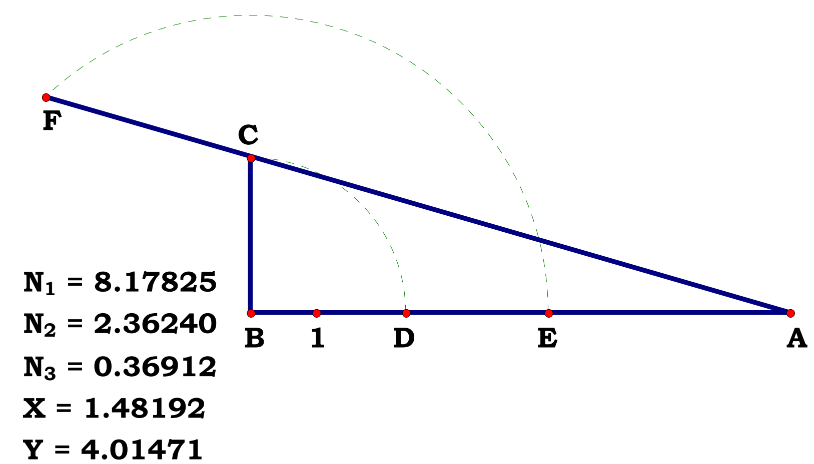
Given.

$N_1 := 8.17825 \quad AB := N_1$

$N_2 := 2.36240 \quad BC := N_2$

$N_3 := .36912$

Given AB, BC and AE as a portion of AD, what is AF?



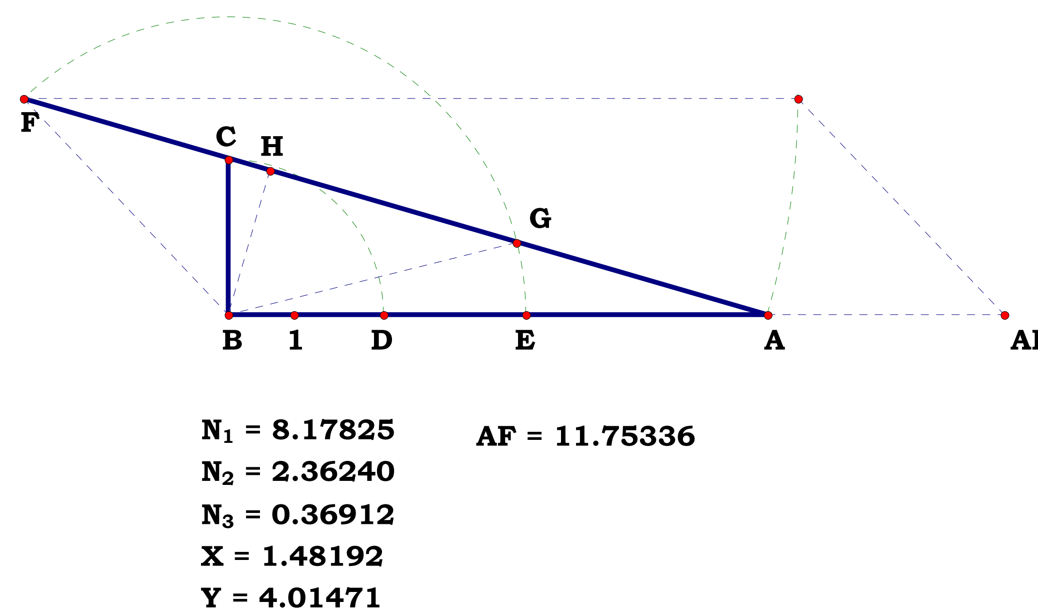
Descriptions.

$AD := N_1 - N_2 \quad DE := AD \cdot N_3 \quad AE := AD - DE$

$BE := AB - AE \quad AC := \sqrt{AB^2 + BC^2} \quad BH := \dfrac{AB \cdot BC}{AC}$

$BG := BE \quad CH := \dfrac{BC^2}{AC} \quad GH := \sqrt{BG^2 - BH^2} \quad FH := GH$

$AF := AC + FH - CH \quad AF = 11.753331$



Definitions.

$AD - (N_1 - N_2) = 0 \quad DE - (N_1 - N_2) \cdot N_3 = 0$

$AE - [(N_3 - 1) \cdot (N_2 - N_1)] = 0 \quad BE - (N_2 + N_1 \cdot N_3 - N_2 \cdot N_3) = 0$

$AC - \sqrt{N_1^2 + N_2^2} = 0 \quad BH - \dfrac{N_1 \cdot N_2}{\sqrt{N_1^2 + N_2^2}} = 0$

$BG - (N_2 + N_1 \cdot N_3 - N_2 \cdot N_3) = 0 \quad CH - \dfrac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

$$GH - \frac{\sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$$

$$FH - \frac{\sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$$

$$AF - \frac{N_1^2 + \sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$$

060208A

Unit.

Given.

$N_1 := 1.40187 \qquad AB := N_1$

$N_2 := 2.31398 \qquad AC := N_2$

$N_3 := 1.13348 \qquad CD := N_3$

For a straight line ellipse and three givens.

a: AB, AC, CD

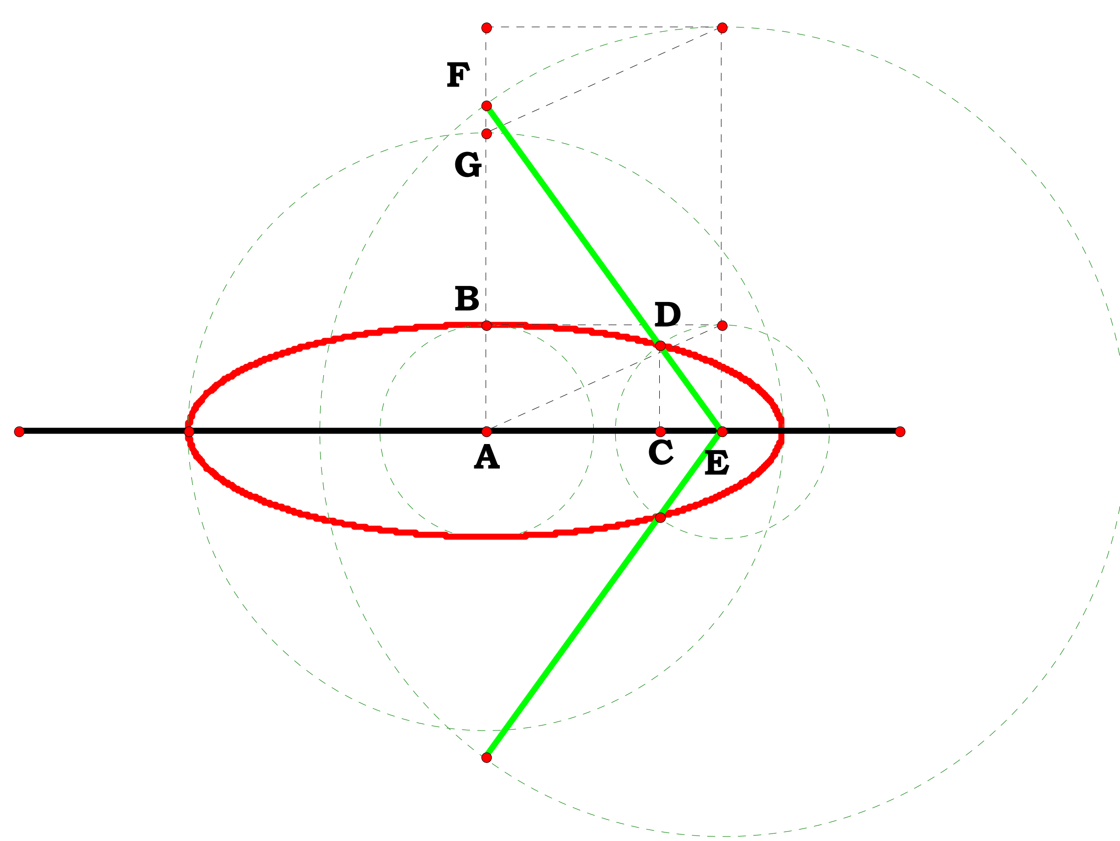
Descriptions.

$DE := AB \qquad CE := \sqrt{DE^2 - CD^2}$

$DF := \frac{DE \cdot AC}{CE} \qquad BG := DF - AB$

Definitions.

$$BG - N_1 \cdot \left(\frac{N_2}{\sqrt{N_1^{\,2} - N_3^{\,2}}} - 1 \right) = 0$$



060208B

Unit.

Given.

$N_1 := .8249$ $\quad CE := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := 1.13348$ $\quad CD := N_3$

For a straight line ellipse and three givens.

b: CE, AC, CD.

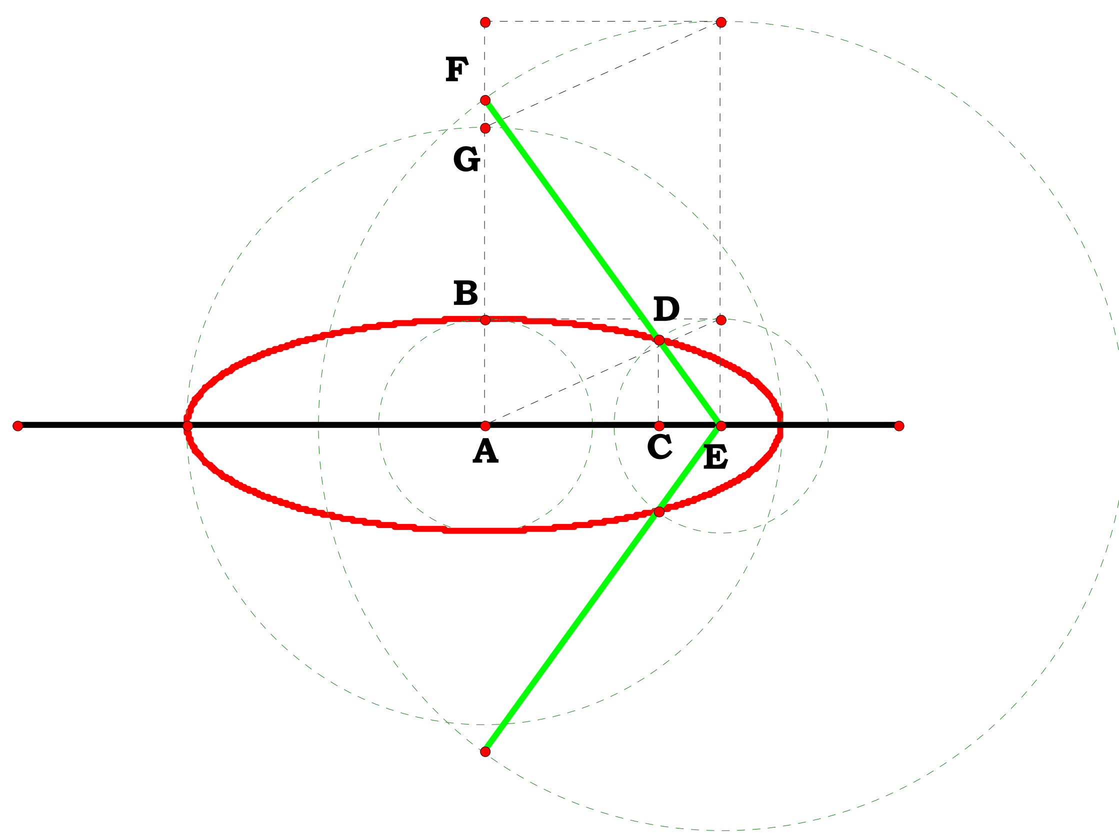
Descriptions.

$AB := \sqrt{CD^2 + CE^2}$ $\quad DE := AB$

$DF := \dfrac{DE \cdot AC}{CE}$ $\quad BG := DF - AB$

Definitions.

$$BG - \frac{\sqrt{N_1^{\,2} + N_3^{\,2}} \cdot (N_2 - N_1)}{N_1} = 0$$



060208C

Unit.

Given.

$N_1 := 1.40187$ $\quad AB := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := .8249$ $\quad CE := N_3$

For a straight line ellipse and three givens.

Descriptions.

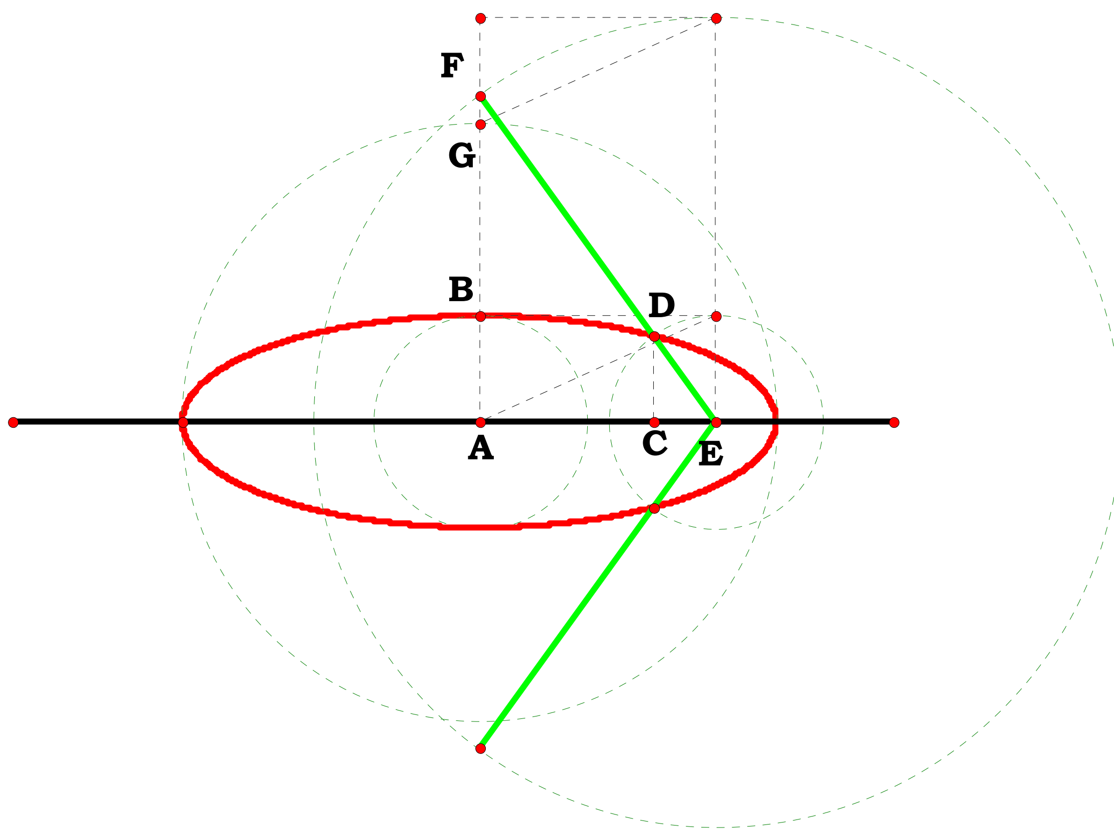
$DE := AB \quad DF := \dfrac{DE \cdot AC}{CE}$

$BG := DF - AB$

Definitions.

$$BG - \frac{N_1 \cdot (N_2 - N_3)}{N_3} = 0$$

c: AB, AC, CE



Unit.
Given.

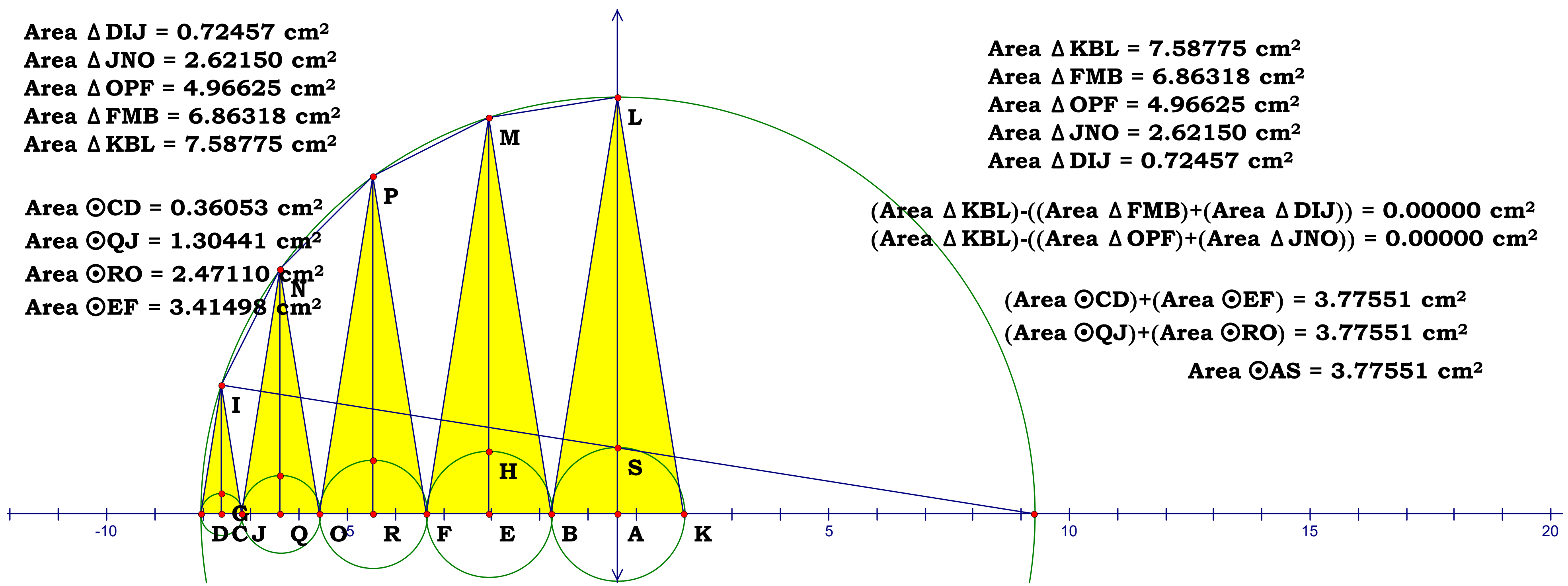
Descriptions.
Definitions.

Procrastinated Write up for 060308

Angles are expressible as an elliptical progression, and they show very arithmetic properties to one another.

Area ∆DIJ = 0.72457 cm²
Area ∆JNO = 2.62150 cm²
Area ∆OPF = 4.96625 cm²
Area ∆FMB = 6.86318 cm²
Area ∆KBL = 7.58775 cm²

Area ⊙CD = 0.36053 cm²
Area ⊙QJ = 1.30441 cm²
Area ⊙RO = 2.47110 cm²
Area ⊙EF = 3.41498 cm²



Area ∆KBL = 7.58775 cm²
Area ∆FMB = 6.86318 cm²
Area ∆OPF = 4.96625 cm²
Area ∆JNO = 2.62150 cm²
Area ∆DIJ = 0.72457 cm²

(Area ∆KBL)-((Area ∆FMB)+(Area ∆DIJ)) = 0.00000 cm²
(Area ∆KBL)-((Area ∆OPF)+(Area ∆JNO)) = 0.00000 cm²

(Area ⊙CD)+(Area ⊙EF) = 3.77551 cm²
(Area ⊙QJ)+(Area ⊙RO) = 3.77551 cm²

Area ⊙AS = 3.77551 cm²

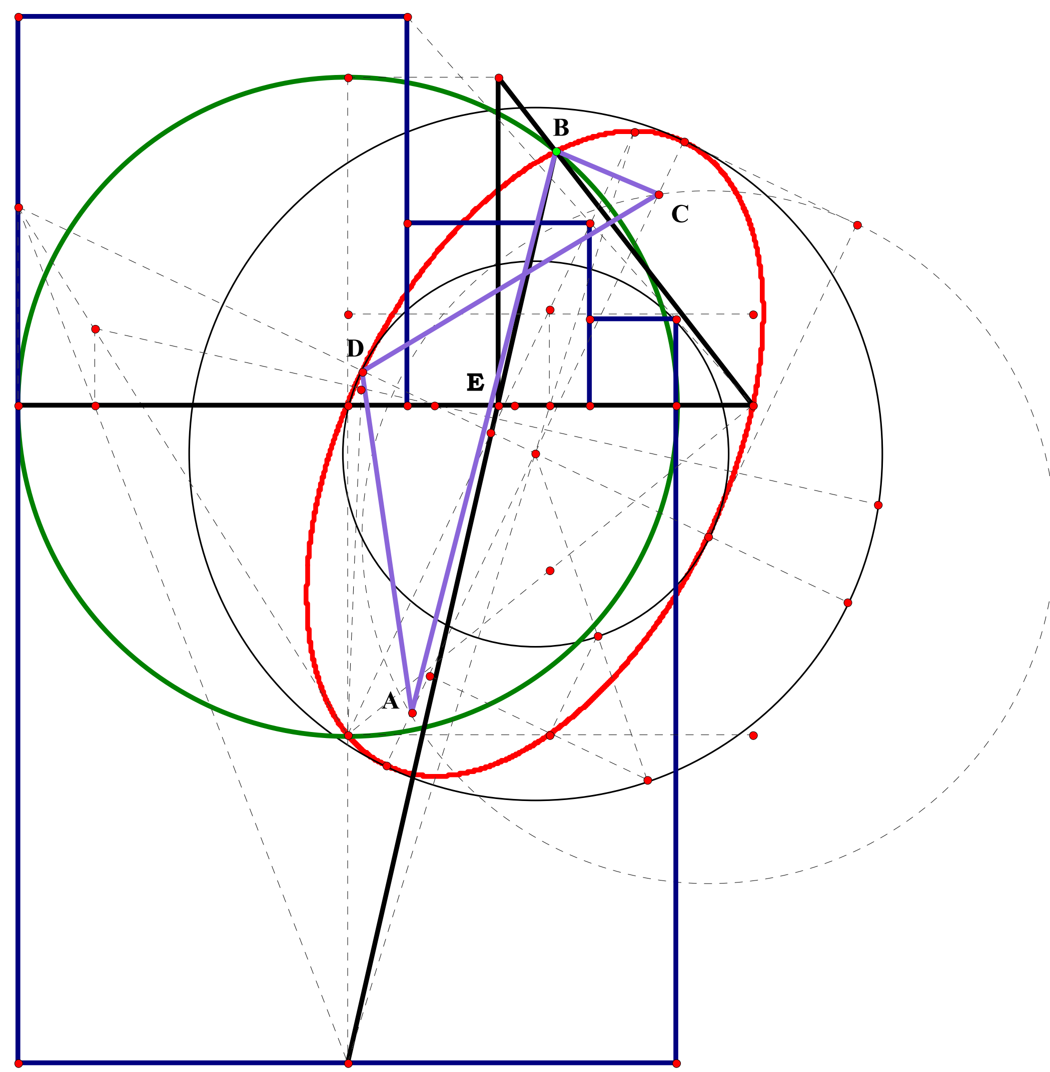
Unit.

Given. Parcing project for 061308a

Descriptions.

Definitions.

See about writing up a proof of the figure using the fact that from the center of the two roots, point E, a simple construct will produce the intersection B for the figure. And chect to see how the point E moves during Gemini roots.

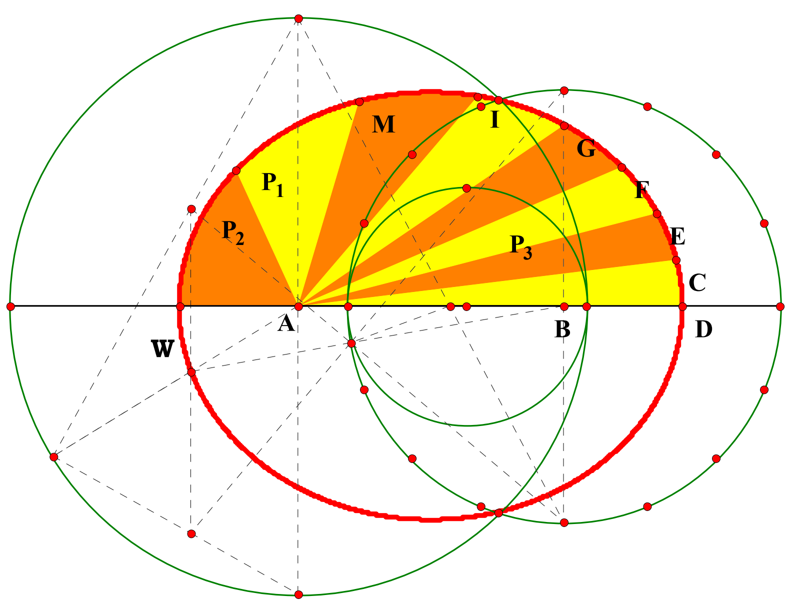


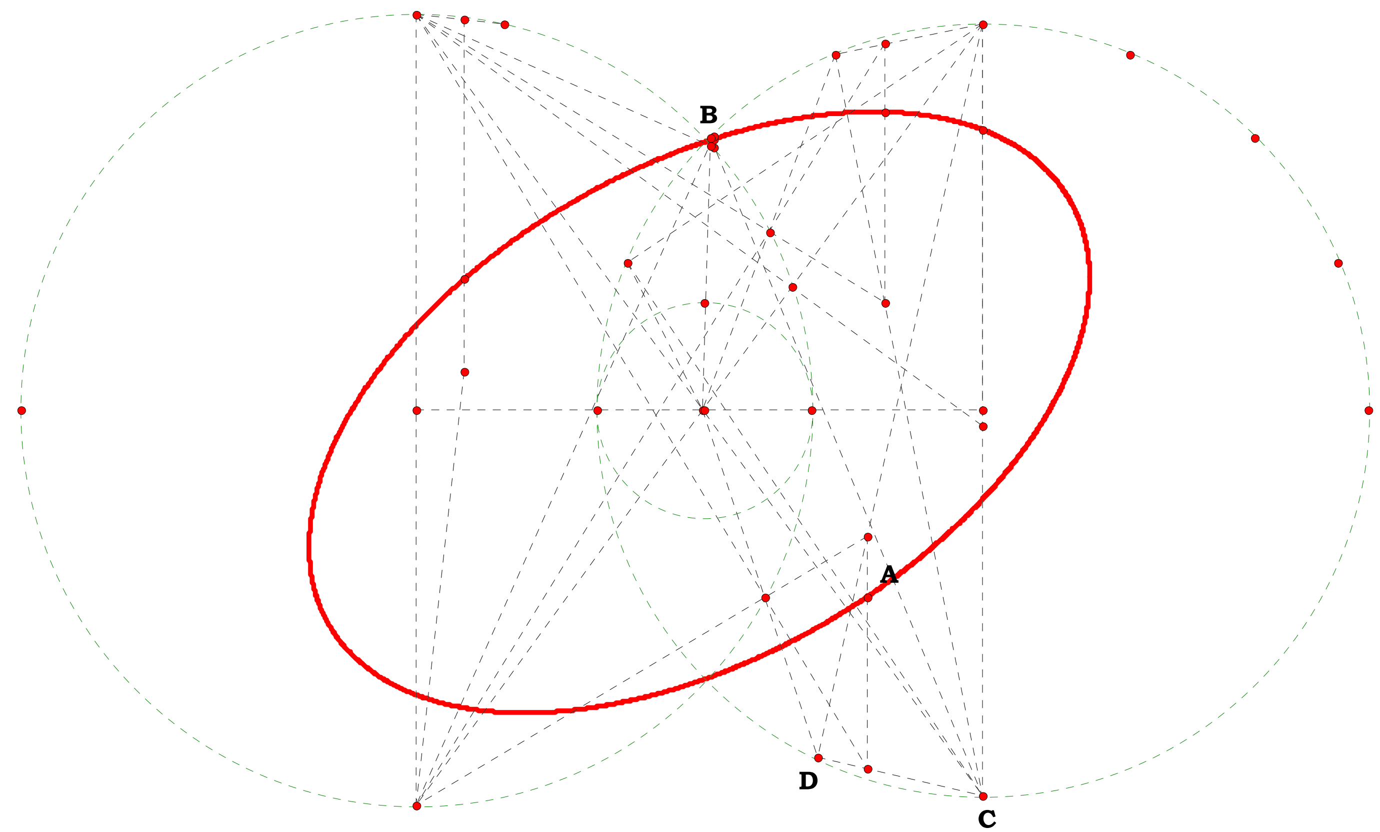
Unit.
Given.

Parcing project for 061308b

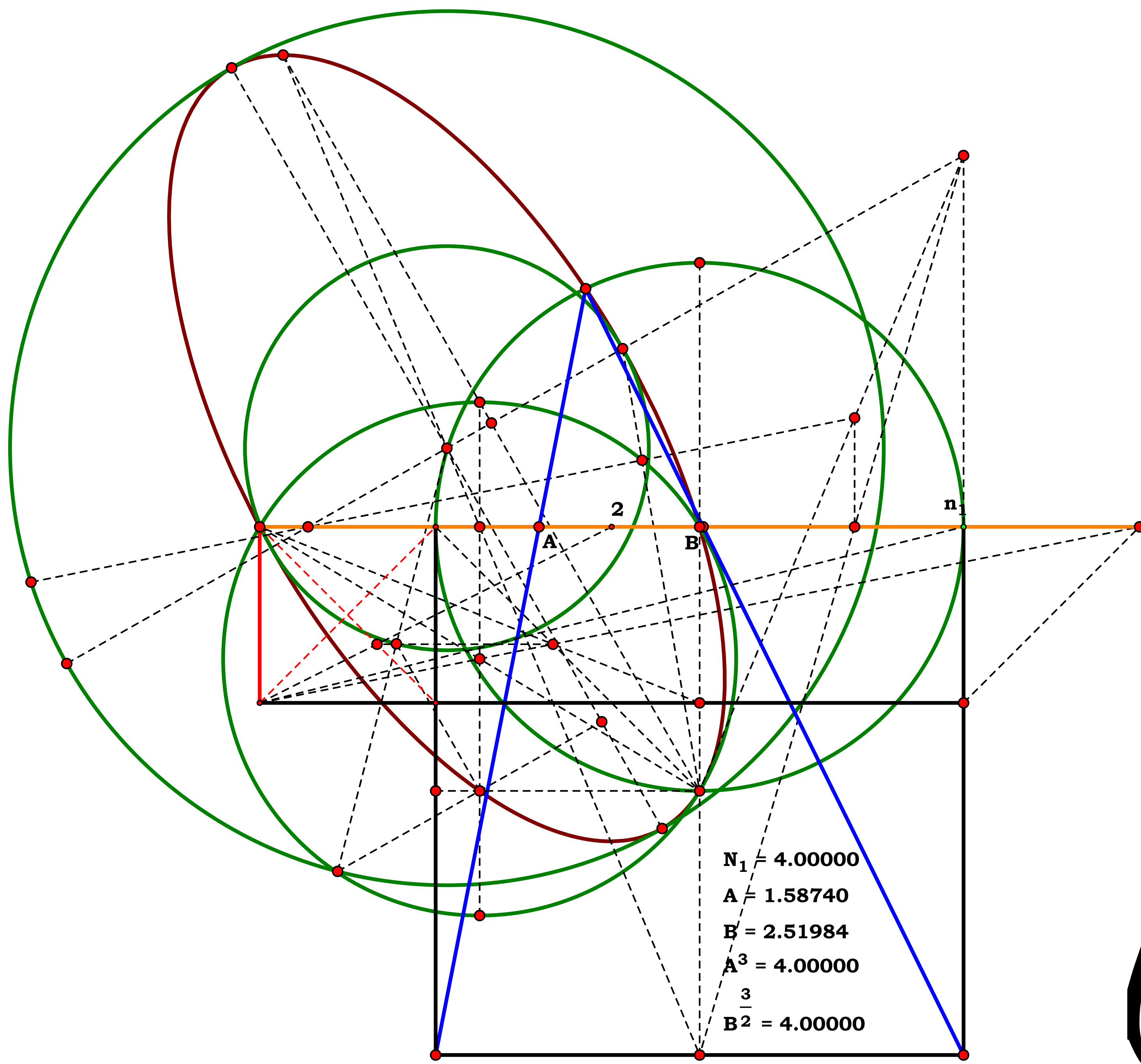
Looking at mass in eliptical motions from a different point of view

Descriptions.
Definitions.





How you understand the ellipes determines how you think an object moving in that orbit ought to be comprehended and written up as a law of nature, however ponder this fact. The velocity of D is a constant; it determines the velocity of A, the object you can see orbiting say the sun. However, the velocity of A exhibits the same characteristics of a planet or asteroid, slingshot effect and all. The velocity is still due to the constant velocity of D. The current understanding of planitary interaction is not correct.



The Delian Quest 2010

John Clark

Unit.
Given.

On angle trisection.

061110

Descriptions.
Definitions.

Parcing project. There is a whole series of plates here, write them up.

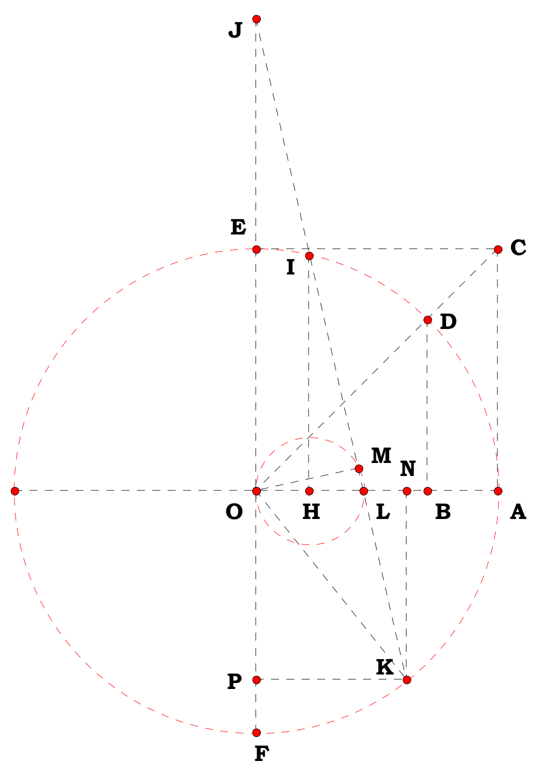
AO = 3.19617 cm
CO = 4.52006 cm
BO = 2.26003 cm

HO = 0.70753 cm

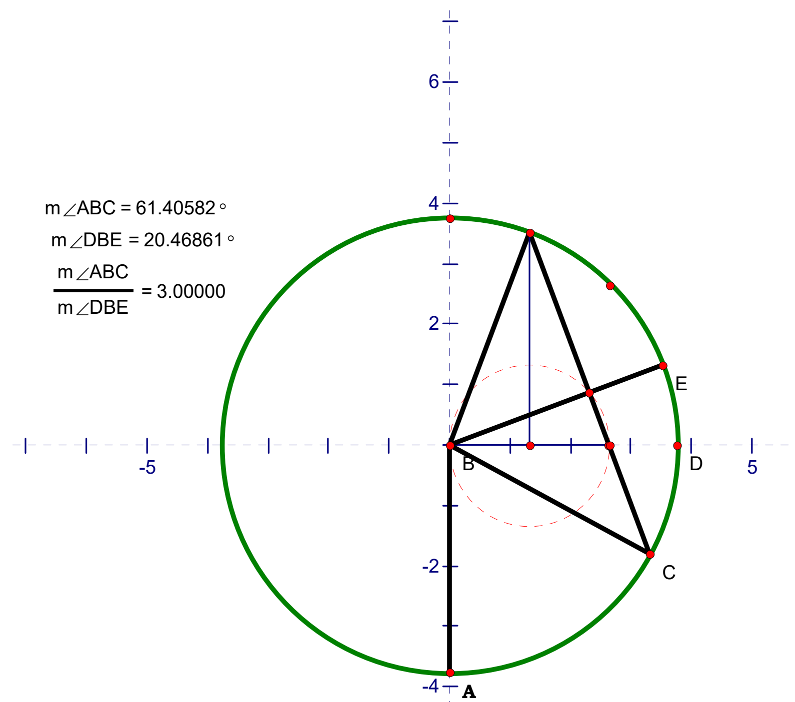
$\frac{\mathbf{BO}}{\mathbf{HO}}$ **= 3.19427**

LM = 0.31325 cm

MO = 1.37995 cm
KM = 2.88292 cm
JK = 8.96201 cm
KP = 1.98389 cm



m∠ABC = 61.40582°
m∠DBE = 20.46861°
$\frac{m\angle ABC}{m\angle DBE}$ = 3.00000

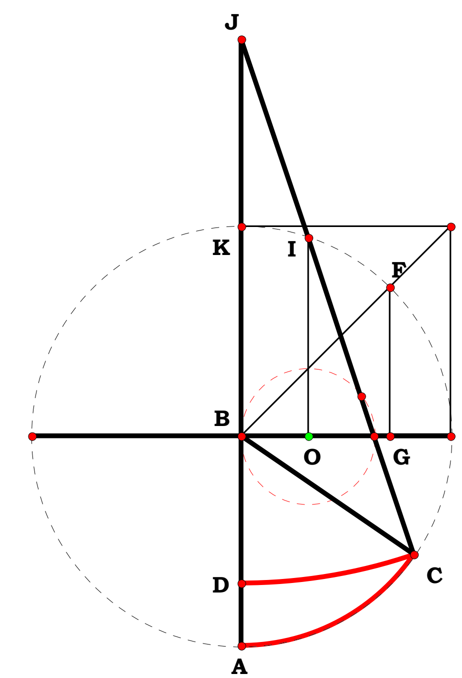


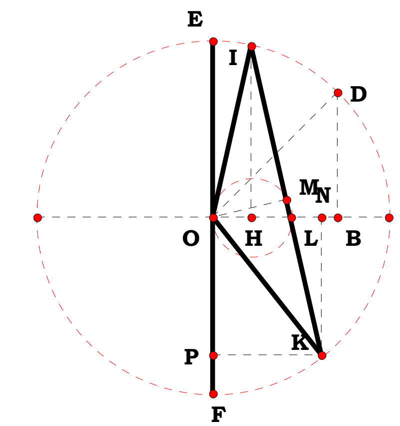
m∠ABC = 55.69074°
m∠IBK = 18.56358°
$\frac{\mathbf{m\angle ABC}}{\mathbf{m\angle IBK}}$ **= 3.00000**

BH = 2.77283 cm
BG = 1.96069 cm

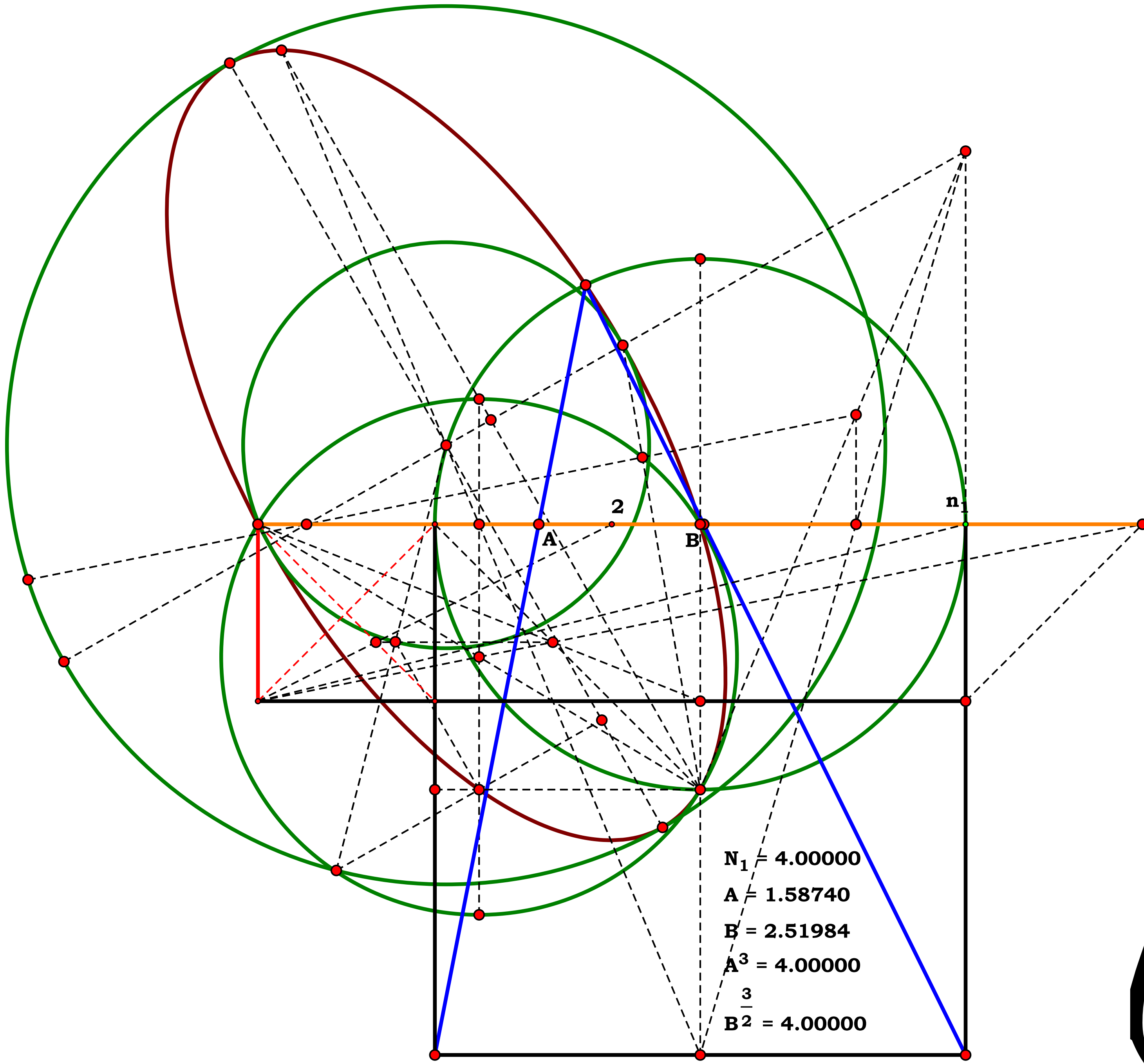
$\frac{\mathbf{BH}}{\mathbf{BG}}$ **= 1.41421**

$\sqrt{2}$ **= 1.41421**





$\frac{\mathbf{m\angle FOK}}{\mathbf{m\angle EOI}}$ **= 3.00000**



The Delian Quest 2011

John Clark

020511

Descriptions.

Rant

Percentages, ratio's, proportions, currency conversion, number conversion, etc. Let us say we have a zillion and one items we which to tanslate from one system of measure to another. How can we do all of the items at one and the same time? Well, if you are a non-Euclidean Geometer, you cannot, you are screwed. In fact, if you are a non-Euclidean Geometer, you are claiming that a unit differs from itself and are way too stupid to realize an obvios fact. So, no, I am not entrusting anything to them. And since they are supported by, if not every educational institution, then almost every-one, I had to drop out of school at an early age. The explicit and the tacit admission of their doctrines by our social structure even made me a social outcast. I say, they can argue with the foundation of their own psychology, Language.

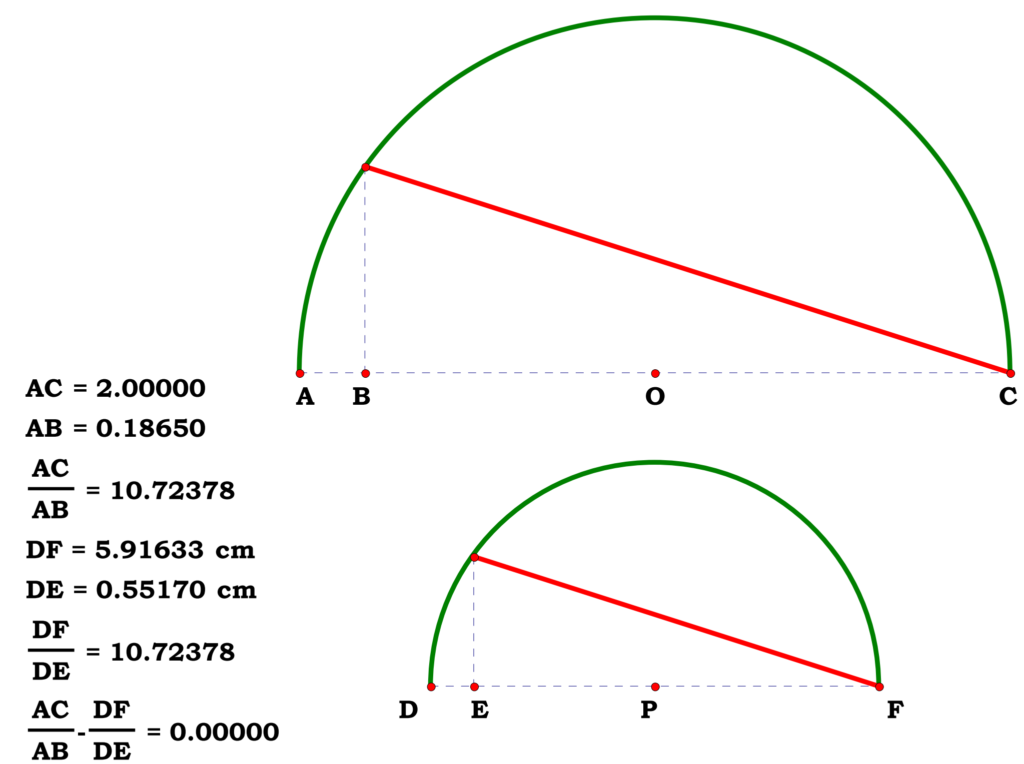
Therefore, I do not believe that what is needed is a write-up demonstrating proportion, again. Maybe I was just entertaining a rant.

What do you think the sentence, As A is to B, so too, C is to D, means? And if you claim that is is not true, citing the mentally lame as authorities for that claim, why is it you prefer a moron over countless examples in your daily life?

Why do you suffer your religious leaders, your political leaders, your teachers and your schools to teach the lies?

It is not because you are mentally functional. All of my ranting can never change the fact that man is still being made, that his mind is still incapable of doing simple operations.

Therefore, in order to put something constructive here, I will put a digitalization of The Science of Absolute Space, a title which is wholly indicative of someone who is illiterate.



THE SCIENCE ABSOLUTE OF SPACE

Bolyai János

THE SCIENCE ABSOLUTE OF SPACE

Independent of the Truth or Falsity of Euclid's Axiom XI

(*which can never be decided a priori*).

JOHN BOLYAI

TRANSLATED FROM THE LATIN

BY DR. GEORGE BRUCE HALSTED

PRESIDENT OF THE TEXAS ACADEMY OF SCIENCE

FOURTH EDITION.

VOLUME THREE OF THE NEOMONIC SERIES

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U. S. A. 1896

TRANSLATOR'S INTRODUCTION.

The immortal *Elements* of Euclid was already in dim antiquity a classic, regarded as absolutely perfect, valid without restriction.

Elementary geometry was for two thousand years as stationary, as fixed, as peculiarly Greek, as the Parthenon. On this foundation pure science rose in Archimedes, in Apollonius, in Pappus; struggled in Theon, in Hypatia; declined in Proclus; fell into the long decadence of the Dark Ages.

The book that monkish Europe could no longer understand was then taught in Arabic by Saracen and Moor in the Universities of Bagdad and Cordova.

To bring the light, after weary, stupid centuries, to western Christendom, an Englishman, Adelhard of Bath, journeys, to learn Arabic, through Asia Minor, through Egypt, back to Spain. Disguised as a Mohammedan student, he got into Cordova about 1120, obtained a Moorish copy of Euclid's *Elements*, and made a translation from the Arabic into Latin.

The first printed edition of Euclid, published in Venice in 1482, was a Latin version from the Arabic. The translation into Latin from the Greek, made by Zamberti from a MS. of Theon's revision, was first published at Venice in 1505.

Twenty - eight years later appeared the *editio princeps* in Greek, published at Basle in 1533 by John Hervagius, edited by Simon Grynaeus. This was for a century and three-quarters the only printed Greek text of all the books, and from it the first English translation (1570) was made by "Henricus Billingsley," afterward Sir Henry Billingsley, Lord Mayor of London in 1591.

And even today, 1895, in the vast system of examinations carried out by the British Government, by Oxford, and by Cambridge, no proof of a theorem in geometry will be accepted which infringes Euclid's sequence of propositions.

Nor is the work unworthy of this extraordinary immortality.

Says Clifford : "This book has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state.

“The encouragement; for it contained a body of knowledge that was really known and could be relied on.

“The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained.”

But Euclid stated his assumptions with the most painstaking candor, and would have smiled at the suggestion that he claimed for his conclusions any other truth than perfect deduction from assumed hypotheses. In favor of the external reality or truth of those assumptions he said no word.

Among Euclid’s assumptions is one differing from the others in prolixity, whose place fluctuates in the manuscripts.

Peyrard, on the authority of the Vatican MS., puts it among the postulates, and it is often called the parallel-postulate. Heiberg, whose edition of the text is the latest and best (Leipzig, 1883–1888), gives it as the fifth postulate.

James Williamson, who published the closest translation of Euclid we have in English, indicating, by the use of italics, the words not in the original, gives this assumption as eleventh among the Common Notions.

Bolyai speaks of it as Euclid’s Axiom XI. Todhunter has it as twelfth of the Axioms.

Clavius (1574) gives it as Axiom 13.

The Harpur Euclid separates it by forty-eight pages from the other axioms.

It is not used in the first twenty-eight propositions of Euclid. Moreover, when at length used, it appears as the inverse of a proposition already demonstrated, the seventeenth, and is only needed to prove the inverse of another proposition already demonstrated, the twenty-seventh.

Now the great Lambert expressly says that Proklus demanded a proof of this assumption because when inverted it is demonstrable.

All this suggested, at Europe’s renaissance, not a doubt of the necessary external reality and exact applicability of the assumption, but the possibility of deducing it from the other assumptions and the twenty-eight propositions already proved by Euclid without it.

Euclid demonstrated things more axiomatic by far. He proves what every dog knows, that any two sides of a triangle are together greater than the third.

Yet after he has finished his demonstration, that straight lines making with a transversal equal alternate angles are parallel, in order to

prove the inverse, that parallels cut by a transversal make equal alternate angles, he brings in the unwieldy assumption thus translated by Williamson (Oxford, 1781) :

"11. And if a straight line meeting two straight lines make those angles which are inward and upon the same side of it less than two right angles, the two straight lines being produced indefinitely will meet each other on the side where the angles are less than two right angles."

As Staeckel says, "it requires a certain courage to declare such a requirement, alongside the other exceedingly simple assumptions and postulates." But was courage likely to fail the man who, asked by King Ptolemy if there were no shorter road in things geometric than through his *Elements*? answered, "To geometry there is no special way for kings!"

In the brilliant new light given by Bolyai and Lobachevski we now see that Euclid understood the crucial character of the question of parallels.

There are now for us no better proofs of the depth and systematic coherence of Euclid's masterpiece than the very things which, their cause unappreciated, seemed the most noticeable blots on his work.

Sir Henry Savile, in his Praelectiones on Euclid, Oxford, 1621, p. 140, says : “In pulcherrimo Geometriae corpore duo sunt naevi, duae labes . . . ” etc., and these two blemishes are the theory of parallels and the doctrine of proportion; the very points in the Elements which now arouse our wondering admiration. But down to our very nineteenth century an ever renewing stream of mathematicians tried to wash away the first of these supposed stains from the most beauteous body of Geometry.

The year 1799 finds two extraordinary young men striving thus

“To gild refined gold, to paint the lily,
To cast a perfume o’er the violet.”

At the end of that year Gauss from Braunschweig writes to Bolyai Farkas in Klausenburg (Kolozsvár) as follows : [Abhandlungen der Koeniglichen Gesellschaft der Wissenschaften zu Goettingen, Bd. 22, 1877.]

“I very much regret, that I did not make use of our former proximity, to find out *more* about your investigations in regard to the first grounds of geometry; I should certainly thereby have spared myself much vain labor, and would have become more restful than any one, such

as I, can be, so long as on such a subject there yet remains so much to be wished for.

In my own world thereon I myself have advanced far (though my other wholly heterogeneous employments leave me little time therefore) but *the* way, which I have hit upon, leads not so much to the goal, which one wishes, as much more to making doubtful the truth of geometry.

Indeed I have core upon much, which with most no doubt would pass for a proof, but which in my eyes proves as good as *nothing*.

For example, if one could prove, that a rectilineal triangle is possible, whose content may be greater, than any given surface, then I am in condition, to prove with perfect rigor all geometry.

Most would indeed let that pass as an axiom; I not; it might well be possible, that, how far apart soever one took the three vertices of the triangle in space, yet the content was always under a given limit.

I have more such theorems, but in none do I find anything satisfying.”

From this letter we clearly see that in 1799 Gauss was still trying to prove that Euclid’s is the only non-contradictory system of geometry,

and that it is the system regnant in the external space of our physical experience.

The first is false; the second can never be proven.

Before another quarter of a century, Bolyai János, then unborn, had created another possible universe; and, strangely enough, though nothing renders it impossible that the space of our physical experience may, this very year, be satisfactorily shown to belong to Bolyai János , yet the same is not true for Euclid.

To decide our space is Bolyai’s, one need only show a single rectilineal triangle whose angle-sum measures less than a straight angle. And this could be shown to exist by imperfect measurements, such as human measurements must always be. For example, if our instruments for angular measurement could be brought to measure an angle to within one millionth of a second, then if the lack were as great as two millionths of a second, we could make certain its existence.

But to prove Euclid’s system, we must show that a triangle’s angle-sum is *exactly* a straight angle, which nothing human can ever do.

However this is anticipating, for in 1799 it seems that the mind of the elder Bolyai, Bolyai Farkas, was in precisely the same state as

that of his friend Gauss. Both were intensely trying to prove what now we know is indemonstrable. And perhaps Bolyai got nearer than Gauss to the unattainable. In his "Kurzer Grundriss eines Versuchs," etc., p. 46, we read : "Koennten jede 3 Punkte, die nicht in einer Geraden sind, in eine Sphaere fallen, so waere das Eucl. Ax. XI. bewiesen." Frischauf calls this "das anschaulichste Axiom." But in his Autobiography written in Magyar, of which my Life of Bolyai contains the first translation ever made, Bolyai Farkas says : "Yet I could not become satisfied with my different treatments of the question of parallels, which was ascribable to the long discontinuance of my studies, or more probably it was due to myself that I drove this problem to the point which robbed my rest, deprived me of tranquillity."

It is well-nigh certain that Euclid tried his own calm, immortal genius, and the genius of his race for perfection, against this self-same question. If so, the benign intellectual pride of the founder of the mathematical school of the greatest of universities, Alexandria, would not let the question cloak itself in the obscurities of the infinitely great or the infinitely small. He would say to himself : "Can I prove

this plain, straightforward, simple theorem : "those straights which are produced indefinitely from less than two right angles meet." [This is the form which occurs in the Greek of Eu. I. 29.]

Let us not underestimate the subtle power of that old Greek mind. We can produce no Venus of Milo. Euclid's own treatment of proportion is found as flawless in the chapter which Stolz devotes to it in 1885 as when through Newton it first gave us our present continuous number-system.

But what fortune had this genius in the fight with its self-chosen simple theorem? Was it found to be deducible from all the definitions, and the nine "Common Notions," and the five other Postulates of the immortal Elements? Not so. But meantime Euclid went ahead without it through twenty-eight propositions, more than half his first book. But at last came the practical pinch, then as now the triangle's angle-sum.

He gets it by his twenty-ninth theorem : "A straight falling upon two parallel straights makes the alternate angles equal."

But for the proof of this he needs that recalcitrant proposition which has how long been keeping him awake nights and waking

him up mornings? Now at last, true man of science, he acknowledges it indemonstrable by spreading it in all its ugly length among his postulates.

Since Schiaparelli has restored the astronomical system of Eudoxus, and Hultsch has published the writings of Autolycus, we see that Euclid knew surface-spherics, was familiar with triangles whose angle-sum is more than a straight angle. Did he ever think to carry out for himself the beautiful system of geometry which comes from the contradiction of his indemonstrable postulate; which exists if there be straights produced indefinitely from less than two right angles yet nowhere meeting; which is real if the triangle's angle-sum is less than a straight angle?

Of how naturally the three systems of geometry flow from just exactly the attempt we suppose Euclid to have made, the attempt to demonstrate his postulate fifth, we have a most romantic example in the work of the Italian priest, Saccheri, who died the twenty-fifth of October, 1733. He studied Euclid in the edition of Clavius, where the fifth postulate is given as Axiom 13. Saccheri says it should not be called an axiom, but ought to be demonstrated. He tries this seemingly simple

task; but his work swells to a quarto book of 101 pages.

Had he not been overawed by a conviction of the absolute necessity of Euclid’s system, he might have anticipated Bolyai János, who ninety years later not only discovered the new world of mathematics but appreciated the transcendent import of his discovery.

Hitherto what was known of the Bolyais came wholly from the published works of the father Bolyai Farkas, and from a brief article by Architect Fr. Schmidt of Budapest “Aus dem Leben zweier ungarischer Mathematiker, Johann und Wolfgang Bolyai von Bolya.” Grunert’s Archiv, Bd. 48, 1868, p. 217.

In two communications sent me in September and October 1895, Herr Schmidt has very kindly and graciously put at my disposal the results of his subsequent researches, which I will here reproduce. But meantime I have from entirely another source come most unexpectedly into possession of original documents so extensive, so precious that I have determined to issue them in a separate volume devoted wholly to the life of the Bolyais; but these are not used in the sketch here given.

Bolyai Farkas was born February 9th, 1775, at Bolya, in that part of Transylvania

(Erdély) called Székelyföld. He studied first at Enyed, afterward at Klausenburg (Kolozsvár), then went with Baron Simon Kemény to Jena and afterward to Goettingen. Here he met Gauss, then in his 19th year, and the two formed a friendship which lasted for life.

The letters of Gauss to his friend were sent by Bolyai in 1855 to Professor Sartorius von Walterhausen, then working on his biography of Gauss. Everyone who met Bolyai felt that he was a profound thinker and a beautiful character.

Benzenberg said in a letter written in 1801 that Bolyai was one of the most extraordinary men he had ever known.

He returned home in 1799, and in January, 1804, was made professor of mathematics in the Reformed College of Maros-Vásárhely. Here for 47 years of active teaching he had for scholars nearly all the professors and nobility of the next generation in Erdély.

Sylvester has said that mathematics is poesy.

Bolyai's first published works were dramas.

His first published book on mathematics was an arithmetic :

Az arithmetica eleje. 8vo. i- xvi, 1–162 pp. The copy in the library of the Reformed College is enriched with notes by Bolyai János .

Next followed his chief work, to which he constantly refers in his later writings. It is in Latin, two volumes, 8vo, with title as - follows :

TENTAMEN | JUVENTUTEM STUDIOSAM IN ELEMENTA MATHESEOS PURAE, ELEMENTARIS AC | SUBLIMIORIS, METHODO INTUITIVA, EVIDENTIA— | QUE HUIC PROPRIA, INTRODUCENDI. |

CUM APPENDICE TRIPLICI. | Auctore Professore Matheseos et Physices Chemiaeque | Publ. Ordinario. | Tomus Primus. | *Maros Vasarhelyini*. 1832. | Typis Collegii Reformatorum per JOSEPHUM, et | SIMEONEM KALI de felsö Vist. | At the back of the title : Imprimatur. | M. Vásárhelyini Die | 12 Octobris, 1829. [Paulus Horváth m. p. | Abbas, Parochus et Censor | Librorum.

Tomus Secundus. | *Maros Vasarhelyini*. 1833.

The first volume contains :

Preface of two pages : *Lectori salutem*.

A folio table : *Explicatio signorumn*.

Index rerum (I–XXXII). *Errata* (XXXIII–XXXVII).

Pro tyronibus prima vice legentibus notanda sequentia (XXXVIII–LII).

Errores (LIII–LXVI).

Scholion (LXVII - LXXIV).

Pluriium errorum haud animadversorum numerous minuitur (LXXV–LXXVI). Recensio per auctorem ipsum facta (LXXVII–LXXVIII).

Errores recentius detecti (L X X V–XCVIII).

Now comes the body of the text (pages 1–502).

Then, with special paging, and a new title page, comes the immortal Appendix, here given in English.

Professors Staeckel and Engel make a mistake in their "Parallellinien" in supposing that this Appendix is referred to in the title of " Tentamen." On page 241 they quote this title, including the words "Cum appendice triplici," and say : "In dem dritten Anhange, der nur 28 Seiten umfasst, hat Johann Bolyai seine neue Geometrie entwickelt."

It is not a third Appendix, nor is it referred to at all in the words "Cum appendice triplici."

These words, as explained in a prospectus in the Magyar language, issued by Bolyai Farkas, asking for subscribers, referred to a real triple Appendix, which appears, as it

should, at the end of the book Tomus Secundus, pp. 265 - 322.

The now world renowned Appendix by Bolyai János was an afterthought of the father, who prompted the son not "to occupy himself with the theory of parallels," as Staeckel says, but to translate from the German into Latin a condensation of his treatise, of which the principles were discovered and properly appreciated in 1823, and which was given in writing to Johann Walter von Eckwehr in 1825.

The father, without waiting for Vol. II, inserted this Latin translation, with separate paging (1 - 26), as an Appendix to his Vol. I, where, counting a page for the title and a page "Explicatio signorum," it has twenty-six numbered pages, followed by two unnumbered pages of Errata.

The treatise itself, therefore, contains only twenty-four pages—the most extraordinary two dozen pages in the whole history of thought! Milton received but a paltry £5 for his Paradise Lost; but it was at least plus £5.

Bolyai János, as we learn from Vol. II, p. 384, of "*Tentamen*," contributed for the

printing of his eternal twenty-six pages, 104 florins 50 kreuzers.

That this Appendix was finished considerably before the Vol. I, which it follows, is seen from the references in the text, breathing a just admiration for the Appendix and the genius of its author.

Thus the father says, p. 452 : Elegans est conceptus *similium*, quem J. B. *Appendicis Auctor* dedit. Again, p. 489 : *Appendicis Actor*, rem acumine singulari aggressus, Geometriam pro omni casu absolute veram posuit; quamvis e magna mole, tantum summe necessaria, in Appendice hujus tomi exhibuerit, multis (ut tetraedri resolutione generali, pluribusque aliis disquisitionibus elegantibus) brevitatis studio omissis.

And the volume ends as follows, p. 502 : Nec operae pretium est plura referre; quum res tota exaltiori contemplationis puncto, in ima penetranti oculo, tractetur in Appendice sequente, a quovis fideli veritatis purae alumno diagna legi.

The father gives a brief resumé of the results of his own determined, life-long, desperate efforts to do that at which Saccheri, J. H. Lambert, Gauss also had failed, to establish Euclid' theory of parallels *a priori*.

He says, p. 490 : “Tentamina idcirco quae olim feceram, breviter exponenda veniunt; ne saltem alius quis operam eandem perdat.” He anticipates J. Delboeuf’s “Prolégoménes philosophiques de la géométrie et solution des postulats,” with the full consciousness in addition that it is *not* the solution,—that the final solution has crowned not his own intense efforts, but the genius of his son.

This son’s Appendix which makes all preceding space only a special case, only a species under a genus, and so requiring a descriptive adjective, *Euclidean*, this wonderful production of pure genius, this strange Hungarian flower, was saved for the world after more than thirty-five years of oblivion, by the rare erudition of Professor Richard Baltzer of Dresden, afterward professor in the University of Giessen. He it was who first did justice publicly to the works of Lobachevski and Bolyai.

Incited by Baltzer, in 1866 J. Hoüel issued a French translation of Lobachevski’s Theory of Parallels, and in a note to his Preface says : “M. Richard Baltzer, dans la seconde édition de ses excellents *Elenents de Geometrie*, a, le premier, introduit ces notions exactes à la place qu’elles doivent occuper,” Honor to

Baltzer! But alas! father and son were already in their graves!

Fr. Schmidt in the article cited (1868) says : “It was nearly forty years before these profound views were rescued from oblivion, and Dr. R. Baltzer, of Dresden, has acquired imperishable titles to the gratitude of all friends of science as the first to draw attention to the works of Bolyai, in the second edition of his excellent *Elemente der Mathematik* (1866–67). Following the steps of Baltzer, Professor Hoüel, of Bordeaux, in a brochure entitled, Essai critique sur les principes fondamentaux de la Géométrie élémentaire, has given extracts from Bolyai’s book, which will help in securing for these new ideas the justice they merit.”

The father refers to the son’s Appendix again in a subsequent book, Urtan elemei kezdöknek [Elements of the science of space for beginners] (1850 – 51), pp. 48. In the College are preserved three sets of figures for this book, two by the author and one by his grandson, a son of János.

The last work of Bolyai Farkas; the only one composed in German, is entitled,

Kurzer Grundriss eines Versuchs

I. Die Arithmetik, durch zvekmässig konstruirte

Begriffe, von eingebildeten und unendlich-kleinen Grössen gereinigt, anschaulich und logisch-streng darzustellen.

II. In der Geometrie, die Begriffe der geraden Linie, der Ebene, des Winkels allgemein, der winkellosen Formen, und der Krummen, der verschiedenen Arten der Gleichheit u. d. gl. nicht nur scharf zu bestimmen; sondern auch ihr Seyn im Raume zu beweisen : und da die Frage, *ob zwey von der dritten geschnittene Geraden, wenn die summe der inneren Winkel nicht = 2R, sich schneiden oder nicht?* neimand auf der Erde ohne ein Axiom (wie Euklid das XI) aufzustellen, beantworten wird; die davon unabhängige Geometrie abzusondern; und eine auf die *Ja*—Antwort, andere auf das *Nein* so zu bauen, das die Formeln der letzten, auf ein Wink auch in der ersten gültig seyen.

Nach ein lateinischen Werke von 1829, M. Vásérhely, und eben daselbst gedruckten ungrischen.

Maros Vásárhely 1851. 8vo. pp. 88.

In this book he says, referring to his son's Appendix : "Some copies of the work published here were sent at that time to Vienna, to Berlin, to Goettingen . . . . From Goettingen the giant of mathematics, who from

his pinnacle embraces in the same view the stars and the abysses, wrote that he was surprised to see accomplished what he had begun, only to leave it behind in his papers.”

This refers to 1832. The only other record that Gauss ever mentioned the book is a letter from Gerling, written October 31st, 1851, to Wolfgang Boylai, on receipt of a copy of “Kurzer Grundriss.” Gerling, a scholar of Gauss, had been from 1817 Professor of Astronomy at Marburg. He writes : “I do not mention my earlier occupation with the theory of parallels, for already in the year 1810–1812 with Gauss, as earlier 1809 with J. F. Pfaff I had learned to perceive how all previous attempts to prove the Euclidean axiom had miscarried. I had then also obtained preliminary knowledge of your works, and so, when I first [1820] had to print something of my view thereon, I wrote it exactly as it yet stands to read on page 187 of the latest edition.

‘‘We had about this time [1819] here a law professor, Schweikart, who was formerly in Charkov, and had attained to similar ideas, since without help of the Euclidean axiom he developed in its beginnings a geometry which he called Astralgeometry. What he communicated to me thereon I sent to Gauss, who

then informed me how much farther already had been attained on this way, and later also expressed himself about the great acquisition, which is offered to the few expert judges in the Appendix to your book."

The "latest edition" mentioned appeared in 1851, and the passage referred to is : "This proof [of the parallel-axiom] has been sought in manifold ways by acute mathematicians, but yet until now not found with complete sufficiency. So long as it fails, the theorem, as all founded on it, remains a hypothesis, whose validity for our life indeed is sufficiently proven by *experience*, whose *general*, *necessary exactness*, however, could be doubted without absurdity."

Alas! that this feeble utterance should have seemed sufficient for more than thirty years to the associate of Gauss and Schweikart, the latter certainly one of the independent discoverers of the non-Euclidean geometry. But then, since neither of these sufficiently realized the transcendent importance of the matter to publish any of their thoughts on the subject, a more adequate conception of the issues at stake could scarcely be expected of the scholar and colleague. How different with Bolyai János and Lobachévski, who claimed

at once, unflinchingly, that their discovery marked an epoch in human thought so momentous as to be unsurpassed by anything recorded in the history of philosophy or of science, demonstrating as had never been proven before the supremacy of pure reason at the very moment of overthrowing what had forever seemed its surest possession, the axioms of geometry.

On the 9th of March, 1832, Bolyai Farkas was made corresponding member in the mathematics section of the Magyar Academy.

As professor he exercised a powerful influence in his country.

In his private life he was a type of true originality. He wore roomy black Hungarian pants, a white flannel jacket, high boots, and a broad hat like an old-time planter’s. The smoke-stained wall of his antique domicile was adorned by pictures of his friend Gauss, of Schiller, and of Shakespeare, whom he loved to call the child of nature. His violin was his constant solace.

He died November 20th, 1856. It was his wish that his grave should bear no mark. The mother of Bolyai János , née, Arkosi Benkö Zsuzsanna, was beautiful, fascinating,

of extraordinary mental capacity, but always nervous.

János , a lively, spirited boy, was taught mathematics by his father. His progress was marvelous. He required no explanation of theorems propounded, and made his own demonstrations for them, always wishing his father to go on. “Like a demon, he always pushed me on to tell him more.”

At 12, having passed the six classes of the Latin school, he entered the philosophic-curriculum, which he passed in two years with great distinction.

When about 13, his father, prevented from meeting his classes, sent his son in his stead. The students said they liked the lectures of the son better than those of the father. He already played exceedingly well on the violin.

In his fifteenth year he went to Vienna to K. K. Ingenieur-Akademie.

In August, 1823, he was appointed “souslieutenant” and sent to Temesvár, where he was to present himself on the 2nd of September.

From Temesvár, on November 3rd, 1823, János wrote to his father a letter in Magyar, of which a French translation was sent me by Professor Koncz József on February 14th,

1895. This will be given in full in my life of Bolyai; but here an extract will suffice :

'"*My Dear and Good Father*. "I have so much to write about my new inventions that it is impossible for the moment to enter into great details, so I write you only on one-fourth of a sheet. I await your answer to my letter of two sheets; and perhaps I would not have written you before receiving it, if 1 had not wished to address to you the letter I am writing to the Baroness, which letter I pray you to send her.

"First of all I reply to you in regard to the binominal.

* * * * * * * * * *

"Now to something else, so far as space permits. I intend to write, as soon as I have put it into order, and when possible to publish, a work on parallels.

"At this moment it is not yet finished, but the way which I have followed promises me with certainty the attainment of the goal, if it in general is attainable. It is not yet attained, but I have discovered such magnificent things that I am myself astonished at them.

"It would be damage eternal if they were

lost. When you see them, my father, you yourself will acknowledge it. Now I can not say more, only so much : *that from nothing I have created another wholly new world*. All that I have hitherto sent you compares to this only as a house of cards to a castle.

"P. S.—I dare to judge absolutely and with conviction of these works of my spirit before you, my father; I do not fear from you any false interpretation (that certainly I would not merit), which signifies that, in certain regards, I consider you as a second self."

Prom the Bolyai MSS., now the property of the College at Maros-Vásárhely, Fr. Schmidt has extracted the following statement by János :

"First in the year 1823 have I pierced through the problem in its essence, though also afterwards completions yet were added.

"I communicated in the year 1825 to my former teacher, Herr Johann Walter von Eckwehr (later k. k. General) [in the Austrian Army], a written treatise, which is still in his hands.

"On the prompting of my father I translated my treatise into the Latin language, and

it appeared as *Appendix* to the *Tentamen*, 1832."

The profound mathematical ability of Bolyai János showed itself physically not only in his handling of the violin, where he was a master, but also of arms, where he was unapproachable.

It was this skill, combined with his haughty temper, which caused his being retired as Captain on June 16th, 1833, though it saved him from the fate of a kindred spirit, the lamented Galois, killed in a duel when only 19. Bolyai, when in garrison with cavalry officers, was provoked by thirteen of them and accepted all their challenges on condition that he be permitted after each duel to play a bit on his violin. He came out victor from his thirteen duels, leaving his thirteen adversaries on the square.

He projected a universal language for speech as we have it for music and for mathematics.

He left parts of a book entitled : Principia doctrinae novae quantitatum imaginariarum perfectae uniceque satisfacientis, aliaeque disquisitiones analyticae et analytico-geometricae cardinales gravissimaeque; auctore

Johan. Bolyai de eadem, C. R. austriaco castrensium captaneo pensionato.

Vindobonae vel Maros Vásárhelyini, 1853.

Bolyai Farkas was a student at Goettingen from 1796 to 1799.

In 1799 he returned to Kolozsvár, where Bolyai János was born December 18th, 1802.

He died January 27th, 1860, four years after his father.

In 1894 a monumental stone was erected on his long-neglected grave in Maros-Vásárhely by the Hungarian Mathematico-Physical Society.

APPENDIX.

SCIENTIAM SPATII *absolute veram* exhibens :

a veritate aut falsitate Axiomatis XI *Euclidei*

(*a priori haud unquam decidenda*)

independentemn. adjecta ad casum falsitatis,

quadratura circuli

geometrica.

Auctore JOHANNE BOLYAI de eadem, Geometrarum

in Exercitu Caesareo Regio Austriaco

Castrensium Capitaneo.

EXPLANATION OF SIGNS.

The straight AB means the aggregate of all points situated in the same straight line with A and B.

The sect AB means that piece of the straight AB between the points A and B.

The ray AB means that half of the straight AB which commences at the point A and contains the point B.

The plane ABC means the aggregate of all points situated in the same plane as the three points (not in a straight) A, B, C.

The hemi-plane ABC means that half of the plane *ABC* which starts from the straight AB and contains the point C.

ABC means the smaller of the pieces into which the plane ABC is parted by the rays BA, BC, or the non-reflex angle of which the sides are the rays BA, BC.

ABCD (the point D being situated within ∠ABC, and the straights BA, CD not intersecting) means the portion of ∠ABC comprised between ray BA, sect BC, ray CD; while BACD designates the portion of the plane ABC comprised between the straights AB and CD.

⊥ is the sign of perpendicularity.

‖ is the sign of parallelism.

∠ means angle.

rt.∠ is right angle.

st.∠ is straight angle.

≅ is the sign of congruence, indicating that two magnitudes are superposable.

AB Π CD means ∠CAB = ∠ACD.

x Y *a* means *x* converges toward the limit *a*.

Δ is triangle.

⊙*r* means the [circumference of the] circle of radius r.

area ⊙*r* means the area of the surface of the circle of radius *r*.

[Not Mentioned: □ , ∞]

THE SCIENCE ABSOLUTE OF SPACE.

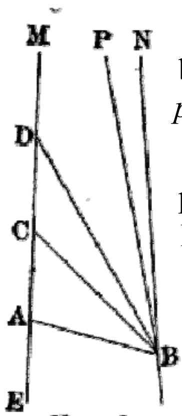

FIG. 1.

§1. If the ray AM is not cut by the ray [3] BN, situated in the same plane, but is cut by every ray BP comprised in the angle ABN, we will call ray BN *parallel* to ray AM; this is designated by BN ║ AM;

It is evident that there *is one such ray BN*, *and only one*, passing through any point B (taken outside of the straight AM), and that the sum of the angles BAM, ABN can not exceed a st. ∠; for in moving BC around B until BAM + ABC = st. ∠, somewhere ray BC *first* does not cut ray AM, and it is then BC ║ AM. It is clear that BN ║ EM, wherever the point E be taken on the straight AM (supposing in all such cases AM > AE).

If while the point C goes away to infinity on ray AM, always CD = CB, we will have constantly CDB = (CBD < NBC); but NBC Y 0; and so also ADB Y 0.

§ 2. If BN ‖ AM, we will have also CN ‖ AM. For take D anywhere in MACN. If C is on ray BN, ray BD cuts ray AM, since BN ‖ AM, and so also ray CD cuts ray AM. But if C is on ray BR take BQ ‖ CD; BQ falls within the ∠ABN (§1), and cuts ray AM; and so also ray CD cuts ray AM. Therefore every ray CD (in ACN) cuts, in each case, the ray AM, without CN itself cutting ray AM. Therefore always CN ‖ AM.

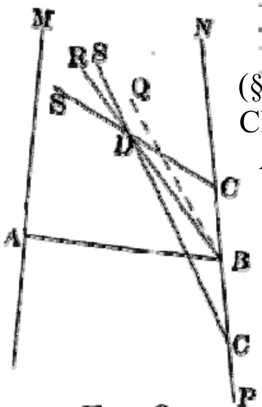

FIG. 2.

§ 3. (Fig. 2.) If BR and CS and each ‖ AM, and C is not on the ray BR, then ray BR and ray CS do not intersect. For if ray BR and ray CS had a common point D, then (§ 2) DR and DS would be each ‖ AM, and ray DS (§ 1) would fall on ray DR, and C on the ray BR (contrary to the hypothesis).

§ 4. If MAN > MAB, we will have for every point B of ray AB, a point C of ray AM, such that BCM = NAM.

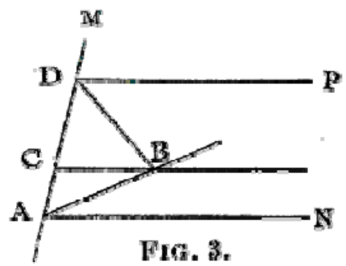

FIG. 3.

For (by § 1) is granted BDM > NAM, and so that MDP = MAN, and B falls in

NADP. If therefore NAM is carried along AM until ray AN arrives on ray DP, ray AN will somewhere have necessarily passed through B, and some BCM = NAM.

§ 5. If BN ‖ AM, there is on the straight [4] AM a point F such that FM Π BN. For by § 1 is granted BCM > CBN; and if CE = CB, and so EC Π BC; evidently BEM < EBN. The point P is moved on EC, the angle BPM always being called *u*, and the angle PBN always *v*, evidently *u* is at first less than the corresponding *v*, but afterwards greater. Indeed *u* increases *continuously* from BEM to BCM; since (by ~ 4) there exists no angle > BEM and < BCM, to which u does not at some time become equal. Likewise v decreases continuously from EBN to CBN. There is therefore on EC a point F such that BFM = FBN.

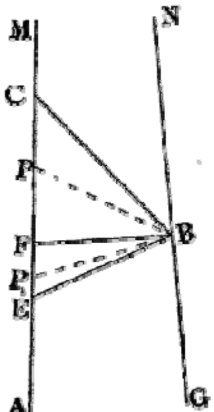

FIG. 4.

§ 6. If BN ‖ AM and E anywhere in the straight AM, and G in the straight BN; then GN ‖ EM and EM ‖ GN. For (by § 1) BN ‖ EM, whence (by § 2) GN ‖ EM. If moreover FM Π BN (§ 5); then MFBN ≅ NBFM, and consequently (since BN ‖ FM) also FM ‖ BN, and (by what precedes) EM ‖ GN.

§ 7. If BN and CP are each ‖ AM, and C not on the straight BN; also BN ‖ CP. For the rays BN and CP do not intersect (§ 3); but AM, BN and CP either are or are not in the same plane; and in the first case, AM either is or is not within BNCP.

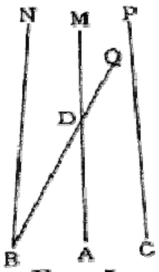

FIG. 5.

If AM, BN, CP are coplanar, and AM falls within BNCP; then every ray BQ (in NBC) cuts the ray AM in some point D (since BN ‖ AM); moreover, since DM ‖ CP (§ 6), the ray DQ will cut the ray CP, and so BN ‖ CP.

But if BN and CP are on the same side of AM; then one of them, for example CP, falls between the two other straights BN, AM : but every ray BQ (in NBA) cuts the ray AM, and so also the straight CP. Therefore BN ‖ CP.

FIG. 6.

If the planes MAB, MAC make *an angle*; then CBN and ABN have in common nothing but the ray BN, while the ray AM (in ABN) and the ray BN, and so also NBC and the ray AM have nothing in common.

But hemi-plane BCD, drawn through any ray BD (in NBA), cuts the ray AM, since ray

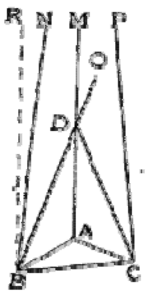

FIG. 7.

BQ cuts ray AM (as BN ∥ AM). Therefore in revolving the hemi-plane BCD around BC until it *begins* to leave the ray AM, the hemi-plane BCD at last will fall upon the hemi-plane BCN. For the same reason this same will fall upon hemi-plane BCP. Therefore BN falls in BCP. Moreover, if BR ∥ CP; then (because also AM ∥ CP) by like reasoning, BR falls in BAM, and also (since BR ∥ CP) in BCP. Therefore the straight BR, being common to the two planes MAB, PCB, of course is the straight BN, and hence BN ∥ CP.*

If therefore CP ∥ AM, and B exterior to the plane CAM; then the intersection BN of the planes BAM, BCP is ∥ as well to AM as to CP.

§ 8. If BN ∥ and Π CP (or more briefly BN ∥ Π CP), and AM (in NBCP) bisects ⊥ the sect BC; then BN ∥ AM.

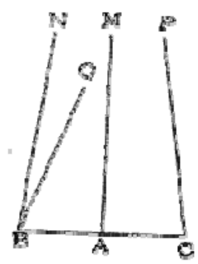

FIG. 8.

For if ray BN cut ray AM, also -ray CP would cut ray AM at the same point (because MABNB ≅ MACP), and this would be common to the rays BN, CP themselves,

* The third case being put before the other two, these can be demonstrated together with more brevity and elegance, like case 2 of ~10. [Author's note.]

although BN ‖ CP. But every ray BQ (in CBN) cuts ray CP; and so ray BQ cuts also ray AM. Consequently BN ‖ AN.

§ 9. If BN II AM, and MAP ⊥ MAB, and the ∠, which NBD makes with NBA (on that side of MABN, where MAP is) is < rt.∠; then MAP and NBD intersect.

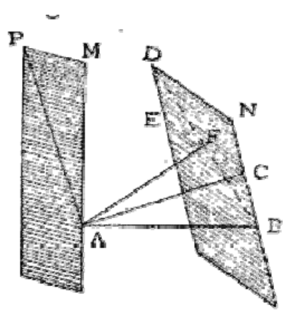

FIG. 9.

For let ∠BAM = rt.∠, and AC ⊥ BN (whether or not C falls on B), and CE ⊥ BN (in NBD); by hypothesis ∠ACE < rt.∠, and AF (⊥ CE) will fall in ACE.

Let ray AP be the intersection of the hemi-planes ABF, AMP (which have the point A common); since BAM ⊥ MAP, ∠BAP = ∠BAM = rt.∠.

If finally the hemi-plane ABF is placed upon the hemi-plane ABM (A and B remaining), ray AP will fall on ray AM; and since AC ⊥ BN, and sect AF < sect AC, evidently sect AF will terminate within ray BN, and so BF falls in ABN. But in *this* position, ray BF cuts ray AP (because BN ‖ AM); and so ray AP and ray BF intersect also *in the* original position; and the point of section is common to the hemi-planes MAP and NBD. Therefore the hemi-planes MAP and NBD intersect. Hence follows

easily that the hemi-planes MAP and NBD intersect if the sum of the interior angles which they make with MABN is < st.∠.

§ 10. If both BN and CP ∥ Π AM; also is BN ∥ Π CP.

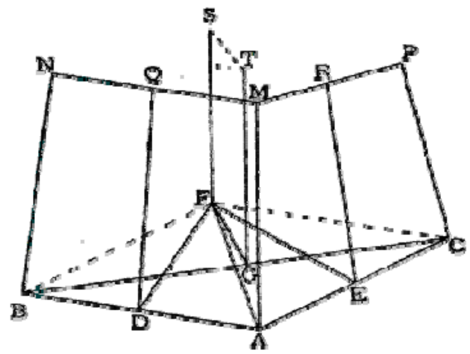

FIG. 10.

For either MAB and MAC make an angle, or they are in a plane.

If the first; let the hemi-plane QDF bisect ⊥ sect AB; then DQ ⊥ AB, and so DQ ∥ AM (§ 8); likewise if hemi-plane ERS bisects ⊥ sect AC, is ER ∥ AM; whence (§ 7) DQ ∥ ER.

Hence follows easily (by § 9), the hemi-planes QDF and ERS intersect, and have (§ 7) their intersection FS ∥ DQ, and (on account of BN ∥ DQ) also FS ∥ BN. Moreover (for any point of FS) FB = FA = FC, and the straight FS falls in the plane TGF, bisecting ⊥ sect BC. But (by § 7) (since FS ∥ BN) also GT ∥ BN. In the same way is proved GT ∥ CP. Meanwhile GT bisects ⊥ sect BC; and so TGBN ≅ TGCP (§ 1), and BN ∥ Π CP.

If BN, AM and CP are in a plane, let (falling without this plane) FS ∥ Π AM; then (from

what precedes) FS ‖ ⊥ both to BN and to CP, and so also BN ‖ ⊥ CP.

§ 11. Consider the aggregate of the point A, and *all* points of which any one B is such, that if BN ‖ AM, also BN Π AM; call it F; but the intersection of F with any plane containing the sect AM call L.

F has a point, and one only, on any straight ‖ AM; and evidently L is divided by ray AM into two congruent parts.

Call the ray AM *the axis* of L. Evidently also, in any plane containing the sect AM, there is for the *axis* ray AM a single L. Call any L of this sort the L of this ray AM (in the plane considered, being understood). Evidently by revolving L around AM we describe the F of which ray AM is called the axis, and in turn F *may be ascribed to the axis ray AM*.

§ 12. If B is anywhere on the L of ray AM, and BN ‖ Π AM (§ 11); then the L of ray AM and the L of ray BN *coincide*. For suppose, in distinction, L′ the L of ray BN. Let C be anywhere in L′, and CP ‖ Π BN (§ 11). Since BN ‖ Π AM, so CP ‖ Π AM (§ 10), and so C also will fall on L. And if C is anywhere on L, and CP ‖ Π AM; then CP ‖ Π BN (§ 10); and C also falls on L′ (§ 11). Thus L and L′ are the

same; and every ray BN is also axis of L, and between all axes of this L, is Π.

The same is evident in the same way of F.

§ 13. If BN ∥ AM, and CP ∥ DQ, and ∠BAM + ∠ABN = st.∠; then also ∠DCP + ∠CDQ = st.∠.

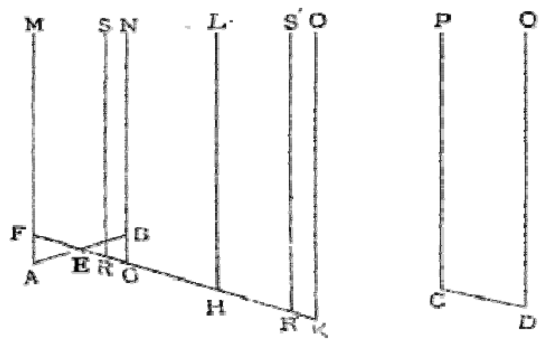

FIG. 11.

For let EA = EB, and EFM = DCP (§ 4). Since ∠BAM + ∠ABN = st. ∠ = ∠ABN + ∠ ABG, we have ∠EBG = ∠EAF; and so if also BG = AF, then ΔEBG ≅ ΔEAF, ∠BEG = ∠AEF and G will fall on the ray FE. Moreover ∠GFM + ∠FGN = st. ∠ (since ∠EGB = ∠EFA).

Also GN ∥ FM (§ 6).

Therefore if MFRS ≅ PCDQ, then RS ∥ GN (§ 7), and R falls within or without the sect FG (unless sect CD = sect FG, where the thing now is evident).

I. In the first case ∠FRS is not > (st. ∠ – ∠RFIM = ∠FGN), since RS ∥ FM. But as RS ∥ GN, also ∠FRS is not < ∠FGN; and so ∠FRS = ∠FGN, and ∠RFM + ∠FRS = ∠GFM +

∠FGN = st.∠. Therefore also ∠DCP + ∠CDQ = st. ∠.

II. If R falls without the sect FG; then ∠NGR = ∠MFR, and let MFGN ≅ NGHL ≅ LHKO, and so on, until FK = FR or begins to be > FR. Then KO ∥ HL ∥ FM (§7).

If K falls on R, then KO falls on RS (§ 1); and so ∠RFM + ∠FRS = ∠KFM + ∠FKO = ∠KFM + ∠FGN = st.∠; but if R falls within the sect HK, then (by I) ∠RHL + ∠KRS = st.∠ = ∠RFM + ∠FRS = ∠DCP + ∠CDQ.

§ 14. If BN ∥ AM, and CP ∥ DQ, and ∠BAM + ∠ABN < st.∠; then also ∠DCP + ∠CDQ < st.∠.

For if ∠DCP + ∠CDQ were not < st.∠, and so (by § 1) were = st.∠, then (by § 13) also ∠BAM + ∠ABN = st.∠ (contra hyp.).

15. Weighing §§ 13 and 14, *the System of Geometry resting on the hypothesis of the truth of Euclid's Axiom XI is called Σ*; *and the system founded on the contrary hypothesis is S*.

All things which are not expressly said to be in Σ or in S, *it is understood are enunciated absolutely*, *that is are asserted true whether Σ or S is reality*.

§16. If AM is the axis of any L; then L, in Σ is a straight ⊥ AM.

FIG. 12.

For suppose BN an axis from any point B of L; in Σ, ∠BAM + ∠ABN = st.∠, and so ∠BAM = rt.∠.

And if C is any point of the straight AB, and CP ∥ AM; then (by § 13) CP Π AM, and so C on L (§ 11).

But in S, no three points A, B, C on L or on F are in a straight. For some one of the axes AM, BN, CP (e.g. AM) falls between the two others; and then (by § 14) ∠BAM and ∠CAM are each < rt.∠.

§ 17. *L in S also is a line*, *and F a surface*. For (by § 11) any plane ⊥ to the axis ray AM (through any point of F) cuts F in [the circumference of] a circle, of which the plane (by § 14) is ⊥ to no other axis ray BN. If we revolve F about BN, any point of F (by § 12) will remain on F, and the section of F with a plane not ⊥ ray BN will describe a surface; and whatever be the points A, B taken on it, F can so be congruent to itself that A falls upon B (by § 12); therefore F is a *uniform surface*.

Hence evidently (by §§ 11 and 12) L is a uniform line.*

§ 18. *The intersection* with F of *any plane*, drawn through a point A of F obliquely to the axis AM, is, in S, *a circle*.

For take A, B, C, three points of this section, and BN, CP, axes; AMBN and AMCP make an angle, for otherwise the plane determined by A, B, C (from § 16) would contain AM, (contra hyp.). Therefore the planes bisecting ⊥ the sects AB, AC intersect (§ 10) in some *axis* ray FS (of F), and FB = FA = FC.

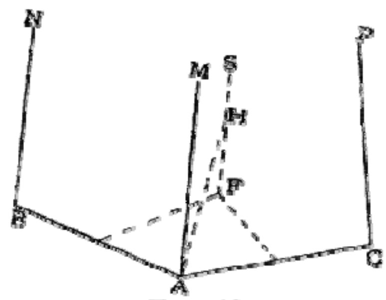

FIG. 13.

Make AH ⊥ FS, and revolve FAH about FS; A will describe a circle of radius HA, passing, through B and C, and situated *both* in F and in the plane ABC; nor have F and the plane ABC anything in common but ⊙ HA (§ 16).

It is also evident that in revolving the portion FA of the line L (as radius) in F around F, its extremity will describe ⊙ HA.

* It is not necessary to restrict the demonstration to the system S; since it may easily be so set forth, that it holds absolutely for S and for Σ.

§ 19. *The perpendicular* BT to the axis BN of L (falling in the plane of L) is, in S, N *tangent* to L. For L has in ray BT no point except B (§ 14), but if BQ falls in TBN, then the center of the section of the plane through BQ perpendicular to TBN with the F of ray BN (§ 18) is evidently located on ray BQ; and if sect BQ is a diameter, evidently ray BQ cuts in Q the line L of ray BN.

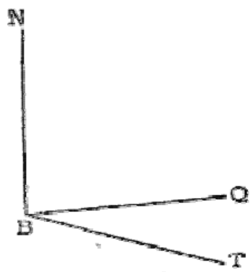

FIG. 14.

§ 20. Any two points of F determine a line L (§§ 11 and 18); and since (from §§ 16 and 19) L is ⊥ to all its axes, every ∠ of lines L in F is equal to the ∠ of the planes drawn through its sides perpendicular to F.

21. Two L form lines, ray AP and ray BD, in the same F, making with a third L form AB, a sum of interior angles < st.∠, intersect.

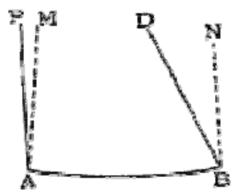

FIG. 15.

(By line AP in F, is to be understood the line L drawn through A and P, but by ray AP that half of this line beginning at A, in which P falls.)

For if AM, BN are axes of F, then the hemi-planes AMP, BND intersect (§ 9); and F cuts

their intersection (by §§ 7 and 11); and so also ray AP and ray BD intersect.

From this it is evident that Euclid's Axiom XI and all things which are claimed in geometry and plane trigonometry hold good *absolutely* in F, L lines being substituted in place of straights : therefore the trigonometric functions are taken here in the same sense as in Σ; and the circle of which the L form radius = r in F, is $2\pi r$; and likewise area of ⊙r (in F) = πr^2 (by π understanding ½ ⊙ 1 in F, or the known 3.1415926 . . .)

§ 22. If ray AB were the L of ray AM, and C on ray AM; and the ∠CAB (formed by the straight ray AM and the L form line ray AB), carried first along the ray AB, then along the ray BA, always forward to infinity : the path CD of C will be the line L of CM.

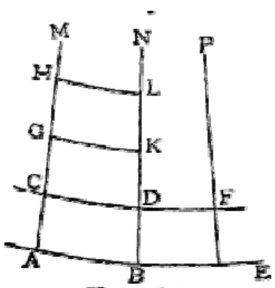

FIG. 16.

For let D be any point in line CD (called later L′, let DN be ∥ CM, and B the point of L falling on the straight DN. We shall have BN Π AM, and sect AC = sect BD, and so DN Π CM, consequently D in L′. But if D in L′ and DN ∥ CM, and B the point of L on the straight DN; we shall have AM Π BN and CM Π DN, whence manifestly sect BD = sect AC,

and D will fall on the path of the point C, and L′ and the line CD are the same. Such an L′ is designated by L′ 8 L.

§ 23. If the L form line CDF 8 ABE (§ 22), and AB = BE, and the rays AM, BN, EP are axes; manifestly CD = DF; and if any three points A, B, E are of line AB, and AB = n.CD, we shall also have AE = n.CF; and so (manifestly even for AB, AE, DC incommensurable), AB : CD = AE : CF, and AB : CD is *independent of AB*, *and completely determined by AC*.

This ratio AB : CD is designated by the capital letter (as X) corresponding to the small letter (as X) by which we represent the sect AC.

§ 24. Whatever be *x* and *y*, (§23), $Y = X^{\frac{x}{y}}$.

For, one of the quantities *x*, *y* is a multiple of the other (e. g. *y* of *x*), or it is not.

If *y* = n.*x*, take *x* = AC = CG = GH = &c., until we get AH = *y*.

Moreover, take CD 8 GK 8 HL.

We have ((§ 23) X = AB : CD – CD : GK = GK : HL; and so

$$\frac{AB}{HL} = \left(\frac{AB}{CD}\right)^n$$

or $Y = X^n = X^{\frac{y}{x}}$.

If *x*, *y* are multiples of *i*, suppose *x* = *mi*, and *y* = *ni*; (by the preceding) $X = I^m$, $Y = I^n$, consequently

$$Y = X^{\frac{n}{m}} = X^{\frac{y}{x}}$$

The same is easily extended to the case of the incommensurability of x and y.

But if $q = y - x$, manifestly $Q = Y : X$. It is also manifest that in Σ, for any x, we have $X = 1$, but in S is X > 1, and for any AB and ABE there is such a CDF 8 AB, that CDF = AB, whence AMBN ≅ AMEP, though the first be any multiple of the second; which indeed is singular, but evidently does not prove the absurdity of S.

§ 25. *In any rectilineal triangle*, *the circles with radii equal to its sides are as the sines of the opposite angles*.

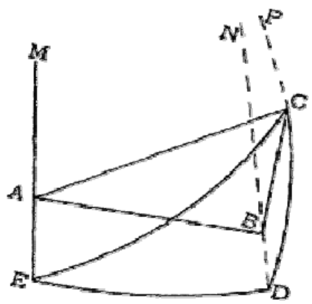

FIG. 17.

For take ∠ABC = rt.∠, and AM ⊥ BAC, and BN and CP ∥ AM; we shall have CAB ⊥ AMBN, and so (since CB ⊥ AMBN, consequently CPBN ⊥ AMBN.

Suppose the F of ray CP cuts the straights BN, AM respectively in D and E, and the bands CPBN, CPAM, BNAM along the L form lines CD, CE, DE. Then (§ 20) ∠CDE = the angle of NDC, NDE, and so = rt.∠; and by like reasoning ∠CED = CAB. But (by § 21) in the L line ΔCDE (supposing always here the radius = 1),

EC : DC = 1 : sin DEC = : sin CAB.

Also (by § 21)

EC : DC = ⊙EC : ⊙DC (in F) = ⊙AC : ⊙BC (§ 18); and so is also

$$\odot AC : \odot BC - 1 : \sin CAB;$$

whence the theorem is evident for any triangle.

§ 26. *In any spherical triangle, the sines of the sides are as the sines of the angles opposite.*

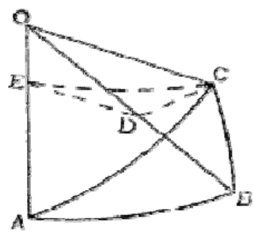

FIG. 18.

For take ∠ABC = rt.∠, and CED ⊥ to the radius OA of the sphere. We shall have CED ⊥ AOB, and (since also BOC ⊥ BOA), CD ⊥ OB. But in the triangles CEO, CDO (by § 25) ⊙EC : ⊙OC : ⊙DC = sin COE : 1 : sin COD = sin AC : 1 : sin BC; meanwhile also (§ 25) ⊙EC : ⊙DC = sin CDE : sin CED. Therefore, sin AC : sin BC = sin CDE : sin CED; but CDE = rt.∠ = CBA, and CED = CAB. Consequently

$$\sin AC : \sin BC = 1 : \sin A.$$

Spherical trigonometry, lowing from this, is thus established independently of Axiom XI.

§ 27. If AC and BD are ⊥ AB, and CAB is carried along the straight AB; we shall have, designating by CD the path of the point C,

$$CD : AB = \sin u : \sin v.$$

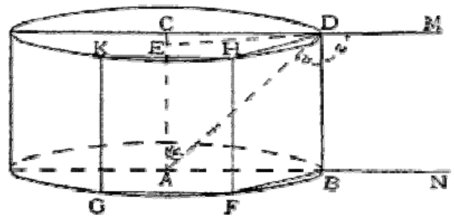

FIG. 19.

For take DE ⊥ CA; in the triangles ADE, ADB (by § 25) ⊙ED : ⊙AD : ⊙AB = sin u : 1 : sin v.

In revolving BACD about AC, B describes ⊙AB, and D describes ⊙ED; and designate here by s⊙CD the path of the said CD. Moreover, let there be any polygon BFG . . . inscribed in ⊙AB.

Passing through all the sides BF, FG, &c., planes ⊥ to ⊙AB we form also a polygonal figure of the same number of sides in s⊙CD, and we may demonstrate, as in § 23, that CD : AB = DH : BF = HK : FG, &c., and so

DH + HK &c. : BF + FG &c. : = CD : AB.

If each of the sides BF, FG . . . approaches the limit zero, manifestly

BF + FG + . . . Y⊙AB and

DH + HK + . . . ⊙ED.

Therefore also ⊙ED : ⊙AB = CD : AB. But we had ⊙ED : ⊙AB = sin u : sin v. Consequently

$$CD : AB = \sin u : \sin v.$$

If AC goes away from BD to infinity, CD : AB, and so also sin u : sin v remains *constant*; but u Y rt.∠ (§1), and if DM ∥ BN, v Y z; whence CD : AB = 1 : sin z.

The path called CD will be denoted by CD 8 AB.

§ 28. If BN ‖ Π AM, and C in ray AM, and AC = x. we shall have (§ 23)

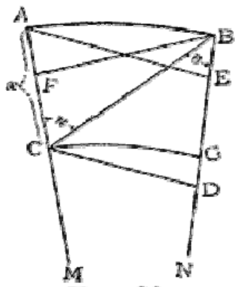

FIG. 20.

$$X = \sin u : \sin v.$$

For if CD and AE are ⊥ BN, and BF ⊥ AM; we shall have (as in § 27)

$$\odot BF : \odot DC = \sin u : \sin v.$$

But evidently BF = AE : therefore

$$\odot EA : \odot CD = \sin u : \sin v.$$

But in the F form surfaces of AM and CM (cutting AMBN in AB and CG) (by § 21)

$$\odot EA : \odot DC = AB : CG = X.$$

Therefore also

$$X = \sin u : \sin v.$$

§ 29. If ∠BAM – rt.∠, and sect AB = y, and BN ‖ AM, we shall have in S

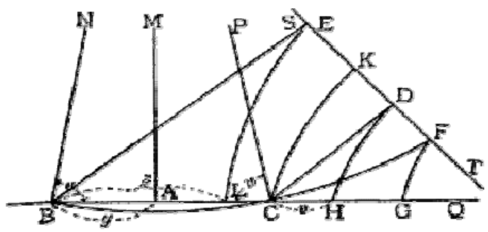

FIG. 21.

$$Y = \cot \tfrac{1}{2} u.$$

For, if sect AB = sect AC, and CP ‖ AM (and so BN ‖ CP), and ∠PCD = ∠QCD; there is given (§19) DS ⊥ ray CD, so that DS ‖ CP, and so (§ 1) DT ‖ CQ. Moreover, if BE ⊥ ray DS, then (§ 7) DS ‖ BN, and so (§ 6)

BN ‖ ES, and (since DT ‖ CG) BQ ‖ ET; consequently (§1) ∠EBN = ∠EBQ. Let BCF be an L-line of BN, and FG, DH, CK, EL, L form lines of FT, DT, CQ and ET; evidently (§ 22) HG = DF = DK = HC; therefore,

$$CG = 2CH = 2v.$$

Likewise it is evident BG – 2BL = 2*z*.

But BC = BG – CG; wherefore $y = z - v$, and so (§24) Y = Z : V.

Finally (§ 28)

Z = : sin ½ u,

and V = : sin (rt.∠ – ½ *u*),

consequently Y - cotan ½ *u*.

§ 30. However, it is easy to see (by § 25) that the solution of the problem of Plane Trigonometry, in S, requires the expression of the circle in terms of the radius; but this can by obtained by the rectification of L.

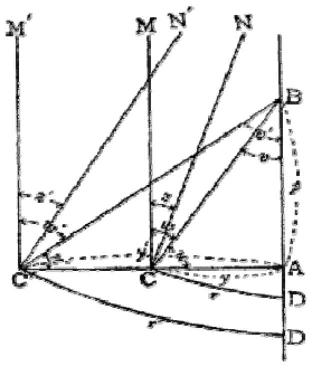

FIG. 22.

Let AB, CM, C'M′ be ⊥ ray AC, and B anywhere in ray AB; we shall have (§ 25)

$$\sin u : \sin v = \odot p : \odot y,$$

$$\text{and } \sin u' : \sin v' = \odot p' : \odot y';$$

$$\text{and so } \frac{\sin u}{\sin v} \cdot \odot y = \frac{\sin u'}{\sin v'} \cdot \odot y'.$$

But (by § 27) $\sin v : \sin v' = \cos u : \cos u'$;

consequently $\frac{\sin u}{\sin u} \cdot \odot y = \frac{\sin u'}{\sin u'} \cdot \odot y'$; or $\odot y : \odot y' = : \tan u' : \tan u = \tan w : \tan w'$.

Moreover, take CN and C'N′ ∥ AB, and CD, C'D' L-form lines ⊥ straight AB; we shall have also (§21)

$$\odot y : \odot y' = r : r', \text{ and so}$$

$$r : r' = \tan w : \tan w'.$$

Now let p beginning from A increase to infinity; then w Y z, and w' Y z', whence also $r : r' = \tan z : \tan z'$.

Designate by i the *constant*

$r : \tan z$ (independent of r);

whilst y Y 0,

$\frac{r}{y} = \frac{i \tan z}{y}$ Y 1, and so

$$\frac{y}{\tan z} \text{ Y } i. \text{ From §29, } \tan z = ½ (\text{Y} – \text{Y}^{-1});$$

therefore $\frac{2y}{\text{Y} - \text{Y}^{-1}}$ Y i,

or (§ 24). $$\frac{2y \cdot \text{I}^{\frac{y}{1}}}{\text{I}^{\frac{2y}{1}} - 1} \text{ Y } i.$$

But we know the limit of this expression (where y Y 0) is.

$$\frac{i}{\text{nat. log I}} \qquad \text{Therefore}$$

$$\frac{i}{\text{nat. log I}} = i, \text{ and}$$

$$\text{I} = e = 2.7182818 \ldots,$$

which noted quantity shines forth here also.

If obviously henceforth i denote that sect of which the I = e, we shall have

$$r = i \tan z.$$

But (§ 21) $\odot y = 2\pi r$; therefore

$$\odot y = 2\pi i \tan z = \pi i\,(\text{Y} - \text{Y}^{-1}) = \pi i \left(e^{\frac{y}{i}} - e^{\frac{y}{-i}}\right)$$

$$= \frac{\pi y}{\text{nat. log Y}}\,(\text{Y} - \text{Y}^{-1}) \text{ (by § 24).}$$

§ 31. For the trigonometric solution of all right-angled rectilineal *triangles* (whence the resolution of all *triangles* is easy, in S, three equations suffice : indeed (a, b denoting the sides, c the hypothenuse, and α, β the angles opposite the sides) an equation expressing the relation

1st, between a, c, α;

2d, between a, α, β;

3d, between a, b, c;

of course from these equations emerge three others by elimination.

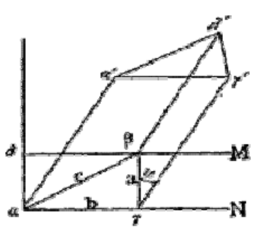

FIG. 23.

From §§ 25 and 30

$$1 : \sin \text{a} = (\text{C} - \text{C'}) : (\text{A} - \text{A}^{-1}) = \left(e^{\frac{\text{e}}{\text{i}}} - e^{\frac{-\text{e}}{\text{i}}}\right) : \left(e^{\frac{\text{a}}{\text{i}}} - e^{\frac{-\text{a}}{\text{i}}}\right)$$

(equation for c, a and α).

II. From § 27 follows (if βM $\parallel$ γN)

cos α : sin β = 1 : sin u, but from § 29

$$1 : \sin u = \tfrac{1}{2}\left(\mathrm{A} + \mathrm{A}^{-1}\right);$$

therefore $\cos\alpha : \sin\beta = \tfrac{1}{2}\left(\mathrm{A} + \mathrm{A}^{-1}\right) = \tfrac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)$

(equation for α, β and a).

III. If $\alpha\alpha' \perp \beta\alpha\gamma$, and $\beta\beta'$ and $\gamma\gamma' \parallel \alpha\alpha'$ (§ 27), and i $\beta'\alpha'\gamma' \perp \alpha\alpha'$; manifestly (as in § 27),

$$\frac{\beta\beta'}{\gamma\gamma'} = \frac{1}{\sin u} = \tfrac{1}{2}\left(\mathrm{A} + \mathrm{A}^{-1}\right);$$

$$\frac{\gamma\gamma'}{\alpha\alpha'} = \tfrac{1}{2}\left(\mathrm{B} + \mathrm{B}^{-1}\right);$$

$$\text{and } \frac{\beta\beta'}{\alpha\alpha'} = \tfrac{1}{2}\left(\mathrm{C} + \mathrm{C}^{-1}\right); \text{ consequently}$$

$$\tfrac{1}{2}\left(\mathrm{C} + \mathrm{C}^{-1}\right) = \tfrac{1}{2}\left(\mathrm{A} + \mathrm{A}^{-1}\right) \cdot \tfrac{1}{2}\left(\mathrm{B} + \mathrm{B}^{-1}\right), \text{ or}$$

$$\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right) = \tfrac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right) \cdot \left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)$$

(equation for a, b and c).

If $\gamma\alpha\delta$ = rt.$\angle$, and $\beta\delta \perp \alpha\delta$;

$\odot\, c : \odot\, a = 1 : \sin\alpha$, and

$\odot\, c : \odot\, (d = \beta\delta) = 1 : \cos\alpha$,

and so (denoting by $\odot\, x^2$, for any x, the product $\odot\, x \cdot \odot\, x$) manifestly

$\odot\, a^2 + \odot\, d^2 - \odot\, c^2$.

But (by § 27 and II)

$$\odot\, d = \odot\, b \,.\, \tfrac{1}{2}\left(\mathrm{A} + \mathrm{A}^{-1}\right), \text{ consequently}$$

$$\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right)^2 = \tfrac{1}{4}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)^2 \cdot \left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)^2 + \left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)^2$$

another equation for a, b and c (the second

member of which may be easily reduced to a form *symmetric* or *invariable*).

Finally, from

$$\frac{\cos\alpha}{\sin\beta} = \tfrac{1}{2}(A + A^{-1}), \text{ and } \frac{\cos\beta}{\sin\alpha} = \tfrac{1}{2}(B + B^{-1}), \text{ we get}$$

(by III)

$$\cot\alpha\cot\beta = \tfrac{1}{2}\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right)$$

(equation for α, β, and c.)

§ 32. It still remains to show briefly the mode of resolving *problems* in S, which being accomplished (through the more obvious examples), finally will be candidly said what this theory shows.

I. Take AB a line in a plane, and $y = f(x)$ its equation in rectangular coordinates, call dz any increment of z, and respectively dx, dy, du the increments of x, of y, and of the area u,

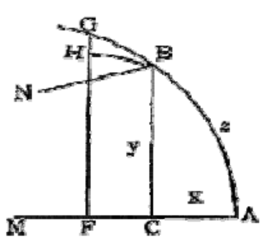

FIG. 24.

corresponding to this dz; take BH 8 CF, and express (from § 31) $\frac{BH}{dx}$ by means of y, and seek the *limit* of $\frac{dy}{dx}$ when dx tends towards the limit zero (which is understood where a limit of this sort is sought) : then will become known also the limit of $\frac{dy}{BH}$ and so tan HBG; and

(since HBC manifestly is neither > nor < , and so = rt∠.), the *tangent* at B of BG will be determined by *y*.

II. It can be demonstrated

$$\frac{dz^2}{dy^2 + \overline{\mathrm{BH}}^{\,2}} \;\mathrm{Y}\; 1.$$

Hence is found the *limit* of $\frac{dz}{dx}$ and thence, by integration, *z* (expressed in terms of *x*.)

And of any line *given in the concrete*, the equation in *S* can be found; e. g., of L. For if ray AM be the axis of L; then any ray CB from ray AM cuts L [since (by § 19) any straight from A except the straight AM will cut L]; but (if BN is axis)

$$\mathrm{X} = 1 : \sin \mathrm{CBN}\ (\S 28),$$

$$\text{and } \mathrm{Y} = \cot\tfrac{1}{2}\,\mathrm{CBN}\ (\S\ 29), \text{ whence}$$

$$\mathrm{Y} = \mathrm{X} + \sqrt{\mathrm{X}^2 - 1}.$$

$$\text{or}\quad e^{\frac{y}{i}} = e^{\frac{x}{i}} + \sqrt{e^{\frac{2x}{i}} - 1},$$

the equation sought.

Hence we get

$$\frac{dy}{dy} \;\mathrm{Y}\; \mathrm{X}(\mathrm{X}^2 - 1)^{-\frac{1}{2}};$$

and $\frac{\mathrm{BH}}{dx}$ Y1 : sin CBN = X; and so

$$\frac{dy}{\mathrm{BH}} \;\mathrm{Y}\; (\mathrm{X}^2 - 1)^{-\frac{1}{2}};$$

$$1 + \frac{dy^2}{BH^2} \text{ Y } X^2(X^2 - l)^{-1},$$

$$\frac{dz^2}{BH^2} \text{ Y } X^2(X^2 - l)^{-1},$$

and $\frac{dz}{BH}$ Y $X (X^2 - l)^{\frac{1}{2}}$ and

$\frac{dz}{dx}$ Y $X^2 (X^2 - l)^{-\frac{1}{2}}$, whence, by integration, we get (as in § 30)

$$z = i\,(X^2 - l)^{\frac{1}{2}} = i \cot CBN.$$

III. Manfestly

$$\frac{du}{dx} \text{ Y } \frac{HFCBH}{dx}$$

which (unless given in y) now first is to be expressed in terms of y; whence we get u by integrating.

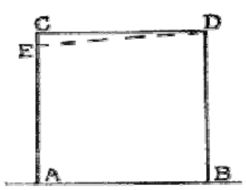

FIG. 25.

If AB = p, AC = q, CD = r, and CABDC = s; we might show (as in II) that

$\frac{ds}{dq}$ Y r, which $= -\frac{1}{2}\, p \left(e^{\frac{q}{i}} - e^{\frac{-q}{i}}\right)$, and, integrating, $s = \frac{1}{2}\, pi \left(e^{\frac{q}{i}} - e^{\frac{-q}{i}}\right)$.

This can also be deduced apart from integration.

For example, the equation of the circle (from § 31, III), of the straight (from § 31, II), of a conic (by what precedes), being expressed, the

areas bounded by these lines could also be expressed.

We know, that a surface *t*, 8 to a plane figure *p* (at the distance *q*), is to *p* in the ratio of the second powers of homologous lines, or as

$$\tfrac{1}{4}\left(e^{\frac{q}{i}} - e^{\frac{-q}{i}}\right)^2 : 1.$$

It is easy to see, moreover, that the calculation of volume, treated in the same manner, requires two integrations (since the differential itself here is determined only by integration); and before all must be investigated the volume contained between *p* and *t*, and the aggregate of all the straights $\perp p$ and joining the boundaries of *p* and *t*.

We find for the volume of this solid (whether by integration or without it)

$$\tfrac{1}{8}\,pi\left(e^{\frac{2q}{i}} - e^{\frac{-2q}{i}}\right) + \tfrac{1}{2}\,pq.$$

The surfaces of bodies may also be determined in S, as well as the *curvatures*, the *involutes*, and *evolutes* of any lines, etc.

As to curvature; this in S either is the curvature of L, or is determined either by the radius of a circle, or by the *distance* to a straight from the curve 8 to this straight; since from what precedes, it may easily be shown, that in a plane there are no uniform lines other than L-lines, circles and curves 8 to a straight.

IV. For the circle (as in III) $\frac{d \text{ area} \square x}{dx}$ Y ⊙ x, whence (by § 29), integrating,

$$\text{area} \odot x = \pi i^2 \left(e^{\frac{x}{i}} - 2 + e^{\frac{-x}{i}}\right).$$

V. For the area CABDC = u (inclosed by an L form line AB = r, the 8 to this, CD = y, and the sects AC = BD = x) $\frac{du}{dx}$ Y y; and (§ 24) y = $re^{\frac{-x}{i}}$, and so (integrating)

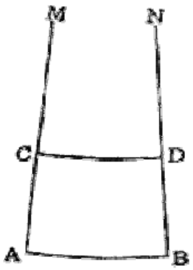

FIG. 26.

$$u = ri\left(1 - e^{\frac{x}{i}}\right).$$

If x increases to infinity, then, in S, $e^{\frac{-x}{i}}$ Y 0, and so u Y ri. By the *size* of MABN, in future this limit is understood.

In like manner is found, if p is a figure on F, the space included by p and the aggregate of axes drawn from the boundaries of p is equal to ½ pi.

VI. If the angle at the center of a segment z of a sphere is $2u$, and a great circle is p, and x the arc FC (of the angle u); (§25)

$$1 : \sin u = p : \odot \text{BC},$$

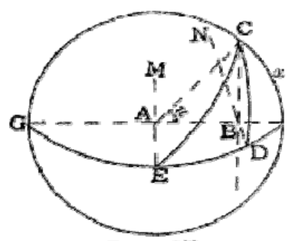

FIG. 27.

and hence ⊙ BC = $p \sin u$.

Meanwhile $x = \frac{pu}{2\pi}$, and $dx = \frac{pdu}{2\pi}$.

Moreover, $\dfrac{dz}{dx}$ Y ⊙ BC, and hence

$$\frac{dz}{du} \text{ Y } \frac{p^2}{2\pi} \sin u, \text{ whence (integrating)}$$

$$z = \frac{\text{ver sin } u}{2\pi} p^2.$$

The F may be conceived on which P falls (passing through the middle F of the segment); through AF and AC the planes FEM, CEM are placed, perpendicular to F and cutting F along FEG and CE; and consider the L form CD (from C ⊥ to FEG), and the L form CF; (§ 20) CEF = u, and (§ 21)

$$\frac{\text{FD}}{p} = \frac{\text{ver sin } u}{2\pi}, \text{ and so } z - \text{FD} \cdot p.$$

But (§ 21) $p = \pi \cdot \text{FGD}$; therefore

$z = \pi \cdot \text{FD} \cdot \text{FDG}$. But (§ 21)

$\text{FD} \cdot \text{FDG} = \text{FC} \cdot \text{FC}$; consequently

$z = \pi \cdot \text{FC} \cdot \text{FC} =$ area ⊙ FC, in F.

Now let BJ = CJ = r; (§ 30) $2r = i\,(\text{Y} - \text{Y}^{-1})$, and so (§ 21)

area ⊙ $2r$ (in F) $= \pi i^2(\text{Y} - \text{Y}^{-1})^2$.

Also (IV) area ⊙ $2y = \pi i^2(\text{Y}^2 - 2 + \text{Y}^{-2})$; therefore, area ⊙ $2r$ (in F) = area ⊙ $2y$, and so *the surface z of a segment of a sphere is equal to the surface of the circle described with the chord* FC *as a radius*.

Hence the whole surface of the sphere

$$= \text{area} \odot \text{FG} = -\text{FDG} \cdot p - \frac{p^2}{\pi},$$

and the surfaces of spheres are to each other as the second powers of their great circles.

VII. In like manner, in S, the volume of the sphere of radius x is found

$$= \tfrac{1}{2}\pi i^3 (\text{X}^2 - \text{X}^{-2}) - 2\pi i^2 x;$$

the surface generated by the revolution of the line CD about AB

$$= \tfrac{1}{2}\pi i p (\text{Q}^2 - \text{Q}^{-2}),$$

and the body described by CABDC

$$= \tfrac{1}{4}\pi i^2 p (\text{Q}^2 - \text{Q}^{-2})^2.$$

But in what manner all things treated from (IV) *even to here, also may be reached apart from integration, for the sake of brevity is suppressed.*

It can be demonstrated that the limit of every expression containing the letter i (and so resting upon the hypothesis that i is given), *when i increases to infinity, expresses the quantity simply for* Σ (and so for the hypothesis of no i), if indeed the equations do not become identical.

But beware lest you understand to be supposed, that the system itself may be varied (for it is entirely determined in itself and by itself); but only *the hypothesis*, which may be

done successively, as long as we are not conducted to an absurdity. *Supposing* therefore that, in *such* an expression, the letter i, in case S is reality, designates that unique quantity whose I = e; but if Σ is actual, the said limit is supposed to be taken in place of the expression : manifestly *all the expressions originating from the hypothesis of the reality of* S (*in this sense*) *will be true absolutely*, a*lthough it be completely unknown whether or not* Σ *is reality*.

So e. g. from the expression obtained in § 30 easily (and as well by aid of differentiation as apart from it) emerges the known value in Σ,

$$\odot\, x = 2\pi x;$$

from I (§ 31) suitably treated, follows

$$1 : \sin\alpha = c : a,$$

but from II

$$\frac{\cos\alpha}{\sin\beta} = 1, \text{ and so}$$

$$\alpha + \beta = \text{rt.}\angle;$$

the first equation in III becomes identical, and so is true in Σ, although it there determines nothing; but from the second follows

$$c^2 = a^2 + b^2.$$

These are the known fundamental equations of plane trigonometry in Σ.

Moreover, we find (from § 32) in Σ, the area and the volume in III each = *pq*; from IV area ⊙ $x = \pi x^2$;

(from VII) the globe of radius x

$$= \frac{4}{3}\pi x^3, \text{ etc.}$$

The theorems enunciated at the end of VI are manifestly *true unconditionally*.

§ 33. It still remains to set forth (as promised in § 32) what this theory means.

I. Whether Σ or some one S is reality, remains undecided.

II. All things deduced from the hypothesis of the falsity of Axiom *XI* (always to be understood in the sense of § 32) are *absolutely true*, and so in this sense, depend upon no hypothesis.

There is therefore *a plane trigonometry a priori*, *in which the system alone really remains unknown*; and so where remain unknown solely the *absolute* magnitudes in the expressions, but where a *single* known case would manifestly fix the whole system. But spherical trigonometry is established absolutely in § 26.

(And we have, on F, a geometry wholly analogous to the plane geometry of Σ.)

III. If it were agreed that Σ exists, nothing more would be unknown in this respect; but

if it were *established* that Σ does not exist, then (§ 31), (e. g.) from the sides x, y, and the rectilineal angle they include being given in a special case, manifestly it would be impossible in itself and by itself to solve absolutely the triangle, that is, to determine *a priori* the other angles and *the ratio of the third side* to the two given; unless X, Y were determined, for which it would be necessary to have in concrete form a certain sect a whose A was known; and then i would be *the natural unit for length* (just as e is the base of *natural* logarithms).

If the existence of this i is determined, it will be evident how it could be constructed, at least very exactly, for practical use.

IV. In the sense explained (I and II), it is evident that all things in space can be solved by the modern analytic method (within just limits strongly to be praised).

V. Finally, to friendly readers will not be unacceptable; that for that case wherein not Σ but S is reality, a rectilineal figure is constructed equivalent to a circle.

§ 34. Through D we may draw DM ∥ AN in the following manner. From D drop DB ⊥ AN; from any point A of the straight AB erect AC ⊥ AN (in DBA), and let fall DC ⊥ AC. We

will have (§ 27) ⊙ CD : ⊙ AB – 1 : sin z, provided that DM ∥ BN. But sin z is not > 1; and so AB is not > DC. Therefore a quadrant described from the center A in BAC, with a radius = DC, will have a point B or O in common with ray BD. In the first case, manifestly $z = rt.\angle$; but in the second case (§ 25)

$$(\odot\ AO = \odot\ CD) : \odot\ AB = 1 \sin AOB,$$

and so z = AOB.

If therefore we take z = AOB, then DM will be ∥ BN.

§ 35. If S were reality; we may, as follows, draw a straight ⊥ to one arm of an acute angle, which is ∥ to the other.

Take AM ⊥ BC, and suppose AB = BC so small (by § 19), that if we draw BN ∥ AM (§ 34), ABN > the given angle.

Moreover draw CP ∥ AM (§ 34); and take NBG and PCD each equal to the given angle; rays BG and CD will cut; for if ray BG (falling *by construction* within NBC) cuts ray CP in E; we shall have (since BN Π CP), ∠EBC < ∠ECB, and so EC < EB. Take EF = EC, EFR

= ECD, and FS ∥ EP; then FS will fall within BFR. For since BN ∥ CP, and so BN ∥ EP, and BN ∥ FS; we shall have (§ 14)

$$\angle FBN + \angle BFS < (\text{st. } \angle = FBN + BFR);$$

therefore, BFS < BFR. Consequently, ray FR cuts ray EP, and so ray CD also cuts ray EG in some point D. Take now DG = DC and DGT = DCP = GBN; we shall have (since CD Π GD) BN Π GT Π CP. Let K (§ 19) be the point of the L-form line of BN falling in the ray BG, and KL the axis; we shall have BN Π KL, and so BKL = BGT = DCP; but also KL Π CP : therefore manifestly K fall on G, and GT ∥ BN. But if HO bisects ⊥ BG, we shall have constructed HO ∥ BN.

§36. Having given the ray CP and the plane MAB, take CB ⊥ the plane MAB, BN (in plane BCP) ⊥ BC, and CQ ∥ BN (§ 34); the intersection of ray CP (if this ray falls within BCQ) with ray BN (in the plane CBN), and so with the plane MAB is found. And if we are given the two planes PCQ, MAB, and we have CB ⊥ to plane MAB, CR ⊥ plane PCQ; and (in plane BCR) BN ⊥ BC, CS ⊥ CR, BN will fall in plane MAB, and CS in plane PCQ; and the

intersection of the straight BN with the straight CS (if there is one) having been found, the perpendicular drawn through this intersection, in PCQ, to the straight CS will manifestly be the intersection of plane MAB and plane PCQ.

§ 37. On the straight AM ‖ BN, is found such an A, that AM Π BN. If (by § 34) we construct outside of the plane NBM, GT ‖ BN, and make BG ⊥GT, GC = GB, and CP ‖ GT; and so place the hemi-plane TGD that it makes with hemi-plane TGB an angle equal to that which hemi-plane PCA makes with hemi-plane PCB; and is sought (by § 36) the intersection straight DQ of hemi-plane TGD with hemi-plane NBD; and BA is made ⊥ DQ.

We shall have indeed, on account of the similitude of the triangles of L lines produced on the F of BN (§ 21), manifestly DB = DA, and AM Π BN.

Hence easily appears (L-lines being given by their extremities alone) we may also find a fourth proportional, or a mean proportional, and execute in this way in F, apart from Axiom XI, all the geometric constructions made

on the plane in Σ. Thus e. g. a perigon can be geometrically divided into any special number of equal parts, if it is permitted to make this special partition in Σ.

§ 38. If we construct (by § 37) for example, NBQ = ⅓ rt.∠, and make (by. § 35), in S, AM ⊥ ray BQ and ∥ BN, and determine (by §37) IM Π BN; we shall have, if IA = x, (§ 28), X = 1 : sin ⅓ rt. ∠ = 2, and x will be constructed *geometrically*.

And NBQ may be so computed, that IA differs from i less than by anything given, which happens for sin NBQ = $1/e$.

§ 39. If (in a plane) PQ and ST are 8 to the straight MN (§27), and AB, CD are equal perpendiculars to MN; manifestly ΔDEC ≅ ΔBEA; and so the angles (perhaps mixtilinear) ECP, EAT will fit, and EC = EA. If, moreover, CF = AG, then ΔACF ≅ ΔCAG, and each is half of the *quadrilateral* FAGC.

If FAGC, HAGK are two quadrilaterals of this sort on AG, between PQ and ST; their equivalence (as in Euclid) is evident, as also

the equivalence of the triangles AGC, AGH, standing on the same AG, and having their vertices on the line PQ. Moreover, ACF = CAG, GCQ = CGA, and ACF + ACG + GCQ = st.∠ (§ 32); and so also CAG + ACG + CGA = st.∠; therefore, in any triangle ACG of this sort, the sum of the three angles = st.∠. But whether the straight AG may have fallen upon AG (which 8 MN), or not; *the equivalence* of the rectilineal triangles AGC, AGH, as well of themselves, *as of the sums of their angles*, is evident.

§ 40. *Equivalent triangles* ABC, ABD, (henceforth rectilineal), *having one side equal*, *have the sums of their angles equal*. For let MN bisect AC and BC, and take (through C) PQ 8 MN; the point D will fall on line PQ.

For, if ray BD cuts the straight MN in the point E, and so (§ 39) the line PQ at the distance EF = EB; we shall have ΔABC = ΔABF, and so also ΔABD = ΔABF, whence D falls at F.

But if ray BD has not cut the straight MN, let C be the point, where the perpendicular bisecting the straight AB cuts the line PQ, and

let GS = HT, so, that the line ST meets the ray BD prolonged in a certain K (which it is evident can be made in a way like as in § 4); moreover take SR = SA, RO 8 ST, and O the intersection of ray BK with RO; then ΔABR = ΔABO (§39), and so ΔABC > ΔABD (contra hyp.).

§ 41. *Equivalent triangles ABC*, *DEF have the sums of their triangles equal*.

For let MN bisect AC and BC, and PQ bisect DF and FE; and take RS 8 MN, and TO 8 PQ; the perpendicular AG to RS will equal the perpendicular DH to TO, or one for example DH will be the greater.

In each case, the ⊙ DF, from center A, has with line-ray GS some point K in common, and (§ 39) ΔABK = ΔABC = ΔDEF. But the ΔAKB (by § 40) has the same angle-sum as ΔDFE, and (by § 39) as ΔABC. Therefore also the triangles ABC, DEF have each the same angle-sum.

In S the inverse of this theorem is true.

For take ABC, DEF two triangles having equal angle-sums, and ΔBAL = ΔDEF; these will have (by what precedes) equal angle-sums,

and so also will ΔABC and ΔABL, and hence manifestly

$$\text{BCL} + \text{BLC} + \text{CBL} = \text{st.} \angle.$$

However (by § 31), the angle-sum of any triangle, in S, is < st.$\angle$.

Therefore L falls on C.

§ 42. Let u be the supplement of the angle-sum of the ΔABC, but v of ΔDEF; then is $\Delta\text{ABC} : \Delta\text{DEF} = u : v$. F For if p be the area of each of the triangles ACG, GCH, HCB, DFK, KFE; and $\Delta\text{ABC} = m \cdot p$, and $\Delta\text{DEF} = n \cdot p$; and s the angle-sum of any triangle equivalent to p, manifestly

st. $\angle - u = m \cdot s - (m-1)$st. $\angle =$ st. $\angle - m($st. $\angle - s)$; and $u = m($st.$\angle - s)$; and in like manner $v = n($st.$\angle - s)$.

Therefore $\Delta\text{ABC} : \Delta\text{DEF} = m : n = u : v$. It is evidently also easily extended to the case of the incommensurability of the triangles ABC, DEF.

In the same way is demonstrated that triangles on a sphere are as the excesses of the sums of their angles above a st. < .

If two angles of the spherical Δ are right, the third z will be the said excess. But

(a great circle being called p) this Δ is manifestly

$$= \frac{z}{2\pi} \, \frac{p^2}{2\pi} \text{ (§ 32, VI);}$$

consequently, any triangle of whose angles the excess is z, is

$$= \frac{zp^2}{4\pi^2}.$$

§ 43. Now, in S, the area of a rectilineal Δ is expressed by means of the sum of its angles.

If AB increases to infinity; (§ 42) Δ ABC : (rt.∠ − u − v) will be constant. But Δ ABC Y BACN (§ 32, V), and rt.∠ − u − v Y z (§ 1); and so BACN : z = Δ ABC : (rt. ∠− u − v) = BAC'N' : z'.

Moreover, manifestly (§ 30) BDCN : BD'C'N' = r : r' tan z : tan z'.

But for y' Y 0, we have

$$\frac{\text{BD'C'N'}}{\text{BAC'N'}} \doteq 1 \text{ and also } \frac{\tan z'}{z'} \text{ Y } 1;$$

consequently,

$$\text{BDCN : BACN} = \tan z : z.$$

But (§ 32)

$$\text{BDCN} = r \cdot i = i^2 \tan z;$$

therefore,

$$\text{BACN} = z \cdot i^2.$$

Designating henceforth, for brevity, any triangle the supplement of whose angle-sum is z by Δ, we will therefore have $\Delta = z \cdot i^2$.

Hence it readily flows that, if OR ‖ AM and RO ‖ AB, the *area* comprehended between the straights OR, ST, BC (which is manifestly the absolute limit of the area of rectilineal triangles increasing without bound, or of Δ for z Y st. $\angle$), is $= \pi i^2 =$ area $\odot i$, in F.

This limit being denoted by $\square$, moreover (by § 30) $\pi r^2 = \tan^2 z \cdot \square =$ area $\odot r$ in F (§ 21) = area $\odot s$ (by §32, VI) if the chord CD is called s.

If now, bisecting at right angles the given radius s of the circle in a plane (or the L form radius of the circle in F), we construct (by § 34) DB ‖ Y CN; by dropping CA $\perp$ DB, and erecting CM $\perp$ CA, we shall get z; whence (by § 37), assuming at pleasure an L form radius for unity, $\tan^2 z$ *can be determined geometrically by means of two uniform lines of the same curvature* (which, their extremities alone being given and their axes

constructed, manifestly may be compared like straights, and in this respect considered equivalent to straights). Moreover, a quadrilateral, ex. gr. regular = □ is constructed as follows :

Take ABC = rt.∠, BAC = ½ rt. ∠, ACB = ¼ rt. ∠, and BC = *x*.

By mere square roots, X (from § 31, II) can be expressed and (by 37) constructed; and having X (by § 38 or also §§ 29 and 35), *x* itself can be determined. And octuple Δ ABC is manifestly = □, and by this *a plane circle of radius s is geometrically squared by means of a rectilinear figure and uniform lines of the same species* (*equivalent to straights as to comparison inter se*); *but an F form circle is planified in the same manner*: *and we have either the Axiom* XI *of Euclid true or the geometric quadrature of the circle*, although thus far it has remained undecided, which of these two has place in reality.

Whenever $\tan^2 z$ is either a whole number, or a rational fraction, whose denominator (reduced to the simplest form) is either a prime number of the form $2^m + 1$ (of which is also $2 = 2^0 + 1$), or a product of however many prime numbers of this form, of which each (with the

exception of 2, which alone may occur any number of times) occurs only once as factor, we can, by the theory of polygons of the illustrious Gauss (remarkable invention of our, nay of every age) (and only for such values of z), construct a rectilineal figure = $\tan^2 z$ □ = area ⊙ s. For the division of □ (the theorem of § 42 extending easily to any polygons) manifestly requires the partition of a st. ∠, which (as can be shown) can be achieved geometrically only under the said condition.

But in all such cases, what precedes conducts easily to the desired end. And any rectilineal figure can be converted geometrically into a regular polygon of n sides, if n falls under the Gaussian form. It remains, finally (that the thing may be completed in every respect), to demonstrate the impossibility (apart from any supposition), of deciding *a priori*, whether Σ, or some S (and which one) exists. This, however, is reserved for a more suitable occasion.

APPENDIX I.

REMARKS ON THE PRECEDING TREATISE, BY BOLYAI FARKAS.

[From Vol. II of Tentamen, pp. 380 – 383.]

Finally it may be permitted to add something appertaining to the author of the *Appendix* in the first volume, who, however, may pardon me if something I have not touched with his acuteness.

The thing consists briefly in this : *the formulas of spherical trigonometry* (demonstrated in the said *Appendix* independently of Euclid's Axiom XI) *coincide with the formulas of plane trigonometry*, *if* (in a way provisionally speaking) *the sides of a spherical triangle are accepted as reals*, *but of a rectilineal triangle as imaginaries*; so that, as to trigonometric formulas, the plane may be considered as an imaginary sphere, if for real, that is accepted in which sin rt. $\angle = 1$.

Doubtless, of the Euclidean axiom has been said in volume first enough and to spare : for

the case if it were not true, is demonstrated (Tom. I. App., p. 13), that there is given a certain *i*, for which the I there mentioned is = e (the base of natural logarithms), and for this case are established also (*ibidem*, p. 14) the formulas of plane trigonometry, and indeed so, that (by the side of p. 19, ibidem) the formulas are still valid for the case of the verity of the said axiom; indeed if the limits of the values are taken, supposing that *i* Y ∞; truly the Euclidean system is as if the limit of the anti-Euclidean (for *i* Y ∞).

Assume for the case of *i* existing, the unit = *i*, and extend the concepts sine and cosine also to imaginary arcs, so that, *p* designating an arc whether real or imaginary,

$$\frac{e^{p\sqrt{-1}} + e^{-p\sqrt{-1}}}{2} \text{ is called the } \textit{cosine} \text{ of } p\text{, and}$$

$$\frac{e^{p\sqrt{-1}} - e^{-p\sqrt{-1}}}{2\sqrt{-1}} \text{ is called the } \textit{sine} \text{ of } p \text{ (as Tom. I., p. 177).}$$

Hence for *q* real

$$\frac{e^{q} - e^{-q}}{2\sqrt{-1}} = \frac{e^{-q\sqrt{-1}\cdot\sqrt{-1}} - e^{q\sqrt{-1}\cdot\sqrt{-1}}}{2\sqrt{-1}} = \sin(-q\sqrt{-1})$$

$$= -\sin(q\sqrt{-1}).$$

$$\frac{e^{q} + e^{-q}}{2} = \frac{e^{-q\sqrt{-1}\cdot\sqrt{-1}} + e^{q\sqrt{-1}\cdot\sqrt{-1}}}{2} = \cos(-q\sqrt{-1})$$

$$= \cos(q\sqrt{-1});$$

if of course also in the imaginary circle, the sine of a negative arc is the same as the sine of a positive arc otherwise equal to the first, except that it is negative, and the cosine of a positive arc and of a negative (if otherwise they be equal) the same.

In the said Appendix, § 25, is demonstrated absolutely, that is, independently of the said axiom; that, in any rectilineal triangle *the sines of the circles are as the circles of radii equal to the sides opposite*.

Moreover is demonstrated for the case of i existing, that the circle of radius y is

$$= \pi\, i \left(e^{\frac{y}{i}} - e^{\frac{-y}{i}}\right) \text{ which, for } i = 1, \text{ becomes}$$

$$\pi\,(e^{y} - e^{-y}).$$

Therefore (§ 31 *ibidem*), for a right-angled rectilineal triangle of which the sides are a and b, the hypothenuse c, and the angles opposite to the sides a, b, c are α, β, rt. $\angle$, (for i = 1), in I,

$$1 : \sin\ \alpha = \pi\,(e^{c} - e^{-c}) : \pi\,(e^{a} - e^{-a});$$

and so

$$1 : \sin \alpha = \frac{e^{c} - e^{-c}}{2\sqrt{-1}} : \frac{e^{a} - e^{-a}}{2\sqrt{-1}}\,.$$

Whence

$$1 : \sin \alpha = -\sin (c\sqrt{-1}) : -\sin (a\sqrt{-1}).$$

And hence

$$1 : \sin \alpha = \sin (c\sqrt{-1}) : \sin (a\sqrt{-1}).$$

In II becomes

$$\cos \alpha : \sin \beta = \cos (a\sqrt{-1}) : 1;$$

in III becomes

$$\cos (c\sqrt{-1}) = \cos (a\sqrt{-1}) \cdot \cos (b\sqrt{-1}).$$

These, as all the formulas of plane trigonometry deducible from them, coincide completely with the formulas of spherical trigonometry; except that if, ex. gr., also the sides and the angles opposite them of a right-angled spherical triangle and the hypothenuse bear the same names, the sides of the rectilineal triangle are to be divided by $\sqrt{-1}$ to obtain the formulas for the spherical triangle.

Obviously we get (clearly as Tom.,II., p. 252),

from I, $1 : \sin \alpha = \sin c : \sin a$;

from II, $1 : \cos a = \sin \beta : \cos \alpha$;

from III, $\cos c = \cos a \cos b$.

Though it be allowable to pass over other things; yet I have learned that the reader may be offended and impeded by the deduction omitted, (Tom. I., App., p. 19) [in § 32 at end] : it will not be irrelevant to show how, ex. gr., from

$$e^{\frac{c}{i}} + e^{\frac{-c}{i}} = \frac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)\left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)$$

follows

$$c^2 = a^2 + b^2.$$

(the theorem of Pythagoras for the Euclidean system); probably thus also the author deduced it, and the others also follow in the same manner.

Obviously we have, the powers of e being expressed by series (like Tom. I., p. 168),

$$e^{\frac{k}{i}} = 1 + \frac{k}{i} + \frac{k^2}{2i^2} + \frac{k^3}{2\cdot 3\cdot i^3} + \frac{k^4}{2\cdot 3\cdot 4\cdot i^4} \ldots ,$$

$$e^{\frac{-k}{i}} = 1 - \frac{k}{i} + \frac{k^2}{2i^2} - \frac{k^3}{2\cdot 3\cdot i^3} + \frac{k^4}{2\cdot 3\cdot 4\cdot i^4} \ldots , \text{ and so}$$

$$e^{\frac{k}{i}} + e^{\frac{-k}{i}} = 2 + \frac{k^2}{i^2} + \frac{k^4}{3\cdot 4\cdot i^4} + \frac{k^6}{3\cdot 4\cdot 5\cdot 6\cdot i^6} \ldots ,$$

$$= 2 + \frac{k^2 + u}{i^2}, \text{ (designating by}$$

$\frac{u}{i^2}$ the sum of all the terms after $\left.\frac{k^2}{i^2}\right)$; and we have u Y 0, while i Y ∞. For all the terms which follow $\frac{k^2}{i^2}$, are divided by i^2; the first term will be $\frac{k^4}{3\cdot 4\cdot i^4}$; and any ratio $< \frac{k^2}{i^2}$; and though the ratio everywhere should remain this, the sum would be (Tom. I., p. 131),

$$\frac{k^4}{3\cdot 4\cdot i^2} : \left(1 - \frac{k^2}{i^2}\right) = \frac{k^4}{3\cdot 4\cdot (i^4 - k^2)}$$

which manifestly Y 0, while i Y ∞.

And from

$$e^{\frac{c}{i}} + e^{\frac{-c}{i}} = \frac{1}{2}\left(e^{\frac{(a+b)}{i}} + e^{\frac{-(a+b)}{i}} + e^{\frac{(a-b)}{i}} + e^{\frac{-(a-b)}{i}}\right)$$

follows (for w, v, λ taken like u)

$$2 + \frac{c^2 + w}{i^2} = \frac{1}{2}\left(2 + \frac{(a+b)^2 + v}{i^2} + 2 + \frac{(a+b)^2 + \lambda}{i^2}\right)$$

And hence

$$c^2 = \frac{a^2 + 2ab + b^2 + a^2 - 2ab + b^2 + v + \lambda - w}{2}$$

which Y $a^2 + b^2$.

APPENDIX II.

SOME POINTS IN JOHN BOLYAI'S

COMPARED WITH LOBACHEVSKI,

BY WOLFGANG BOLYAI.

[From *Kurzer Grundriss*, p. 82.]

Lobachevski and the author of the Appendix each consider two points A, B, of the sphere-limit, and the corresponding axes ray AM, ray BN (§ 23).

They demonstrate that, if α, β, γ designate the arcs of the circle limit AB, CD, HL, separated by segments of the axis AC = 1, AH = x, we have

$$\frac{\alpha}{\gamma} = \left(\frac{\alpha}{\beta}\right)^{x}.$$

Lobachevski represents the value of $\frac{\gamma}{\alpha}$ by e^{-x}, e having some value > 1, dependent on the unit for length that we have chosen, and able to be supposed equal to the Naperian base.

The author of the Appendix is led directly to introduce the base of natural logarithms.

If we put $\dfrac{\alpha}{\beta} = \delta$, and γ, γ' are arcs situated at the distances y, i from α, we shall have

$$\frac{\alpha}{\gamma} = \delta^{y} = \mathrm{Y}, \frac{\alpha}{\gamma'} = \delta^{i} = \mathrm{I}, \text{ whence } \mathrm{Y} = \mathrm{I}^{\frac{y}{i}}.$$

He demonstrates afterward (§ 29) that, if u is the angle which a straight makes with the perpendicular y to its parallel, we have

$$\mathrm{Y} = \cot \tfrac{1}{2} u.$$

Therefore, if we put $z = \dfrac{\pi}{2} - u$, we have

$$\mathrm{Y} = \tan (z + \tfrac{1}{2} u) = \frac{\tan z + \tan - \frac{1}{2} u}{1 - \tan z \tan \frac{1}{2} u},$$

whence we get, having regard to the value of $\tan \frac{1}{2} - u = \mathrm{Y}^{-1}$,

$$\tan z - \tfrac{1}{2} (\mathrm{Y} - \mathrm{Y}^{-1}) = \tfrac{1}{2} \left(\mathrm{I}^{\frac{y}{i}} - \mathrm{I}^{\frac{-y}{i}} \right) (\S 30).$$

If now y is the semi-chord of the arc of circle-limit $2r$, we prove (§30) that $\dfrac{r}{\tan z}$ = constant.

Representing this constant by i, and making y tend toward zero, we have

$$\frac{2r}{2y} \text{ Y } 1, \text{ whence}$$

$$2y \text{ Y } 2i \tan z \text{ Y } i \frac{\mathrm{I}^{\frac{2y}{i}} - 1}{\mathrm{I}^{\frac{y}{i}}},$$

or putting $\frac{2y}{i} = k$, I $= el$,

$$k \text{ I I } \tfrac{y}{i} \text{ Y } e^{kl} - 1 \text{ Y } kl\,(1 + \lambda),$$

λ being infinitesimal at the same time as k. Therefore, for the limit, $1 = l$ and consequently I $= e$.

The circle traced on the sphere-limit with the arc r of the curve-limit for radius, has for length $2\pi r$. Therefore,

$$(\odot y = 2\pi r = 2\pi i \tan z = \pi i\,(\text{Y} - \text{Y}^{-1}).$$

In the rectilineal Δ where α, β designate the angles opposite the sides a, b, we have (§ 25)

$$\sin\alpha : \sin\beta = \odot\ a : \odot\ b = \pi\ i\,(\text{A} - \text{A}^{-1}) : \pi\ i\,(\text{B} - \text{B}^{-1})$$
$$= \sin(a\sqrt{-1}) : \sin(b\sqrt{-1}).$$

Thus in plane trigonometry as in spherical trigonometry, the sines of the angles are to each other as the sines of the opposite sides, only that on the sphere the sides are reals, and in the plane we must consider them as imaginaries, just as if the plane were an imaginary sphere.

We may arrive at this proposition without a preceding determination of the value of I.

If we designate the constant $\frac{r}{\tan z}$ by q, we shall have, as before

$$\odot\ y = \pi\ q\,(\text{Y} - \text{Y}^{-1}),$$

whence we deduce the same proportion as above, taking for i the distance for which the ratio I is equal to e.

If axiom XI is not true, there exists a determinate, which must be substituted in the formulas.

If, on the contrary, this axiom is true, we must make in the formulas $i = \infty$. Because, in this case, the quantity $\frac{\alpha}{\gamma} = Y$ is always $= 1$, the sphere-limit being a plane, and the axes being parallel in Euclid's sense.

The exponent $\frac{y}{i}$ must therefore be zero, and consequently $i = \infty$.

It is easy to see that Bolyai's formulas of plane trigonometry are in accord with those of Lobachevski.

Take for example the formula of § 37,

$$\tan \parallel (a) = \sin B \tan \parallel (p),$$

a being the hypothenuse of a right-angled triangle, p one side of the right angle, and B the angle opposite to this side.

Bolyai's formula of § 31, I, gives

$$1 : \sin B = (A - A^{-1}) : (P - P^{-1}).$$

Now, putting for brevity, $\frac{1}{2} \parallel (k) = k'$, we have $\tan 2p' : \tan 2a' = (\cot a' - \tan a') : (\cot p' - \tan p') = (A - A^{-1}) : (P - P^{-1}) : \sin B$.

APPENDIX III.

LIGHT FROM NON-EUCLIDEAN SPACES ON THE

TEACHING OF ELEMENTARY GEOMETRY.

BY G. B. HALSTED.

As foreshadowed by Bolyai and Riemann, founded by Cayley, extended and interpreted for hyperbolic, parabolic, elliptic spaces by Klein, recast and applied to mechanics by Sir Robert Ball, projective metrics may be looked upon as characteristic of what is highest and most peculiarly modern in all the bewildering range of mathematical achievement. Mathematicians hold that number is wholly a creation of the human intellect, while on the contrary our space has an empirical element. Of possible geometries we can not say a priori which shall be that of our actual space, the space in which we move. Of course an advance so important, not only for mathematics but for philosophy, has had some metaphysical opponents, and as long ago as 1878 I mentioned in my Bibliography of Hyper-Space

and Non-Euclidean Geometry (American Journal of Mathematics, Vol. I, 1878, Vol. II, 1879) one of these, Schmitz-Dumont, as a sad paradoxer, and another, J. C. Becker, both of whom would ere this have shared the oblivion of still more antiquated fighters against the light, but that Dr. Schotten, praiseworthy for the very attempt at a comparative planimetry, happens to be himself a believer in the *a priori* founding of geometry, while his American reviewer, Mr. Ziwet, was then also an anti-non-Euclidean, though since converted.

He says, “we find that some of the best German text books do not try at all to define what is space, or what is a point, or even what is a straight line.” Do any German geometries define space? I never remember to have met one that does.

In experience, what comes first is a bounded surface, with its boundaries, lines, and their boundaries, points. Are the points whose definitions are omitted anything different or better?

Dr. Schotten regards the two ideas “direction” and “distance” as intuitively given in the mind and as so simple as to not require definition.

When we read of two jockeys speeding

around a track in opposite directions, and also on page 87 of Richardson's Euclid, 1891, read, "The sides of the figure must be produced in the same direction of rotation; . . . going round the figure always in the same direction," we do not wonder that when Mr. Ziwet had written : "he therefore bases the definition of the straight line on these two ideas," he stops, modifies, and rubs that out as follows, "or rather recommends to elucidate the intuitive idea of the straight line possessed by any well-balanced mind by means of the still simpler ideas of direction" [in a circle] "and distance" [on a curve.

But when we come to geometry as a science, as foundation for work like that of Cayley and Ball, I think with Professor Chrystal : "It is essential to be careful with our definition of a *straight line*, for it will be found that virtually the properties of the straight line determine the nature of space.

"Our definition shall be that two points in *general* determine a straight line."

We presume that Mr. Ziwet glories in that unfortunate expression "a straight line is the shortest distance between two points," still occurring in Wentworth (New Plane Geometry, page 33), even after he has said, page 5,

"the length of the straight line is called the *distance* between two points." If the *length* of the one straight line between two points is the distance between those points, how can the straight line itself be the *shortest* distance? If there is only one distance, it is the longest as much as the shortest distance, and if it is the *length* of this shorto-longest distance which is the distance then it is not the straight line itself which is the longo-shortest distance. But Wentworth also says : "Of all lines joining two points the *shortest* is the straight line."

This general comparison involves the measurement of curves, which involves the theory of limits, to say nothing of ratio. The very ascription of length to a curve involves the idea of a limit. And then to introduce this general axiom, as does Wentworth, only to prove a very special case of itself, that two sides of a triangle are together greater than the third, is surely bad logic, bad pedagogy, bad mathematics.

This latter theorem, according to the first of Pascal's rules for demonstrations, should not be proved at all, since every dog knows it. But to this objection, as old as the sophists, Simson long ago answered for the science of

geometry, that the number of assumptions ought not to be increased without necessity; or as Dedekind has it : “*Was beweisbar ist*, *soll in der Wissenschaft nicht ohne Beweis geglaubt werden*.”

Professor W. B. Smith (Ph. D., Goettingen), has written : “Nothing could be more unfortunate than the attempt to lay the notion of Direction at the bottom of Geometry.”

Was it not this notion which led so good a mathematician as John Casey to give as a demonstration of a triangle’s angle-sum the procedure called “ a practical demonstration” on page 87 of Richardson’s Euclid, and there described as “laying a ‘straight edge’ along one of the sides of the figure, and then turning it round so as to coincide with each side in turn.”

This assumes that a segment of a straight line, a sect, may be translated without rotation, which assumption readily comes to view when you try the procedure in two-dimensional spherics. Though this fallacy was exposed by so eminent a geometer as Olaus Henrici in so public a place as the pages of ‘Nature,’ yet it has just been solemnly reproduced by Professor G. C. Edwards, of the University of California, in his Elements of Geometry : MacMillan, 1895.

It is of the greatest importance for every teacher to know and connect the commonest forms of assumption equivalent to Euclid's Axiom XI. If in a plane two straight lines perpendicular to a third nowhere meet, are there others, not both perpendicular to any third, which nowhere meet? Euclid' s Axiom XI is the assumption *No*. Playfair's answers *no* more simply. But the very same answer is given by the common assumption of our geometries, usually unnoticed, that a circle may be passed through any three points not costraight.

This equivalence was pointed out by Bolyai Parkas, who looks upon this as the simplest form of the assumption. Other equivalents are, the existence of any finite triangle whose angle-sum is a straight angle; or the existence of a plane rectangle; or that, in triangles, the angle-sum is constant.

One of Legendre's forms was that through every point within an angle a straight line may be drawn which cuts both arms.

But Legendre never saw through this matter because he had hot, as we have, the eyes of Bolyai and Lobachevski to see with. The same lack of their eyes has caused the author of the charming book " Euclid and His Modern

Rivals," to give us one more equivalent form : " In any circle, the inscribed equilateral tetragon is greater than any one of the segments which lie outside it." (A New Theory of Parallels by C. L. Dodgson, 3d. Ed., 1890.)

Any attempt to define a straight line by means of "direction" is simply a case of "argumentum in circulo." In all such attempts the loose word "direction" is used in a sense which presupposes the straight line. The directions from a point in Euclidean space are only the ∞^2 rays from that point.

Rays not costraight can be said to have the same direction only after a theory of parallels is presupposed, assumed.

Three of the exposures of Professor G. C. Edwards' fallacy are here reproduced. The first, already referred to, is from Nature, Vol. XXIX, p. 453, March 13, 1884.

"I select for discussion the 'quaternion proof" given by Sir William Hamilton. . . . Hamilton's proof consists in the following : "One side AB of the triangle ABC is turned about the point B till it lies in the continuation of BC; next, the line BC is made to slide along BC till B comes to C, and is then turned about C till it comes to lie in the continuation of AC.

“ It is now again made to slide along CA till the point B comes to A, and is turned about A till it lies in the line AB. Hence it follows, *since rotation is independent of translation*, that the line has performed a whole revolution, that is, it has been turned through four right angles. But it has also described in succession the three exterior angles of the triangle, hence these are together equal to four right angles, and from this follows at once that the interior angles are equal to two right angles.

“To show how erroneous this reasoning is—in spite of Sir William Hamilton and in spite of quaternions—I need only point out that it holds exactly in the same manner for a triangle on the surface of the sphere, from which it would follow that the sum of the angles in a spherical triangle equals two right angles, whilst this sum is known to be always greater than two right angles. The proof depends only on the fact, that any line can be made to coincide with any other line, that two lines do so coincide when they have two points in common, and further, that a line may be turned about any point in it without leaving the surface. But if instead of the plane we take a spherical surface, and instead of a line a great

circle on the sphere, all these conditions are again satisfied.

"The reasoning employed must therefore be fallacious, and the error lies in the words printed in italics; for these words contain an assumption which has not been proved.

"O. HENRICI."

Perronet Thompson, of Queen's College, Cambridge, in a book of which the third edition is dated 1830, says:

"Professor Playfair, in the Notes to his 'Elements of Geometry' [1813], has proposed another demonstration, founded on a remarkable *non causa pro causa*.

"It purports to collect the fact [Eu. I., 32, Cor., 2] that (on the sides being successively prolonged to the same hand) the exterior angles of a rectilinear triangle are together equal to four right angles, from the circumstance that a straight line carried round the perimeter of a triangle by being applied to all the sides in succession, is brought into its old situation again; the argument being, that because this line has made the sort of somerset it would do by being turned through four right angles about a fixed point, the exterior

angles of the triangle have necessarily been equal to four right angles.

"The answer to which is, that there is no connexion between the things at all, and that the result will just as much take place where the exterior angles are avowedly not equal to four right angles.

"Take, for example, the plane triangle formed by three small arcs of the same or equal circles, as in the margin; and it is manifest that an arc of this circle may be carried round precisely in the way described and return to its old situation, and yet there be no pretense for inferring that the exterior angles were equal to four right angles.

"And if it is urged that these are curved lines and the statement made was of straight; then the answer is by demanding to know, what property of straight lines has been laid down or established, which determines that what is not true in the case of other lines is

true in theirs. It has been shown that, as a general proposition, the connexion between a line returning to its place and the exterior angles having been equal to four right angles, is a *non sequitur*; that it is a thing that may be or may not be; that the notion that it returns to its place *because* the exterior angles have been equal to four right angles, is a mistake. From which it is a legitimate conclusion, that if it had pleased nature to make the exterior angles of a triangle greater or less than four right angles, this would not have created the smallest impediment to the line's returning to its old situation after being carried round the sides; and consequently the line's returning is no evidence of the angles not being greater or less than four right angles."

Charles L. Dodgson, of Christ Church, Oxford, in his "Curiosa Mathematica," Part I, pp. 70 – 71, 3d Ed., 1890, says:

"Yet another process has been invented—quite fascinating in its brevity and its elegance—which, though involving the same fallacy as the Direction-Theory, proves Euc. I, 32, without even mentioning the dangerous word 'Direction.'

"We are told to take any triangle ABC; to produce CA to D; to make part of CD, viz., AD, revolve, about A, into the position ABE; then to make part of this line, viz., BE, revolve, about B, into the position BCF; and lastly to make part of this line, viz., CF, revolve, about C, till it lies along CD, of which it originally formed a part. We are then assured that it must have revolved through four right angles : from which it easily follows that the interior angles of the triangle are together equal to two right angles.

"The disproof of this fallacy is almost as brief and elegant as the fallacy itself. We first quote the general principle that we can not reasonably be told to make a line fulfill two conditions, either of which is enough by itself to fix its position : e. g., given three points X, Y, Z, we can not reasonably be told to draw a line from X which shall pass through Y and Z : we can make it pass through Y, but it must then take its chance of passing through Z; and *vice versa*.

" Now let us suppose that, while one part of

AE, viz., BE, revolves into the position BF, another little bit of it, viz., AG, revolves, through an equal angle, into the position AH; and that, while CF revolves into the position of lying along CD, AH revolves—and here comes the fallacy.

"You must not say 'revolves, through an equal angle, into the position of lying along AD,' for this would be to make AH *fulfill two conditions at once*.

"If you say that the one condition involves the other, you are virtually asserting that the lines CF, AH are equally inclined to CD—and this in *consequence* of AH having been so drawn that these same lines are equally inclined to AE.

"That is, you are asserting, 'A pair of lines which are equally inclined to a certain transversal, are so to any transversal.' [Deducible from Euc. I, 27, 28, 29.]"

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I found this in Chapter 2 of THE PARABOLA, ELLIPSE, AND HYPERBOLA, TREATED GEOMETRICALLY. BY
ROBERT WILLIAM GRIFFIN, A.M., LL.D.,
Now a Lemma is a given, but why? Could the author not show the construction? I remembered the figure from Heath's translation of The Elements by Euclid, but alas, the construction was not there either. So, I did a work up of the figure.

LEMMA.

If a right line AB be divided internally at O in any ratio, and externally at O' in the same ratio, and a circle be described on OO' as diameter, the right lines joining any point P on this circle with the extremities of the line AB will have the same ratio.

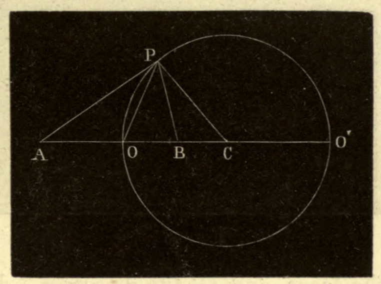


Fig. 1.

Bisect OO' in C; join CP, PO.

Then $AO' : O'B = AO : OB$;

$\therefore\ AO' + AO : AO' - AO = O'B + OB : O'B - OB$;

$\therefore\ 2AC : 2OC = 2OC : 2BC$;

$\therefore\ AC : CP = CP : CB$;

$\therefore\ \triangle ACP$ is similar to $\triangle PCB$; (6 VI. Euclid.)

$\therefore\ \angle CPB = \angle CAP$;

but $\angle CPO = \angle COP$ (5 I. Euclid.)

$= \angle OAP + \angle OPA$ (32 I. Euclid.)

$= \angle CPB + \angle OPA$;

$\therefore\ \angle BPO = \angle OPA$;

$\therefore\ AP : PB = AO : OB.$ (3 VI. Euclid.)

and also, as BD is to DC, so is BA to AE: for AD has been drawn parallel to EC, one of the sides of the triangle BCE: [VI. 2]

therefore also, as BA is to AC, so is BA to AE. [V. 11]

Therefore AC is equal to AE, [V. 9]

so that the angle AEC is also equal to the angle ACE. [I. 5]

But the angle AEC is equal to the exterior angle BAD, [I. 29]

and the angle ACE is equal to the alternate angle CAD; [*id.*]

therefore the angle BAD is also equal to the angle CAD.

Therefore the angle BAC has been bisected by the straight line AD.

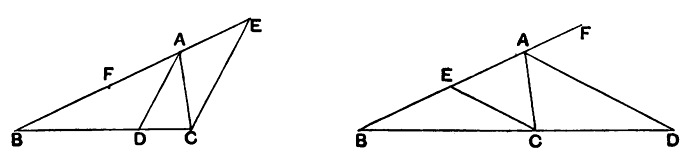
Therefore etc.

Q. E. D.

The demonstration assumes that CE *will* meet BA produced in some point E. This is proved in the same way as it is proved in VI. 4 that BA, ED will meet if produced. The angles ABD, BDA in the figure of VI. 3 are together less than two right angles, and the angle BDA is equal to the angle BCE, since DA, CE are parallel. Therefore the angles ABC, BCE are together less than two right angles; and BA, CE must meet, by I. Post. 5.

The corresponding proposition about the segments into which BC is divided *externally* by the bisector of the *external angle* at A when that bisector meets BC produced (i.e. when the sides AB, AC are not equal) is important. Simson gives it as a separate proposition, A, noting the fact that Pappus assumes the result without proof (Pappus, VII. p. 730, 24).

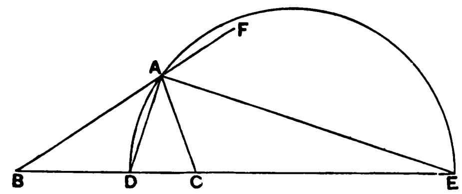
The best plan is however, as De Morgan says, to combine Props. 3 and A in one proposition, which may be enunciated thus: *If an angle of a triangle be bisected internally or externally by a straight line which cuts the opposite side or the opposite side produced, the segments of that side will have the same ratio as the other sides of the triangle; and, if a side of a triangle be divided internally or externally so that its segments have the same ratio as the other sides of the triangle, the straight line drawn from the point of section to the angular point which is opposite to the first mentioned side will bisect the interior or exterior angle at that angular point.*



Let AC be the smaller of the two sides AB, AC, so that the bisector AD of the exterior angle at A may meet BC produced beyond C. Draw CE through C parallel to DA, meeting BA in E.

Then, if FAC is the exterior angle bisected by AD in the case of external bisection, and if a point F is taken on AB in the figure of VI. 3, the proof of

VI. 3 can be used almost word for word for the other case. We have only to speak of the angle "FAC" for the angle "BAC," and of the angle "FAD" for the angle "BAD" wherever they occur, to say "let BA, or BA produced, meet CE in E," and to substitute "BA or BA produced" for "BAE" lower down.



If AD, AE be the internal and external bisectors of the angle A in a triangle of which the sides AB, AC are unequal, AC being the smaller, and if AD, AE meet BC and BC produced in D, E respectively,

the ratios of BD to DC and of BE to EC are alike equal to the ratio of BA to AC.

Therefore BE is to EC as BD to DC,

that is, BE is to EC as the difference between BE and ED is to the difference between ED and EC,

whence BE, ED, EC are in *harmonic progression*, or DE is a *harmonic mean* between BE and EC, or again B, D, C, E is a *harmonic range*.

Since the angle DAC is half of the angle BAC,

and the angle CAE half of the angle CAF,

while the angles BAC, CAF are equal to two right angles,

the angle DAE is a right angle.

Hence the circle described on DE as diameter passes through A.

Now, if the ratio of BA to AC is given, and if BC is given, the points D, E on BC and BC produced are given, and therefore so is the circle on D, E as diameter. Hence *the locus of a point such that its distances from two given points are in a given ratio (not being a ratio of equality) is a circle.*

This locus was discussed by Apollonius in his *Plane Loci*, Book II., as we know from Pappus (VII. p. 666), who says that the book contained the theorem that, if from two given points straight lines inflected to another point are in a given ratio, the point in which they meet will lie on either a straight line or a circumference of a circle. The straight line is of course the locus when the ratio is one of equality. The other case is quoted in the following form by Eutocius (Apollonius, ed. Heiberg, II. pp. 180—4).

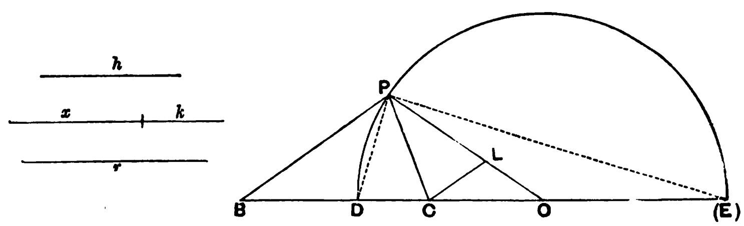
Given two points in a plane and a proportion between unequal straight lines, it is possible to describe a circle in the plane so that the straight lines inflected from the given points to the circumference of the circle shall have a ratio the same as the given one.

Apollonius' construction, as given by Eutocius, is remarkable because he makes no use of either of the points D, E. He finds O, the centre of the required circle, and the length of its radius directly from the data BC and the given ratio which we will call $h : k$. But the construction was not discovered by Apollonius; it belongs to a much earlier date, since it appears in exactly

the same form in Aristotle, *Meteorologica* III. 5, 376 a 3 sqq. The analysis leading up to the construction is, as usual, not given either by Aristotle or Eutocius. We are told to take three straight lines x, CO (a length measured along BC produced beyond C, where B, C are the points at which the greater and smaller of the inflected lines respectively terminate), and r, such that, if $h : k$ be the given ratio and $h > k$,

$$k : h = h : k + x, \quad \ldots\ldots\ldots\ldots(\alpha)$$

$$x : BC = k : CO = h : r. \quad \ldots\ldots\ldots\ldots(\beta)$$



This determines the position of O, and the length of r, the radius of the required circle. The circle is then drawn, *any* point P is taken on it and joined to B, C respectively, and it is proved that

$$PB : PC = h : k.$$

We may conjecture that the analysis proceeded somewhat as follows.

It would be seen that B, C are "conjugate points" with reference to the circle on DE as diameter. (Cf. Apollonius, *Conics*, I. 36, where it is proved, in terms, for a circle as well as for an ellipse and a hyperbola, that, if the polar of B meets the diameter DE in C, then $EC : CD = EB : BD$.)

If O be the middle point of DE, and therefore the centre of the circle, D, E may be eliminated, as in the *Conics*, I. 37, thus.

Since $$EC : CD = EB : BD,$$

it follows that $$EC + CD : EC \sim CD = EB + BD : EB \sim BD,$$

or $$2OD : 2OC = 2OB : 2OD,$$

that is, $$BO \cdot OC = OD^2 = r^2, \text{ say.}$$

If therefore P be any point on the circle with centre O and radius r,

$$BO : OP = OP : OC,$$

so that BOP, POC are similar triangles.

In addition, $h : k = BD : DC = BE : EC$

$$= BD + BE : DE = BO : r.$$

Hence we require that

$$BO : r = r : OC = BP : PC = h : k. \quad \ldots\ldots\ldots (\delta)$$

Therefore, alternately,

$$k : CO = h : r,$$

which is the second relation in (β) above.

Now assume a length x such that each of the last ratios is equal to $x : BC$, as in (β).

Then $x : BC = k : CO = h : r.$

Therefore $x + k : BO = h : r,$

and, alternately, $x + k : h = BO : r$

$= h : k$, from (δ) above;

and this is the relation (α) which remained to be found.

Apollonius' proof of the construction is given by Eutocius, who begins by saying that it is manifest that r is a mean proportional between BO and OC. This is seen as follows.

From (β) we derive

$$x : BC = k : CO = h : r = (k + x) : BO,$$

whence $BO : r = (k + x) : h$

$= h : k$, by (α),

$= r : CO$, by (β),

and therefore $r^2 = BO \,.\, CO.$

But the triangles BOP, POC have the angle at O common, and, since $BO : OP = OP : OC$, the triangles are similar and the angles OPC, OBP are equal.

[Up to this point Aristotle's proof is exactly the same; from this point it diverges slightly.]

If now CL be drawn parallel to BP meeting OP in L, the angles BPC LCP are equal also.

Therefore the triangles BPC, PCL are similar, and

$$BP : PC = PC : CL,$$

whence $BP^2 : PC^2 = BP : CL$

$= BO : OC$, by parallels,

$= BO^2 : OP^2$ (since $BO : OP = OP : OC$).

Therefore $BP : PC = BO : OP$

$= h : k$ (for $OP = r$).

[Aristotle infers this more directly from the similar triangles POB, COP. Since these triangles are similar,

$$OP : CP = OB : BP,$$

whence $BP : PC = BO : OP$

$= h : k.$]

Apollonius proves lastly, by ***reductio ad absurdum***, that the last equation cannot be true with reference to any point P which is not on the circle so described.

PROPOSITION 4.

In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

052211 Plate 1

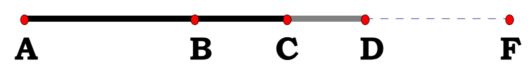
Unit.

Given.

$N_1 := 5 \quad AC := N_1$

$N_2 := 2 \quad CD := N_2$

Given AC is not one half of AD, find AF, DF such that AC : CD :: AF : DF.



Descriptions.

The author of The PAKABOLA, ELLIPSE, AND HYPEKBOLA by R. W. Griffin, gave this figure as a Lemma, and Book VI Prop. 6 of Euclid only has it in the notes, but no construction.

Basically it is how to construct a circle with a given ratio, which is why it is probably the first thing in Chapt. 2 of Griffin's work. So, I will first demonstrate how to construct the figure from the internal ratio of AC : CD.

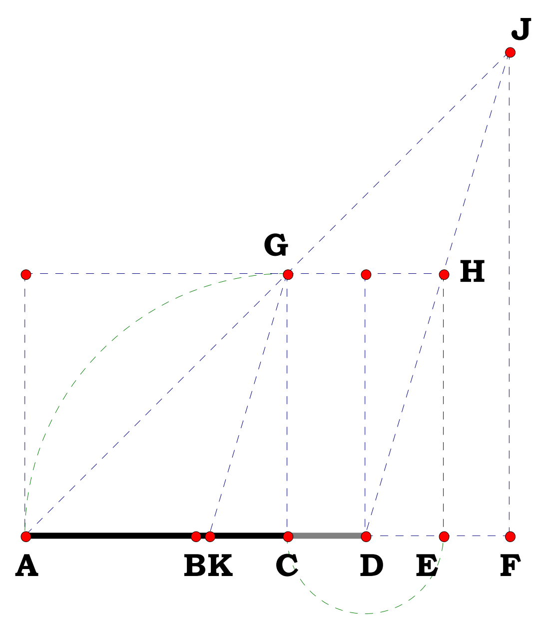
$CK := CD$

$AK := AC - CK \quad AG := \sqrt{2 \cdot AC^2}$

$AD := AC + CD \quad AJ := \dfrac{AG \cdot AD}{AK}$

$AF := \dfrac{AC \cdot AJ}{AG} \quad DF := AF - AD$

$CF := AF - AC$



Definitions.

$$AF - \left(N_1 \cdot \frac{N_1 + N_2}{N_1 - N_2}\right) = 0 \qquad \frac{AC}{CD} - \frac{AF}{DF} = 0$$

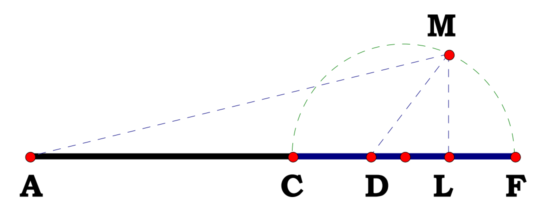
$$DF - \left(N_1 \cdot \frac{N_1 + N_2}{N_1 - N_2} - N_1 - N_2\right) = 0$$

$$\left(\frac{N_1 \cdot \dfrac{N_1 + N_2}{N_1 - N_2}}{N_1 \cdot \dfrac{N_1 + N_2}{N_1 - N_2} - N_1 - N_2}\right) - \frac{N_1}{N_2} = 0$$

Next demonstrate that from the circumference of CM, the proportion remains.

$$N_3 := 1 \qquad N_4 := 4$$

$$CL := CF \cdot \frac{N_3}{N_4} \qquad FL := CF - CL$$



$$AL := AC + CL \qquad ML := \sqrt{CL \cdot FL}$$

$$AM := \sqrt{ML^2 + AL^2}$$

$$DL := CL - CD$$

$$DM := \sqrt{ML^2 + DL^2}$$

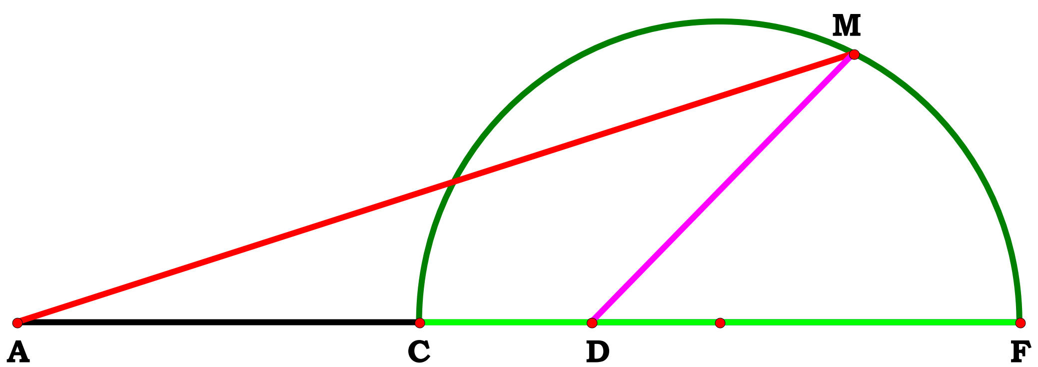
$$\frac{AC}{CD} - \frac{AM}{DM} = 0$$

$$DM - \left[\left|\frac{N_2}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4}\right)^{\frac{1}{2}}\right|\right] = 0$$

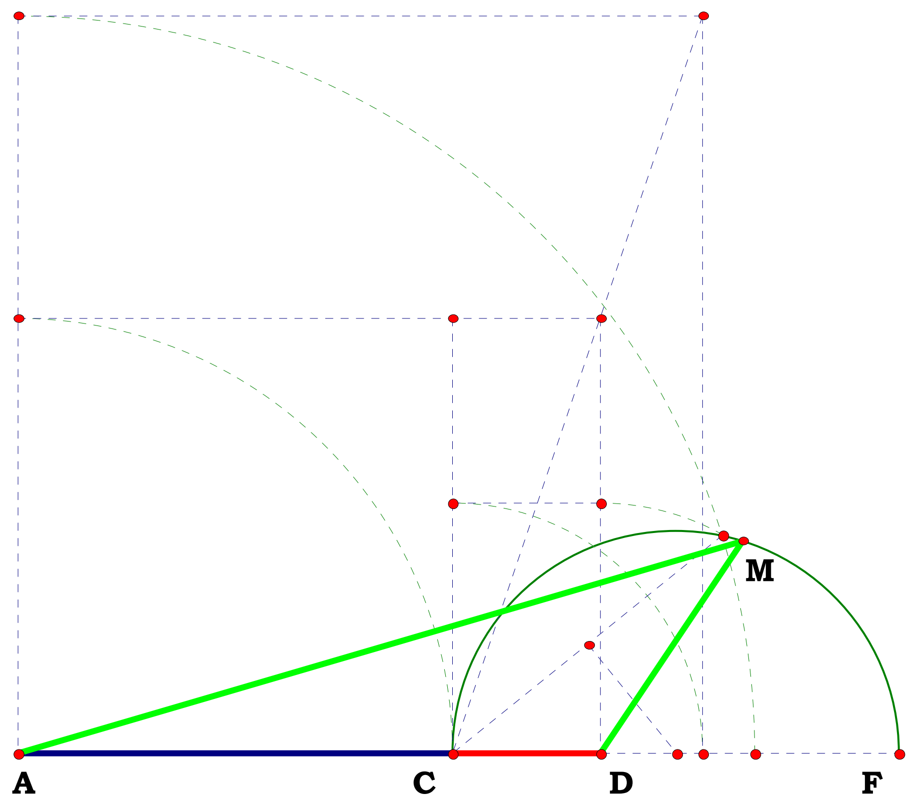
$$AM - \left[\left|\frac{N_1}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4}\right)^{\frac{1}{2}}\right|\right] = 0$$

$$\left[\frac{\frac{N_1}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4}\right)^{\frac{1}{2}}}{\frac{N_2}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4}\right)^{\frac{1}{2}}}\right] - \frac{N_1}{N_2} = 0$$

Therefore the three ratios are equal. AC : CD :: AF : DF :: AM : DM, and one can construct a circle using a given ratio.



Alternate Construction


M
A
C
D
F

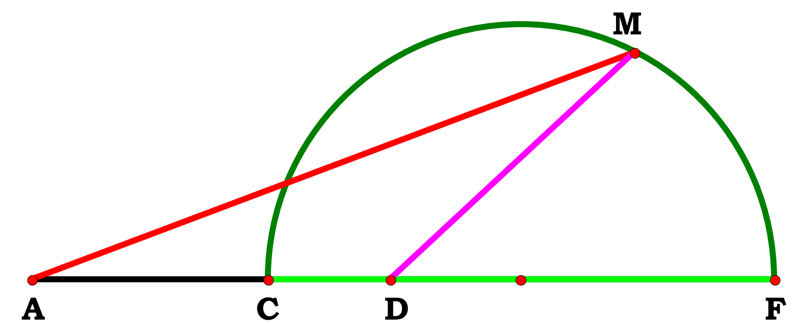
Unit.

Given.

$N_1 := 3.22792 \quad AC := N_1$

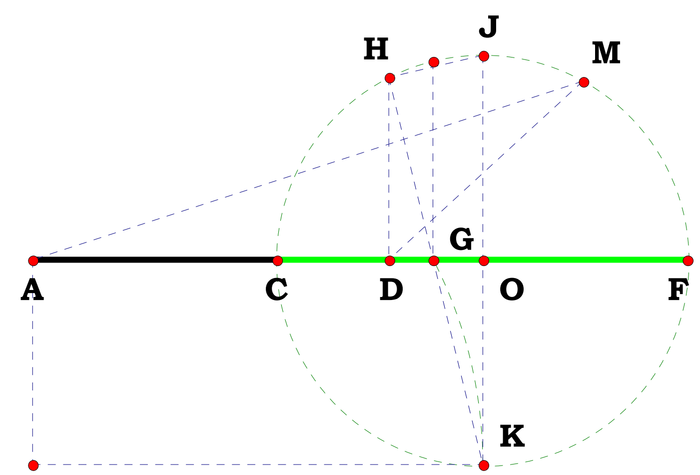
$N_2 := 8.65187 \quad AF := N_2$

052211 Plate 2



Descriptions.

For the given figure, find D if only AC and AF are given.



$CF := AF - AC$

$AG := \sqrt{AC \cdot AF} \qquad OK := \dfrac{CF}{2}$

$AO := AC + OK$

$GO := AO - AG \qquad GK := \sqrt{OK^2 + GO^2}$

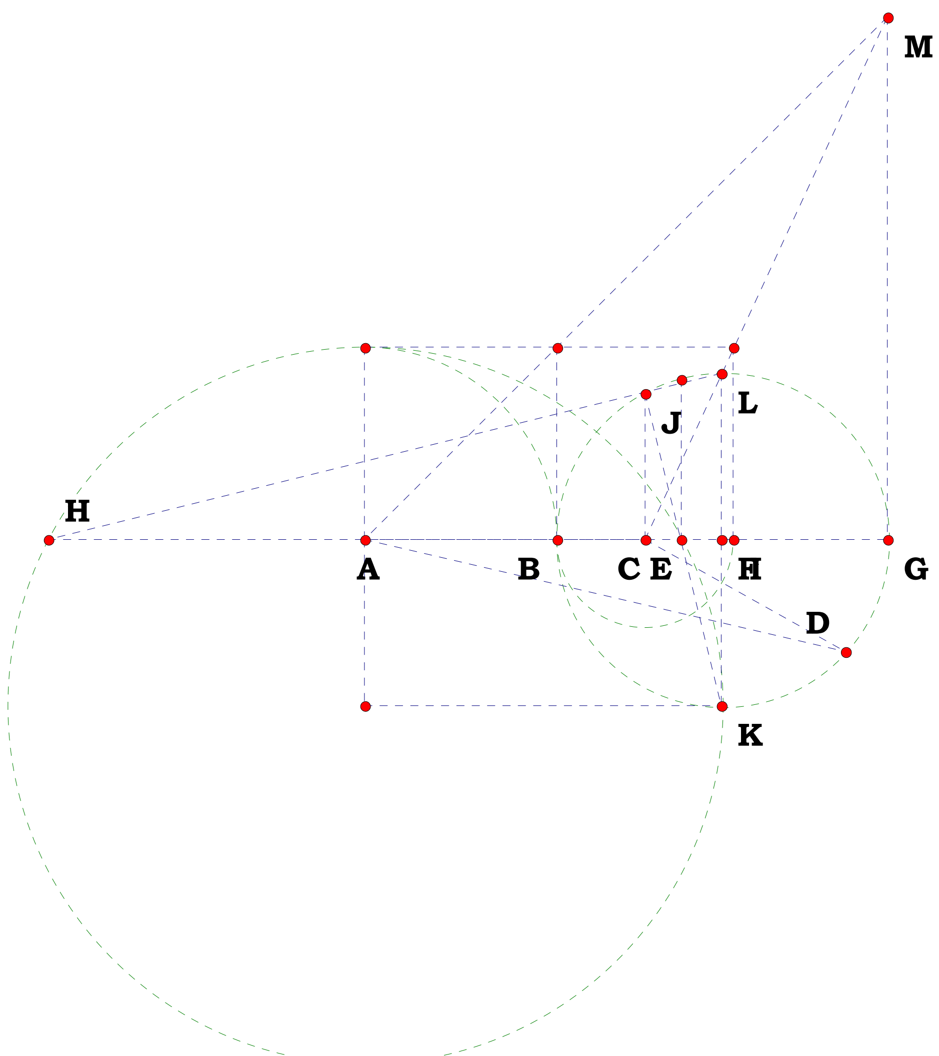
$HK := \dfrac{OK \cdot CF}{GK} \qquad GH := HK - GK \qquad DG := \dfrac{GO \cdot GH}{GK}$

$AD := AG - DG \qquad DF := AF - AD \qquad CD := AD - AC$

Definitions.

$$\frac{AC}{CD} - \frac{AF}{DF} = 0 \qquad AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0$$

$$DF - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \qquad CD - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$$



M
L
J
H
A
B
C
E
H
G
D
K

Lemma Plate 3

052211

Descriptions.

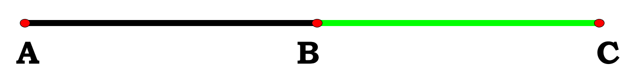
Unit.

Given.

$N_1 := 3.86292 \quad AB := N_1$

$N_2 := 7.59354 \quad AC := N_2$

Simplify Plate 2.

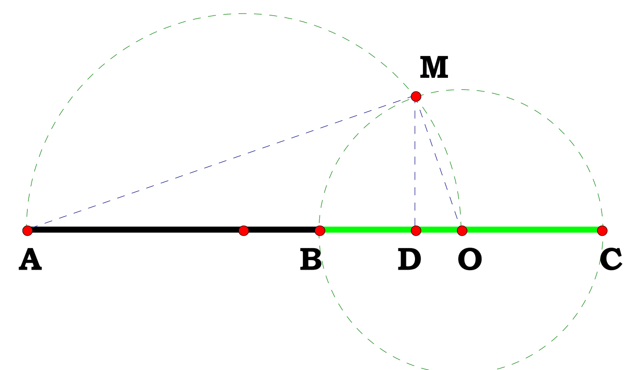


$BC := AC - AB \quad BO := \dfrac{BC}{2} \quad MO := BO$

$AO := AB + BO \quad AM := \sqrt{AO^2 - MO^2}$

$DO := \dfrac{MO^2}{AO} \quad BD := BO - DO$

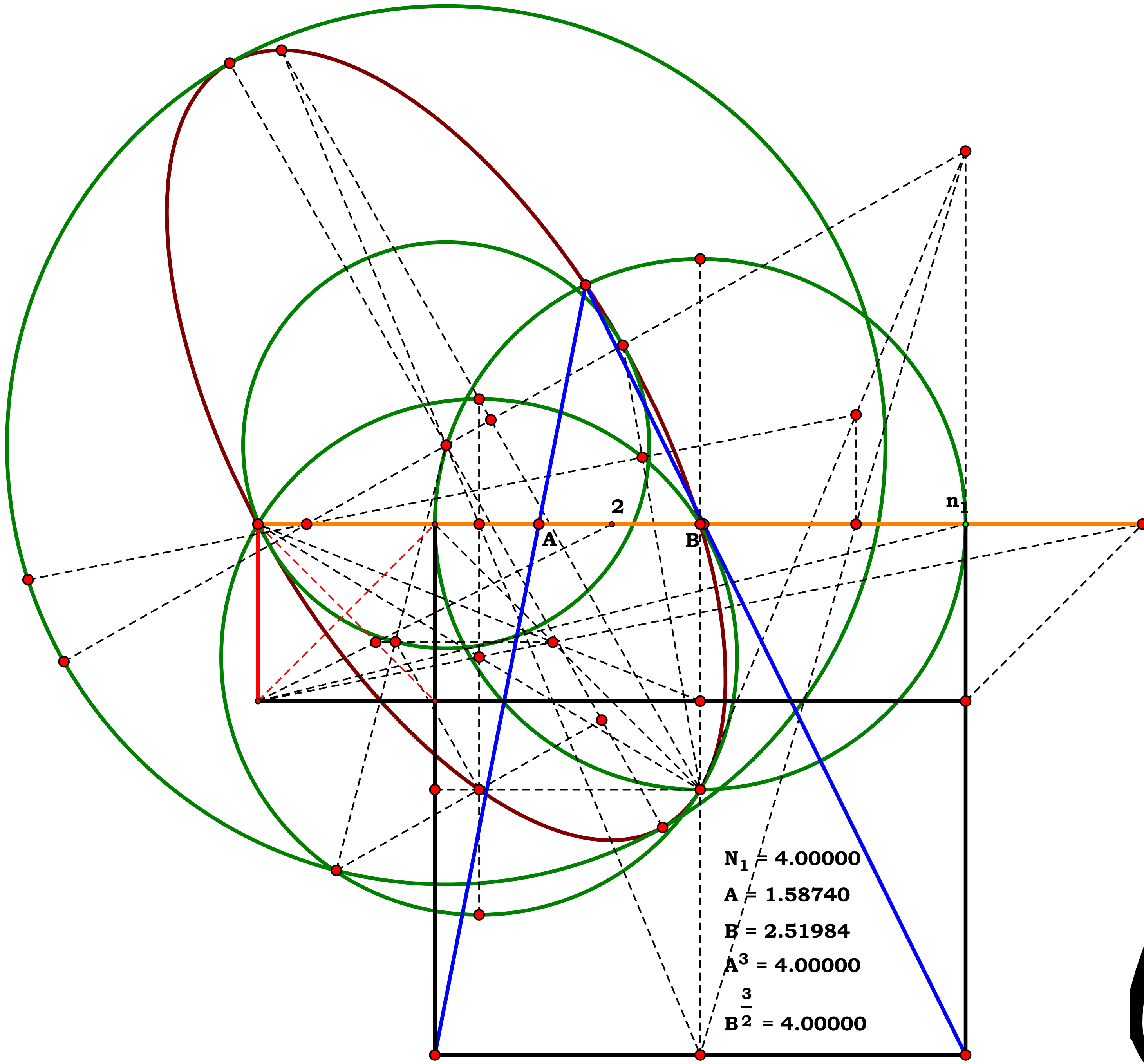
$CD := BC - BD \quad AD := AB + BD$



Definitions.

$$\frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} - BD = 0 \qquad \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} - CD = 0$$

$$\frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} - AD = 0$$



The Delian Quest 2015

John Clark

Unit.

Given.

$AB := 6.20183$

$AC := 4.89358$

$BC := 9.20468$

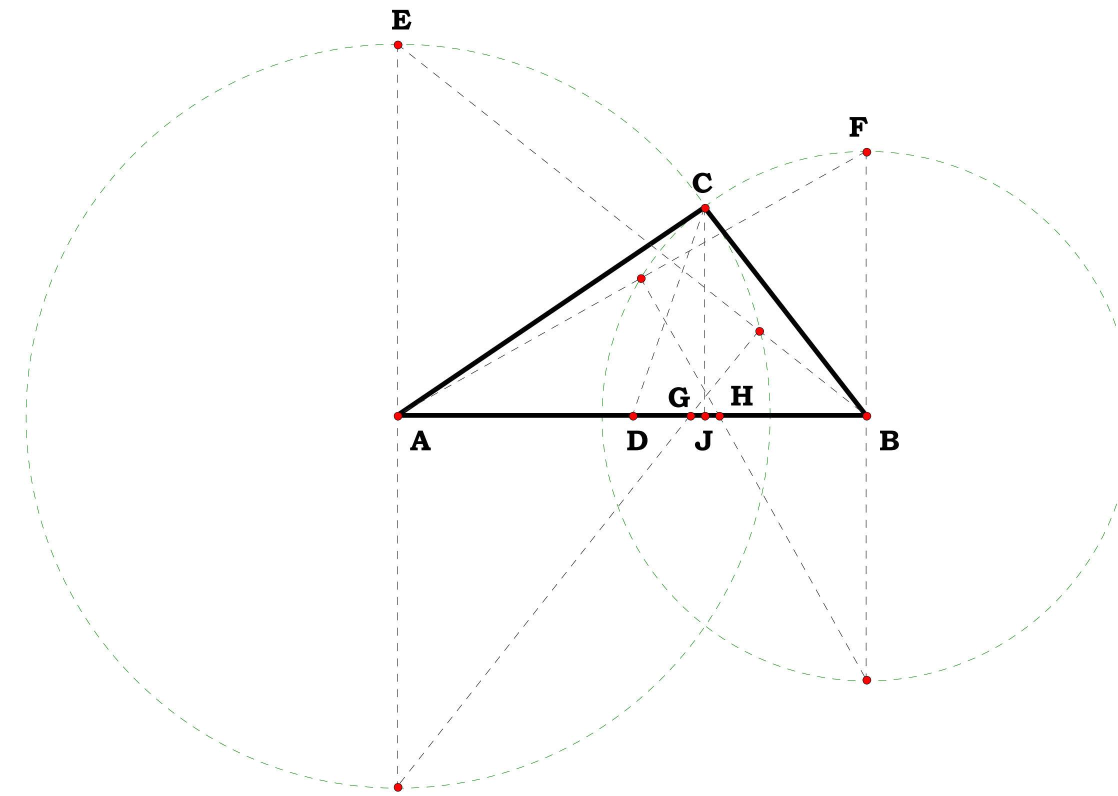
08092015

Descriptions.

$GA := \frac{AC^2}{AB}$ $\quad HB := \frac{BC^2}{AB}$ $\quad GH := AB - (GA + HB)$ $\quad JA := GA + \frac{GH}{2}$

$JB := HB + \frac{GH}{2}$ $\quad CJ := \sqrt{AC^2 - JA^2}$ $\quad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$

One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.



Pythagoras Revisited Again!

Definitions.

$$CJ - \frac{\sqrt{(AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)}}{2\cdot AB} = 0$$

$$CD - \frac{\sqrt{2\cdot AC^2 - AB^2 + 2\cdot BC^2}}{2} = 0 \qquad JA - \frac{AB^2 + AC^2 - BC^2}{2\cdot AB} = 0$$

$$JB - \frac{AB^2 - AC^2 + BC^2}{2\cdot AB} = 0$$

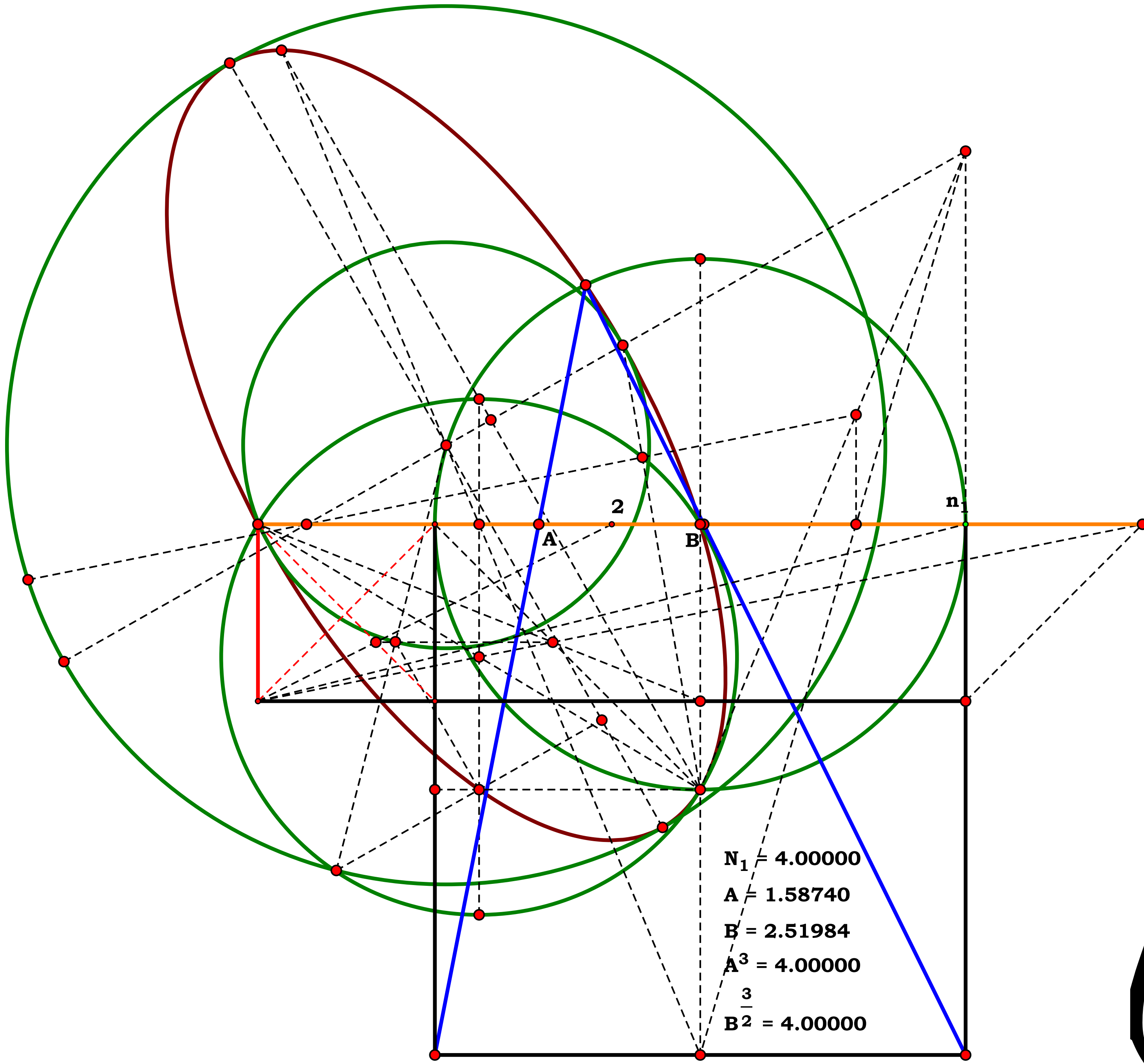
Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.

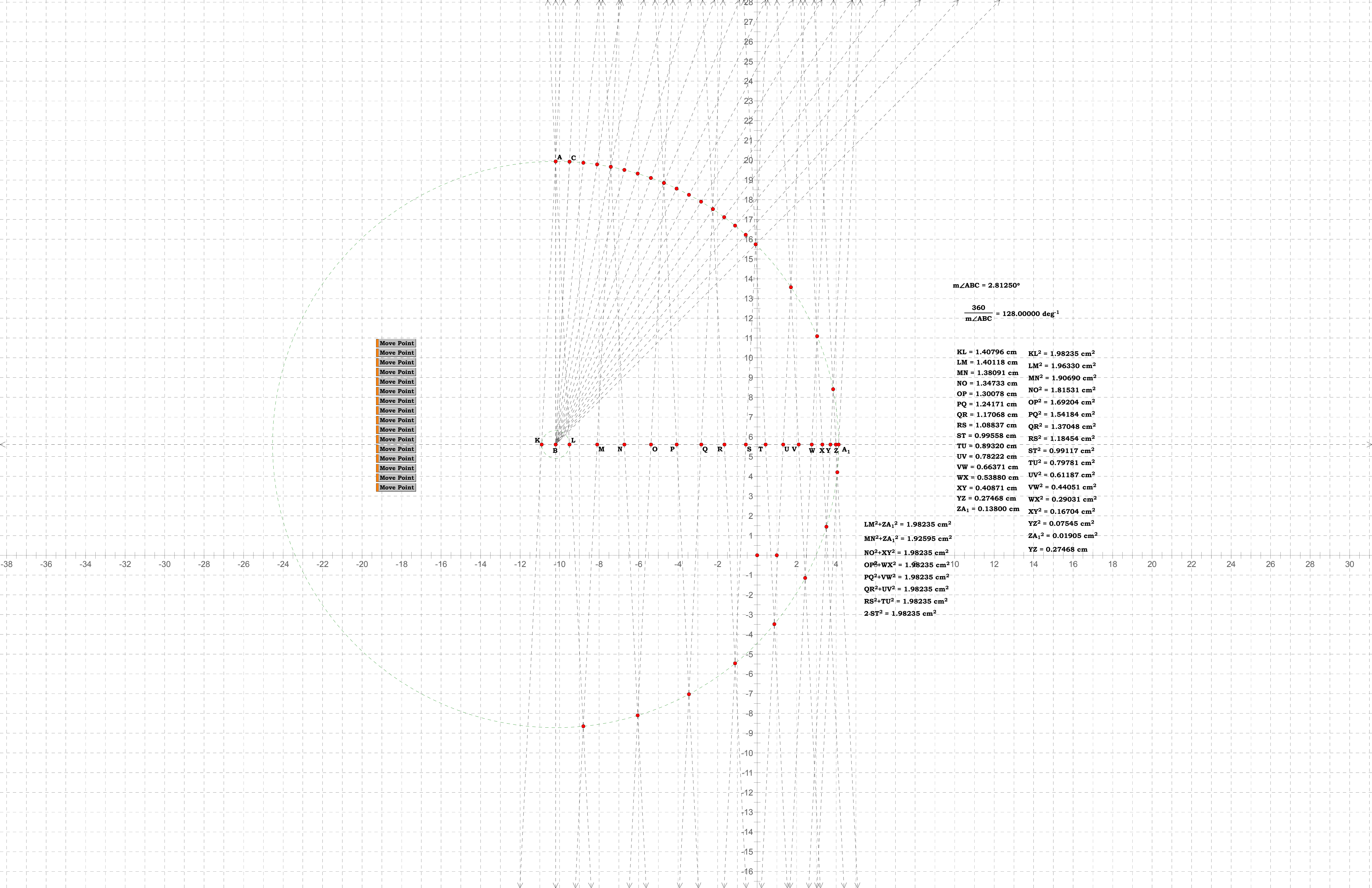
$$AJ := \sqrt{AC^2 - CJ^2} \qquad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2\cdot AB} = 0$$

$$BJ := \sqrt{BC^2 - CJ^2} \qquad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2\cdot AB} = 0$$

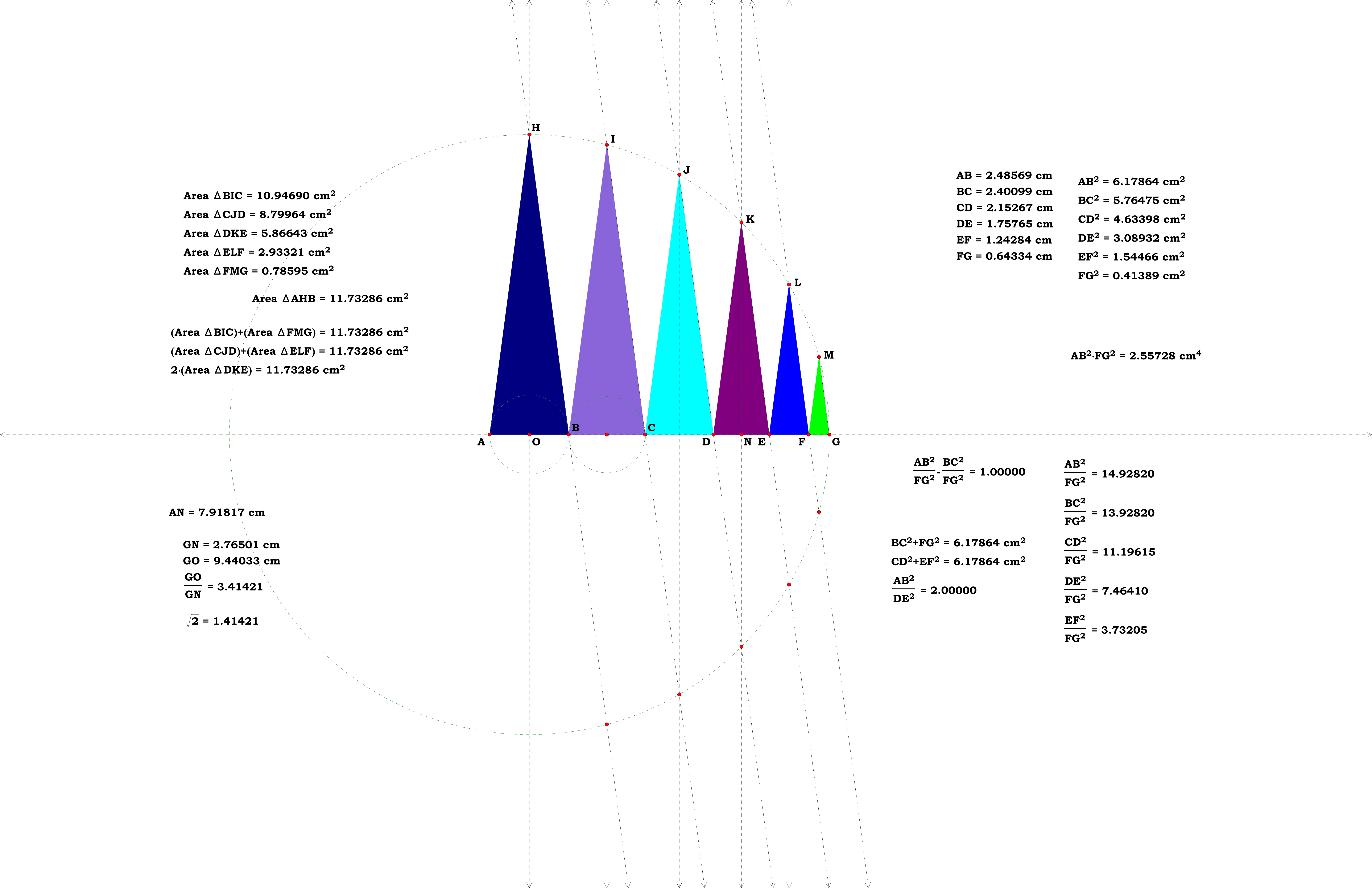
The Delian Quest 2017

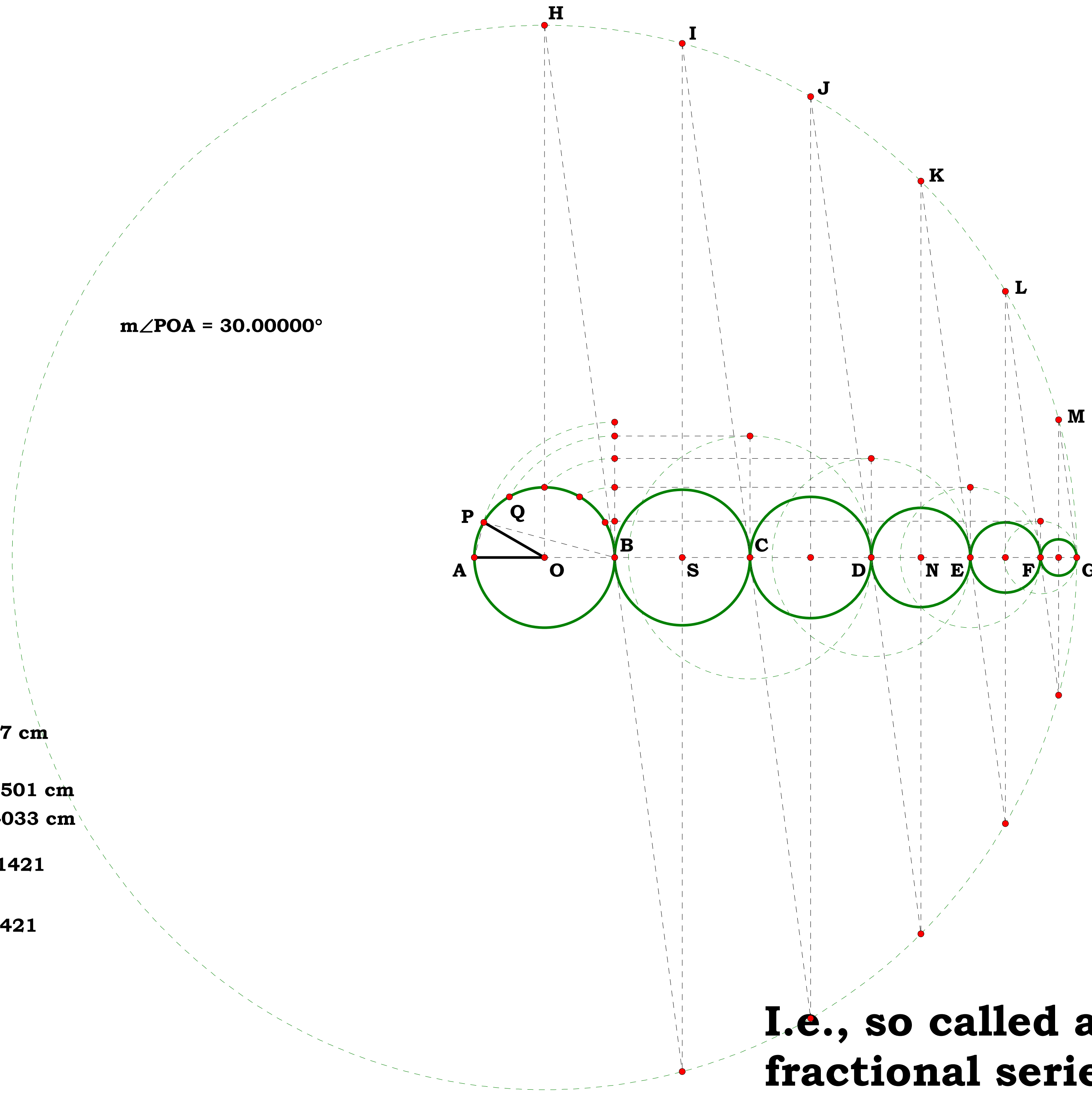
John Clark





A
C
K
B
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
A_1
Move Point
$m\angle ABC = 2.81250°$
$\frac{360}{m\angle ABC} = 128.00000\ deg^{-1}$
KL = 1.40796 cm
LM = 1.40118 cm
MN = 1.38091 cm
NO = 1.34733 cm
OP = 1.30078 cm
PQ = 1.24171 cm
QR = 1.17068 cm
RS = 1.08837 cm
ST = 0.99558 cm
TU = 0.89320 cm
UV = 0.78222 cm
VW = 0.66371 cm
WX = 0.53880 cm
XY = 0.40871 cm
YZ = 0.27468 cm
ZA_1 = 0.13800 cm
$KL^2 = 1.98235\ cm^2$
$LM^2 = 1.96330\ cm^2$
$MN^2 = 1.90690\ cm^2$
$NO^2 = 1.81531\ cm^2$
$OP^2 = 1.69204\ cm^2$
$PQ^2 = 1.54184\ cm^2$
$QR^2 = 1.37048\ cm^2$
$RS^2 = 1.18454\ cm^2$
$ST^2 = 0.99117\ cm^2$
$TU^2 = 0.79781\ cm^2$
$UV^2 = 0.61187\ cm^2$
$VW^2 = 0.44051\ cm^2$
$WX^2 = 0.29031\ cm^2$
$XY^2 = 0.16704\ cm^2$
$YZ^2 = 0.07545\ cm^2$
$ZA_1^2 = 0.01905\ cm^2$
YZ = 0.27468 cm
$LM^2+ZA_1^2 = 1.98235\ cm^2$
$MN^2+ZA_1^2 = 1.92595\ cm^2$
$NO^2+XY^2 = 1.98235\ cm^2$
$OP^2+WX^2 = 1.98235\ cm^2$
$PQ^2+VW^2 = 1.98235\ cm^2$
$QR^2+UV^2 = 1.98235\ cm^2$
$RS^2+TU^2 = 1.98235\ cm^2$
$2 \cdot ST^2 = 1.98235\ cm^2$


H
I
J
K
L
M
A
O
B
C
D
N
E
F
G
Area $\triangle BIC$ = 10.94690 cm^2
Area $\triangle CJD$ = 8.79964 cm^2
Area $\triangle DKE$ = 5.86643 cm^2
Area $\triangle ELF$ = 2.93321 cm^2
Area $\triangle FMG$ = 0.78595 cm^2
Area $\triangle AHB$ = 11.73286 cm^2
(Area $\triangle BIC$)+(Area $\triangle FMG$) = 11.73286 cm^2
(Area $\triangle CJD$)+(Area $\triangle ELF$) = 11.73286 cm^2
2·(Area $\triangle DKE$) = 11.73286 cm^2
AB = 2.48569 cm
BC = 2.40099 cm
CD = 2.15267 cm
DE = 1.75765 cm
EF = 1.24284 cm
FG = 0.64334 cm
AB^2 = 6.17864 cm^2
BC^2 = 5.76475 cm^2
CD^2 = 4.63398 cm^2
DE^2 = 3.08932 cm^2
EF^2 = 1.54466 cm^2
FG^2 = 0.41389 cm^2
$AB^2 \cdot FG^2$ = 2.55728 cm^4
$\frac{AB^2}{FG^2}-\frac{BC^2}{FG^2}$ = 1.00000
$\frac{AB^2}{FG^2}$ = 14.92820
$\frac{BC^2}{FG^2}$ = 13.92820
$\frac{CD^2}{FG^2}$ = 11.19615
$\frac{DE^2}{FG^2}$ = 7.46410
$\frac{EF^2}{FG^2}$ = 3.73205
BC^2+FG^2 = 6.17864 cm^2
CD^2+EF^2 = 6.17864 cm^2
$\frac{AB^2}{DE^2}$ = 2.00000
AN = 7.91817 cm
GN = 2.76501 cm
GO = 9.44033 cm
$\frac{GO}{GN}$ = 3.41421
$\sqrt{2}$ = 1.41421



$m\angle POA = 30.00000°$

AB = 2.48569 cm
BC = 2.40099 cm
CD = 2.15267 cm
DE = 1.75765 cm

EF = 1.24284 cm
QA = 1.24284 cm
FG = 0.64334 cm
PA = 0.64334 cm

$AB^2 = 6.17864\ cm^2$
$BC^2 = 5.76475\ cm^2$
$CD^2 = 4.63398\ cm^2$
$DE^2 = 3.08932\ cm^2$
$EF^2 = 1.54466\ cm^2$
$FG^2 = 0.41389\ cm^2$

$AB^2 \cdot FG^2 = 2.55728\ cm^4$

AN = 7.91817 cm

GN = 2.76501 cm
GO = 9.44033 cm

$\frac{GO}{GN} = 3.41421$

$\sqrt{2} = 1.41421$

$\frac{AB^2}{FG^2} - \frac{BC^2}{FG^2} = 1.00000$

$BC^2+FG^2 = 6.17864\ cm^2$
$CD^2+EF^2 = 6.17864\ cm^2$

$\frac{AB^2}{DE^2} = 2.00000$

$\frac{AB^2}{FG^2} = 14.92820$

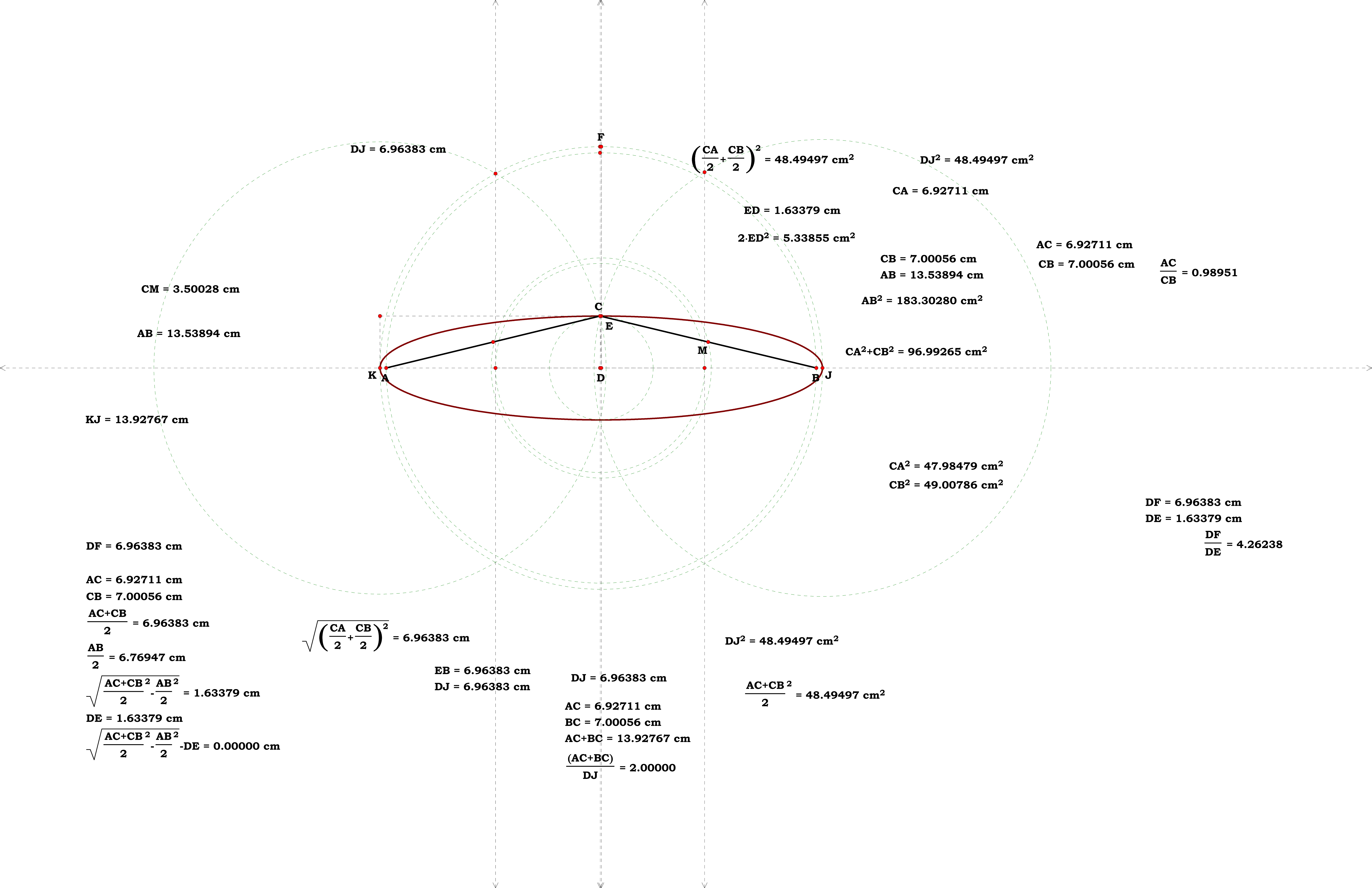
$\frac{BC^2}{FG^2} = 13.92820$

$\frac{CD^2}{FG^2} = 11.19615$

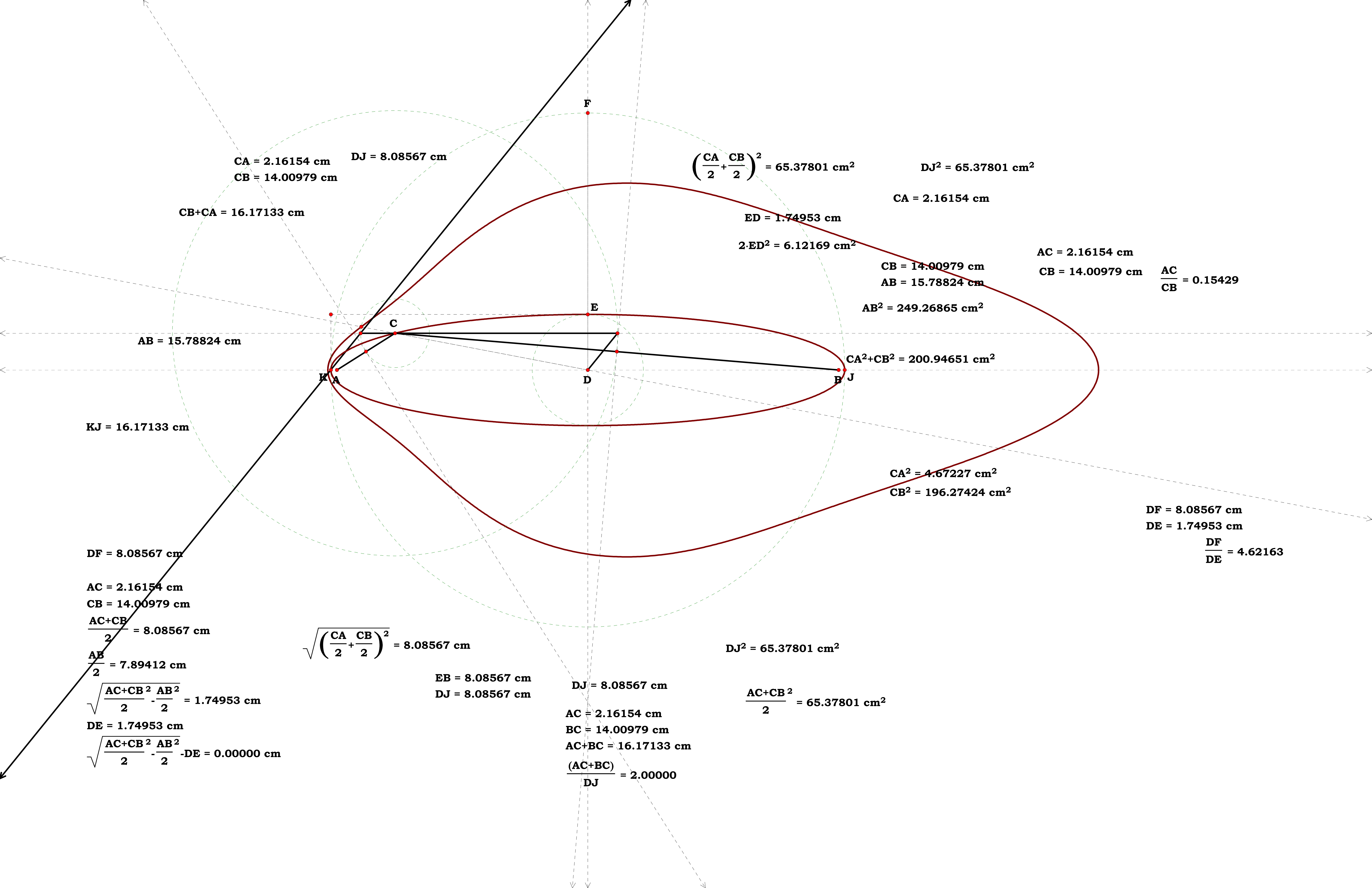
$\frac{DE^2}{FG^2} = 7.46410$

$\frac{EF^2}{FG^2} = 3.73205$

I.e., so called angular division is also a fractional series. One can call fractional series elliptical functions.



F
C
E
M
K
A
D
B
J
DJ = 6.96383 cm
$\left(\frac{CA}{2}+\frac{CB}{2}\right)^2$ = 48.49497 cm^2
DJ^2 = 48.49497 cm^2
CA = 6.92711 cm
ED = 1.63379 cm
2·ED^2 = 5.33855 cm^2
AC = 6.92711 cm
CB = 7.00056 cm
$\frac{AC}{CB}$ = 0.98951
CB = 7.00056 cm
AB = 13.53894 cm
AB^2 = 183.30280 cm^2
CM = 3.50028 cm
AB = 13.53894 cm
CA^2+CB^2 = 96.99265 cm^2
KJ = 13.92767 cm
CA^2 = 47.98479 cm^2
CB^2 = 49.00786 cm^2
DF = 6.96383 cm
DE = 1.63379 cm
$\frac{DF}{DE}$ = 4.26238
DF = 6.96383 cm
AC = 6.92711 cm
CB = 7.00056 cm
$\frac{AC+CB}{2}$ = 6.96383 cm
$\sqrt{\left(\frac{CA}{2}+\frac{CB}{2}\right)^2}$ = 6.96383 cm
$\frac{AB}{2}$ = 6.76947 cm
$\sqrt{\frac{AC+CB}{2}^2-\frac{AB}{2}^2}$ = 1.63379 cm
DE = 1.63379 cm
$\sqrt{\frac{AC+CB}{2}^2-\frac{AB}{2}^2}$-DE = 0.00000 cm
DJ^2 = 48.49497 cm^2
EB = 6.96383 cm
DJ = 6.96383 cm
DJ = 6.96383 cm
$\frac{AC+CB}{2}^2$ = 48.49497 cm^2
AC = 6.92711 cm
BC = 7.00056 cm
AC+BC = 13.92767 cm
$\frac{(AC+BC)}{DJ}$ = 2.00000


F
E
C
K A
D
B J
CA = 2.16154 cm
CB = 14.00979 cm
DJ = 8.08567 cm
CB+CA = 16.17133 cm
$\left(\frac{CA}{2}+\frac{CB}{2}\right)^2$ = 65.37801 cm^2
DJ^2 = 65.37801 cm^2
CA = 2.16154 cm
ED = 1.74953 cm
$2 \cdot ED^2$ = 6.12169 cm^2
AC = 2.16154 cm
CB = 14.00979 cm
$\frac{AC}{CB}$ = 0.15429
CB = 14.00979 cm
AB = 15.78824 cm
AB^2 = 249.26865 cm^2
AB = 15.78824 cm
CA^2+CB^2 = 200.94651 cm^2
KJ = 16.17133 cm
CA^2 = 4.67227 cm^2
CB^2 = 196.27424 cm^2
DF = 8.08567 cm
DE = 1.74953 cm
$\frac{DF}{DE}$ = 4.62163
DF = 8.08567 cm
AC = 2.16154 cm
CB = 14.00979 cm
$\frac{AC+CB}{2}$ = 8.08567 cm
$\frac{AB}{2}$ = 7.89412 cm
$\sqrt{\frac{AC+CB}{2}^2-\frac{AB}{2}^2}$ = 1.74953 cm
DE = 1.74953 cm
$\sqrt{\frac{AC+CB}{2}^2-\frac{AB}{2}^2}$-DE = 0.00000 cm
$\sqrt{\left(\frac{CA}{2}+\frac{CB}{2}\right)^2}$ = 8.08567 cm
EB = 8.08567 cm
DJ = 8.08567 cm
DJ = 8.08567 cm
DJ^2 = 65.37801 cm^2
$\frac{AC+CB}{2}^2$ = 65.37801 cm^2
AC = 2.16154 cm
BC = 14.00979 cm
AC+BC = 16.17133 cm
$\frac{(AC+BC)}{DJ}$ = 2.00000

Index is Base 0

Indexs: $I_{ndx} = 1$ $C_{indx} = 13.00000$

Number of div. by difference at an index.

$$\frac{(I_{ndx} \cdot N_2 - N_1 \cdot N_3) \cdot ((I_{ndx} \cdot N_2 + N_2) - N_1 \cdot N_3)}{N_1 \cdot N_2} = 105.00000$$

Total number of fractions.

$$\frac{N_1 \cdot (N_3 - 1)}{N_2} = 14.00000 \qquad \frac{N_1 \cdot N_3 - N_1}{N_2} = 14.00000$$

Fraction at Index:

Num: $N_1 \cdot N_3 - I_{ndx} \cdot N_2 = 30.00000$

Den: $N_1 = 4.00000$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} = 7.50000$$

Fraction at Compliment:

$$\frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 1.50000$$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} + \frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 9.00000$$

N[1] -> 0 | N[1] -> 1 | N[1] -> 2 | N[1] -> 3 | N[1] -> 4 | N[1] -> 5 | N[1] -> 6 | N[1] -> 7 | N[1] -> 8 | N[1] -> 9 | N[1] -> 10 | N[1] -> 11

N[2] -> 0 | N[2] -> 1 | N[2] -> 2 | N[2] -> 3 | N[2] -> 4 | N[2] -> 5 | N[2] -> 6 | N[2] -> 7 | N[2] -> 8 | N[2] -> 9 | N[2] -> 10 | N[2] -> 11

$N_1 = 4.00000$

$N_2 = 2.00000$

$N_3 = 8.00000$

Present 2 Actions

$$\frac{N_3}{N_3 + 1} = 0.88889$$

N[2] -> 0 | N[2] -> 1 | N[2] -> 2 | N[2] -> 3 | N[2] -> 4 | N[2] -> 5 | N[2] -> 6 | N[2] -> 7 | N[2] -> 8 | N[2] -> 9 | N[2] -> 10 | N[2] -> 11

$$\frac{N_1 \cdot N_3 - N_1 - I_{ndx} \cdot N_2}{N_2} = 13.00000$$

$\frac{N_3}{1} = 8.00000$ $\frac{N_3}{H} = 4.00000$

$\frac{N_3}{A} = 7.50000$ $\frac{N_3}{I} = 3.50000$

$\frac{N_3}{B} = 7.00000$ $\frac{N_3}{J} = 3.00000$

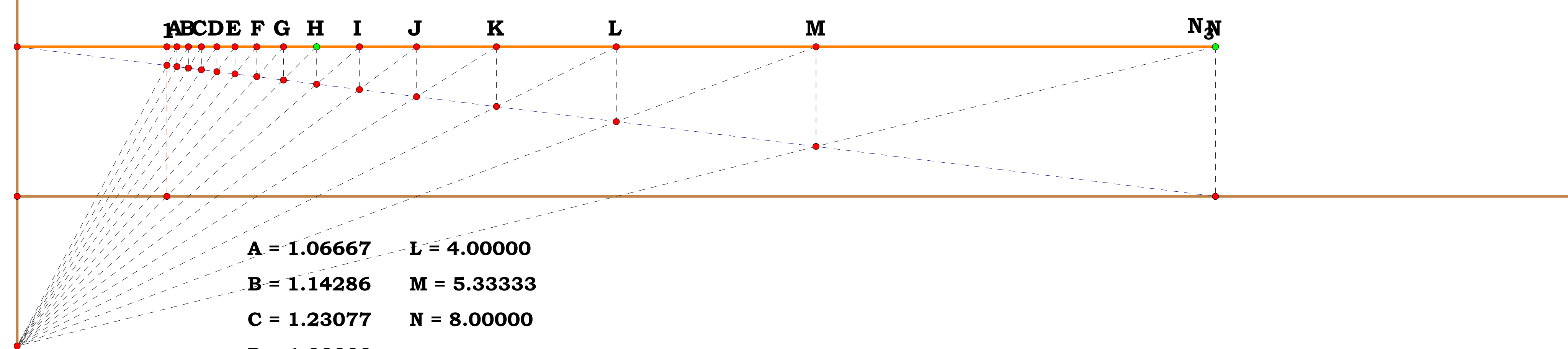
$\frac{N_3}{C} = 6.50000$ $\frac{N_3}{K} = 2.50000$

$\frac{N_3}{D} = 6.00000$ $\frac{N_3}{L} = 2.00000$

$\frac{N_3}{E} = 5.50000$ $\frac{N_3}{M} = 1.50000$

$\frac{N_3}{F} = 5.00000$ $\frac{N_3}{N} = 1.00000$

$\frac{N_3}{G} = 4.50000$



A = 1.06667
B = 1.14286
C = 1.23077
D = 1.33333
E = 1.45455
F = 1.60000
G = 1.77778
H = 2.00000
I = 2.28571
J = 2.66667
K = 3.20000
L = 4.00000
M = 5.33333
N = 8.00000

Indexs: Index = 0 C_{indx} = 8.00

Number of div. by difference at an index.

$$\frac{(Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)\cdot((N_3+Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)-1)}{N_1\cdot N_2\cdot(N_3-1)} = 81.14286$$

len of frac. $\frac{N_1\cdot N_2\cdot N_3}{N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1)} = 1.00000$

Total number of fractions.

$N_1\cdot N_2 = 8.00000$

Fraction at Index:

Num: $N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1) = 64.00000$

Den: $N_1\cdot N_2 = 8.00000$

$$\frac{(N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1))}{(N_1\cdot N_2)} = 8.00000$$

Fraction at Compliment:

$$\frac{N_1\cdot N_2\cdot N_3-C_{indx}\cdot(N_3-1)}{N_1\cdot N_2} = 1.00000$$

$$\frac{(N_3-C_{indx}\cdot N_3-Index\cdot N_3)+2\cdot N_1\cdot N_2\cdot N_3}{N_1\cdot N_2} = 9.00000$$

N[1] -> 0
N[1] -> 1
N[1] -> 2
N[1] -> 3
N[1] -> 4
N[1] -> 5
N[1] -> 6
N[1] -> 7
N[1] -> 8
N[1] -> 9
N[1] -> 10
N[1] -> 11

N[2] -> 0
N[2] -> 1
N[2] -> 2
N[2] -> 3
N[2] -> 4
N[2] -> 5
N[2] -> 6
N[2] -> 7
N[2] -> 8
N[2] -> 9
N[2] -> 10
N[2] -> 11

$N_1 = 4.00000$

$N_2 = 2.00000$

$N_3 = 8.00000$

N[3] -> 0
N[3] -> 1
N[3] -> 2
N[3] -> 3
N[3] -> 4
N[3] -> 5
N[3] -> 6
N[3] -> 7
N[3] -> 8
N[3] -> 9
N[3] -> 10
N[3] -> 11

$\frac{N_3}{1} = 8.00000$

$\frac{N_3}{A} = 7.12500$

$\frac{N_3}{B} = 6.25000$

$\frac{N_3}{C} = 5.37500$

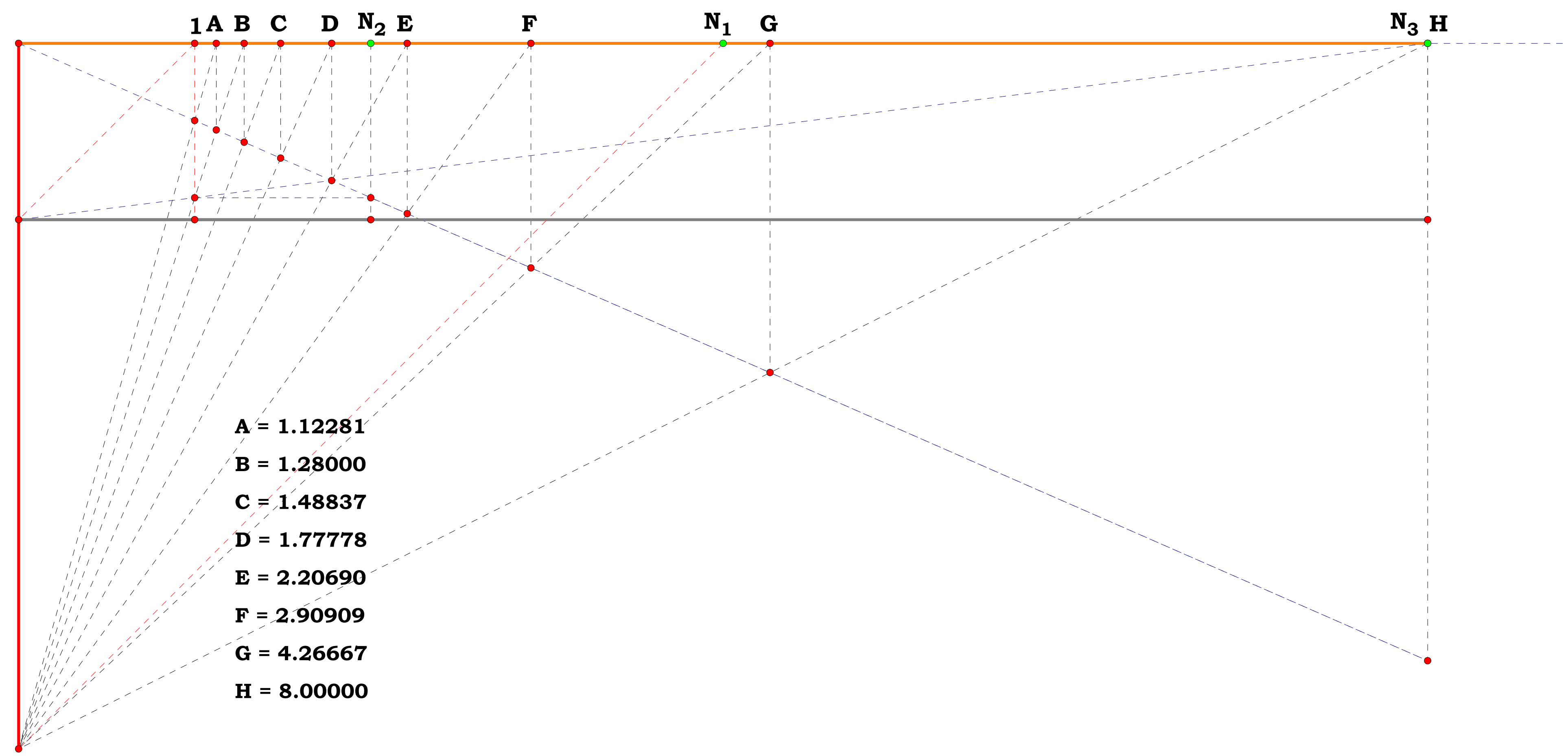
$\frac{N_3}{D} = 4.50000$

$\frac{N_3}{E} = 3.62500$

$\frac{N_3}{F} = 2.75000$

$\frac{N_3}{G} = 1.87500$

$\frac{N_3}{H} = 1.00000$



A = 1.12281
B = 1.28000
C = 1.48837
D = 1.77778
E = 2.20690
F = 2.90909
G = 4.26667
H = 8.00000

$i_{dx} = 1$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)\cdot(((N_2-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)-N_0\cdot N_1\cdot N_3)}{N_1\cdot N_3\cdot(((N_2+i_{dx}\cdot(N_2-N_0\cdot N_2))-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)} = 60.00000$$

$$\frac{N_0\cdot N_1\cdot N_3}{i_{dx}\cdot(N_2-N_2\cdot N_0)+N_0\cdot N_1\cdot N_3} = 1.06667$$

$$\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 1.25000$$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)}+\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 5.00000$$

Total number of fractions.

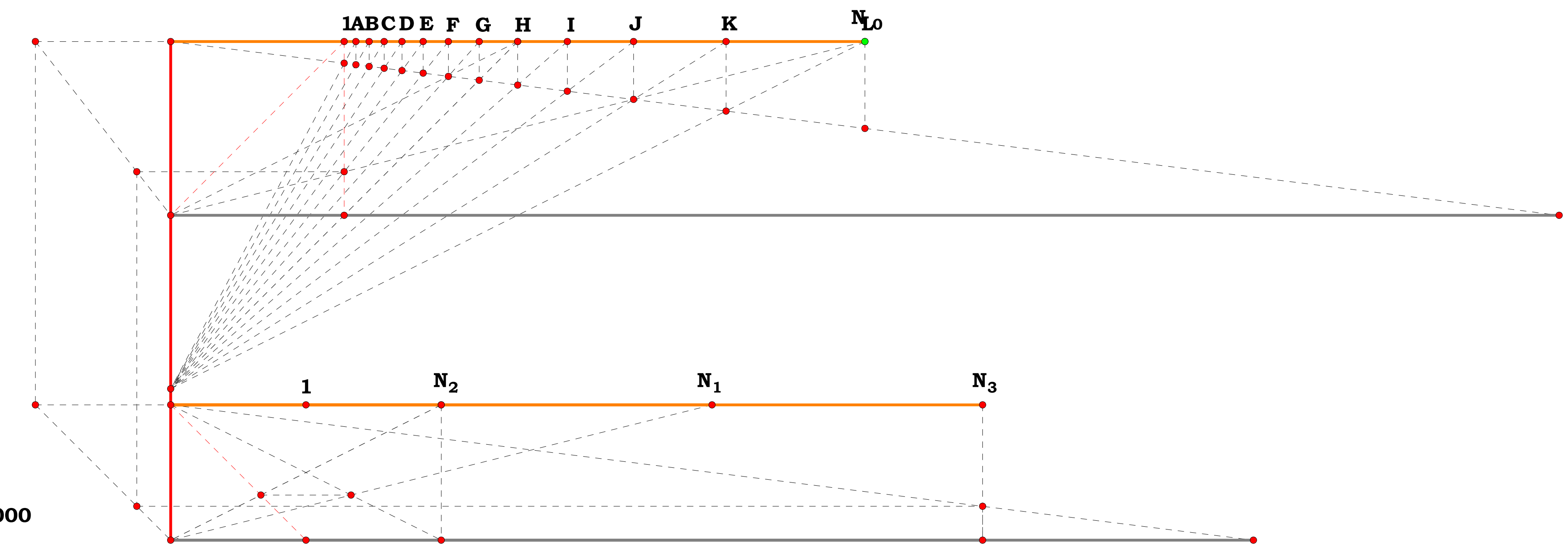
$$\frac{N_1\cdot N_3}{N_2} = 12.00000$$

Fraction at Index:

Num: $i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3 = 90.00000$

Den: $N_1\cdot N_3 = 24.00000$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)} = 3.75000$$



$N_0 = 4.00000$ $N_2 = 2.00000$

$N_1 = 4.00000$ $N_3 = 6.00000$ $N_1\cdot N_3 = 24.00000$

N[1] -> 0	N[2] -> 0	N[3] -> 0	N [0] -> 0
N[1] -> 1	N[2] -> 1	N[3] -> 1	N [0] -> 1
N[1] -> 2	N[2] -> 2	N[3] -> 2	N [0] -> 2
N[1] -> 3	N[2] -> 3	N[3] -> 3	N [0] -> 3
N[1] -> 4	N[2] -> 4	N[3] -> 4	N [0] -> 4
N[1] -> 5	N[2] -> 5	N[3] -> 5	N [0] -> 5
N[1] -> 6	N[2] -> 6	N[3] -> 6	N [0] -> 6
N[1] -> 7	N[2] -> 7	N[3] -> 7	N [0] -> 7
N[1] -> 8	N[2] -> 8	N[3] -> 8	N [0] -> 8
N[1] -> 9	N[2] -> 9	N[3] -> 9	N [0] -> 9
N[1] -> 10	N[2] -> 10	N[3] -> 10	N [0]-> 10
N[1] -> 11	N[2] -> 11	N[3] -> 11	N [0] -> 11

A = 1.06667
B = 1.14286
C = 1.23077
D = 1.33333
E = 1.45455
F = 1.60000
G = 1.77778
H = 2.00000
I = 2.28571
J = 2.66667
K = 3.20000
L = 4.00000

$\frac{N_0}{A} = 3.75000$ $\frac{N_0}{G} = 2.25000$

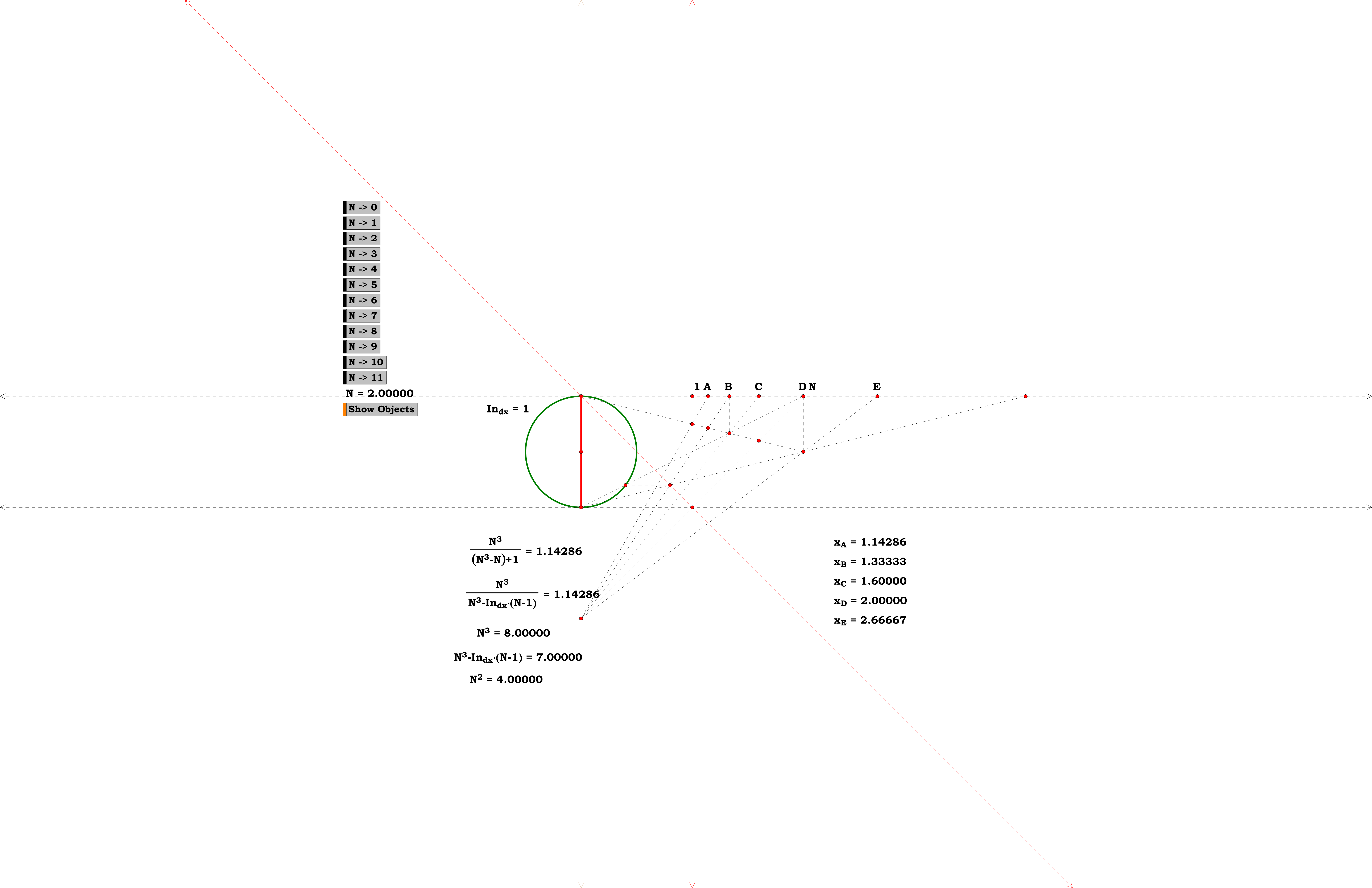
$\frac{N_0}{B} = 3.50000$ $\frac{N_0}{H} = 2.00000$

$\frac{N_0}{C} = 3.25000$ $\frac{N_0}{I} = 1.75000$

$\frac{N_0}{D} = 3.00000$ $\frac{N_0}{J} = 1.50000$

$\frac{N_0}{E} = 2.75000$ $\frac{N_0}{K} = 1.25000$

$\frac{N_0}{F} = 2.50000$ $\frac{N_0}{L} = 1.00000$



N -> 0
N -> 1
N -> 2
N -> 3
N -> 4
N -> 5
N -> 6
N -> 7
N -> 8
N -> 9
N -> 10
N -> 11
N = 2.00000
Show Objects
$In_{dx} = 1$
1 A B C D N E
$\frac{N^3}{(N^3-N)+1} = 1.14286$
$\frac{N^3}{N^3-In_{dx}\cdot(N-1)} = 1.14286$
$N^3 = 8.00000$
$N^3-In_{dx}\cdot(N-1) = 7.00000$
$N^2 = 4.00000$
$x_A = 1.14286$
$x_B = 1.33333$
$x_C = 1.60000$
$x_D = 2.00000$
$x_E = 2.66667$

Circle Points Circle Points Circle Points
Show Lines Hide Objects Show Lines Hide Objects Show Lines Hide Objects

0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9
10	10	10	10	10	10
11	11	11	11	11	11
12	12	12	12	12	12
13	13	13	13	13	13
14	14	14	14	14	14
15	15	15	15	15	15

$In_{dx} = 0$

$\frac{26.25000}{26.25000} = 1.00000$

$B_3 = 6.17647$

$\frac{N_1}{B_3 - N_1} + 1 = 5.25000$

$\frac{7}{3.75} = 1.86667$

A = 1.03960	J = 1.61538	S = 3.62069
B = 1.08247	K = 1.72131	T = 4.20000
C = 1.12903	L = 1.84211	U = 5.00000
D = 1.17978	M = 1.98113	
E = 1.23529	N = 2.14286	
F = 1.29630	O = 2.33333	
G = 1.36364	P = 2.56098	
H = 1.43836	Q = 2.83784	
I = 1.52174	R = 3.18182	

$N_1 = 5.00000$

$N_2 = 0.00000$

$n_a = 7.00000$

$n_b = 5.00000$

$r_{da} = 1.40000$

$\frac{n_a}{n_b} = 1.40000$

$N_1 \cdot \frac{n_a}{n_b} = 7.00000$

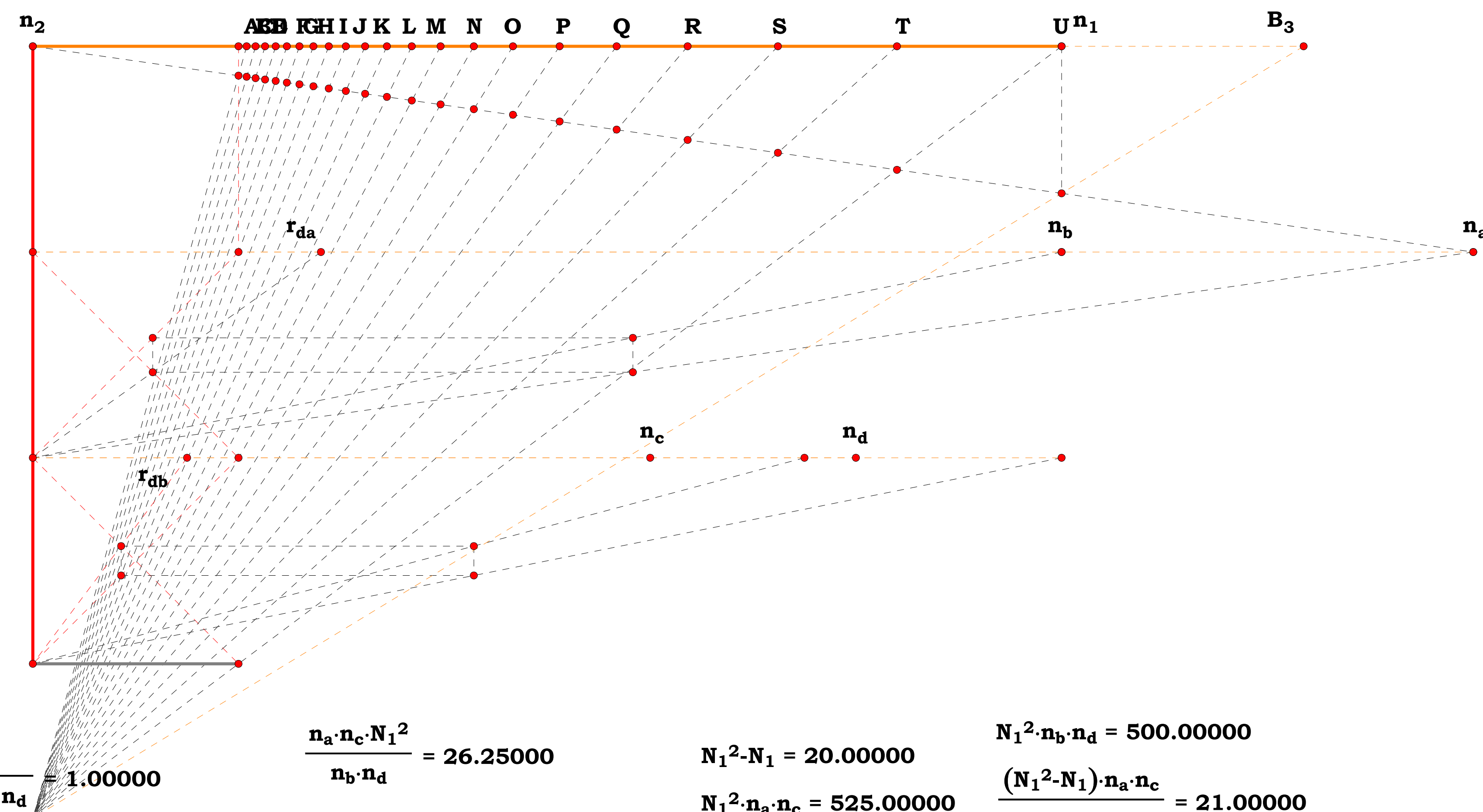
$n_c = 3.00000$

$n_d = 4.00000$

$r_{db} = 0.75000$

$\frac{n_c}{n_d} = 0.75000$

$\frac{n_c}{n_d} \cdot N_1 = 3.75000$



$\frac{N_1{}^2 \cdot n_a \cdot n_c}{N_1{}^2 \cdot n_a \cdot n_c - In_{dx} \cdot n_b \cdot n_d} = 1.00000$

$\frac{n_a \cdot n_c \cdot N_1{}^2}{n_b \cdot n_d} = 26.25000$

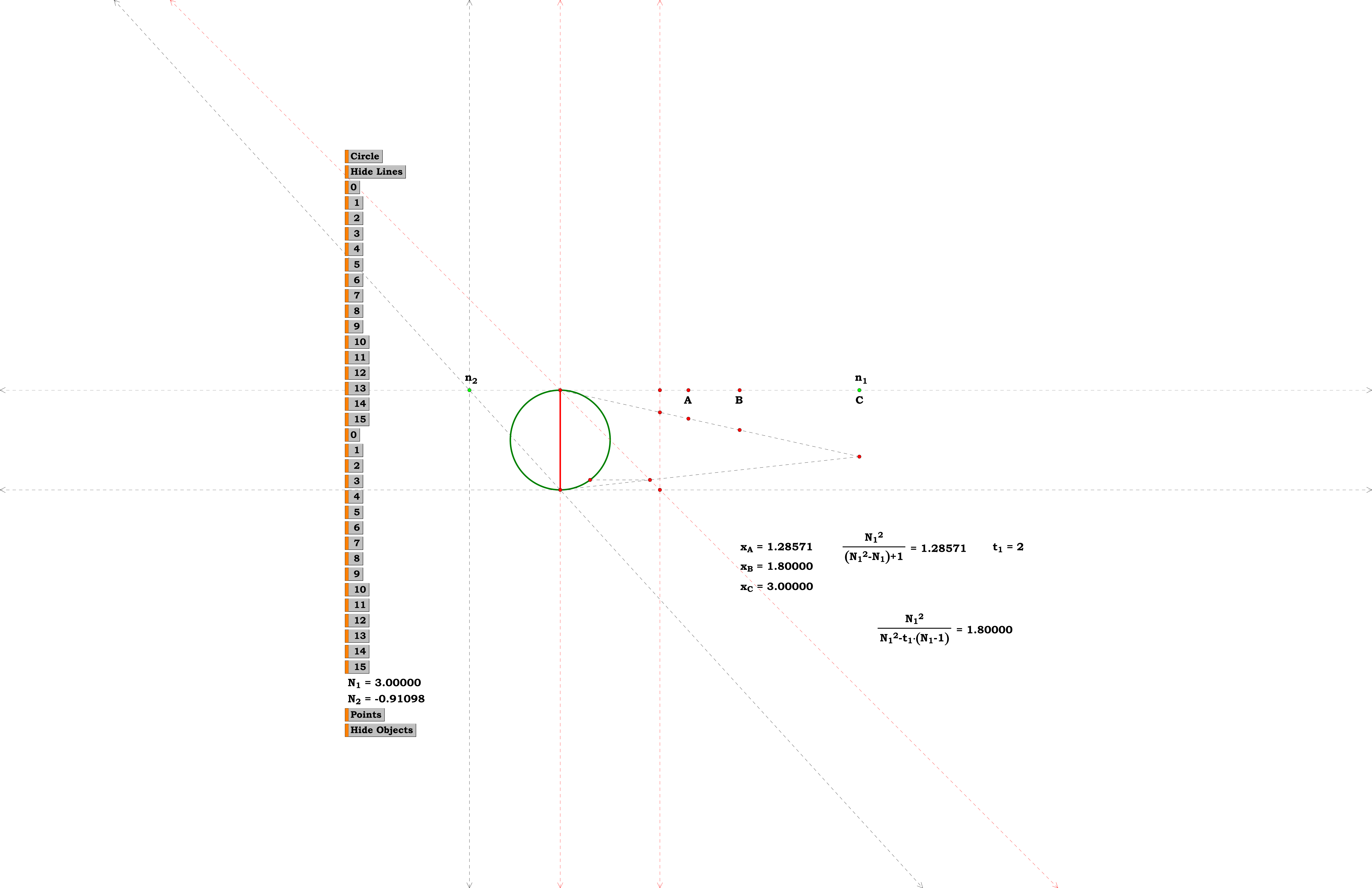
$\frac{n_a \cdot n_c \cdot N_1{}^2}{n_b \cdot n_d} - In_{dx} = 26.25000$

$N_1{}^2 - N_1 = 20.00000$

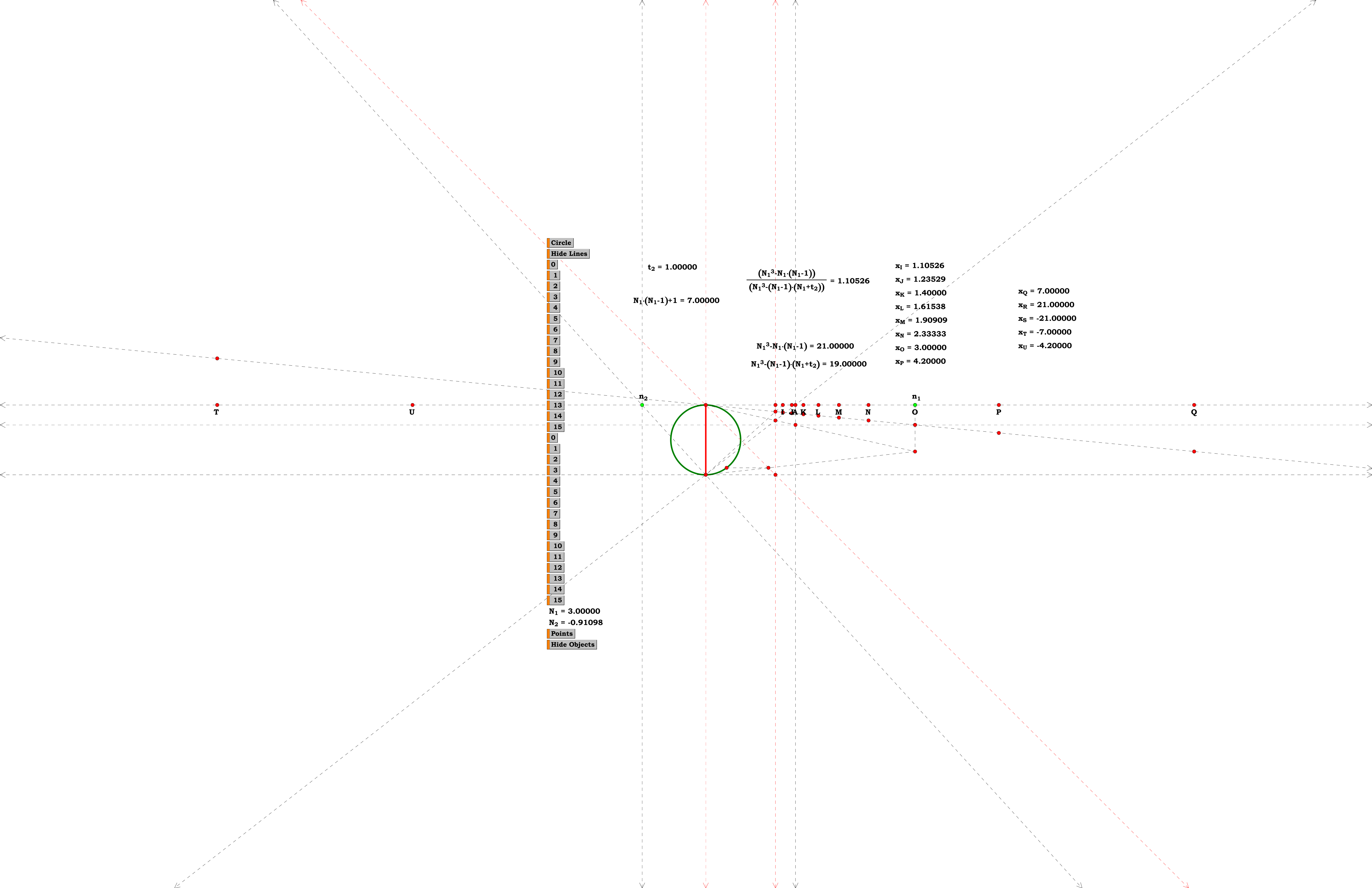
$N_1{}^2 \cdot n_a \cdot n_c = 525.00000$

$N_1{}^2 \cdot n_b \cdot n_d = 500.00000$

$\frac{(N_1{}^2 - N_1) \cdot n_a \cdot n_c}{n_b \cdot n_d} = 21.00000$



Circle
Hide Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
$N_1 = 3.00000$
$N_2 = -0.91098$
Points
Hide Objects
n_2
n_1
A
B
C
$x_A = 1.28571$
$x_B = 1.80000$
$x_C = 3.00000$
$\frac{N_1^2}{(N_1^2 - N_1) + 1} = 1.28571$
$t_1 = 2$
$\frac{N_1^2}{N_1^2 - t_1 \cdot (N_1 - 1)} = 1.80000$


Circle
Hide Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
N_1 = 3.00000
N_2 = -0.91098
Points
Hide Objects
t_2 = 1.00000
$N_1 \cdot (N_1-1)+1$ = 7.00000
$\frac{(N_1^3 - N_1 \cdot (N_1-1))}{(N_1^3 - (N_1-1) \cdot (N_1+t_2))}$ = 1.10526
$N_1^3 - N_1 \cdot (N_1-1)$ = 21.00000
$N_1^3 - (N_1-1) \cdot (N_1+t_2)$ = 19.00000
x_I = 1.10526
x_J = 1.23529
x_K = 1.40000
x_L = 1.61538
x_M = 1.90909
x_N = 2.33333
x_O = 3.00000
x_P = 4.20000
x_Q = 7.00000
x_R = 21.00000
x_S = -21.00000
x_T = -7.00000
x_U = -4.20000
n_2
n_1
T
U
K
L
M
N
O
P
Q

Index is Base 0

Indexs: $I_{ndx} = 1$ $C_{indx} = 13.00000$

Number of div. by difference at an index.

$$\frac{(I_{ndx} \cdot N_2 - N_1 \cdot N_3) \cdot ((I_{ndx} \cdot N_2 + N_2) - N_1 \cdot N_3)}{N_1 \cdot N_2} = 105.00000$$

Total number of fractions.

$$\frac{N_1 \cdot (N_3 - 1)}{N_2} = 14.00000 \qquad \frac{N_1 \cdot N_3 - N_1}{N_2} = 14.00000$$

Fraction at Index:

Num: $N_1 \cdot N_3 - I_{ndx} \cdot N_2 = 30.00000$

Den: $N_1 = 4.00000$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} = 7.50000$$

Fraction at Compliment:

$$\frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 1.50000$$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} + \frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 9.00000$$

N[1] -> 0
N[1] -> 1
N[1] -> 2
N[1] -> 3
N[1] -> 4
N[1] -> 5
N[1] -> 6
N[1] -> 7
N[1] -> 8
N[1] -> 9
N[1] -> 10
N[1] -> 11

N[2] -> 0
N[2] -> 1
N[2] -> 2
N[2] -> 3
N[2] -> 4
N[2] -> 5
N[2] -> 6
N[2] -> 7
N[2] -> 8
N[2] -> 9
N[2] -> 10
N[2] -> 11

$N_1 = 4.00000$
$N_2 = 2.00000$
$N_3 = 8.00000$

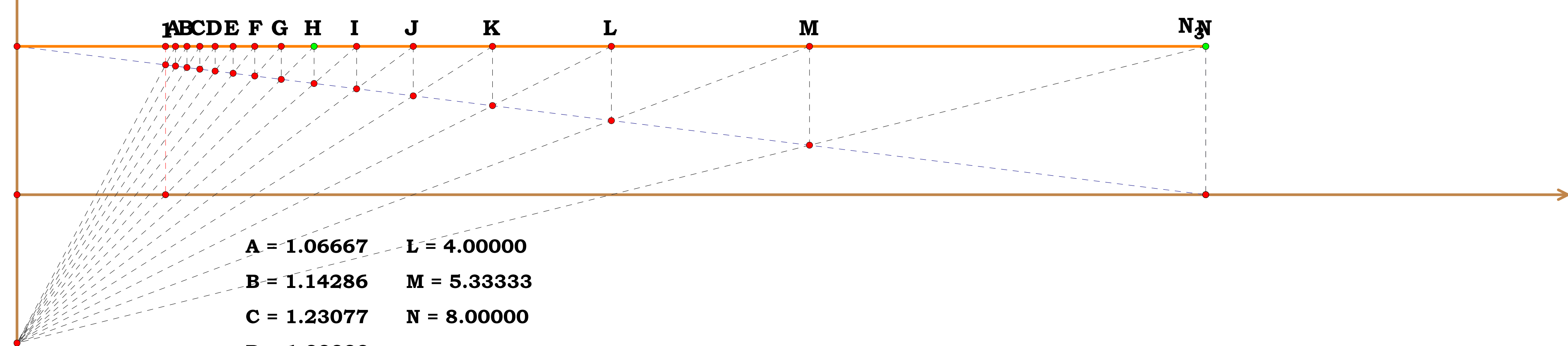
Present 2 Actions

$$\frac{N_3}{N_3 + 1} = 0.88889$$

N[2] -> 0
N[2] -> 1
N[2] -> 2
N[2] -> 3
N[2] -> 4
N[2] -> 5
N[2] -> 6
N[2] -> 7
N[2] -> 8
N[2] -> 9
N[2] -> 10
N[2] -> 11

$$\frac{N_1 \cdot N_3 - N_1 - I_{ndx} \cdot N_2}{N_2} = 13.00000$$

$\frac{N_3}{1} = 8.00000$	$\frac{N_3}{H} = 4.00000$
$\frac{N_3}{A} = 7.50000$	$\frac{N_3}{I} = 3.50000$
$\frac{N_3}{B} = 7.00000$	$\frac{N_3}{J} = 3.00000$
$\frac{N_3}{C} = 6.50000$	$\frac{N_3}{K} = 2.50000$
$\frac{N_3}{D} = 6.00000$	$\frac{N_3}{L} = 2.00000$
$\frac{N_3}{E} = 5.50000$	$\frac{N_3}{M} = 1.50000$
$\frac{N_3}{F} = 5.00000$	$\frac{N_3}{N} = 1.00000$
$\frac{N_3}{G} = 4.50000$	



A = 1.06667
B = 1.14286
C = 1.23077
D = 1.33333
E = 1.45455
F = 1.60000
G = 1.77778
H = 2.00000
I = 2.28571
J = 2.66667
K = 3.20000
L = 4.00000
M = 5.33333
N = 8.00000

Indexs: Index = 0 C_{indx} = 8.00

Number of div. by difference at an index.

$$\frac{(Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)\cdot((N_3+Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)-1)}{N_1\cdot N_2\cdot(N_3-1)} = 81.14286$$

len of frac. $\frac{N_1\cdot N_2\cdot N_3}{N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1)} = 1.00000$

Total number of fractions.

$N_1\cdot N_2 = 8.00000$

Fraction at Index:

Num: $N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1) = 64.00000$

Den: $N_1\cdot N_2 = 8.00000$

$$\frac{(N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1))}{(N_1\cdot N_2)} = 8.00000$$

Fraction at Compliment:

$$\frac{N_1\cdot N_2\cdot N_3-C_{indx}\cdot(N_3-1)}{N_1\cdot N_2} = 1.00000$$

$$\frac{(N_3-C_{indx}\cdot N_3-Index\cdot N_3)+2\cdot N_1\cdot N_2\cdot N_3}{N_1\cdot N_2} = 9.00000$$

N[1] -> 0
N[1] -> 1
N[1] -> 2
N[1] -> 3
N[1] -> 4
N[1] -> 5
N[1] -> 6
N[1] -> 7
N[1] -> 8
N[1] -> 9
N[1] -> 10
N[1] -> 11

N[2] -> 0
N[2] -> 1
N[2] -> 2
N[2] -> 3
N[2] -> 4
N[2] -> 5
N[2] -> 6
N[2] -> 7
N[2] -> 8
N[2] -> 9
N[2] -> 10
N[2] -> 11

$N_1 = 4.00000$

$N_2 = 2.00000$

$N_3 = 8.00000$

N[3] -> 0
N[3] -> 1
N[3] -> 2
N[3] -> 3
N[3] -> 4
N[3] -> 5
N[3] -> 6
N[3] -> 7
N[3] -> 8
N[3] -> 9
N[3] -> 10
N[3] -> 11

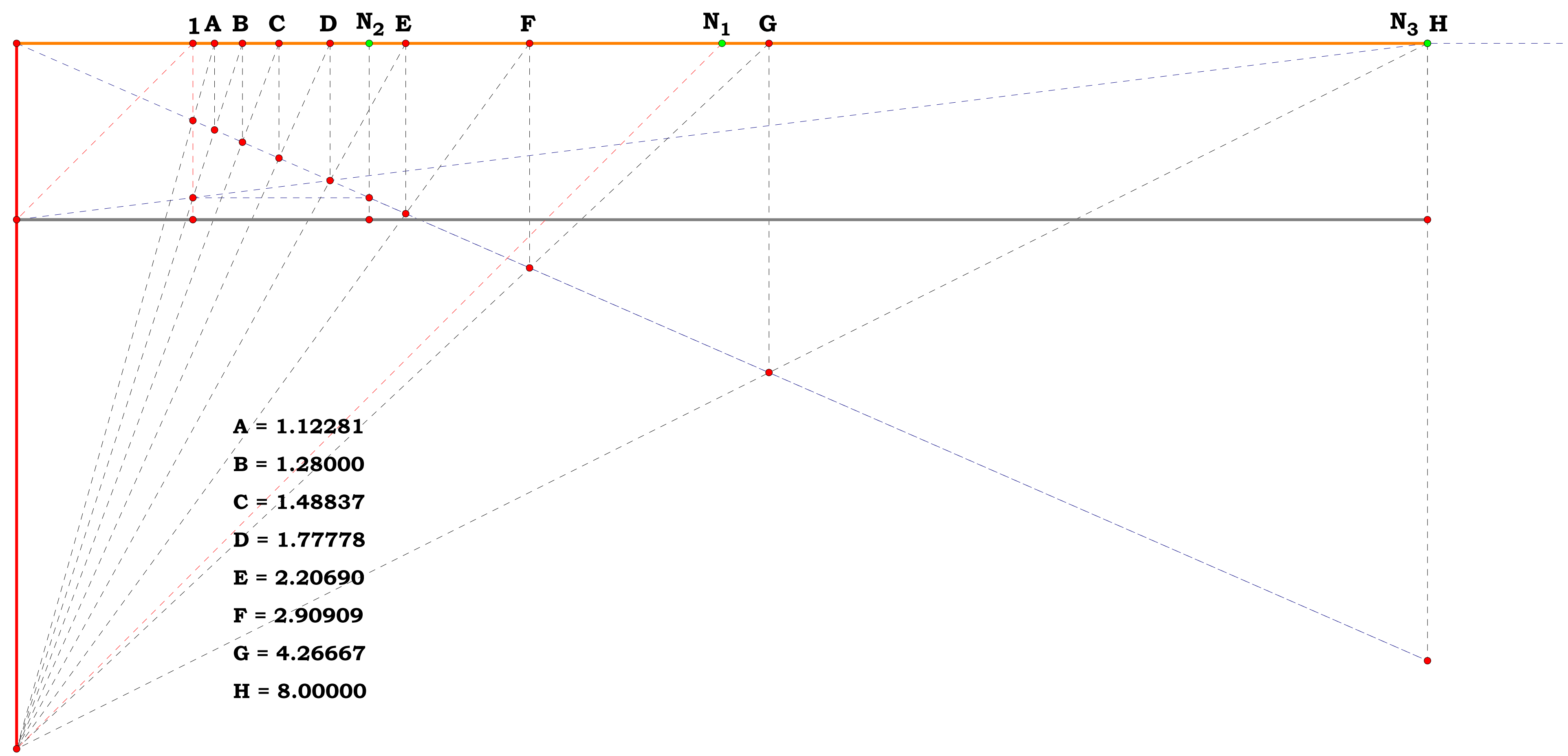
$\frac{N_3}{1} = 8.00000$ $\frac{N_3}{E} = 3.62500$

$\frac{N_3}{A} = 7.12500$ $\frac{N_3}{F} = 2.75000$

$\frac{N_3}{B} = 6.25000$ $\frac{N_3}{G} = 1.87500$

$\frac{N_3}{C} = 5.37500$ $\frac{N_3}{H} = 1.00000$

$\frac{N_3}{D} = 4.50000$



A = 1.12281

B = 1.28000

C = 1.48837

D = 1.77778

E = 2.20690

F = 2.90909

G = 4.26667

H = 8.00000

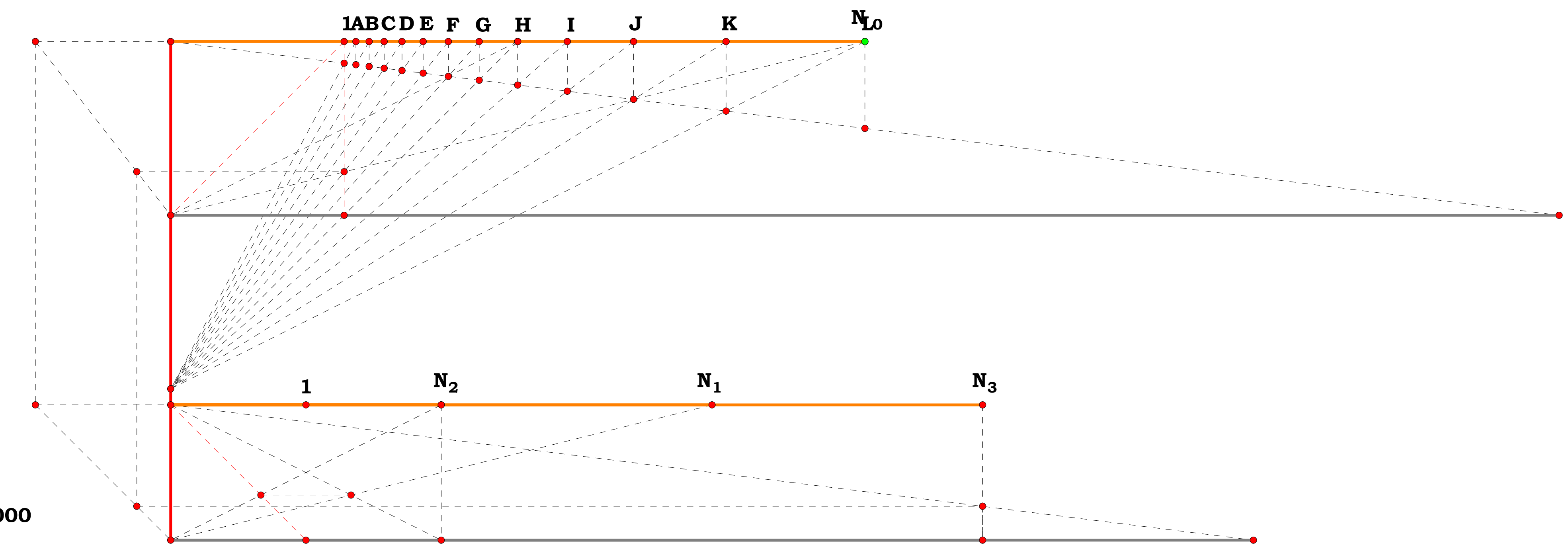
$i_{dx} = 1$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)\cdot(((N_2-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)-N_0\cdot N_1\cdot N_3)}{N_1\cdot N_3\cdot(((N_2+i_{dx}\cdot(N_2-N_0\cdot N_2))-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)} = 60.00000$$

$$\frac{N_0\cdot N_1\cdot N_3}{i_{dx}\cdot(N_2-N_2\cdot N_0)+N_0\cdot N_1\cdot N_3} = 1.06667$$

$$\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 1.25000$$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)}+\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 5.00000$$



Total number of fractions.

$$\frac{N_1\cdot N_3}{N_2} = 12.00000$$

Fraction at Index:

Num: $i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3 = 90.00000$

Den: $N_1\cdot N_3 = 24.00000$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)} = 3.75000$$

$N_0 = 4.00000$ $N_2 = 2.00000$

$N_1 = 4.00000$ $N_3 = 6.00000$ $N_1\cdot N_3 = 24.00000$

N[1] -> 0	N[2] -> 0	N[3] -> 0	N [0] -> 0
N[1] -> 1	N[2] -> 1	N[3] -> 1	N [0] -> 1
N[1] -> 2	N[2] -> 2	N[3] -> 2	N [0] -> 2
N[1] -> 3	N[2] -> 3	N[3] -> 3	N [0] -> 3
N[1] -> 4	N[2] -> 4	N[3] -> 4	N [0] -> 4
N[1] -> 5	N[2] -> 5	N[3] -> 5	N [0] -> 5
N[1] -> 6	N[2] -> 6	N[3] -> 6	N [0] -> 6
N[1] -> 7	N[2] -> 7	N[3] -> 7	N [0] -> 7
N[1] -> 8	N[2] -> 8	N[3] -> 8	N [0] -> 8
N[1] -> 9	N[2] -> 9	N[3] -> 9	N [0] -> 9
N[1] -> 10	N[2] -> 10	N[3] -> 10	N [0]-> 10
N[1] -> 11	N[2] -> 11	N[3] -> 11	N [0] -> 11

A = 1.06667 K = 3.20000
B = 1.14286 L = 4.00000
C = 1.23077
D = 1.33333
E = 1.45455
F = 1.60000
G = 1.77778
H = 2.00000
I = 2.28571
J = 2.66667

$\frac{N_0}{A} = 3.75000$ $\frac{N_0}{G} = 2.25000$

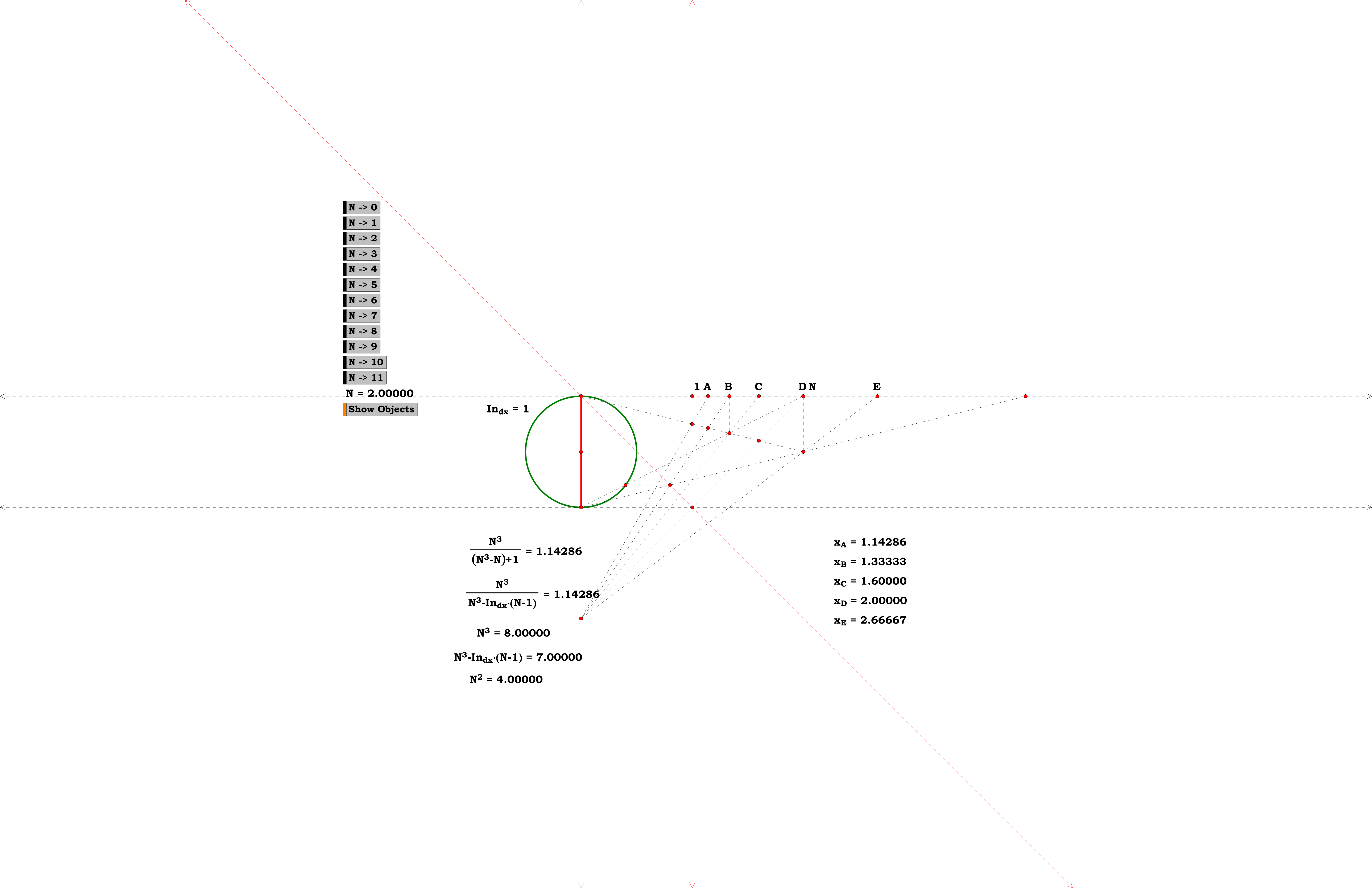
$\frac{N_0}{B} = 3.50000$ $\frac{N_0}{H} = 2.00000$

$\frac{N_0}{C} = 3.25000$ $\frac{N_0}{I} = 1.75000$

$\frac{N_0}{D} = 3.00000$ $\frac{N_0}{J} = 1.50000$

$\frac{N_0}{E} = 2.75000$ $\frac{N_0}{K} = 1.25000$

$\frac{N_0}{F} = 2.50000$ $\frac{N_0}{L} = 1.00000$



N -> 0
N -> 1
N -> 2
N -> 3
N -> 4
N -> 5
N -> 6
N -> 7
N -> 8
N -> 9
N -> 10
N -> 11
N = 2.00000
Show Objects
In_dx = 1
1 A B C D N E
N^3 / ((N^3-N)+1) = 1.14286
N^3 / (N^3-In_dx·(N-1)) = 1.14286
N^3 = 8.00000
N^3-In_dx·(N-1) = 7.00000
N^2 = 4.00000
x_A = 1.14286
x_B = 1.33333
x_C = 1.60000
x_D = 2.00000
x_E = 2.66667

Circle Points Circle Points Circle Points

Show Lines Hide Objects Show Lines Hide Objects Show Lines Hide Objects

0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9
10	10	10	10	10	10
11	11	11	11	11	11
12	12	12	12	12	12
13	13	13	13	13	13
14	14	14	14	14	14
15	15	15	15	15	15

$In_{dx} = 0$

$\frac{26.25000}{26.25000} = 1.00000$

$B_3 = 6.17647$

$\frac{N_1}{B_3 - N_1} + 1 = 5.25000$

$\frac{7}{3.75} = 1.86667$

$A = 1.03960$ $B = 1.08247$ $C = 1.12903$ $D = 1.17978$ $E = 1.23529$ $F = 1.29630$ $G = 1.36364$ $H = 1.43836$ $I = 1.52174$

$J = 1.61538$ $K = 1.72131$ $L = 1.84211$ $M = 1.98113$ $N = 2.14286$ $O = 2.33333$ $P = 2.56098$ $Q = 2.83784$ $R = 3.18182$

$S = 3.62069$ $T = 4.20000$ $U = 5.00000$

$N_1 = 5.00000$

$N_2 = 0.00000$

$n_a = 7.00000$

$n_b = 5.00000$

$r_{da} = 1.40000$

$\frac{n_a}{n_b} = 1.40000$

$N_1 \cdot \frac{n_a}{n_b} = 7.00000$

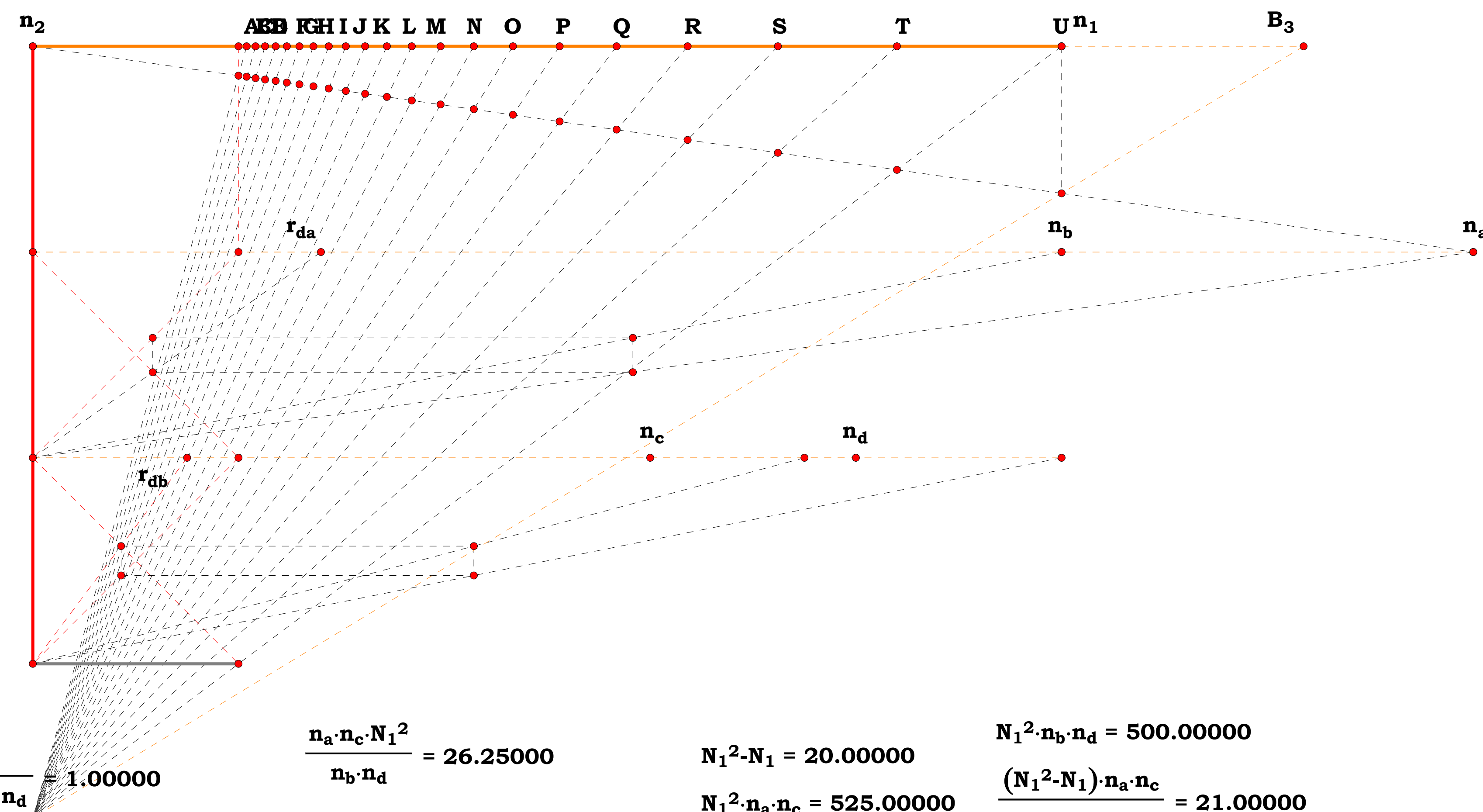
$n_c = 3.00000$

$n_d = 4.00000$

$r_{db} = 0.75000$

$\frac{n_c}{n_d} = 0.75000$

$\frac{n_c}{n_d} \cdot N_1 = 3.75000$



$\frac{N_1{}^2 \cdot n_a \cdot n_c}{N_1{}^2 \cdot n_a \cdot n_c - In_{dx} \cdot n_b \cdot n_d} = 1.00000$

$\frac{n_a \cdot n_c \cdot N_1{}^2}{n_b \cdot n_d} = 26.25000$

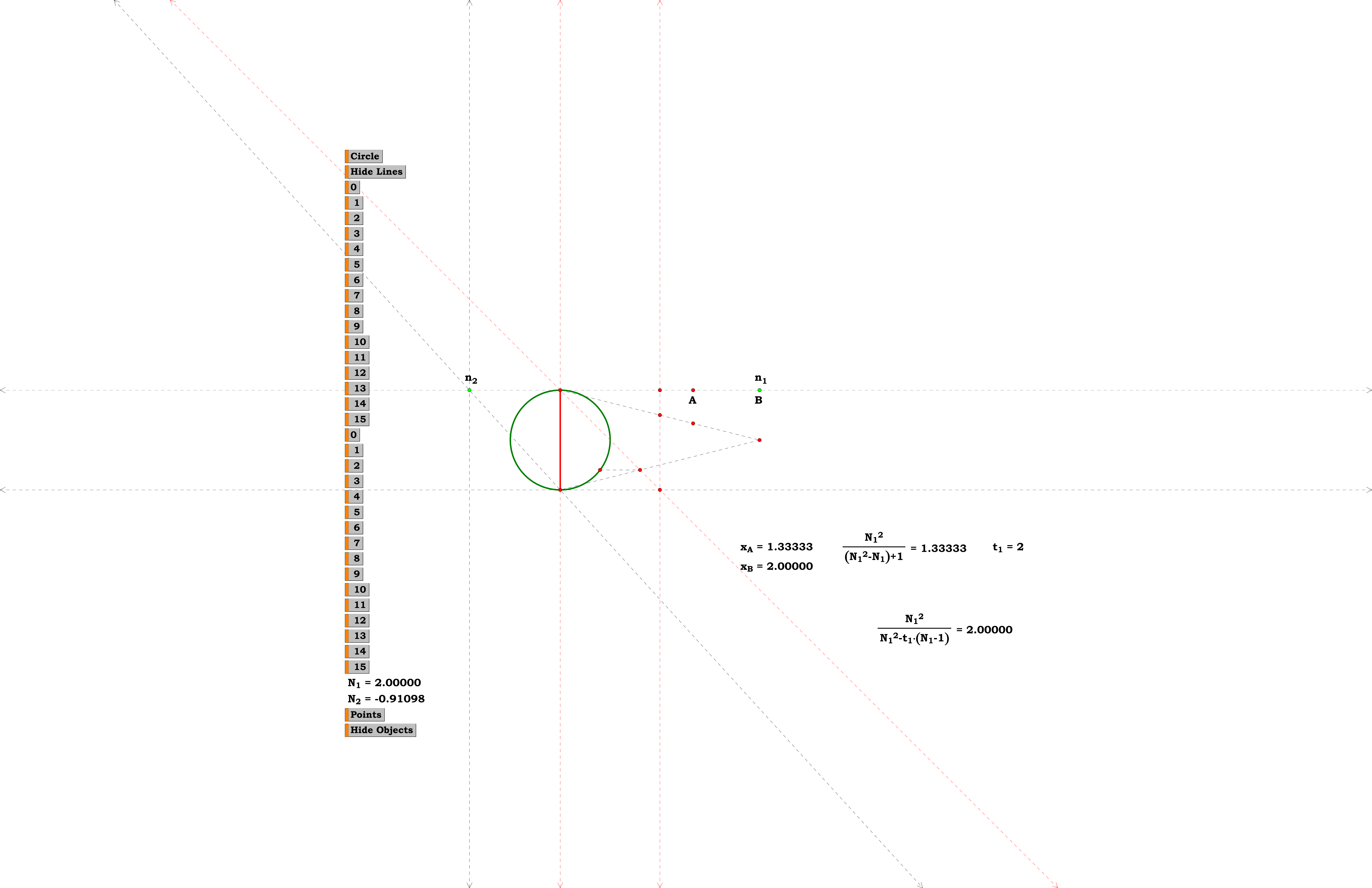
$\frac{n_a \cdot n_c \cdot N_1{}^2}{n_b \cdot n_d} - In_{dx} = 26.25000$

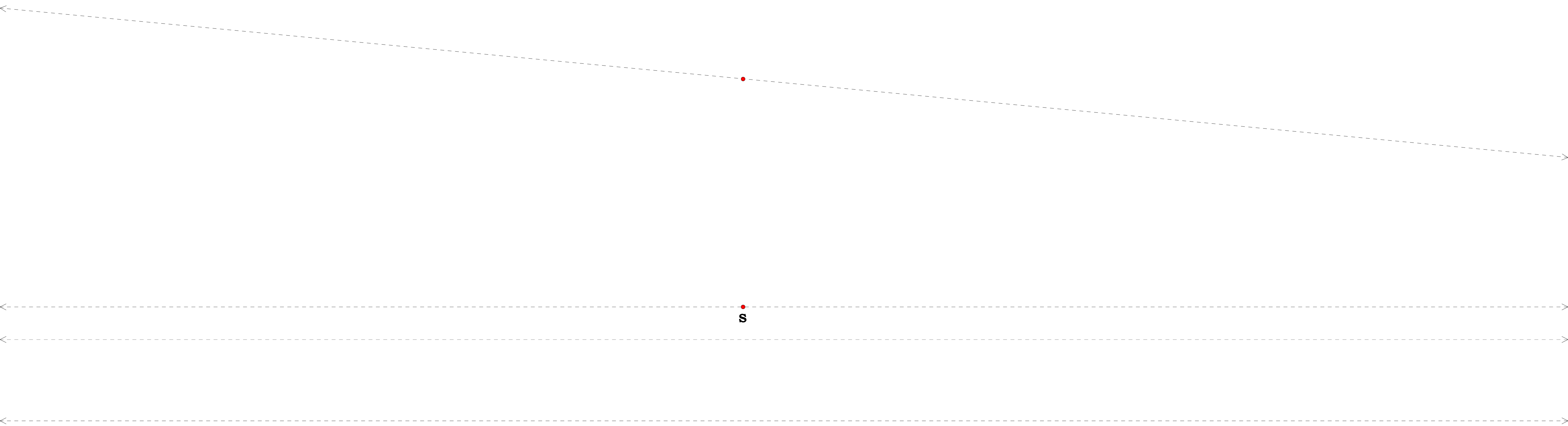
$N_1{}^2 - N_1 = 20.00000$

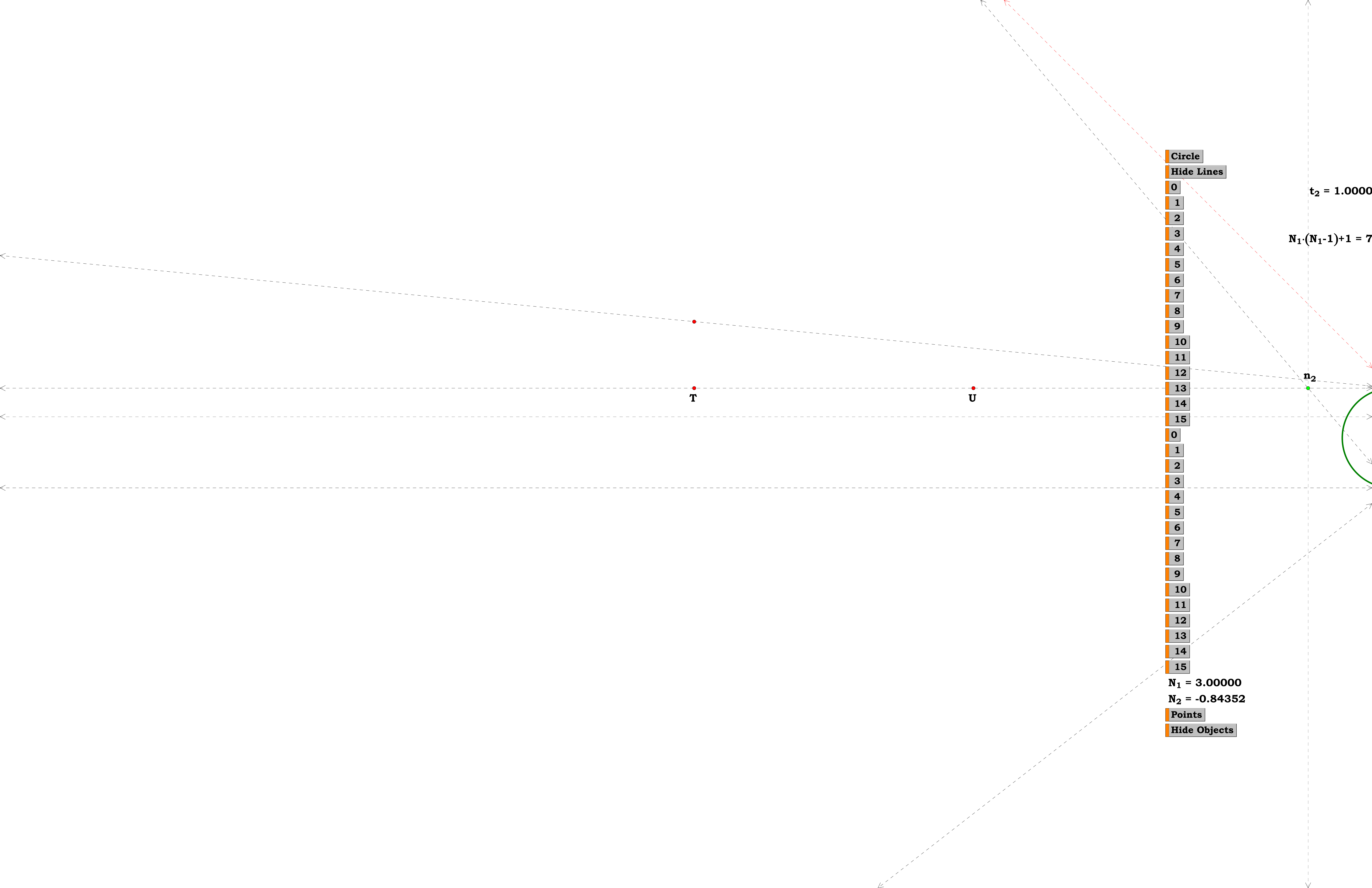
$N_1{}^2 \cdot n_a \cdot n_c = 525.00000$

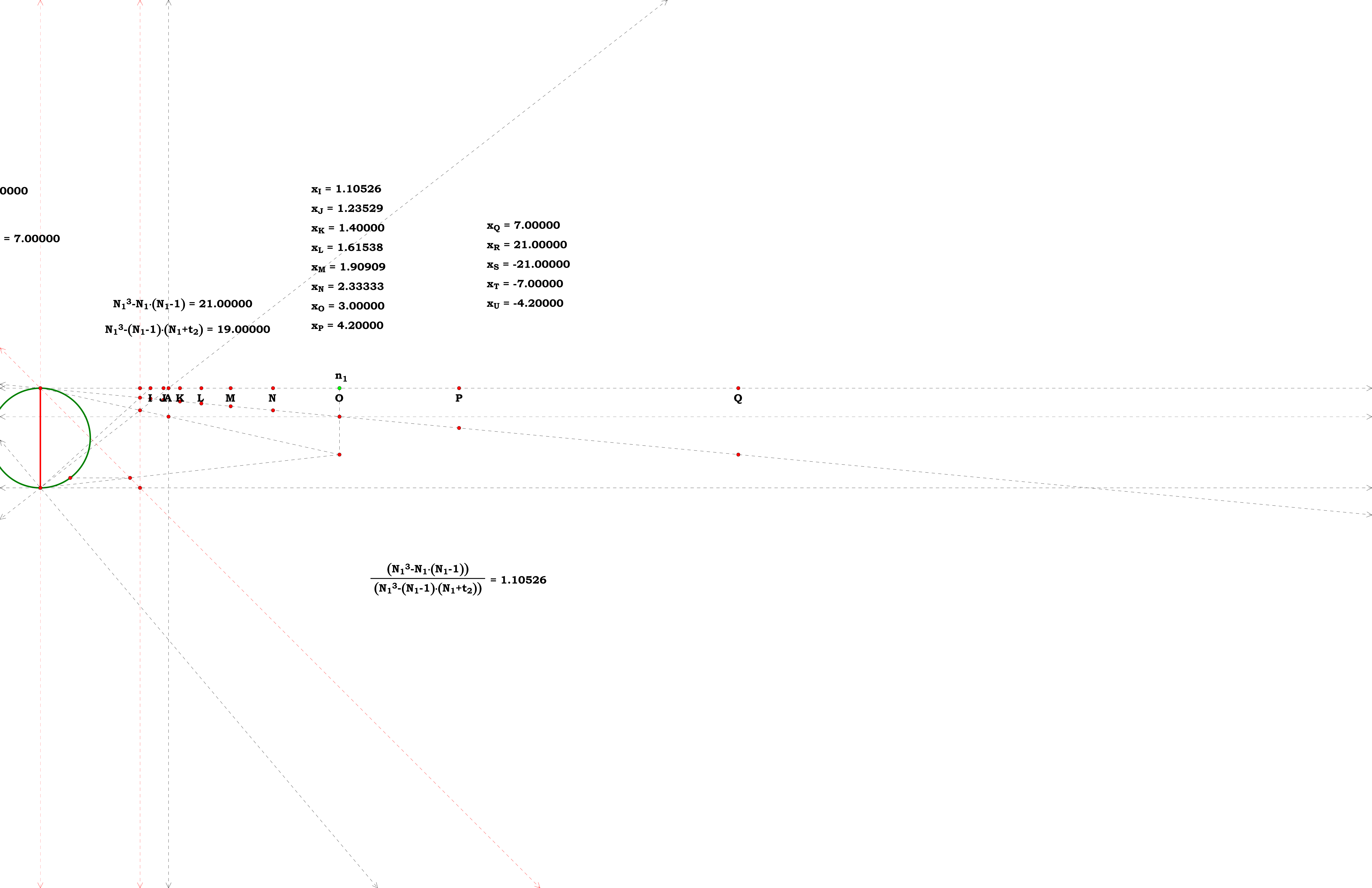
$N_1{}^2 \cdot n_b \cdot n_d = 500.00000$

$\frac{(N_1{}^2 - N_1) \cdot n_a \cdot n_c}{n_b \cdot n_d} = 21.00000$

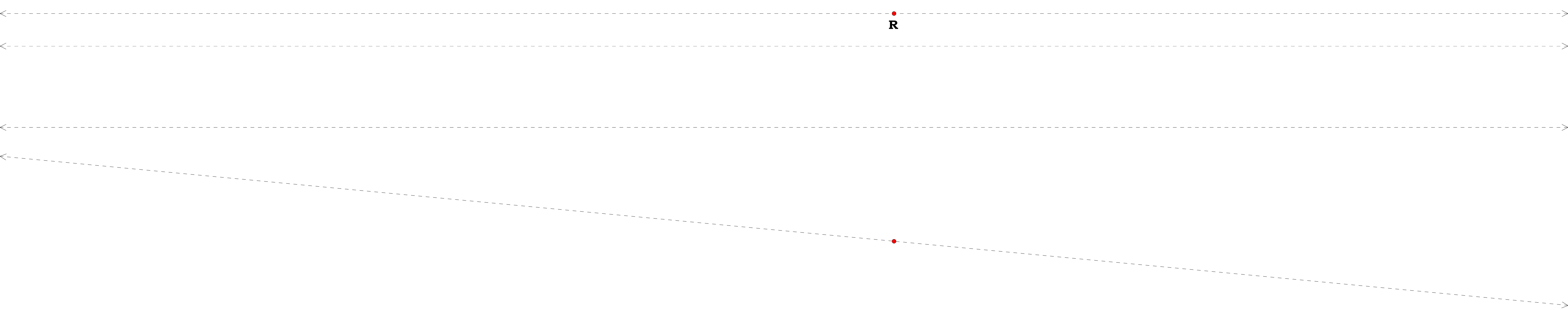

Circle
Hide Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
N_1 = 2.00000
N_2 = -0.91098
Points
Hide Objects
n_2
n_1
A
B
x_A = 1.33333
x_B = 2.00000
$\frac{N_1^2}{(N_1^2-N_1)+1}$ = 1.33333
t_1 = 2
$\frac{N_1^2}{N_1^2-t_1\cdot(N_1-1)}$ = 2.00000


S


Circle
Hide Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
N_1 = 3.00000
N_2 = -0.84352
Points
Hide Objects
t_2 = 1.0000
$N_1 \cdot (N_1-1)+1$ = 7
n_2
T
U



0000
= 7.00000
x_I = 1.10526
x_J = 1.23529
x_K = 1.40000
x_L = 1.61538
x_M = 1.90909
x_N = 2.33333
x_O = 3.00000
x_P = 4.20000
x_Q = 7.00000
x_R = 21.00000
x_S = -21.00000
x_T = -7.00000
x_U = -4.20000
$N_1^3 - N_1 \cdot (N_1 - 1) = 21.00000$
$N_1^3 - (N_1 - 1) \cdot (N_1 + t_2) = 19.00000$
n_1
I
J
A
K
L
M
N
O
P
Q
$\frac{(N_1^3 - N_1 \cdot (N_1 - 1))}{(N_1^3 - (N_1 - 1) \cdot (N_1 + t_2))} = 1.10526$

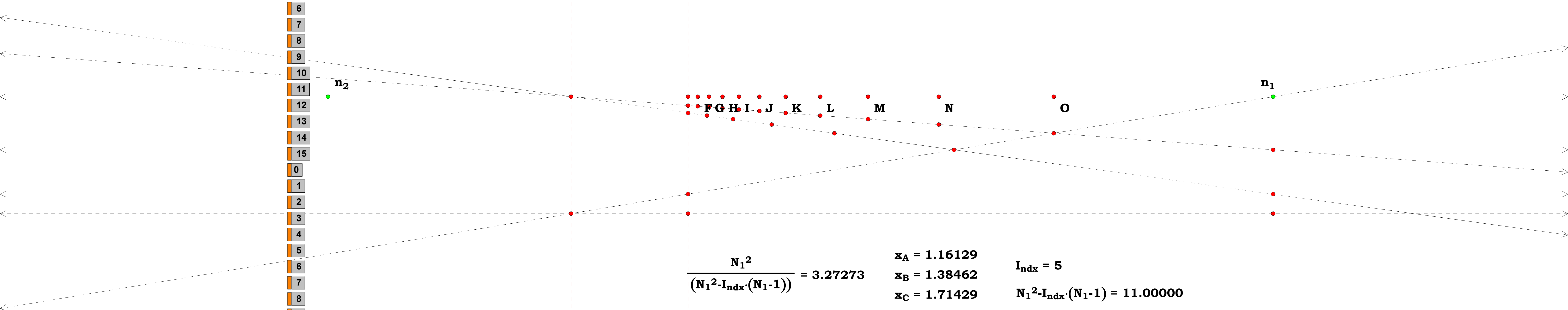

R

Circle

Hide Lines

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Show Objects



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$N_1 = 6.00000$

$N_2 = -2.07732$

Points

Hide Objects

$$\frac{N_1^2}{(N_1^2 - I_{ndx} \cdot (N_1 - 1))} = 3.27273$$

$x_A = 1.16129$

$x_B = 1.38462$

$x_C = 1.71429$

$x_D = 2.25000$

$x_E = 3.27273$

$I_{ndx} = 5$

$N_1^2 - I_{ndx} \cdot (N_1 - 1) = 11.00000$

$x_F = 1.08197$

$x_G = 1.17857$

$x_H = 1.29412$

$x_I = 1.43478$

$x_J = 1.60976$

$x_K = 1.83333$

$x_L = 2.12903$

$x_M = 2.53846$

$x_N = 3.14286$

$x_O = 4.12500$

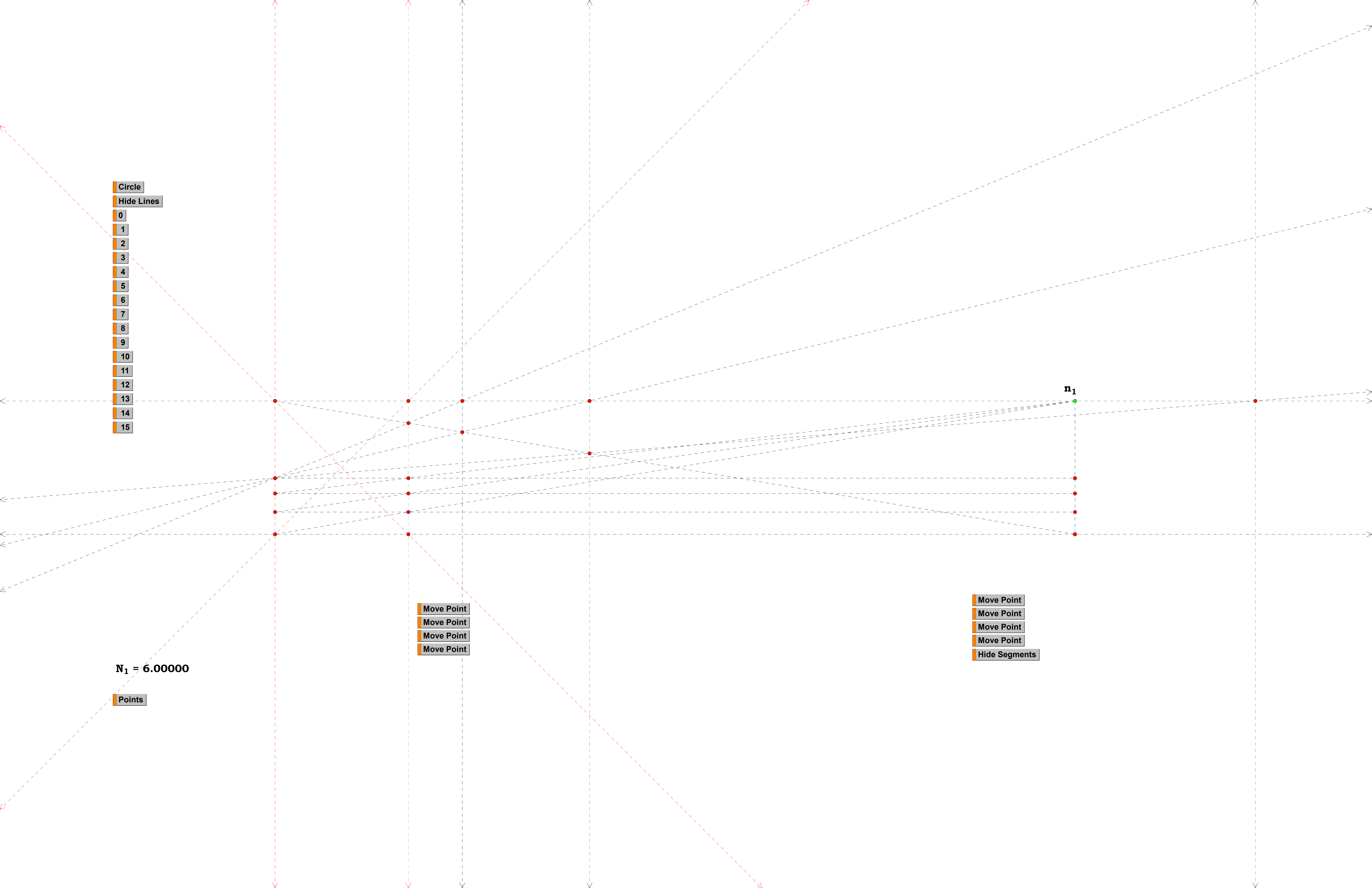
$N_1^2 + N_1 \cdot (N_1 - 1) = 66.00000$

$2 \cdot N_1 - 1 = 11.00000$

$$\frac{(N_1^2 + N_1 \cdot (N_1 - 1))}{(N_1^2 + (N_1 - 1) \cdot (N_1 - I_{ndx}))} = 1.60976$$

$N_1^2 + (N_1 - 1) \cdot (N_1 - I_{ndx}) = 41.00000$

$(N_1^2 + N_1 \cdot (N_1 - 1)) - I_{ndx} \cdot (N_1 - 1) = 41.00000$


Circle
Hide Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
n1
Move Point
Move Point
Move Point
Move Point
Move Point
Move Point
Move Point
Move Point
Hide Segments
N1 = 6.00000
Points

Circle
Show Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

$N_1 = 5.00000$

Points

Circle
Show Lines
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Points

$I_{ndx} = 3$

$n_3 = 3.00000$

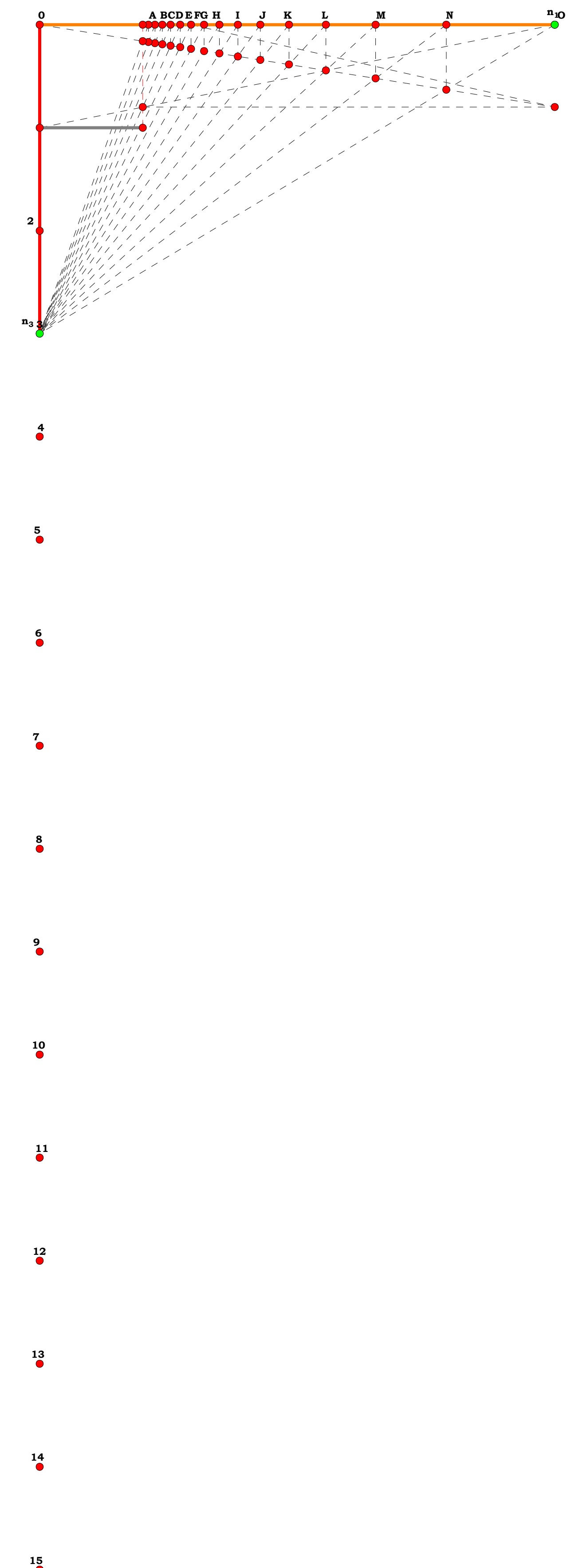
$N_1^2 \cdot n_3 = 75.00000$

$N_1^2 \cdot n_3 - I_{ndx} \cdot (N_1 - 1) = 63.00000$

$$\frac{(N_1^2 \cdot n_3)}{(N_1^2 \cdot n_3 - I_{ndx} \cdot (N_1 - 1))} = 1.19048$$

$N_1 \cdot n_3 = 15.00000$

$x_A = 1.05634$
$x_B = 1.11940$
$x_C = 1.19048$
$x_D = 1.27119$
$x_E = 1.36364$
$x_F = 1.47059$
$x_G = 1.59574$
$x_H = 1.74419$
$x_I = 1.92308$
$x_J = 2.14286$
$x_K = 2.41935$
$x_L = 2.77778$
$x_M = 3.26087$
$x_N = 3.94737$
$x_O = 5.00000$

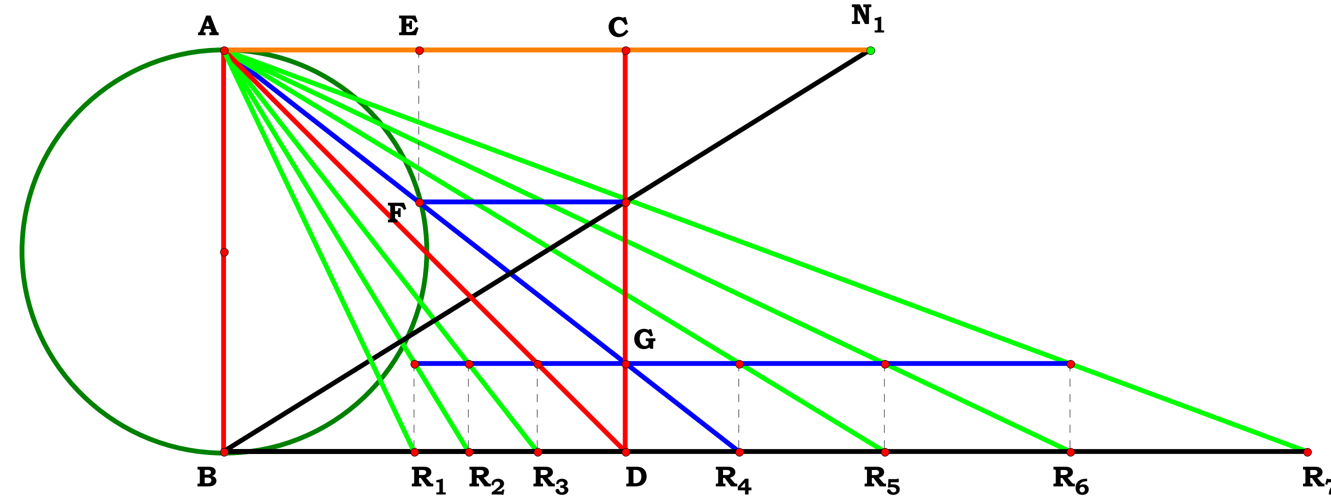


Circles-Plate 2

$AB := 1$

$N_1 := 1.60907$

$N_1 = 1.60907$
$R_1 = 0.47533$
$R_2 = 0.60907$
$R_3 = 0.78043$
$R_4 = 1.28135$
$R_5 = 1.64185$
$R_6 = 2.10379$
$R_7 = 2.69569$



$$EF := 1 - \frac{1}{N_1} \qquad AE := \sqrt{EF \cdot (1 - EF)}$$

$$R_4 := \frac{AE}{EF} \qquad CG := \frac{1}{R_4} \qquad R_3 := CG \qquad R_2 := R_3 \cdot CG$$

$$R_1 := R_2 \cdot CG \qquad R_5 := \frac{R_4}{CG} \qquad R_6 := \frac{R_5}{CG} \qquad R_7 := \frac{R_6}{CG}$$

$$R_1 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-3} = 0 \qquad R_2 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-2} = 0 \qquad R_3 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-1} = 0 \qquad 1 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{0} = 0$$

$$R_4 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{1} = 0 \qquad R_5 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{2} = 0 \qquad R_6 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{3} = 0 \qquad R_7 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{4} = 0$$

$R_1 = 0.475336$

$R_2 = 0.60907$

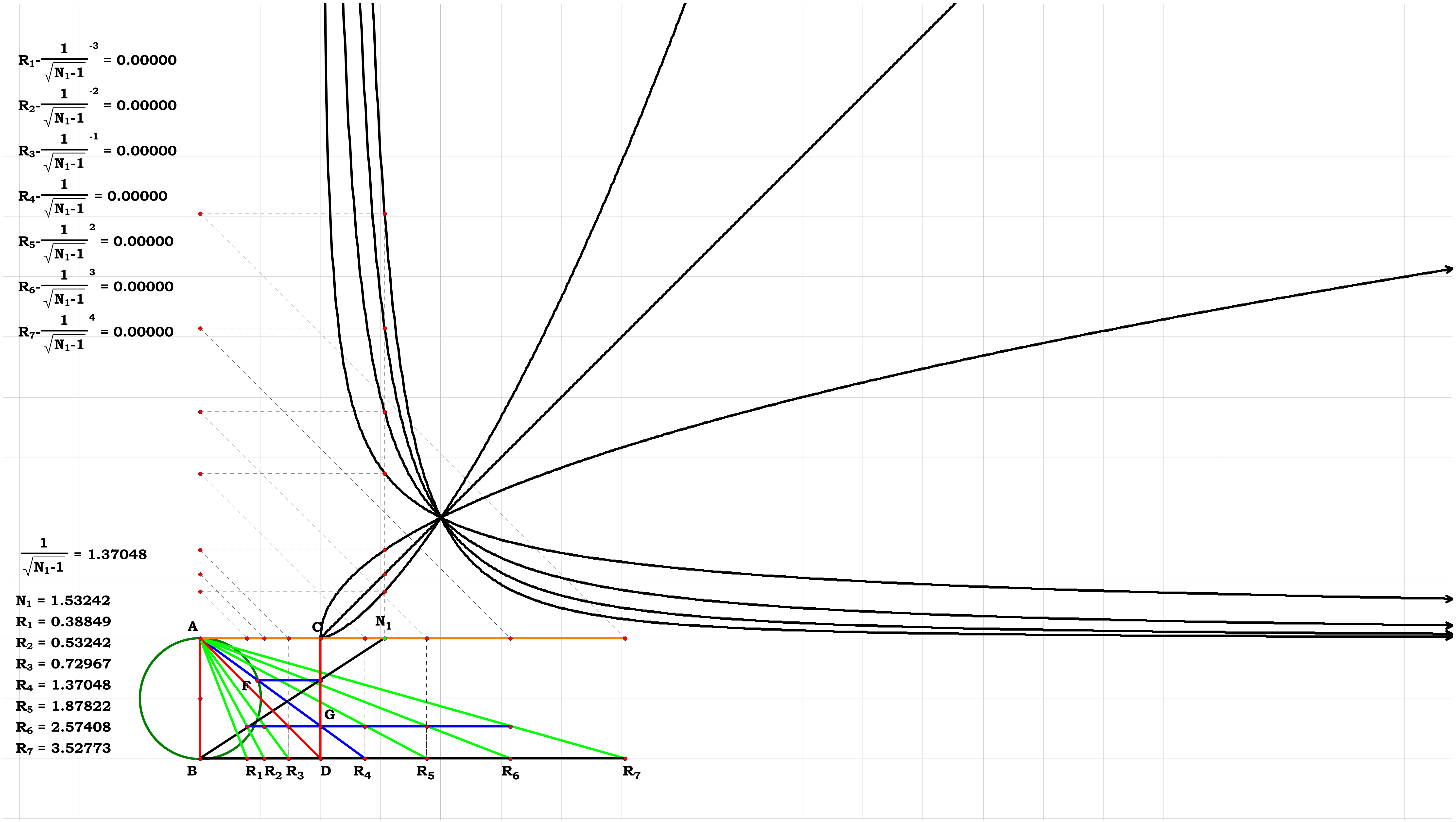
$R_3 = 0.780429$

$R_4 = 1.281346$

$R_5 = 1.641847$

$R_6 = 2.103775$

$R_7 = 2.695663$



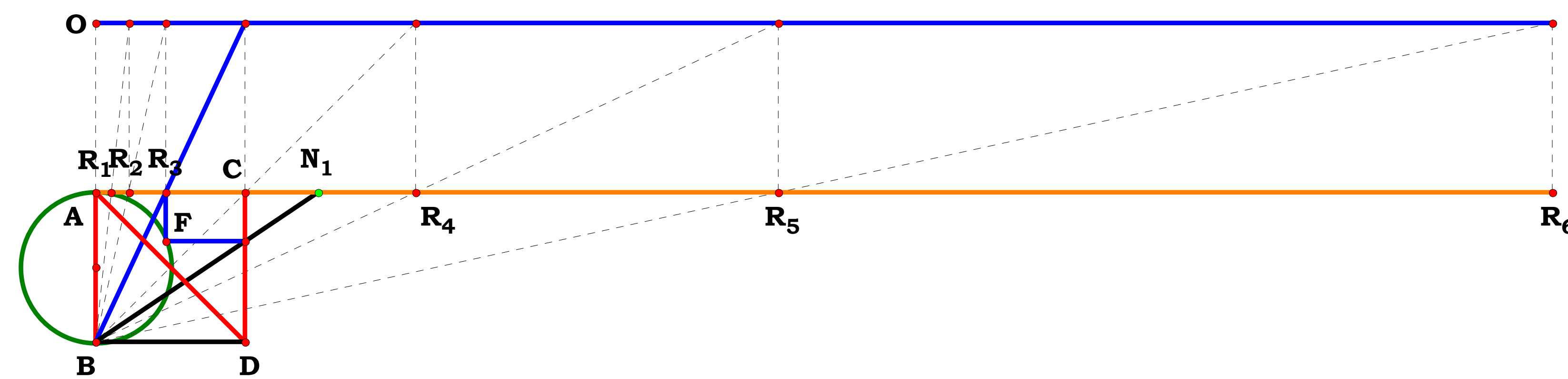
$R_1 - \frac{1}{\sqrt{N_1 - 1}}^{-3} = 0.00000$
$R_2 - \frac{1}{\sqrt{N_1 - 1}}^{-2} = 0.00000$
$R_3 - \frac{1}{\sqrt{N_1 - 1}}^{-1} = 0.00000$
$R_4 - \frac{1}{\sqrt{N_1 - 1}} = 0.00000$
$R_5 - \frac{1}{\sqrt{N_1 - 1}}^{2} = 0.00000$
$R_6 - \frac{1}{\sqrt{N_1 - 1}}^{3} = 0.00000$
$R_7 - \frac{1}{\sqrt{N_1 - 1}}^{4} = 0.00000$
$\frac{1}{\sqrt{N_1 - 1}} = 1.37048$
$N_1 = 1.53242$
$R_1 = 0.38849$
$R_2 = 0.53242$
$R_3 = 0.72967$
$R_4 = 1.37048$
$R_5 = 1.87822$
$R_6 = 2.57408$
$R_7 = 3.52773$
A
C
N_1
F
G
B
R_1
R_2
R_3
D
R_4
R_5
R_6
R_7

Circles-Plate 3

$$AB := 1$$

$$N_1 := 1.48165$$

$N_1 = 1.48165$
$R_1 = 0.10277$
$R_2 = 0.21940$
$R_3 = 0.46840$
$R_4 = 2.13491$
$R_5 = 4.55784$
$R_6 = 9.73059$



$$RF := \frac{N_1 - 1}{N_1} \qquad R_3 := \sqrt{RF \cdot (1 - RF)}$$

$$R_3 - \frac{\sqrt{N_1 - 1}}{N_1} = 0$$

$$R_2 := R_3{}^{2} \quad R_1 := R_3{}^{3} \quad R_4 := R_3{}^{-1} \quad R_5 := R_3{}^{-2} \quad R_6 := R_3{}^{-3}$$

$$R_1 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{-3} = 0 \qquad R_2 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{-2} = 0 \qquad R_3 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{-1} = 0$$

$$AB - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{0} = 0 \quad R_4 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{1} = 0 \quad R_5 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{2} = 0 \quad R_6 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{3} = 0$$

$R_1 = 0.102769$

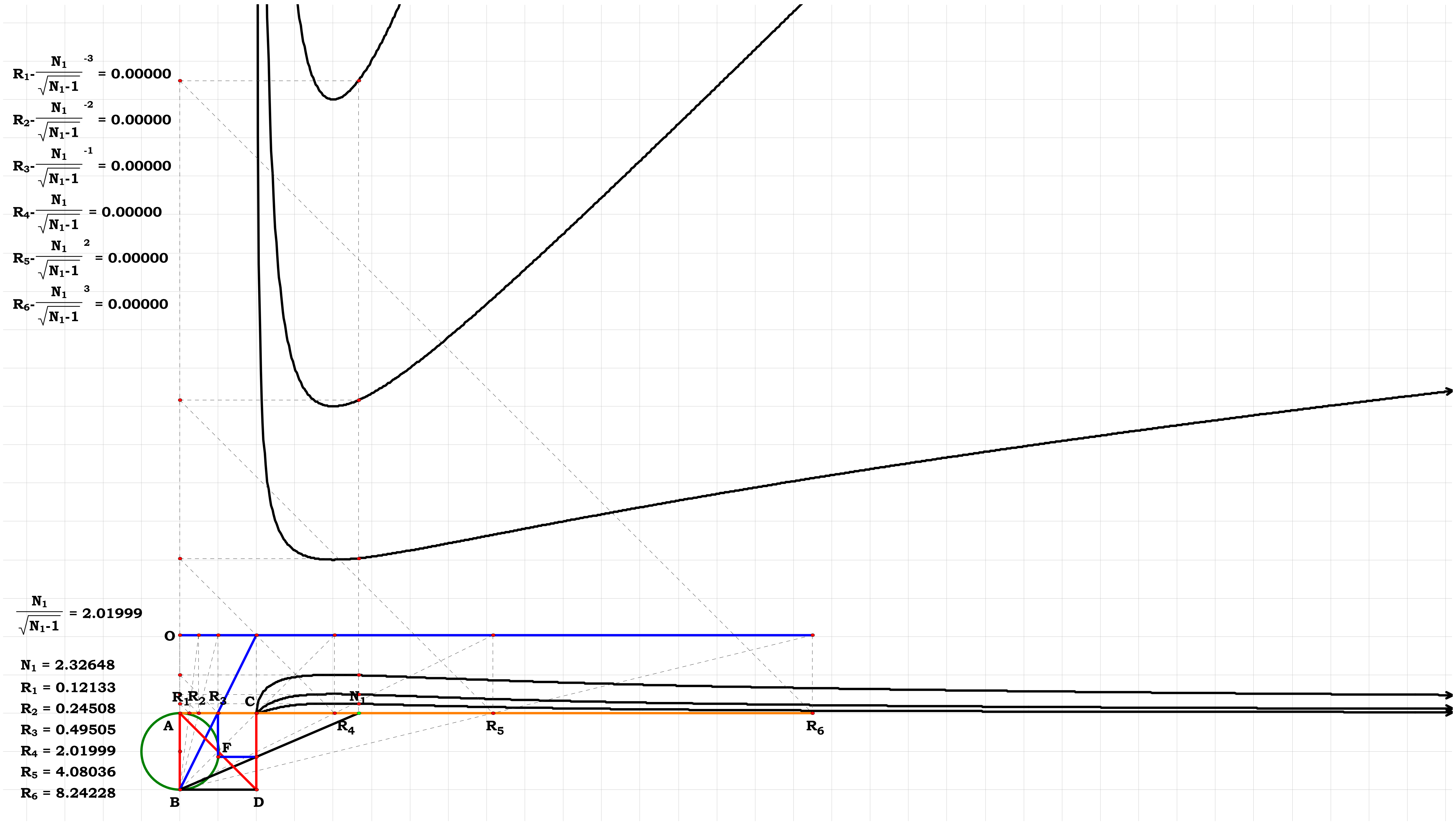
$R_2 = 0.219402$

$R_3 = 0.468404$

$R_4 = 2.134911$

$R_5 = 4.557846$

$R_6 = 9.730598$



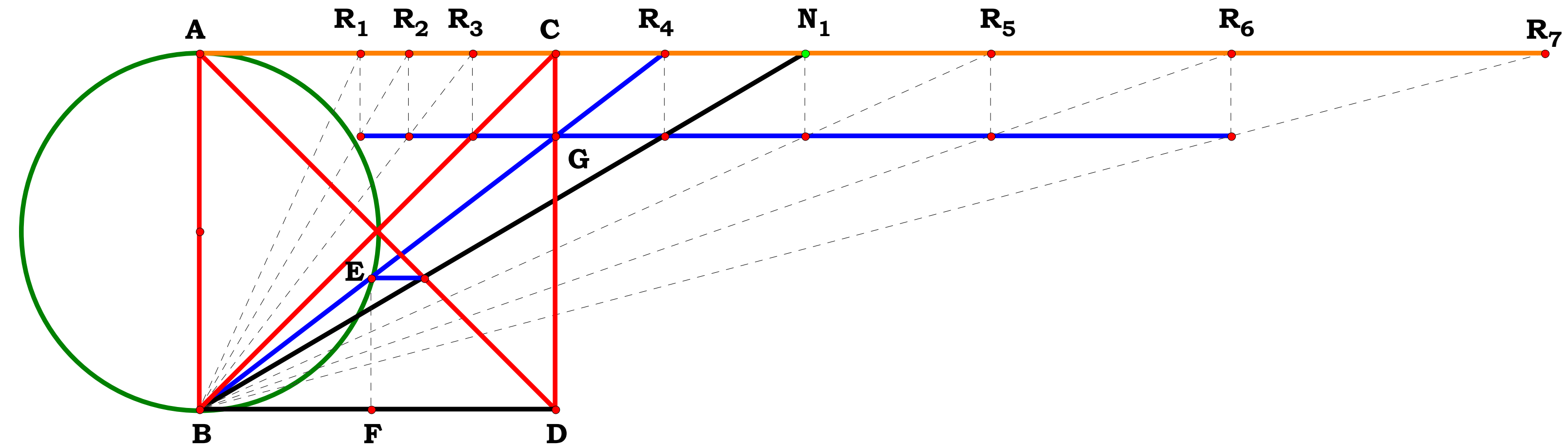
$R_1-\frac{N_1}{\sqrt{N_1-1}}^{-3} = 0.00000$
$R_2-\frac{N_1}{\sqrt{N_1-1}}^{-2} = 0.00000$
$R_3-\frac{N_1}{\sqrt{N_1-1}}^{-1} = 0.00000$
$R_4-\frac{N_1}{\sqrt{N_1-1}} = 0.00000$
$R_5-\frac{N_1}{\sqrt{N_1-1}}^{2} = 0.00000$
$R_6-\frac{N_1}{\sqrt{N_1-1}}^{3} = 0.00000$
$\frac{N_1}{\sqrt{N_1-1}} = 2.01999$
$N_1 = 2.32648$
$R_1 = 0.12133$
$R_2 = 0.24508$
$R_3 = 0.49505$
$R_4 = 2.01999$
$R_5 = 4.08036$
$R_6 = 8.24228$
O
R_1 R_2 R_3
C
N_1
A
F
R_4
R_5
R_6
B
D

$\mathbf{AB := 1}$

$\mathbf{N_1 := 1.70189}$

Circles-Plate 4

$N_1 = 1.70189$
$R_1 = 0.45041$
$R_2 = 0.58758$
$R_3 = 0.76654$
$R_4 = 1.30456$
$R_5 = 2.22022$
$R_6 = 2.89641$
$R_7 = 3.77855$



$$EF := \frac{1}{N_1 + 1} \qquad BF := \sqrt{EF \cdot (1 - EF)}$$

$$DG := \frac{EF}{BF} \qquad R_4 := \frac{1}{DG} \qquad R_4 - \frac{\sqrt{N_1} \cdot (N_1 + 1)}{\sqrt{(N_1 + 1)^2}} = 0 \qquad R_4 - \sqrt{N_1} = 0$$

$$R_3 := \frac{1}{R_4} \qquad R_2 := \frac{1}{R_4^{\,2}} \qquad R_1 := \frac{1}{R_4^{\,3}} \qquad N_1 - R_4^{\,2} = 0 \qquad R_5 := R_4^{\,3} \qquad R_6 := R_4^{\,4} \qquad R_7 := R_4^{\,5}$$

$$R_1 - \left(\sqrt{N_1}\right)^{-3} = 0 \qquad R_2 - \left(\sqrt{N_1}\right)^{-2} = 0 \qquad R_3 - \left(\sqrt{N_1}\right)^{-1} = 0 \qquad AB - \left(\sqrt{N_1}\right)^{0} = 0 \qquad R_4 - \left(\sqrt{N_1}\right)^{1} = 0$$

$$N_1 - \left(\sqrt{N_1}\right)^{2} = 0 \qquad R_5 - \left(\sqrt{N_1}\right)^{3} = 0 \qquad R_6 - \left(\sqrt{N_1}\right)^{4} = 0 \qquad R_7 - \left(\sqrt{N_1}\right)^{5} = 0$$

$R_1 = 0.450405$

$R_2 = 0.587582$

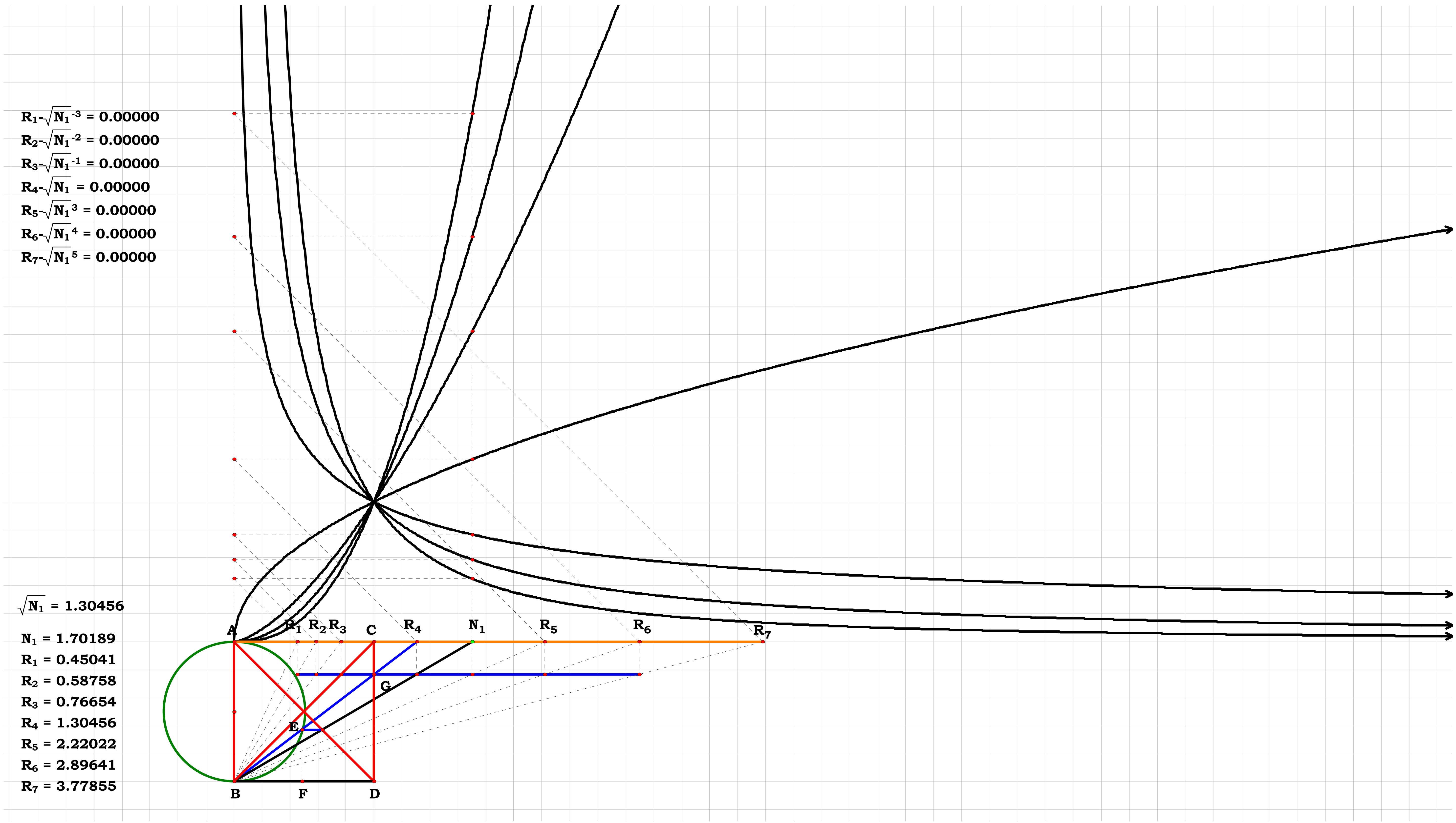
$R_3 = 0.766539$

$R_4 = 1.304565$

$R_5 = 2.220226$

$R_6 = 2.89643$

$R_7 = 3.778581$



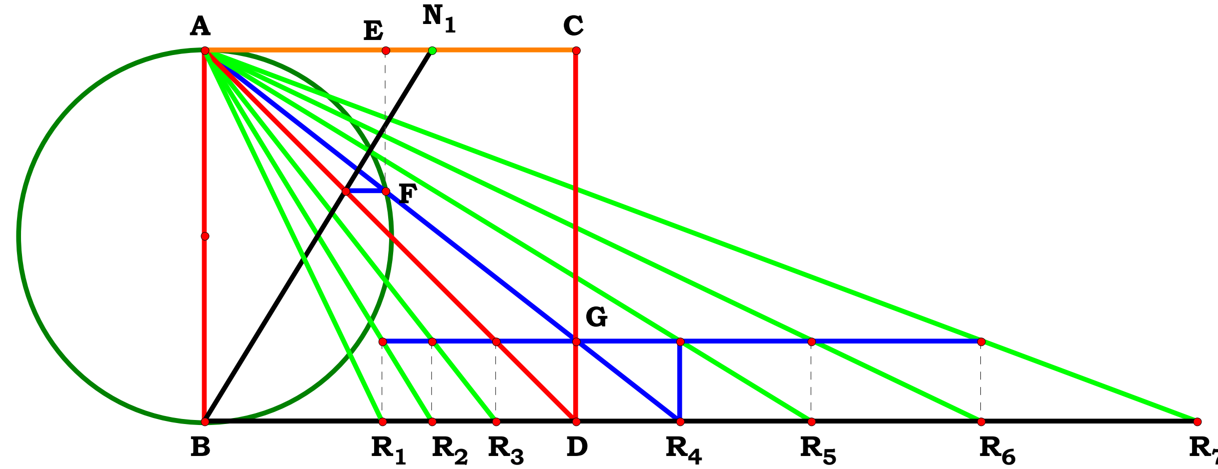
$R_1 - \sqrt{N_1}^{-3} = 0.00000$
$R_2 - \sqrt{N_1}^{-2} = 0.00000$
$R_3 - \sqrt{N_1}^{-1} = 0.00000$
$R_4 - \sqrt{N_1} = 0.00000$
$R_5 - \sqrt{N_1}^{3} = 0.00000$
$R_6 - \sqrt{N_1}^{4} = 0.00000$
$R_7 - \sqrt{N_1}^{5} = 0.00000$
$\sqrt{N_1} = 1.30456$
$N_1 = 1.70189$
$R_1 = 0.45041$
$R_2 = 0.58758$
$R_3 = 0.76654$
$R_4 = 1.30456$
$R_5 = 2.22022$
$R_6 = 2.89641$
$R_7 = 3.77855$
A
R_1
R_2
R_3
C
R_4
N_1
R_5
R_6
R_7
G
E
B
F
D

Circles-Plate 5

$$AB := 1$$

$$N_1 := .61225$$

$N_1 = 0.61225$
$R_1 = 0.47906$
$R_2 = 0.61225$
$R_3 = 0.78246$
$R_4 = 1.27802$
$R_5 = 1.63333$
$R_6 = 2.08742$
$R_7 = 2.66776$



$$EF := \frac{N_1}{N_1 + 1} \qquad AE := \sqrt{EF \cdot (1 - EF)}$$

$$R_4 := \frac{AE}{EF} \qquad CG := \frac{1}{R_4} \qquad R_3 := CG$$

$$R_2 := R_3 \cdot CG \qquad R_1 := R_2 \cdot CG \qquad R_5 := \frac{R_4}{CG} \qquad R_6 := \frac{R_5}{CG} \qquad R_7 := \frac{R_6}{CG}$$

$$R_4 - \frac{N_1 + 1}{\sqrt{N_1} \cdot \sqrt{\left(N_1 + 1\right)^2}} = 0 \qquad R_4 - \frac{1}{\sqrt{N_1}} = 0$$

$$R_1 - \left(\frac{1}{\sqrt{N_1}}\right)^{-3} = 0 \qquad R_2 - \left(\frac{1}{\sqrt{N_1}}\right)^{-2} = 0 \qquad R_3 - \left(\frac{1}{\sqrt{N_1}}\right)^{-1} = 0 \qquad AB - \left(\frac{1}{\sqrt{N_1}}\right)^{0} = 0$$

$$R_4 - \left(\frac{1}{\sqrt{N_1}}\right)^{1} = 0 \qquad R_5 - \left(\frac{1}{\sqrt{N_1}}\right)^{2} = 0 \qquad R_6 - \left(\frac{1}{\sqrt{N_1}}\right)^{3} = 0 \qquad R_7 - \left(\frac{1}{\sqrt{N_1}}\right)^{4} = 0$$

$R_1 = 0.479064$

$R_2 = 0.61225$

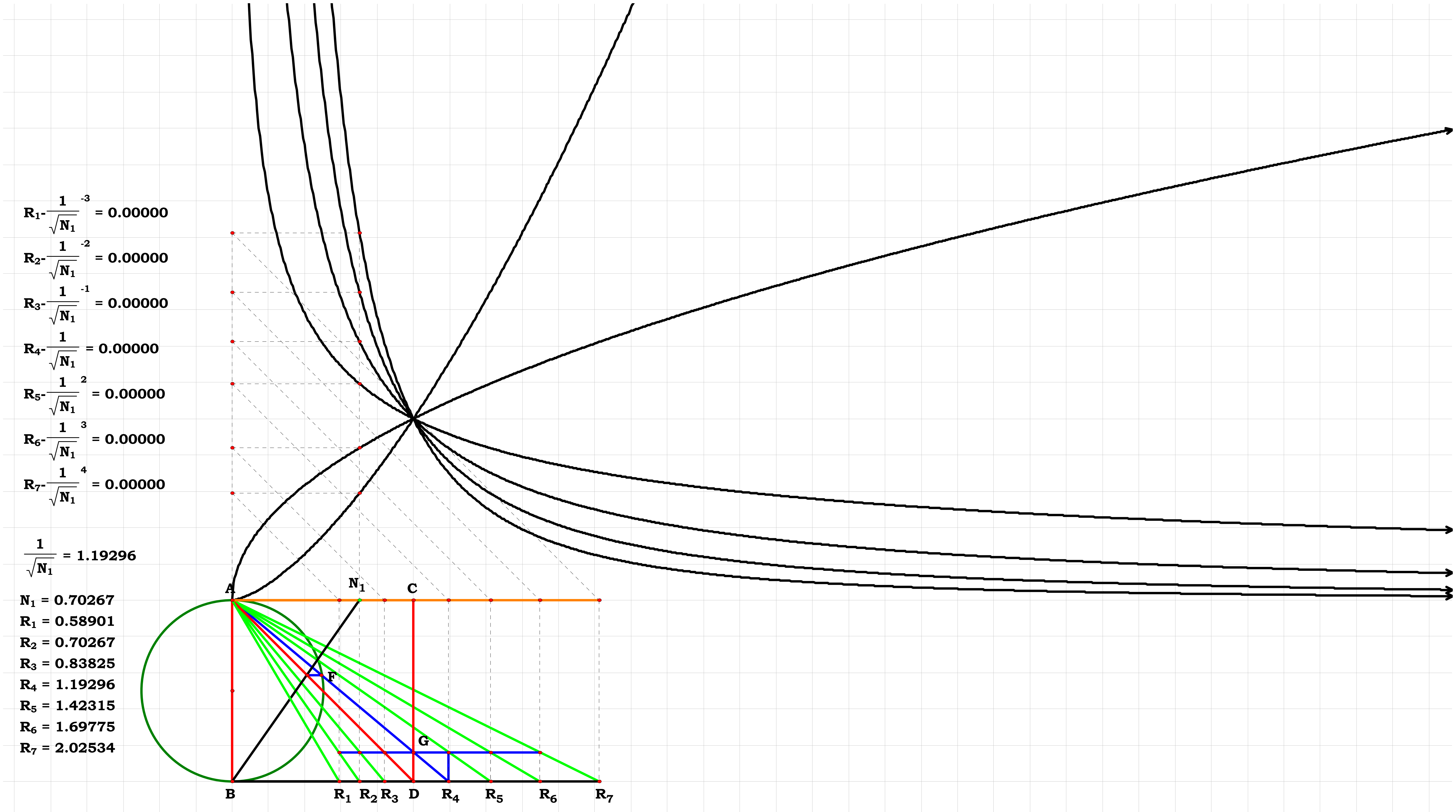
$R_3 = 0.782464$

$R_4 = 1.278014$

$R_5 = 1.63332$

$R_6 = 2.087405$

$R_7 = 2.667733$



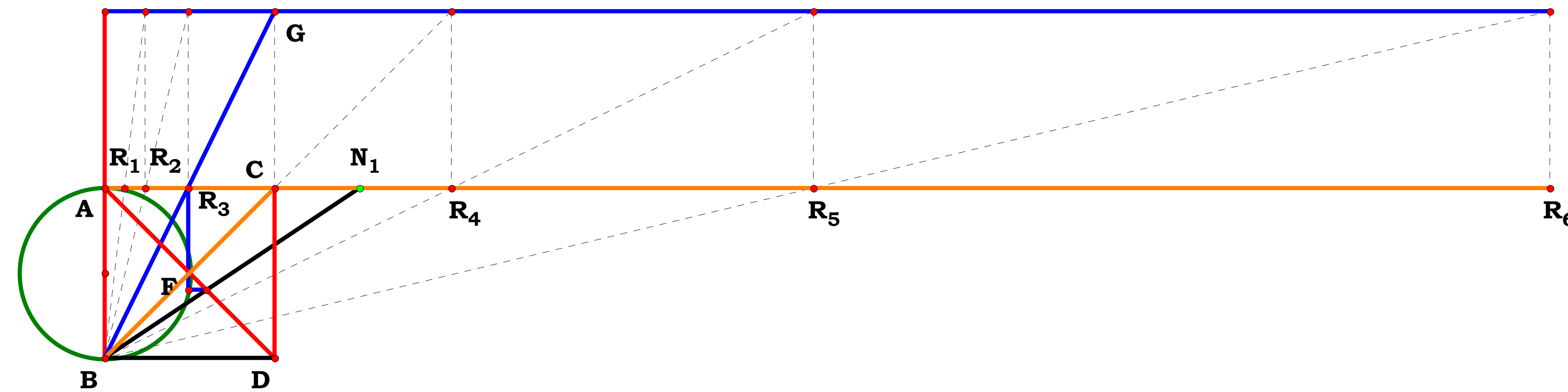
$R_1 - \frac{1}{\sqrt{N_1}}^{-3} = 0.00000$
$R_2 - \frac{1}{\sqrt{N_1}}^{-2} = 0.00000$
$R_3 - \frac{1}{\sqrt{N_1}}^{-1} = 0.00000$
$R_4 - \frac{1}{\sqrt{N_1}} = 0.00000$
$R_5 - \frac{1}{\sqrt{N_1}}^{2} = 0.00000$
$R_6 - \frac{1}{\sqrt{N_1}}^{3} = 0.00000$
$R_7 - \frac{1}{\sqrt{N_1}}^{4} = 0.00000$
$\frac{1}{\sqrt{N_1}} = 1.19296$
$N_1 = 0.70267$
$R_1 = 0.58901$
$R_2 = 0.70267$
$R_3 = 0.83825$
$R_4 = 1.19296$
$R_5 = 1.42315$
$R_6 = 1.69775$
$R_7 = 2.02534$
A
N_1
C
F
G
B
R_1
R_2
R_3
D
R_4
R_5
R_6
R_7

Circles-Plate 6

$$AB := 1$$

$$N_1 := 1.49352$$

$N_1 = 1.49352$
$R_1 = 0.11773$
$R_2 = 0.24021$
$R_3 = 0.49011$
$R_4 = 2.04036$
$R_5 = 4.16308$
$R_6 = 8.49419$



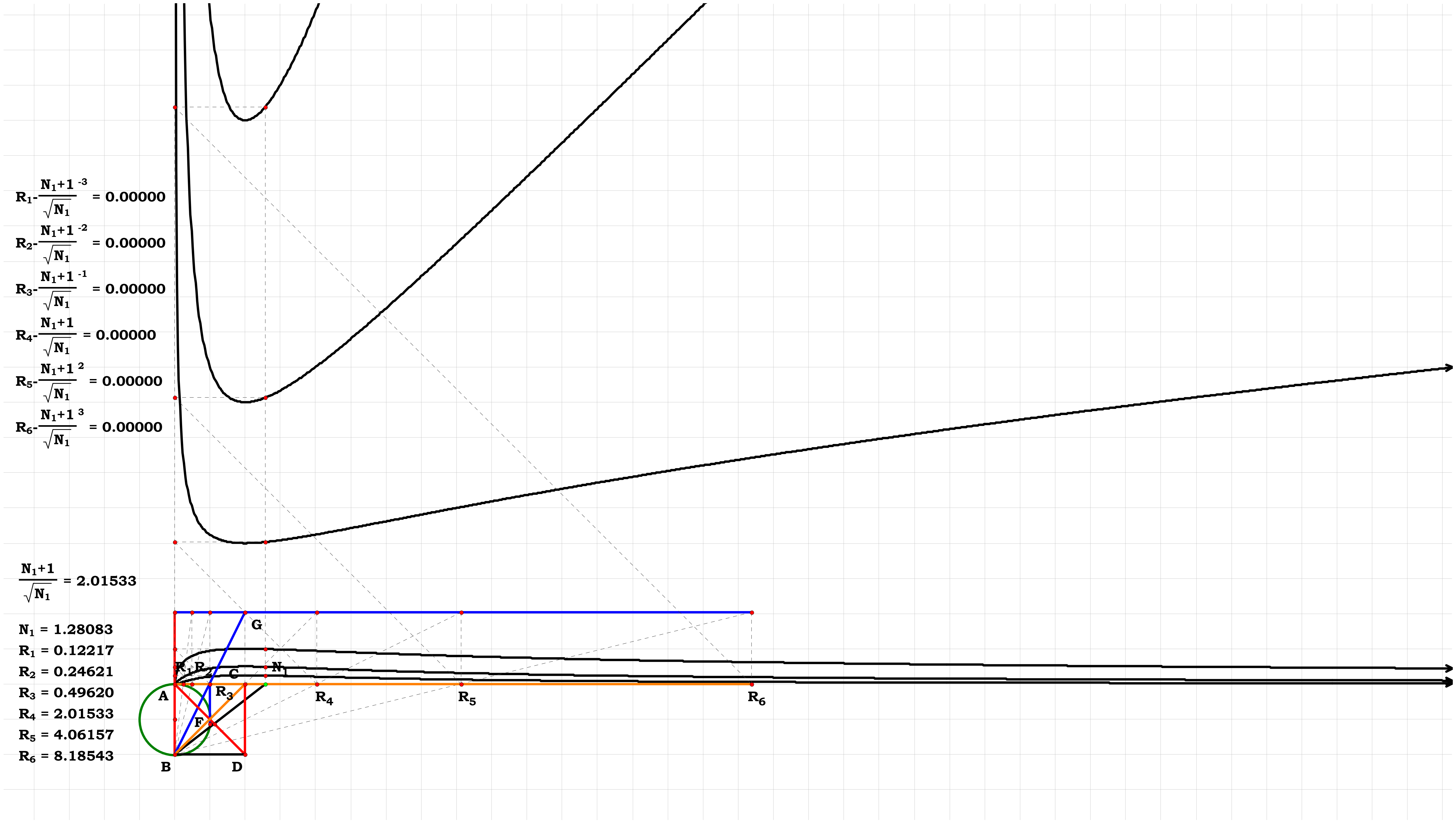
$$FR := \frac{N_1}{N_1 + 1} \qquad R_3 := \sqrt{FR \cdot (1 - FR)}$$

$$R_2 := {R_3}^2 \qquad R_1 := {R_3}^3 \qquad AB - \frac{R_3}{R_3} = 0 \qquad R_4 := \frac{R_3}{{R_3}^2} \qquad R_5 := \frac{R_3}{{R_3}^3} \qquad R_6 := \frac{R_3}{{R_3}^4}$$

$$R_3 - \sqrt{\frac{N_1}{\left(N_1 + 1\right)^2}} = 0$$

$$R_1 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{-3} = 0 \qquad R_2 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{-2} = 0 \qquad R_3 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{-1} = 0$$

$$R_4 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right) = 0 \qquad R_5 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{2} = 0 \qquad R_6 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{3} = 0$$



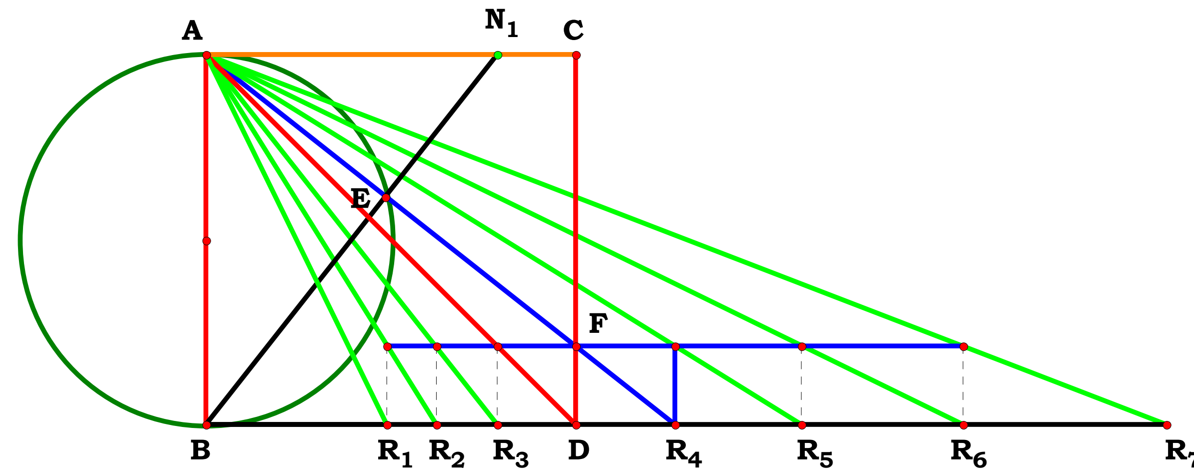
$R_1 - \frac{N_1+1^{-3}}{\sqrt{N_1}} = 0.00000$
$R_2 - \frac{N_1+1^{-2}}{\sqrt{N_1}} = 0.00000$
$R_3 - \frac{N_1+1^{-1}}{\sqrt{N_1}} = 0.00000$
$R_4 - \frac{N_1+1}{\sqrt{N_1}} = 0.00000$
$R_5 - \frac{N_1+1^{2}}{\sqrt{N_1}} = 0.00000$
$R_6 - \frac{N_1+1^{3}}{\sqrt{N_1}} = 0.00000$
$\frac{N_1+1}{\sqrt{N_1}} = 2.01533$
$N_1 = 1.28083$
$R_1 = 0.12217$
$R_2 = 0.24621$
$R_3 = 0.49620$
$R_4 = 2.01533$
$R_5 = 4.06157$
$R_6 = 8.18543$
G
R_1
R_2
C
N_1
A
R_3
F
R_4
R_5
R_6
B
D

Circles-Plate 7

$AB := 1$

$N_1 := .78798$

$N_1 = 0.78798$
$R_1 = 0.48926$
$R_2 = 0.62091$
$R_3 = 0.78798$
$R_4 = 1.26907$
$R_5 = 1.61054$
$R_6 = 2.04389$
$R_7 = 2.59384$



$$R_4 := \frac{1}{N_1}$$

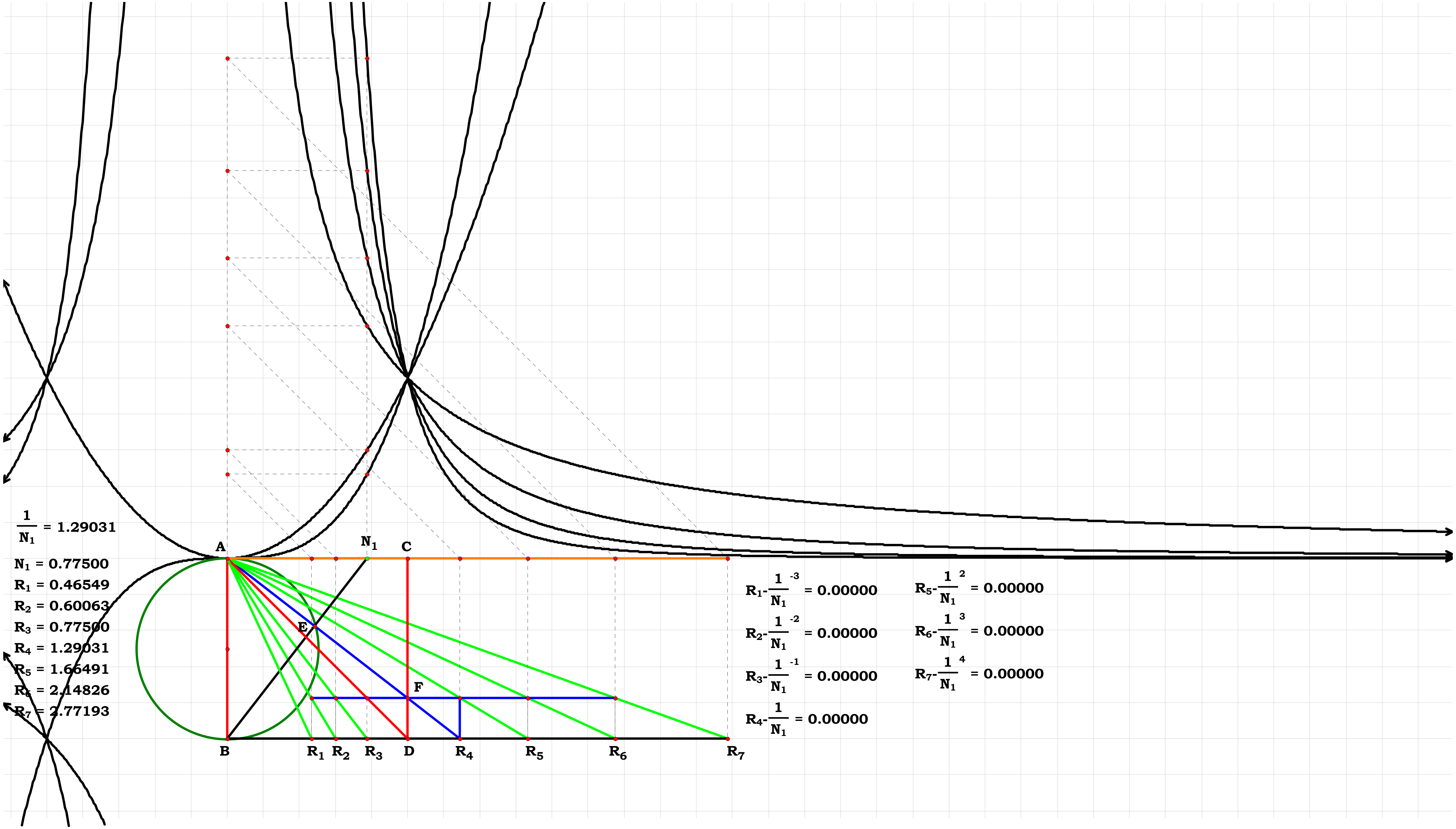
$$BN_1 := \sqrt{1^2 + N_1^2} \qquad EN_1 := \frac{N_1^2}{BN_1} \qquad BE := BN_1 - EN_1$$

$$R_4 := \frac{BN_1 \cdot BE}{N_1} \qquad R_4 - \frac{1}{N_1} = 0$$

$$R_1 := R_4^{-3} \quad R_2 := R_4^{-2} \quad R_3 := R_4^{-1} \quad R_5 := R_4^{2} \quad R_6 := R_4^{3} \quad R_7 := R_4^{4}$$

$$R_1 - \left(\frac{1}{N_1}\right)^{-3} = 0 \qquad R_2 - \left(\frac{1}{N_1}\right)^{-2} = 0 \qquad R_3 - \left(\frac{1}{N_1}\right)^{-1} = 0 \qquad AB - \left(\frac{1}{N_1}\right)^{0} = 0$$

$$R_4 - \left(\frac{1}{N_1}\right)^{1} = 0 \qquad R_5 - \left(\frac{1}{N_1}\right)^{2} = 0 \qquad R_6 - \left(\frac{1}{N_1}\right)^{3} = 0 \qquad R_7 - \left(\frac{1}{N_1}\right)^{4} = 0$$



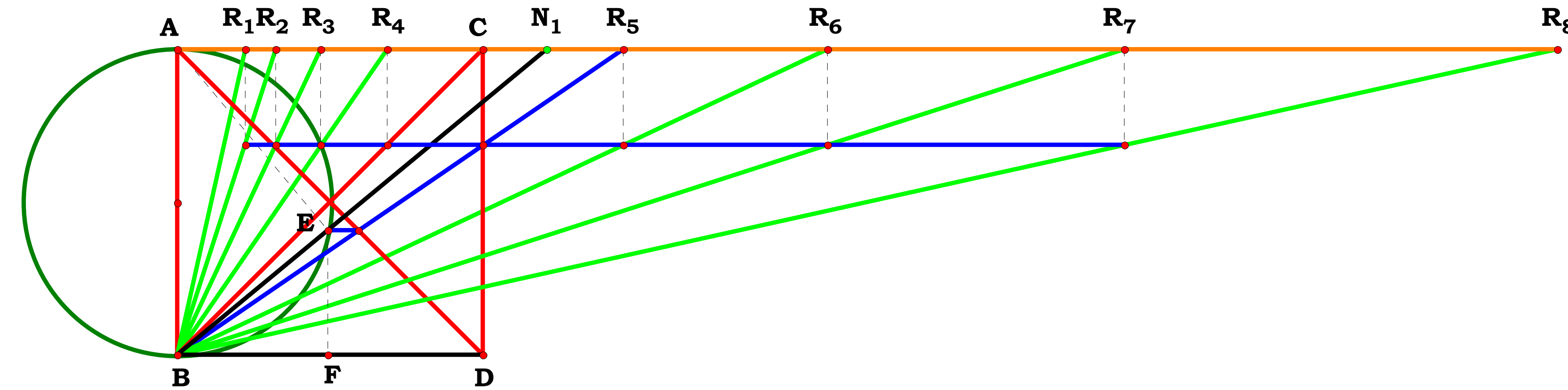
1/N1 = 1.29031
N1 = 0.77500
R1 = 0.46549
R2 = 0.60063
R3 = 0.77500
R4 = 1.29031
R5 = 1.66491
R6 = 2.14826
R7 = 2.77193
A
N1
C
E
F
B
R1
R2
R3
D
R4
R5
R6
R7
R1-(1/N1)^-3 = 0.00000
R2-(1/N1)^-2 = 0.00000
R3-(1/N1)^-1 = 0.00000
R4-1/N1 = 0.00000
R5-(1/N1)^2 = 0.00000
R6-(1/N1)^3 = 0.00000
R7-(1/N1)^4 = 0.00000

Circles-Plate 8

$AB := 1$

$N_1 := 1.20725$

$N_1 = 1.20725$
$R_1 = 0.22162$
$R_2 = 0.32301$
$R_3 = 0.47077$
$R_4 = 0.68613$
$R_5 = 1.45746$
$R_6 = 2.12418$
$R_7 = 3.09591$
$R_8 = 4.51215$



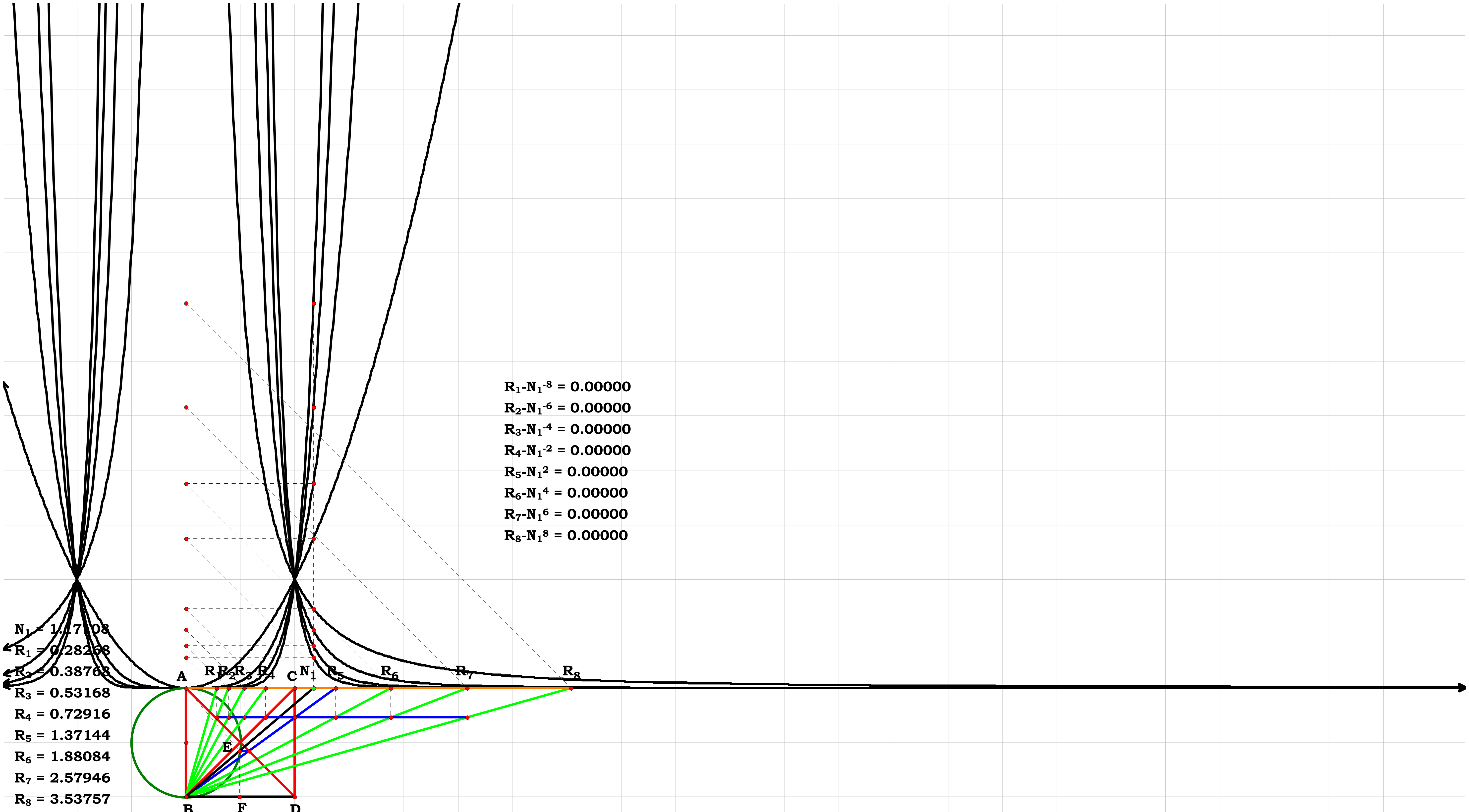
$$EF := \frac{1}{N_1^{\,2} + 1} \qquad R_5 := \frac{1 - EF}{EF} \qquad R_5 - N_1^{\,2} = 0$$

$$R_4 := \frac{1}{R_5} \qquad R_3 := R_4^{\,2} \qquad R_2 := R_4^{\,3} \qquad R_1 := R_4^{\,4}$$

$$R_6 := R_5^{\,2} \qquad R_7 := R_5^{\,3} \qquad R_8 := R_5^{\,4}$$

$$R_1 - N_1^{\,-8} = 0 \qquad R_2 - N_1^{\,-6} = 0 \qquad R_3 - N_1^{\,-4} = 0 \qquad R_4 - N_1^{\,-2} = 0$$

$$R_5 - N_1^{\,2} = 0 \qquad R_6 - N_1^{\,4} = 0 \qquad R_7 - N_1^{\,6} = 0 \qquad R_8 - N_1^{\,8} = 0$$



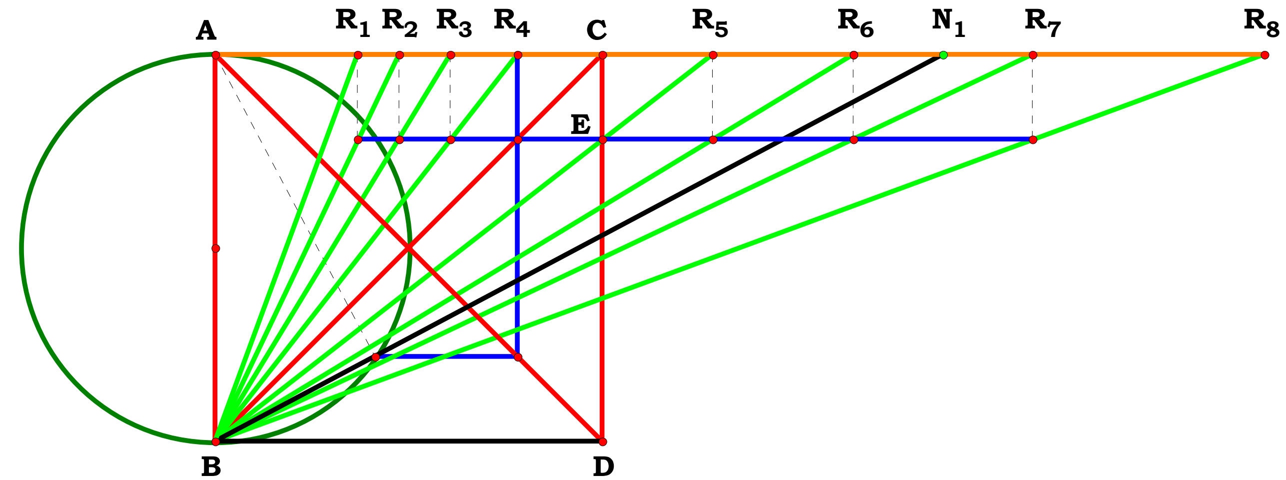
R1-N1-8 = 0.00000
R2-N1-6 = 0.00000
R3-N1-4 = 0.00000
R4-N1-2 = 0.00000
R5-N12 = 0.00000
R6-N14 = 0.00000
R7-N16 = 0.00000
R8-N18 = 0.00000
N1 = 1.17108
R1 = 0.28268
R2 = 0.38768
R3 = 0.53168
R4 = 0.72916
R5 = 1.37144
R6 = 1.88084
R7 = 2.57946
R8 = 3.53757
A
R1
R2
R3
R4
C
N1
R5
R6
R7
R8
E
B
F
D

Circles-Plate 9

$AB := 1$

$N_1 := 1.88007$

$N_1 = 1.88007$
$R_1 = 0.36916$
$R_2 = 0.47360$
$R_3 = 0.60758$
$R_4 = 0.77948$
$R_5 = 1.28291$
$R_6 = 1.64587$
$R_7 = 2.11150$
$R_8 = 2.70888$



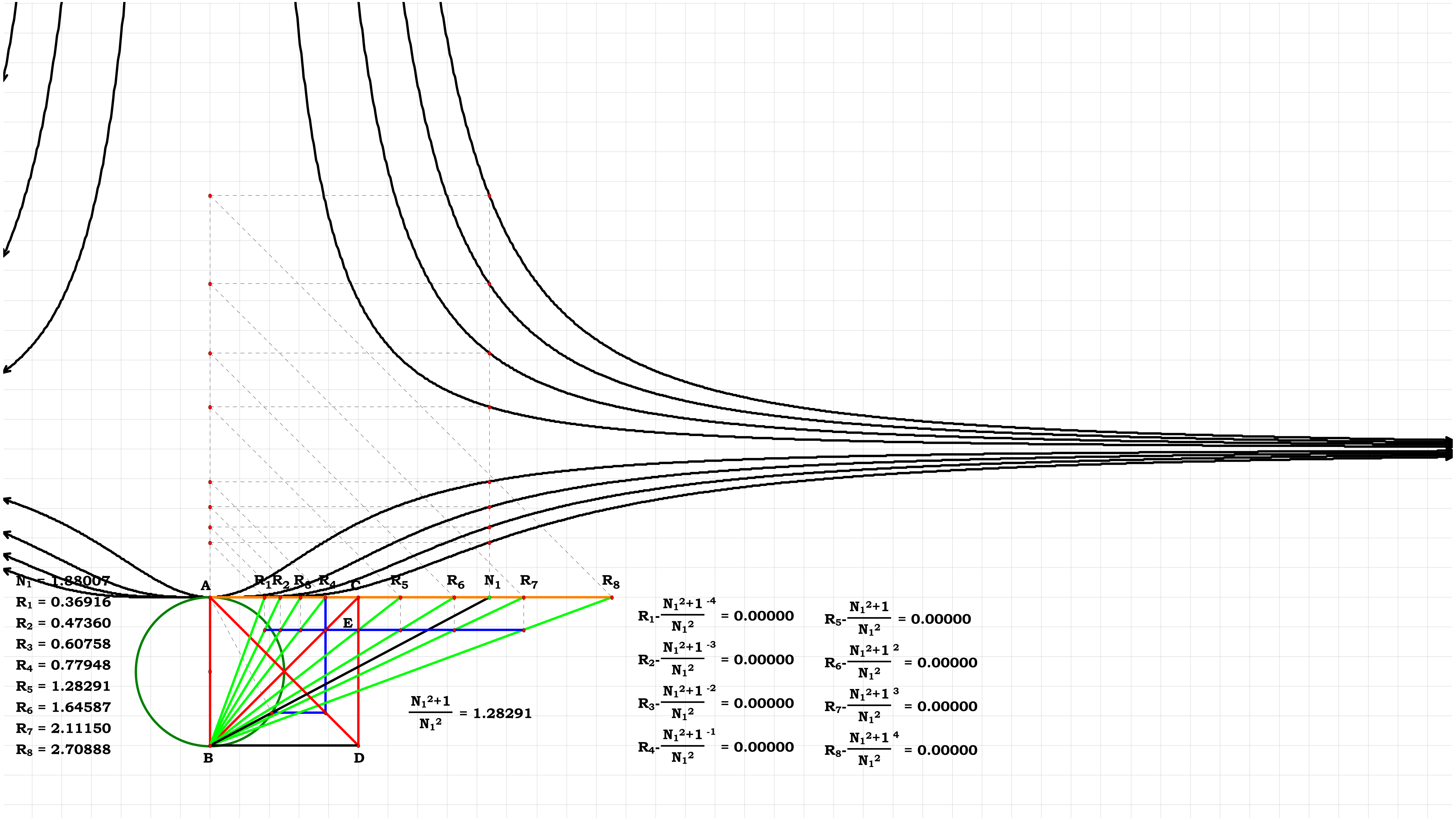
$$CE := \frac{1}{N_1^{\,2}+1} \qquad R_4 := 1 - CE \qquad R_4 - \frac{N_1^{\,2}}{N_1^{\,2}+1} = 0$$

$$R_3 := R_4^{\,2} \qquad R_2 := R_4^{\,3} \qquad R_1 := R_4^{\,4} \qquad R_5 := R_4^{\,-1}$$

$$R_6 := R_4^{\,-2} \qquad R_7 := R_4^{\,-3} \qquad R_8 := R_4^{\,-4}$$

$$R_1 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{-4} = 0 \qquad R_2 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{-3} = 0 \qquad R_3 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{-2} = 0 \qquad R_4 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{-1} = 0$$

$$R_5 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right] = 0 \qquad R_6 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{2} = 0 \qquad R_7 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{3} = 0 \qquad R_8 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1^{\,2}}\right]^{4} = 0$$



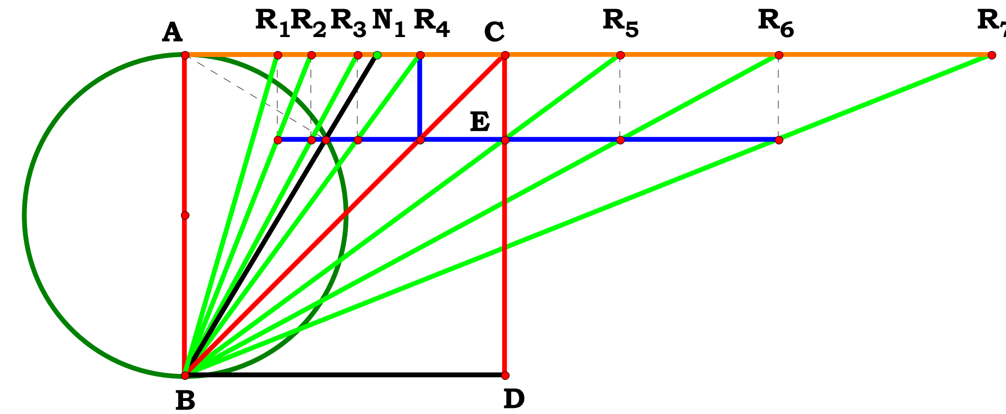
N1 = 1.88007
R1 = 0.36916
R2 = 0.47360
R3 = 0.60758
R4 = 0.77948
R5 = 1.28291
R6 = 1.64587
R7 = 2.11150
R8 = 2.70888
A
B
C
D
E
R1 R2 R3 R4
R5
R6
N1
R7
R8
(N1²+1)/N1² = 1.28291
R1 - (N1²+1)/N1² ^-4 = 0.00000
R2 - (N1²+1)/N1² ^-3 = 0.00000
R3 - (N1²+1)/N1² ^-2 = 0.00000
R4 - (N1²+1)/N1² ^-1 = 0.00000
R5 - (N1²+1)/N1² = 0.00000
R6 - (N1²+1)/N1² ^2 = 0.00000
R7 - (N1²+1)/N1² ^3 = 0.00000
R8 - (N1²+1)/N1² ^4 = 0.00000

Circles-Plate 10

$AB := 1$

$N_1 := .60053$

$N_1 = 0.60053$
$R_1 = 0.29177$
$R_2 = 0.39699$
$R_3 = 0.54016$
$R_4 = 0.73495$
$R_5 = 1.36063$
$R_6 = 1.85131$
$R_7 = 2.51896$



$$DE := \frac{1}{N_1^{\,2} + 1} \qquad R_4 := DE \qquad R_3 := R_4^{\,2} \qquad R_2 := R_4^{\,3}$$

$$R_1 := R_4^{\,4} \qquad R_5 := R_4^{\,-1} \qquad R_6 := R_4^{\,-2} \qquad R_7 := R_4^{\,-3}$$

$$R_1 - \left(N_1^{\,2} + 1\right)^{-4} = 0 \qquad R_2 - \left(N_1^{\,2} + 1\right)^{-3} = 0 \qquad R_3 - \left(N_1^{\,2} + 1\right)^{-2} = 0 \qquad R_4 - \left(N_1^{\,2} + 1\right)^{-1} = 0$$

$$R_5 - \left(N_1^{\,2} + 1\right) = 0 \qquad R_6 - \left(N_1^{\,2} + 1\right)^{2} = 0 \qquad R_7 - \left(N_1^{\,2} + 1\right)^{3} = 0$$

$R_1 = 0.291764$

$R_2 = 0.396985$

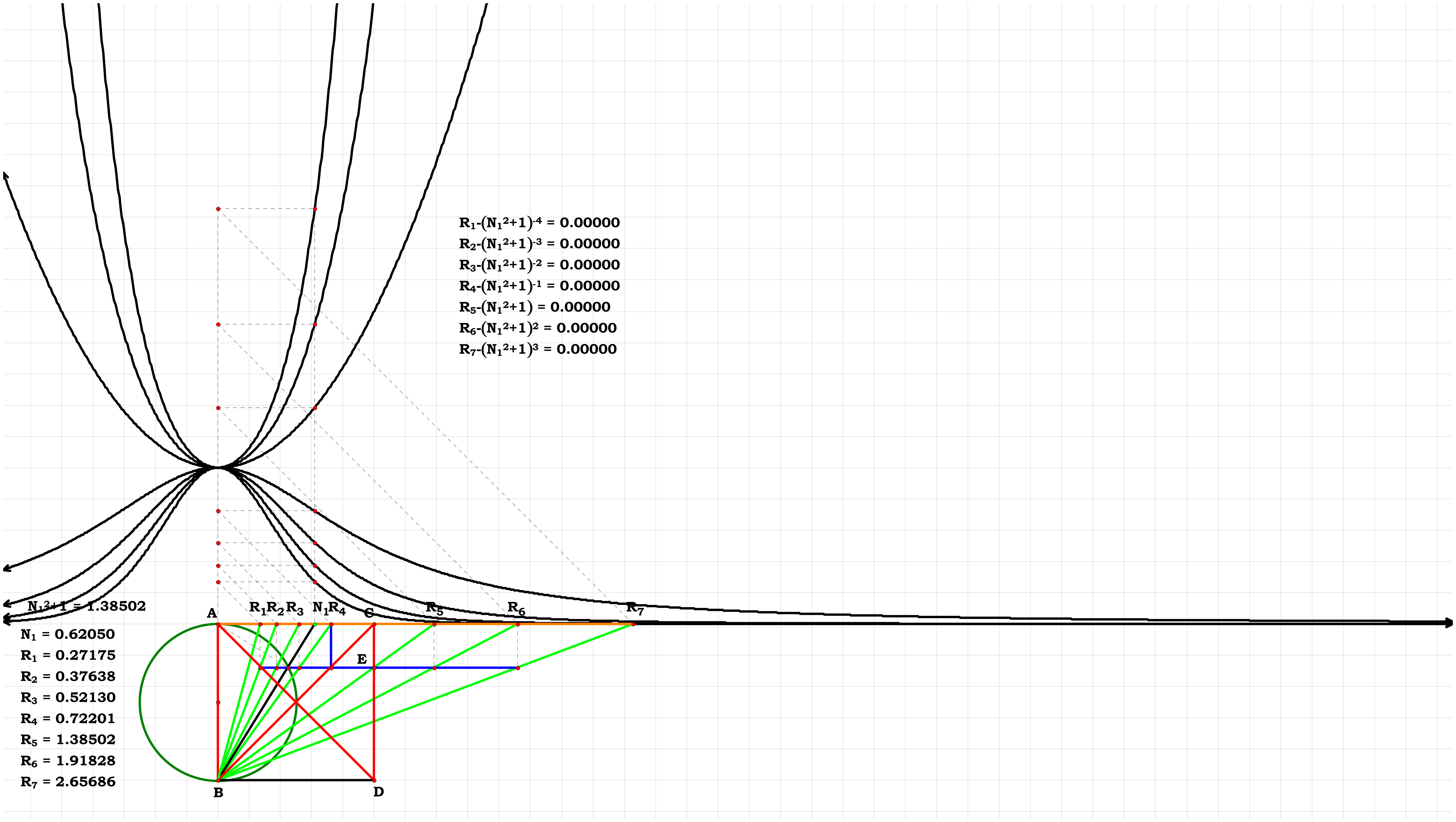
$R_3 = 0.540152$

$R_4 = 0.73495$

$R_5 = 1.360636$

$R_6 = 1.851331$

$R_7 = 2.518988$



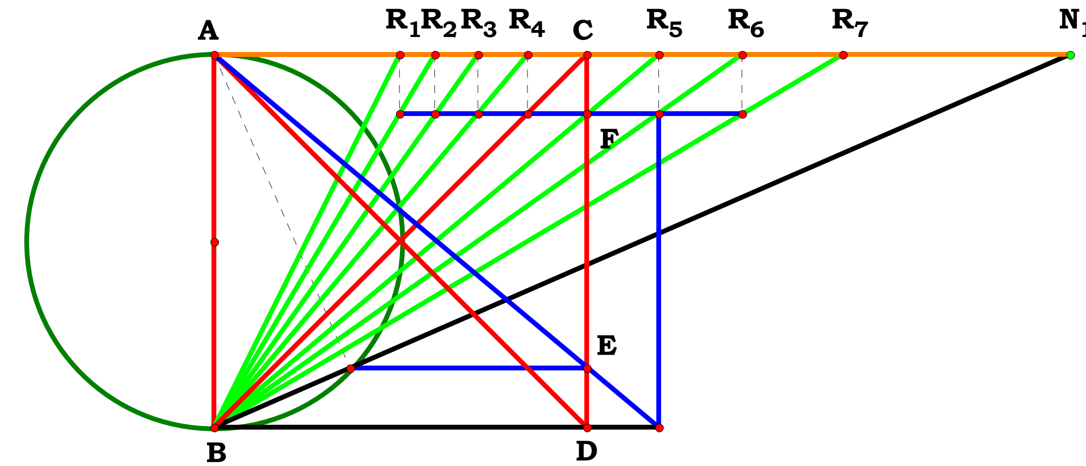
$R_1-(N_1^2+1)^{-4} = 0.00000$
$R_2-(N_1^2+1)^{-3} = 0.00000$
$R_3-(N_1^2+1)^{-2} = 0.00000$
$R_4-(N_1^2+1)^{-1} = 0.00000$
$R_5-(N_1^2+1) = 0.00000$
$R_6-(N_1^2+1)^2 = 0.00000$
$R_7-(N_1^2+1)^3 = 0.00000$
$N_1^2+1 = 1.38502$
A
R_1 R_2 R_3 N_1 R_4
C
R_5
R_6
R_7
E
B
D
$N_1 = 0.62050$
$R_1 = 0.27175$
$R_2 = 0.37638$
$R_3 = 0.52130$
$R_4 = 0.72201$
$R_5 = 1.38502$
$R_6 = 1.91828$
$R_7 = 2.65686$

Circles-Plate 12

$AB := 1$

$N_1 := 2.29403$

$N_1 = 2.29403$
$R_1 = 0.49863$
$R_2 = 0.59339$
$R_3 = 0.70614$
$R_4 = 0.84032$
$R_5 = 1.19002$
$R_6 = 1.41615$
$R_7 = 1.68525$



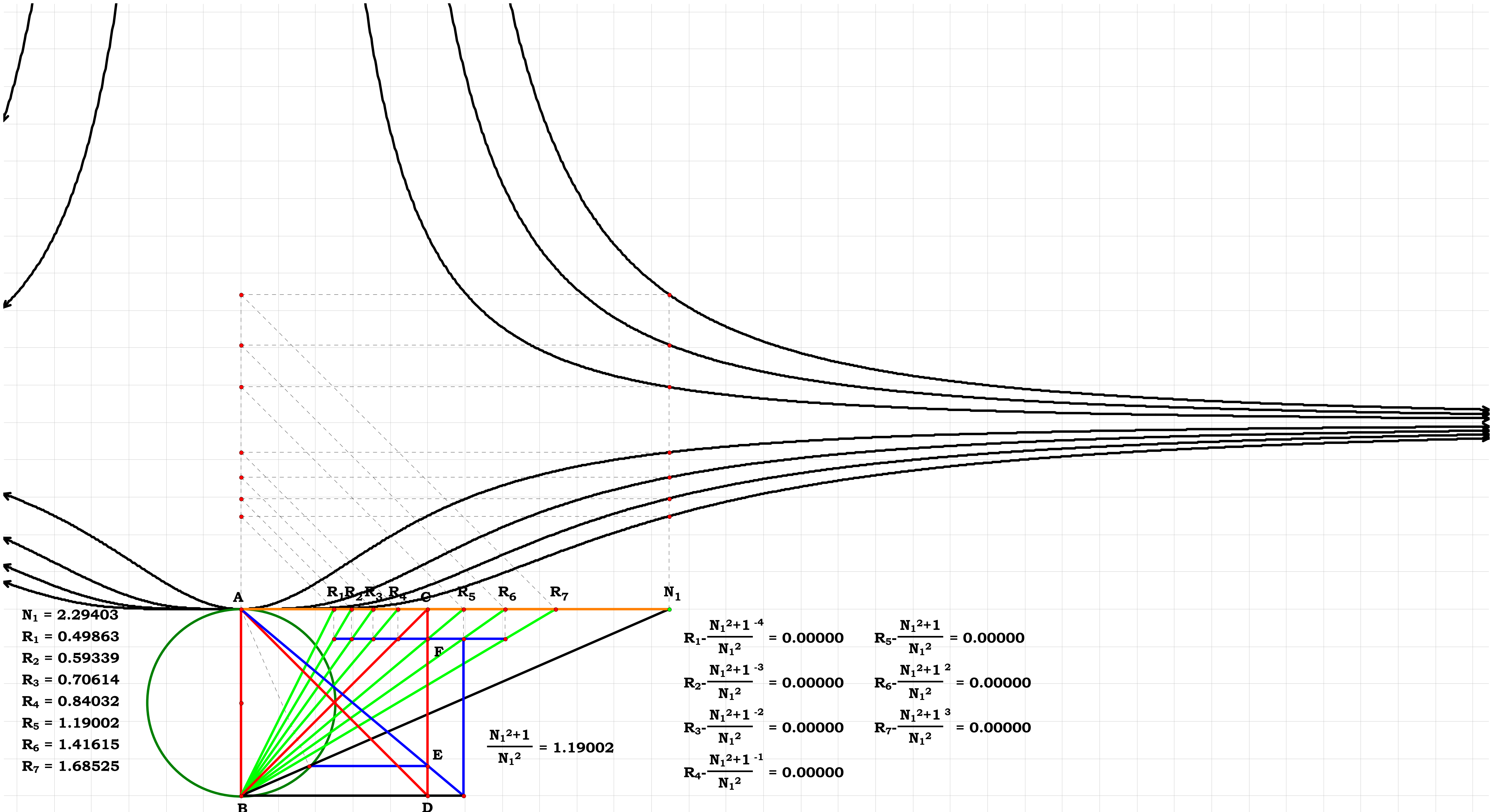
$$DE := \frac{1}{N_1^{\,2} + 1} \qquad R_5 := \frac{1}{1 - DE}$$

$$R_4 := R_5^{\,-1} \qquad R_3 := R_4^{\,2} \qquad R_2 := R_4^{\,3} \qquad R_1 := R_4^{\,4}$$

$$R_6 := R_5^{\,2} \qquad R_7 := R_5^{\,3}$$

$$R_1 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{-4} = 0 \qquad R_2 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{-3} = 0 \qquad R_3 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{-2} = 0 \qquad R_4 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{-1} = 0$$

$$R_5 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{1} = 0 \qquad R_6 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{2} = 0 \qquad R_7 - \left(\frac{N_1^{\,2}+1}{N_1^{\,2}}\right)^{3} = 0$$

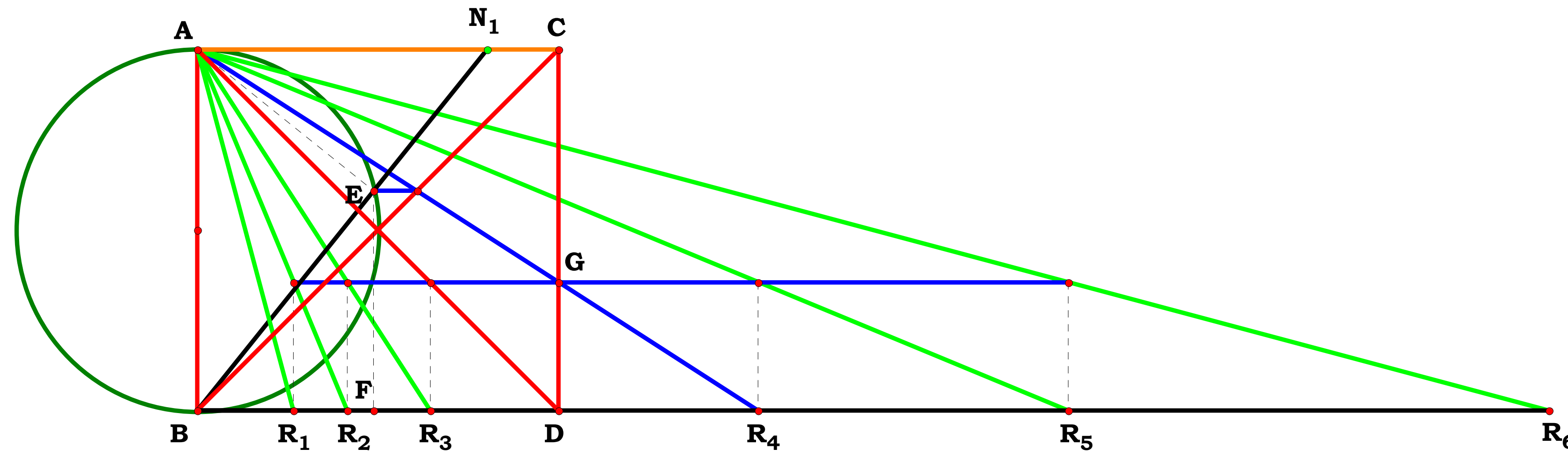


A
B
C
D
E
F
R_1 R_2 R_3 R_4 R_5 R_6 R_7
N_1
N_1 = 2.29403
R_1 = 0.49863
R_2 = 0.59339
R_3 = 0.70614
R_4 = 0.84032
R_5 = 1.19002
R_6 = 1.41615
R_7 = 1.68525
$\frac{N_1^2+1}{N_1^2}$ = 1.19002
$R_1-\frac{N_1^2+1}{N_1^2}^{-4}$ = 0.00000
$R_2-\frac{N_1^2+1}{N_1^2}^{-3}$ = 0.00000
$R_3-\frac{N_1^2+1}{N_1^2}^{-2}$ = 0.00000
$R_4-\frac{N_1^2+1}{N_1^2}^{-1}$ = 0.00000
$R_5-\frac{N_1^2+1}{N_1^2}$ = 0.00000
$R_6-\frac{N_1^2+1}{N_1^2}^{2}$ = 0.00000
$R_7-\frac{N_1^2+1}{N_1^2}^{3}$ = 0.00000

Circles-Plate 13

$AB := 1$

$N_1 := .80275$



$EF := \dfrac{1}{N_1^{\,2} + 1} \qquad R_4 := \dfrac{EF}{1 - EF} \qquad CG := \dfrac{1}{R_4}$

$R_3 := R_4^{\,-1} \quad R_2 := R_4^{\,-2} \quad R_1 := R_4^{\,-3} \quad R_5 := R_4^{\,2} \quad R_6 := R_4^{\,3}$

$R_1 - \left(\dfrac{1}{N_1^{\,2}}\right)^{-3} = 0 \qquad R_2 - \left(\dfrac{1}{N_1^{\,2}}\right)^{-2} = 0 \qquad R_3 - \left(\dfrac{1}{N_1^{\,2}}\right)^{-1} = 0$

$R_4 - \left(\dfrac{1}{N_1^{\,2}}\right) = 0 \qquad R_5 - \left(\dfrac{1}{N_1^{\,2}}\right)^{2} = 0 \qquad R_6 - \left(\dfrac{1}{N_1^{\,2}}\right)^{3} = 0$

$R_1 = 0.267597$

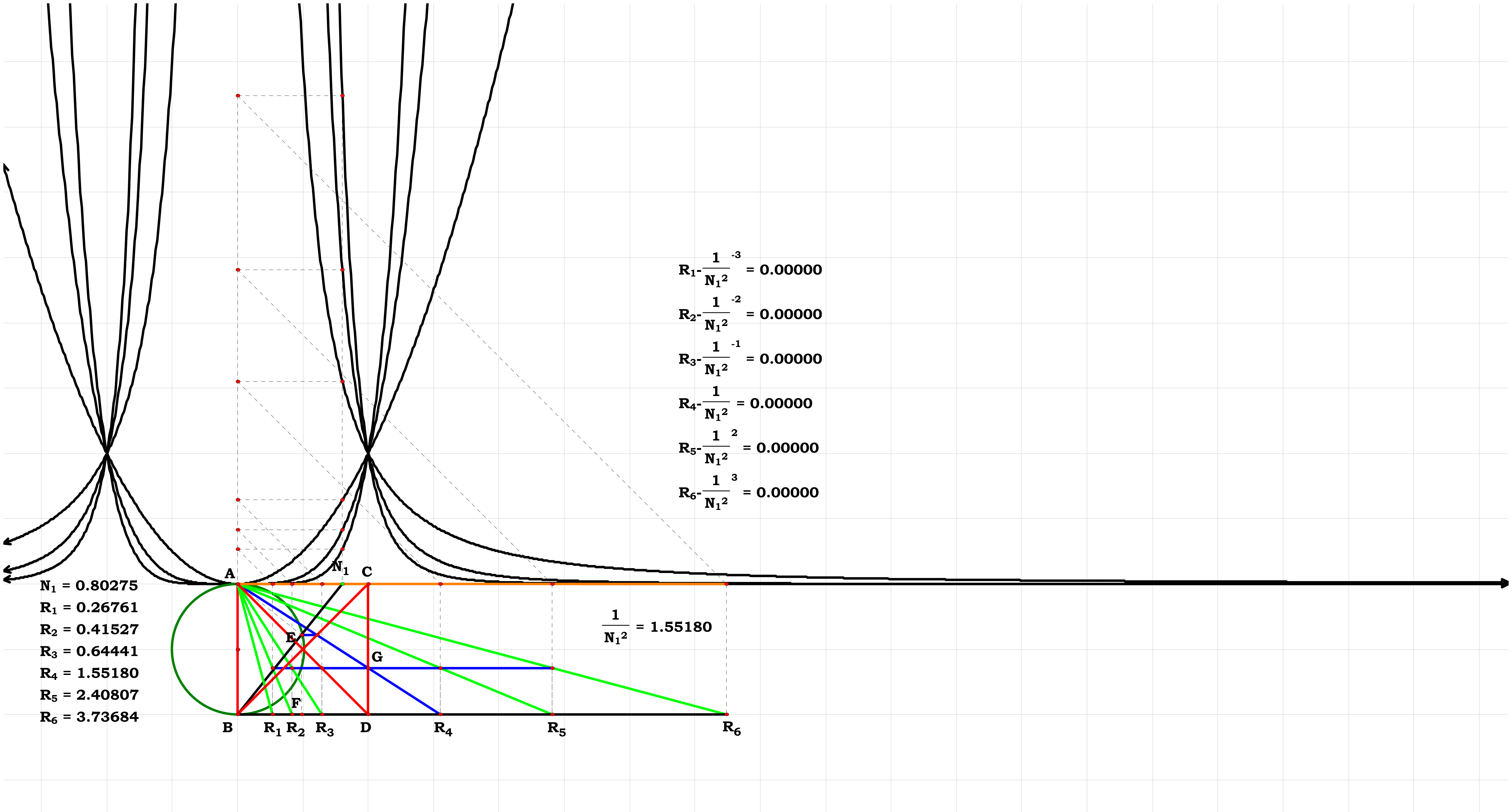
$R_2 = 0.415261$

$R_3 = 0.644408$

$R_4 = 1.551813$

$R_5 = 2.408123$

$R_6 = 3.736957$



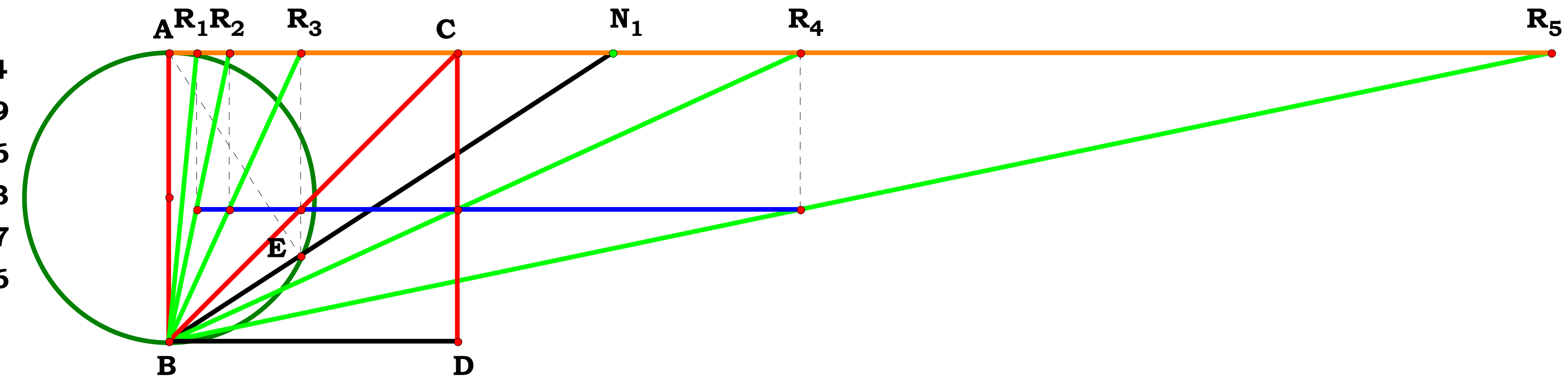
$R_1 - \frac{1}{N_1^2}^{-3} = 0.00000$
$R_2 - \frac{1}{N_1^2}^{-2} = 0.00000$
$R_3 - \frac{1}{N_1^2}^{-1} = 0.00000$
$R_4 - \frac{1}{N_1^2} = 0.00000$
$R_5 - \frac{1}{N_1^2}^{2} = 0.00000$
$R_6 - \frac{1}{N_1^2}^{3} = 0.00000$
$N_1 = 0.80275$
$R_1 = 0.26761$
$R_2 = 0.41527$
$R_3 = 0.64441$
$R_4 = 1.55180$
$R_5 = 2.40807$
$R_6 = 3.73684$
$\frac{1}{N_1^2} = 1.55180$
A
N_1
C
E
G
F
B
R_1
R_2
R_3
D
R_4
R_5
R_6

Circles-Plate 14

$AB := 1$

$N_1 := 1.53604$

$N_1 = 1.53604$
$R_1 = 0.09559$
$R_2 = 0.20906$
$R_3 = 0.45723$
$R_4 = 2.18707$
$R_5 = 4.78326$

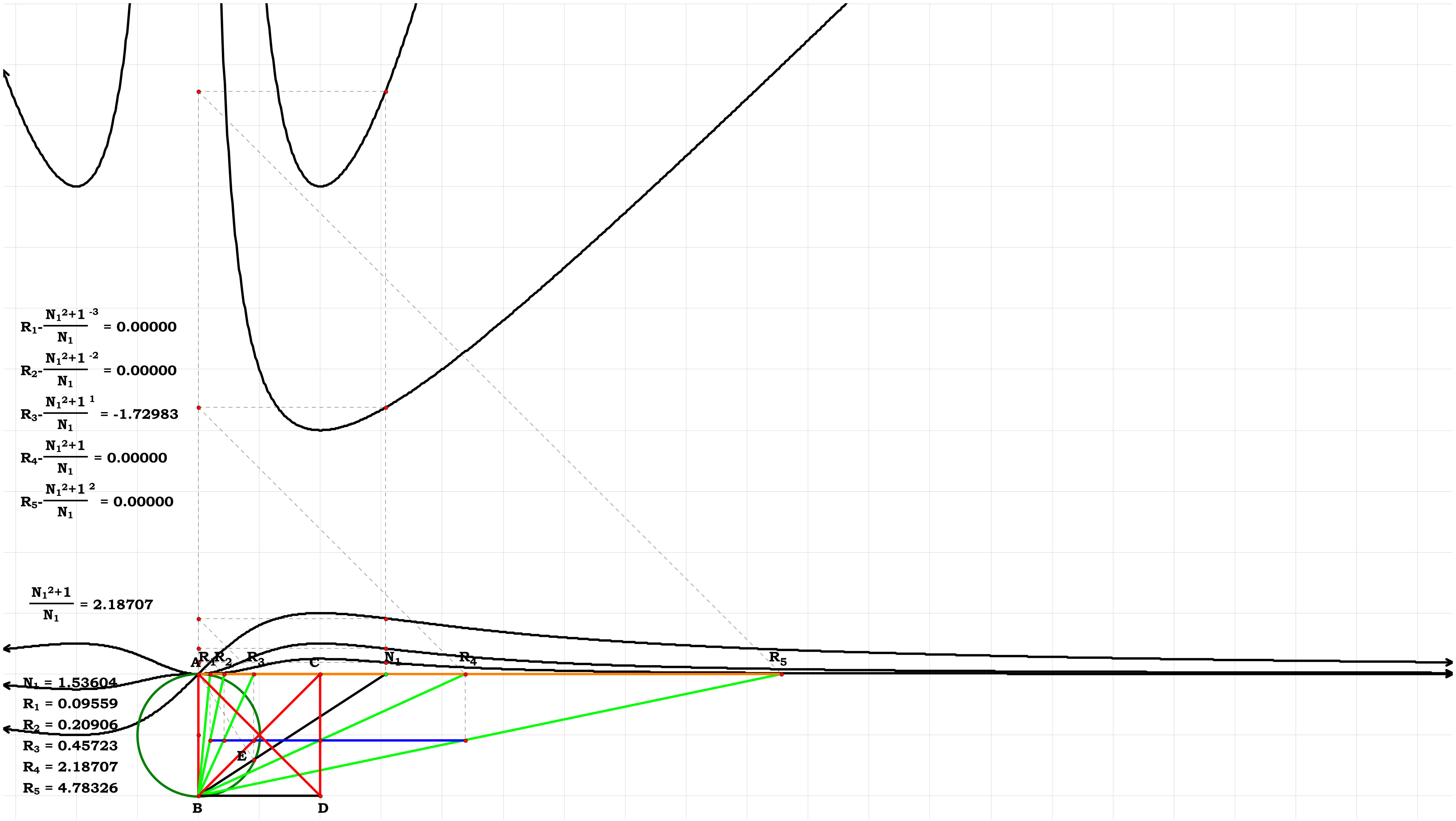


$$R_3 := \frac{N_1}{N_1^{\,2}+1} \qquad R_2 := R_3^{\,2} \qquad R_1 := R_3^{\,3}$$

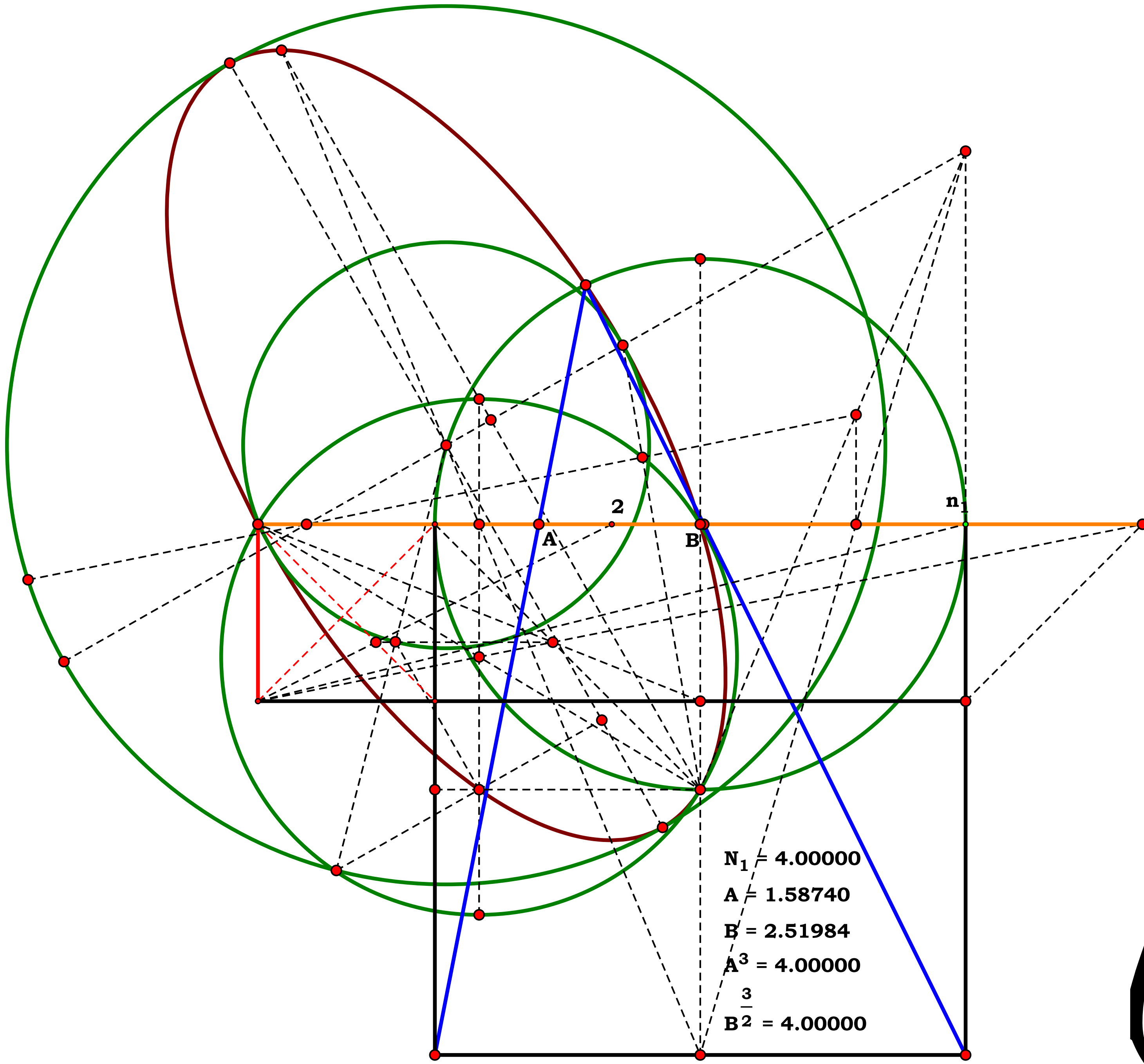
$$R_4 := R_3^{\,-1} \qquad R_5 := R_3^{\,-2}$$

$$R_1 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1}\right]^{-3} = 0 \qquad R_2 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1}\right]^{-2} = 0 \qquad R_3 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1}\right]^{-1} = 0$$

$$R_4 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1}\right] = 0 \qquad R_5 - \left[\frac{\left(N_1^{\,2}+1\right)}{N_1}\right]^{2} = 0$$

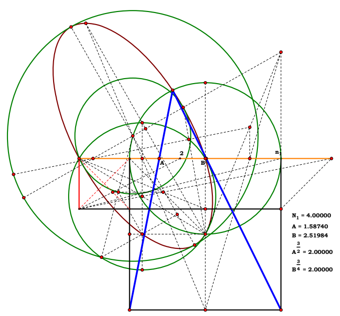


$R_1 - \frac{N_1^2+1^{-3}}{N_1} = 0.00000$
$R_2 - \frac{N_1^2+1^{-2}}{N_1} = 0.00000$
$R_3 - \frac{N_1^2+1^{1}}{N_1} = -1.72983$
$R_4 - \frac{N_1^2+1}{N_1} = 0.00000$
$R_5 - \frac{N_1^2+1^{2}}{N_1} = 0.00000$
$\frac{N_1^2+1}{N_1} = 2.18707$
A
R_1
R_2
R_3
C
N_1
R_4
R_5
E
B
D
$N_1 = 1.53604$
$R_1 = 0.09559$
$R_2 = 0.20906$
$R_3 = 0.45723$
$R_4 = 2.18707$
$R_5 = 4.78326$



The Delian Quest 2019

John Clark

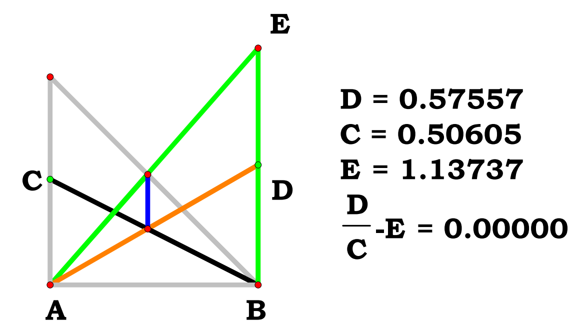


Two Triangles

Saturday, April 6, 2019

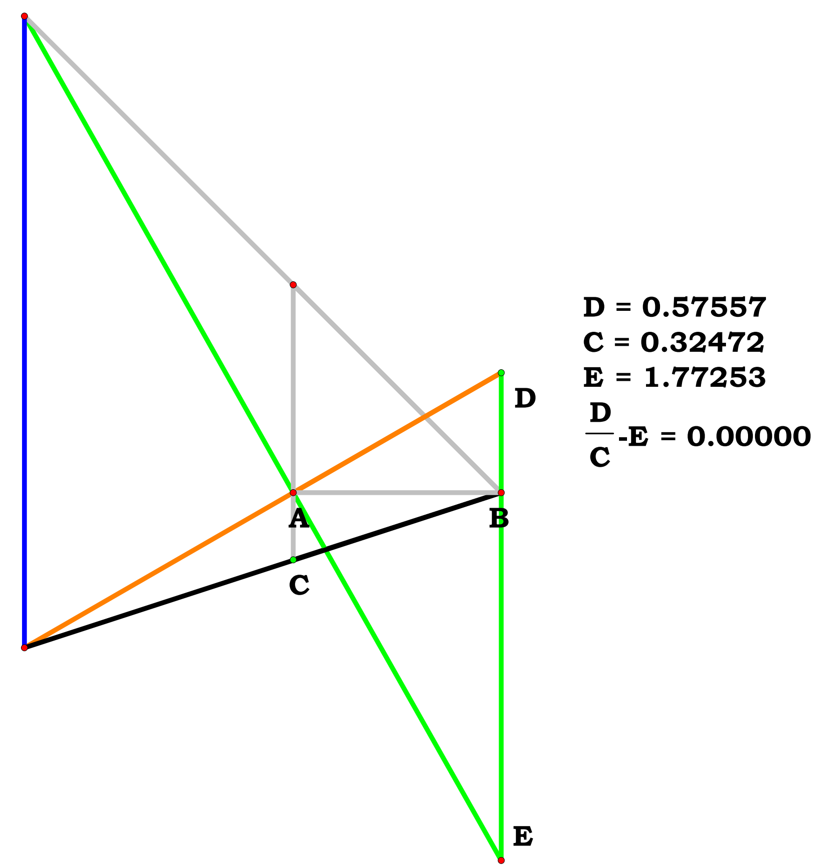
Just the first book or two of Euclid's *Elements* should give one enough information to discover *Basic Analog Mathematics*. However, it apparently did not happen. The reason being is that it is a whole lot easier to repeat and memorize perceptible information than to see the intelligible being expressed.

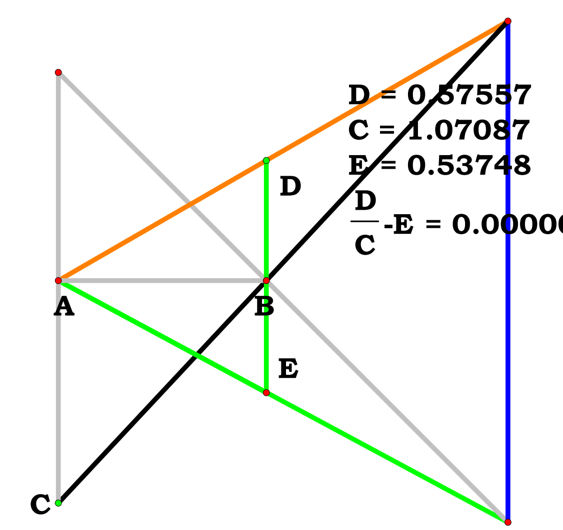
In this little essay, I am not going to say much, I am just going to present a little figure which I call Two Triangles. Just imagine what you can do with two right angles on the same base. I can call one ACB and the other ADB. I will simply present a series of plates which only differ in so far as a point go.

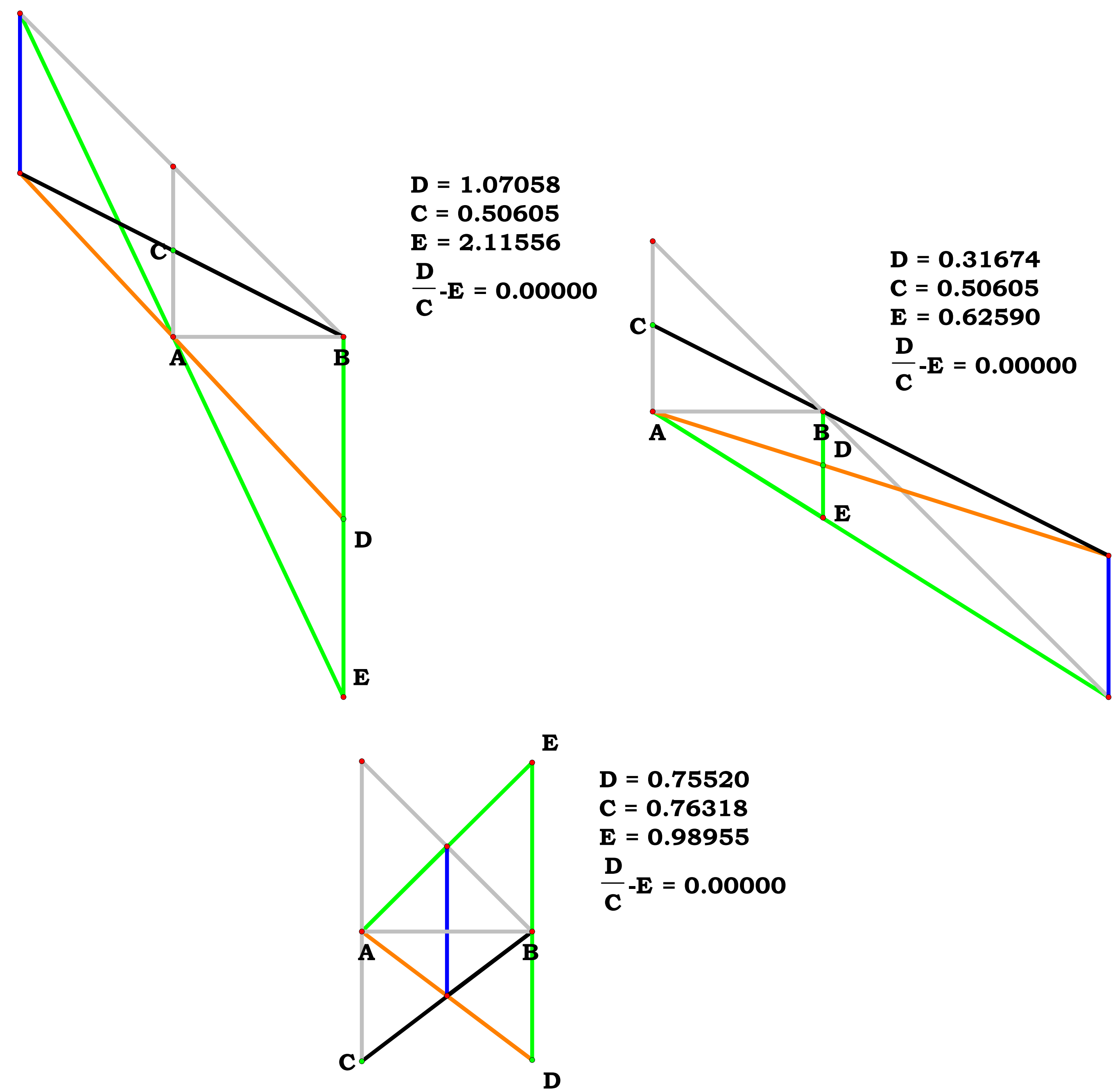


Let us call AB the standard unit by which C and D and their produce E is named. We simply have a right triangle to perform our operation. Call it AB. The intersection of the two triangles perpendicular to the base AB fall on what we can call the segment known to be the square root of 2. We are not, however, interested where it intersects the segment, but only that it intersects the line which contains the segment. We are interested in it only insofar as it expresses and projects a ratio.

Now, we can give C and D any values we like. We can imagine them as triangles ACB and ADB, however, we have no wish to think or speak in terms of the mystical angle. We adhere to the notion that a two-dimensional plane is expressible as a ratio between two units.


A
B
C
D
E
D = 0.57557
C = 0.32472
E = 1.77253
$\frac{D}{C}$-E = 0.00000


A
B
C
D
E
D = 0.57557
C = 1.07087
E = 0.53748
$\frac{D}{C}$-E = 0.00000



One of the things which BAM helps one with, or one of the things Geometry helps one with, is not to set the standard of understanding a figure based on the perceptible, which can be very confusing, but on the intelligible content established by standards. The standard references what is intersected while the mind is looking at where.

One may notice, even in the *Elements* as we have them today, propositions written up by a weaker mind writes up the same proposition in terms of cases based on perceptible location. One can see here, if they were done correctly, the equation never changes. Notice also that any and every other type of triangle can be found using in the figure. I am not interested in obtuse, acute, or any other name one can give to any other expression of a triangle. I am only interested in the fact that a two-dimensional matrix can produce results using an unit whatsoever when compared to another and I do not need Cartesian Geometry, Trigonometry, or Calculus to do it.

The ability to equate an analog to its logical name is not, in any wise apparent to the eye. One has to find and use standards to express it, and comprehend it in the mind. You can call an analog an isosceles right triangle in gray, or a method of dividing two given things of the same relative difference in accordance with a standard unit. All of the other so called triangles are simply parts of a much bigger and better ordered universe.

Let us take our little figure, Lay it on its side and imagine that C and D are on two parallels and AB is just a unit.

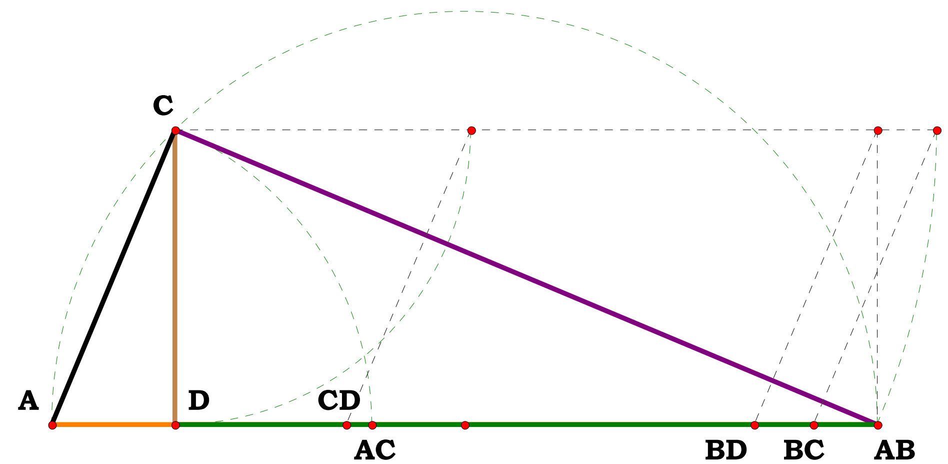
111619

Unit.

Given.

Descriptions.

Definitions.



AB = 1.00000

AC = 0.92195 $\sqrt{BD}$ = 0.92195 AC-$\sqrt{BD}$ = 0.00000

BD = 0.85000 AC²-BD = 0.00000

AC = 0.38730

CD = 0.35707 $\sqrt{AD \cdot BD}$-CD = 0.00000

AD = 0.15000 AC^2 = 0.15000

$\sqrt{AD}$ = 0.38730 AD-AC^2 = 0.00000

Hide Point A
Hide Point B
Show Base Line Points (20)

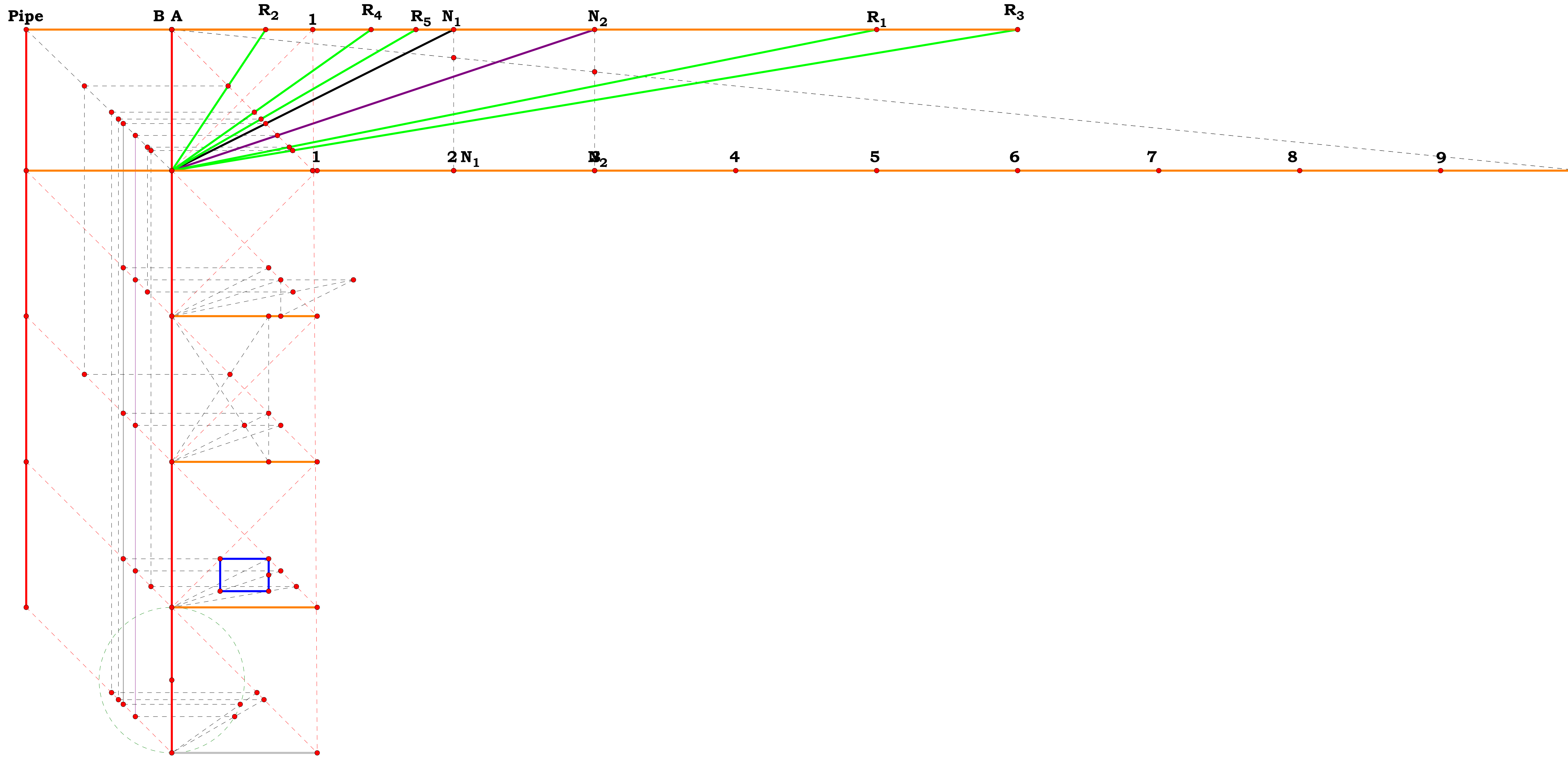
X0
X1
X2
X3
X4
X5
X6
X7
X8
X9
X10
X11
X12
X13
X14
X15
X16
X17
X18
X19
X20
Y0
Y1
Y2
Y3
Y4
Y5
Y6
Y7
Y8
Y9
Y10
Y11
Y12
Y13
Y14
Y15
Y16
Y17
Y18
Y19
Y20

Hide Intersections
1
2
3
4
5
6
7
8
9
10
0

Hide Intersections
1
2
3
4
5
6
7
8
9
10
0
Pipe

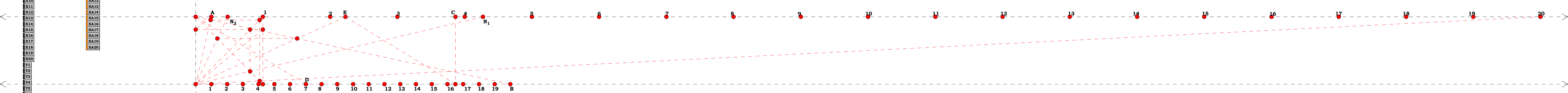
$A = 0.00000$
$N_1 = 2.00000$
$N_2 = 3.00000$
$R_1 = 5.00000$
$R_2 = 0.66667$
$R_3 = 6.00000$
$R_4 = 1.41421$
$R_5 = 1.73205$

$N_1 + N_2 = 5.00000$
$\frac{N_1}{N_2} = 0.66667$
$N_1 \cdot N_2 = 6.00000$
$\sqrt{N_1} = 1.41421$
$\sqrt{N_2} = 1.73205$



$A = 0.00000$
$B = 0.00000$

Constructing a numbered line which is adjustable in steps of 20. N1 will set the range, and N2 one can select the working domain. This will go up to 20 squared.



$N_1 = 4.27256$

$N_2 = 0.47553$

$A = 0.23405$

$B = 4.68103$

$C = 3.86434$

$D = 1.63836$

$E = 2.22598$

$N_1 \cdot 20 = 85.45125$

$\frac{N_2 \cdot B}{A} = 9.51062$

$\frac{C}{A} = 16.51062$

$\frac{D}{A} = 7.00000$

$\frac{20}{A} = 85.45125$

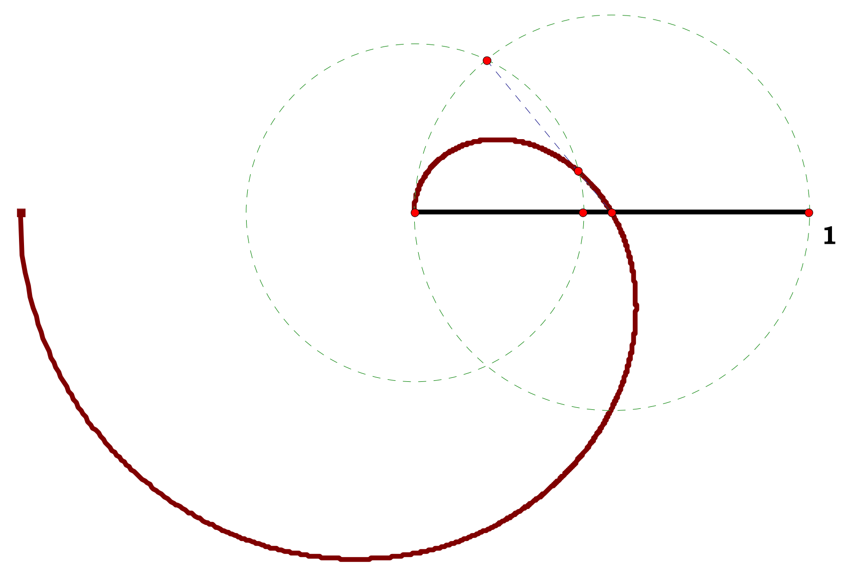
$\frac{B}{A} = 20.00000$

$\frac{E}{A} = 9.51062$

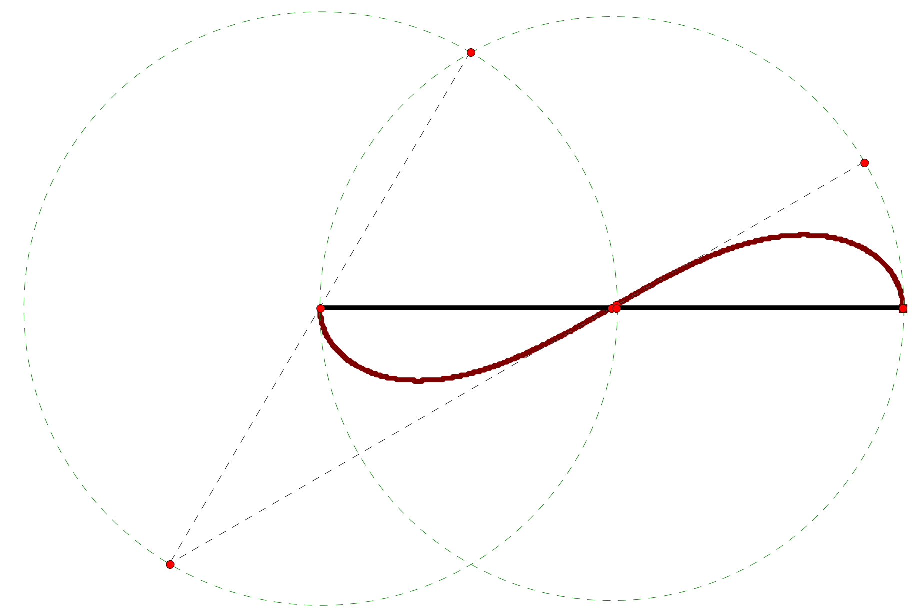
120119

Simple Spirals

Spiral A



Spiral B



120119A Spiral

Unit. $AB := 1$

Given. $X := 12$

$Y := 20$

Descriptions.

$$AC := \frac{AB}{2} \quad CE := AC \quad AD := \frac{X}{Y} \quad AE := AD \quad AH := \frac{AE^2}{AB}$$

$$BD := AB - AD \quad BH := AB - AH \quad EH := \sqrt{(AH \cdot BH)}$$

$$CH := \sqrt{AC^2 - EH^2} \quad EK := \frac{AE}{2} \quad EJ := \frac{EK \cdot AD}{AC}$$

$$EF := 2 \cdot EJ \quad CF := |EF - CE| \quad CG := \frac{CH \cdot CF}{CE}$$

$$FG := \frac{EH \cdot CF}{CE} \cdot \frac{|AC - AD|}{AC - AD}$$

Definitions.

$$AC - \frac{1}{2} = 0 \quad CE - \frac{1}{2} = 0 \quad AD - \frac{X}{Y} = 0 \quad AE - \frac{X}{Y} = 0 \quad AH - \left(\frac{X}{Y}\right)^2 = 0$$

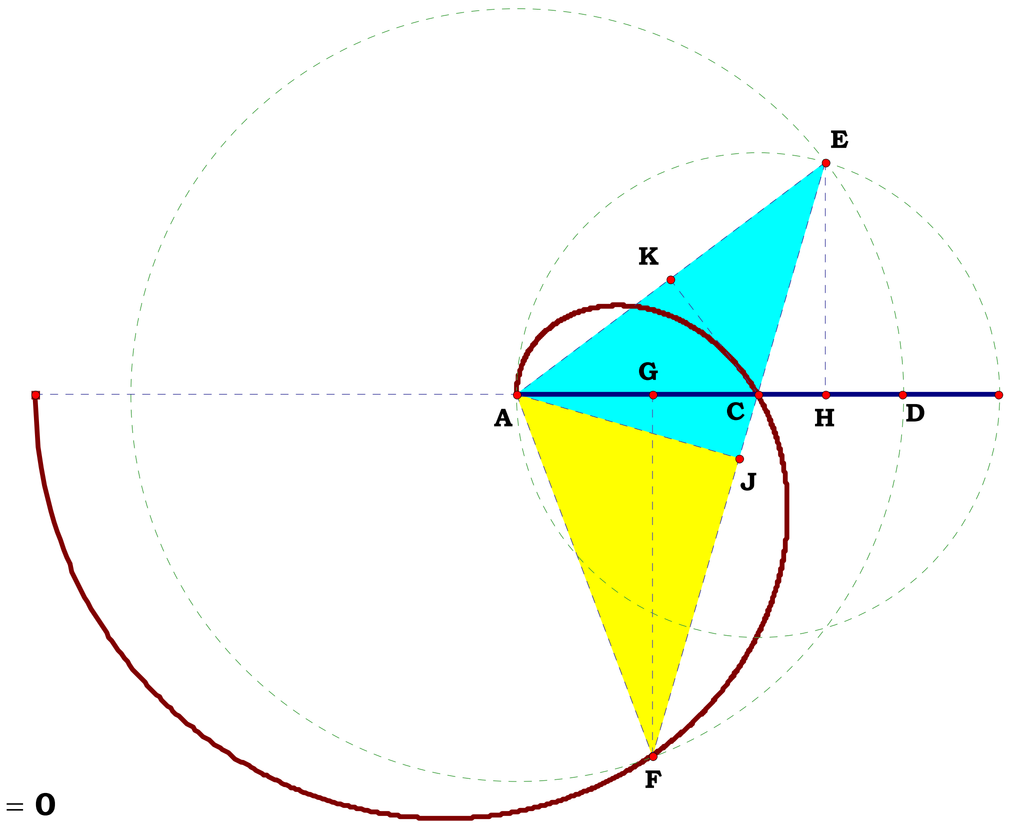
$$BD - \frac{Y - X}{Y} = 0 \quad BH - \frac{(Y - X) \cdot (X + Y)}{Y^2} = 0 \quad EH - \frac{X \cdot \sqrt{(Y - X) \cdot (X + Y)}}{Y^2} = 0$$

$$CH - \frac{|2 \cdot X^2 - Y^2|}{2 \cdot Y^2} = 0 \quad EK - \frac{X}{2 \cdot Y} = 0 \quad EJ - \frac{X^2}{Y^2} = 0 \quad EF - \frac{2 \cdot X^2}{Y^2} = 0$$

$$CF - \frac{|(2 \cdot X - Y) \cdot (2 \cdot X + Y)|}{2 \cdot Y^2} = 0 \quad CG - \frac{|(Y - 2 \cdot X) \cdot (2 \cdot X + Y)| \cdot |2 \cdot X^2 - Y^2|}{2 \cdot Y^4} = 0$$

$$FG - \frac{X \cdot |(Y - 2 \cdot X) \cdot (2 \cdot X + Y)| \cdot |Y - 2 \cdot X| \cdot \sqrt{(Y - X) \cdot (X + Y)}}{Y^4 \cdot (Y - 2 \cdot X)} = 0$$

Spiral A



Unit = 1.00000

XY = 0.80000

X = 16.00000

Y = 20.00000

$$\frac{X \cdot \sqrt{(Y-X) \cdot (X+Y)}}{Y^2} - EH = 0.00000$$

$$\frac{|2 \cdot X^2 - Y^2|}{2 \cdot Y^2} - CH = 0.00000$$

$$\frac{X \cdot |(Y-2 \cdot X) \cdot (2 \cdot X+Y)| \cdot |Y-2 \cdot X| \cdot \sqrt{(Y-X) \cdot (X+Y)}}{Y^4 \cdot (Y-2 \cdot X)} - FG = 0.00000$$

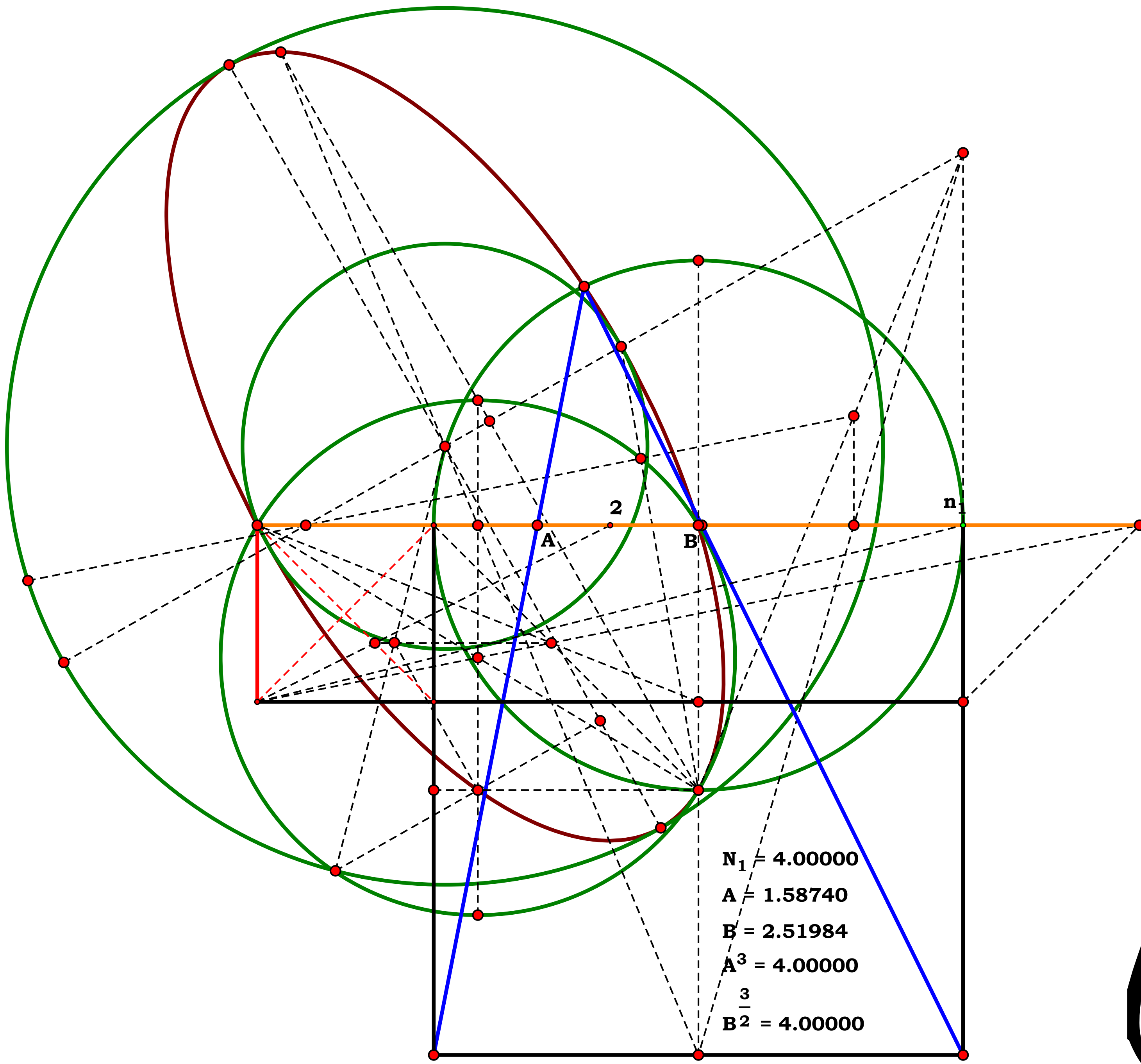
$$\frac{|2 \cdot X^2 - Y^2| \cdot |(2 \cdot X-Y) \cdot (2 \cdot X+Y)|}{2 \cdot Y^4} - CG = 0.00000$$

$$\frac{X \cdot |(Y-2 \cdot X) \cdot (2 \cdot X+Y)| \cdot |Y-2 \cdot X| \cdot \sqrt{(Y-X) \cdot (X+Y)}}{Y^4 \cdot (Y-2 \cdot X)} - FG = 0.00000$$

$$\frac{(\text{Area } \triangle AFJ)}{(\text{Area } \triangle AJE)} = 1.00000$$

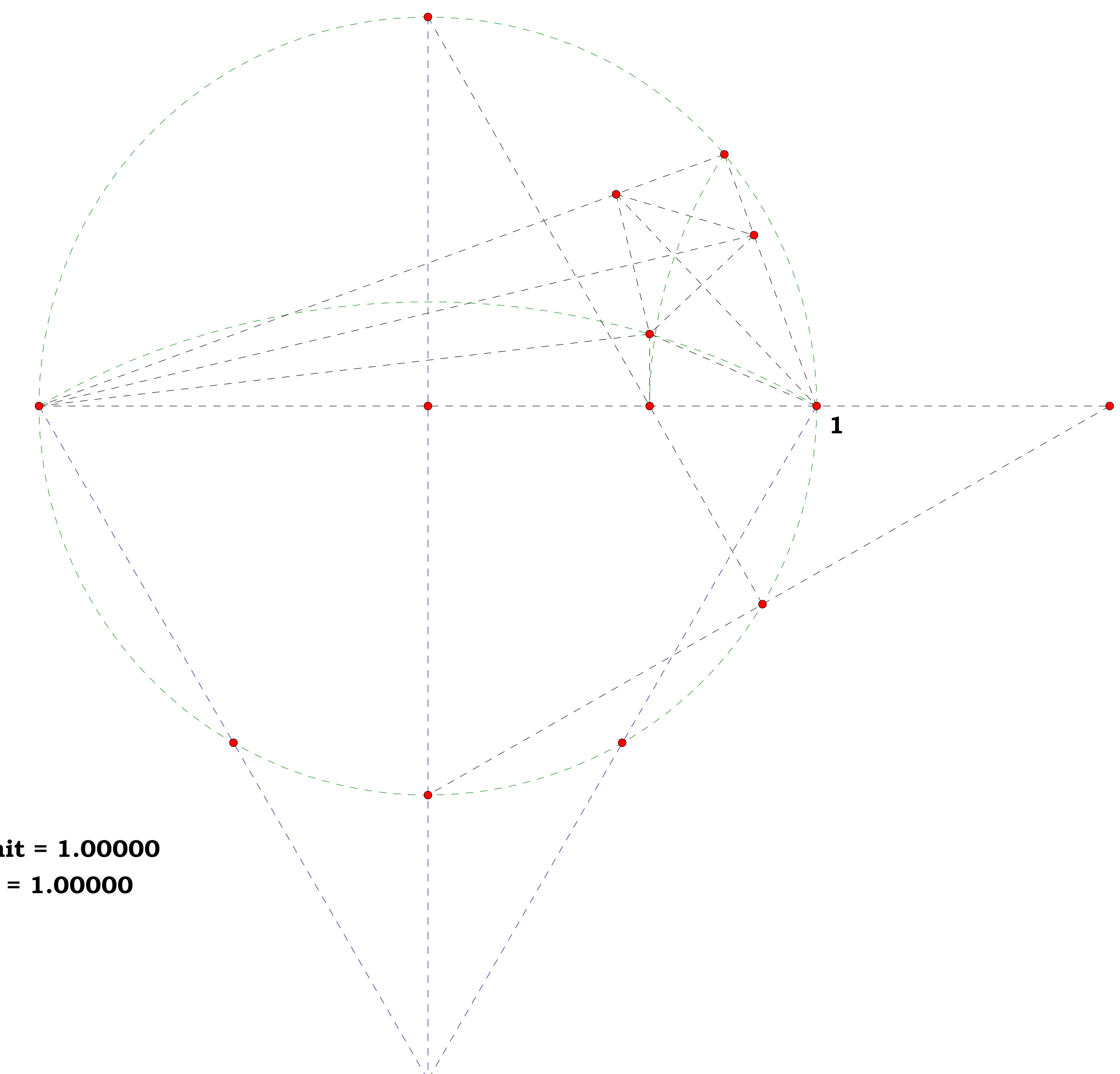
$$\frac{FG}{CG} = 3.42857$$

$$\frac{HE}{CH} - \frac{FG}{CG} = 0.00000$$



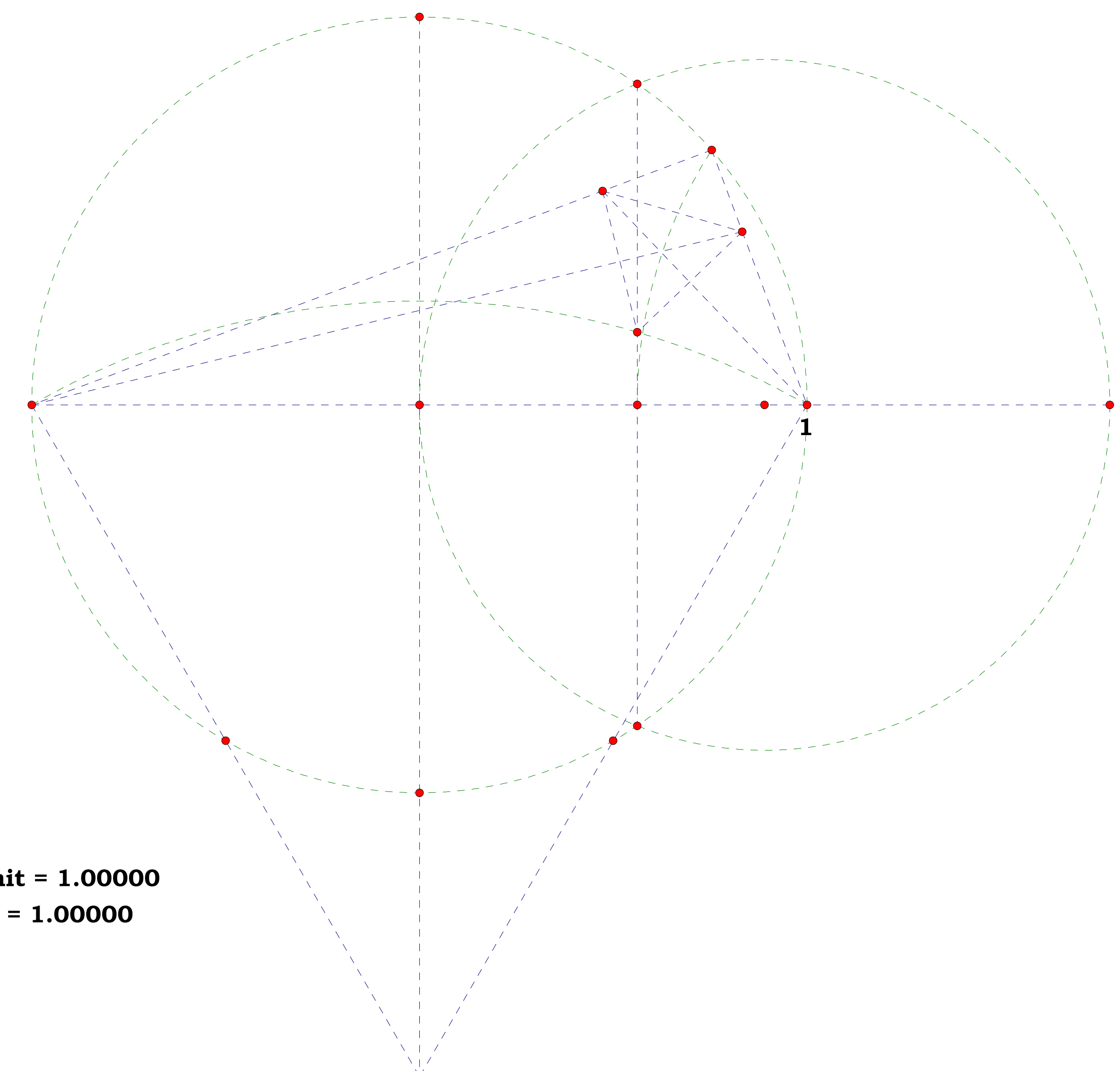
The Delian Quest 2020

John Clark

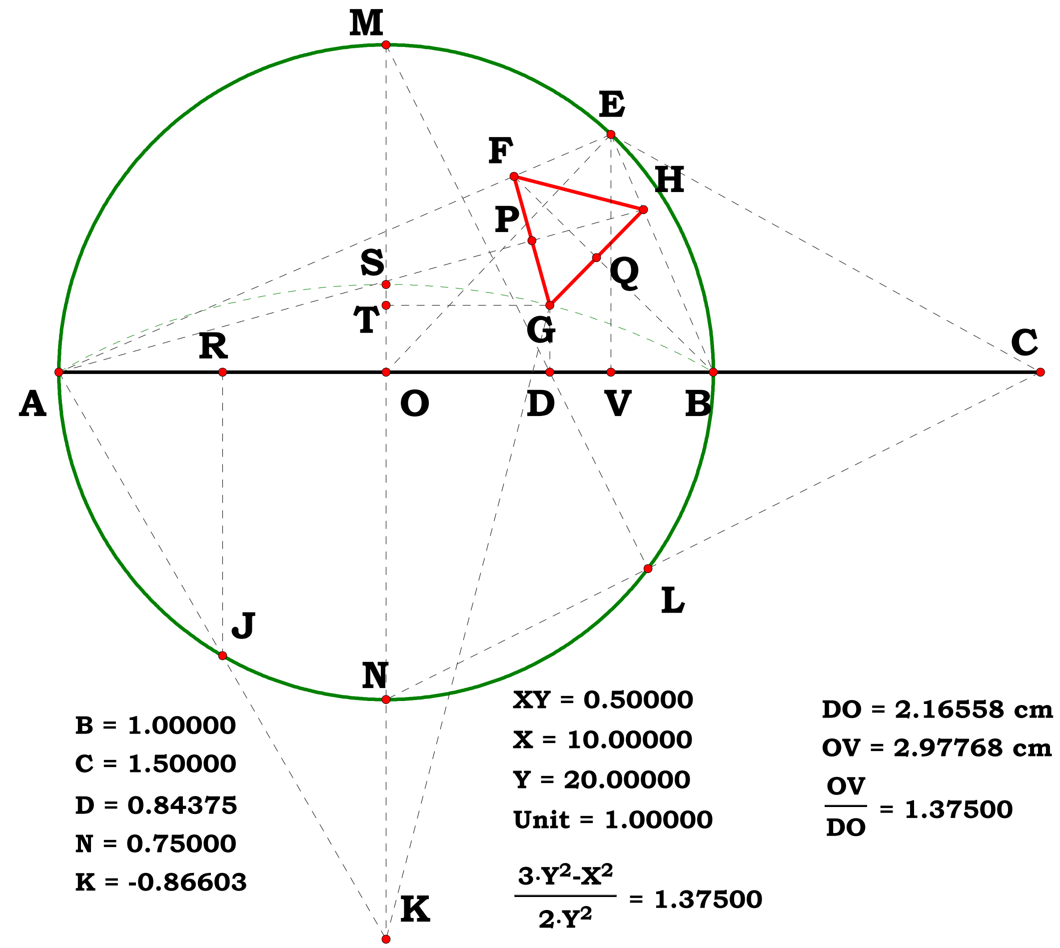


Unit = 1.00000

$\mathbf{x_1}$ **= 1.00000**



Unit = 1.00000
x_1 = 1.00000



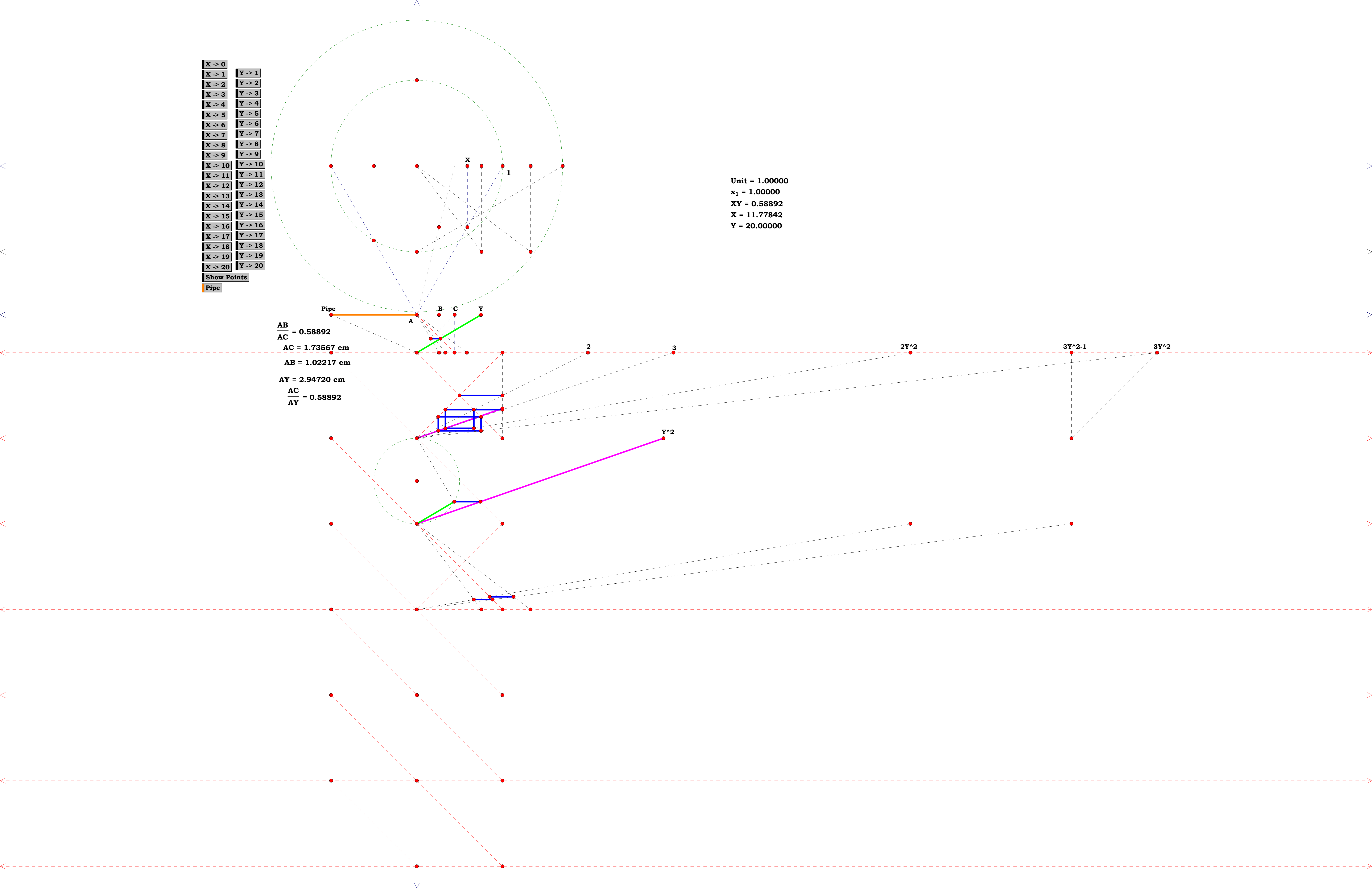
B = 1.00000
C = 1.50000
D = 0.84375
N = 0.75000
K = -0.86603

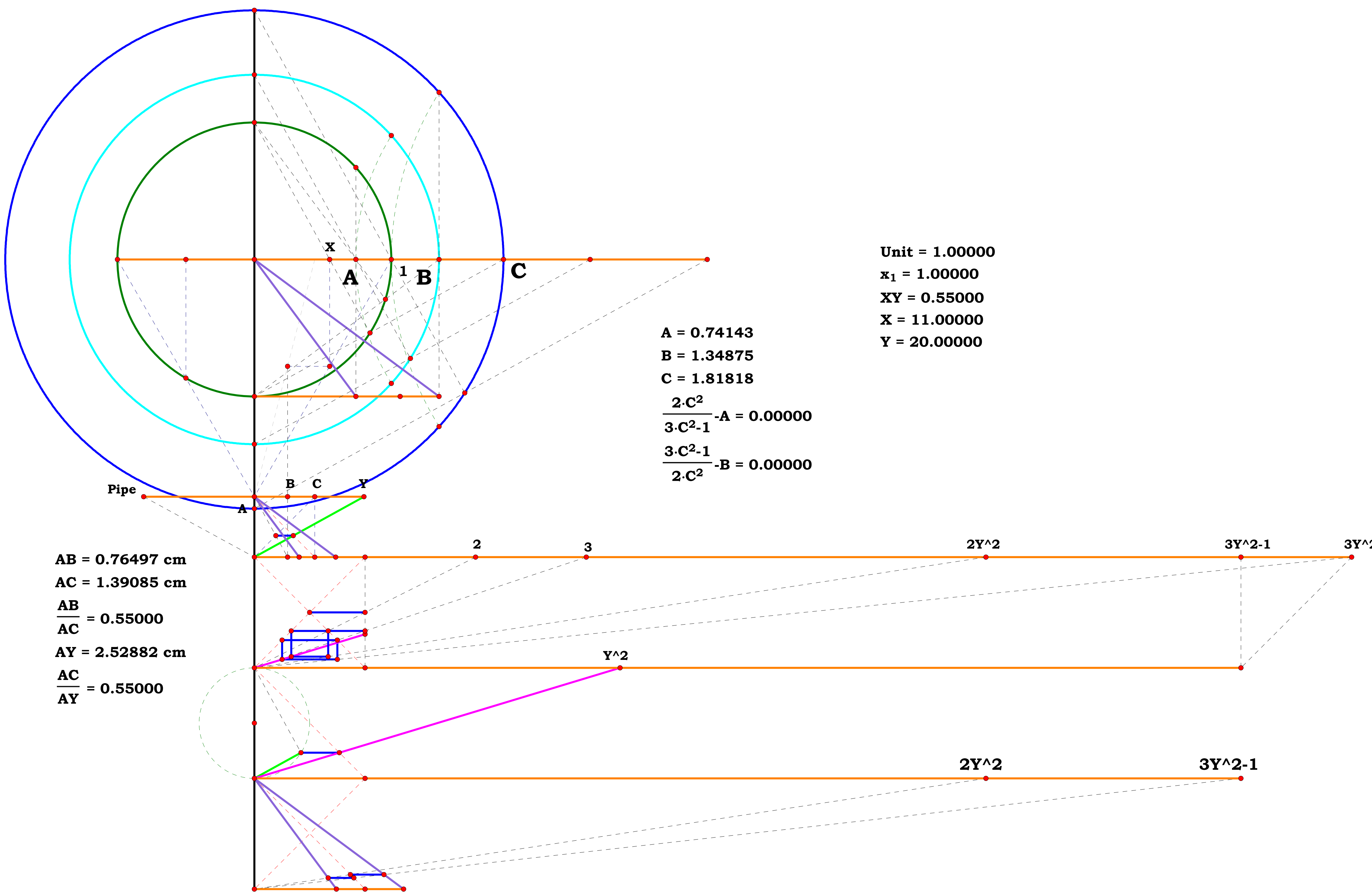
XY = 0.50000
X = 10.00000
Y = 20.00000
Unit = 1.00000

$$\frac{3 \cdot Y^2 - X^2}{2 \cdot Y^2} = 1.37500$$

DO = 2.16558 cm
OV = 2.97768 cm

$$\frac{OV}{DO} = 1.37500$$


X -> 0
X -> 1
X -> 2
X -> 3
X -> 4
X -> 5
X -> 6
X -> 7
X -> 8
X -> 9
X -> 10
X -> 11
X -> 12
X -> 13
X -> 14
X -> 15
X -> 16
X -> 17
X -> 18
X -> 19
X -> 20
Y -> 1
Y -> 2
Y -> 3
Y -> 4
Y -> 5
Y -> 6
Y -> 7
Y -> 8
Y -> 9
Y -> 10
Y -> 11
Y -> 12
Y -> 13
Y -> 14
Y -> 15
Y -> 16
Y -> 17
Y -> 18
Y -> 19
Y -> 20
Show Points
Pipe
X
1
Unit = 1.00000
x_1 = 1.00000
XY = 0.58892
X = 11.77842
Y = 20.00000
Pipe
A
B
C
Y
$\frac{AB}{AC}$ = 0.58892
AC = 1.73567 cm
AB = 1.02217 cm
AY = 2.94720 cm
$\frac{AC}{AY}$ = 0.58892
2
3
2Y^2
3Y^2-1
3Y^2
Y^2

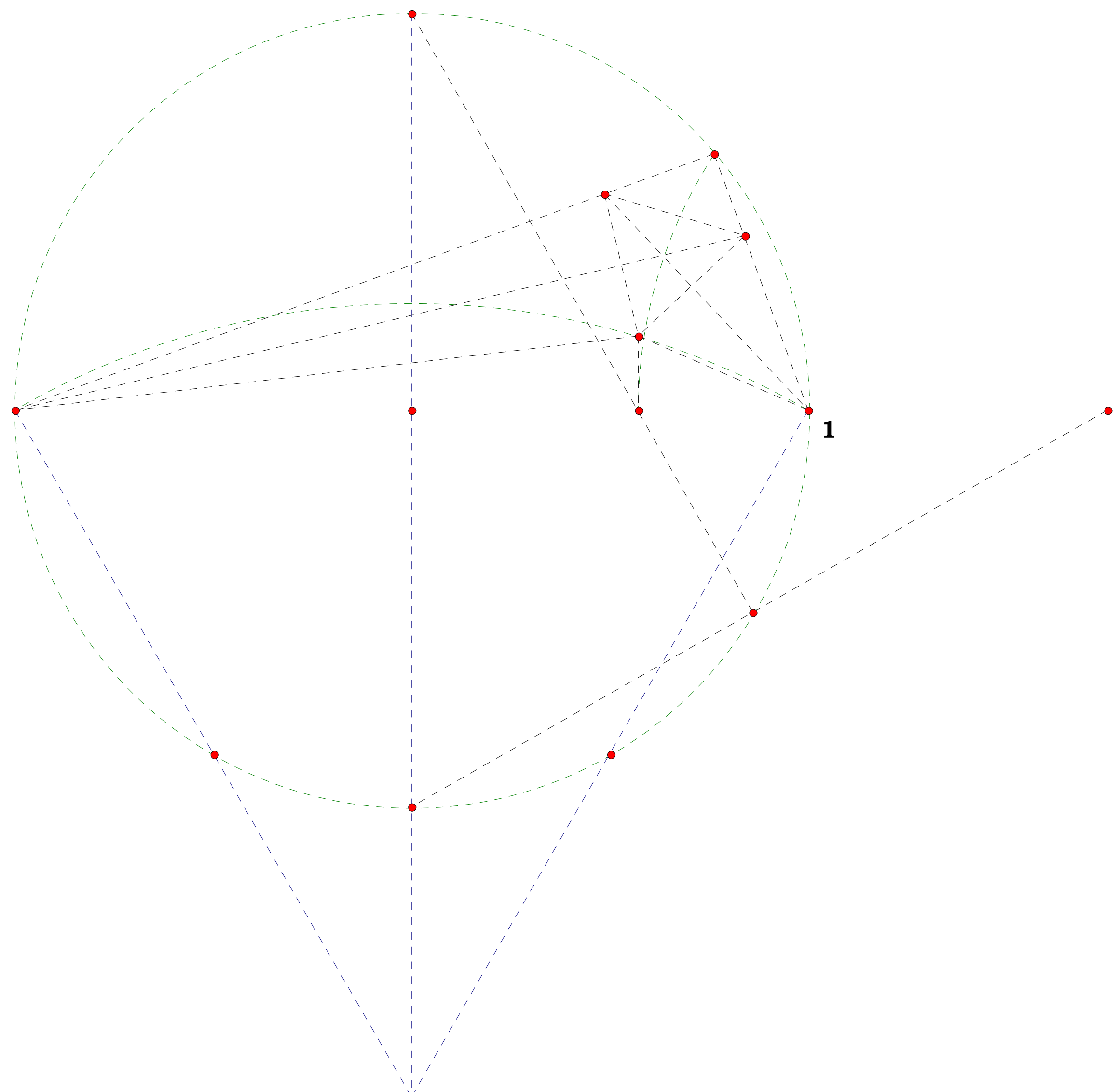

X
A
1
B
C
Unit = 1.00000
x1 = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000
A = 0.74143
B = 1.34875
C = 1.81818
2·C2/3·C2-1 -A = 0.00000
3·C2-1/2·C2 -B = 0.00000
Pipe
A
B
C
Y
2
3
2Y^2
3Y^2-1
3Y^2
AB = 0.76497 cm
AC = 1.39085 cm
AB/AC = 0.55000
AY = 2.52882 cm
AC/AY = 0.55000
Y^2
2Y^2
3Y^2-1

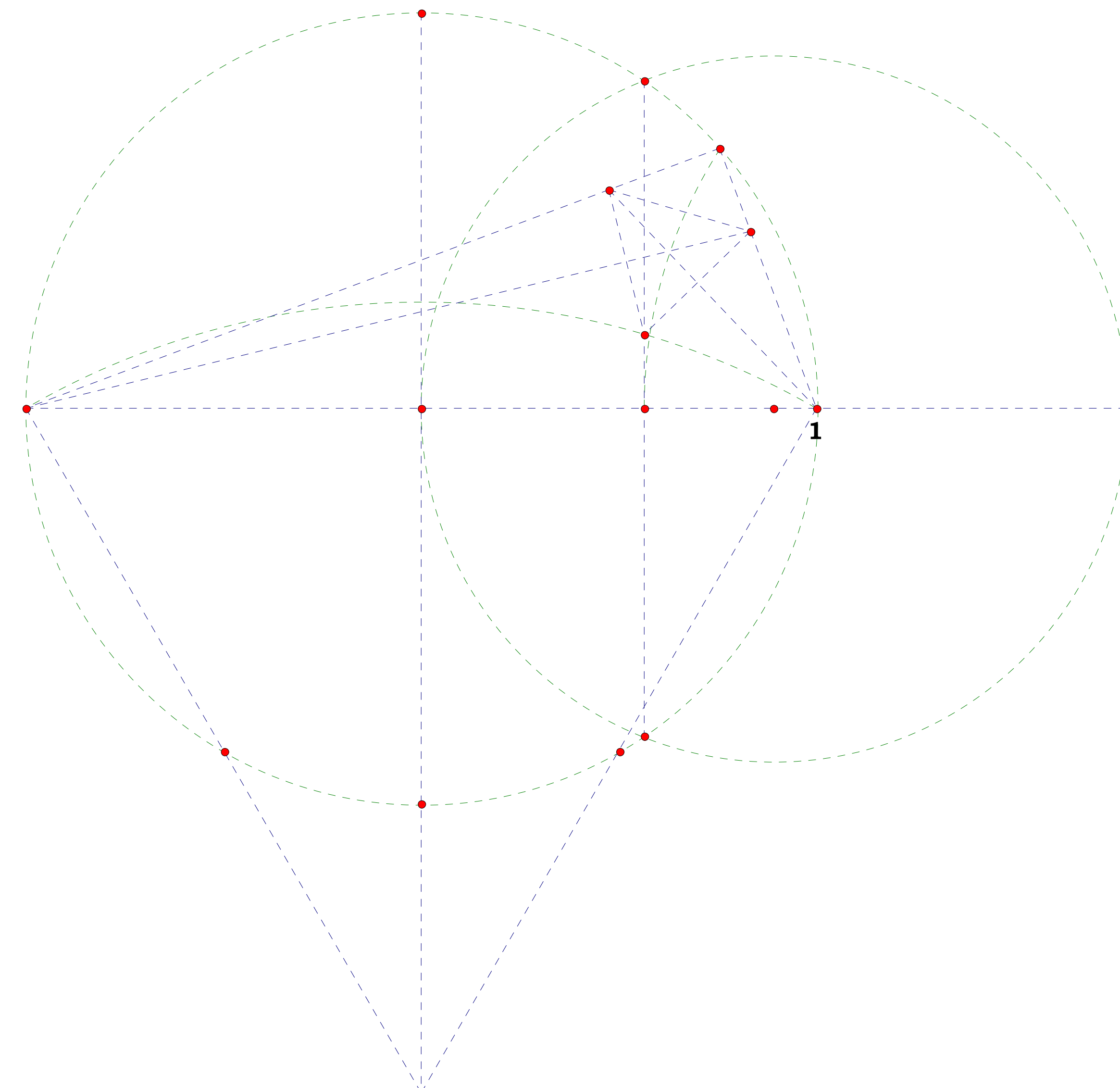
012220

Redo 080400

Construction variation.

Trisecting any angle in Geometry is only possible when you know the difference between the perceptible and the intelleligible.





012220

Descriptions.

Unit.

$AB := 1$

Given.

$X := 10$

$Y := 20$

What is the ratio of DO to OV?

$AO := \frac{AB}{2}$ $\quad AD := AO + AO \cdot \frac{X}{Y}$ $\quad AR := \frac{AO}{2}$ $\quad RJ := \sqrt{AR \cdot (AB - AR)}$

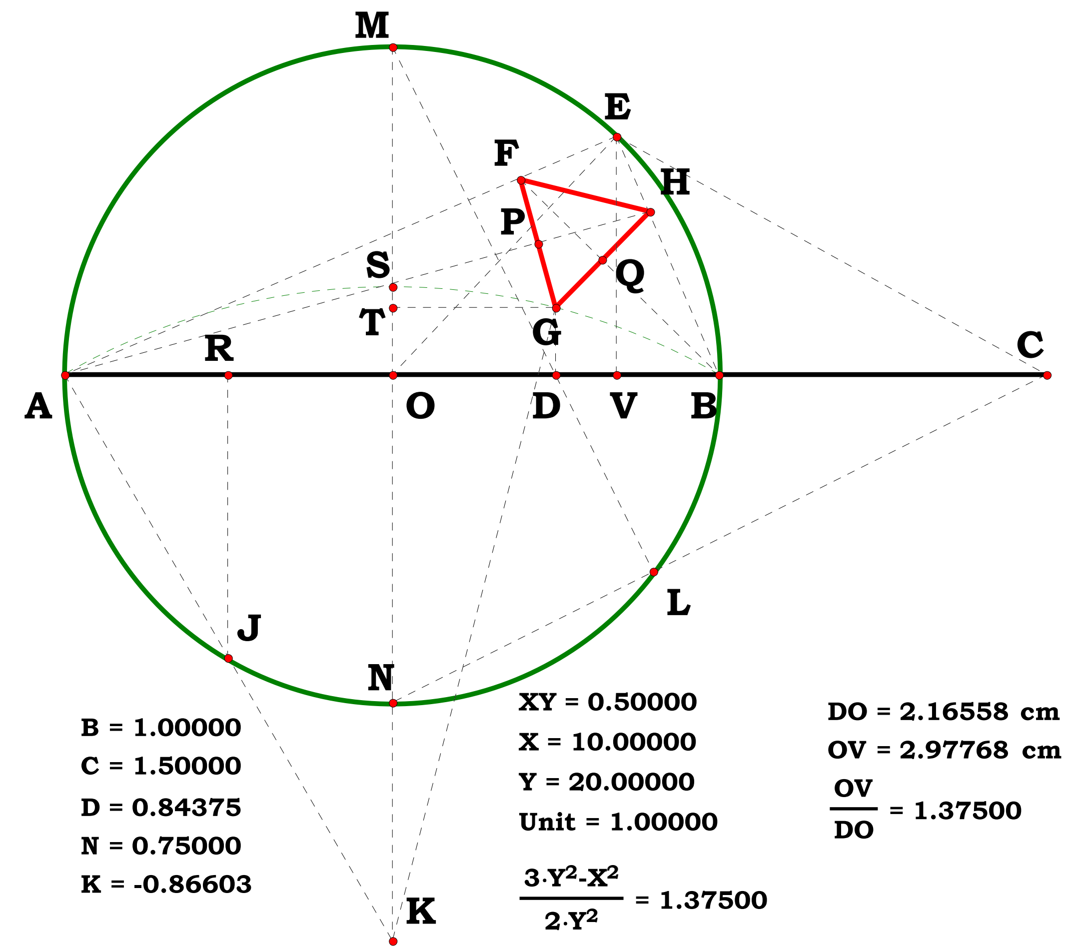
$KO := 2 \cdot RJ$ $\quad DO := AD - AO$ $\quad AJ := AO$ $\quad AK := 2 \cdot AJ$ $\quad EO := AO$

$KS := AK$ $\quad GK := AK$ $\quad KT := \sqrt{GK^2 - DO^2}$ $\quad TO := KT - KO$

$DG := TO$ $\quad MN := AB$ $\quad MO := AO$ $\quad DM := \sqrt{MO^2 + DO^2}$

$LN := \frac{DO \cdot MN}{DM}$ $\quad CO := \frac{MO^2}{DO}$ $\quad CD := CO - DO$ $\quad CE := CD$

$OV := \frac{CO^2 + EO^2 - CE^2}{2 \cdot CO}$ $\quad \frac{OV}{DO} = 1.375$ $\quad \frac{DO}{OV} = 0.727273$



Definitions.

$AO - \frac{1}{2} = 0$ $\quad AD - \frac{X+Y}{2 \cdot Y} = 0$ $\quad AR - \frac{1}{4} = 0$ $\quad RJ - \frac{\sqrt{3}}{4} = 0$

$KO - \frac{\sqrt{3}}{2} = 0$ $\quad DO - \frac{X}{2 \cdot Y} = 0$ $\quad AJ - \frac{1}{2} = 0$ $\quad AK - 1 = 0$

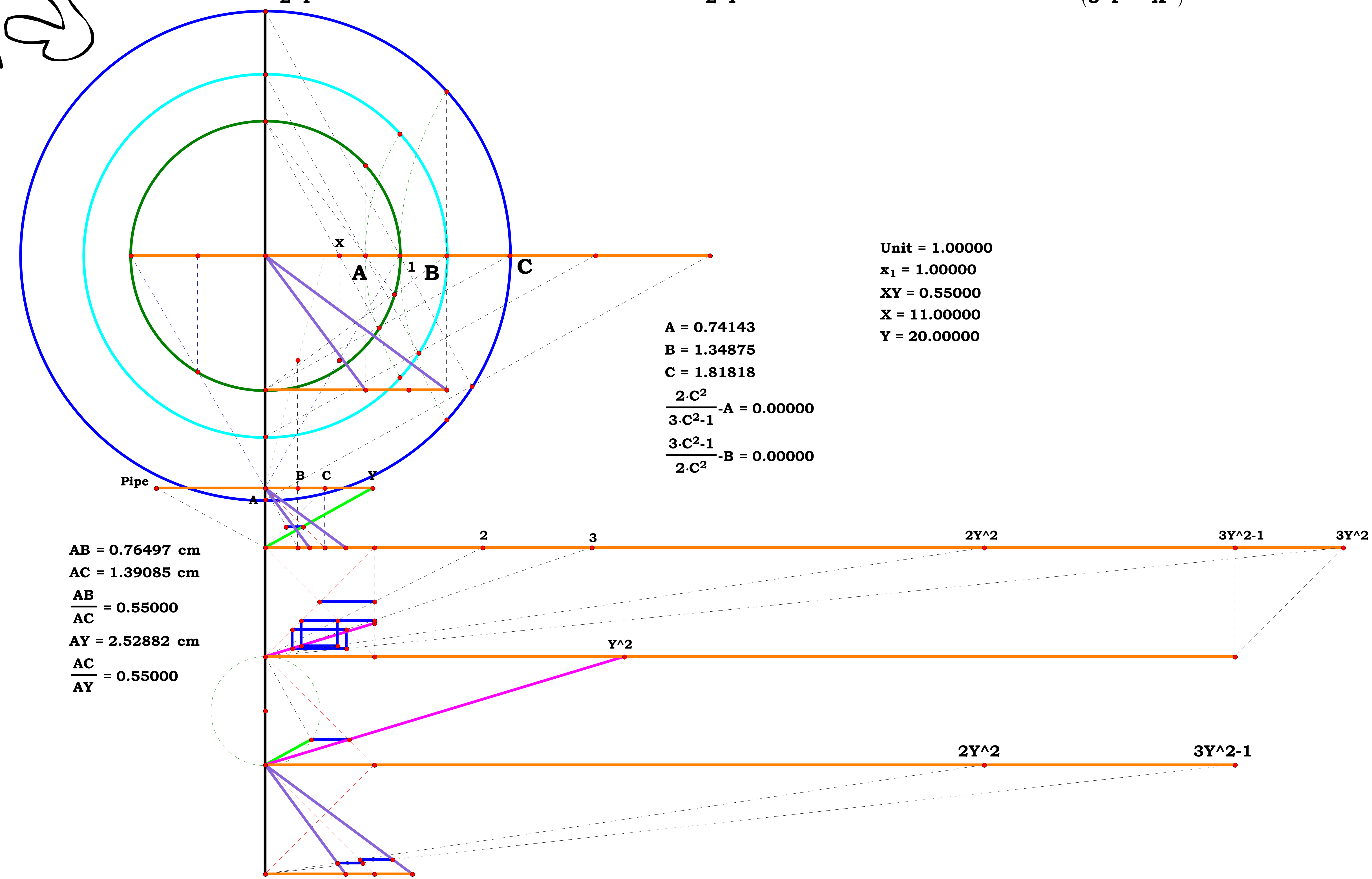
$EO - \frac{1}{2} = 0$ $\quad KS - 1 = 0$ $\quad GK - 1 = 0$ $\quad KT - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{2 \cdot Y} = 0$ $\quad TO - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{2 \cdot Y} = 0$

$DG - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{2 \cdot Y} = 0$ $\quad MN - 1 = 0$ $\quad MO - \frac{1}{2} = 0$ $\quad DM - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$ $\quad LN - \frac{X}{\sqrt{X^2 + Y^2}} = 0$

$CO - \frac{Y}{2 \cdot X} = 0$ $\quad CD - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $\quad CE - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $\quad OV - \frac{X \cdot \left(3 \cdot Y^2 - X^2\right)}{4 \cdot Y^3} = 0$ $\quad \frac{OV}{DO} - \frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} = 0$ $\quad \frac{DO}{OV} - \frac{2 \cdot Y^2}{\left(3 \cdot Y^2 - X^2\right)} = 0$

One can say now that the length of the sides of an equalateral triangle in a right triangle used in trisection is $\frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y}$

$\frac{X}{Y} = 0.5 \quad \frac{(3 \cdot Y^2 - X^2)}{2 \cdot Y^2} = 1.375 \quad OV = 0.34375 \quad DO \cdot \frac{(3 \cdot Y^2 - X^2)}{2 \cdot Y^2} - OV = 0 \quad DO = 0.25 \quad OV \cdot \frac{2 \cdot Y^2}{(3 \cdot Y^2 - X^2)} - DO = 0$



Unit = 1.00000

x_1 = 1.00000

XY = 0.55000

X = 11.00000

Y = 20.00000

A = 0.74143

B = 1.34875

C = 1.81818

$\frac{2 \cdot C^2}{3 \cdot C^2 - 1} - A = 0.00000$

$\frac{3 \cdot C^2 - 1}{2 \cdot C^2} - B = 0.00000$

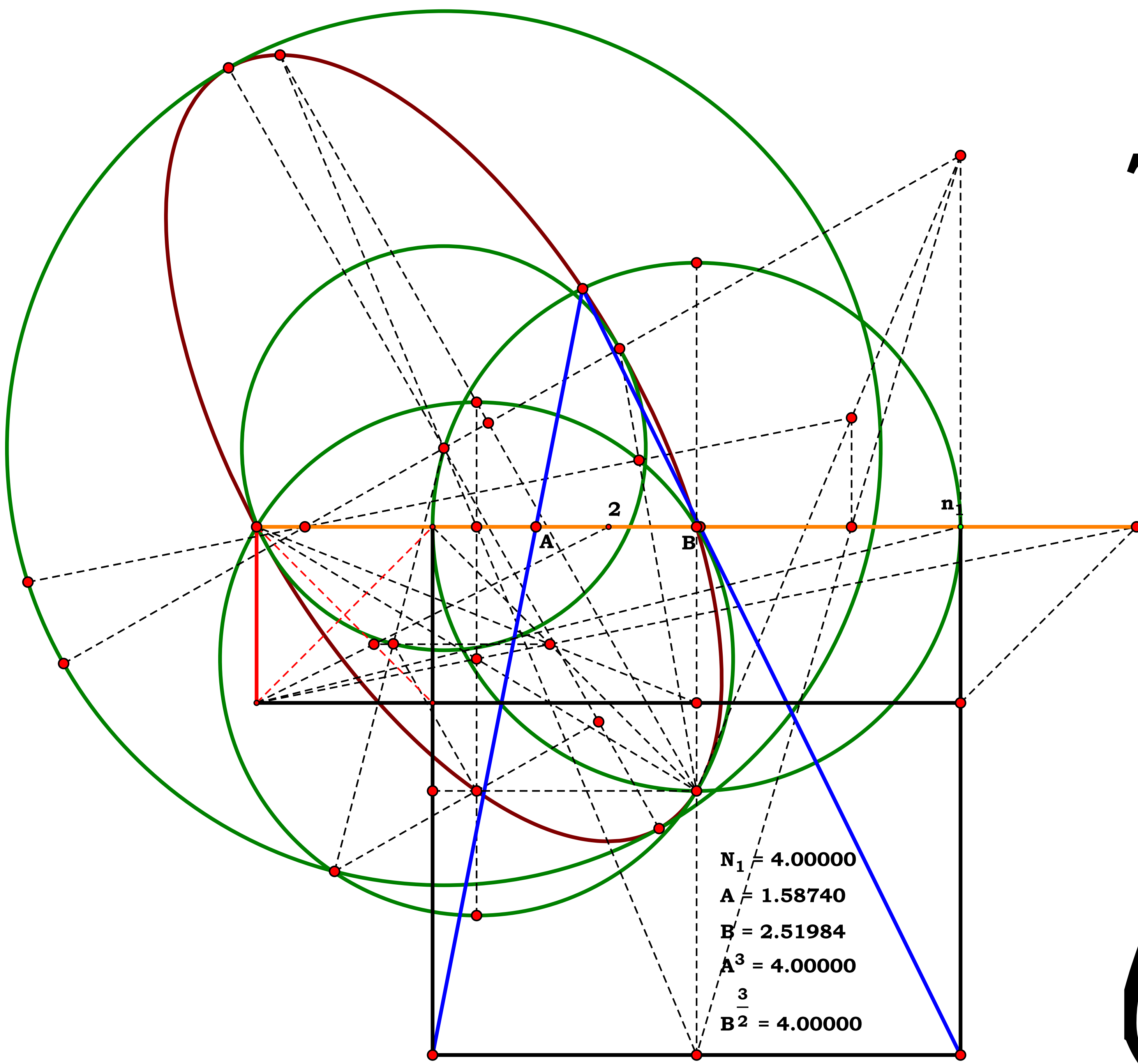
AB = 0.76497 cm

AC = 1.39085 cm

$\frac{AB}{AC} = 0.55000$

AY = 2.52882 cm

$\frac{AC}{AY} = 0.55000$



The Delian Quest Side Jobs

John Clark

Descriptions. **020320 Easy Cube**

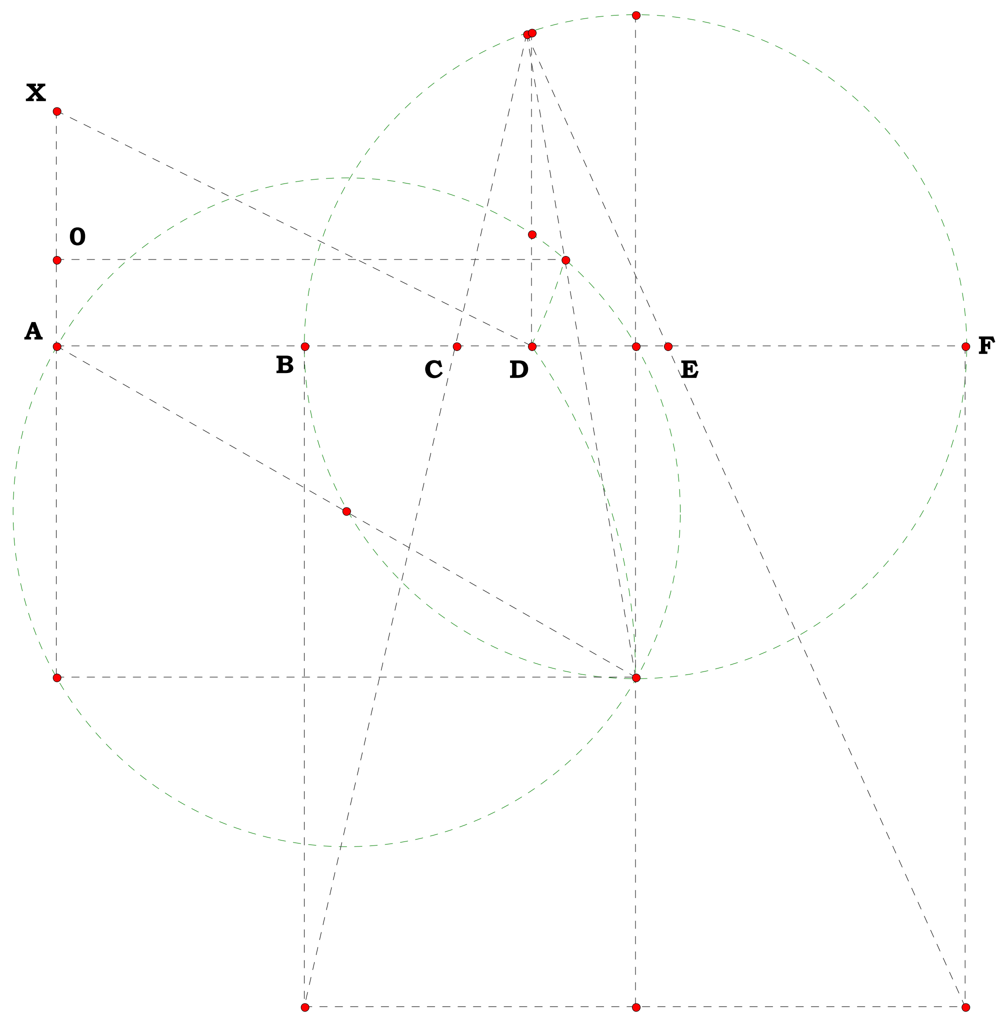
Often, one would like to create an easy plate for cube roots, instead of doing the whole figure. Here is the simplest and most accurate way to achive it.

Draw X to A anywhere and then construct 0 parallel to AF. Have your macro make X seek 0.
You might believe that simple geometry can out Calculus Calculus, maybe in these plates you will change your mind. Calculus is a work frought with grammatical contradictions, Cartesian Geometry, Calculus, Trigonometry are not even grammatically correct. They are not derived from a correct concept of grammar as any possible grammar is afforded by complete induction and deduction of a simple binary unit.

Unit = 1.00000
XY = 3.66072
X = 18.10729
Y = 4.94638

F = 3.66072
E = 2.46101
C = 1.60738
D = 1.91330

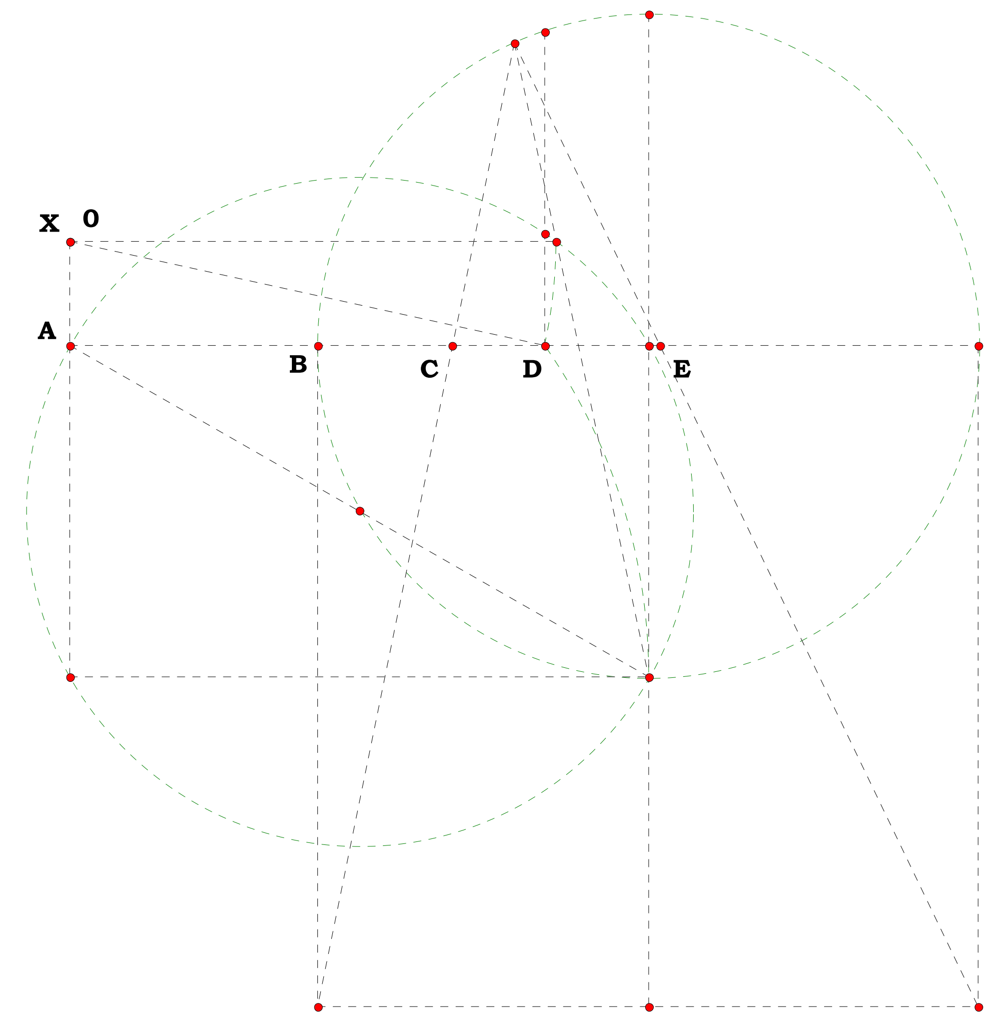
$F^{\frac{1}{2}}-D = 0.00000$
$F^{\frac{1}{3}}-C = -0.06619$
$F^{\frac{2}{3}}-E = -0.08575$



Unit = 1.00000
XY = 3.66072
X = 18.10729
Y = 4.94638

F = 3.66072
E = 2.37526
C = 1.54119
D = 1.91330

$F^{\frac{1}{2}}-D = 0.00000$
$F^{\frac{1}{3}}-C = 0.00000$
$F^{\frac{2}{3}}-E = 0.00000$

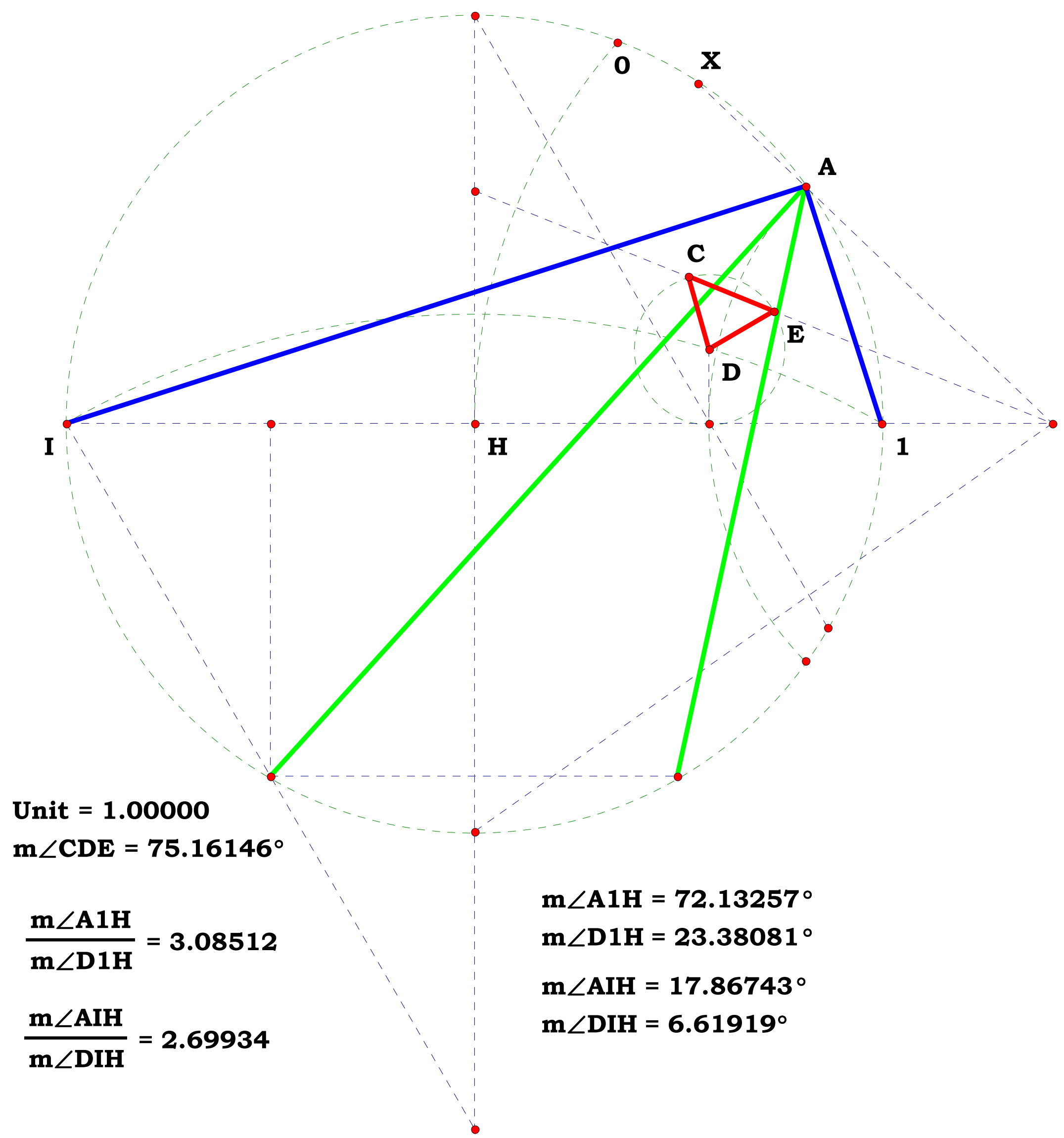


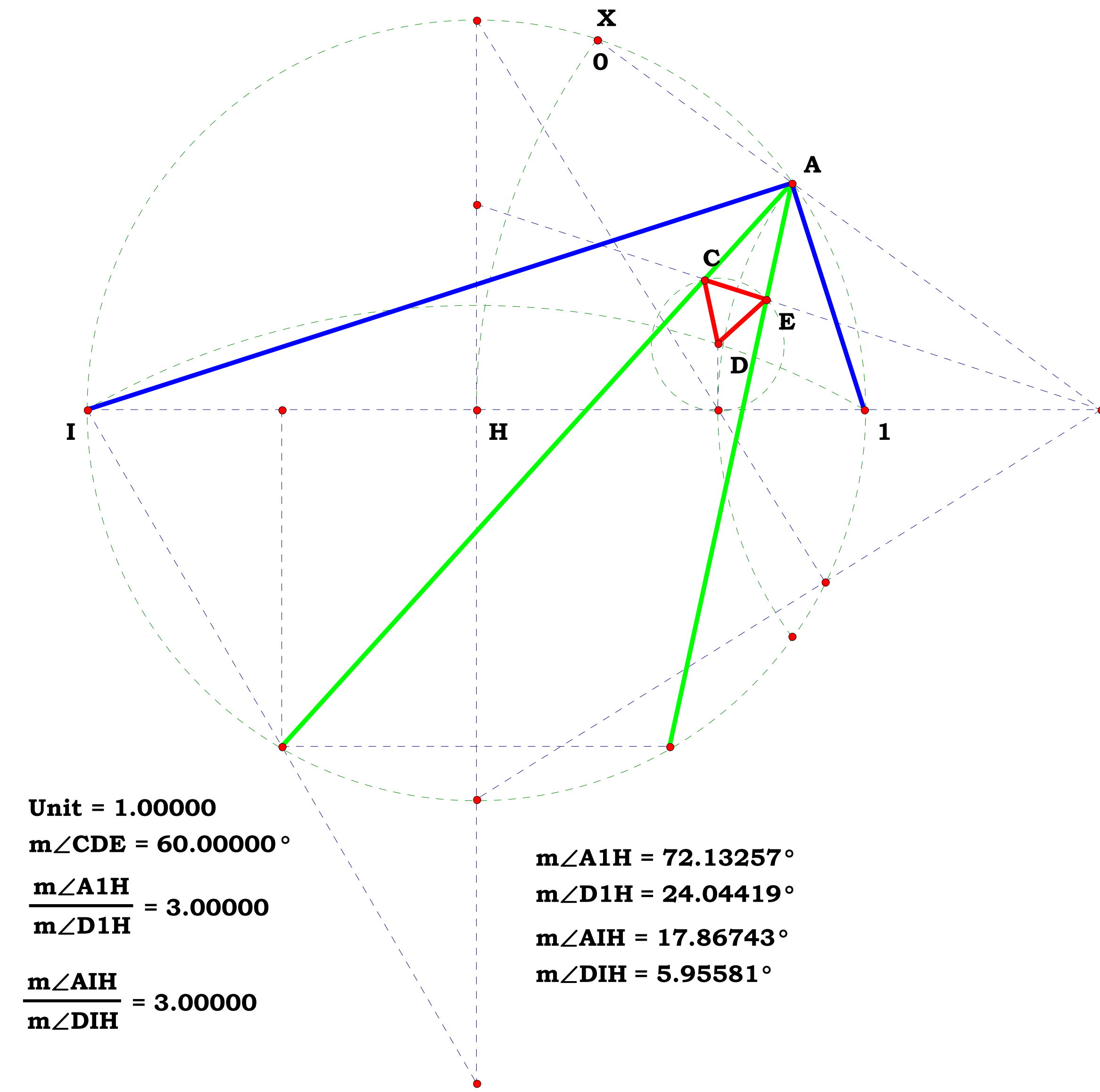
020320 Easy Trisection

Descriptions.

Sometimes one is want to trisect some particular angle the easy way. Here it is. Draw it up with X anywhere and project 0. Have your macro make X seek 0. It is a whole lot neater than sliding a piece of paper.

By knowing the geometric end results, one can write algorithms which are a whole lot more accurate and efficient than by just using the traditional methods.



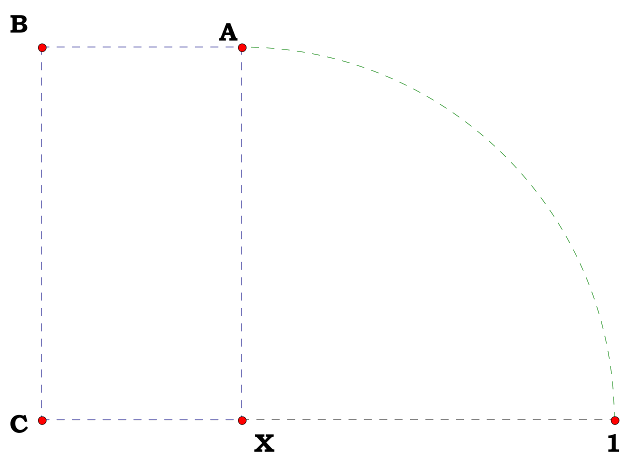


020320 Squaring a Rectangle

Descriptions.

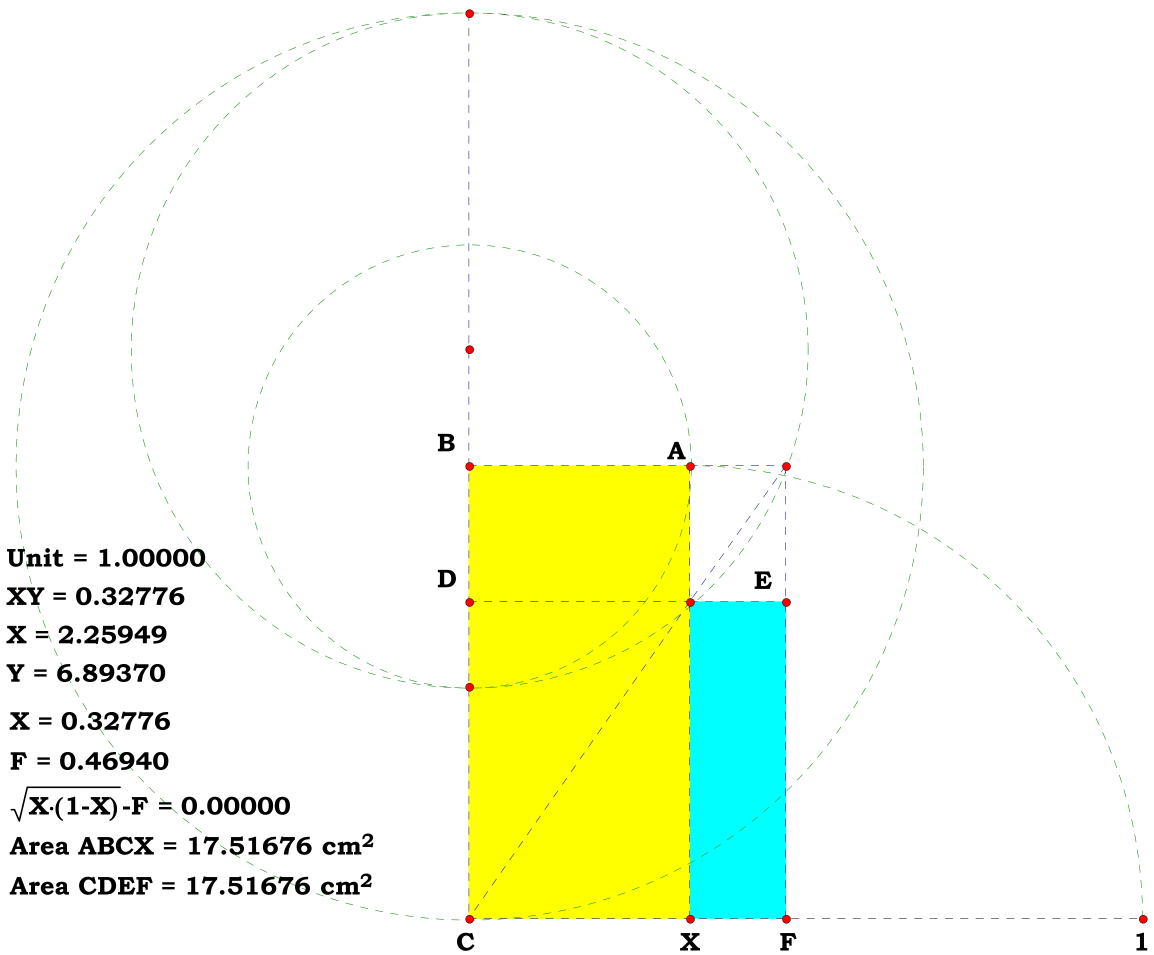
This is just another way to study rectangles, complements and square roots.

Unit = 1.00000
XY = 0.34859
X = 2.40306
Y = 6.89370



Unit = 1.00000
XY = 0.32776
X = 2.25949
Y = 6.89370

X = 0.32776
F = 0.46940
$\sqrt{X\cdot(1-X)}-F = 0.00000$
Area ABCX = 17.51676 cm^2
Area CDEF = 17.51676 cm^2



The Holy Grail

Sunday, February 23, 2020

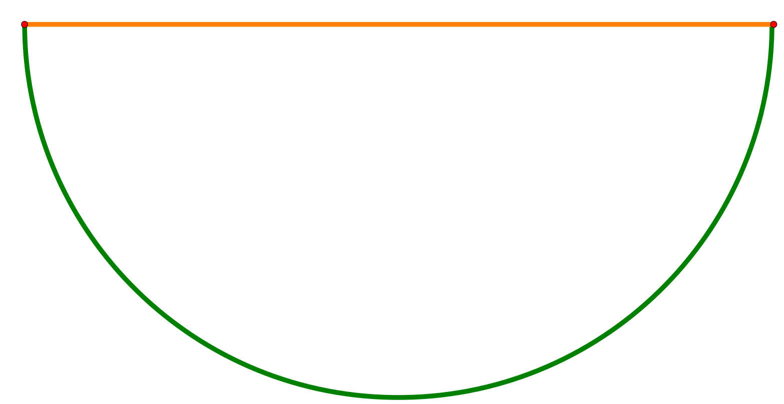
- noun

(the Grail or the Holy Grail) (in medieval legend) the cup or platter used by Christ at the Last Supper, and in which Joseph of Arimathea received Christ's blood at the Cross. Quests for it undertaken by medieval knights are described in versions of the Arthurian legends written from the early 13th century onward.

A thing which is eagerly pursued or sought after: the enterprise society where profit at any cost has become the holy grail.

– origin from Old French graal, from medieval Latin gradalis ‘dish’.

Most people, around the world believe that the Holy Grail, if they believe in the Grail, has been lost to history, but this is decidedly not true:—



There it is, it has also been called the Bowl of Siddhartha and even the Philosopher’s Stone. It is certainly not lost; it is mankind that is lost. Here is the mystery people do not comprehend; perhaps no one has explained it to them. Before I get into that, I need to dispel other myths men tell each other, especially about the Bible and the science of their own evolution.

Did you know that it is written, in several places of the Bible, that man cannot even read that Book until after a certain time in history? That man is still being made and until he reaches a certain point in his making, he will only dream that he is a man, that he has understanding. It is also written that man will be in this condition until a pure language is introduced to him. Today, even scholars still do not know the relationship between Language and Grammar. I can put that relationship into grammar, but your ability to comprehend what I say is determined by how much of a man you are, how much of you is complete, as a man.

A man is measured by his distinction from other forms of life on this planet. That distinction resides in his ability to clearly see Language, which nothing in all of creation can speak. It is an intelligible. Language is a biological inheritance. Every form of life is made from it, and every form of life expresses its comprehension of it, from the most primitive forms of life to the most complex. Language is Universal and Intelligible. Grammar, which is a physical recursion of language, is Particular and Perceptible. A species can only formulate their systems of grammar to the degree that they comprehend language. As scholars, even the current scholars, have and are still, expounding their confusion and lack of comprehension in each of these, all I can do is explain it to you, your own state of creation determines your ability to comprehend what I say. Suffice it to say, both science and religion today are still lost as to what man is, why he is, and what his purpose is in this life, even though everything was long ago put into simple words, words which are decidedly very provable.

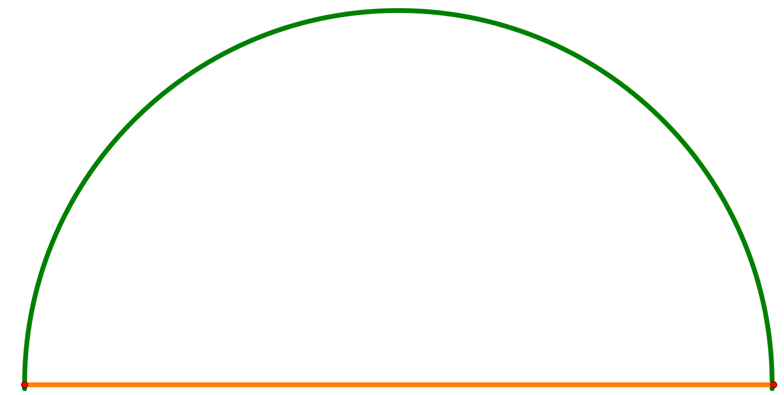
Every life support system of a living organism is designed for the salvation of the life of that organism; every one of them. However, that salvation is particular to that part of the environment that life support system can process. Each of them is particular, and thus, do not have the ability to process time, itself. The mind is one such life support

system, and it is the most powerful life support system possible. It is the most powerful possible, because it is designed to process the intelligible which is over every possible relative difference, even time, itself. When functional, man will even conquer death. A mind is a symbolic information processor constructed to predict the results of every action, every relative difference, and every thing. It achieves this from the Universal of Language, also called the Word of God, which simple minds believe is just some book, but it is factually the Word of Creation itself. Since nothing but God can speak in language, man has to speak in grammar systems which are in the image of that language. Language is, in a metaphor, such as Adam and Eve, A Conjugate Binary Pair which affords even reality itself, complete induction and deduction of every thing.

Today, all one has to do is meditate on their computer to realize that all of information is processed using binary; however, it takes any species a long time to evolve to the stage where it can comprehend this fact with a mind.

One is also, in metaphor, informed that there are exactly four systems of grammar derived from binary, arrived at by simple binary recursion. They are also informed in metaphor not only the fact that every possible grammar is metaphorical, able to use the binary unit for complete induction and deduction with that grammar, but also that of the four grammars three are logical, and the last is analogical, this last grammar can be used to metaphorically to illustrate every possible line of reasoning in every possible grammar, it is called Geometry. The first three grammars are all logical, Common Grammar, Arithmetic, and Algebra.

So, in this little section of my work, I will show you how to understand the philosophers stone, the cup of Siddhartha, the Holy Grail, of the life of mind and body. You use an image of the Cup, like this:—



You turn your world upside down, turn from illiteracy to literacy. Literacy gives a species the ability to turn the past into a future and to bring that future to pass, which, deliberately, is the solution to the name of the Beast, 666. It is a puzzle that those with eyes can easily solve.

List of Plates to use for the story.

The plates used in the outline.

022020

Descriptions.

Homind's Quest for The Holy Grail

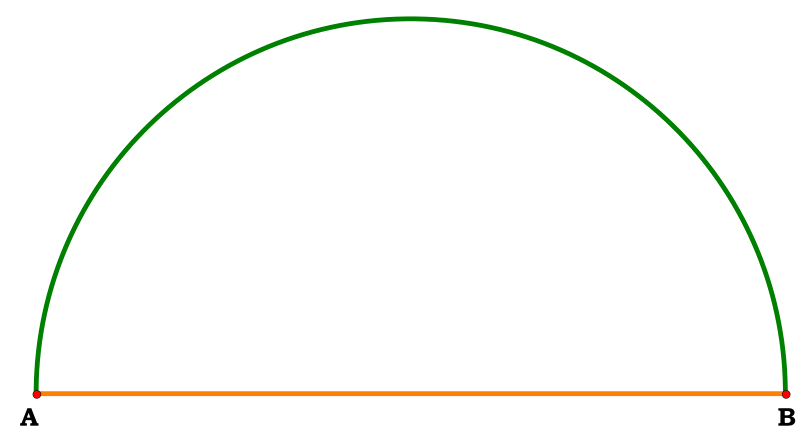
Story project: this is simply an outline of that story. How pedantic can one be and still hold the storyline?

The whole idea is an ordered progression, unlike our real thrashing aboout. This ordering, howevever, is to be implied in the story, and never mentioned. It is to follow step by step the available moves with straightedge and compass with what the figure offers. One can even turn Hominid into several characters over several generations. This project should be done.

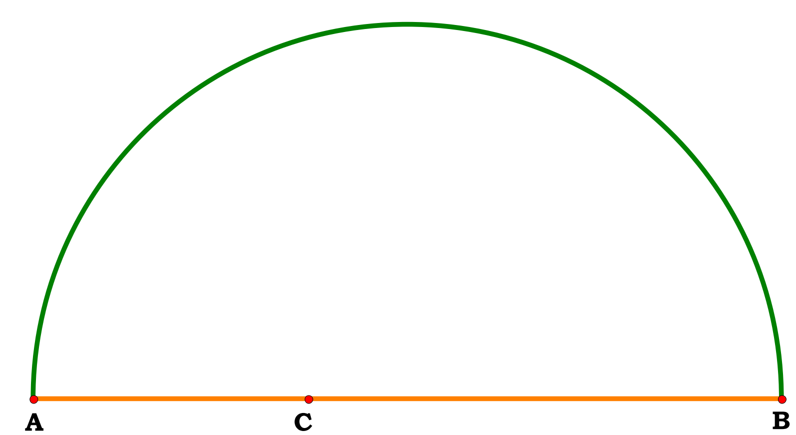
Once upon a time, Hominid took time to ponder the common stick which he found laying on the ground and as he wanted to think about it long and hard, he preserved that stick in memory. He learnt how to draw in the ground with his stick and to follow those drawings in memory.

He found, that in order to ponder this stick, that it might be advantageous to name it, and so he did. He named when it came, A, and when it left, B.

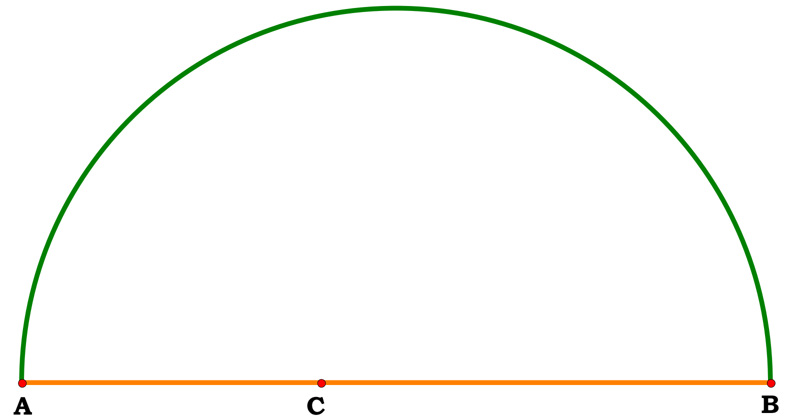
One day, Hominid decided to do for the stick as he had done for himself, build it a shelter.



This shelter reminded him of his own home and on another day, while meditating on his stick, in its home, it seemed that this stick and home might feel better if it had at least one occupant. And so Hominid placed in his home an occupant like himself.



In order to meditate on his little home, with its littlel man, Hominid decided that it might be advantageous to increase his understanding of names. Eventually, Hominid learnt meditation about C and where he was at through a process he called arithmetic. Suddenly, Homid realize that this gave him a lot to think about.



C = 8.00000
B = 20.00000

Given.

$AB := 20$

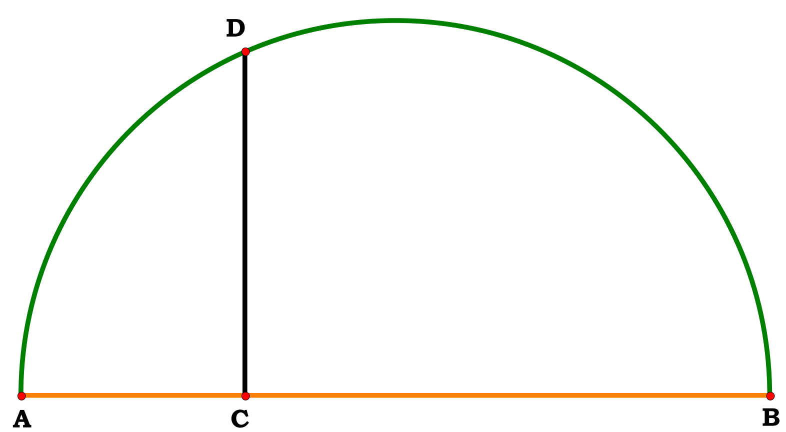
$AC := 8$

$BC := AB - AC \qquad BC = 12$

$\frac{AB}{AC} = 2.5 \qquad \frac{AB}{BC} = 1.666667 \qquad \frac{AC}{AB} = 0.4 \qquad \frac{AC}{AB} = 0.4 \qquad \frac{AC}{BC} = 0.666667 \qquad \frac{BC}{AC} = 1.5 \qquad AC \cdot BC = 96$

All of the places, his arithmetic told him, were in the home of C, except one. This last result troubled Hominid. 96 could not possibly be in the home of C. What is the meaning of this? He began to wonder about the future home of C and about things C could never see.

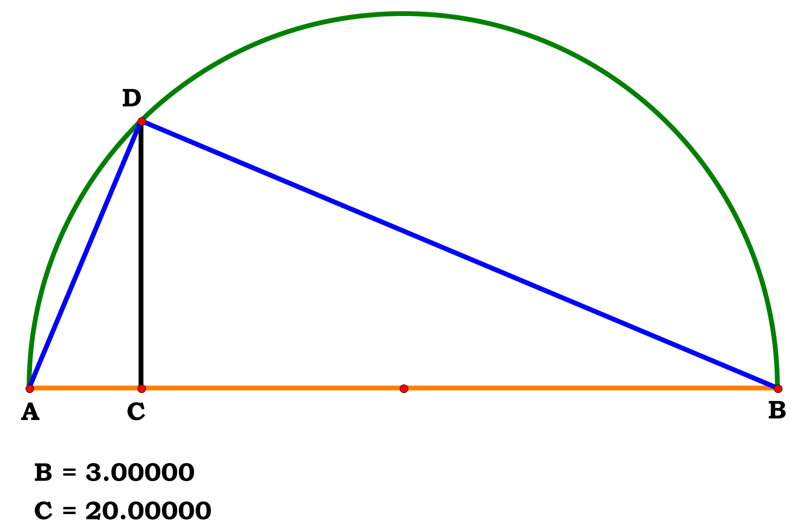
One day, Hominid wanted C to be more like him, and ponder the sky dome and so, he noted it as the following.



C = 6.00000
B = 20.00000

Then Hominid began to wonder, again, what would there be to ponder in CD, in itself, by itself? How would one clime up to heaven, to D to learn?

Hominid built C ladders up into heaven to D
and over time, he learnt from his arithmetic
and his stick, his shelter, its occupant.



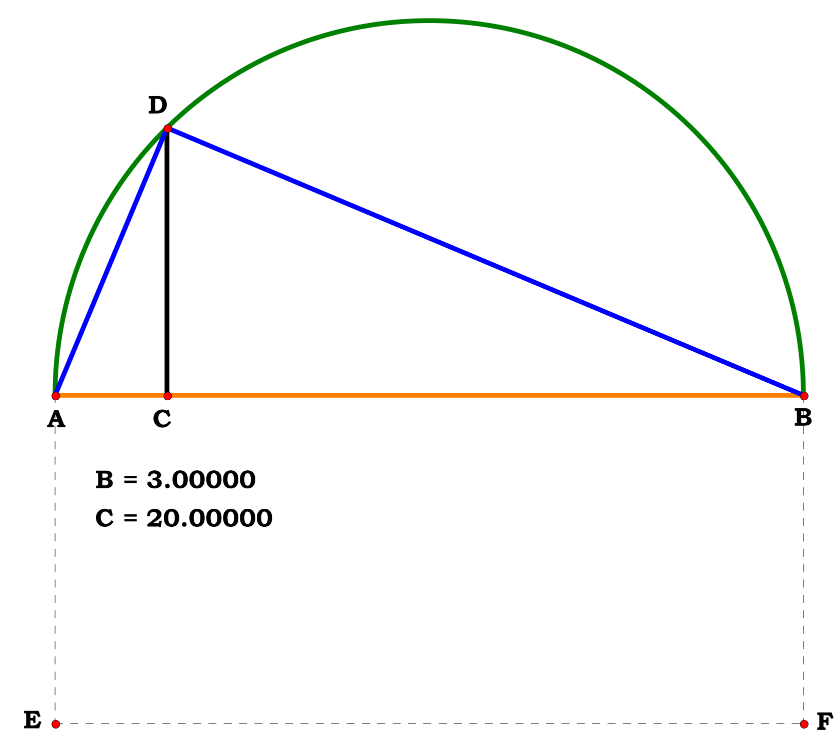
$$CD := \sqrt{AC \cdot BC}$$

$$AD := \sqrt{AC^2 + CD^2} \qquad BD := \sqrt{BC^2 + CD^2}$$

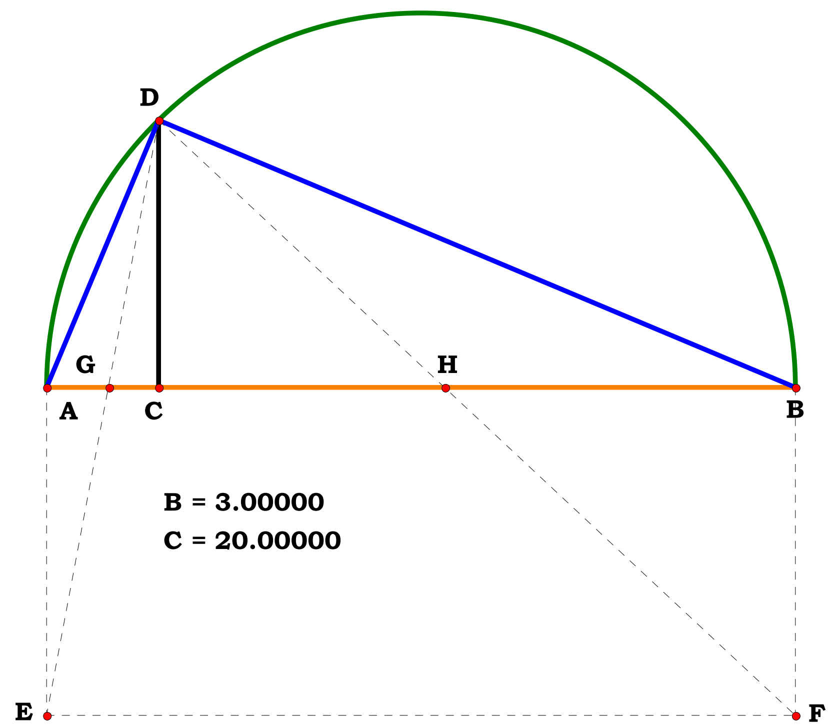
$$\frac{CD^2}{AC} - BC = 0 \qquad \frac{CD^2}{BC} - AC = 0$$

$$\frac{AD^2}{AC} - AB = 0 \qquad \frac{BD^2}{BC} - AB = 0$$

And so on. As the number of things which Homid learnt grew, Homid thought that it was time that C had a basement to put all of his stuff in, the home was getting crowded, which to him was an odd thing to think, as crowd simply means to group into one with the added connotation, a bit much for one place. But, he built his basement anyway.
For a long time Homid and his little man played with all the toys they had found together in the room in his mind using a simple stick.



In our little story, Hominid starts pondering this issue, which will eventually lead to the following.

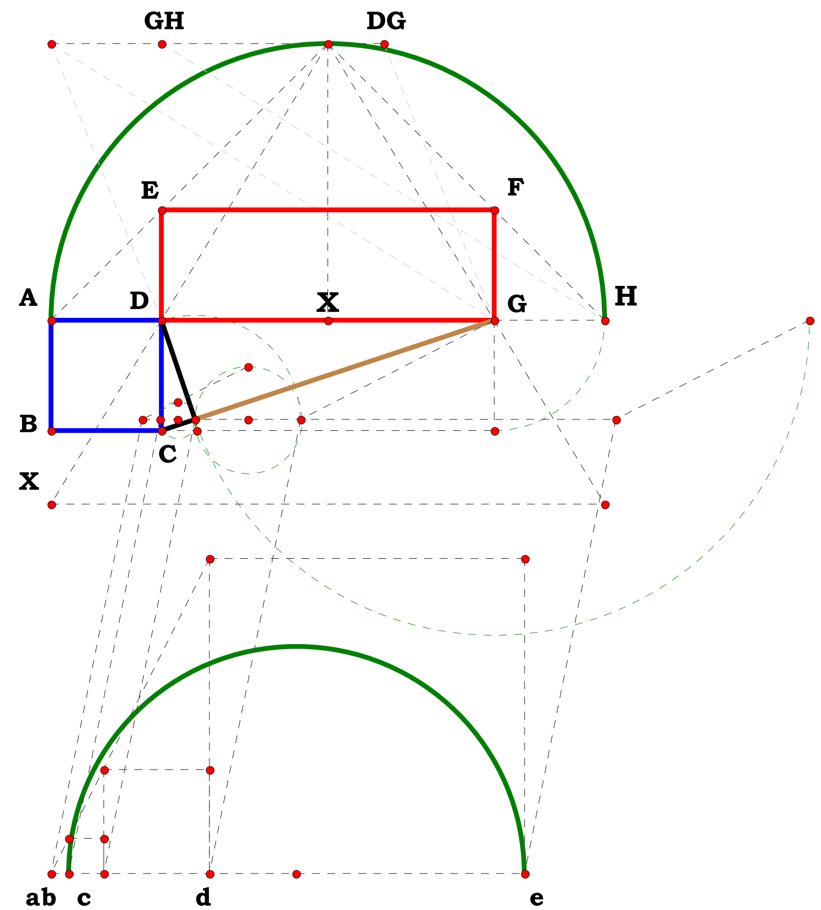


First and second basement archives. Here, X is fixed midpoint. The finishing touch, is in the next plate, to move it and learn the final ratio's.

$\frac{DG}{DE} = 3.00000$

$AD = 0.20000$
$GH = 0.20000$
$DE = 0.20000$
$DG = 0.60000$

$DEFG = 0.12000$
$ABCD = 0.04000$
$\frac{DEFG}{ABCD} = 3.00000$



$a = 1.00000$
$b = 3.00000$
$c = 9.00000$
$d = 27.00000$

$\frac{d}{b} = 9.00000$

$x_b = 0.03162$
$x_c = 0.09487$
$x_d = 0.28460$
$x_e = 0.85381$

$d^{\frac{1}{3}} - b = 0.00000$
$d^{\frac{2}{3}} - c = 0.00000$
$b^3 - d = 0.00000$
$b^2 - c = 0.00000$
$\frac{b}{d} = 0.11111$

$\frac{Z^2}{Y} = 9.00000$
$\frac{Z^3}{Y} = 27.00000$

$\frac{Z}{Y} = 3.00000$

$\frac{x_e}{x_d} = 3.00000$
$\frac{x_d}{x_c} = 3.00000$
$\frac{x_c}{x_b} = 3.00000$

Final basement archive.

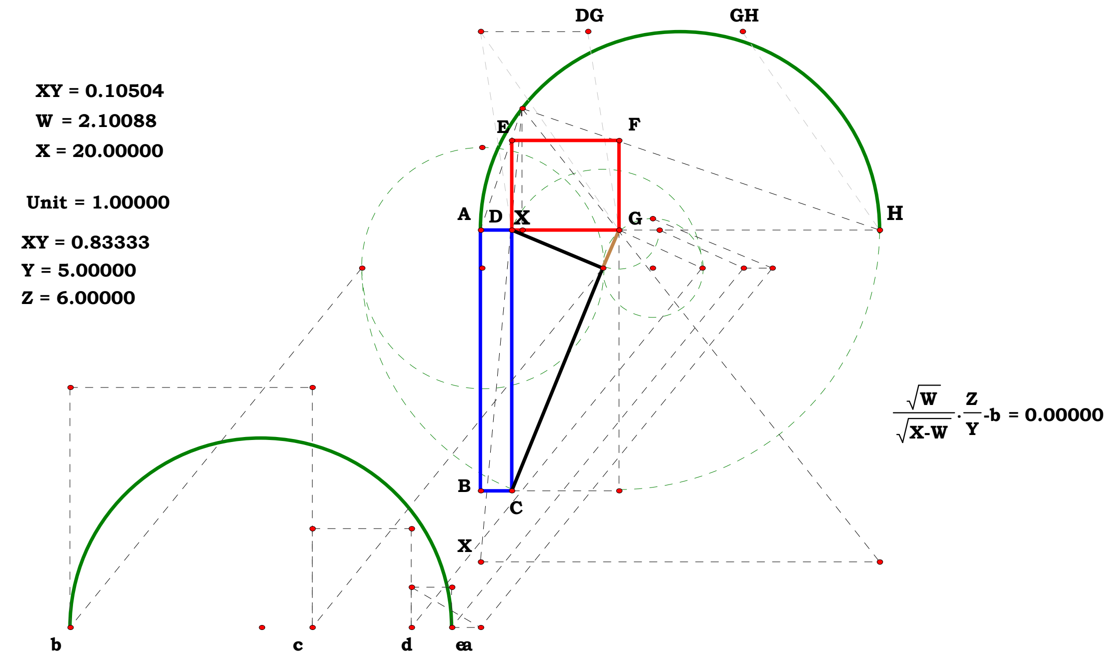
In the above, the aviary, exponential manipuloation and how to locate any bird in the sky has to be incorporated. This story might take some time.

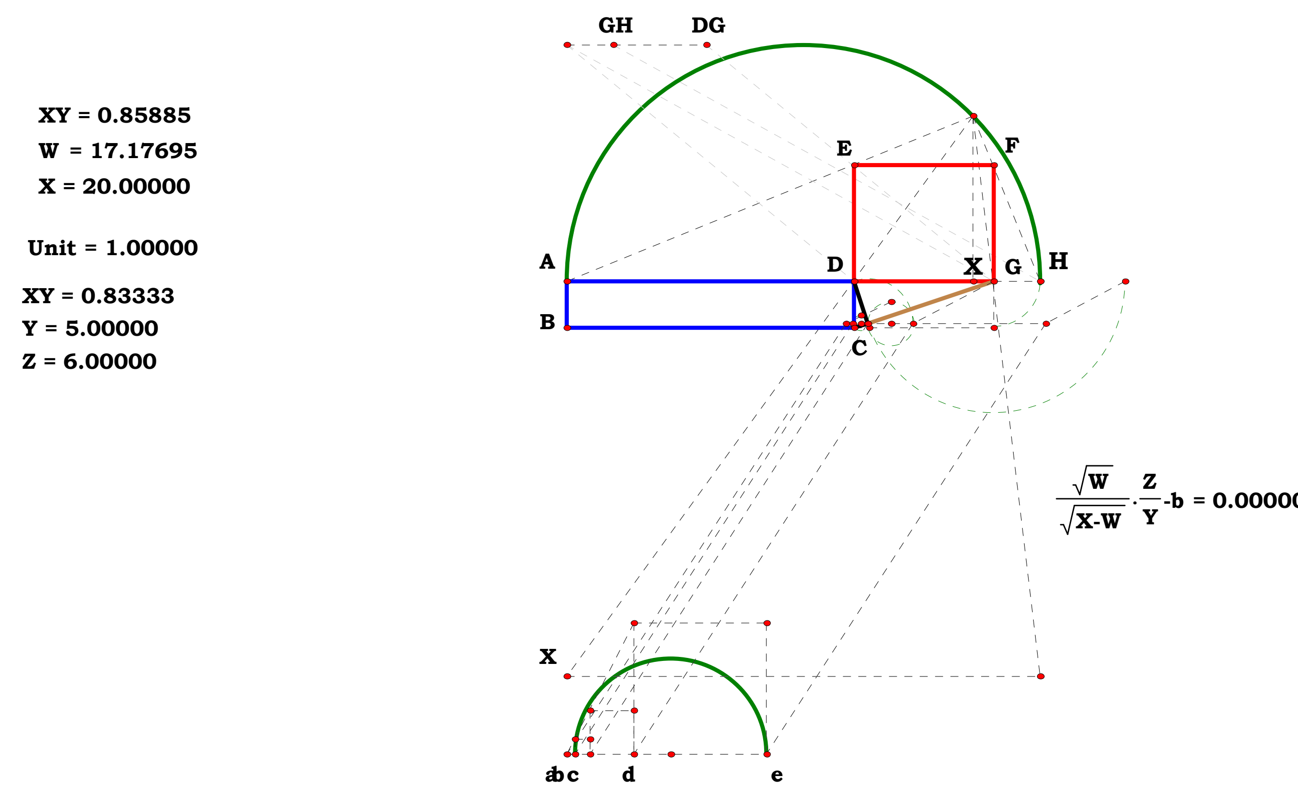
One has to add in induction and deduction, in Hominid's home and how he goes beyond it.

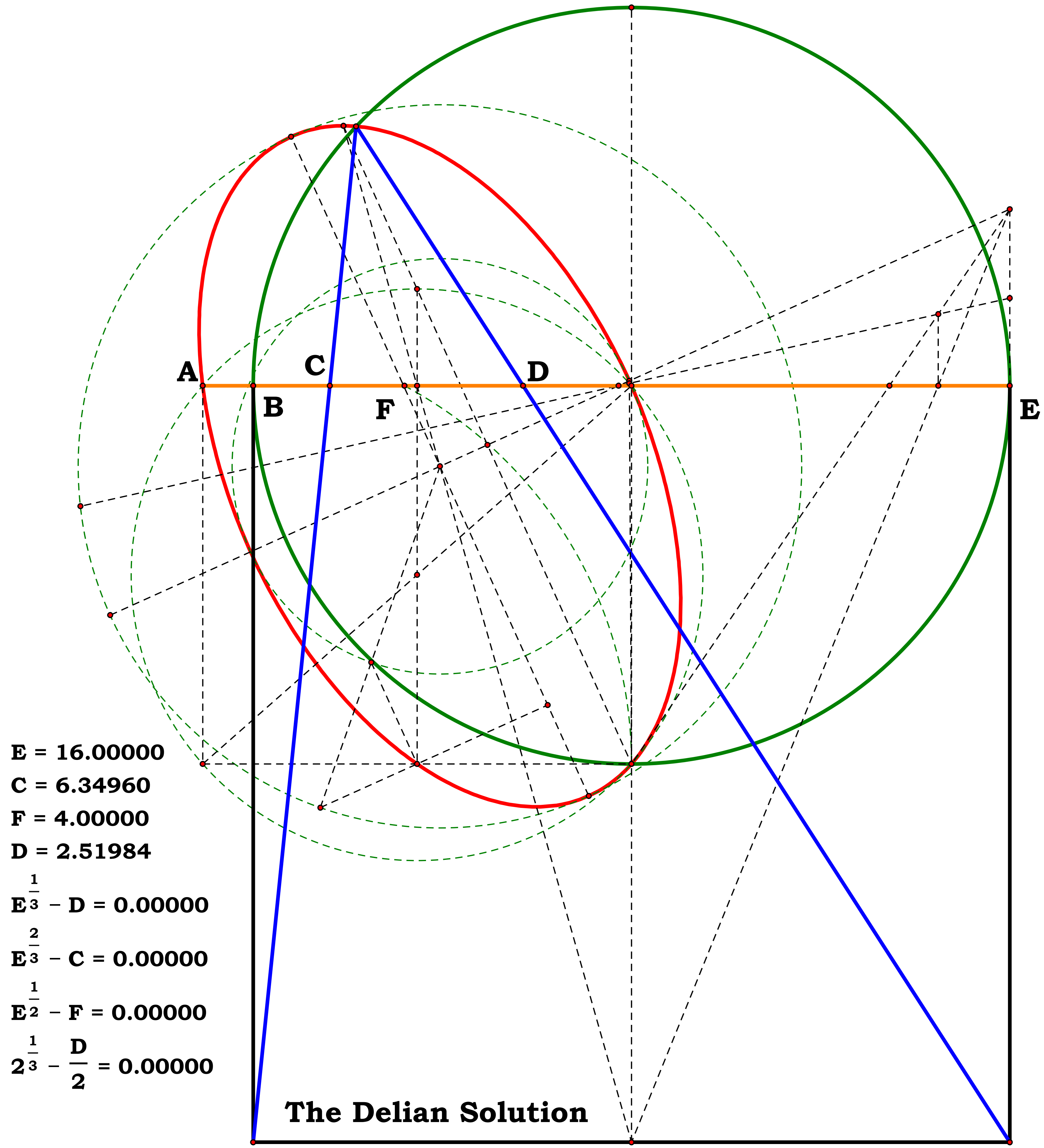
This last plate demonstrates the whole of matematics in a single unit. All the simple relationships formed by it. It all resolves down to simple arithmetic.

Induction and deduction in every grammar system does not change the fact that recursion can never change simple arithmetic or binary progression. The Holy Grail has always been the image of 1.

Nothing like a single equation for the whole of grammar.






A
B
C
D
E
F
E = 16.00000
C = 6.34960
F = 4.00000
D = 2.51984
$E^{\frac{1}{3}} - D = 0.00000$
$E^{\frac{2}{3}} - C = 0.00000$
$E^{\frac{1}{2}} - F = 0.00000$
$2^{\frac{1}{3}} - \frac{D}{2} = 0.00000$
The Delian Solution